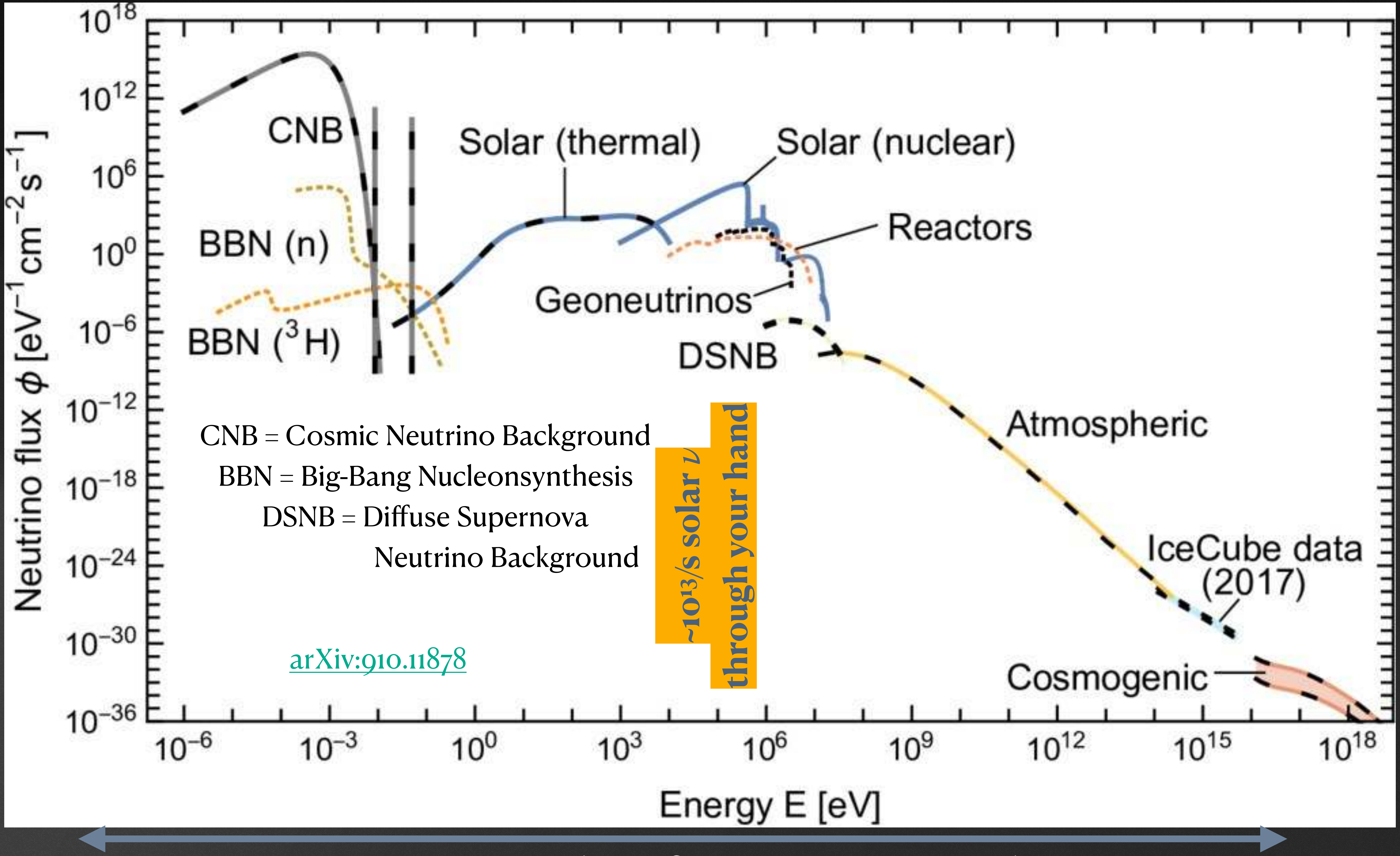
On the Leggett-Garg Inequality Testing with Neutrino Oscillation Measurements

Son Cao, IFIRSE, ICISE

100 Years of Quantum Physics, ICISE, 2025

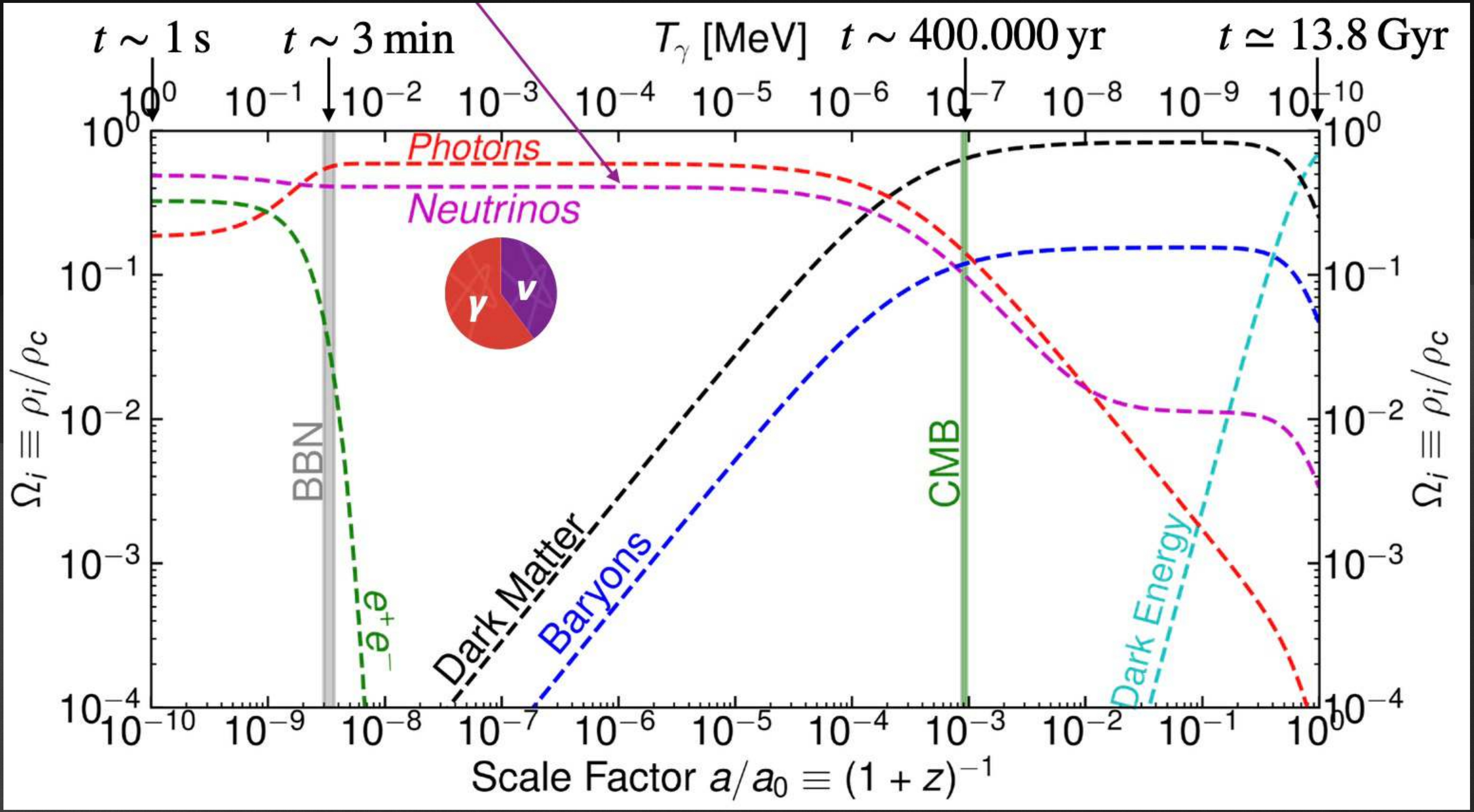
Neutrino is everywhere



Span ~ 24 order of energy magnitude

On average, ~ 340 neutrinos/cm^3, the second most abundant elementary particles

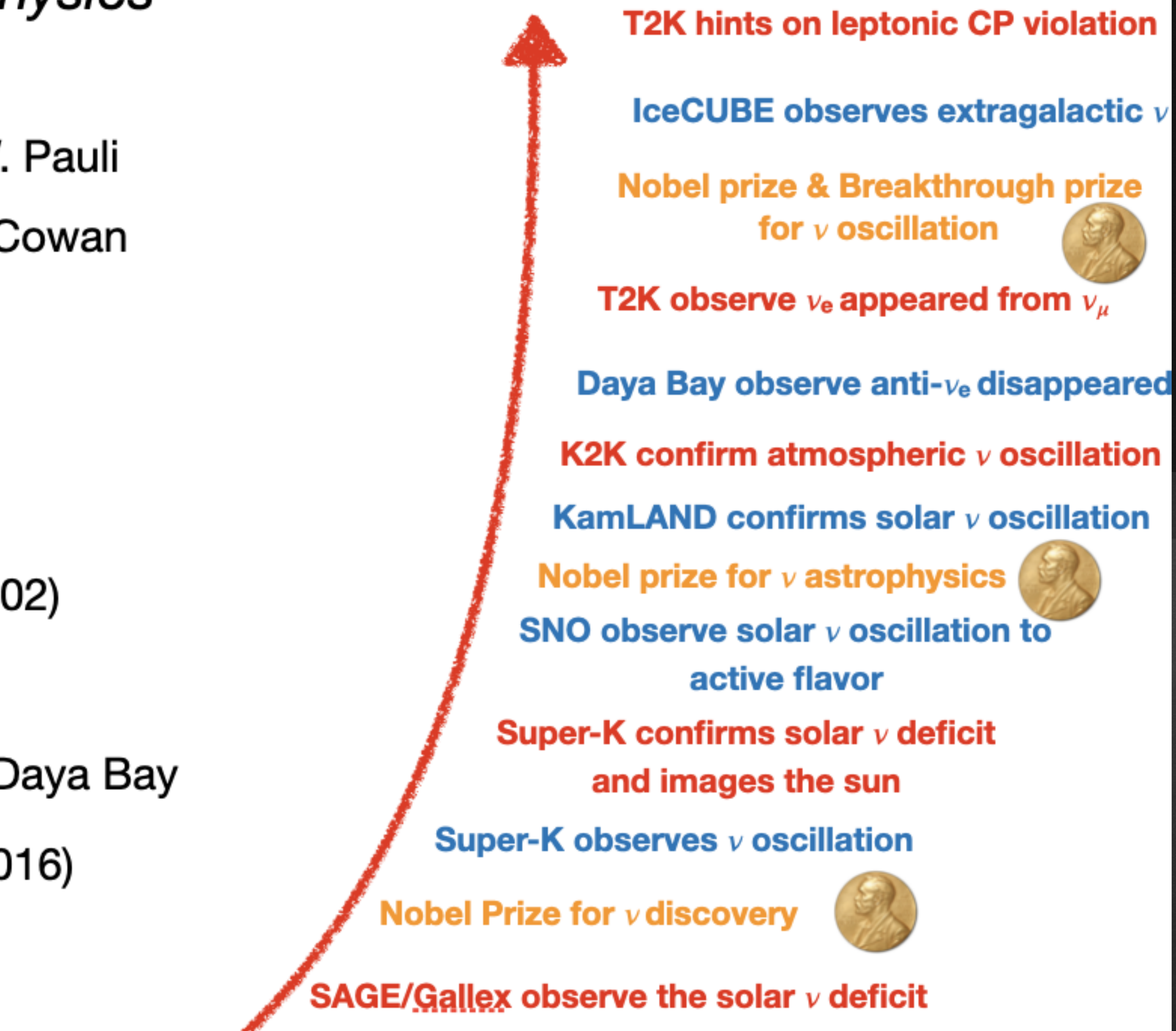
Neutrino is essential part of the Universe evolution

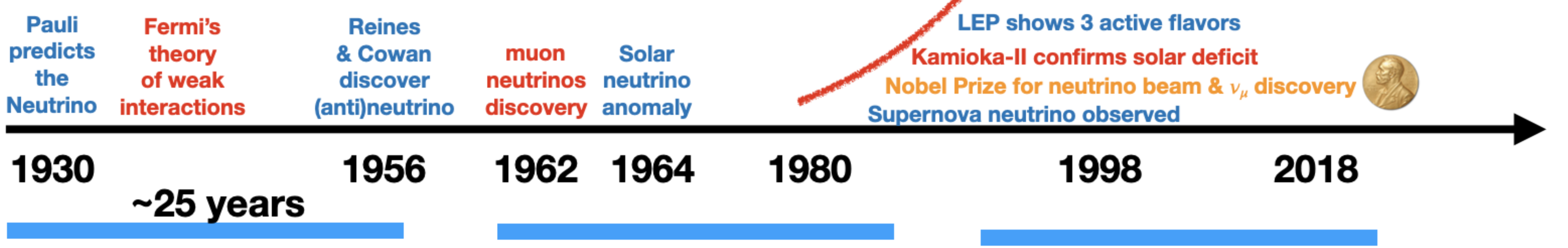


https://ifirse.icise.vn/nugroup/nuview2025/docs/Escudero_BSMNeutrino-Cosmology_Vietnam.pdf

Adapted "The Growing Excitement of Neutrino Physics " by APS

- ★ 1930: On-paper appearance as "desperate" remedy by W. Pauli
- ★ 1956: Anti-ν_e first experimentally discovered by Reines & Cowan
- ★ 1962: ν_μ existence confirmed by Lederman *et al*
- ★ 1986: Existence of ν_τ was established
- ★ 1998: Atmospheric ν oscillations discovered by Super-K
- ★ 2001: Solar ν oscillations detected by SNO (KamLAND 2002)
- ★ 2011: $\nu_\mu \rightarrow \nu_\tau$ transitions observed by OPERA
- ★ 2011-13: $\nu_\mu \rightarrow \nu_e$ observed by T2K and *anti-*$\nu_e \nrightarrow$ *anti-*ν_e by Daya Bay
- ★ 2015: Nobel prize for ν oscillations, Breakthrough prize (2016)
- ★ 2018: T2K hints on leptonic CP violation





More detail: https://neutrino-history.in2p3.fr/neutrinos-milestones-and-historical-events/

Neutrino and Great puzzles of Nature

ph/0502070 (2005))

NEUTRINO PHYSICS

1. Where and what is dark matter?
2. How massive are neutrinos?
3. What are the implications of neutrino mass?
4. What are the origins of mass?
5. Why is there a spectrum of fermion masses?
6. Why is gravity so weak?
7. Is Nature supersymmetric?
8. Why is the Universe made of matter and not antimatter?
9. Where do ultrahigh-energy cosmic rays come from?
10. Did the Universe inflate at birth?

Posed by Chris Quigg, 20 years ago, still waiting for the answers!!!

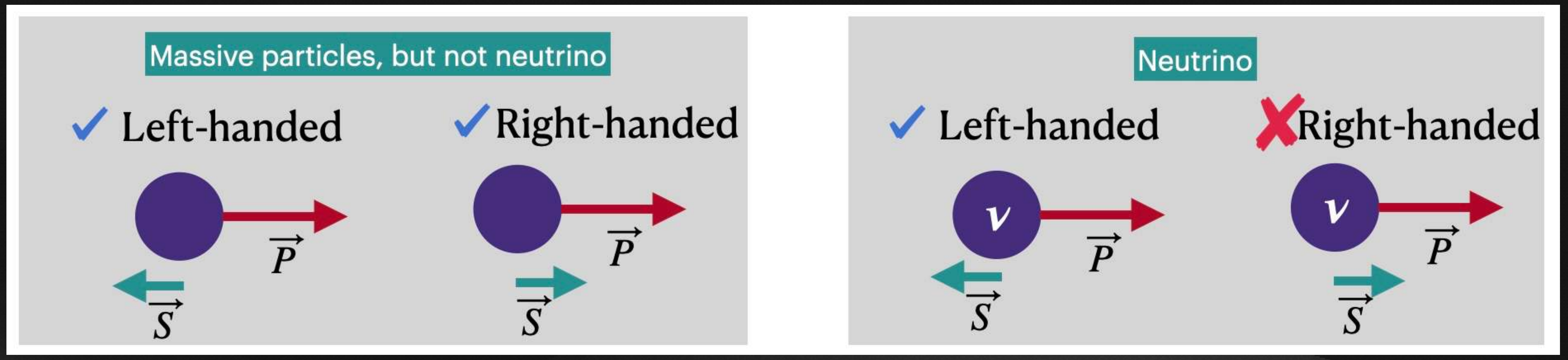
- **Also flavor model**
- **Testing fundamental law (CPT, Lorentz...)**

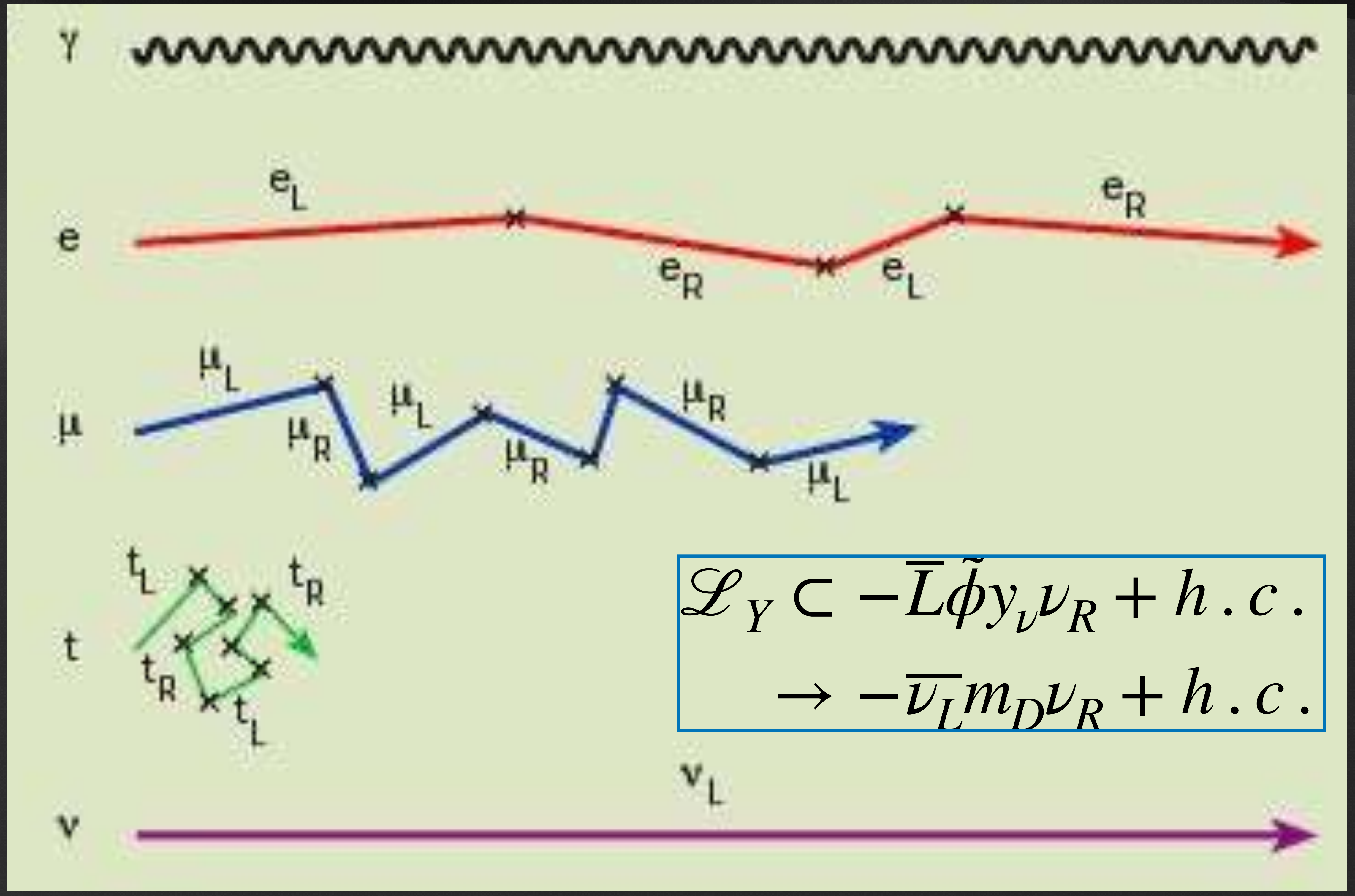
Quyen Phan, SC *et al* PRD 103 (11), 112010

Ngoc Tran, SC *et al* PRD 107 (1), 016013

Basics of neutrino properties

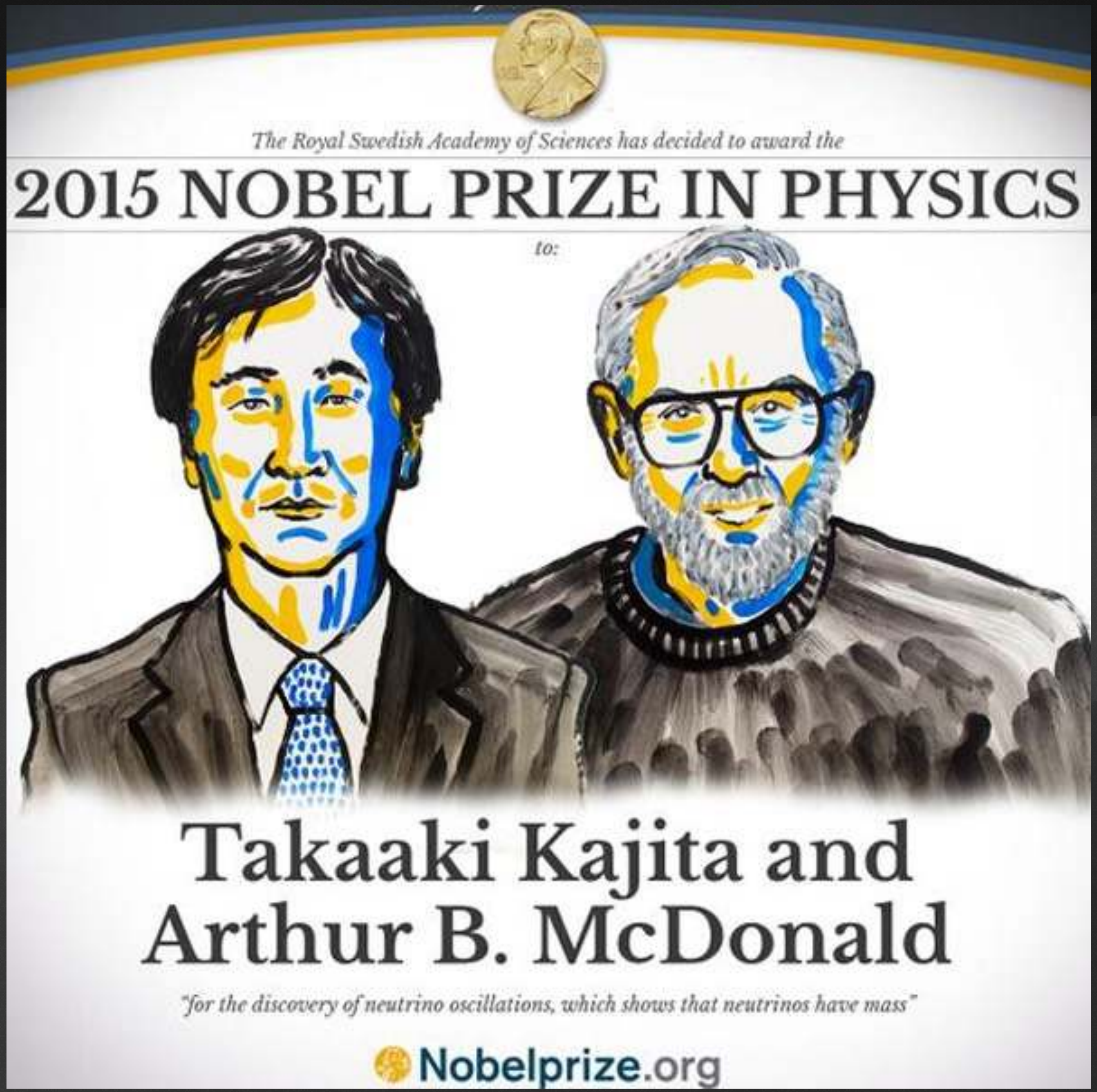
- Neutrino is electrical neutral; spin - 1/2, lepton (*fairly similar to electron*)
- Only experience weak interaction and gravity
- Only left-handed neutrinos are found ➜ zero Dirac mass term in the Standard Model
  - Established **phenomenon of neutrino oscillation** imply that neutrinos are massive ⬅ (only) palpable evidence of new physics beyond Standard Model





http://hitoshi.berkeley.edu/neutrino/neutrino4.html

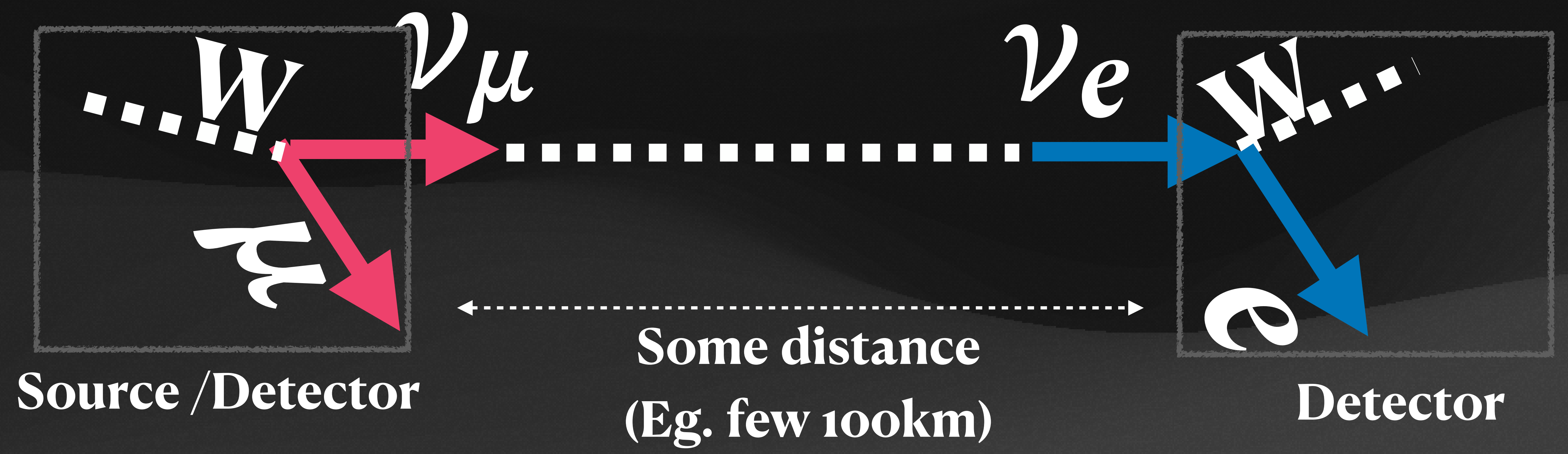
Neutrino oscillations: *A game-changer*



"...for the discovery of ***neutrino oscillations***, which shows that **neutrinos have mass**"

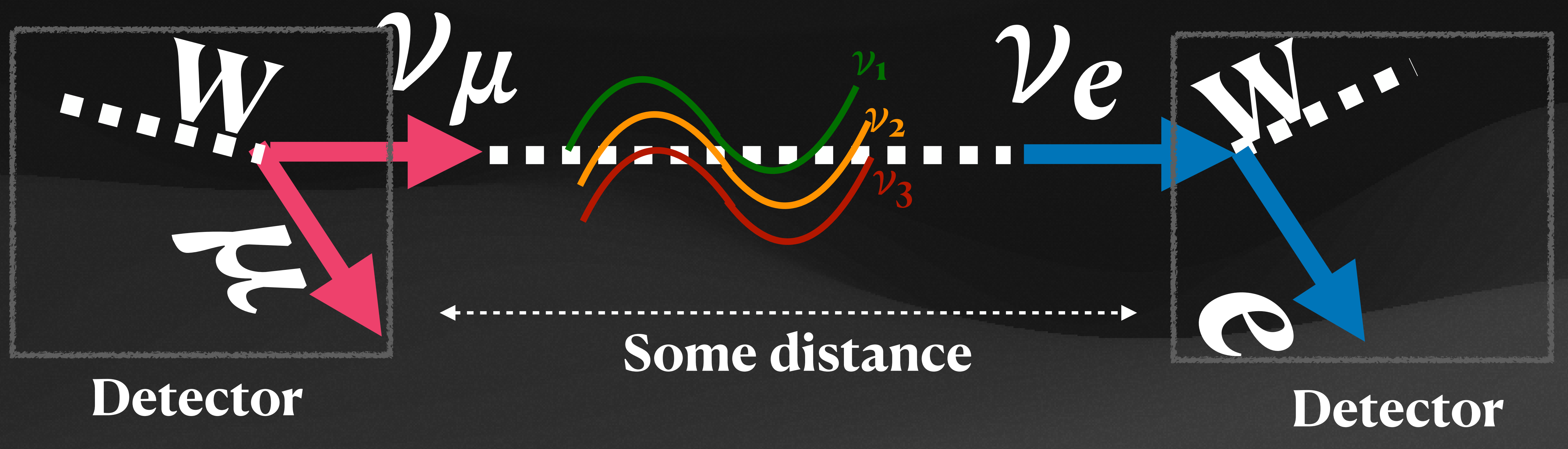
Neutrino oscillations

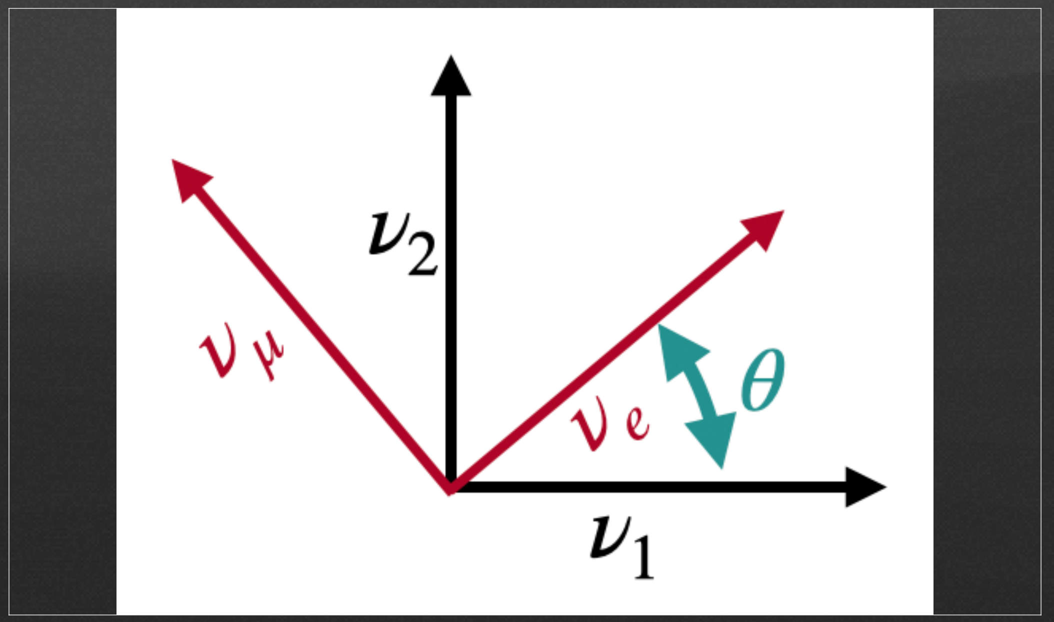
Neutrino can change its flavor when give it time to propagate



Neutrino oscillations

Neutrino can change its flavor when give it time to propagate





- **Neutrino oscillations** require an existence of **neutrino mass spectrum**, i.e mass eigenstate ν_i with definite mass m_i (where i is 1, 2, 3* at least)
- It requires flavor eigenstate with definite flavor, ν_α (where α is e, μ, τ) must be **superpositions** of the mass eigenstates, *a fundamental quantum mechanic phenomenon*

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$$

flavor eigenstate

mass eigenstate

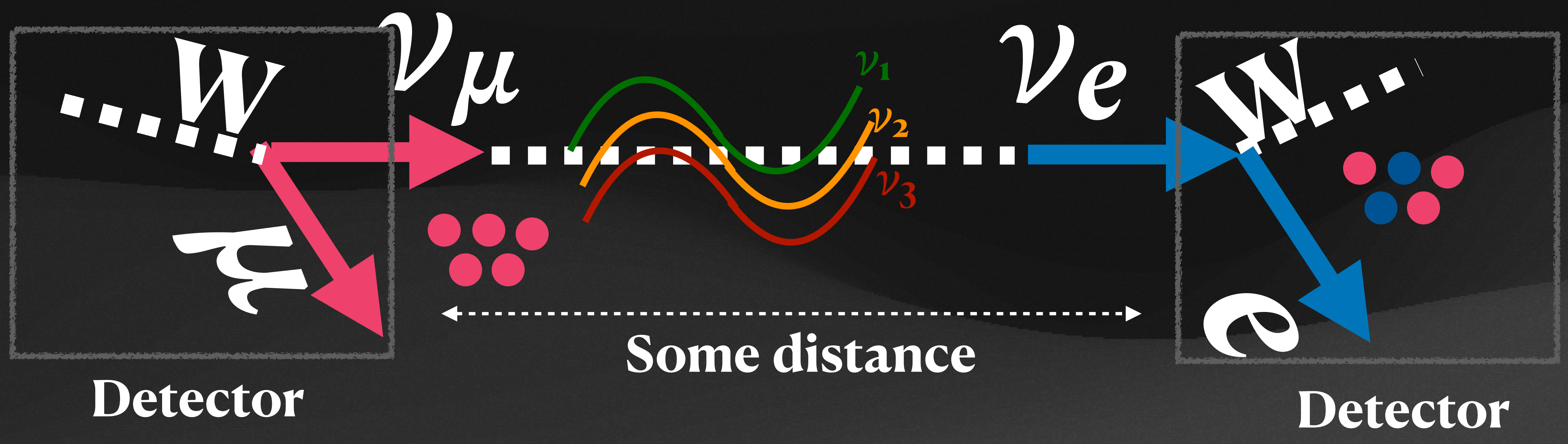
PMNS** leptonic mixing matrix

**It's still possible that there are more than 3 mass eigenstates*

***PMNS is shorted for Pontecorvo-Maki-Nakagawa- Sakata*

A *simple* measurement ... *except neutrino is too shy to show up*

Neutrino can change its flavor when give it time to propagate



1. Need a source of ν_α (eg. ν_μ)
2. Put detector at some distance from ν_α source
3. Look for ν_β *appeared* from the source of ν_α

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} Re(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin^2\left(\Delta m^2_{ij}\frac{L}{4E}\right)$$
$$\pm 2\sum_{i>j} Im(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin\left(\Delta m^2_{ij}\frac{L}{2E}\right)$$

distance/baseline

neutrino energy

where

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

Oscillation probability is driven by the mixing matrix and neutrino mass-squared differences

What revealed with neutrino oscillation?

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle$$

flavor eigenstate

PMNS** leptonic mixing matrix

mass eigenstate

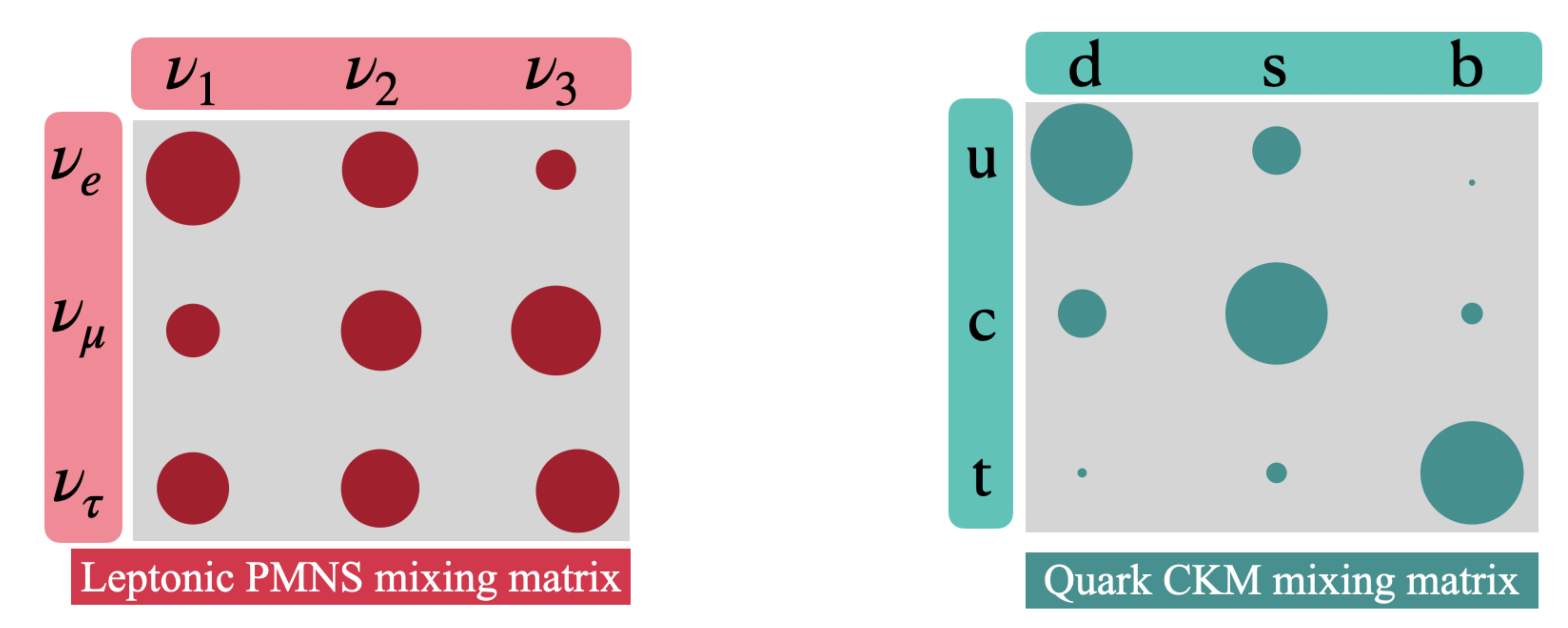
$$c_{ij} = \cos\theta_{ij},\ \ s_{ij} = \sin\theta_{ij}$$

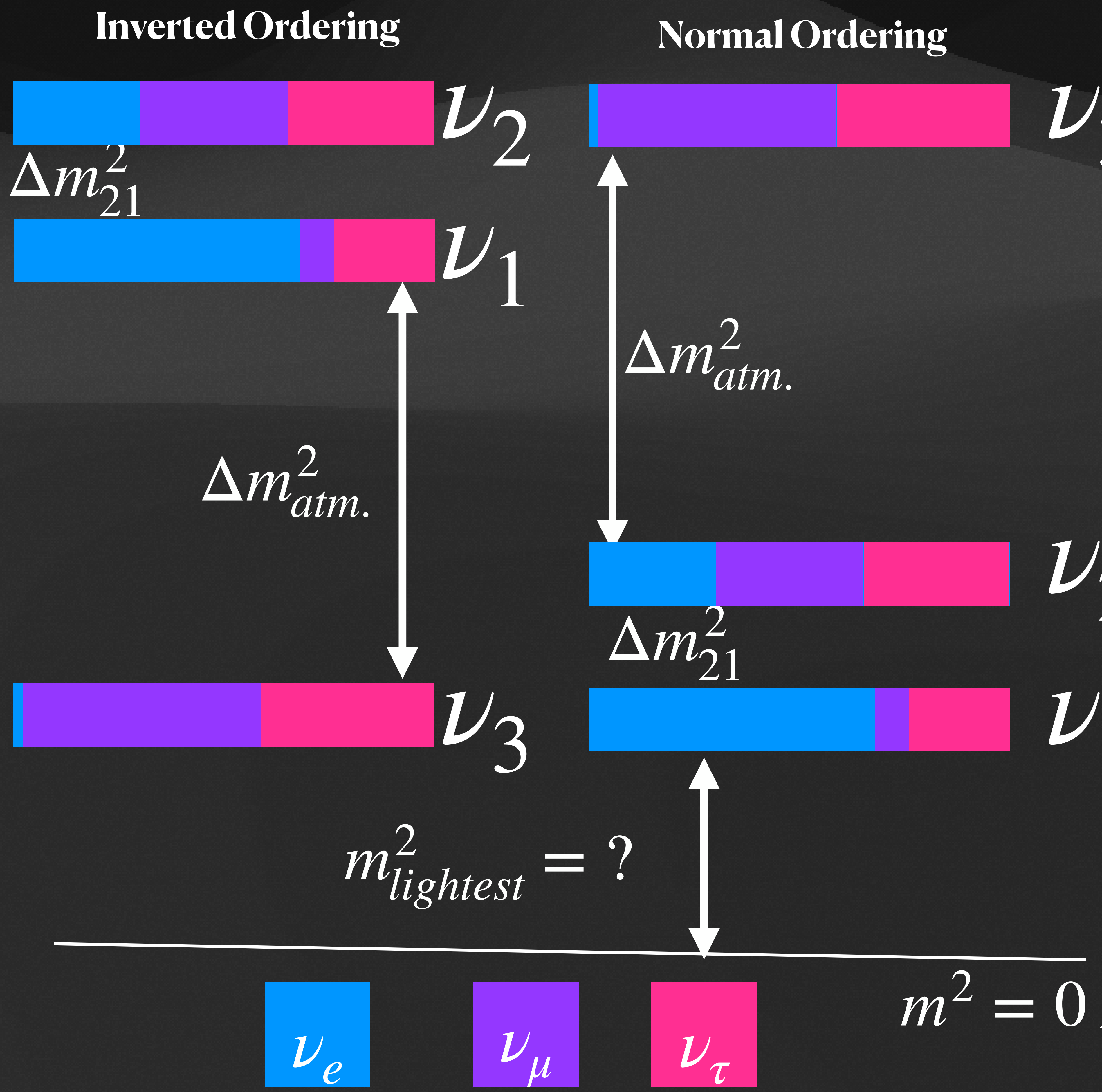
$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \text{Diag}(e^{i\rho_1}, e^{i\rho_2}, 1)$$

- Three mixing angles are non-zero: two ($\theta_{23} \sim 45^\circ$ and $\theta_{12} \sim 33^\circ$) are relatively large (*comparing to the quark mixing*) and $\theta_{13} \sim 9^\circ$
- Two mass-squared splitting $\Delta m^2_{21} \approx 7.5 \times 10^{-5}\text{eV}^2$ and $|\Delta m^2_{31}| \approx 2.5 \times 10^{-3}\text{eV}^2$
- These five fundamental parameters are measured at few percentage of uncertainty

Known unknown in neutrino oscillation

- CP violation phase *(There are some hints from single experiment but not the other)*
- Whether neutrino mass ordering is normal or inverted?
- Precise value of the largest mixing angle θ_{23}





What's behind the *difference in the mixing patterns of quark and lepton* is **unknown and must be understood**

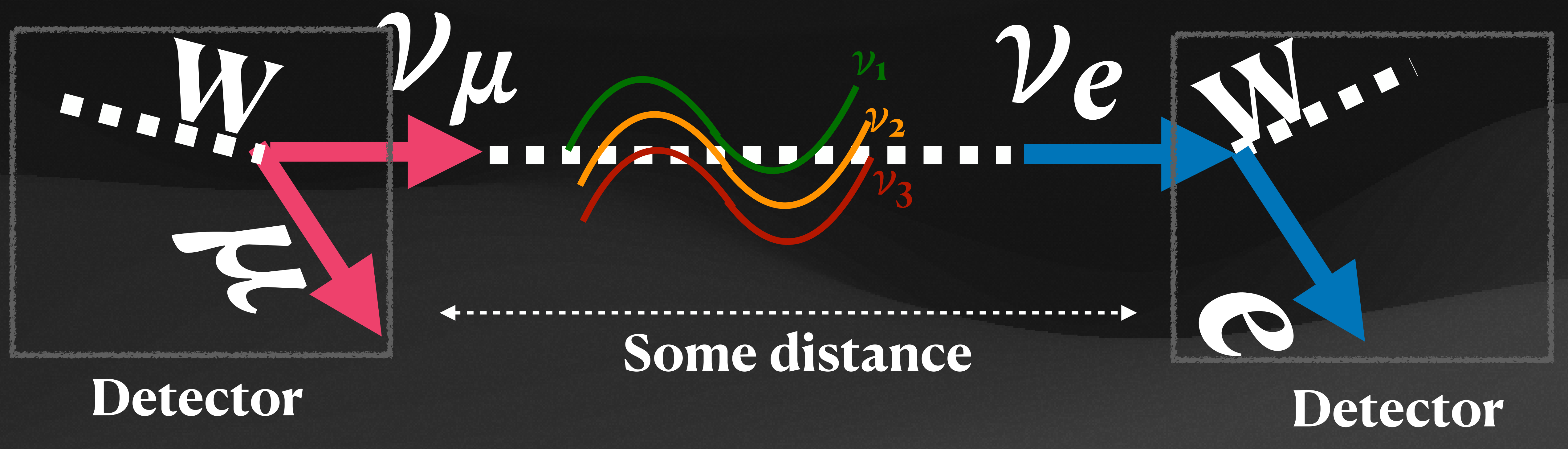
SC et al PRD 103 (11), 112010

SC et al Symmetry 14 (1), 56

Quye Phan, SC et al PTEP (2025): ptaf082

Other assets of neutrino oscillations

Neutrino can change its flavor when give it time to propagate



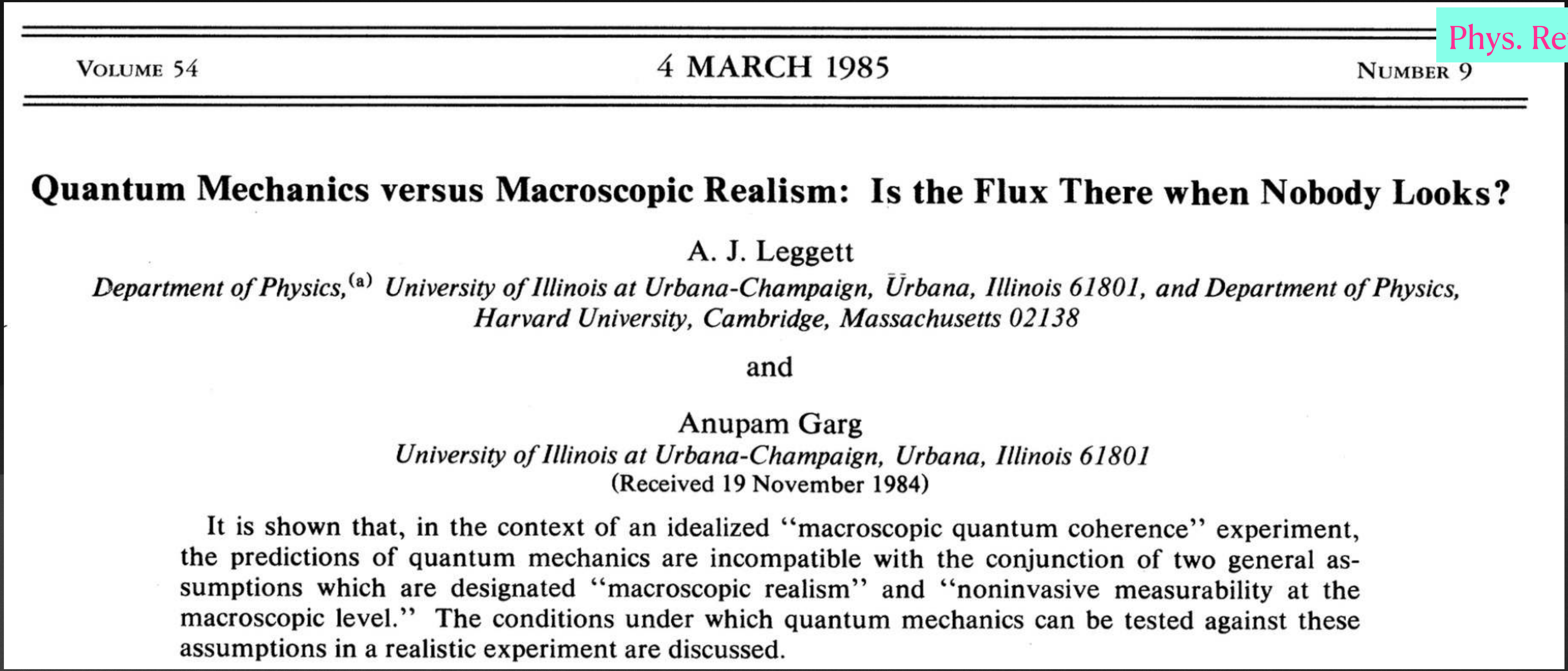
- Neutrino oscillation is a hallmark of quantum superposition and coherence
- Also a bridge btw. microscopic quantum phenomena and macroscopic observability

➜ Unique tool to test quantum foundation

Leggett-Garg inequality

Leggett-Garg inequality: a *time-analogue* Bell iequality

Phys. Rev. Lett. 54, 857


VOLUME 54 4 MARCH 1985 NUMBER 9

Quantum Mechanics versus Macroscopic Realism: Is the Flux There when Nobody Looks?

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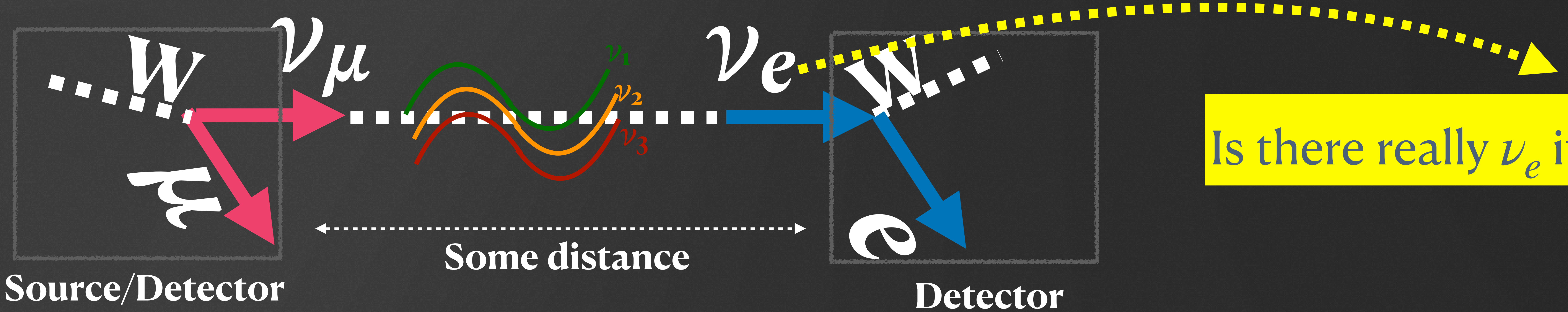
University of Illinois at Urbana-Champaign, Urbana, Illinois 61801
(Received 19 November 1984)

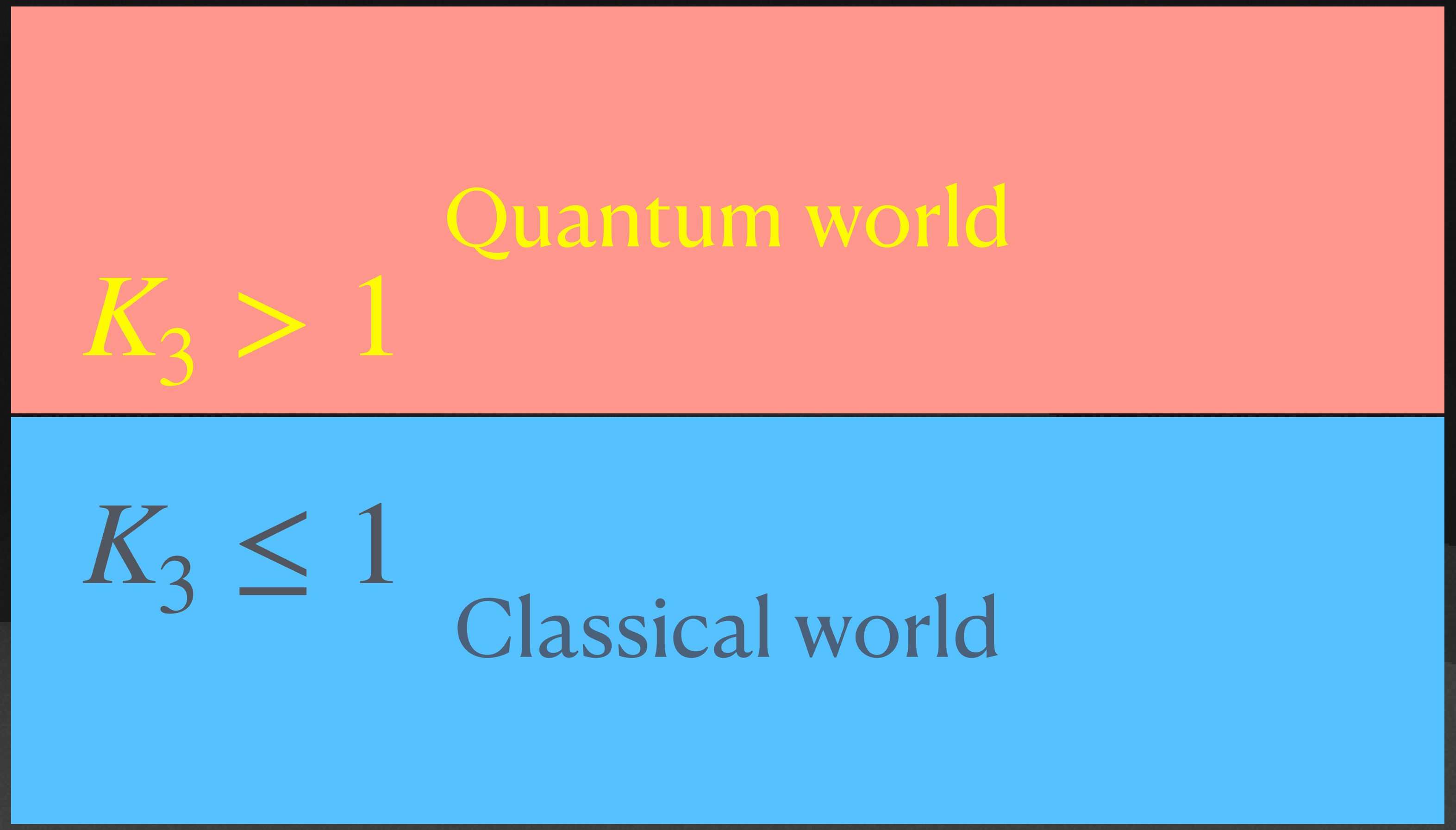
It is shown that, in the context of an idealized "macroscopic quantum coherence" experiment, the predictions of quantum mechanics are incompatible with the conjunction of two general assumptions which are designated "macroscopic realism" and "noninvasive measurability at the macroscopic level." The conditions under which quantum mechanics can be tested against these assumptions in a realistic experiment are discussed.



Is there really ν_e if not detected?

Leggett-Garg inequality: a *time-analogue* Bell iequality

- Based on assumptions:
  - Macroscopic realism *per se*: State has defined value at all times
    $\langle \hat{Q}(t_i)\hat{Q}(t_j)\rangle = \langle \hat{Q}(t_i)\rangle\langle \hat{Q}(t_j)\rangle$
  - Noninvasive measurability: can measure state without disrupting it
    $[\hat{Q}(t_i), \hat{Q}(t_j)] = 0$
  - (*Later added*) Induction: Measurement outcome can't be affected by future possible measurement



- Assume a **dichotomic observables** $\langle \hat{Q}(t)\rangle = \pm 1$ with $C_{ij} = \langle \hat{Q}(t_i)\hat{Q}(t_j)\rangle$, lead to inequalities:
  - For three-measurements $K_3 = C_{12} + C_{23} - C_{31} \leq 1$
  - For n-measurements $K_n = \sum_{i=1}^{n-1} C_{i,i+1} - C_{n,1} \leq n-2$

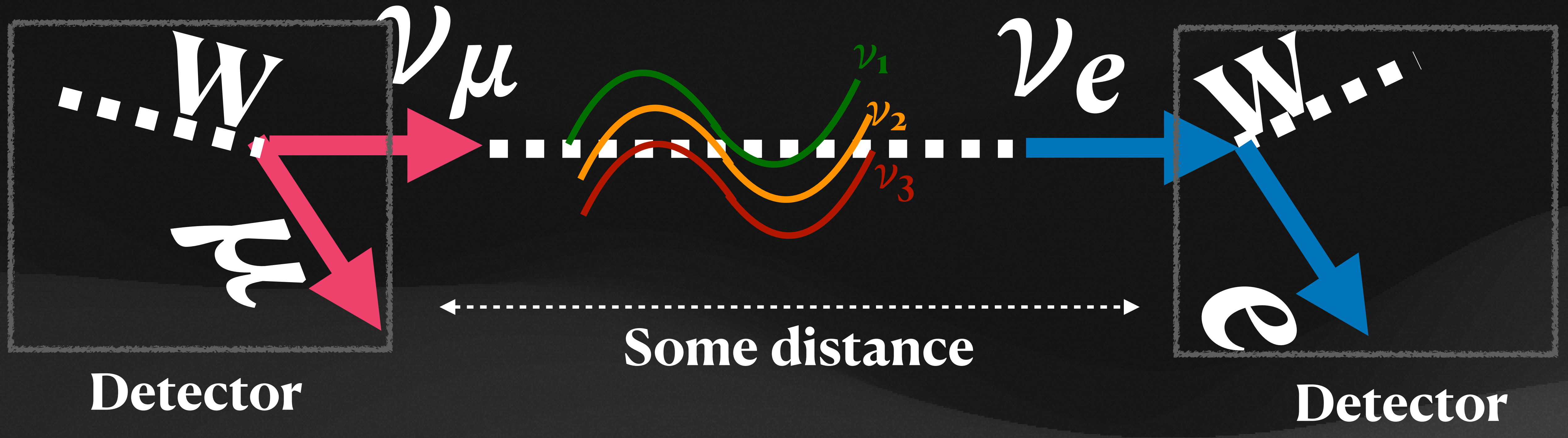
LGI concerns the *temporal* correlation of a single state and impact of measurement on that state evolution ➜ **LGI violation** suggest that state **doesn't** evolve *deterministically* and *independently* of observation

Experimental test (*in microscopic sys.*):

- Use **weakly measurement of photon polarization**, Goggin, M. E., et al PNAS 108, no. 4 (2011): 1256-1261. . "Violation of the Leggett–Garg inequality with weak measurements of photons."
- Use **single spins in a diamond defect center** , Waldherr. G., et al PRL 107.9 (2011): 090401 . "Violation of a temporal Bell inequality for single spins in a diamond defect center."
- Use **superconducting quantum circuit**, Palacios-Laoy, A. Nature Physics, 6.6 (2010): 442-447. "Experimental violation of a Bell's inequality in time with weak measurement."

Formulate LGI test with neutrino oscillation

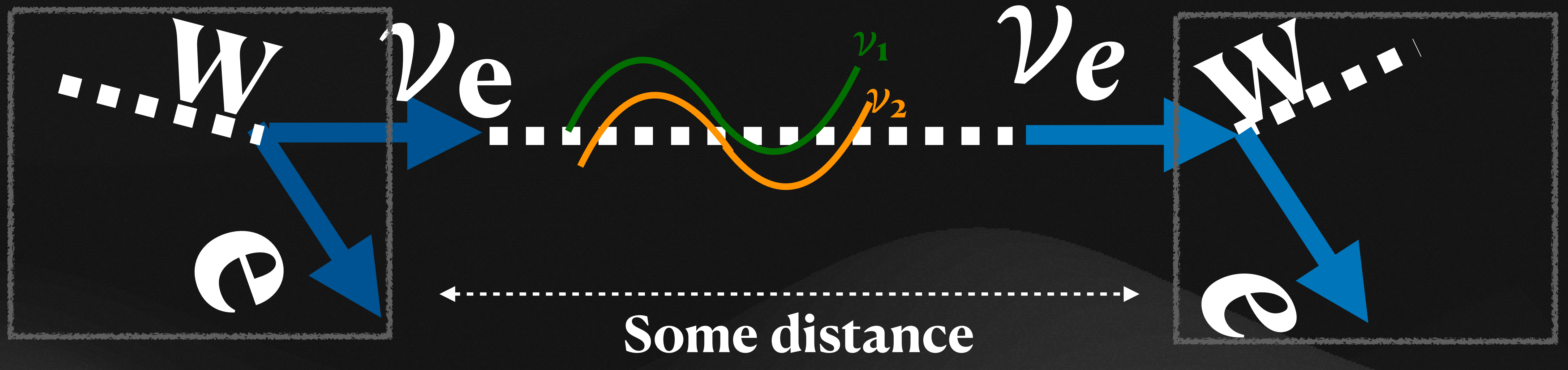
Is neutrino oscillation suitable for LGI test?

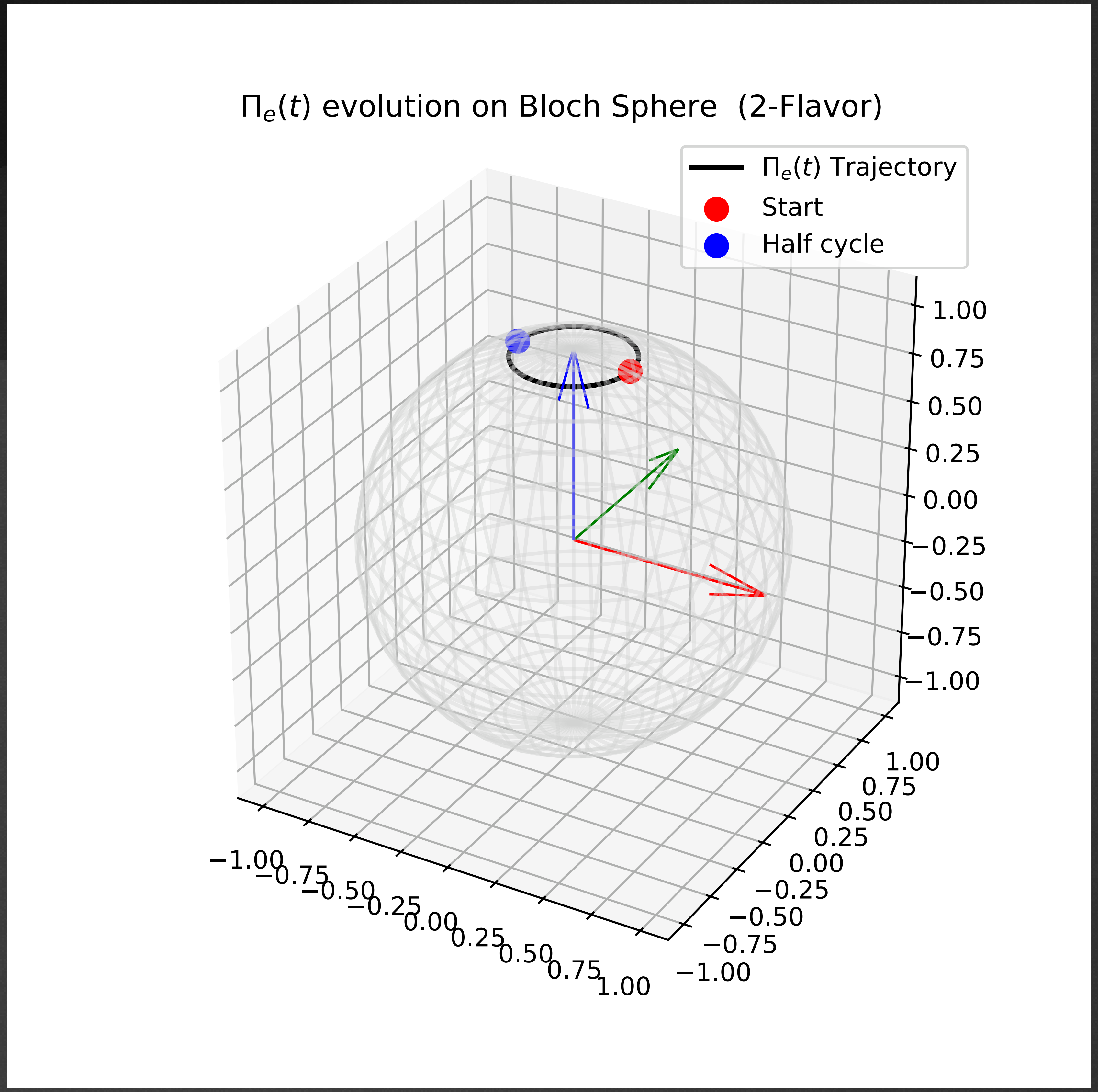


- Small mass-squared difference: quantum effects observed at a macroscopic level $L_{osc} \sim 10^3 - 10^5$ km
- Weak interaction: Coherence maintains for a long distance $L_{coh.} \sim 10^3 - 10^{16}$ km *(depend on energy and wave packet size of neutrino)*
- Distinguished flavors: electron, muon, tau ➜ use for dichotomic observable
  - Measure survival probability $P(\nu_\alpha \to \nu_\alpha)$ and appearance probability $P(\nu_\alpha \to \nu_\beta)$, $\alpha \neq \beta$
- Neutrino state evolution, *in two-flavor approximation*, depend on a single phase $\omega(t) \sim \Delta m^2 \frac{ct}{E} = \Delta m^2 \frac{L}{E}$ ➜ essential for *stationarity* condition
- Neutrino detection: projective measurement on single particle from the ensemble (*unlike other weak measurements*)

Formulate LGI test with neutrino oscillation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$





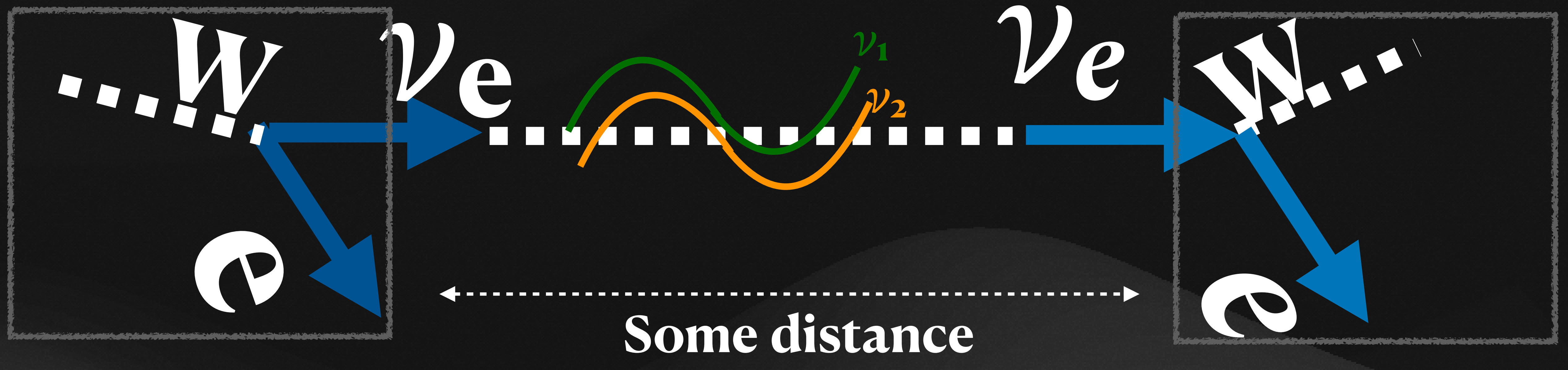
- Relativistic neutrinos $L \propto t$. Assume a ν_e source with *two-flavor approximation*
  - $Q(x) = +1$ if detected as ν_e
  - $Q(x) = -1$ if detected as ν_μ

- Q can be identified as a flavor projector $Q = |\nu_e\rangle\langle\nu_e| - \nu_\mu\rangle\langle\nu_\mu|$ and evolution $Q(t) = U^\dagger(t)QU(t)$ where $U = e^{-iHt}$

$\rightarrow \langle Q(x)\rangle = 2P_{\nu_e\rightarrow\nu_e}(x) - 1$

Formulate LGI test w/ neutrino oscillation

- **Additional assumptions for practical test**
  - Single particle can't be measured at different times ➜ Ensemble-averaged measurement over identically prepared neutrino states
  - "*Stationarity*" assumption, i.e correlation among measurements depends on their time interval
    - Need time-dependent Hamiltonian



- Relativistic neutrinos $L \propto t$. Assume a ν_e source with *two-flavor approximation*
  - $Q(x) = +1$ if detected as ν_e
  - $Q(x) = -1$ if detected as ν_μ

Two-flavor approximation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

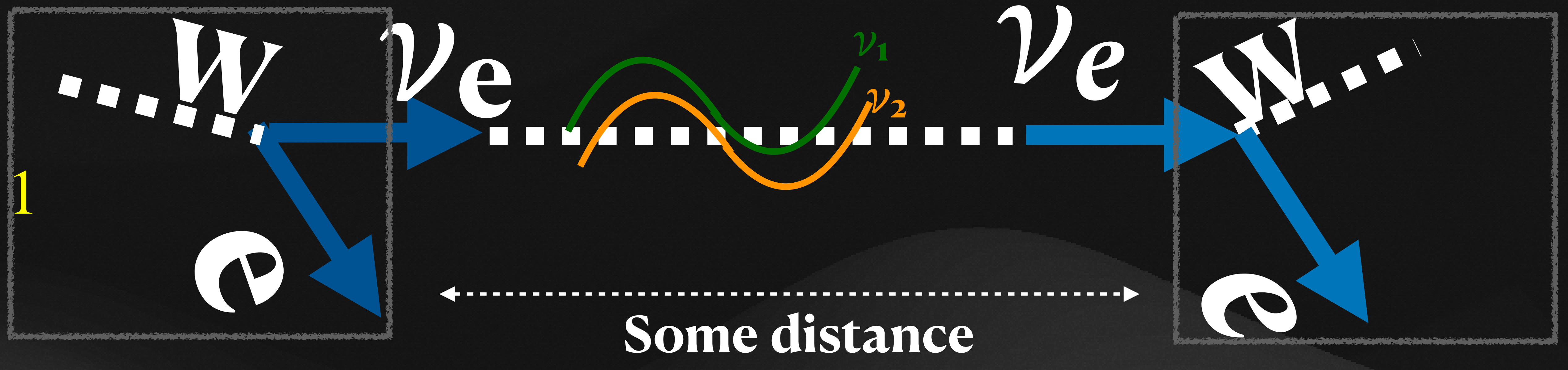
$$P_{\nu_e \to \nu_\mu}(x) = \sin^2 2\theta \cdot \sin^2 1.267 \Delta m^2 [eV^2] \frac{L[km]}{E[GeV]}$$

$$P_{\nu_e \to \nu_e}(x) = 1 - P_{\nu_e \to \nu_\mu}(x)$$

- Q can be identified as a flavor projector $Q = |\nu_e\rangle\langle\nu_e| - \nu_\mu\rangle\langle\nu_\mu|$ and evolution $Q(t) = U^\dagger(t) Q U(t)$ where $U = e^{-iHt}$

➜ $\langle Q(x) \rangle = 2P_{\nu_e \to \nu_e}(x) - 1$

Formulate LGI test w/ neutrino oscillation



- Two-time correlation (w/ stationarity assumption)

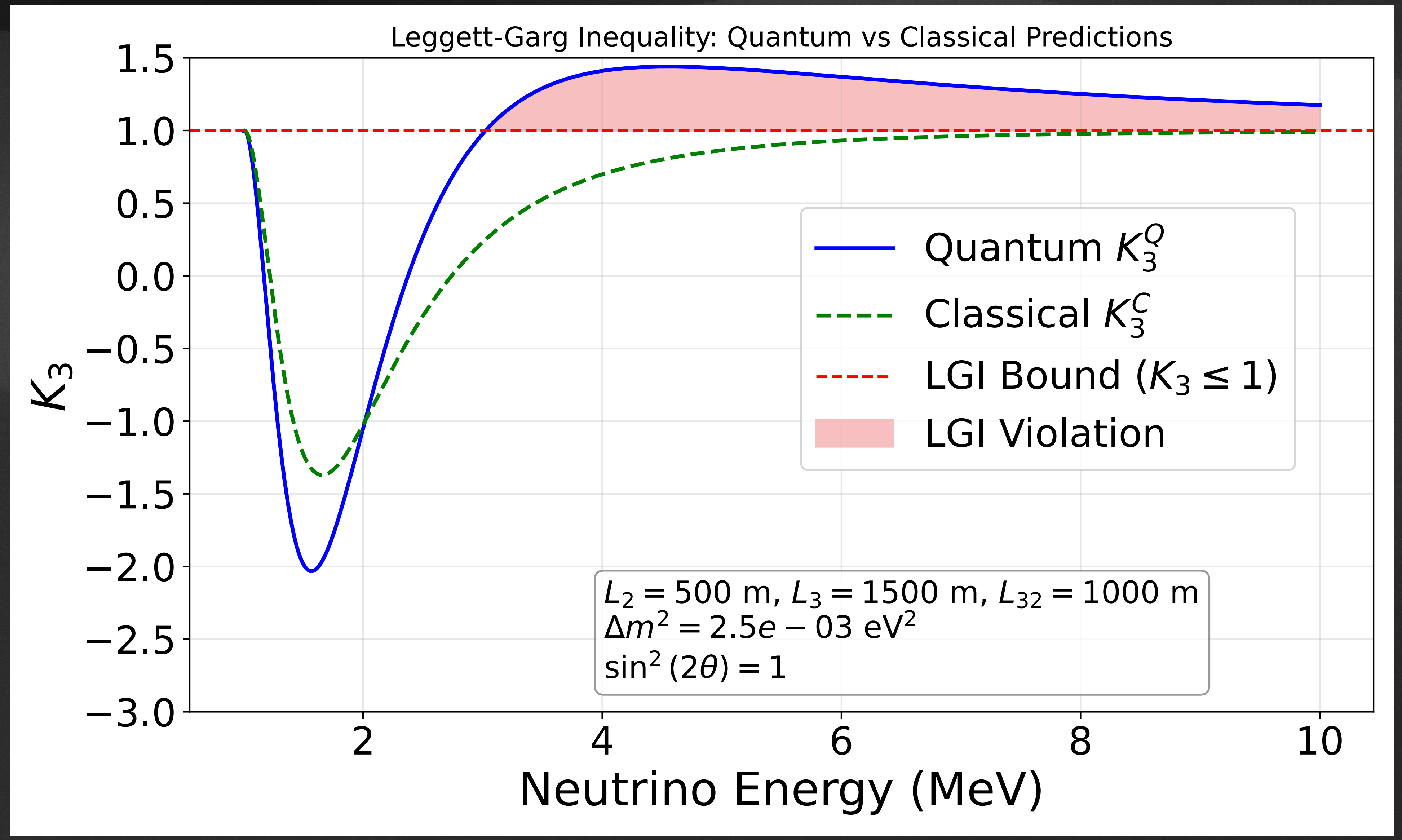
  $C_{ij} = \langle Q(t_i)Q(t_j)\rangle = 2P_{\nu_e\to\nu_e}(t_i - t_j) - 1 = 2P_{\nu_e\to\nu_e}(\Delta L_{ij}) - 1$

- For 3-measurement correlation
  - *"Classical"* (assumed Markov-like evolution)

    $K_3^C = 1 - 4\cdot\left(1 - P_{\nu_e\to\nu_e}(L_2)\right)\cdot\left(1 - P_{\nu_e\to\nu_e}(L_3 - L_2)\right)$

  - *Quantum* (unitary evolution)

    $K_3^Q = 2\left(P_{\nu_e\to\nu_e}(L_2) + P_{\nu_e\to\nu_e}(L_3 - L_2) - P_{\nu_e\to\nu_e}(L_3)\right) - 1$

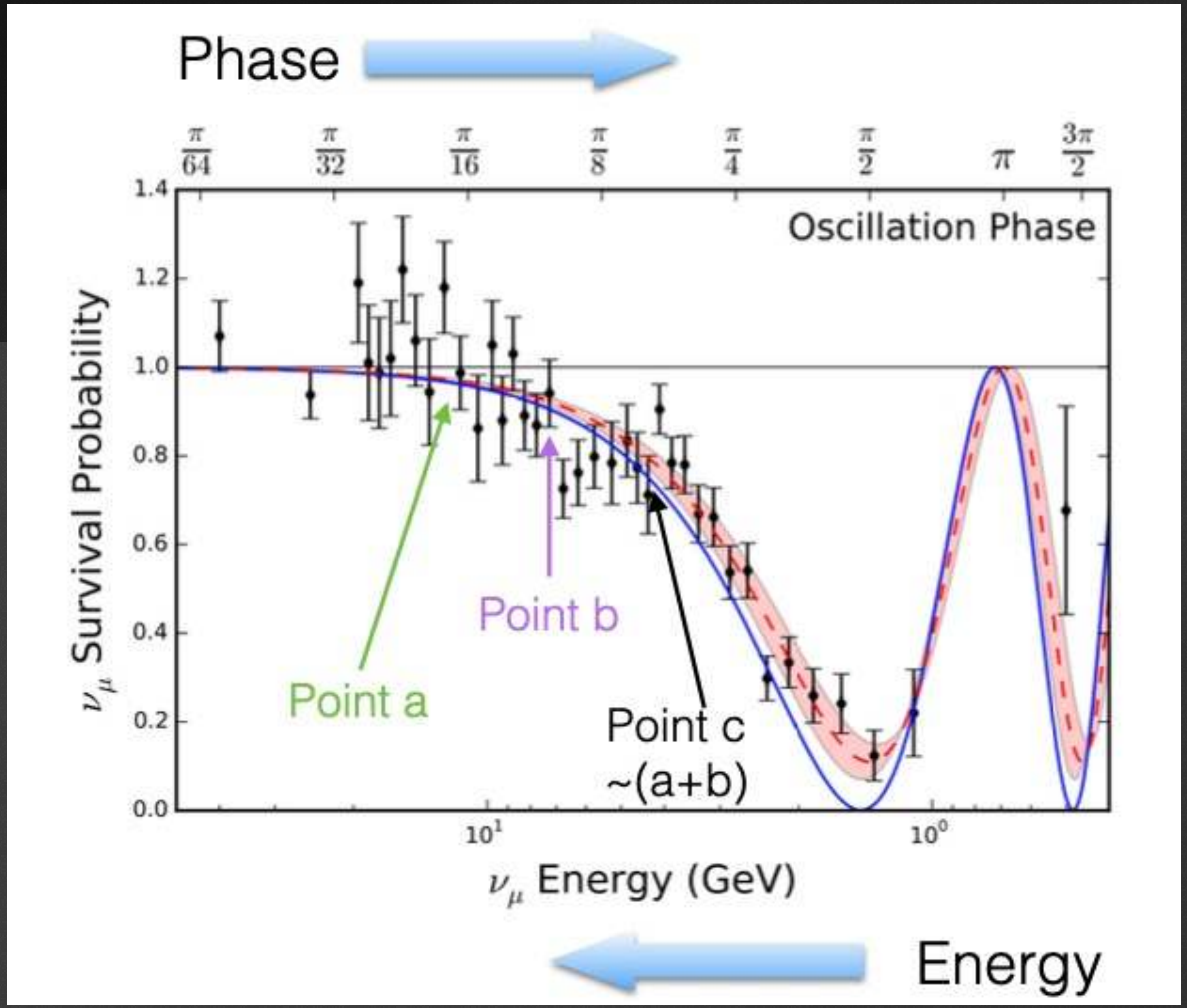


- Ideally, to test K_3 we will need measurements at three baselines: $L_2, L_3, (L_3 - L_2)$ (or two if $L_3 = 2\cdot L_2$)
  - (*Work-in-progress*) Interestingly, neutrino oscillation measurement can explore LGI near at the quantum bound of $K_3^{Q,max} = 1.5$ due to (almost) maximum leptonic mixing

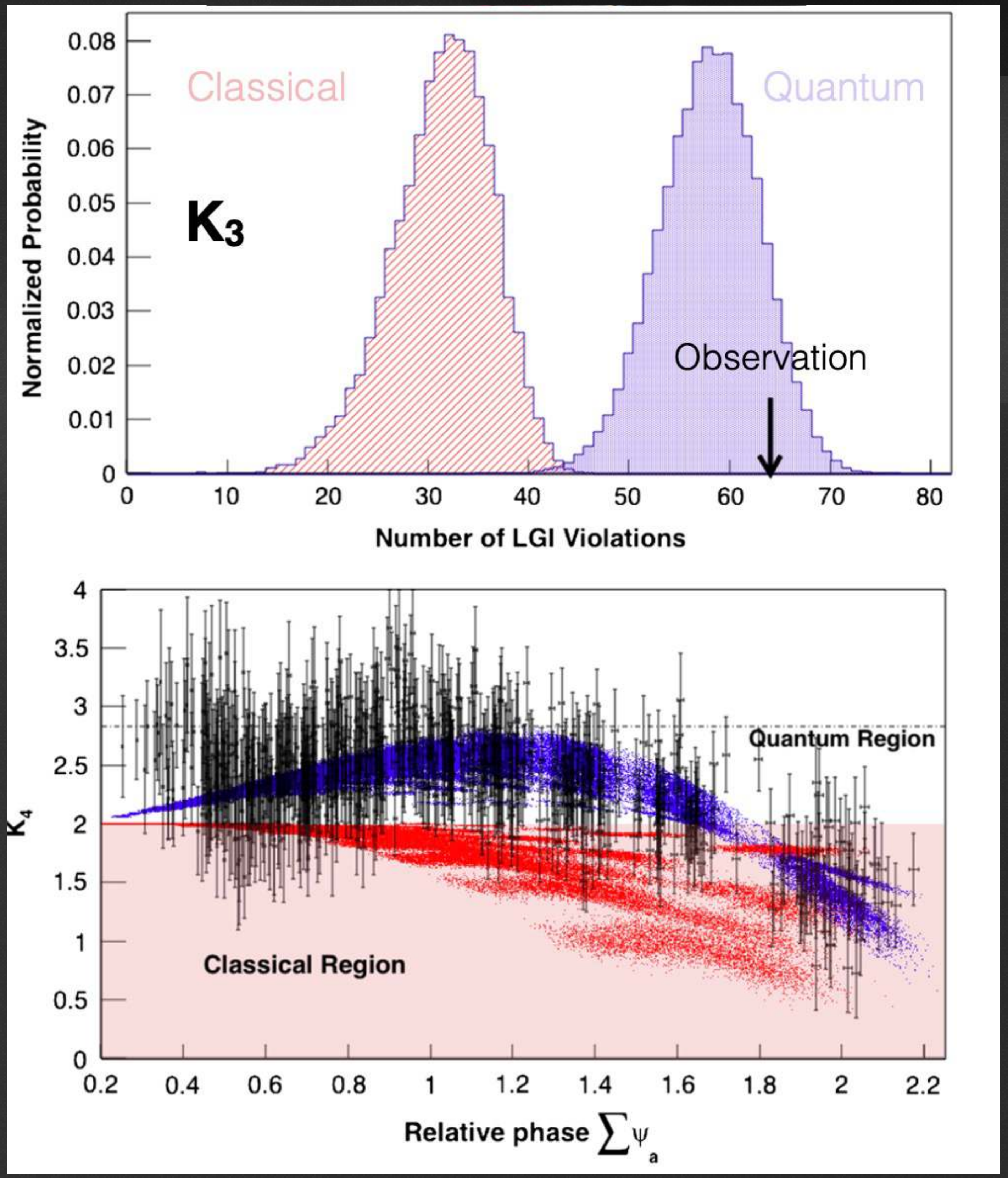
LGI test with neutrino oscillation

PRL 117, 050402 (2016))

- Neutrino oscillation exp. typically uses two detectors: one near (*for no-oscillated reference*) and one far (*for oscillated pattern*)
- Since neutrino state evolves in time depending solely on phase $\omega(t) \sim \Delta m^2 \frac{ct}{E} = \Delta m^2 \frac{L}{E}$, energy is used as time proxy
  - Instead of three detectors, **use only one detector**



Using energy as time proxy



More than 5σ LGI violation

It's not surprising that LGI is violated with neutrino oscillation data. More concerns:

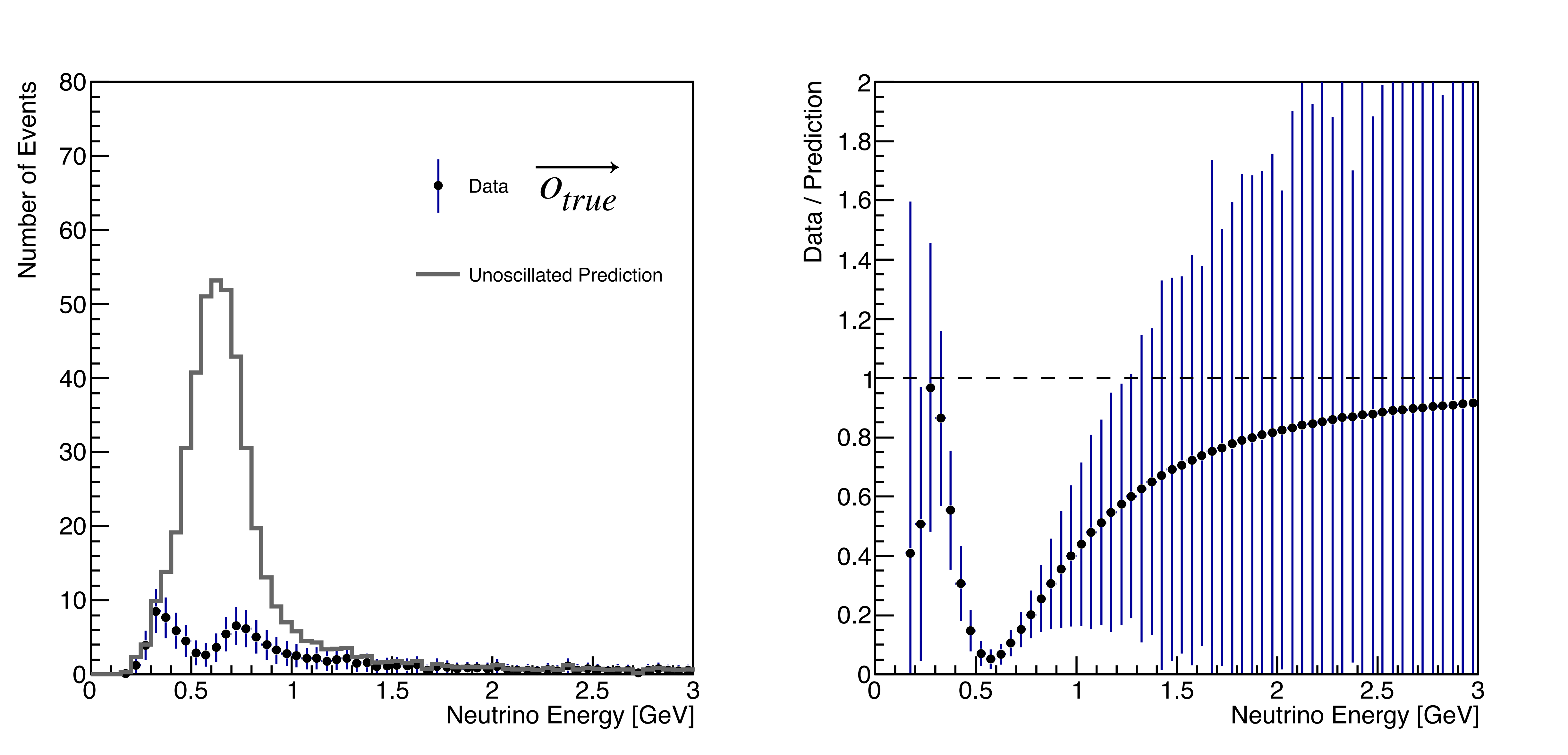
- Robustness of the test
- What can we tell from this kind of test?

Understand the neutrino oscillation measurement

- Do not measure directly probability but ratio of collected data (*w/ some background*) to no-oscillated prediction
  - *Background subtraction is needed*
- Neutrino energy is not known exactly but being constructed with some resolution
  - *Unfolding probability as* $f(E_\nu^{true})$ *is necessary*
- Not NIM measurement but averaged measurement of **ensemble of neutrino in a given energy bin**

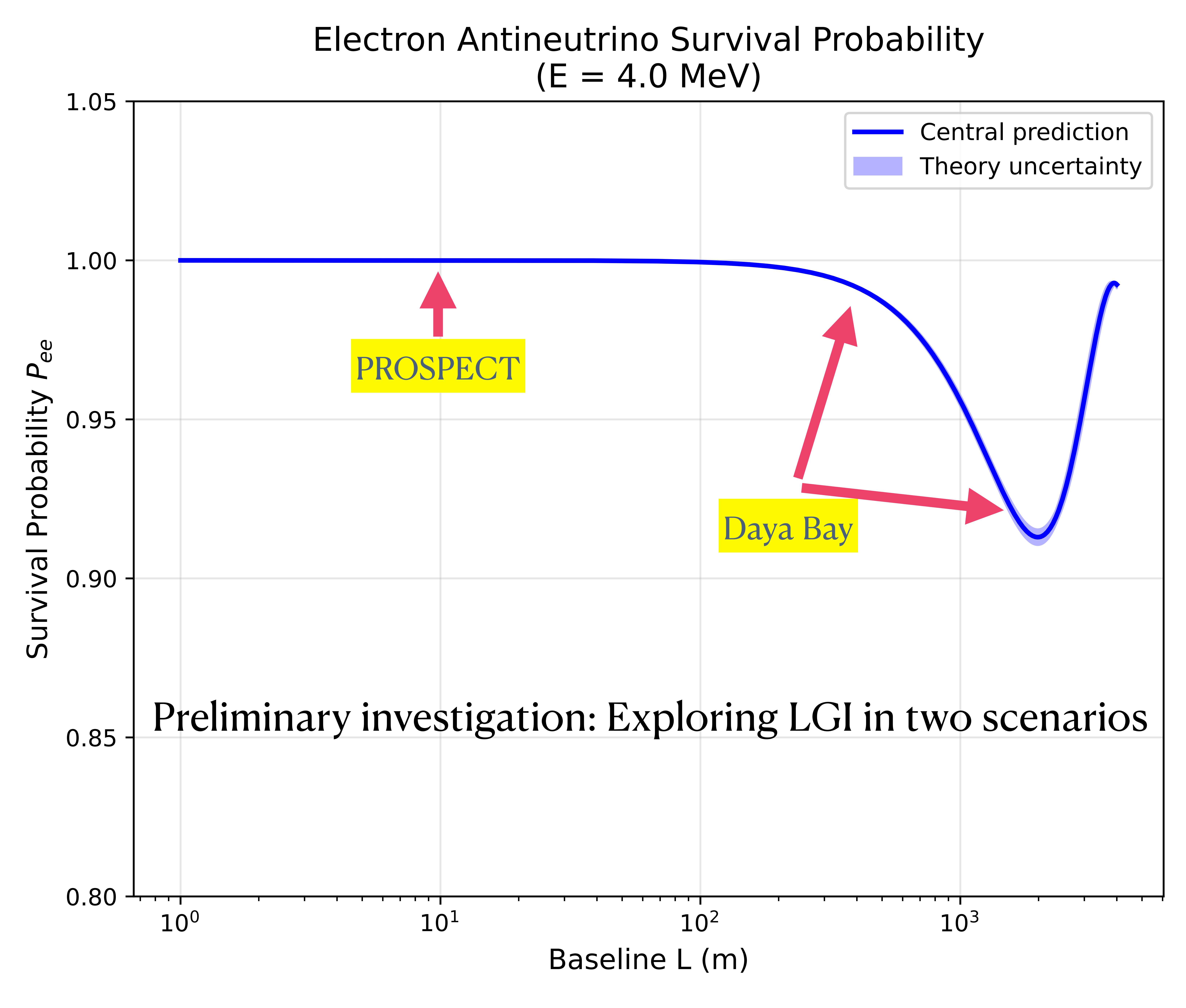
$$N_i(\vec{o}) = \Phi_{flux} \times \sigma \times M_{det.} \times \epsilon \times P(\vec{o})$$

where $\vec{o} = (\Delta m^2_{21}, \Delta m^2_{31}, \theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP})$



Understand what experiments really measure and uncertainty on the extracted/unfolded probability from data is vital. (In addition to additional assumption for formulating the LGI test)

$$P(\nu_\mu \to \nu_\mu) \sim 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{31} L}{4E_\nu}$$


Electron Antineutrino Survival Probability
(E = 4.0 MeV)
Central prediction
Theory uncertainty
PROSPECT
Daya Bay
Preliminary investigation: Exploring LGI in two scenarios
Survival Probability P_{ee}
Baseline L (m)
1.05
1.00
0.95
0.90
0.85
0.80
10^0
10^1
10^2
10^3

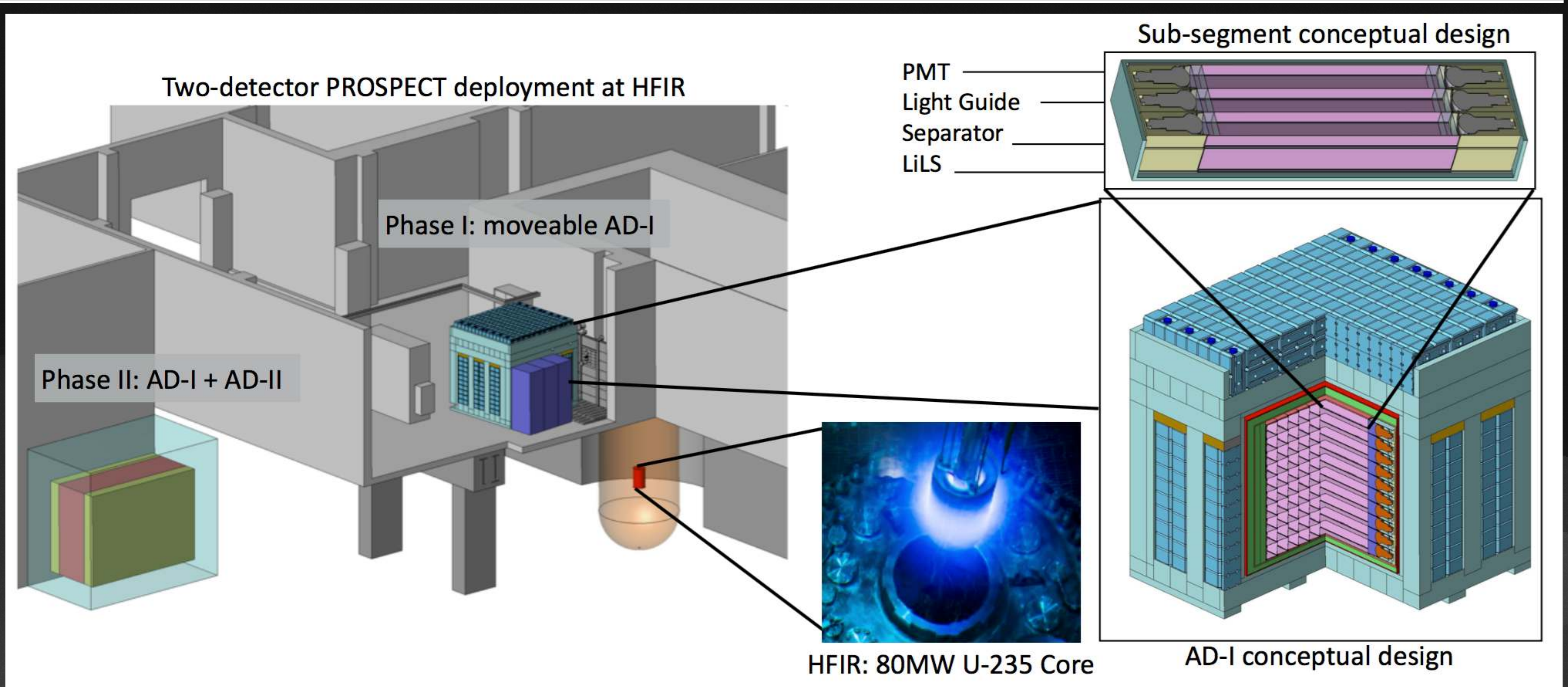
Preliminary results with PROSPECT

Look at where no “standard” oscillation (i.e *classical* behavior) is expected, i.e very short-baseline (*compared to oscillation wavelengths*)

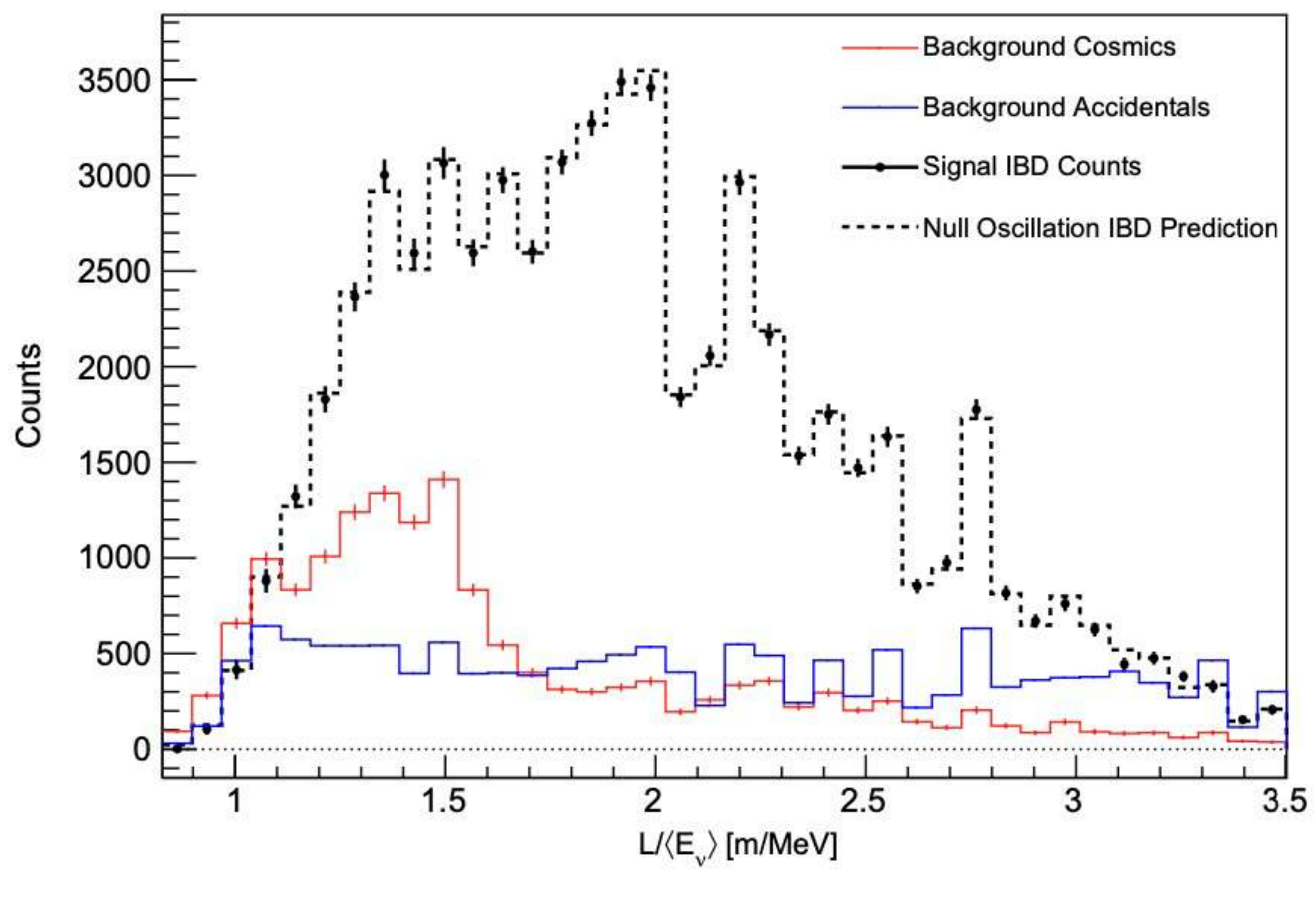
PROSPECT-I Detector at HFIR

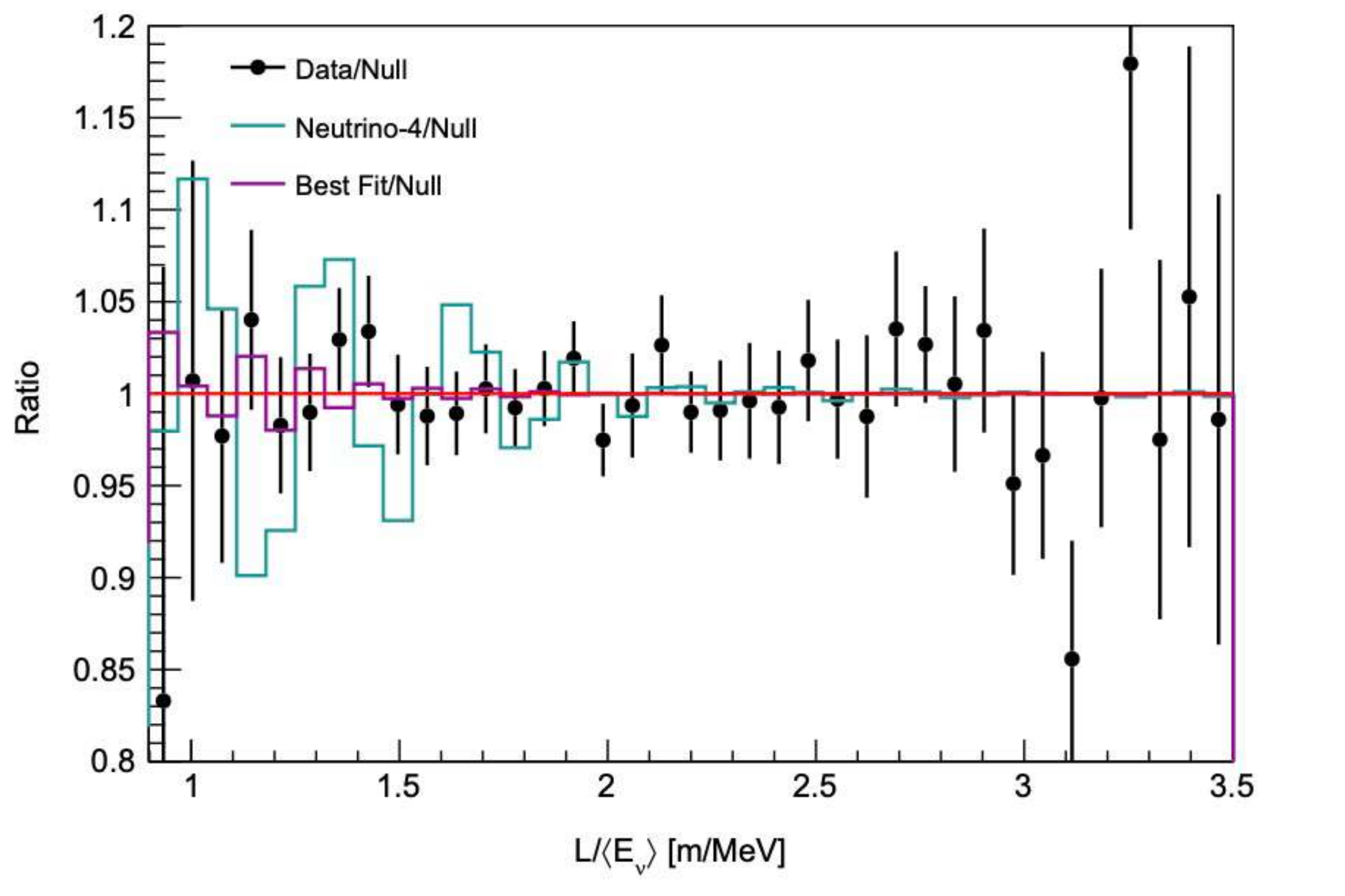
PRL 134, 151802 (2025))

- Compact annular cylindrical core-size (R=0.435m, h = 0.508m)
  - Highly enriched U235
- Baseline 6.7-9.2 m
- High statistics (~60k IBD events)



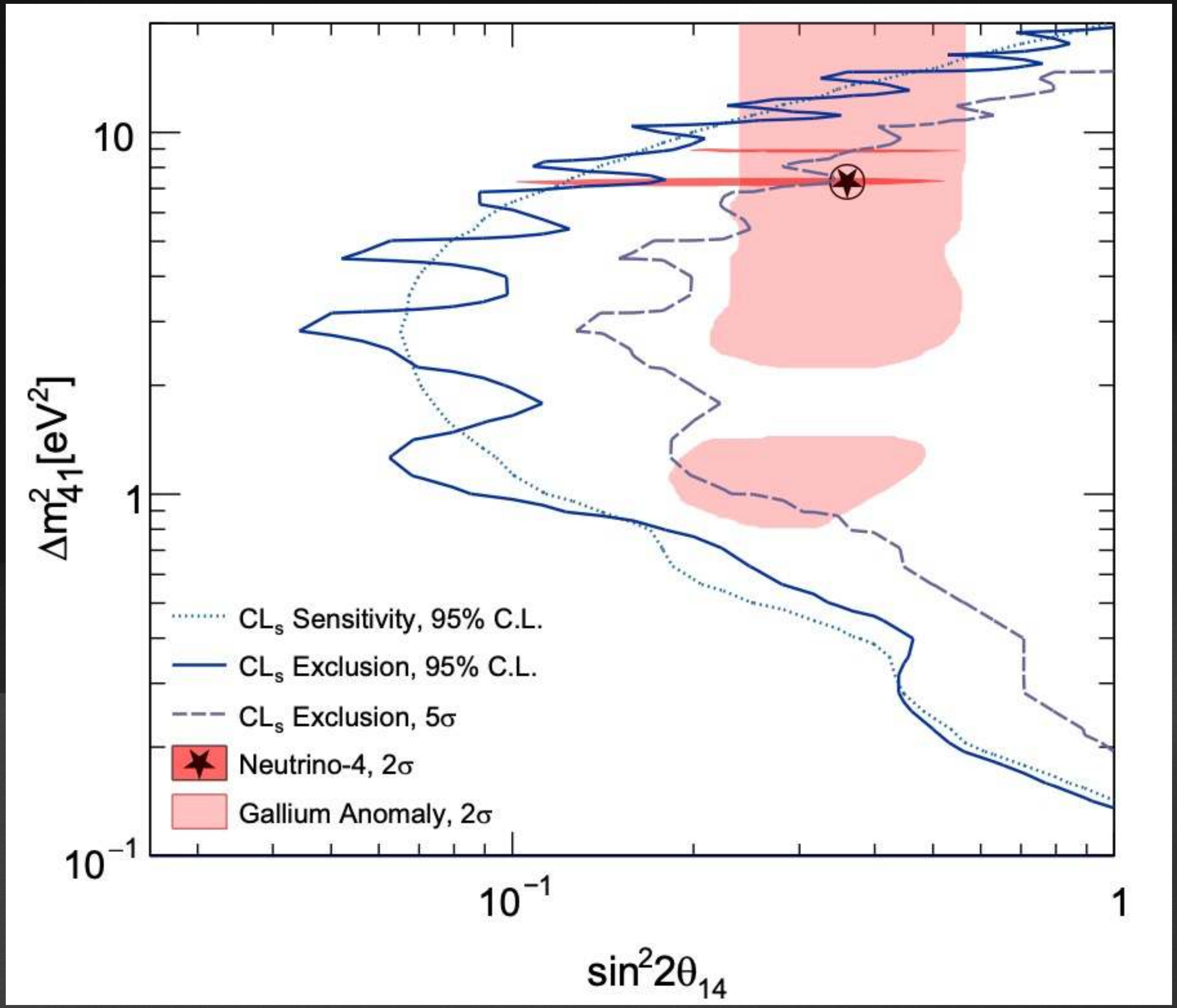
$$P_{\nu_e \to \nu_x}(E, L) = \sin^2 2\theta_{14} \cdot \sin^2 1.267 \Delta m^2_{41}[eV^2] \frac{L[m]}{E[MeV]}$$



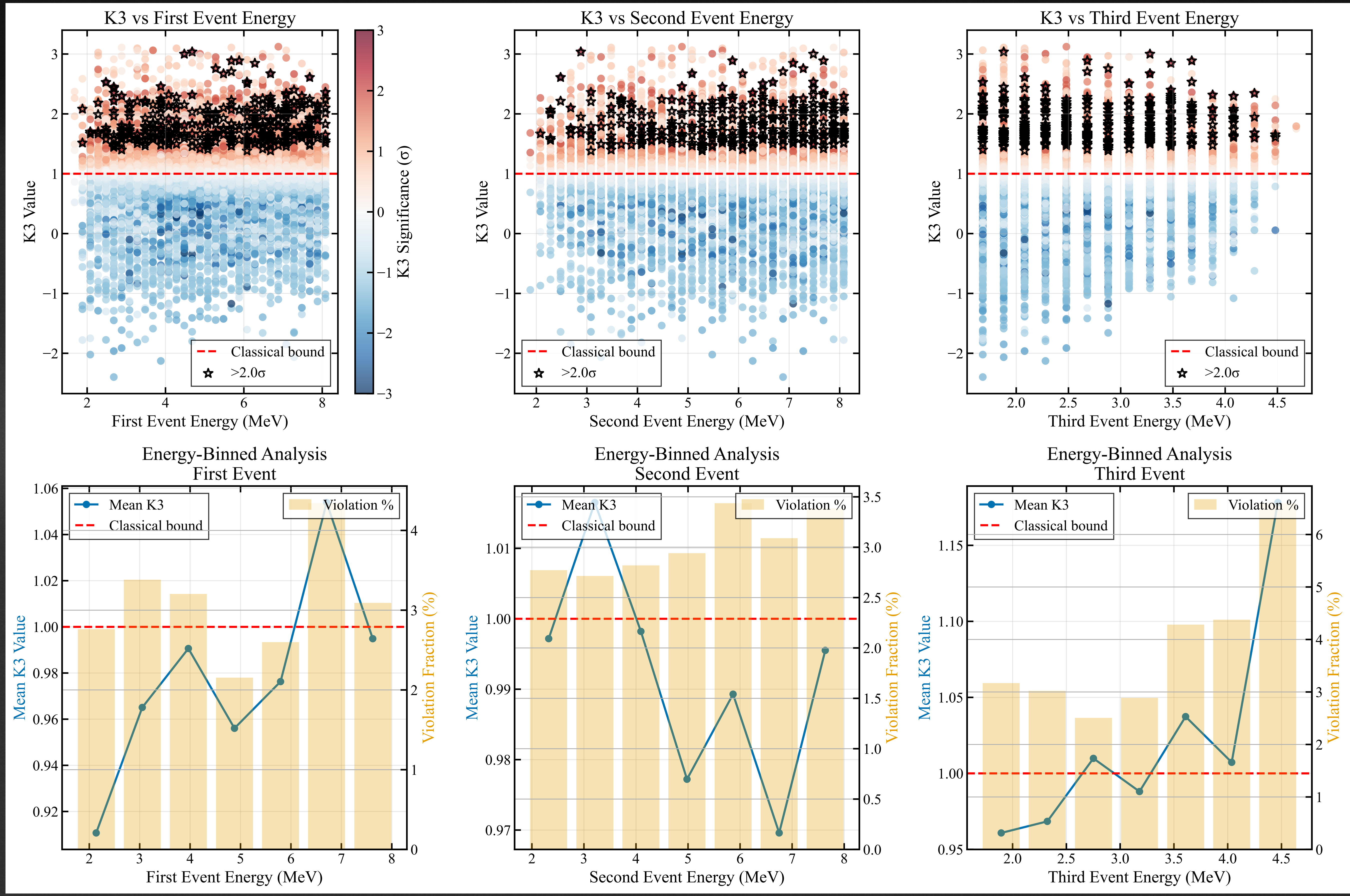


- Conventional approach: parameter fitting
  - Report *excluding region* of parameters if null hypothesis is not excluded significantly

PROSPECT: Alternative view with LGI

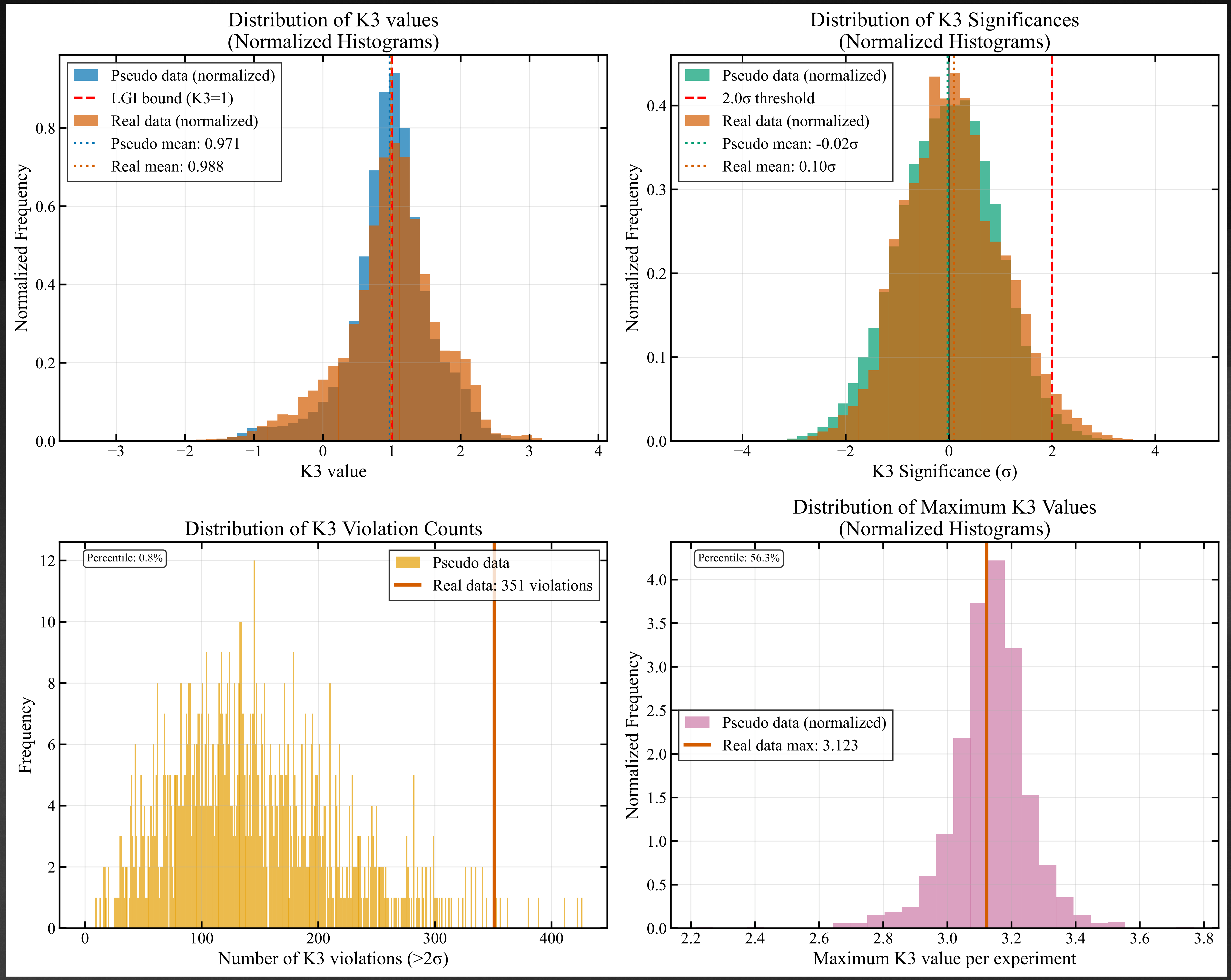


Conventional approach



New approach: use LGI

PROSPECT: Alternative view with LGI



LGI violation found in ~3.6% of phase-matched triplets. Pseudo data are generated to see if this fraction is consistent w. statistical fluctuations. (*Higher statistics of pseudo data are being generated for more precise test but it seems not so significant <3σ*)

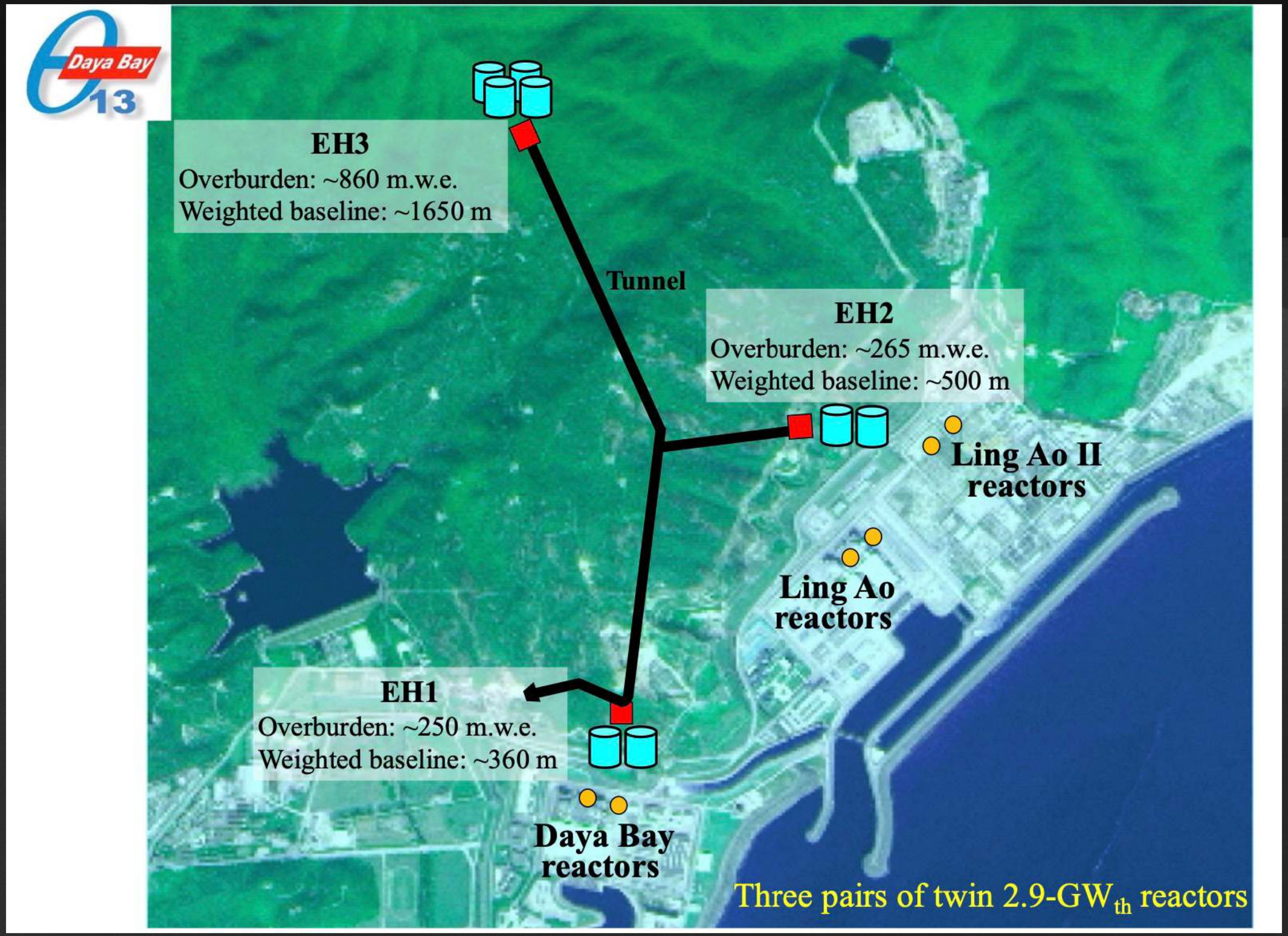
Some preliminary results with Daya Bay

Look where *standard* oscillation (i.e *non-classical* behavior) is expected

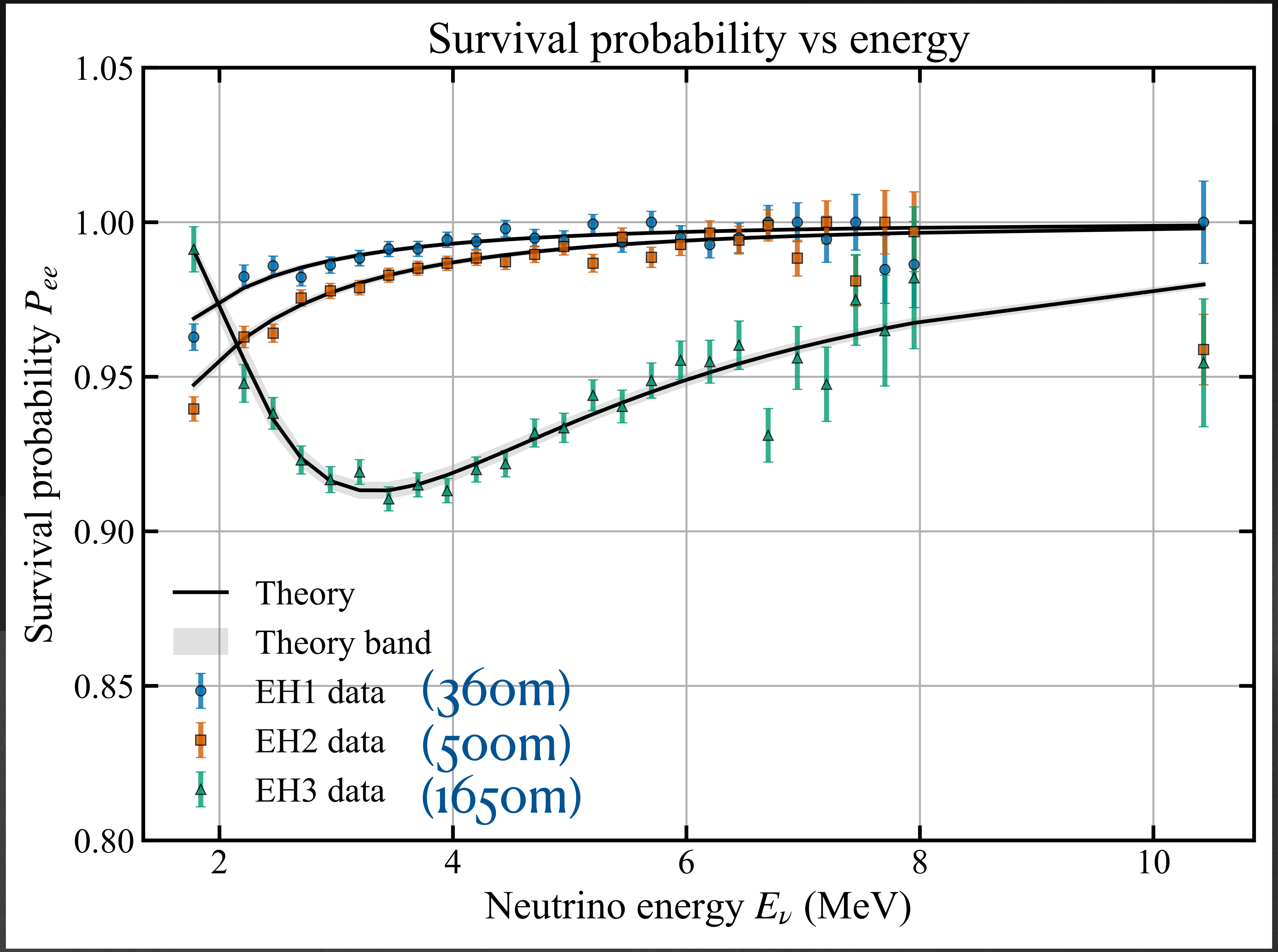
Daya Bay: reactor-based neutrino experiment

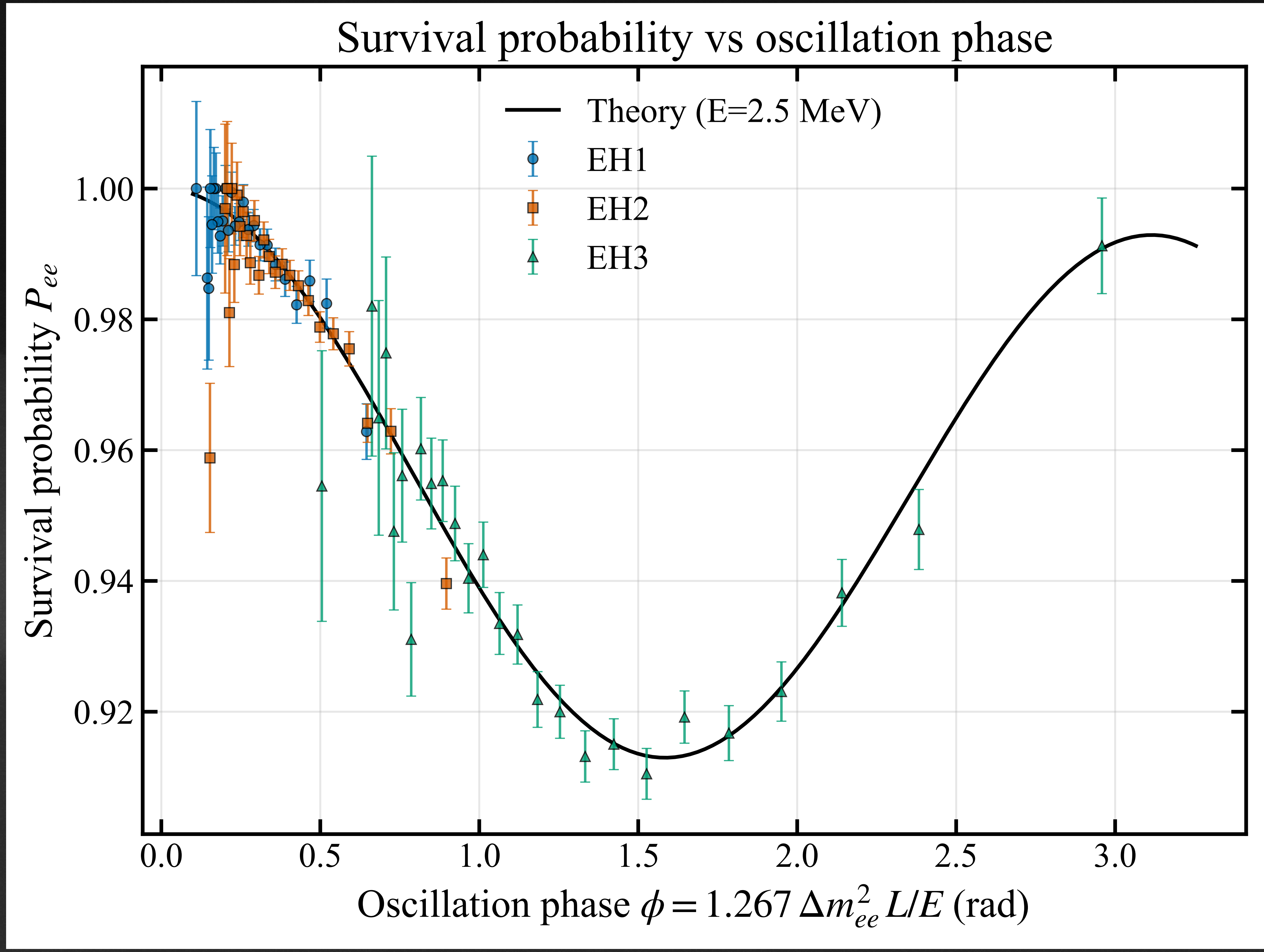
Why Daya Bay?

- Also measure $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ from reactors
- Matter effect is small due to short baseline
- Good approximation for two-flavor single phase evolution; driven by well-constrained mixing angle θ_{13}
- Large and published statistics
- Most important: Data from 3 detectors with identical functionality → examine concept of the 3-detector LGI test



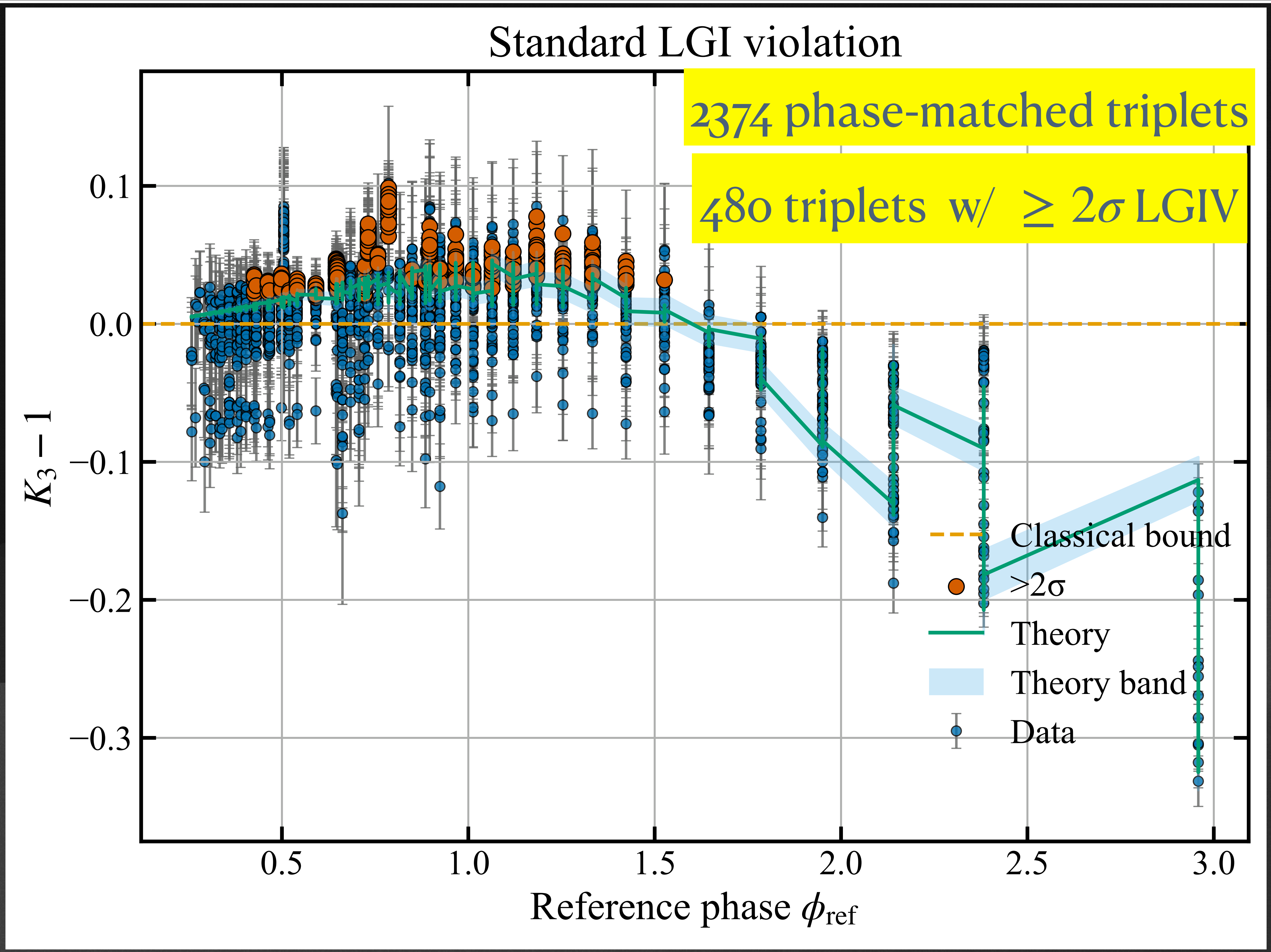
Daya Bay: Background-subtracted vs. prediction

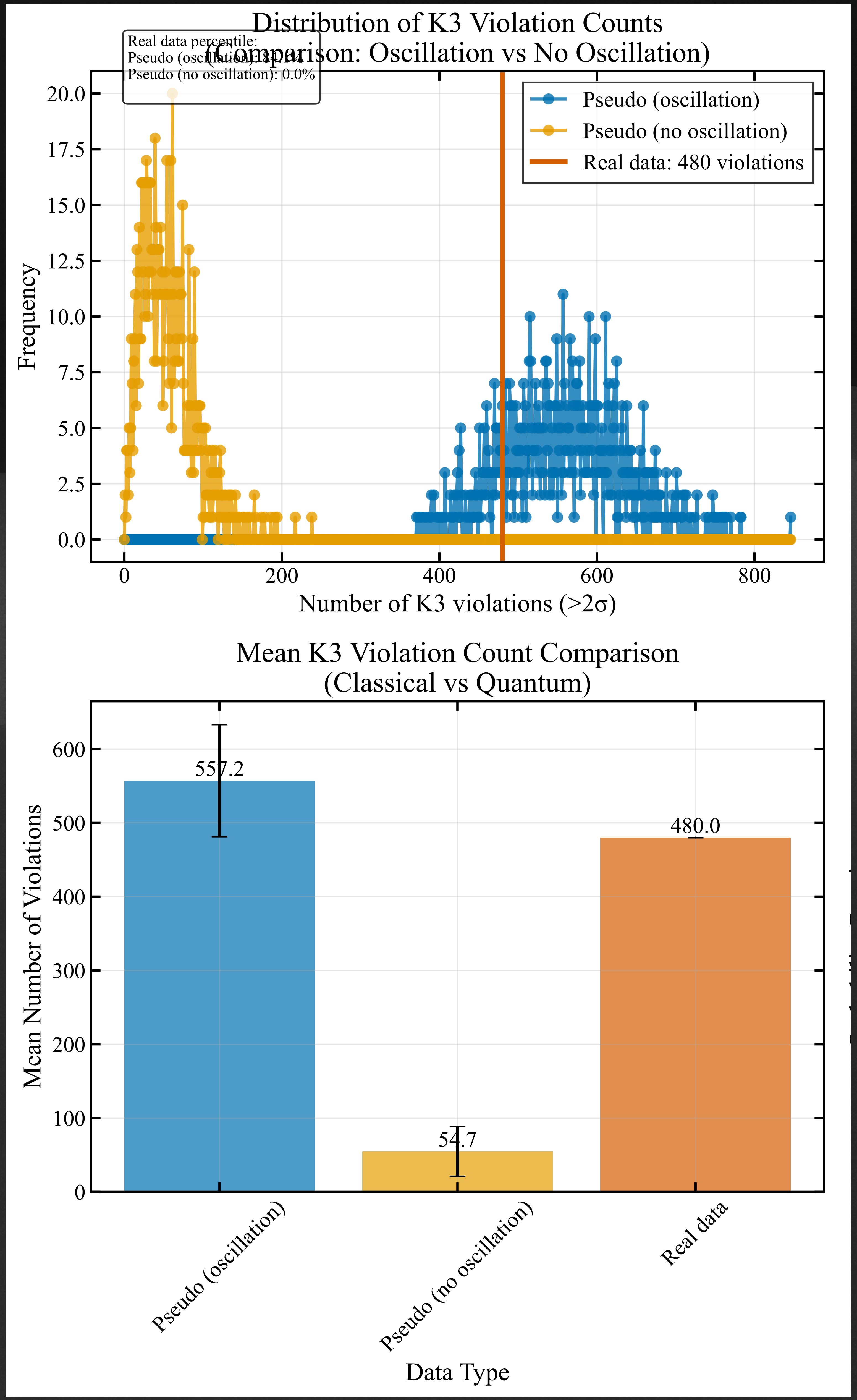




User data released in PRL 130.16 (2023): 161802

Daya Bay: arbitrary phase matching $\phi_1 + \phi_2 \approx \phi_3$

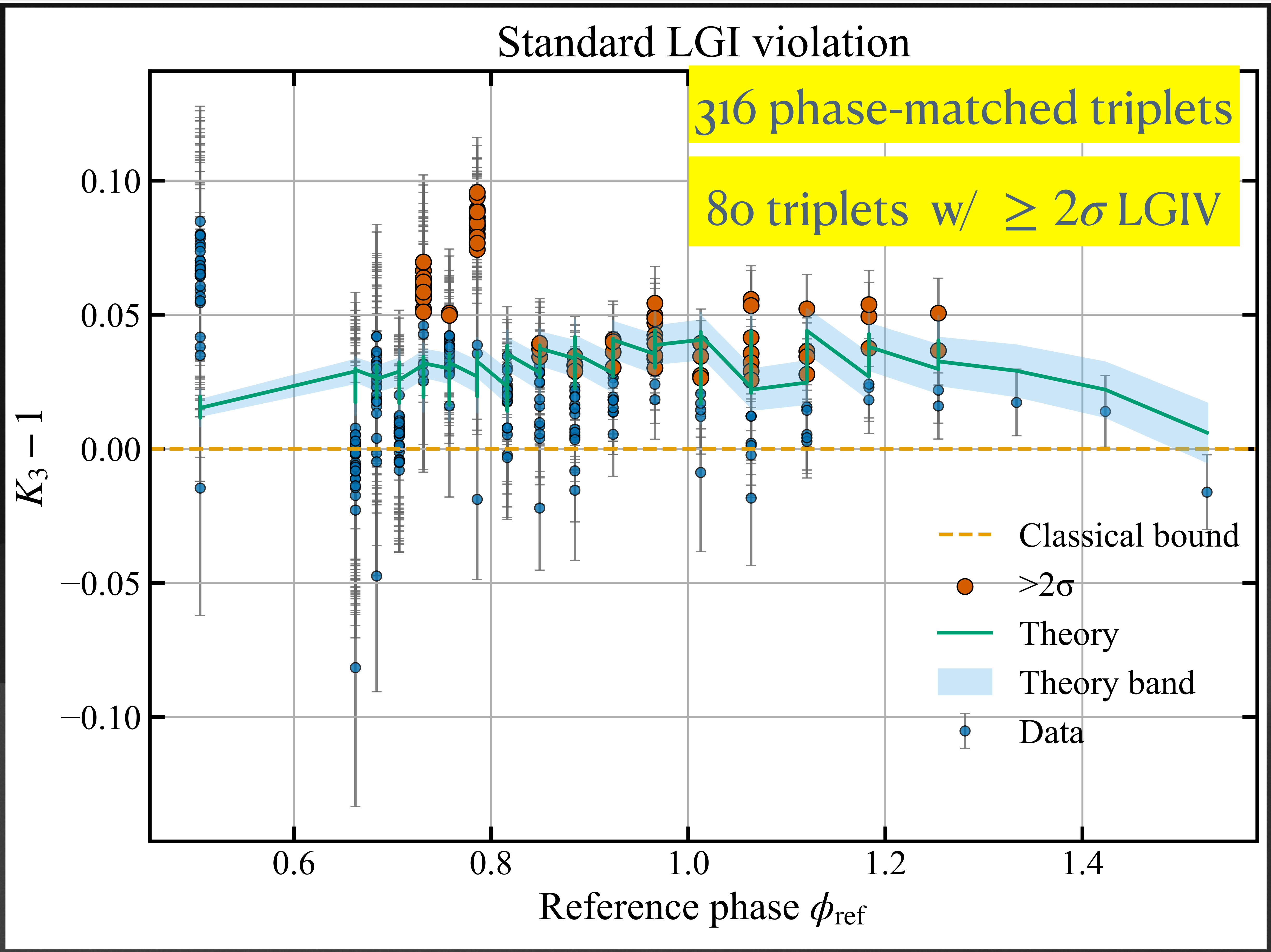


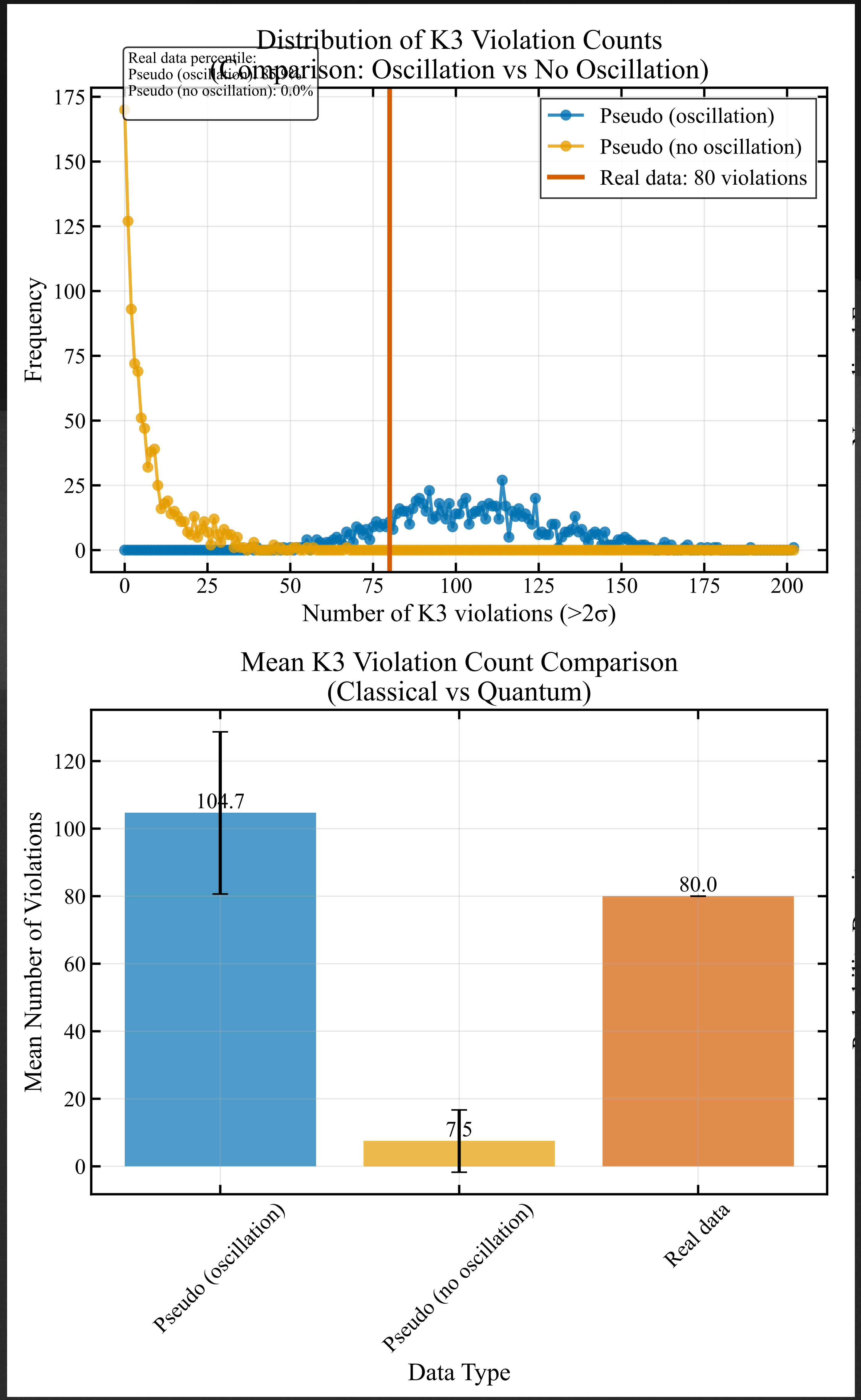


- Consider all three detector measurements as whole, just consider the phase only
- Phase matching tolerance (*2%, also checked w/ different tolerance*)

LGI violation observed with Daya Bay data!

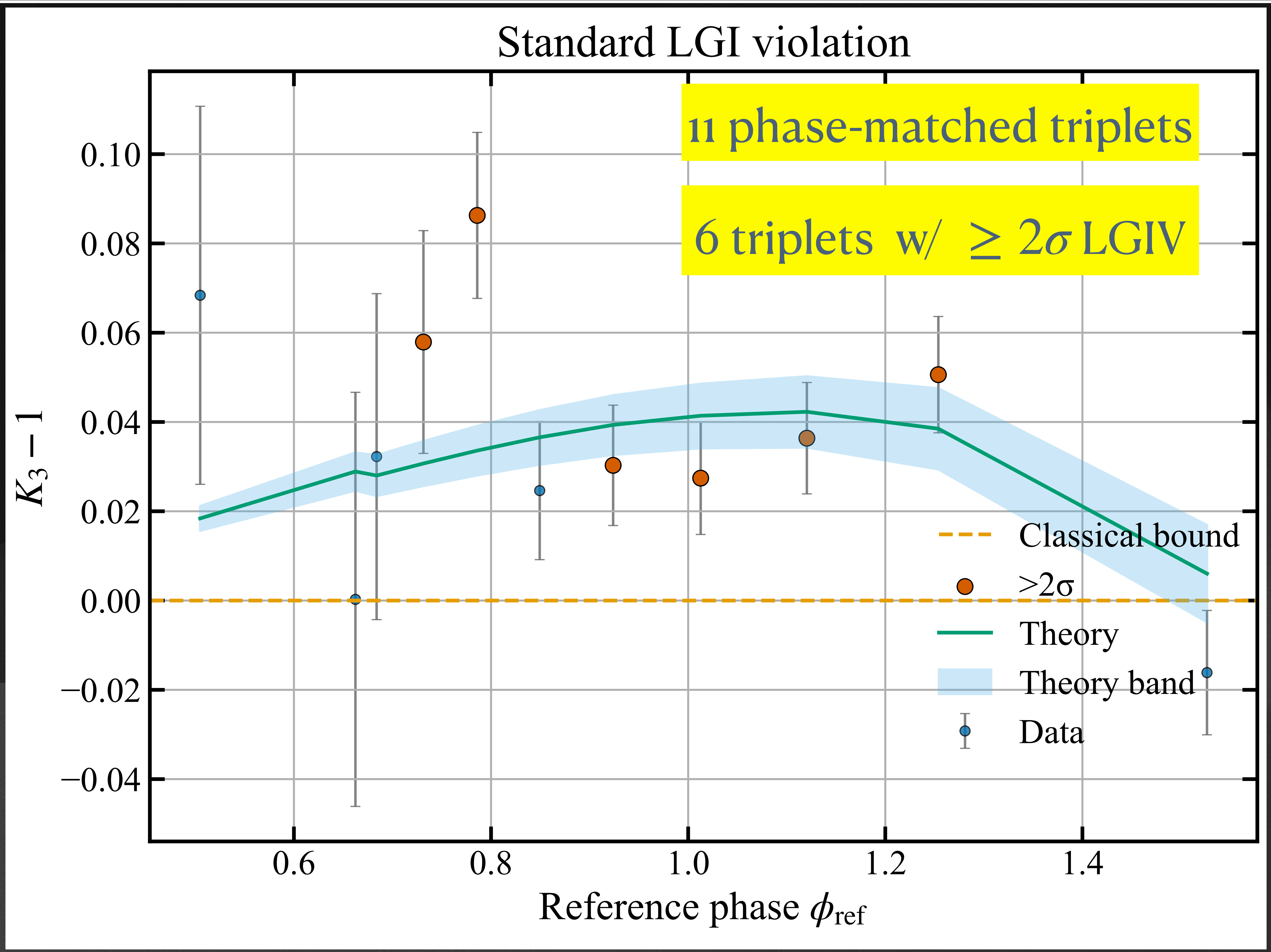
Daya Bay: limit one phase per detector

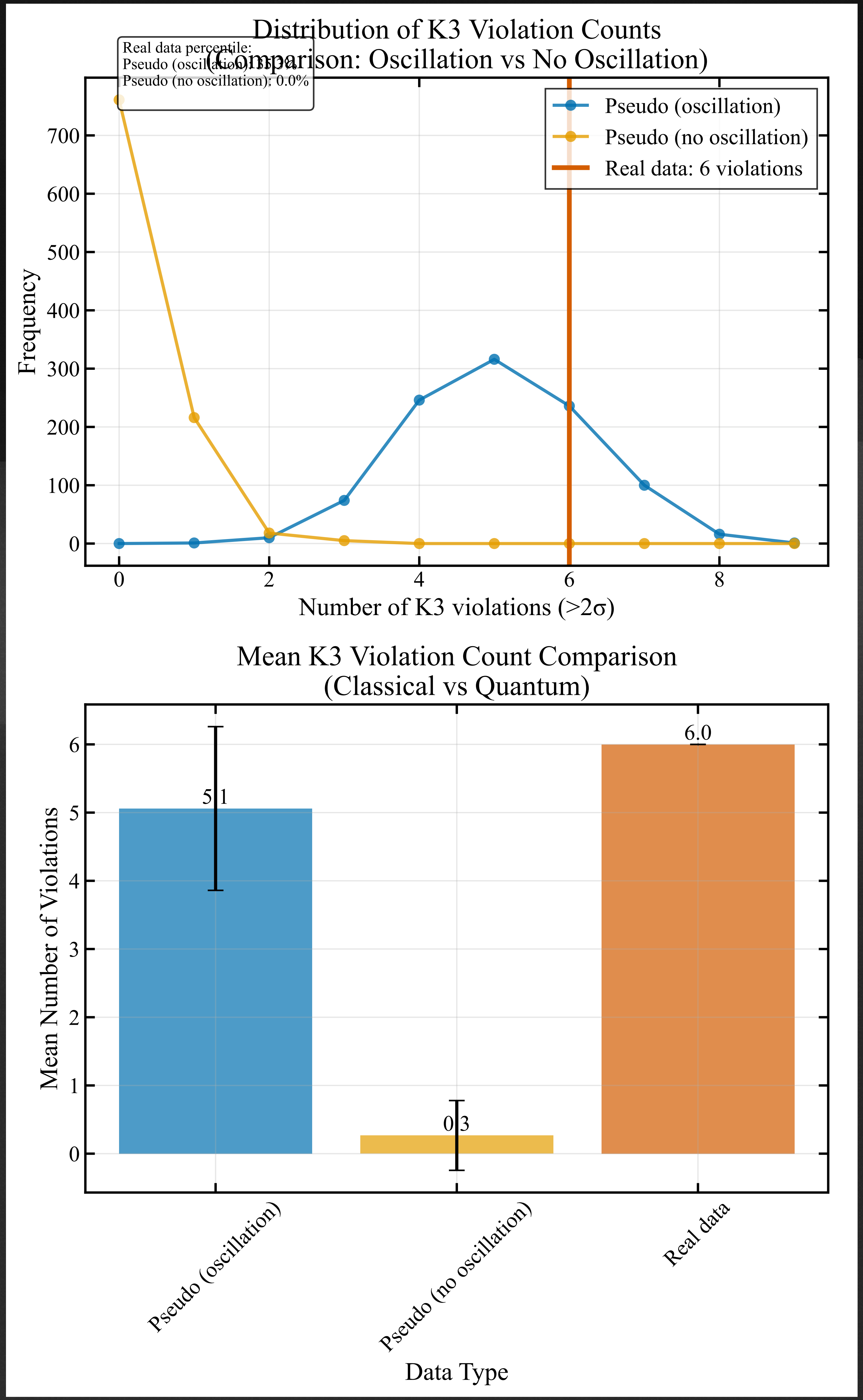




- Select phases in predefined detector (ϕ_1 in EH1, ϕ_2 in EH2, ϕ_3 in EH3)

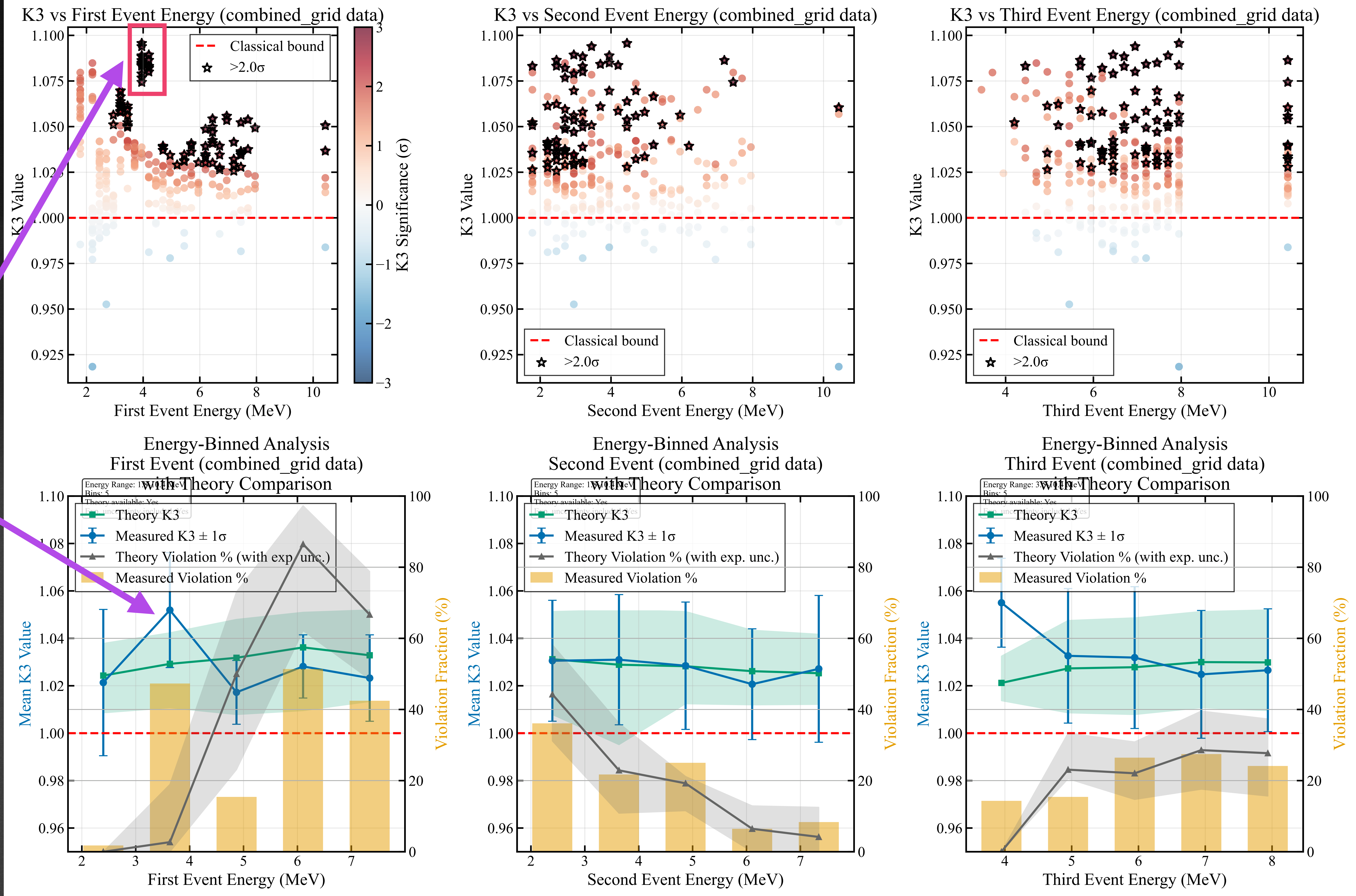
Daya Bay: limit one phase per detector and fix energy in two





- Select phases in predefined detector (ϕ_1 in EH1, ϕ_2 in EH2, ϕ_3 in EH3)
- Energy is the same in (ϕ_1, ϕ_2) but adjusted in ϕ_3

Is there unusual pattern?



Under-investigation

Summary

* Elusive neutrino plays essential role in modern physics, still most mysterious particle which can pave the way for future of particle physics

* Neutrino oscillation, a hallmark of microscopic superposition macroscopic coherence, can be an essential tool for testing the quantum foundations

* Legget-Garg Inequality test adopted for (1) non-classical behavior in sterile neutrino search (2) possibly identify the unusual pattern (*unaccounted systematics or new physics(?)*) and more

* Better LGI test: energy-dependent with multiple detector baseline

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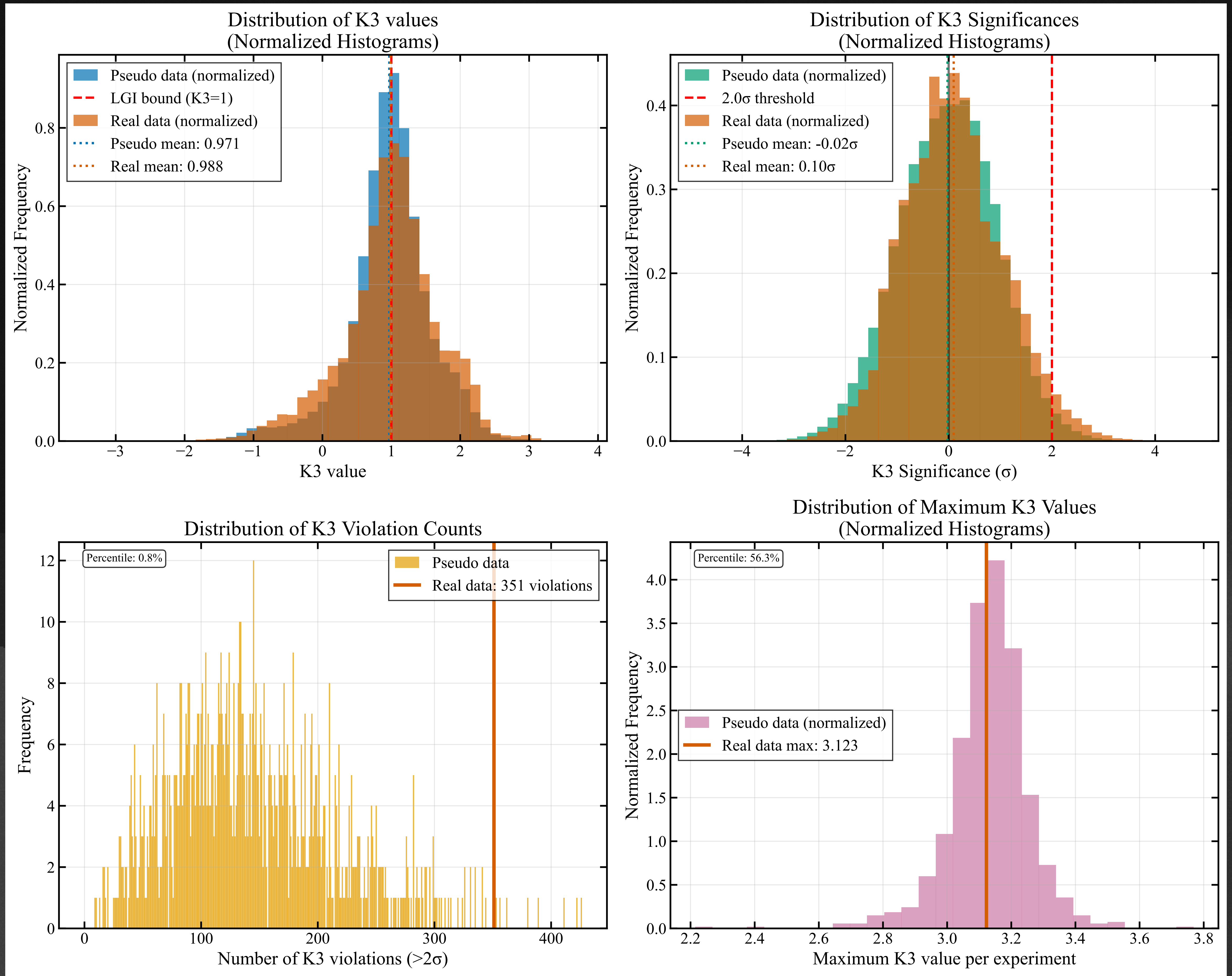
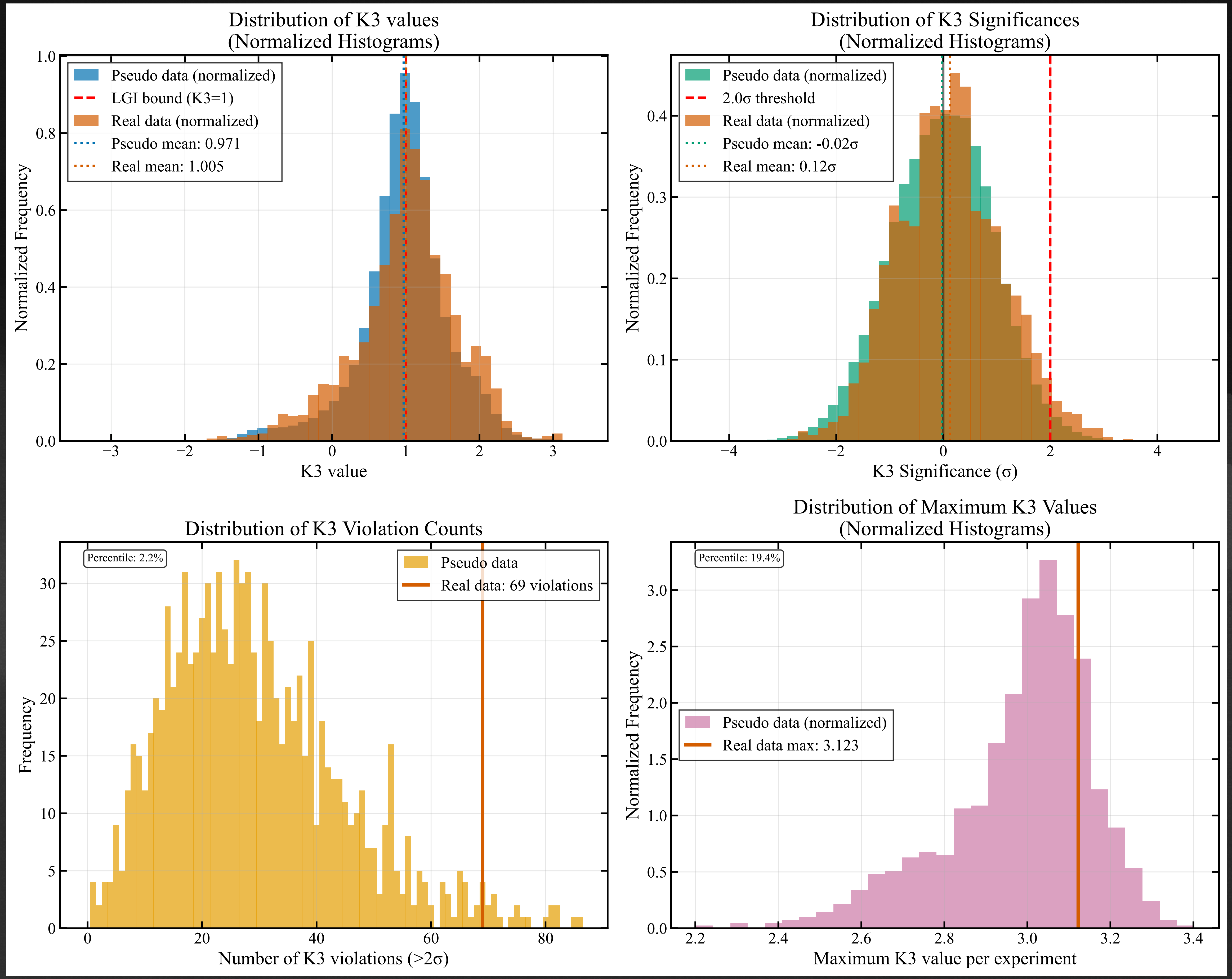
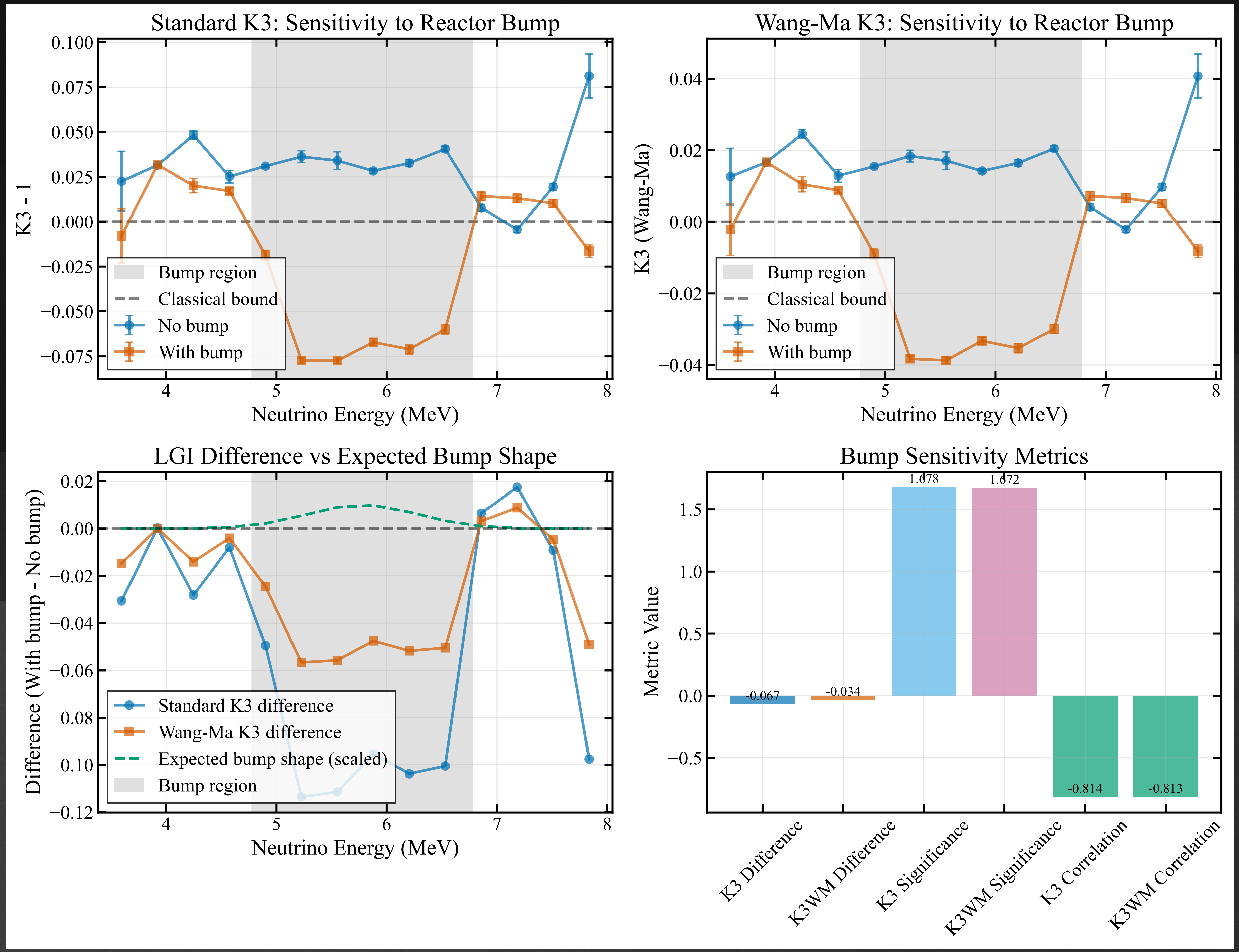


0.5% phase matching tolerance

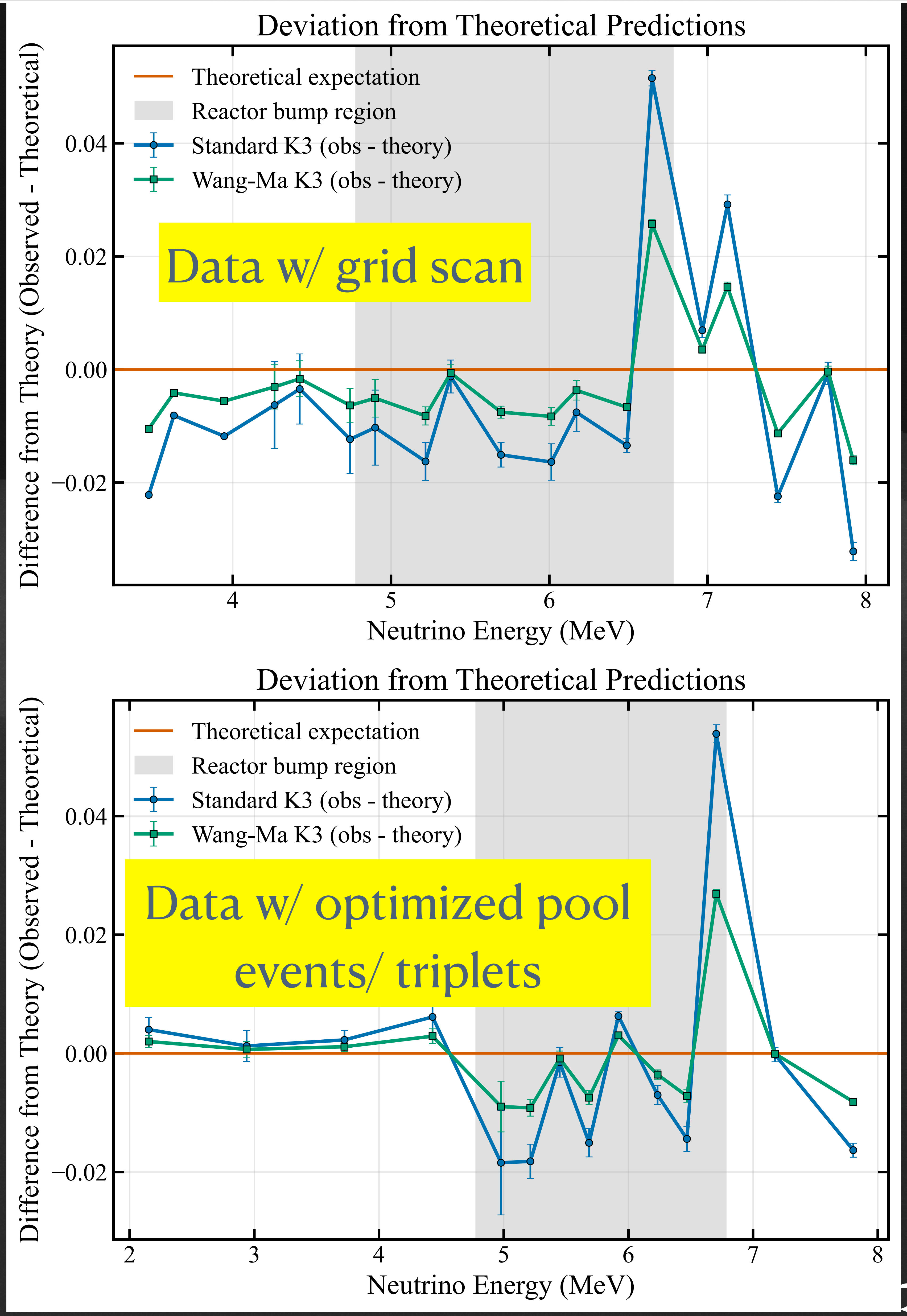


0.1% phase matching tolerance

Energy-depependent LGI?



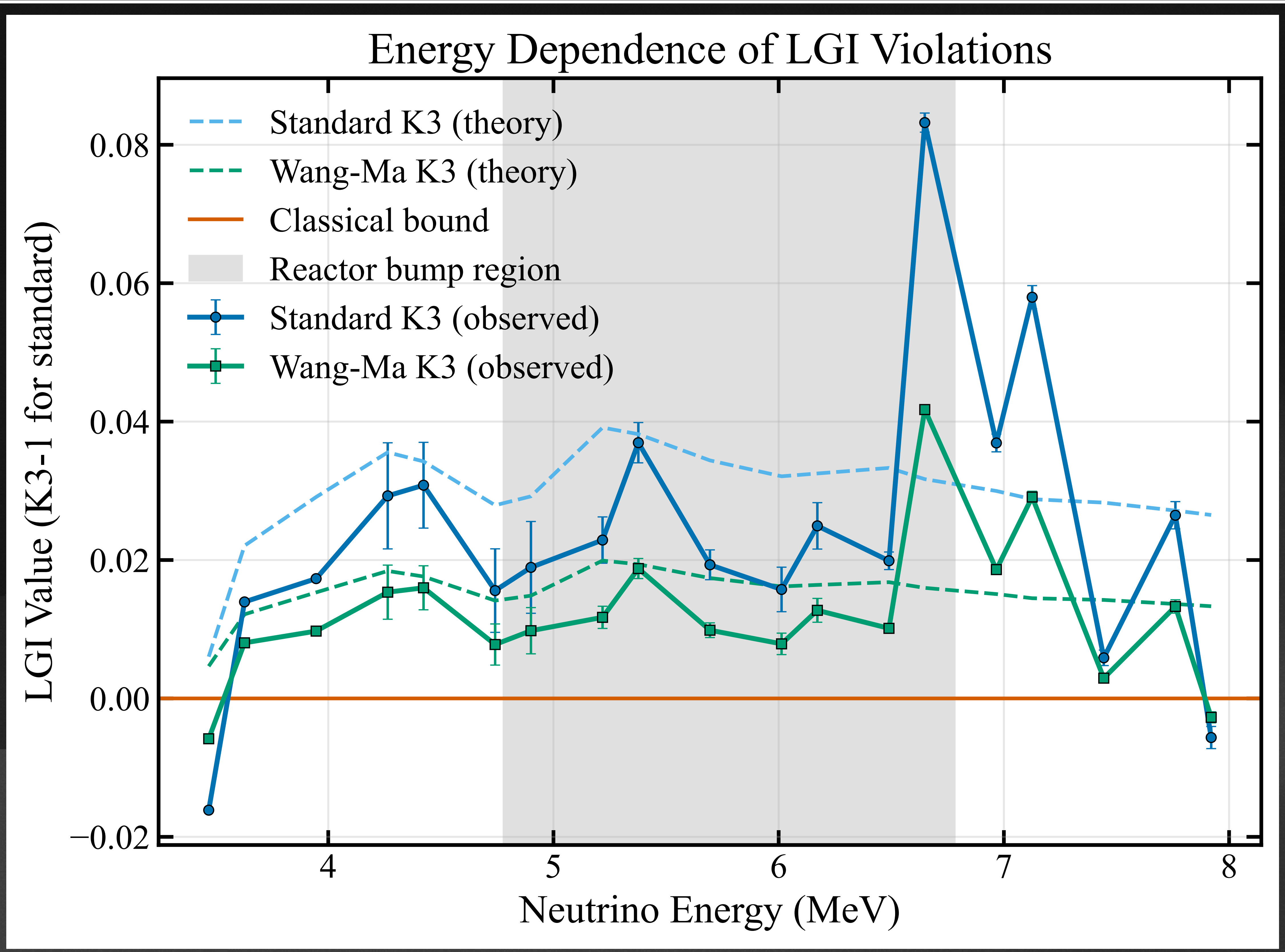
Sensitivity with grid scan



Sensitivity with Grid Scan assume 10%

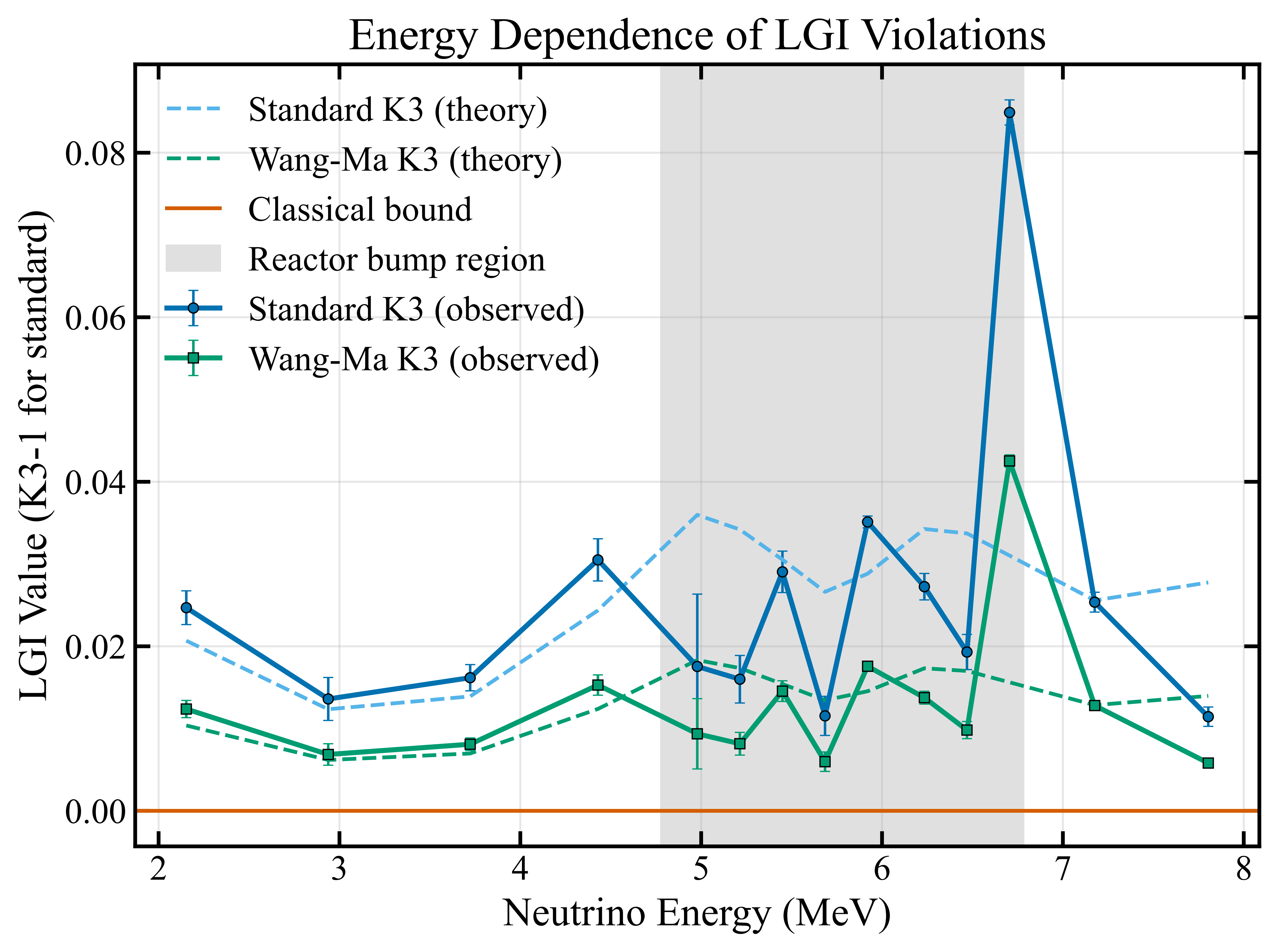
And width of bump

Energy-depependent LGI?



Grid-scan tripltest

$\phi_1 + \phi_2 = \phi_3$



Optimized triplets

Back up

* Common sense: particles carry the properties that are measured

* Locality: there is no information transmitted faster than light, or when I measure particle A, I can't modify instantaneously the result of measuring particle B

    * non-local: opposite

* Local realism: inaccessible deterministic world (due to hidden variables which could be beyond our reach)

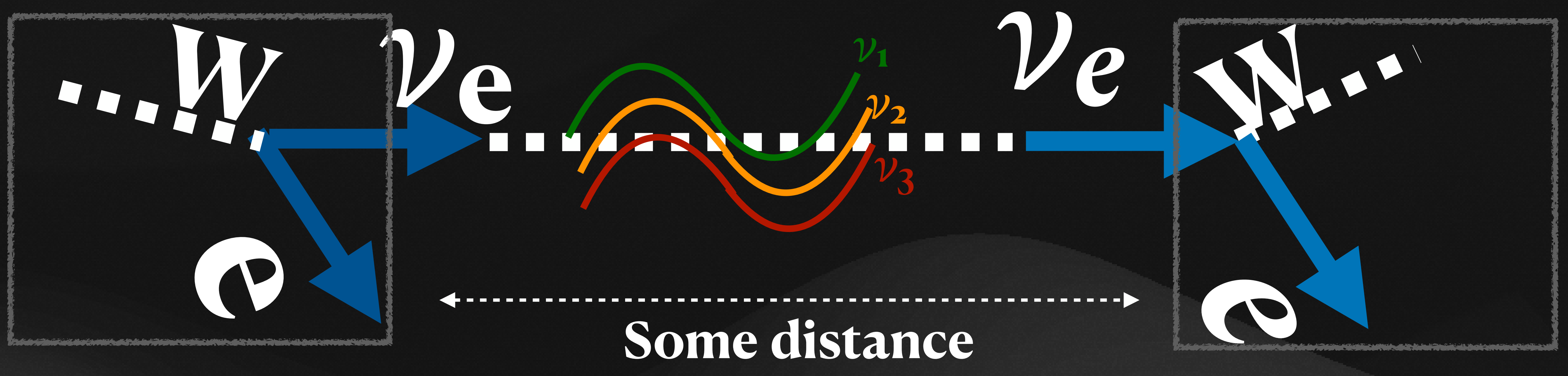
* Macroscopic realism (MR): system is in a definite state at all time

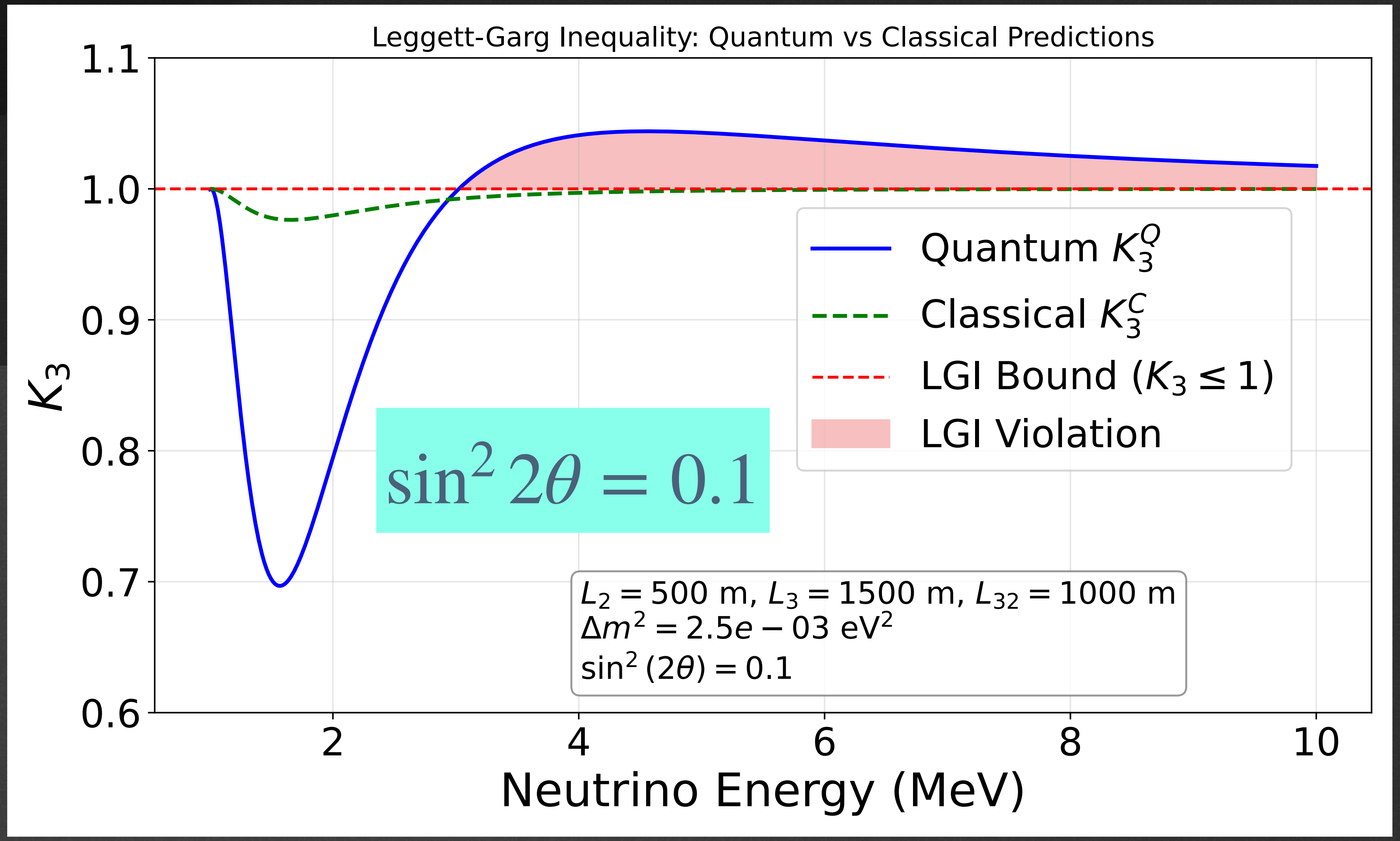
* Noninvasive measurement (NIM): possible to determine the state with arbitrary small perturbation

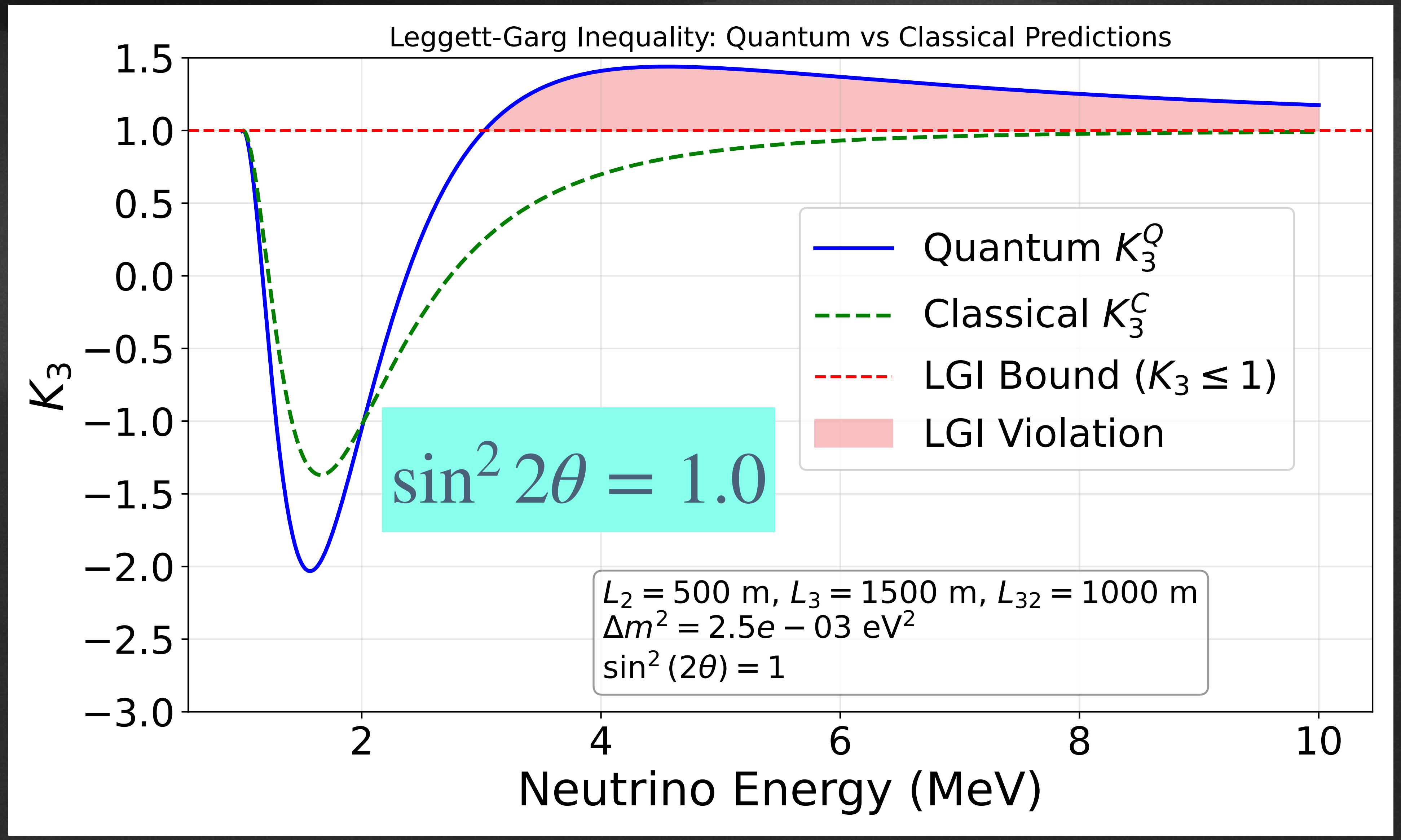
* Decoherence: evolution of a pure state into a mixed state through interaction with the environment

Formulate LGI test with neutrino oscillation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$



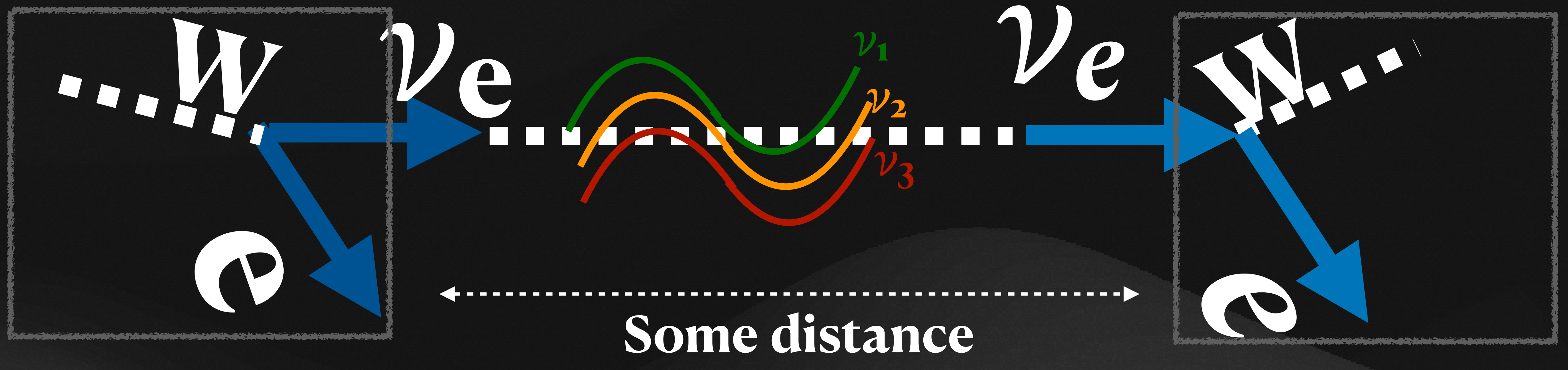


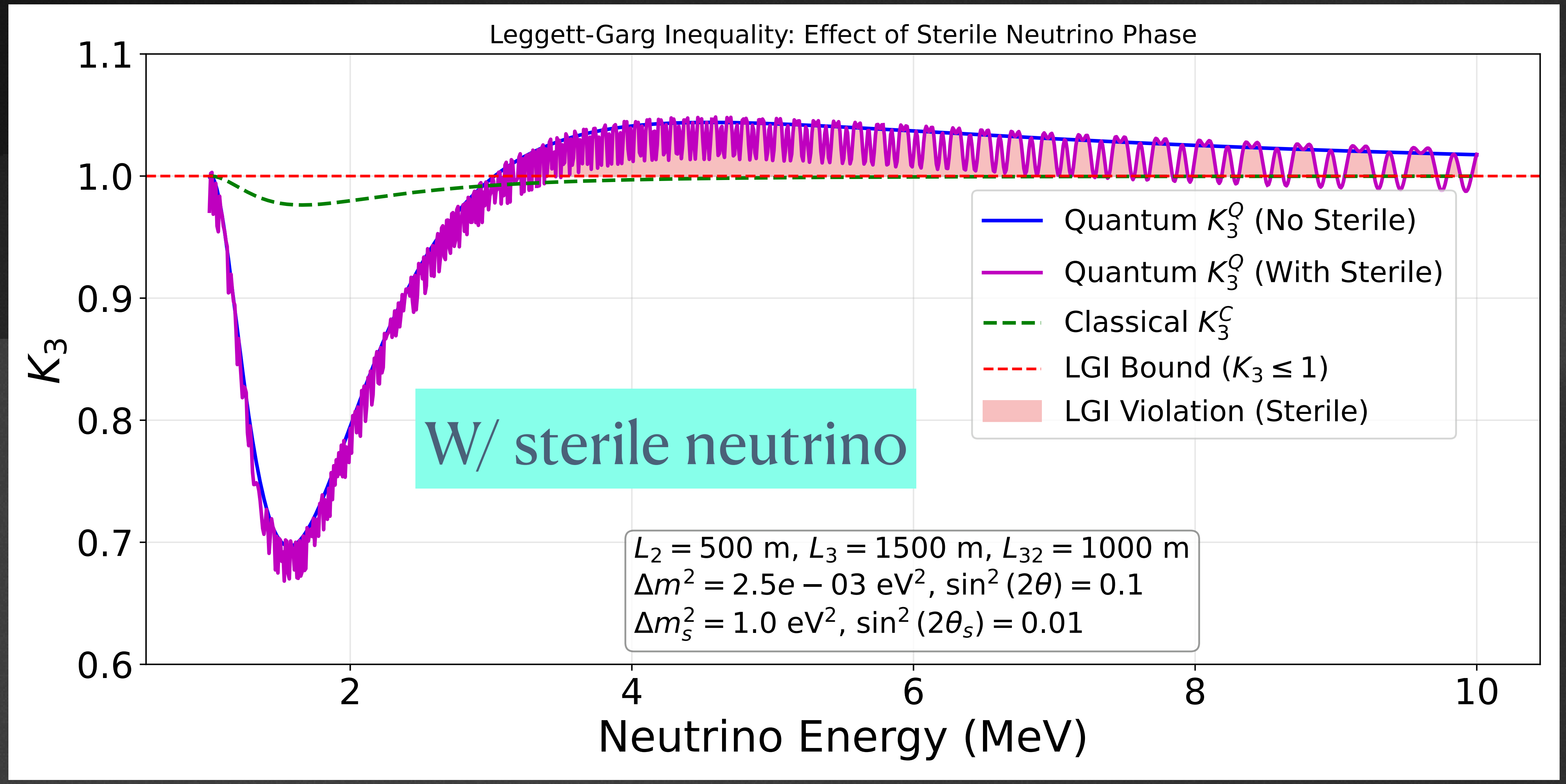


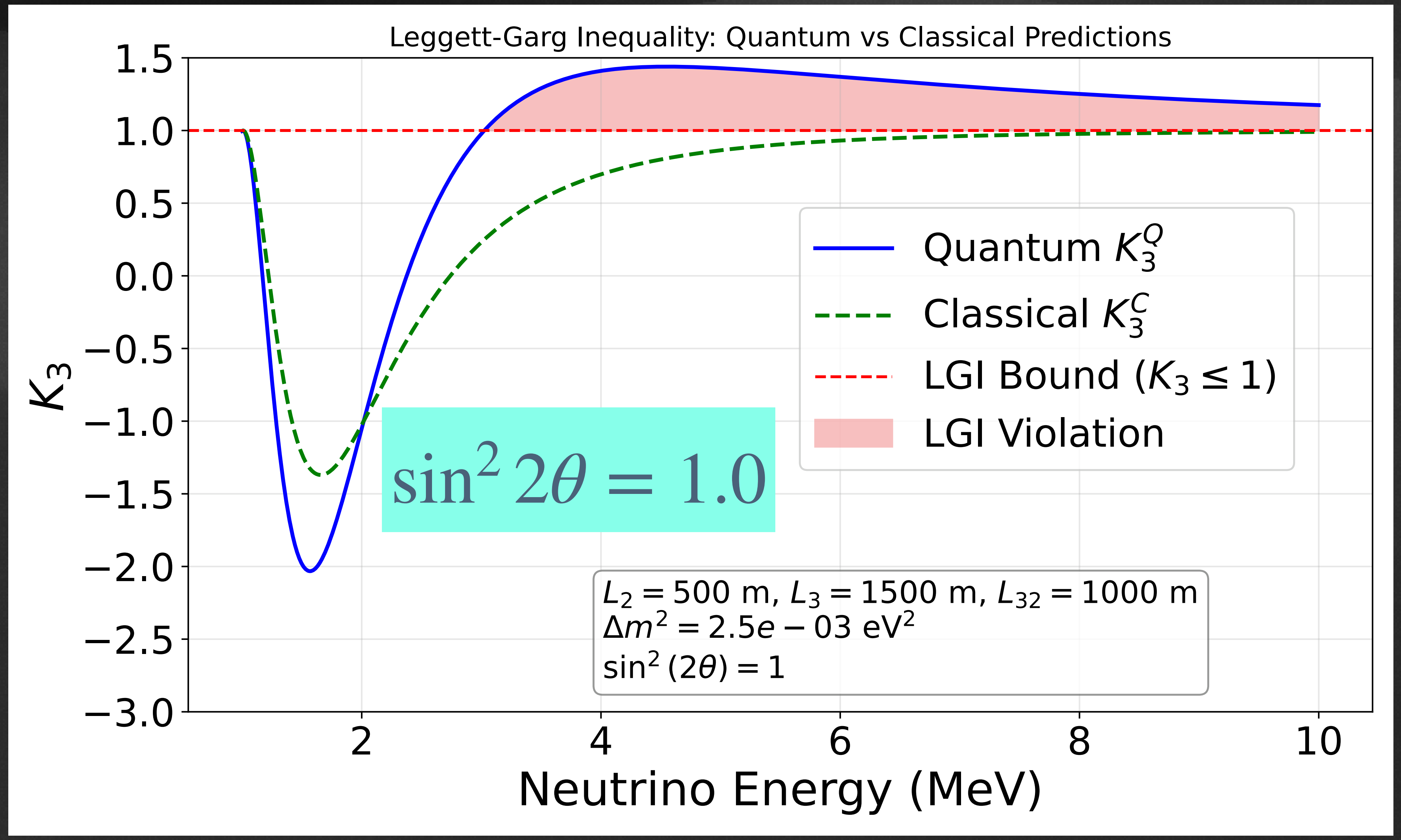
Depending on mixing angles between flavor and mass eigenstate

Formulate LGI test with neutrino oscillation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$







Work-in-progress to use LGI-based test for searching sterile neutrino in short-baseline and mass-ordering measurement via multiple wavelength interference