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100 YEARS OF QUANTUM PHYSICS

USING QUANTUM CLOUD AS A TOOL TO STUDY HARDY TYPE NONLOCALITY

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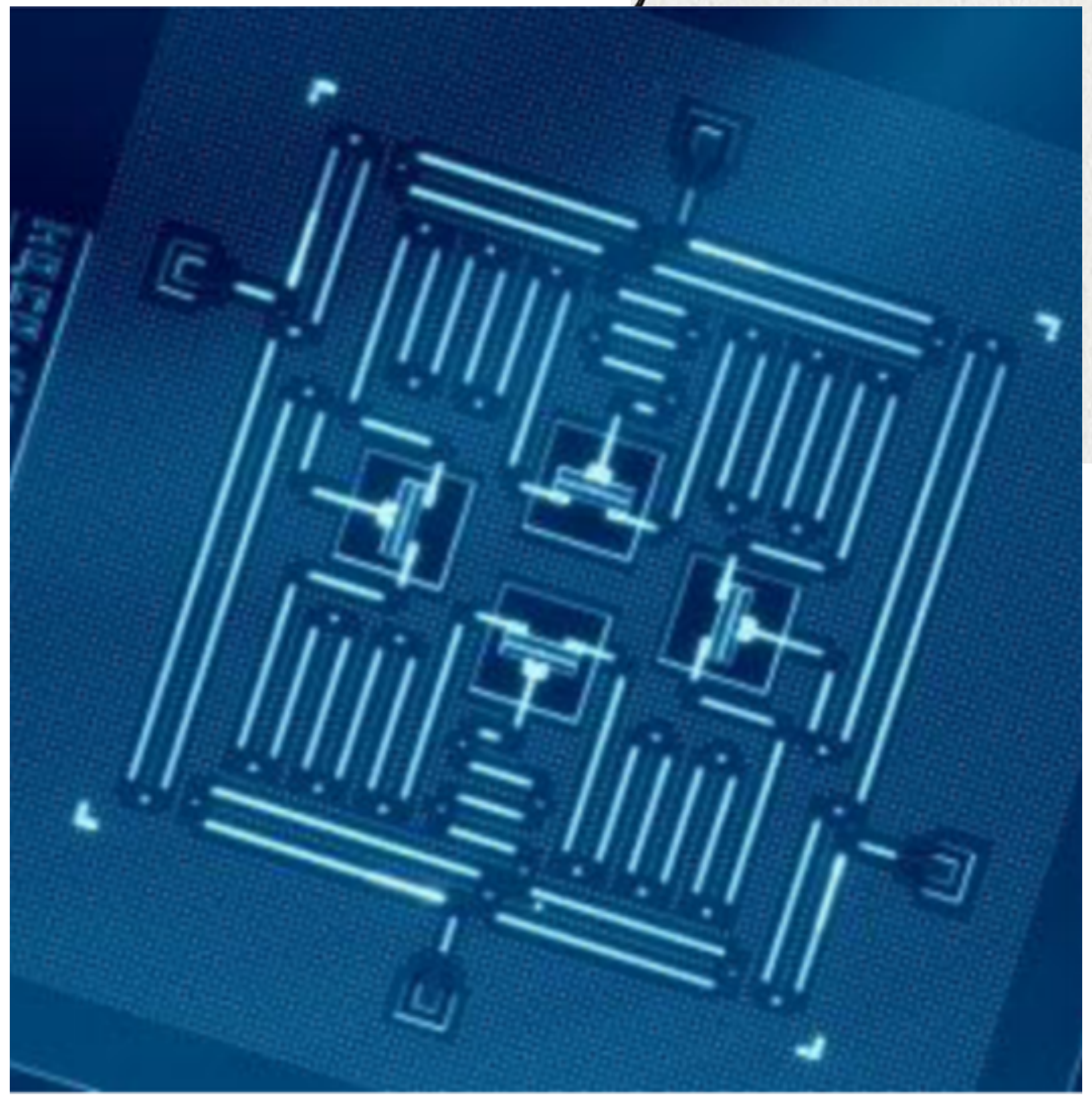
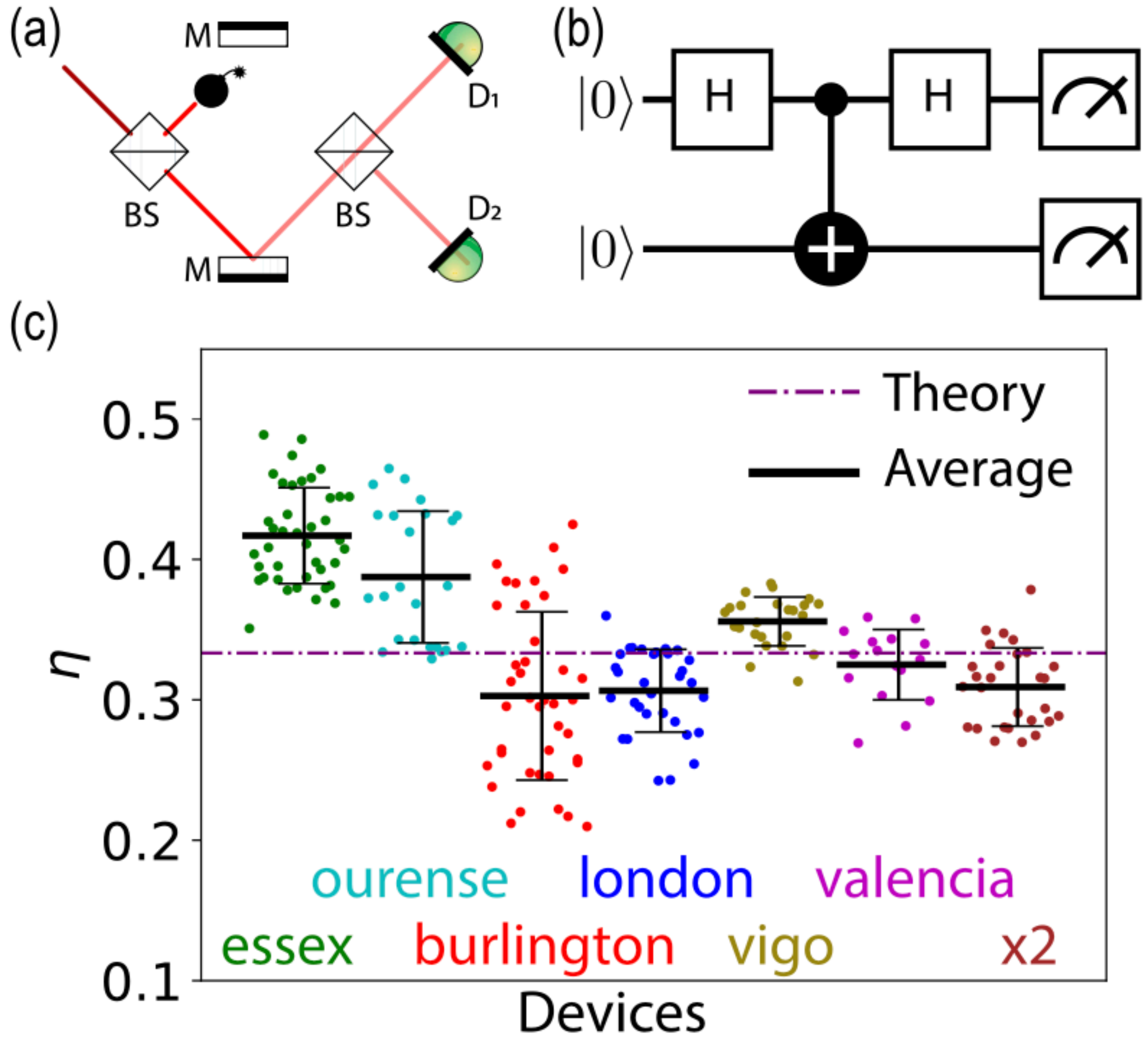
2025 Physics Nobel Prize honored Quantum Engineering Efforts



Quantum circuit on the IBM

- Quantum circuit describes an evolution of a state following some rules (algorithm)
- Read from left to right (time arrow)
- A language that executable on any quantum hardware
- A quantum testbed for experiments



Name	Date mm/yy	T_1 (μs)	T_2 (μs)	CNOT error (%)	Readou error (%
Burlington	08/20	84.88	67.36	1.50	4.64
Essex	08/20	104.31	123.7	1.76	3.59
London	08/20	61.45	62.74	1.75	4.40
Ourense	08/20	93.15	66.43	0.92	2.96
Valencia (Fig. 2)	08/20	84.18	62.78	1.11	2.32
Valencia (Fig. 3)	09/20	100.00	80.49	1.10	2.52
Vigo (Figs. 1, 2)	08/20	73.28	50.73	1.07	1.66
Vigo (Fig. 4)	09/20	107.64	74.04	0.94	1.96
x2	08/20	57.08	45.40	1.82	3.18

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Hardy's nonlocality

Consider 2 physical observables in their bases {u,v} and {c.d}

The quantum state write equivalently

$$
\begin{aligned}
|\Psi\rangle &= N(|c_1\rangle|c_2\rangle - A^2|u_1\rangle|u_2\rangle) \\
&= N(AB|u_1\rangle|v_2\rangle + AB|v_1\rangle|u_2\rangle + B^2|v_1\rangle|v_2\rangle) \\
&= N(|c_1\rangle(A|u_2\rangle + B|v_2\rangle) - A^2(A^*|c_1\rangle - B|d_1\rangle)|u_2\rangle) \\
&= N((A|u_1\rangle + B|v_1\rangle)|c_2\rangle - A^2|u_1\rangle(A^*|c_2\rangle - B|d_2\rangle)) \\
&= N(|c_1\rangle|c_2\rangle - A^2(A^*|c_1\rangle - B|d_1\rangle)(A^*|c_2\rangle - B|d_2\rangle))
\end{aligned}
$$

Without $|d_1\rangle|v_2\rangle$ $\quad P(U_2|D_1) = 1$

$P(U_1|D_2) = 1$

$P(D_1D_2) > 0$, which implies $P(U_1U_2) > 0$

In Hardy's original notation: $$\gamma = \left(\frac{(|\alpha| - |\beta|)|\alpha\beta|}{1 - |\alpha\beta|}\right)^2.$$

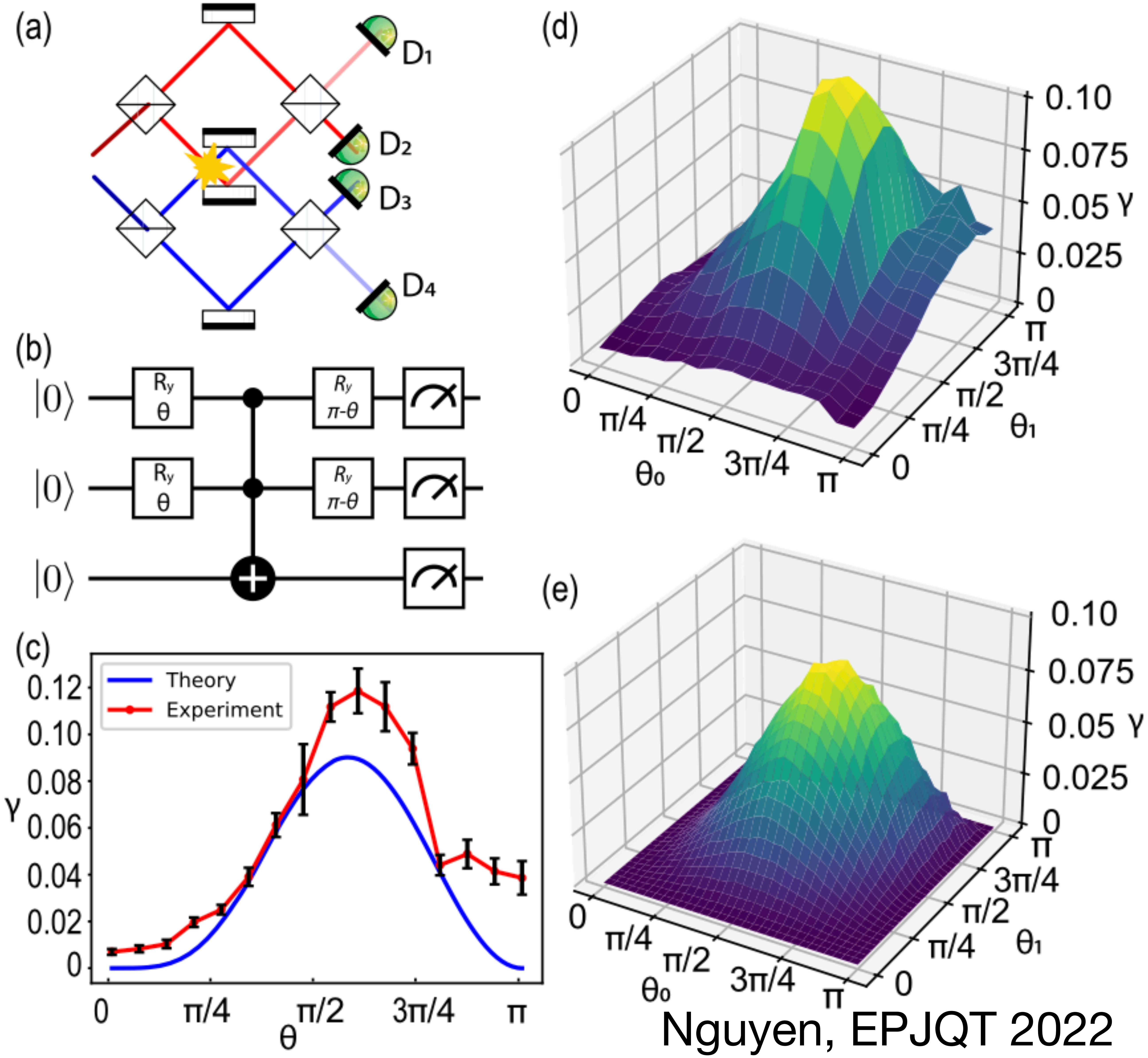
Simulating Hardy's paradox on IBM quantum devices

The equivalent between optical apparatus and quantum circuit

$$
\begin{aligned}
|\psi_f\rangle = &-\frac{1}{4}\sin\theta_1 \sin\theta_0 |000\rangle \\
&+ \frac{1}{2}\sin\theta_1 \sin^2\frac{\theta_0}{2}|100\rangle + \frac{1}{2}\sin^2\frac{\theta_1}{2}\sin\theta_0|010\rangle \\
&+ \frac{1}{4}\left(2\cos\theta_1\sin^2\frac{\theta_0}{2} + \cos\theta_0 + 3\right)|110\rangle \\
&+ \frac{1}{4}\sin\theta_1\sin\theta_0|001\rangle - \frac{1}{2}\sin^2\frac{\theta_0}{2}\sin\theta_1|101\rangle \\
&- \frac{1}{2}\sin^2\frac{\theta_1}{2}\sin\theta_0|011\rangle + \sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_0}{2}|111\rangle.
\end{aligned}
$$

$$
\begin{aligned}
\gamma &= \frac{\sin^2\theta_1 \sin^2\theta_0}{4(2\cos\theta_1\sin^2\frac{\theta_0}{2} + \cos\theta_0 + 3)} \\
&= \frac{2\sin^4\frac{\theta}{2}\cos^2\frac{\theta}{2}}{3-\cos\theta}.
\end{aligned}
$$

Identical to Hardy analysis



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Generalization to the n-particle: 1st condition

$$|u_k\rangle = A_k^*|c_k\rangle - B_k|d_k\rangle,\ \ |v_k\rangle = B_k^*|c_k\rangle + A_k|d_k\rangle,$$

$$|c_k\rangle = A_k|u_k\rangle + B_k|v_k\rangle,\ \ |d_k\rangle = -B_k^*|u_k\rangle + A_k^*|v_k\rangle,$$

$$|\Psi_n\rangle = N\big[|c_1c_2\cdots c_n\rangle - \mathcal{A}_\Omega|u_1u_2\cdots u_n\rangle\big]$$

Our first condition:

$$P(\mathcal{U}_\Omega) \equiv P(U_1U_2\cdots U_n) = 0.$$

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Generalization to the n-particle: 2nd condition

Particle kth is measured in {c,d}, while others are measured in {u,v}

$$|\Psi_n\rangle = N\Big[\big(A_1|u_1\rangle + B_1|v_1\rangle\big) \otimes \cdots \otimes |c_k\rangle \otimes \cdots \otimes \big(A_n|u_n\rangle + B_n|v_n\rangle\big) \\ - \mathcal{A}_\Omega|u_1\cdots u_{k-1}\rangle \otimes \big(A_k^*|c_k\rangle - B_k|d_k\rangle\big) \otimes |u_{k+1}\cdots u_n\rangle\Big]$$

the only term containing d_k is $B_k\mathcal{A}_\Omega|u_1\cdots d_k\cdots u_n\rangle$

If one measure D_k, it infers a complete collapse of \psi to this substate.

U_i measurement at $i \neq k$ would yield $U_i = 1$.

$$P(\mathcal{U}_{\overline{\kappa}}|D_k) = 1,\ \forall k \in \Omega,$$

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Generalization to the n-particle: 3rd condition

$$P(\mathcal{D}_\alpha) > 0, \quad \forall \alpha \in \mathcal{P}(\Omega), |\alpha| \geqslant 2,$$

And success probability

$$P_{\text{success}} = |\mathcal{A}_\Omega|^2 - \frac{|\mathcal{A}_\Omega|^4}{1 - |\mathcal{A}_\Omega|^2} \sum_{k=1}^{n} \frac{1 - |A_k|^2}{|A_k|^2}$$

which maximize when all A equal

$$P_{\text{success}} = A^{2n} - n \frac{A^{4n-2}(1 - A^2)}{1 - A^{2n}}$$

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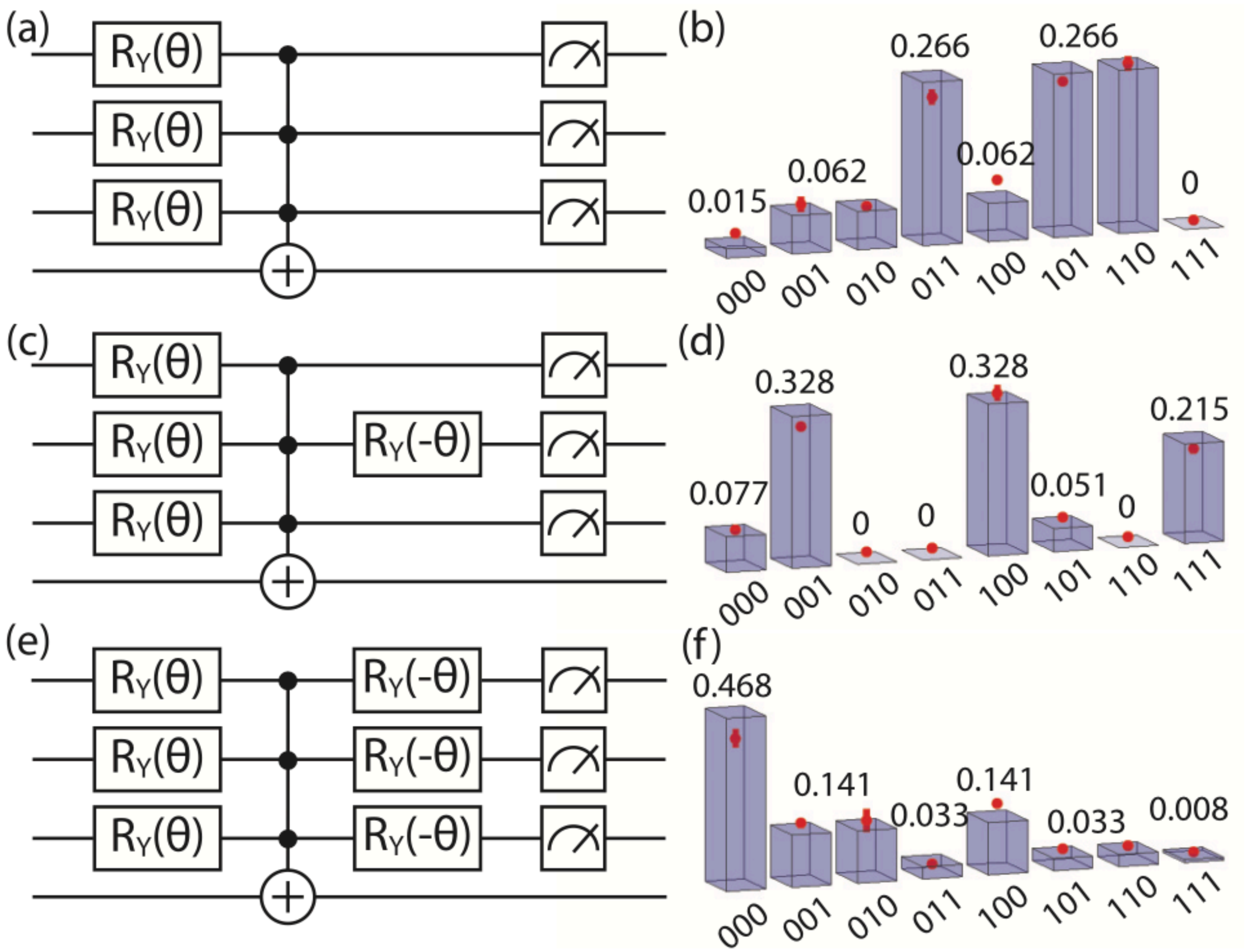
Simulation on quantum circuit

1st condition

2nd condition

3rd condition



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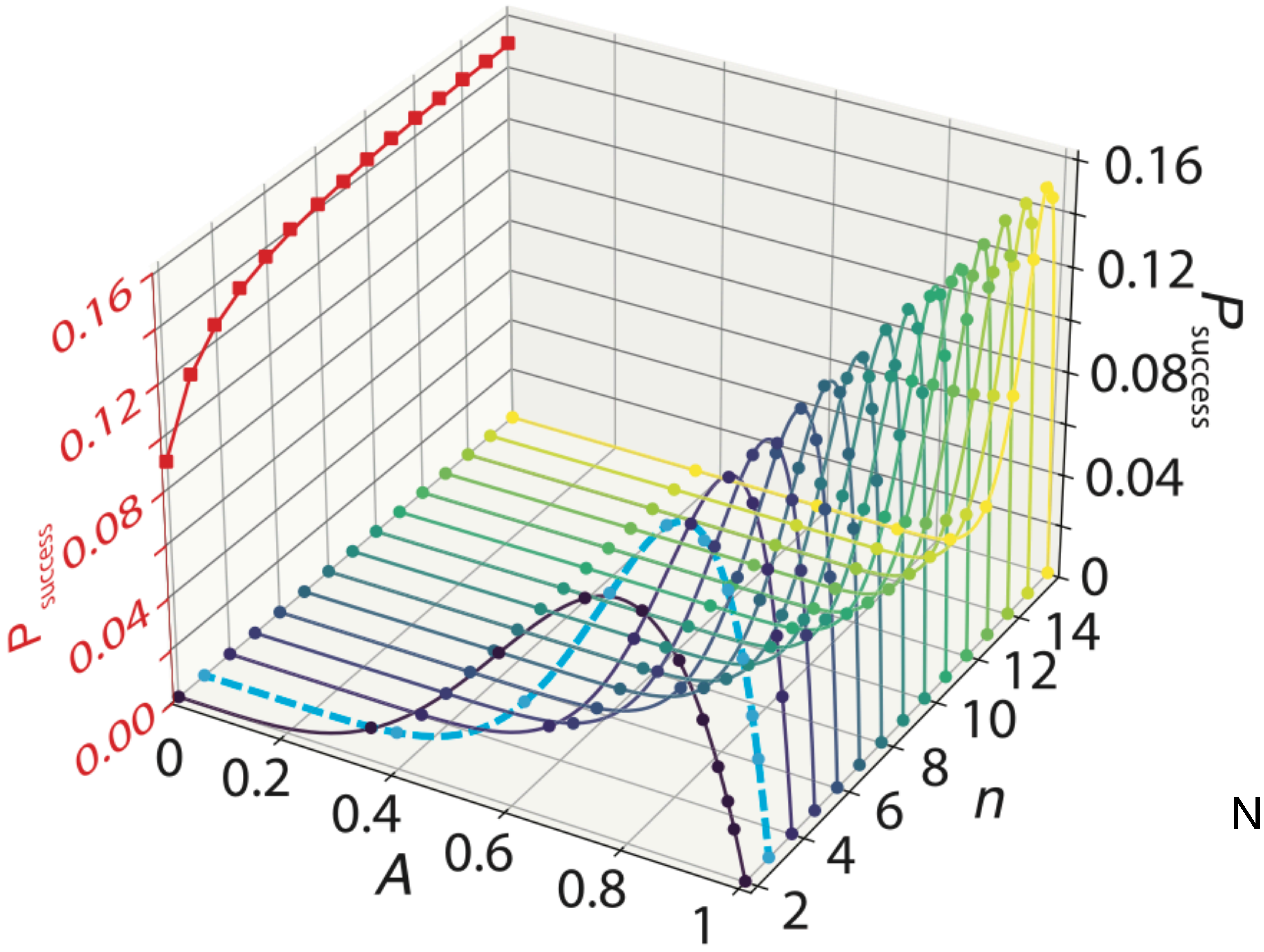
Hardy for n-particle: simulation vs analytical



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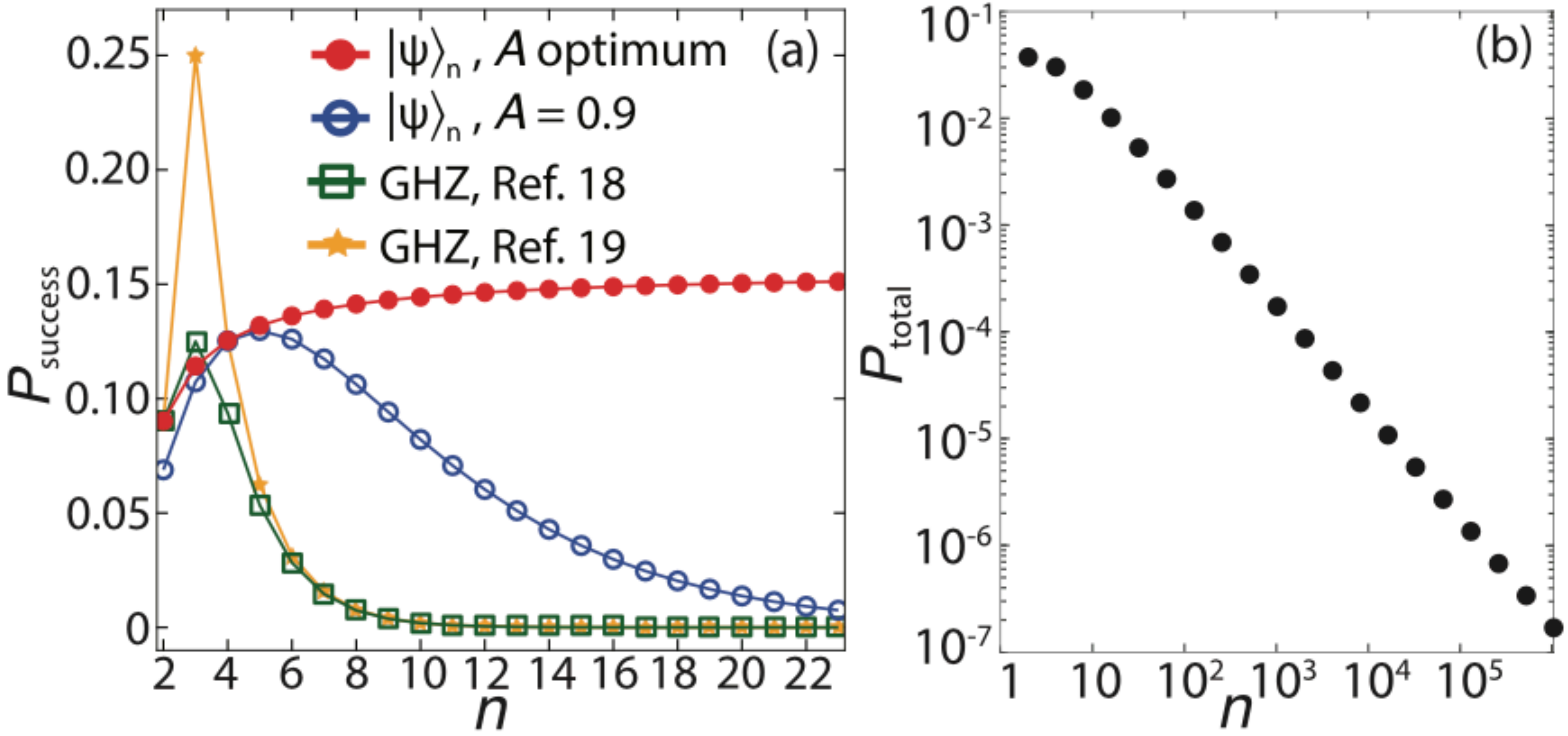
Success probability



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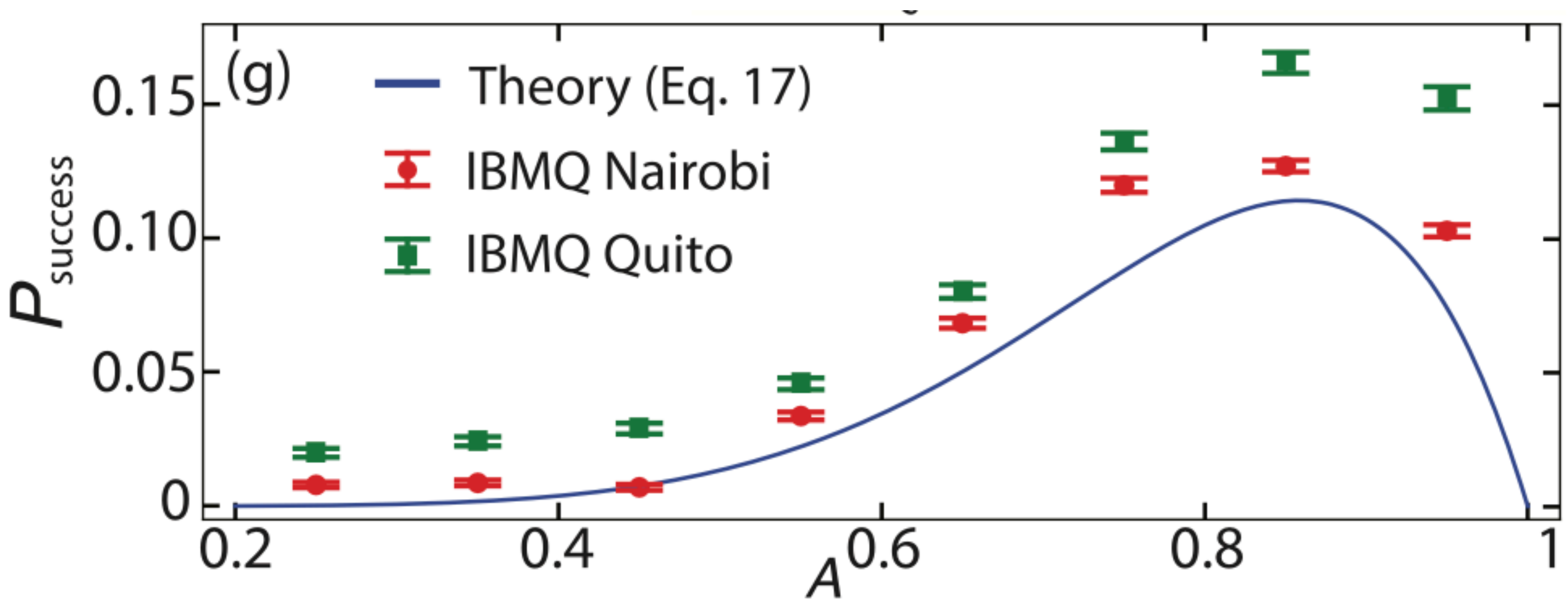
Realization of nonlocality on quantum hardware



Summary

- An insight from quantum circuit symmetry originate the generalization
- Analytic, simulation, and experiment verification
- Complicated quantum nonlocality generated on generic platform with cloud access
- A new testbed for foundation of quantum physics

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