Thermodynamics of Precision in Open Quantum Systems

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TVV, PRX Quantum **6**, 010343 (2025)

TVV, R. Honma, K. Saito, arXiv:2508.21567

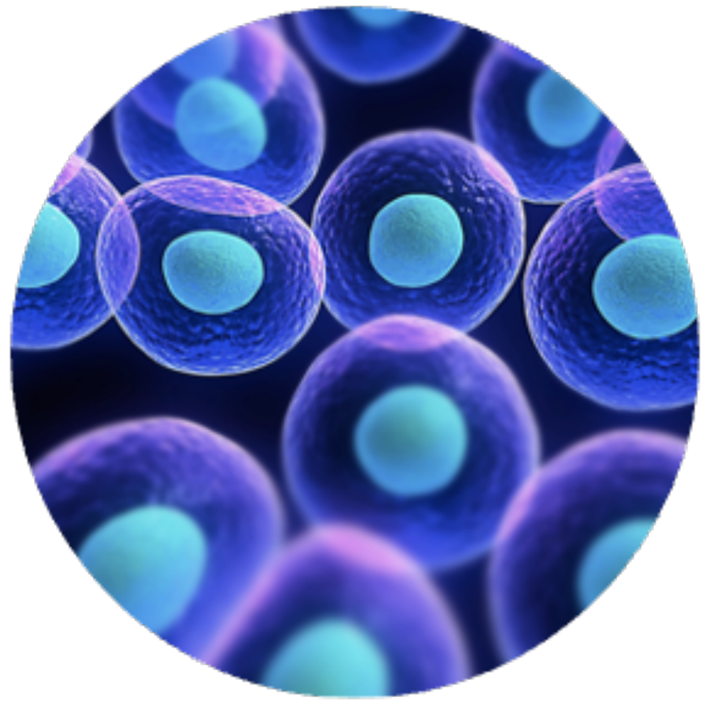
100 Years of Quantum Physics

Oct 6-9 2025, ICISE, Quy Nhon

Stochastic and quantum thermodynamics

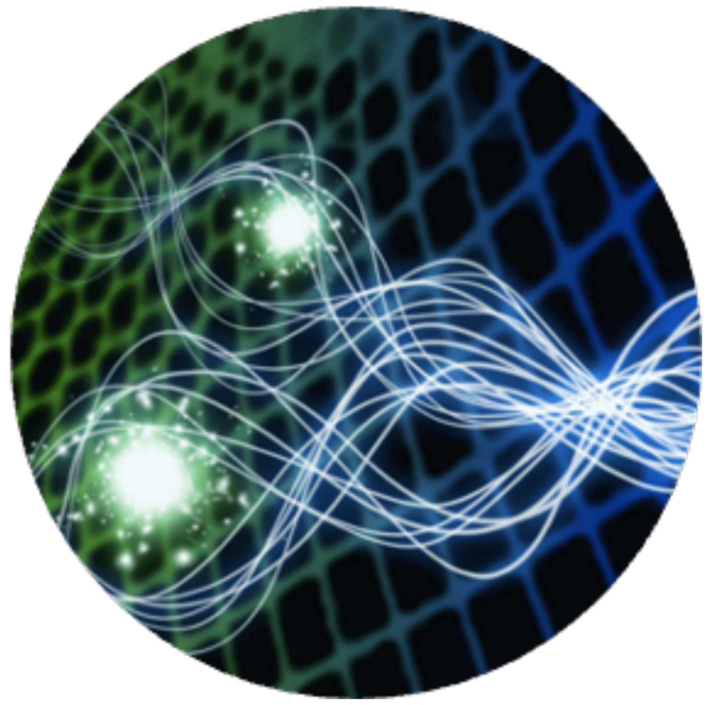
Classical thermodynamics

Neglebile fluctuations



Stochastic thermodynamics

Large fluctuations

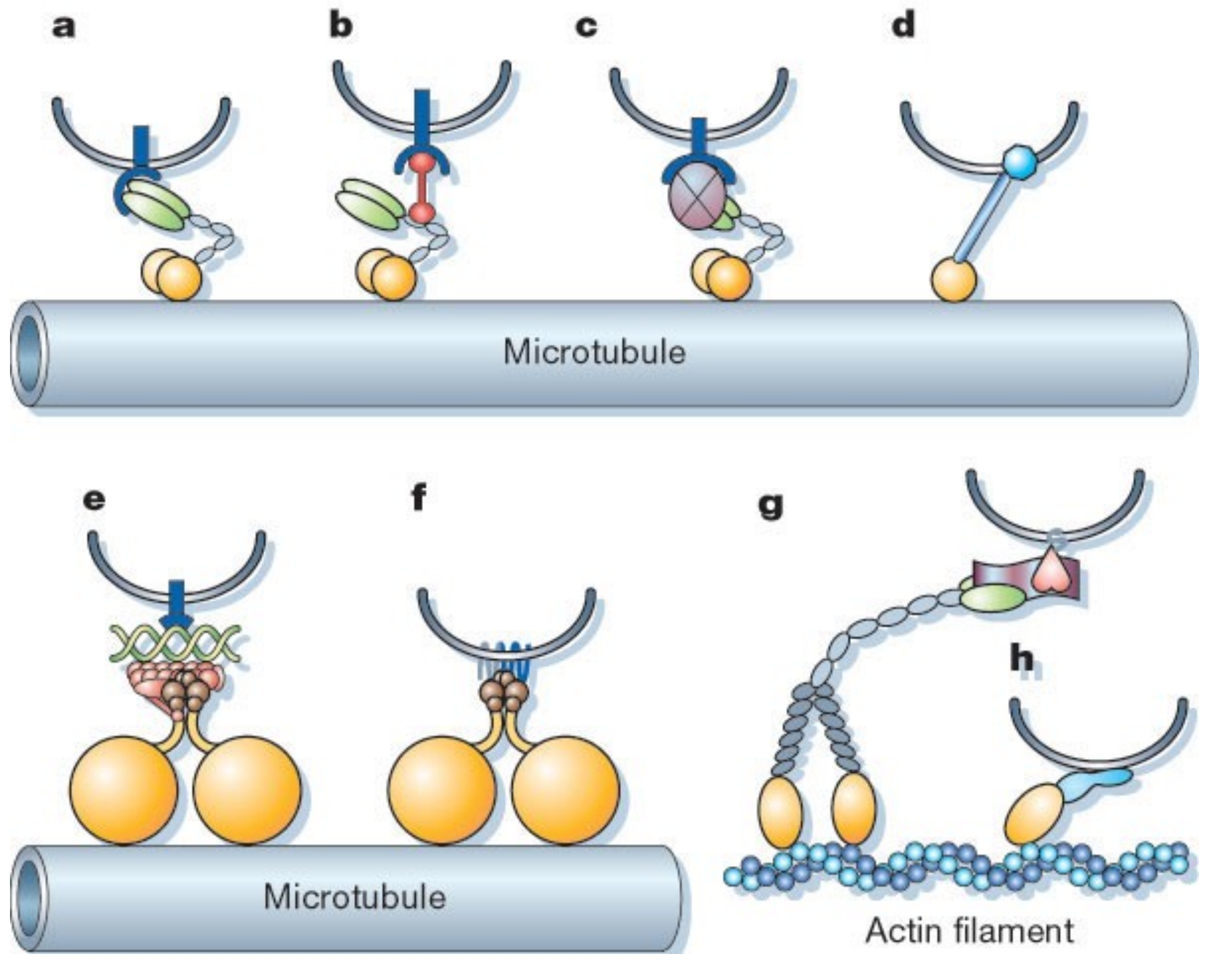


Quantum thermodynamics

Quantum effects

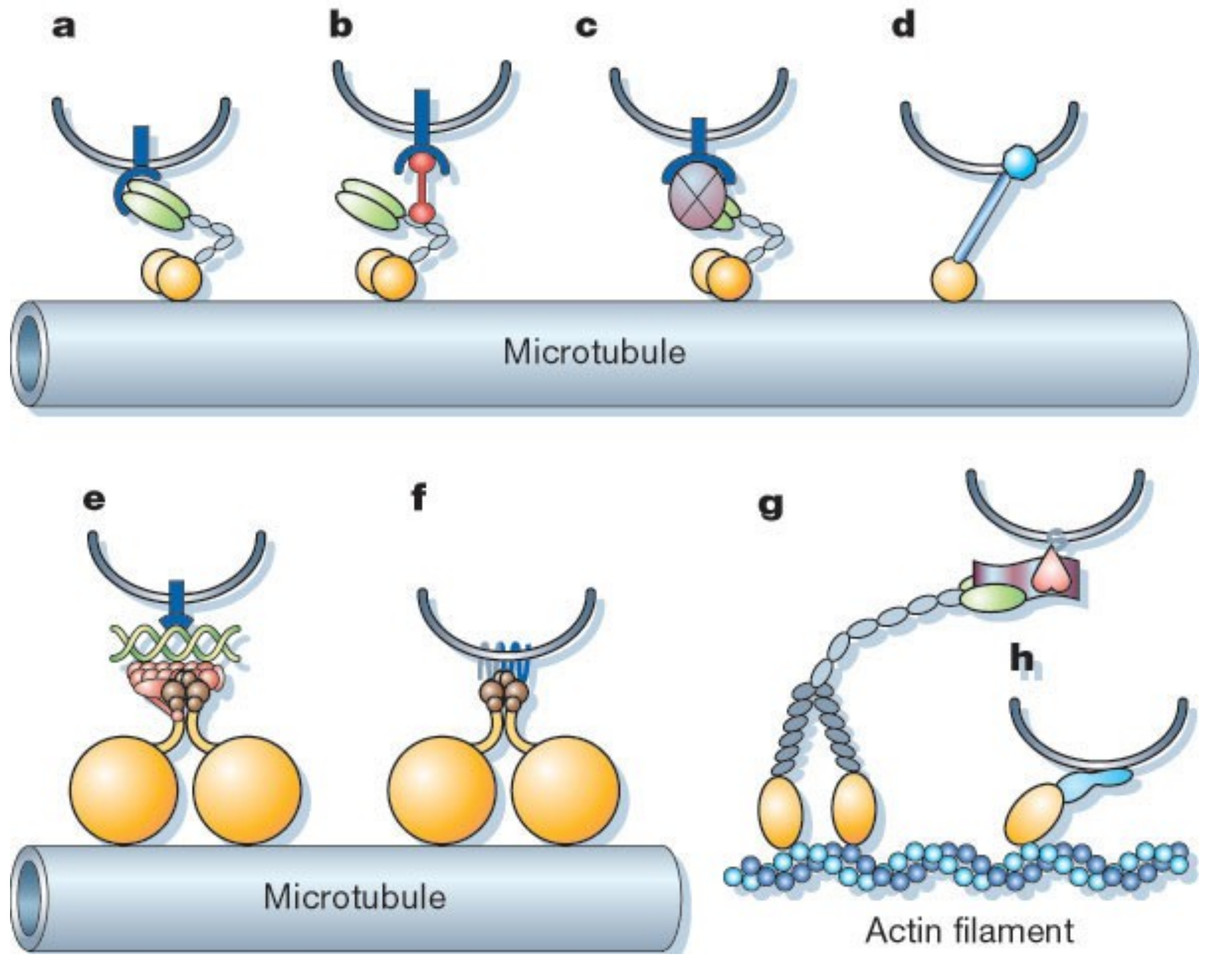
Stochastic and quantum thermodynamics

Stochastic and quantum thermodynamics



Molecular motor, Nature (2003)

Stochastic and quantum thermodynamics



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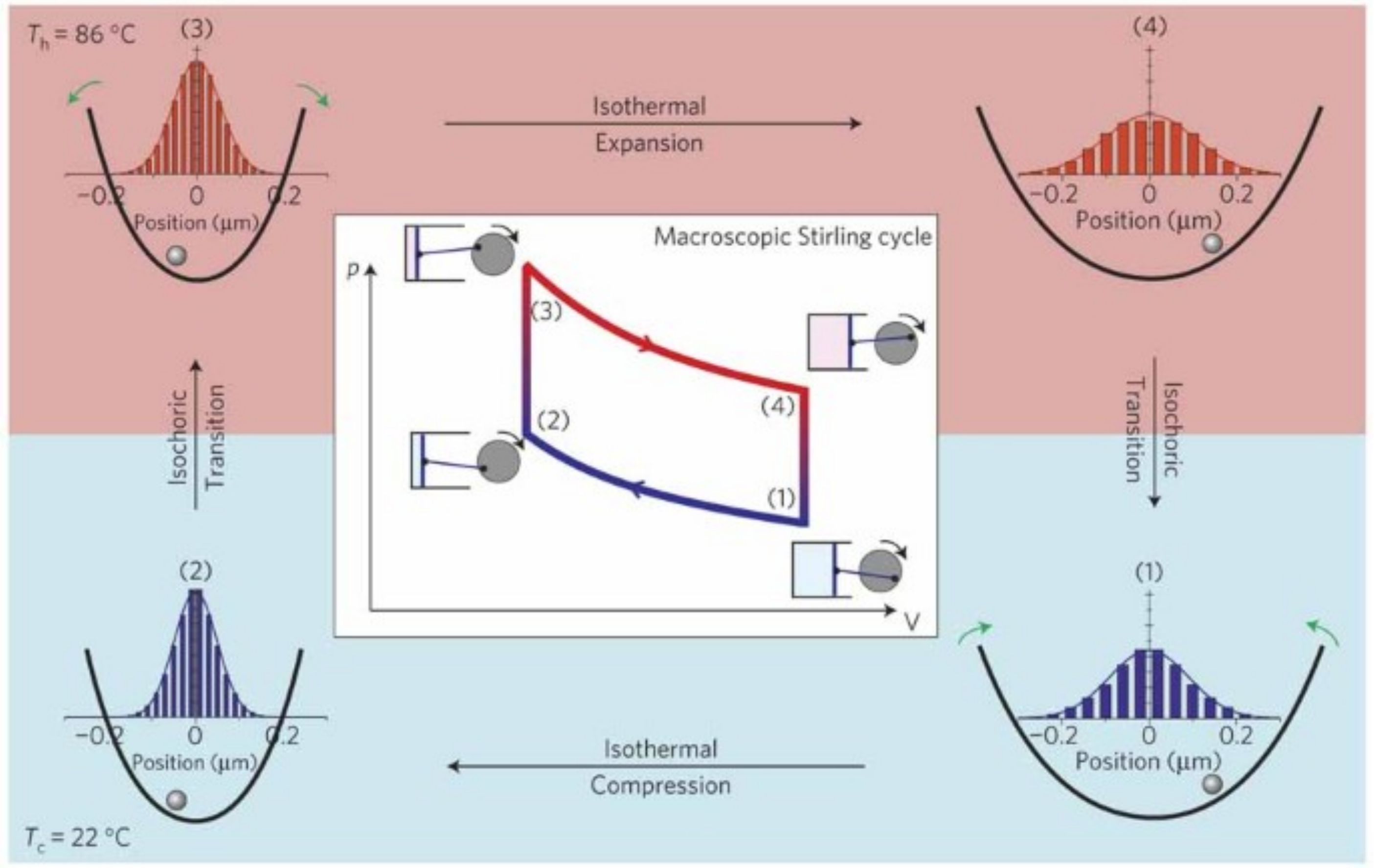

Colloidal heat engine, Nat. Phys. (2012)

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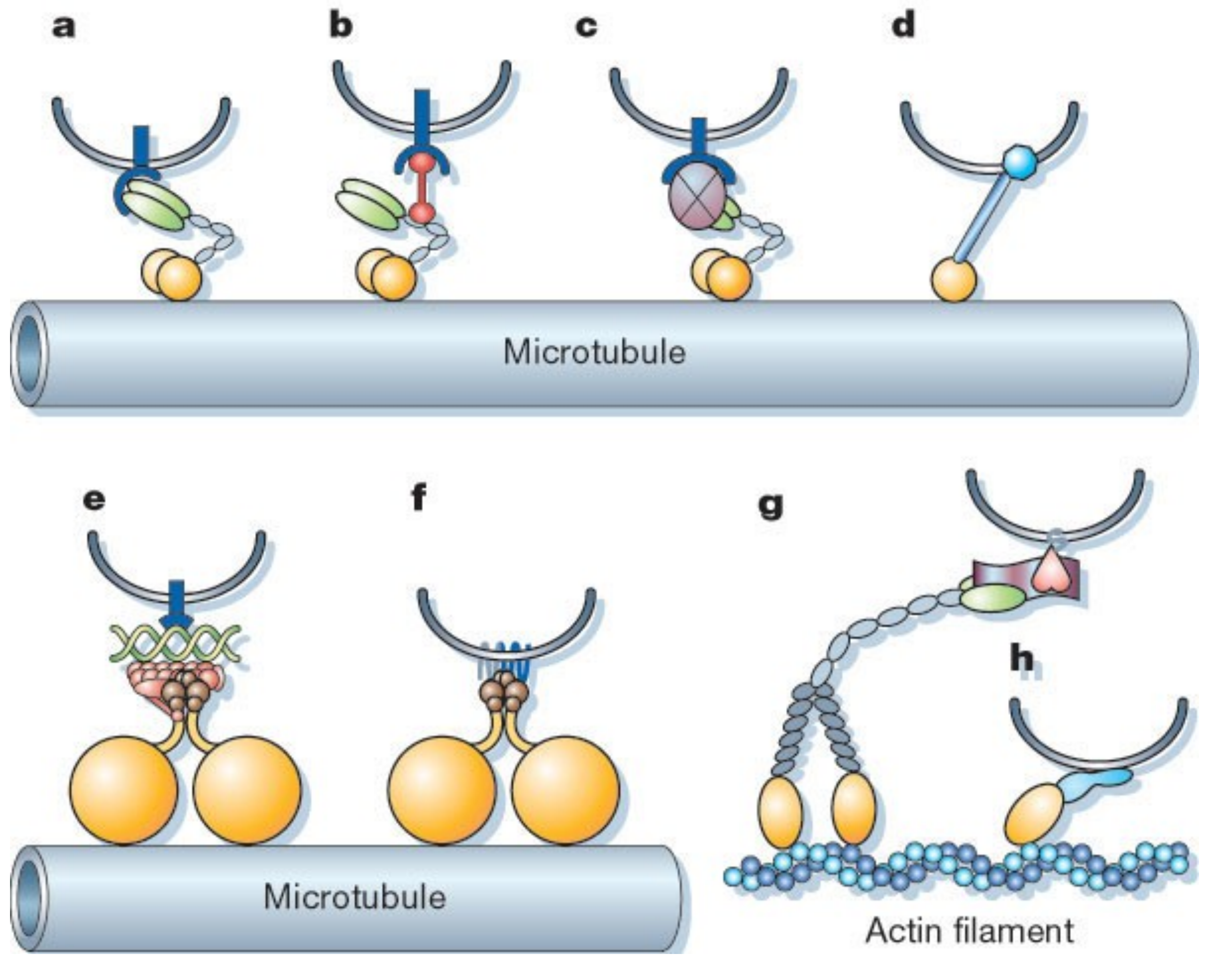

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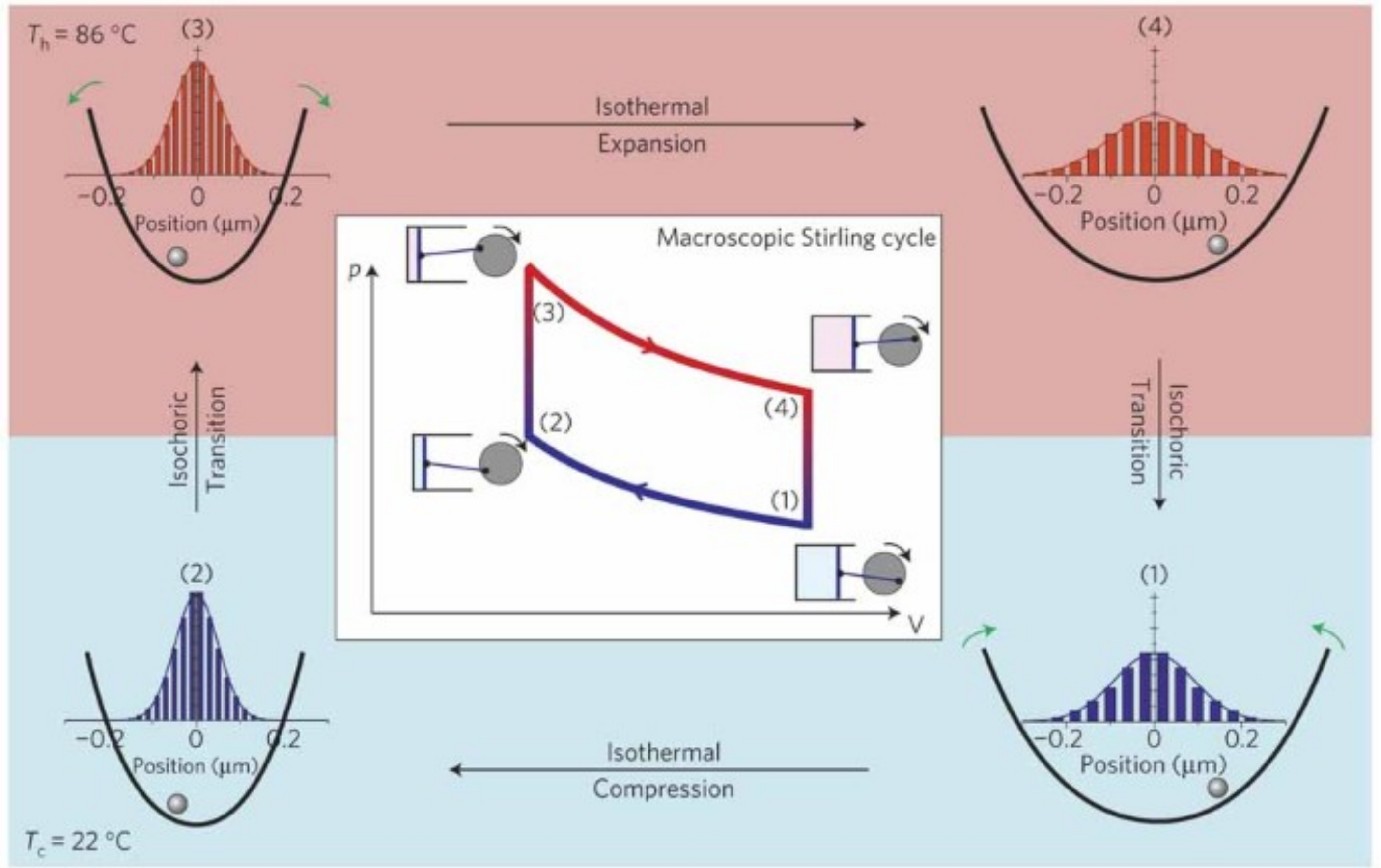

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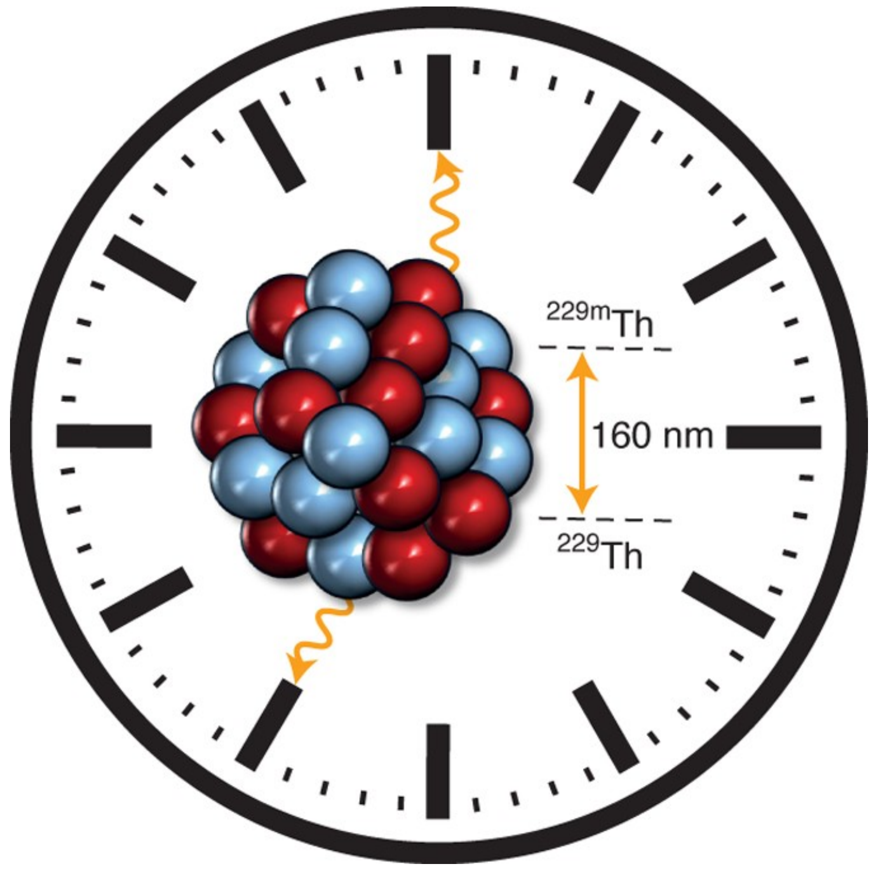

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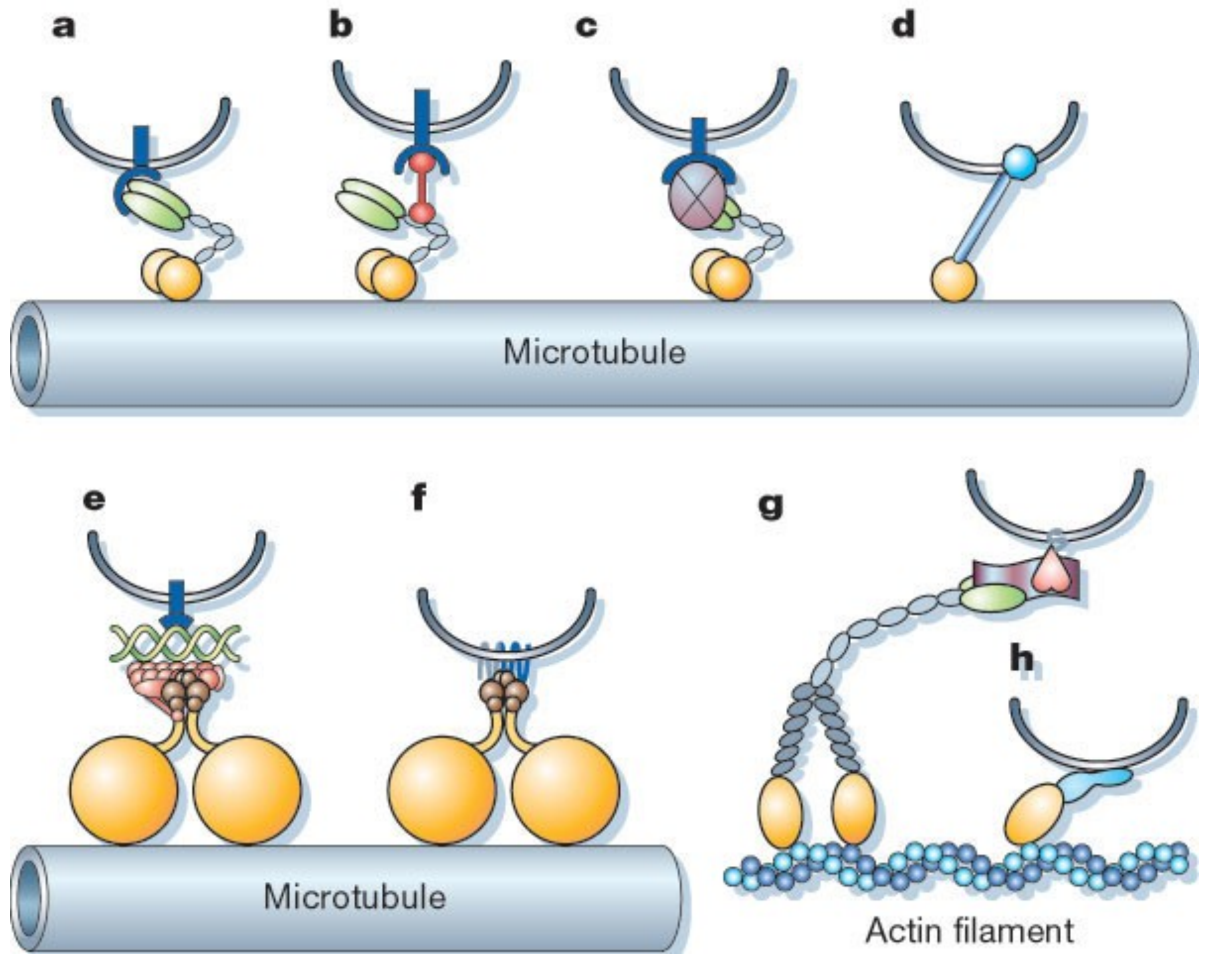

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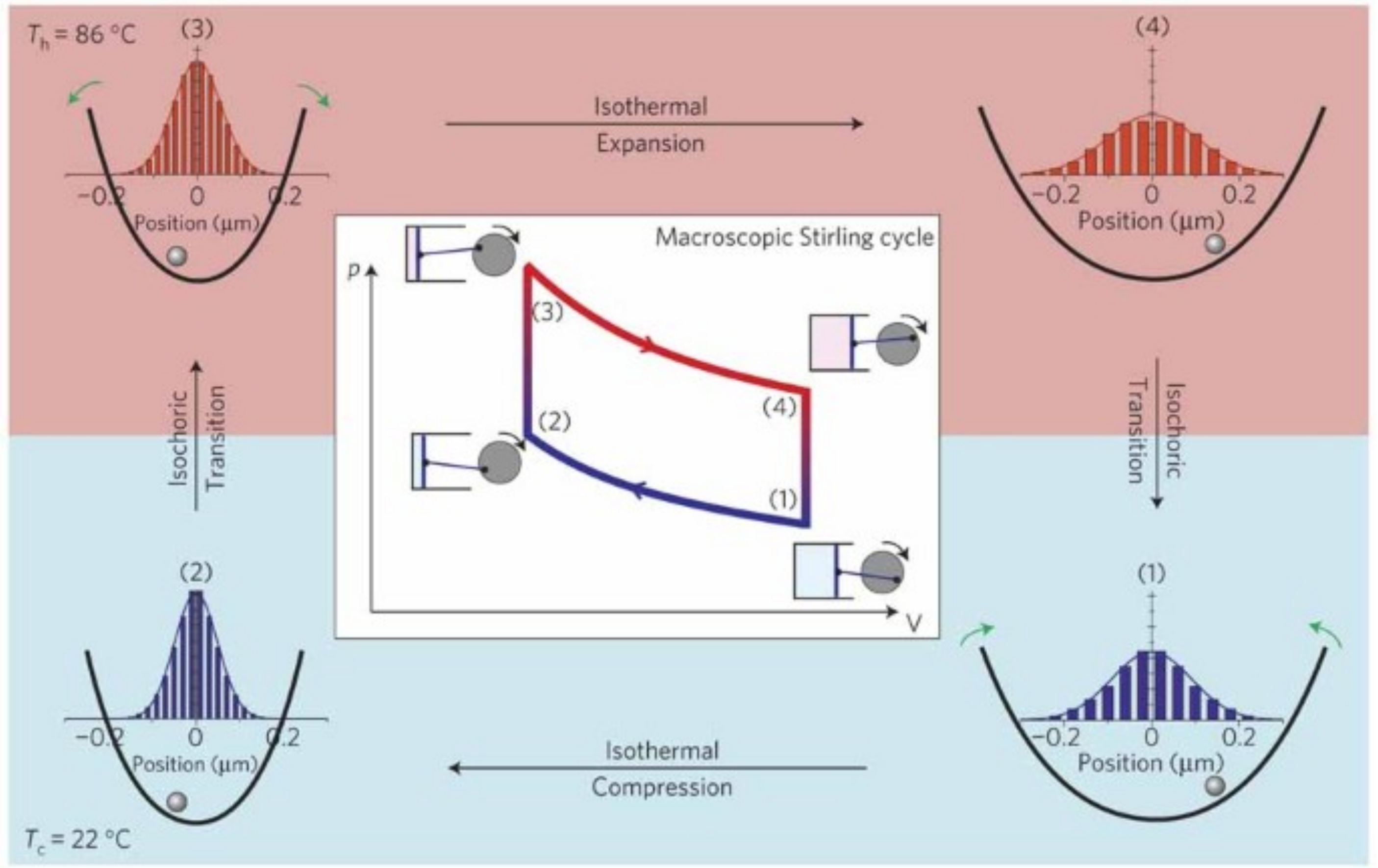
traveled distance



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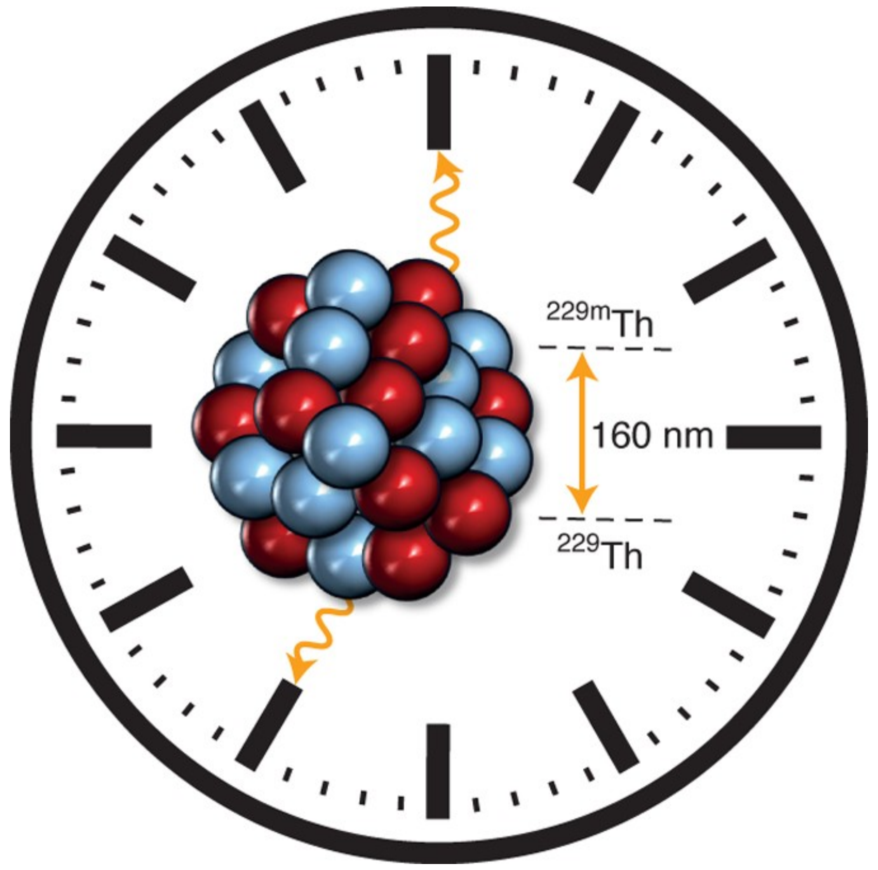
power current



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tick current

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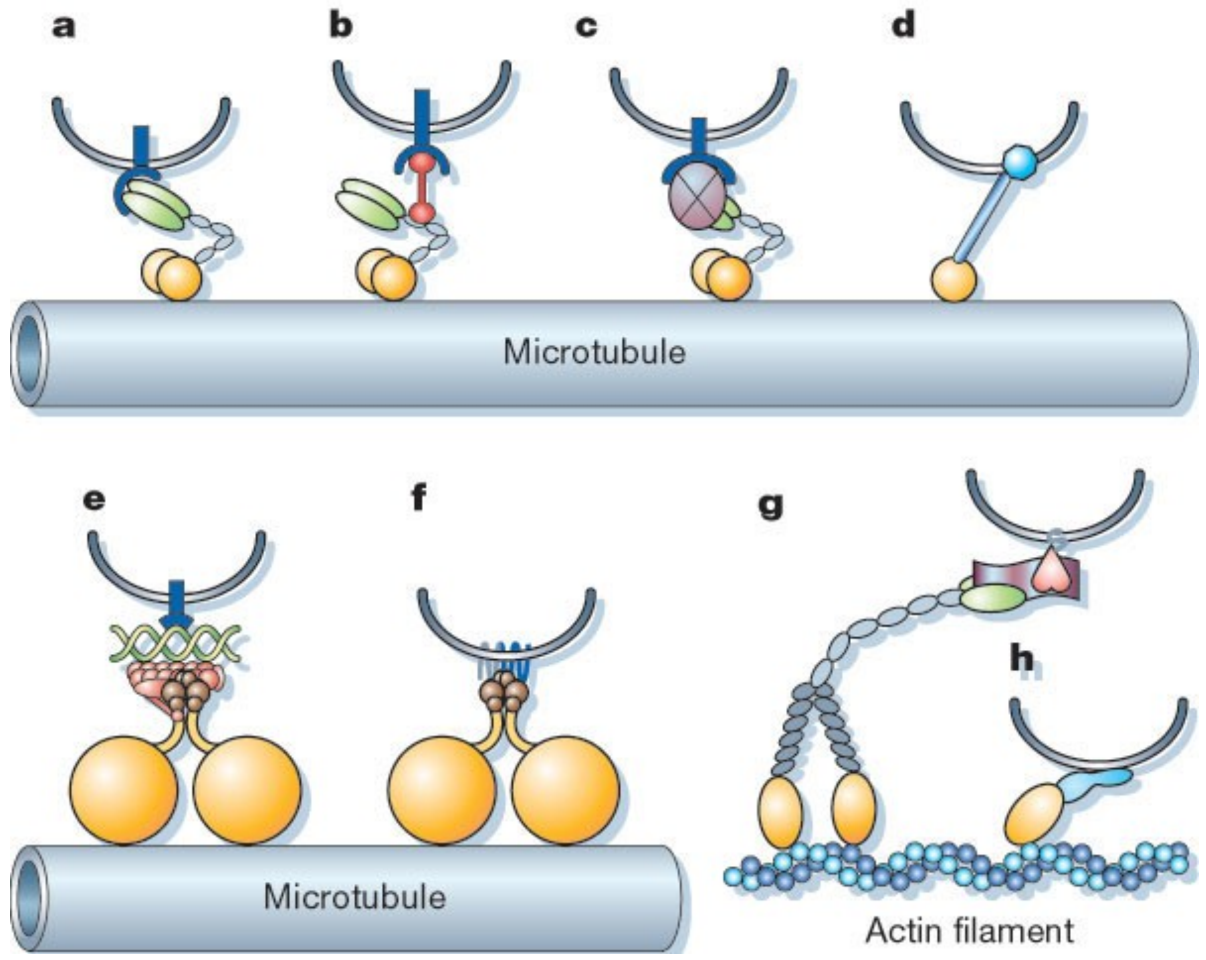

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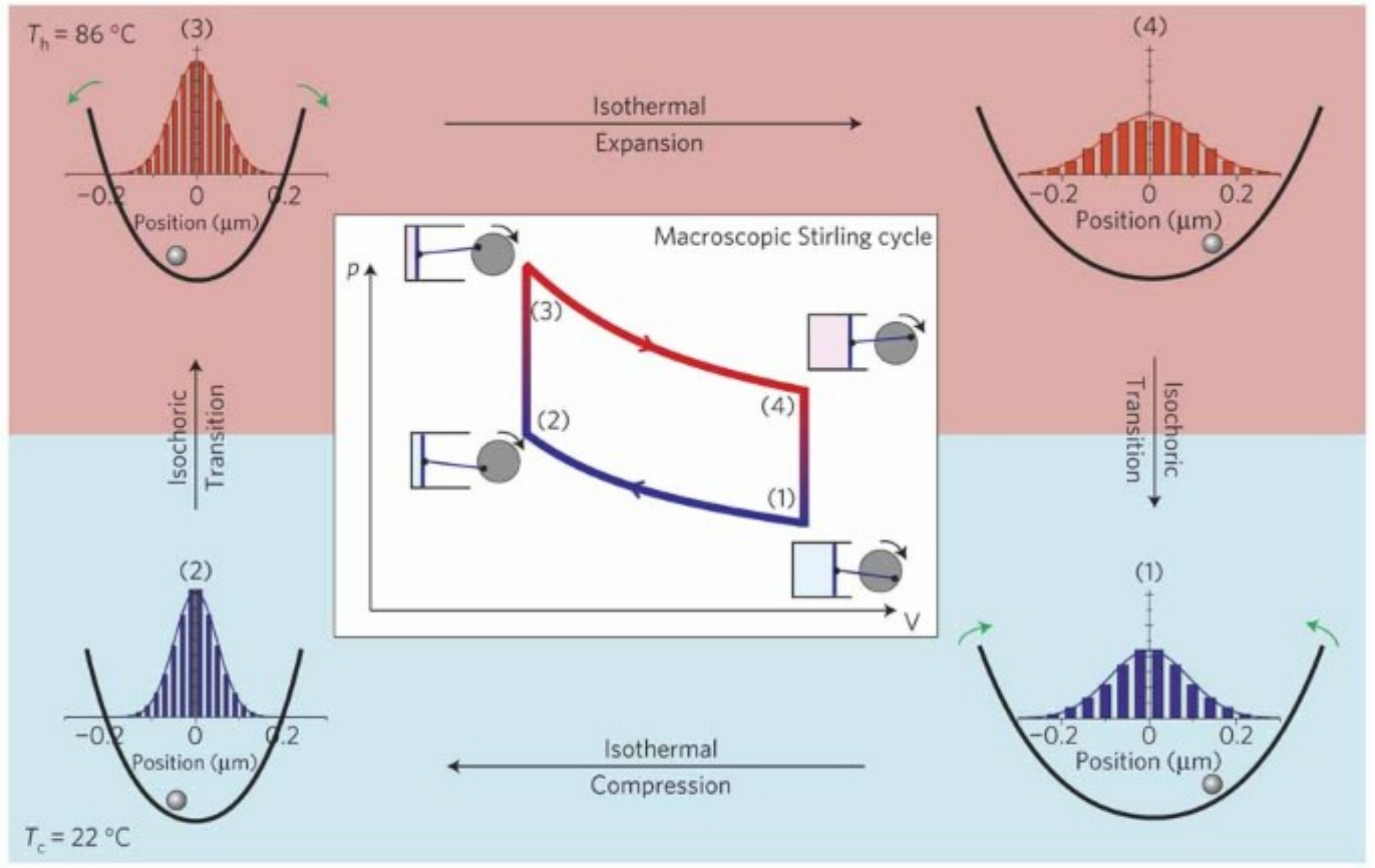

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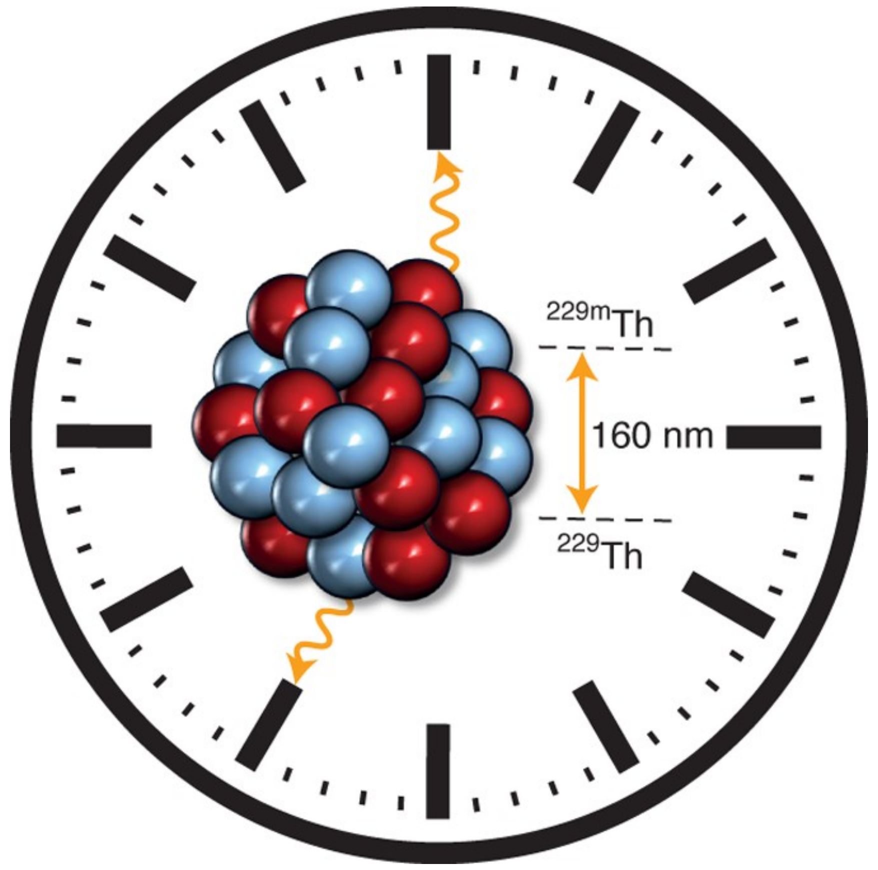

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tick current

- How precise can these currents be?
- What does it cost to achieve high precision?

Classical precision bounds

- Thermodynamic and kinetic uncertainty relations

$$F_\phi := \tau \frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2}$$

ϕ: time-integrated current

e.g., particle & heat current,
motor's displacement

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Garrahan, PRE (2017)

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TKUR: $F_\phi \geq \dfrac{4a}{\sigma^2}\Phi\left(\dfrac{\sigma}{2a}\right)^2 \geq \max\left(\dfrac{2}{\sigma}, \dfrac{1}{a}\right)$

Vo, TVV, Hasegawa, J. Phys. A (2022)

Φ: inverse function of $x \tanh x$

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Vo, TVV, Hasegawa, J. Phys. A (2022)

- Valid only for overdamped and Markov jump processes

Φ: inverse function of $x \tanh x$

Motivation

- TKUR can be violated for quantum systems due to quantum coherence

consecutive occurrence of identical jumps

x classical: $1 \to 0,\ 1 \to 0,\ \dots$

✓ quantum: $|1\rangle \to |0\rangle, |1\rangle \to |0\rangle,\ \dots$

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- How precision limits are modified in the presence of quantum effects?
  - Understand effects of quantum features (coherence & entanglement)
  - Utilize quantum features to enhance precision

Markovian dynamics - setup

- GKSL equation

$$\dot{\varrho}_t = \mathscr{L}(\varrho_t),$$

$$\mathscr{L}(\circ) := -i[H, \circ] + \sum_{k \geq 1} (L_k \circ L_k^\dagger - \{L_k^\dagger L_k, \circ\}/2)$$

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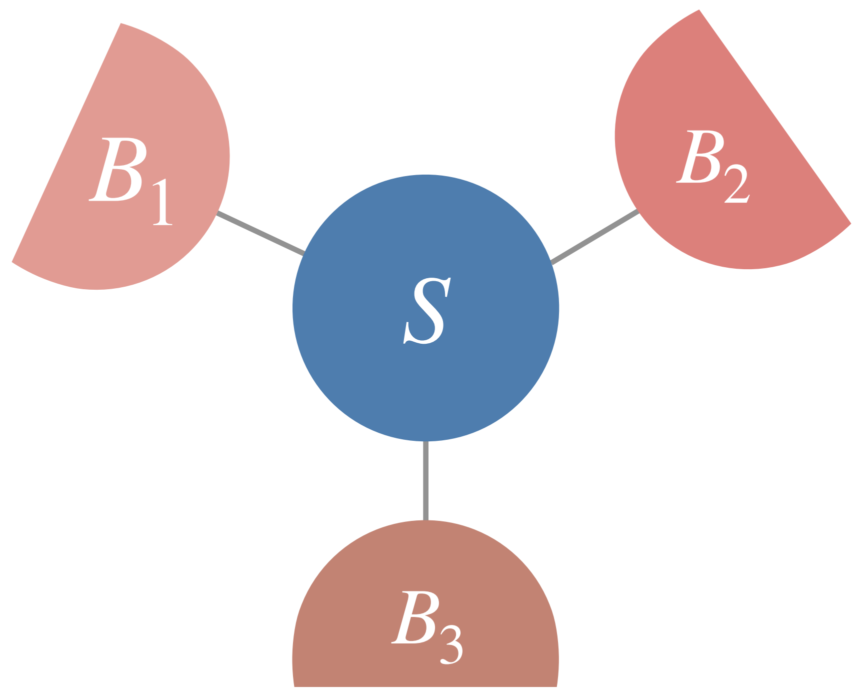
- Thermodynamic processes

local detailed balance condition $L_k = e^{\Delta s_k/2} L_{k^*}^\dagger$

Δs_k: environmental entropy change due to jump L_k



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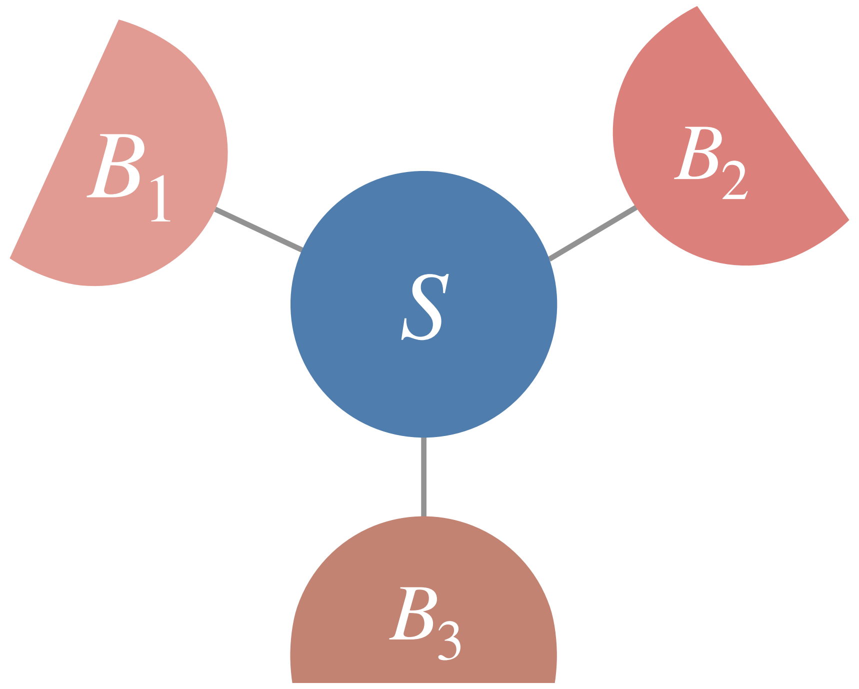

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Δs_k: environmental entropy change due to jump L_k

- Entropy production rate & dynamical activity rate

$$\sigma = \sum_{k\geq 1} \mathrm{tr}(L_k \pi L_k^\dagger) \Delta s_k \qquad a = \sum_{k\geq 1} \mathrm{tr}(L_k \pi L_k^\dagger)$$

π: steady state

Unraveling & observables

- Quantum jump unraveling

$$\varrho_{t+dt} = \sum_{k\geq 0} M_k \varrho_t M_k^\dagger \qquad \underbrace{M_0 = \mathbf{1} - (iH + \sum_{k\geq 1} L_k^\dagger L_k/2)dt}_{\text{no jump}}, \quad \underbrace{M_k = L_k\sqrt{dt}}_{k\text{th jump}}\ (k \geq 1)$$

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- Observables defined for stochastic trajectory $\Gamma = \{(t_1, k_1), \ldots, (t_J, k_J)\}$

$$\phi(\Gamma) := \sum_{i=1}^{J} c_{k_i}$$

c_k: coefficient associated with kth jump

e.g., entropy flux $(c_k = \Delta s_k)$, number of jumps $(c_k = 1)$

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- Currents: $c_k = -c_{k*}$ (time-antisymmetry)

Quantum thermokinetic uncertainty relation

TVV, PRX Quantum (2025)

Quantum thermokinetic uncertainty relation

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$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

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$$\widehat{\mathscr{D}}_\ell = \sum_{k\geq 1}\ell_k\left[L_k\otimes L_k^* - \frac{1}{2}(L_k^\dagger L_k)\otimes\mathbf{1} - \frac{1}{2}\mathbf{1}\otimes(L_k^\dagger L_k)^\top\right]$$

$$\widehat{\mathscr{C}} = \sum_{k\geq 1} c_k L_k\otimes L_k^*$$

$\widehat{\mathscr{L}}^+$: pseudoinverse of $\widehat{\mathscr{L}}$

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- Remarks

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Derivation:

quantum Cramer-Rao inequality

& virtual perturbation

TVV, PRX Quantum (2025)

Application to heat engines

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- Power-efficiency trade-off for steady-state heat engines

$$P\frac{\eta}{\eta_C-\eta}\frac{T_c(1+\delta_P)^2}{D_P}\leq\frac{1}{2}$$

P: power, D_P: power fluctuation

η: efficiency, η_C: Carnot efficiency

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Pietzonka+, PRL (2018)

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$(1+\delta_P)^2=O(\eta_C-\eta)$ $\mapsto$ possible to achieve Carnot efficiency at finite power without divergent power fluctuation?

Application to quantum clocks

- Precision of quantum clocks is limited by both dissipation and coherence

ϕ: number of clock cycles

$\mathcal{N} := \dfrac{\langle\phi\rangle}{\mathrm{Var}[\phi]}$: clock precision

$\Sigma_{\mathrm{tick}} := \dfrac{\tau\sigma}{\langle\phi\rangle}$: entropy production per tick

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$\mapsto$ precision can be arbitrarily large as $1+\delta_\phi \to 0$

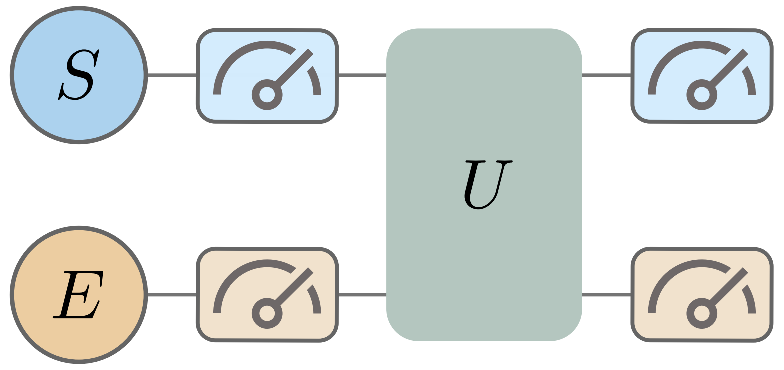
cf. $\mathcal{N} = O(e^{\Sigma_{\mathrm{tick}}})$ Meier+, Nat. Phys. (2025)

General open quantum dynamics - setup

- General unitary dynamics

$$\dot{\varrho}_t = -[H, \varrho_t]$$
$$H = H_S + H_E + H_I$$

(a)



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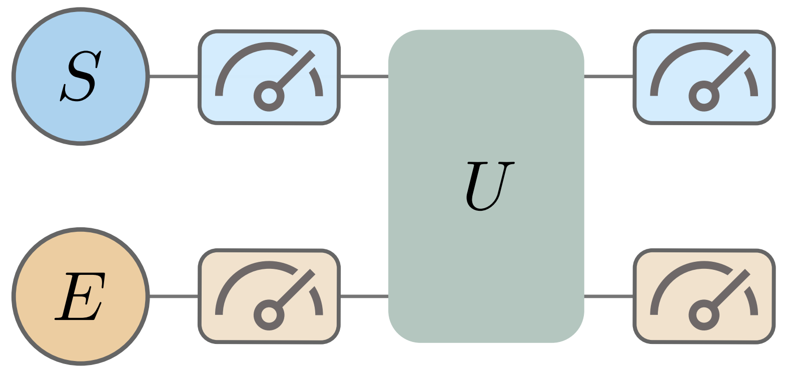

- Projective measurements on both system and environments

$$\varrho_S = \sum_n p_n |n\rangle\langle n|, \quad \varrho_E = \sum_\mu p_\mu |\mu\rangle\langle \mu| \; (= e^{-\beta H_E}/Z)$$

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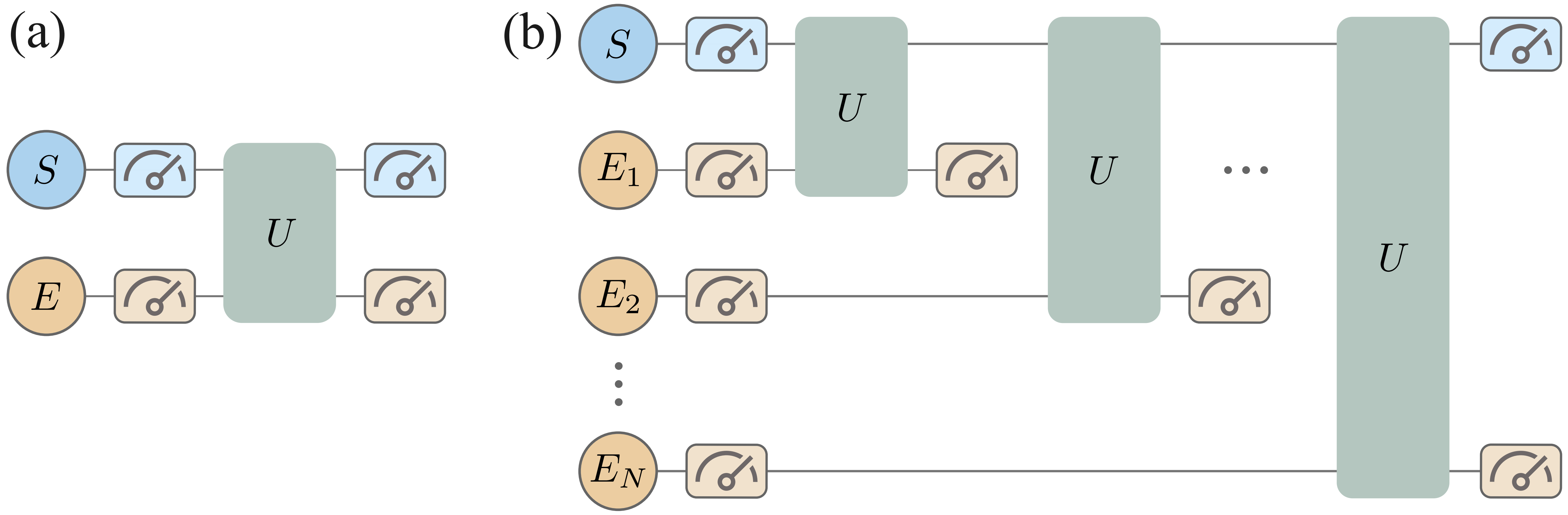

- Projective measurements on both system and environments

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General open quantum dynamics - setup

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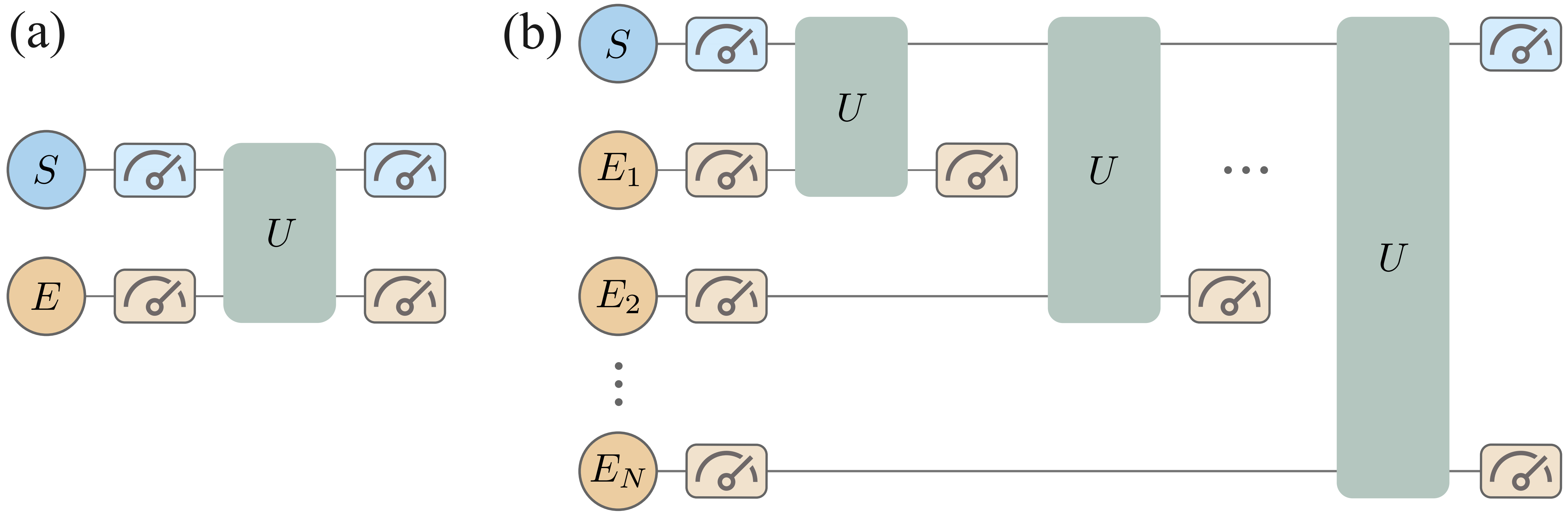

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- Stochastic trajectory $\gamma = \{n, (\nu_1, \mu_1), \ldots, (\nu_N, \mu_N), m\}$

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ΔS: von Neumann entropy change

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- Forward-backward asymmetry

$$\Sigma_* := \left\langle \ln \frac{\mathrm{P}(\gamma)}{\widetilde{\mathrm{P}}(\gamma)} \right\rangle$$

- difference between forward and backward processes
- vanishes when $\mathrm{P} \equiv \widetilde{\mathrm{P}}$
- arises due to dynamical factors

Generalized TUR

TVV, R. Honma, K. Saito, arXiv:2508.21567

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$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq f(\Sigma + \Sigma_*)$$

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✓ tight and saturable

✓ fundamental limit on current precision

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$\mathscr{P} := \sum_{\gamma \in I} \mathrm{P}(\gamma)$: probability of inactivity

$\to \mathscr{P}^{-1} - 1$ can be interpreted as an activity term

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✓ saturable for $\phi(\gamma) = 1 \ \forall \gamma \notin I$

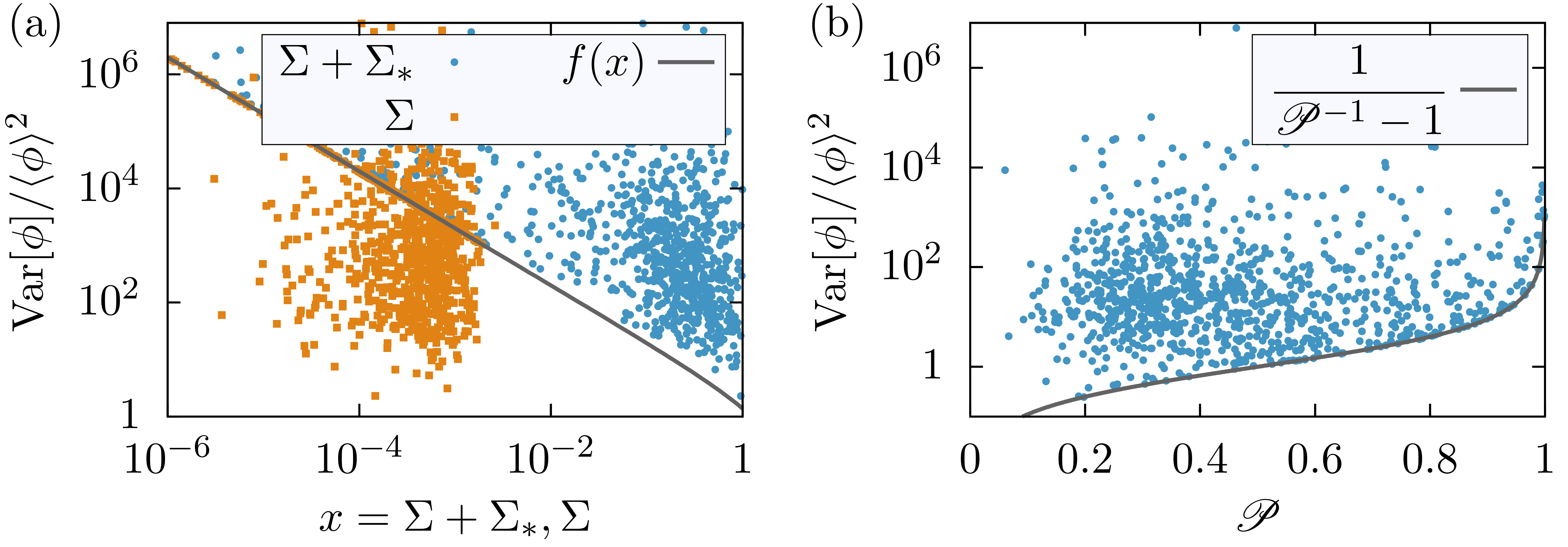
✓ tightens and generalizes previous results

Hasegawa, PRL 126, 010602 (2021)
Hasegawa, PRL 127, 240602 (2021)

TVV, R. Honma, K. Saito, arXiv:2508.21567

Demonstration - Qubit

$$H = \frac{1}{2}(\omega_z \sigma_z + \omega_x \sigma_x) \otimes \mathbf{1}_E + \mathbf{1}_S \otimes H_E + \lambda V_S \otimes V_E$$



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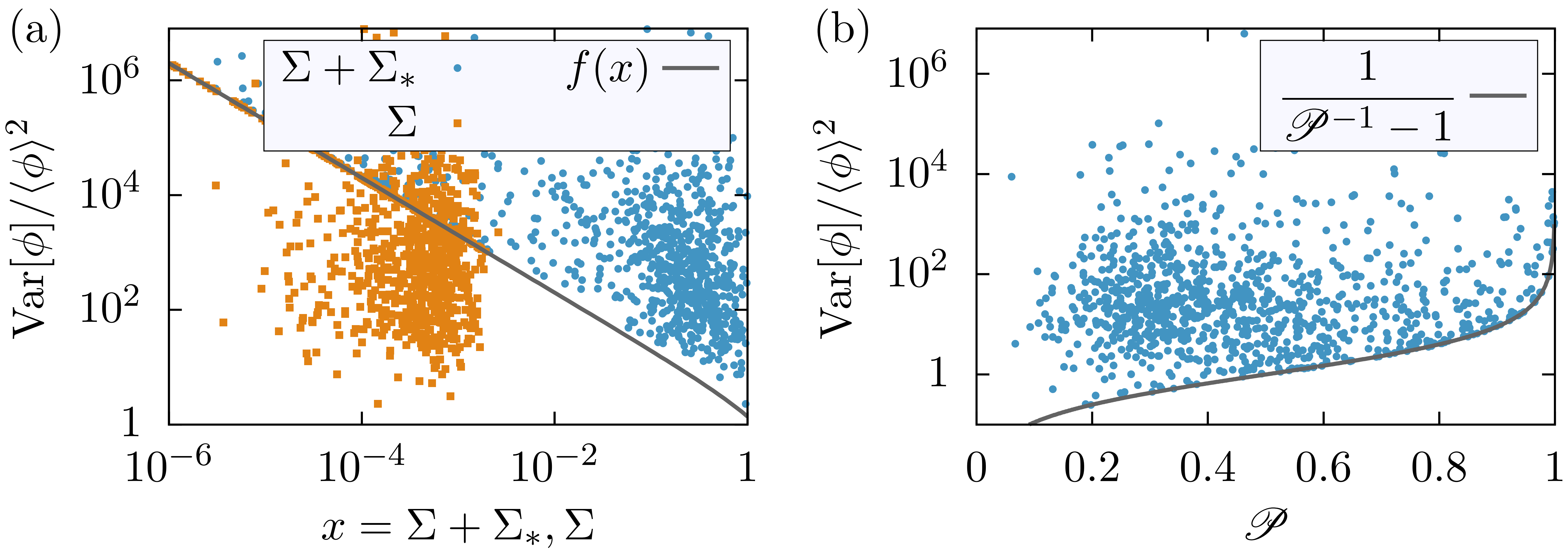

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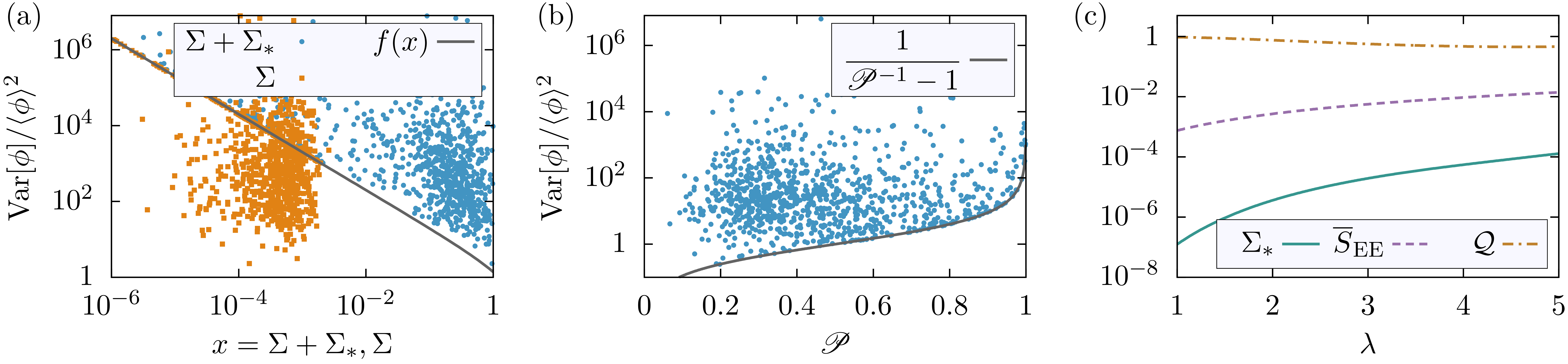

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