Coulomb Interaction-Stabilized Chern Insulators with $C > 1$ *in* Twisted Rhombohedral Trilayer-Bilayer Graphene

arXiv:2505.07981

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Florida State University
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100 Years of Quantum Physics
Quy Nhơn, Việt Nam

Prof. Cyprian Lewandowski

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Quantum Hall Insulators

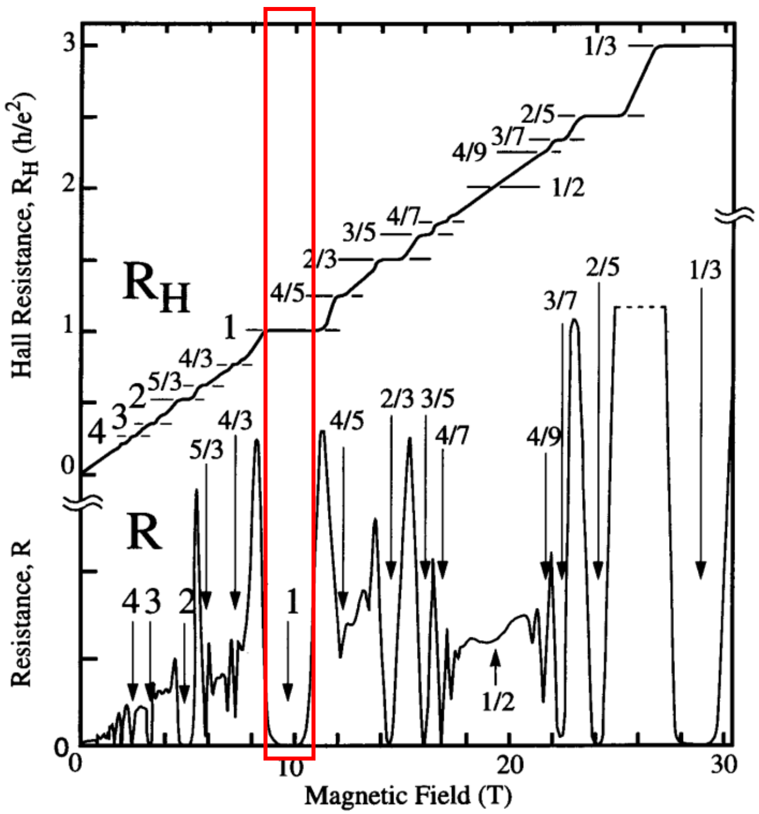

Figure 18. The FQHE as it appears today in ultra-high mobility modulation doped GaAs / AlGaAs 2DESs. Many fractions are visible. The most prominent sequence, ν=p/(2p±1), converges toward ν=1/2 and is discussed in the text.

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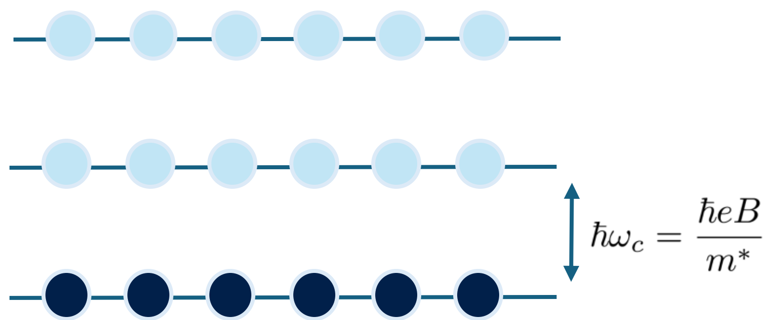

- **Integer** quantum Hall effect = **full** Landau levels filled
  - Used as resistance standard

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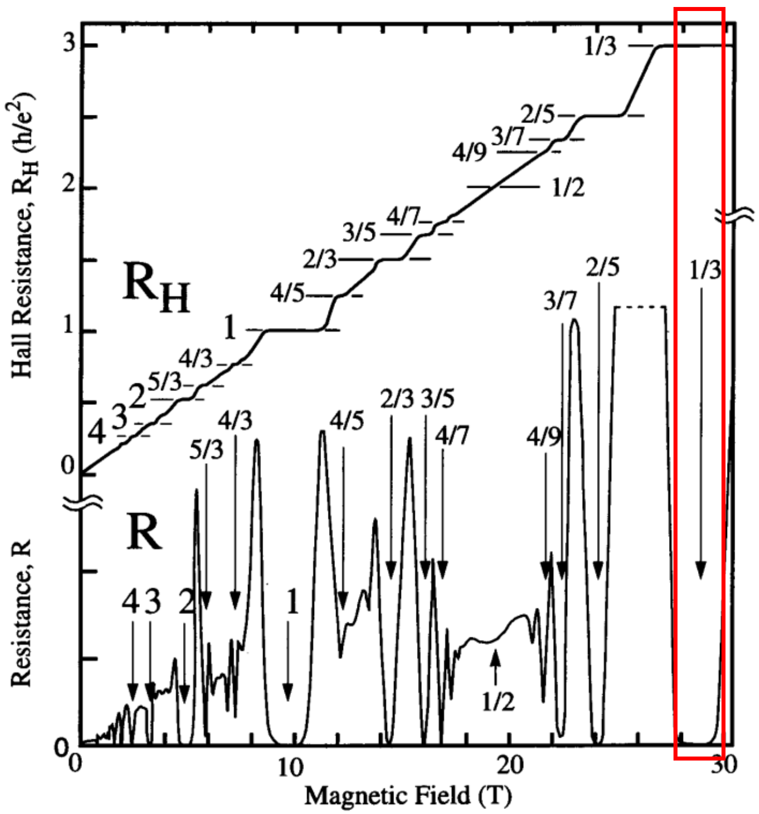

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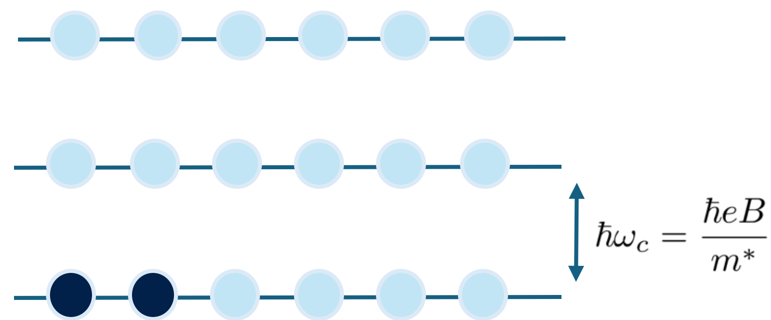

- **Integer** quantum Hall effect = **full** Landau levels filled
  - Used as resistance standard
- **Fractional** quantum Hall effect = **partial** Landau levels filled
  - Host Abelian and non-Abelian anyons potentially useful for topological quantum computation

Topology – a Unifying Viewpoint

Why is quantization of the Hall conductivity so precise?

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- Each Landau level has a Chern number of one

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Fractional Chern Insulators

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The Chern number is defined for any lattice system, completely agnostic of the origin of time-reversal symmetry breaking!

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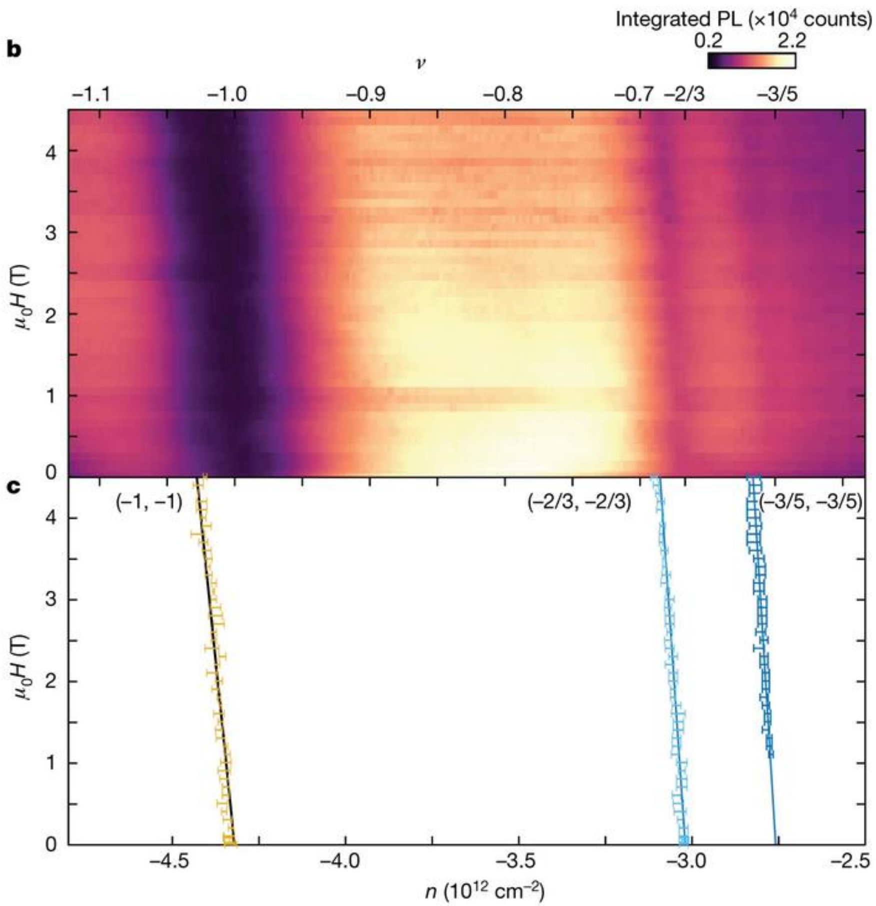
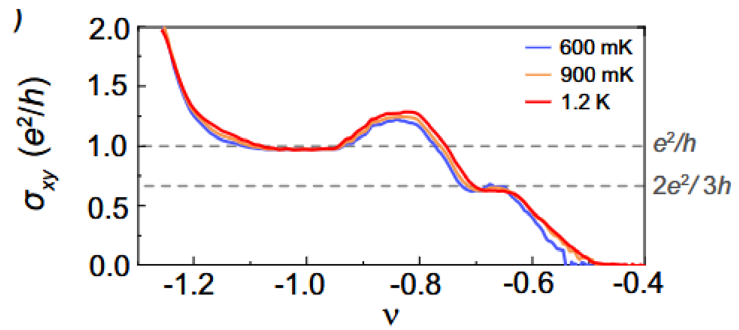
This has been seen in two different classes of materials

Fractional Chern Insulators





Twisted MoTe2

- Fractions seen: 1, 2/3, 3/5

Cai, Jiaqi, et al. *Nature* 622.7981 (2023): 63-68.
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Redekop, Evgeny, et al. *Nature* 635.8039 (2024): 584-589.
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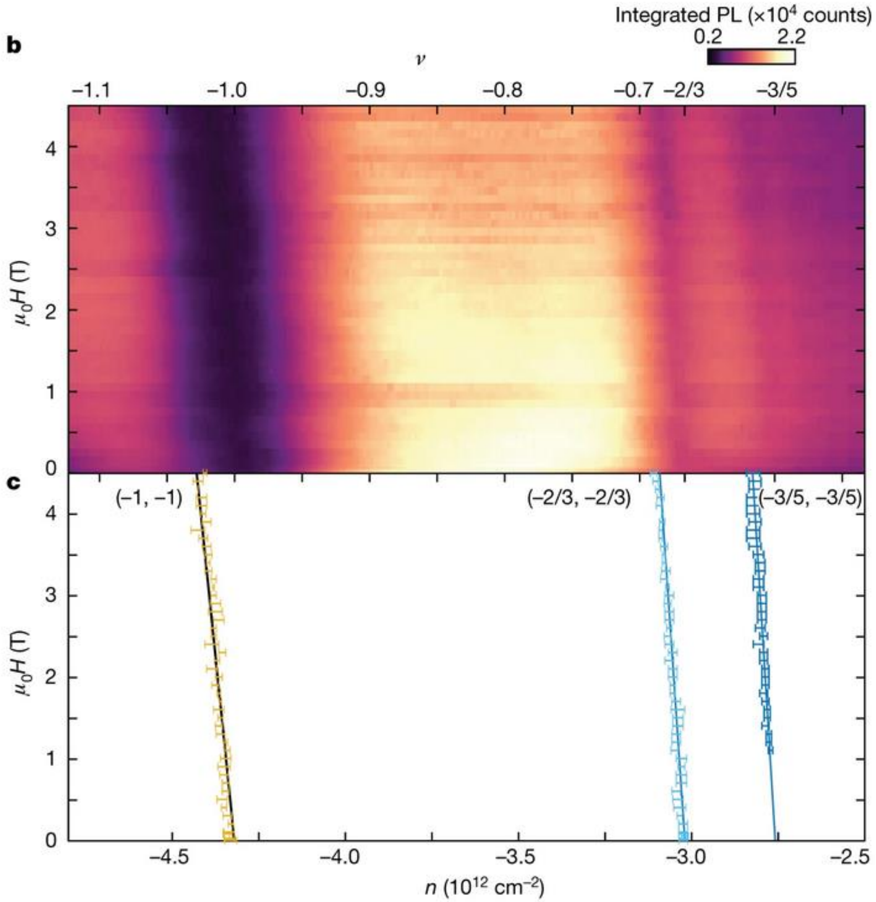
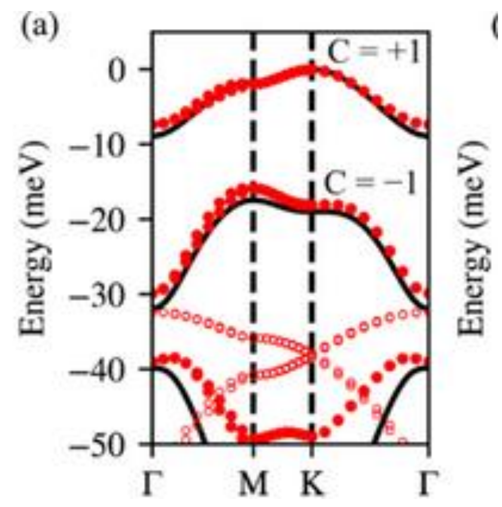
Fractional Chern Insulators





Wang, Chong, et al. *Phys. Rev. Lett.* 132.3 (2024): 036501.
Reddy, Aidan P., et al. *Phys. Rev. B* 108.8 (2023): 085117.
Yu, Jiabin, et al. *Phys. Rev. B* 109.4 (2024): 045147.
Jia, Yujin, et al. *Phys. Rev. B* 109.20 (2024): 205121.

Twisted MoTe2

- Fractions seen: 1, 2/3, 3/5
- Noninteracting band structure contains narrow, isolated Chern bands.
- Chern number = 1

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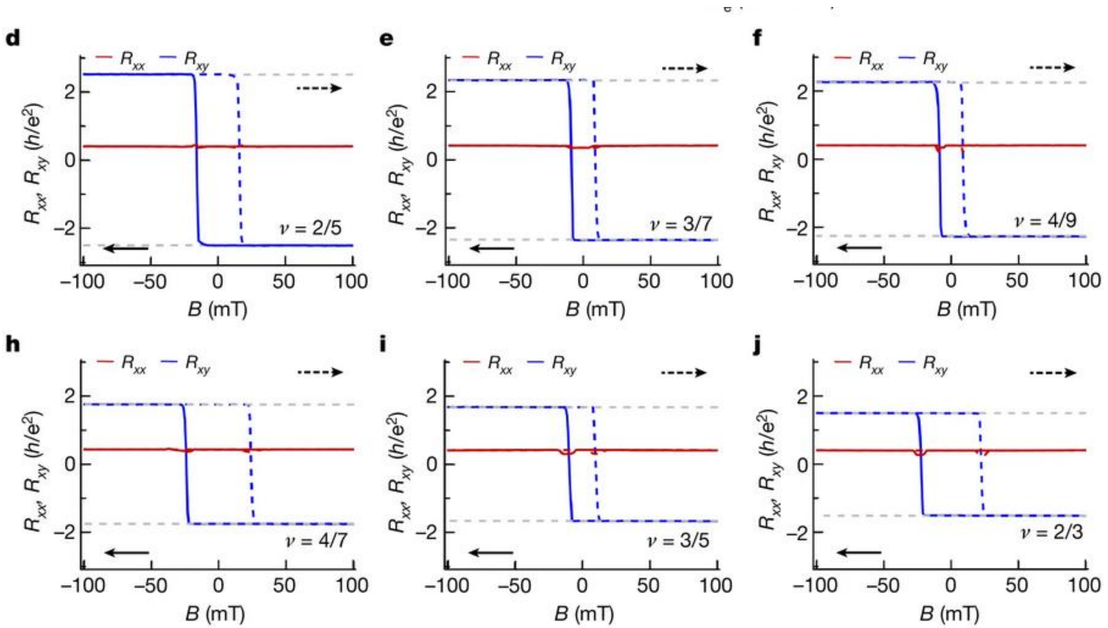

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Multilayer Graphene/hBN

- Fractions seen: 1, 2/3, 3/5, 4/7, 4/9, 3/7, 2/5, 5/9, 5/11
- Number of layers: 4, 5, 6
- At large displacement field

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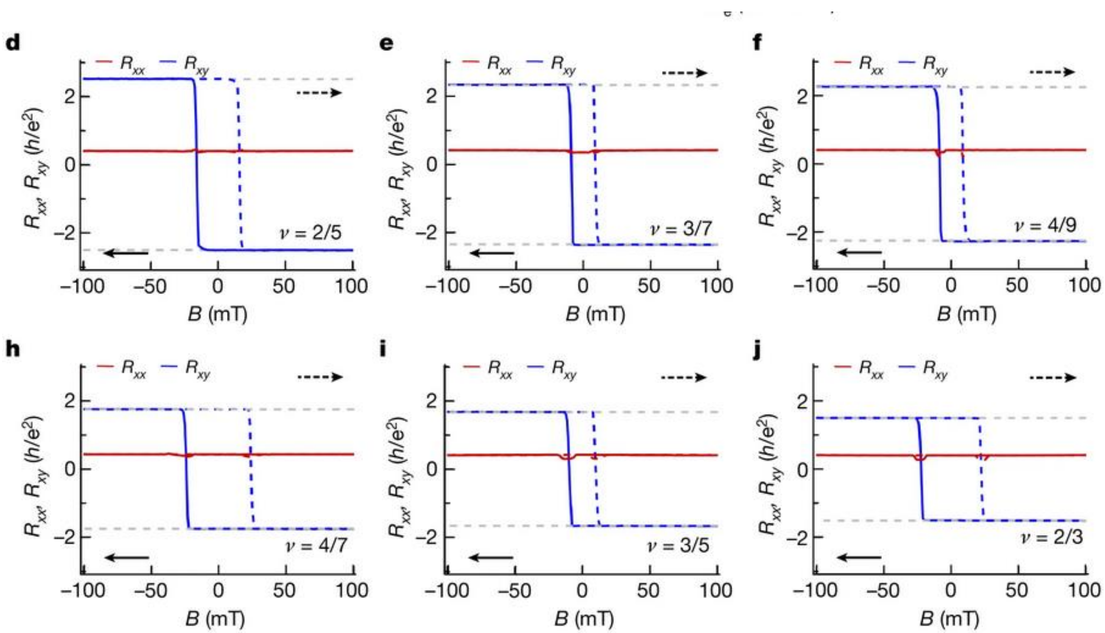

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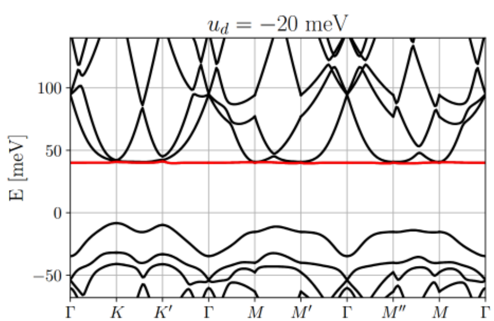

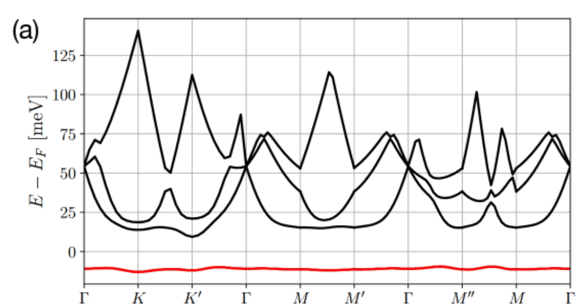

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- Number of layers: 4, 5, 6
- At large displacement field
- Noninteracting band structure contains narrow, but **not** isolated, Berry-curvature rich bands
- Chern number = 1
- Role of substrate is unclear

Dong, Zhihuan, Adarsh S. Patri, and Todadri Senthil. *Phys. Rev. Lett.* 133.20 (2024): 206502.
Herzog-Arbeitman, Jonah, et al. *Phys. Rev. B* 109.20 (2024): 205122.
Dong, Junkai, et al. *Phys. Rev. Lett.* 133.20 (2024): 206503.
Kudo, Koji, Ryota Nakai, and Kentaro Nomura. *Phys. Rev. B* 110.24 (2024): 245135.
Guo, Zhongqing, et al. *Phys. Rev. B* 110.7 (2024): 075109.
Zhou, Boran, Hui Yang, and Ya-Hui Zhang. *Phys. Rev. Lett.* 133.20 (2024): 206504.
Huang, Ke, et al. *Phys. Rev. B* 110.11 (2024): 115146.

Motivation

Now that FCI's from $C = 1$ bands have been experimentally observed, we are motivated to search for

lattice systems that host bands which are:

- **Narrow** and **isolated** post-Hartree Fock renormalization
- Topological with **Chern numbers > 1** – beyond Landau level paradigm
- Possible **candidates for FCI's** upon doping

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Analogy to the zeroth Landau level: Trace condition violation

Quantum metric $$g_{\mu\nu}(\mathbf{k}) = \frac{\langle \partial_\mu u_\mathbf{k} | \partial_\nu u_\mathbf{k} \rangle + \langle \partial_\nu u_\mathbf{k} | \partial_\mu u_\mathbf{k} \rangle}{2} - \langle \partial_\mu u_\mathbf{k} | u_\mathbf{k} \rangle \langle u_\mathbf{k} | \partial_\nu u_\mathbf{k} \rangle$$

The quantum metric measures distances between Bloch states.

$$\lambda = \frac{1}{2\pi} \int d^2\mathbf{k} \left[\mathrm{Tr} g_{\mu\nu}(\mathbf{k}) - |\mathcal{B}_z(\mathbf{k})| \right]$$

This quantity is exactly **zero** for the **zeroth** Landau level, is **two** for the **first** Landau level, and is **four** for the **second** Landau level.

Ledwith, Patrick J., et al. *Phys. Rev. Res.* 2.2 (2020): 023237.

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Quant ... $\partial_\nu u_{\mathbf{k}}\rangle$

Trace condition violation for pentalayer: 0.6-1.45

Trace condition violation for MoTe2: 0.1-0.7

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Th ... rst

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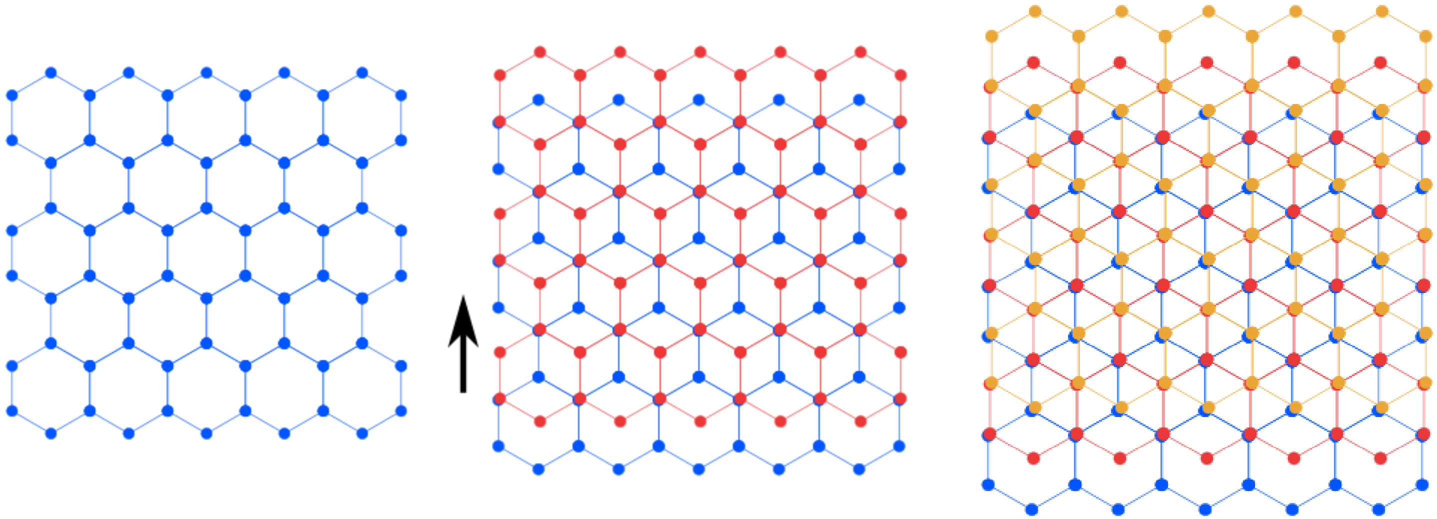
Rhombohedral Graphene Multilayer



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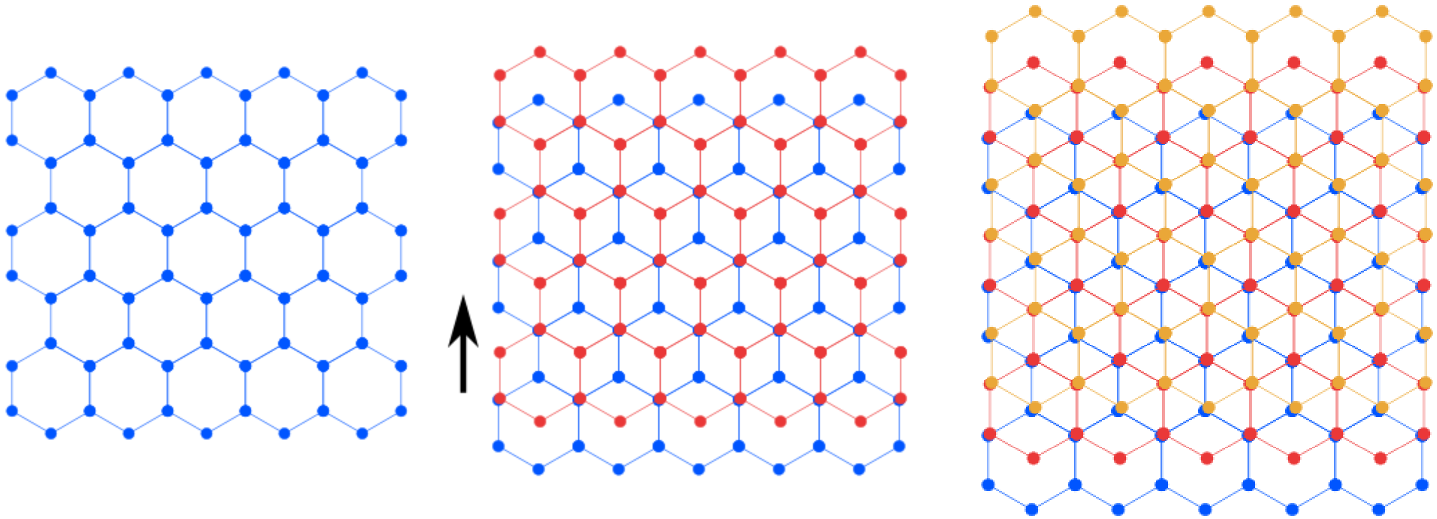

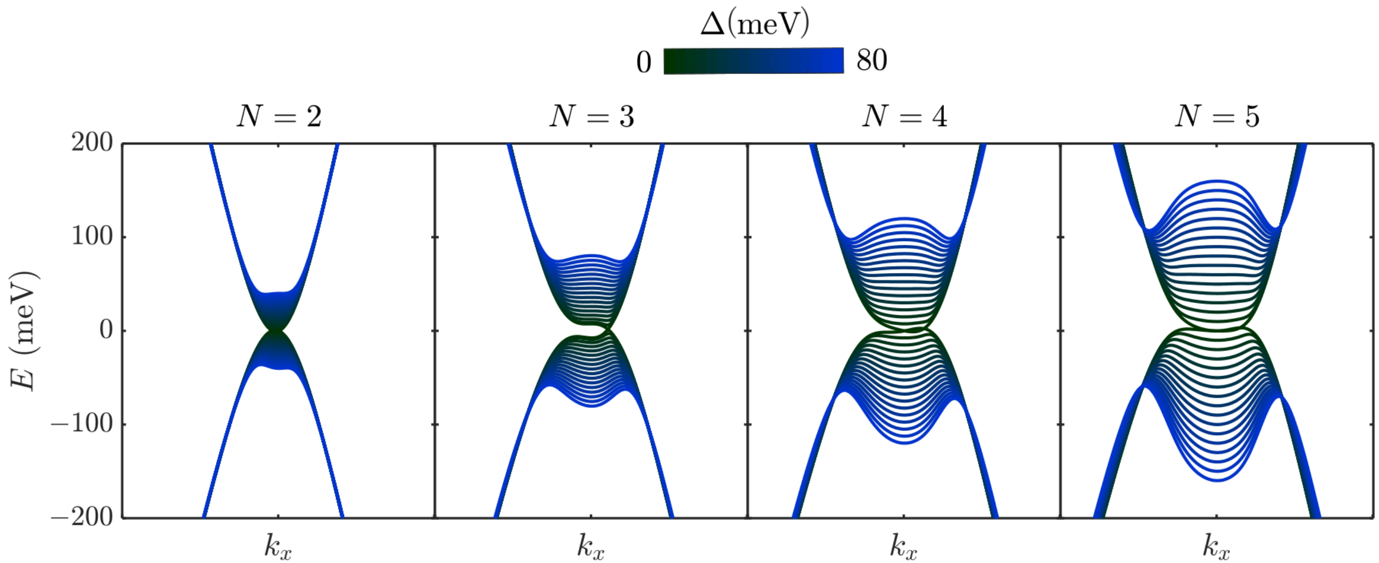

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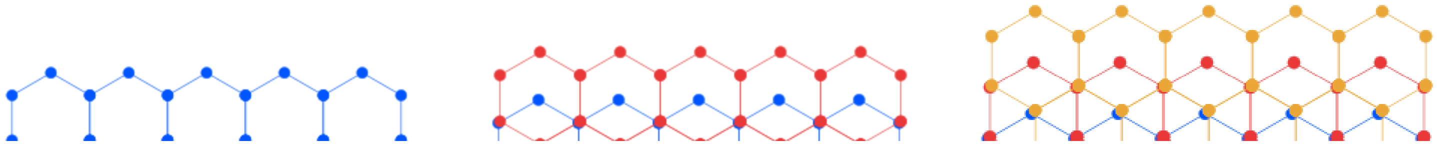
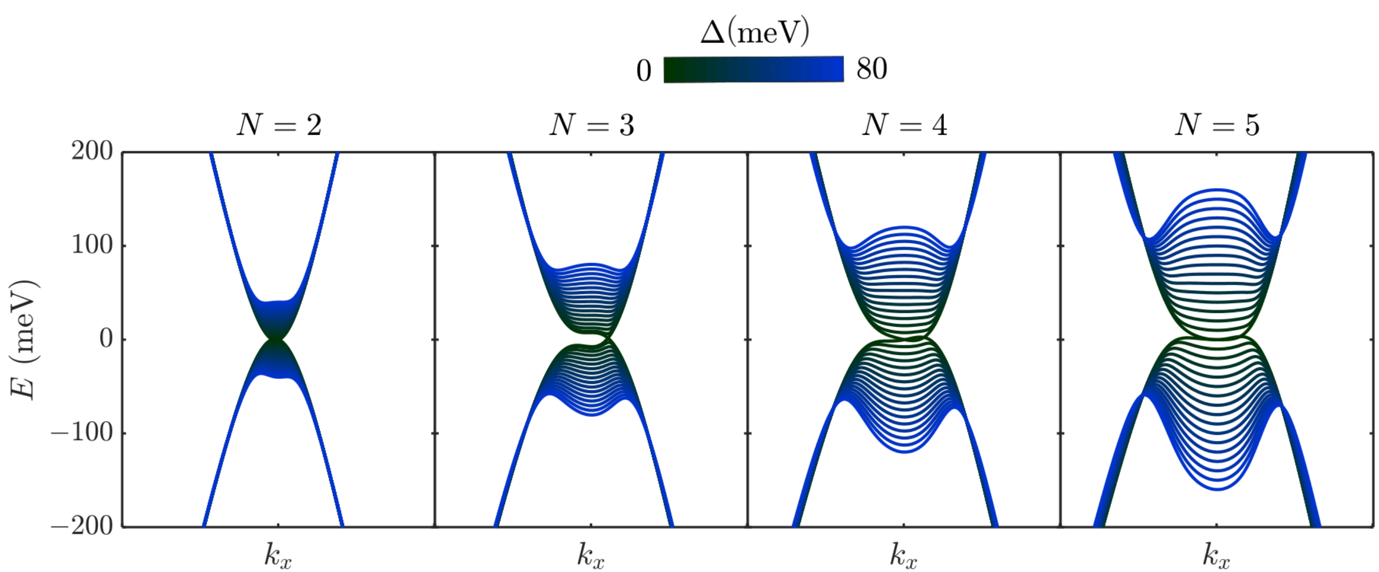
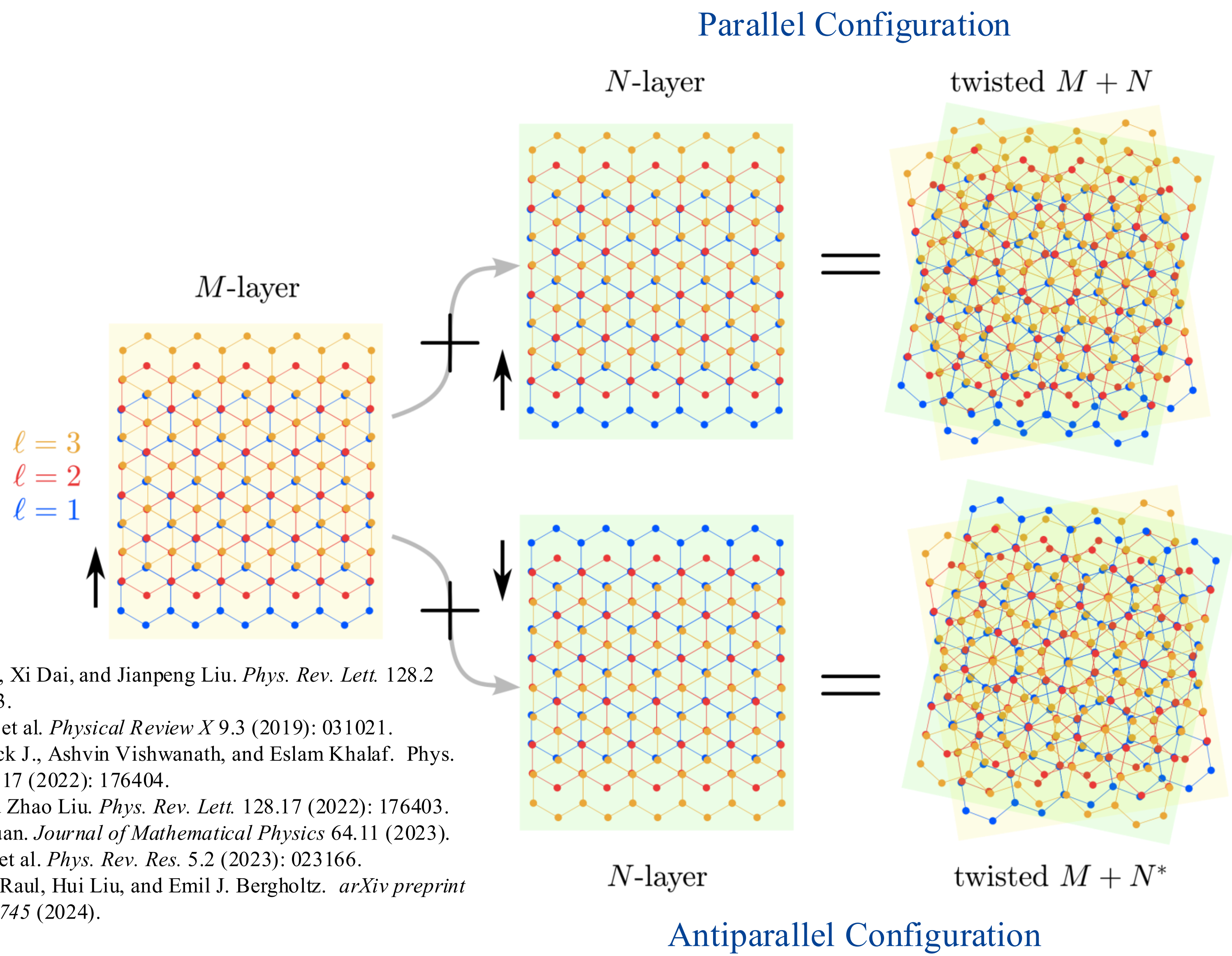


- Band structures can be gapped by interlayer displacement field
- Bottom of conduction band flattens with increasing field
- The more layers, the smaller the field needed to achieve the same gap



Twisted Rhombohedral Graphene Multilayer



Zhang, Shihao, Xi Dai, and Jianpeng Liu. *Phys. Rev. Lett.* 128.2 (2022): 026403.
Liu, Jianpeng, et al. *Physical Review X* 9.3 (2019): 031021.
Ledwith, Patrick J., Ashvin Vishwanath, and Eslam Khalaf. Phys. Rev. Lett. 128.17 (2022): 176404.
Wang, Jie, and Zhao Liu. *Phys. Rev. Lett.* 128.17 (2022): 176403.
Yang, Mengxuan. *Journal of Mathematical Physics* 64.11 (2023).
Dong, Junkai, et al. *Phys. Rev. Res.* 5.2 (2023): 023166.
Perea-Causin, Raul, Hui Liu, and Emil J. Bergholtz. *arXiv preprint arXiv:2412.02745* (2024).

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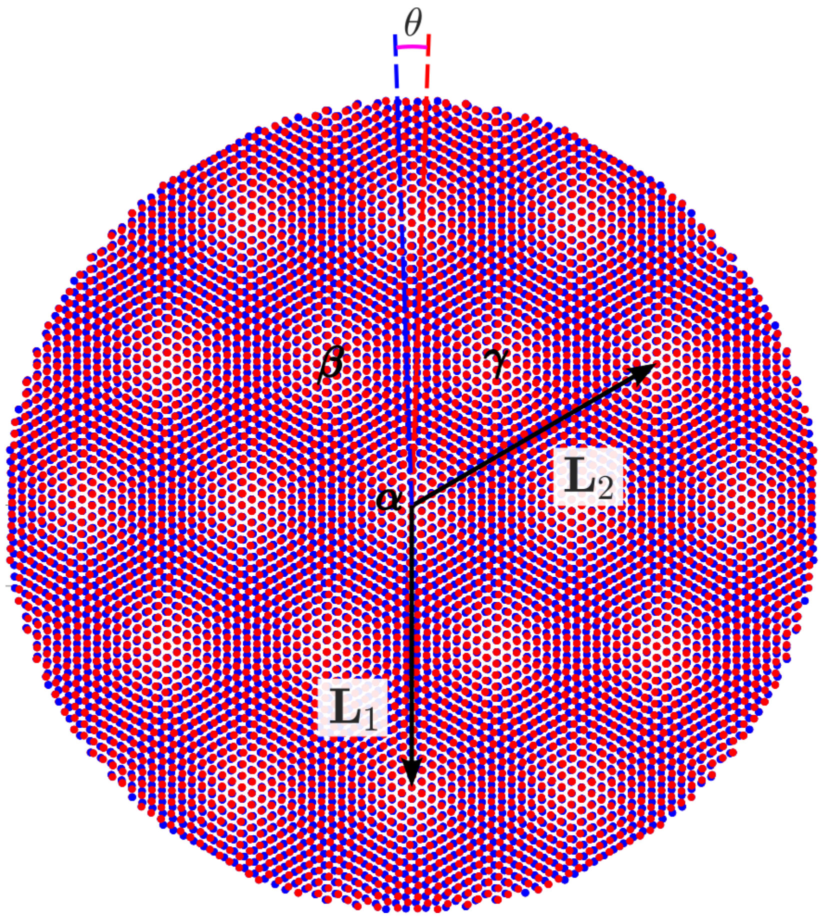


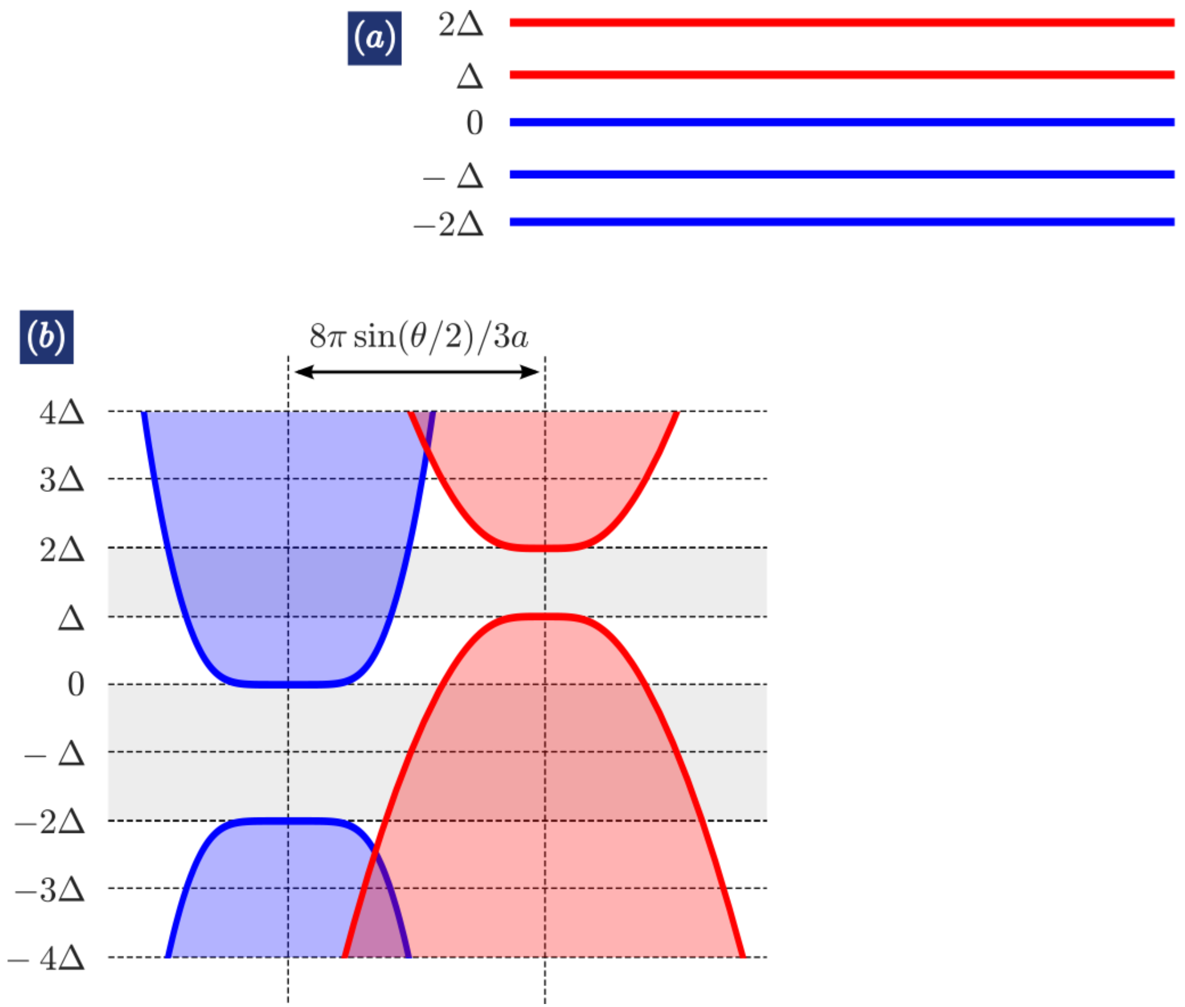
Moiré Structure

Twisted Rhombohedral Graphene Multilayer

For concreteness, we consider twisted 3+2 multilayer graphene for the remainder of this talk.

Twisted Rhombohedral Graphene Multilayer

Twisted 3+2 Multilayer Graphene



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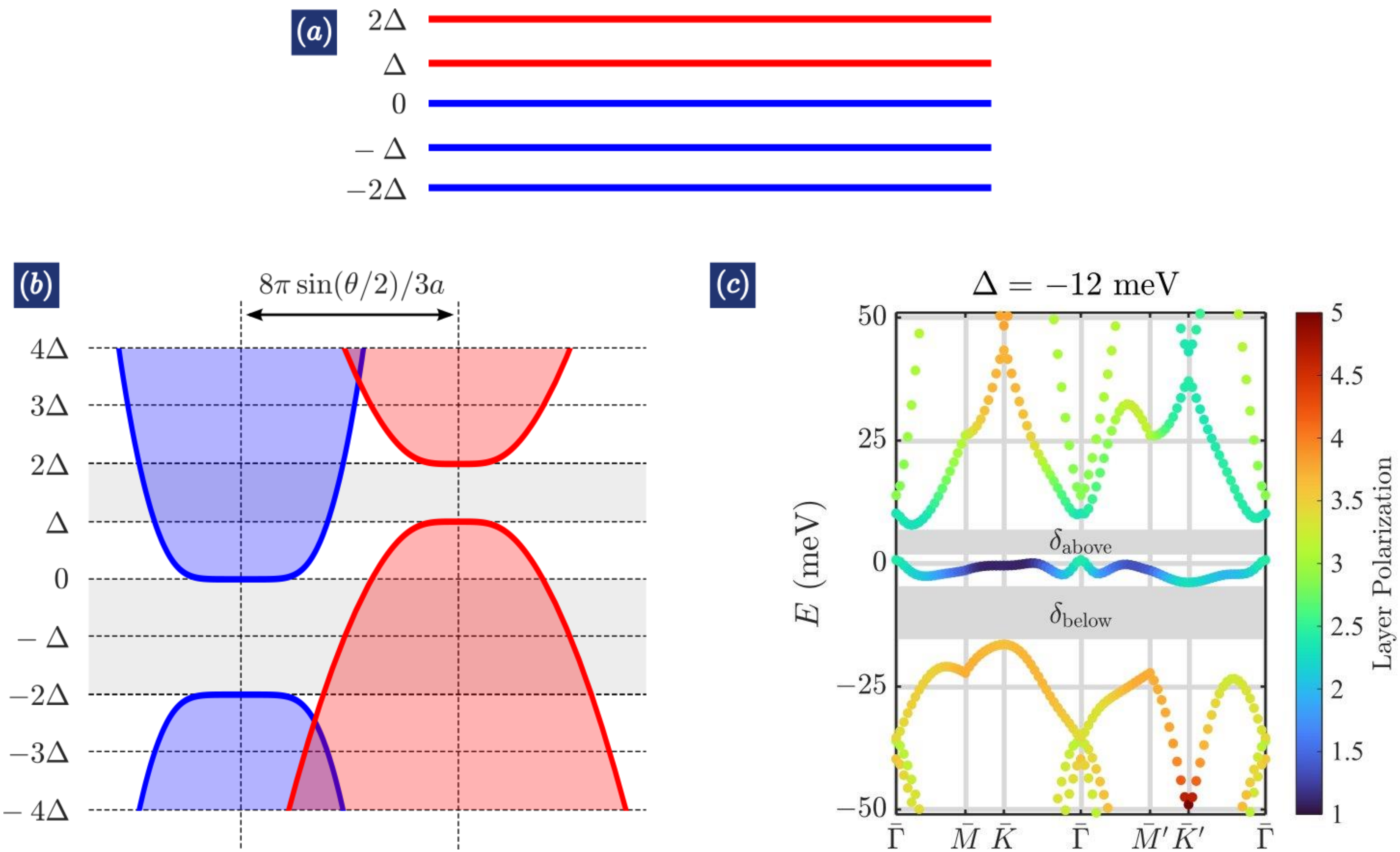
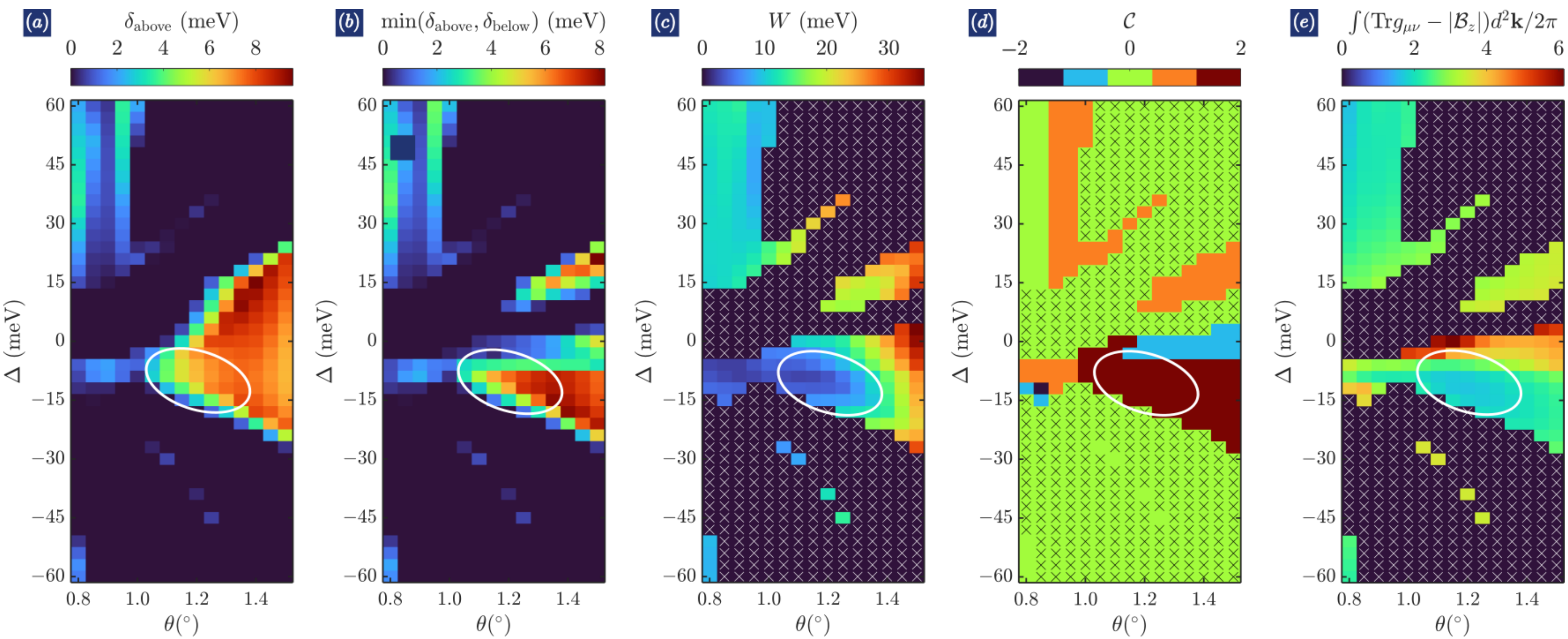


Hybridization is crucial for gap formation, i.e. moiré structure is important!

Non-Interacting Phase Diagram

Antiparallel Stacking Order



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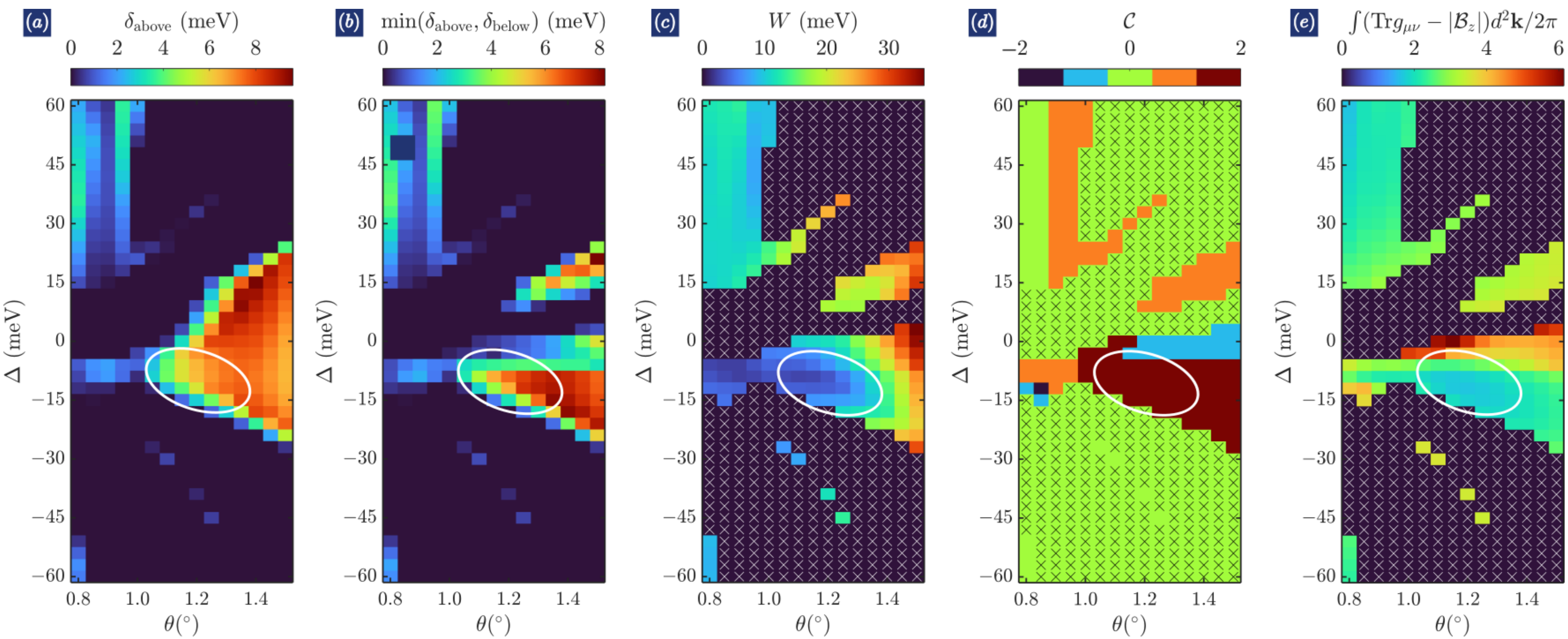
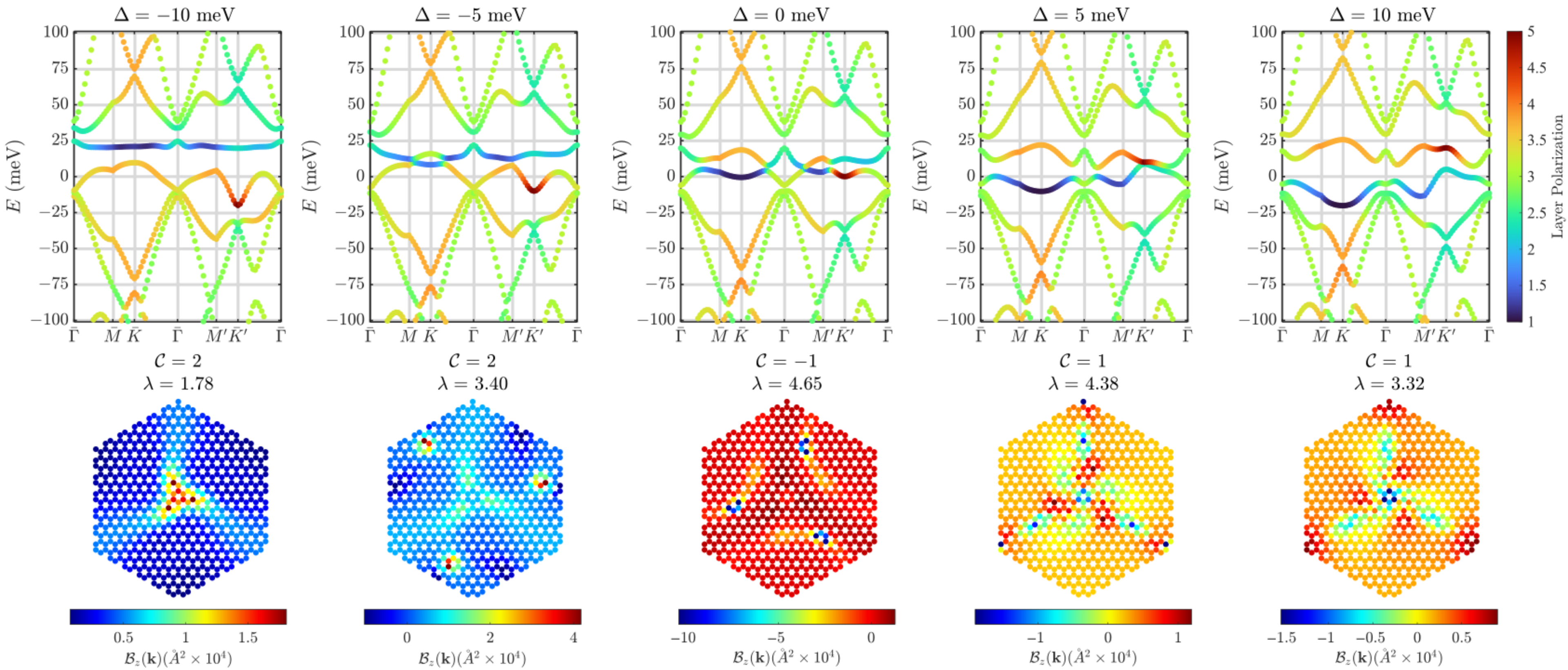


There exists a region in phase space which simultaneously has:

(1) large gaps, (2) small bandwidths, (3) Chern number = 2, and (4) the trace condition is relatively small ~ 1.7

Non-Interacting Phase Diagram

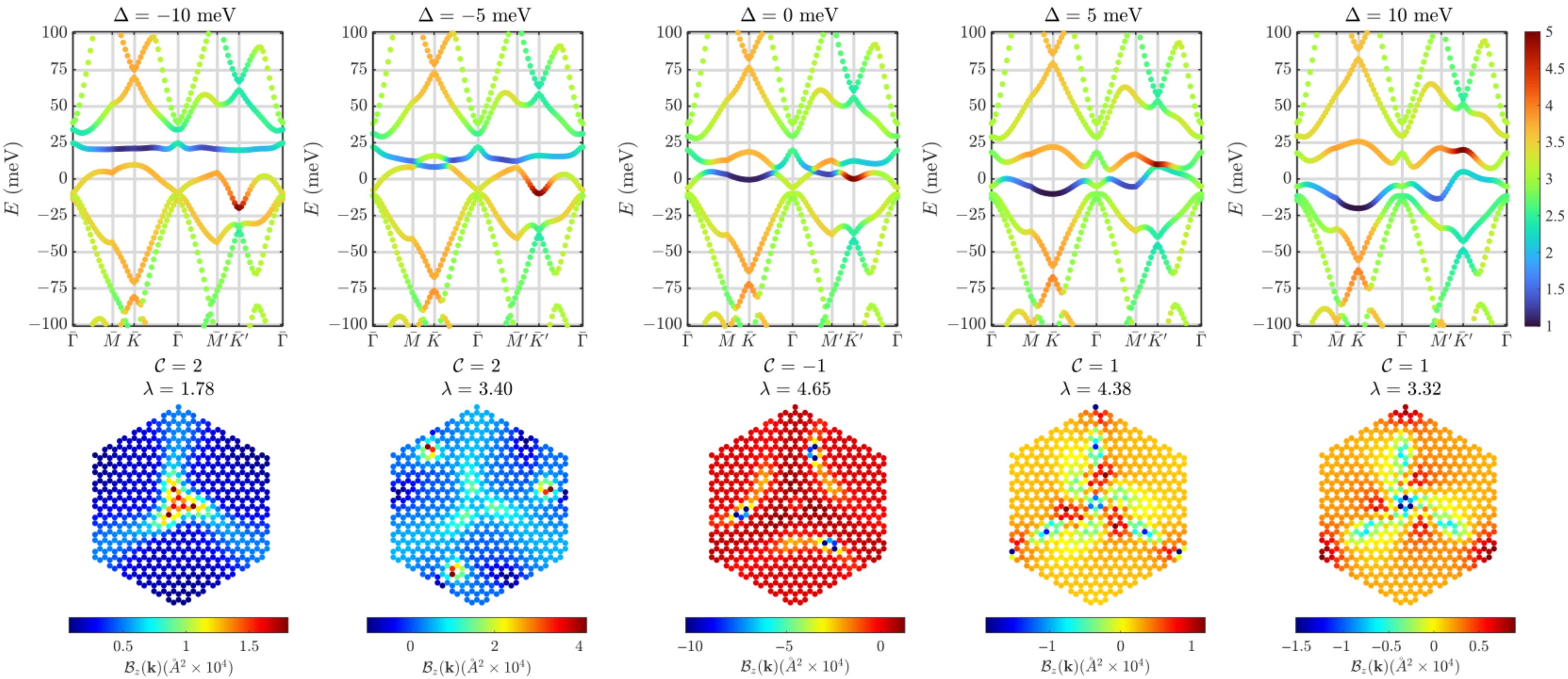
Antiparallel Stacking Order



- As Δ increases in the **negative** direction, a Chern-two band flattens. Charge density is localized on the trilayer substack.

Non-Interacting Phase Diagram

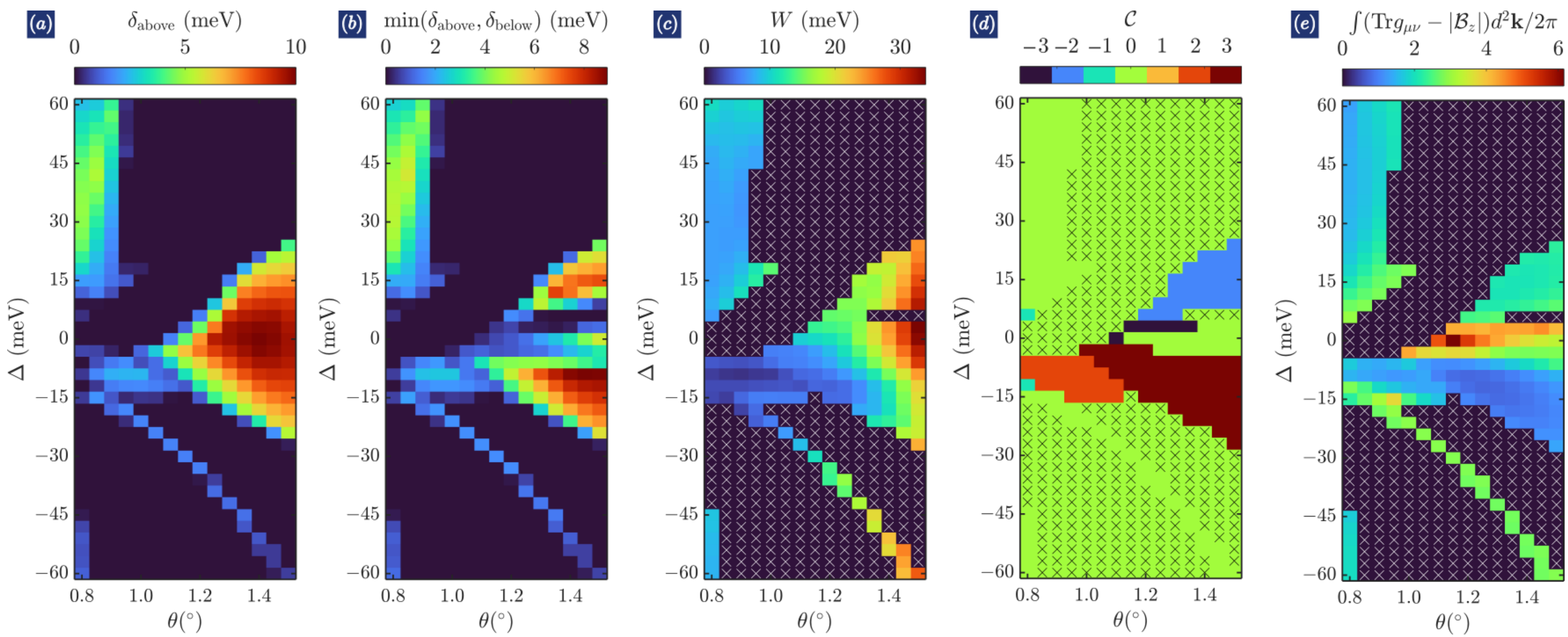
Antiparallel Stacking Order



- As Δ increases in the **negative** direction, a Chern-two band flattens. Charge density is localized on the trilayer substack.
- As Δ increases in the **positive** direction, a Chern-one band develops but is not isolated. Charge density is localized on the bilayer substack.

Non-Interacting Phase Diagram

Parallel Stacking Order



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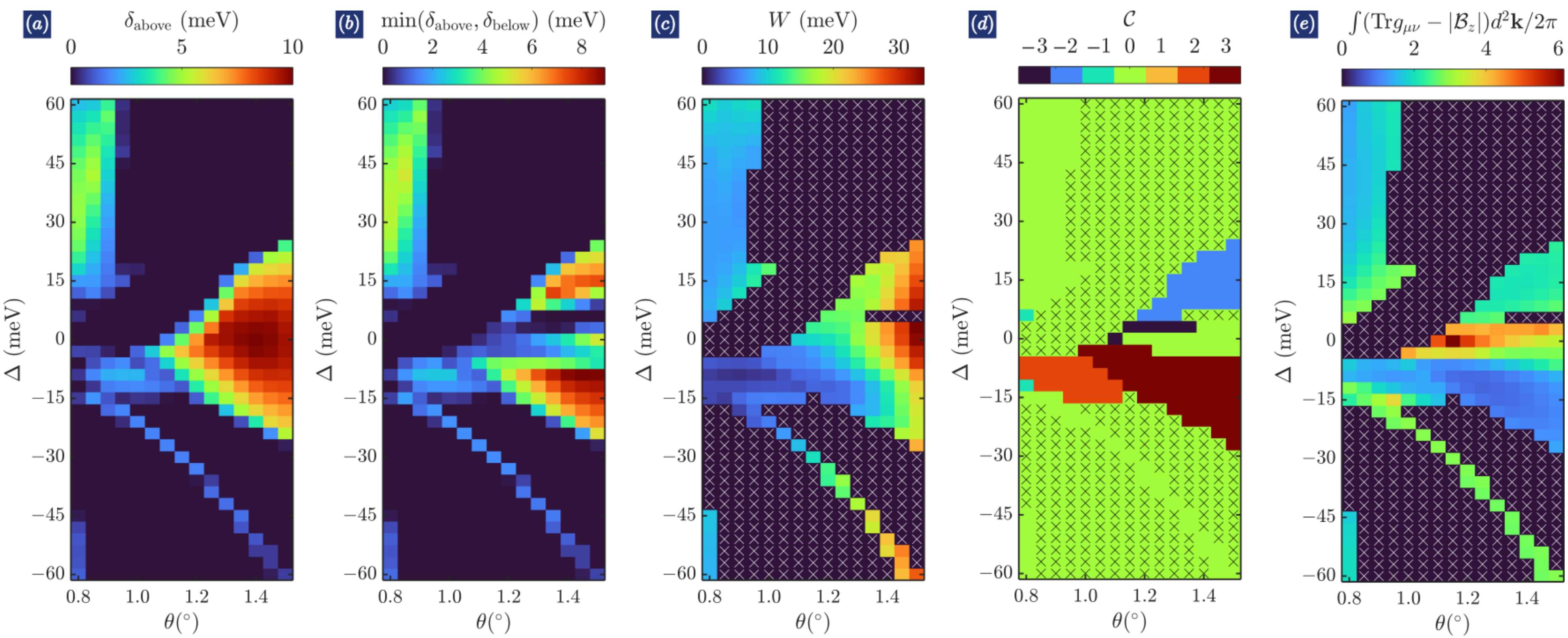


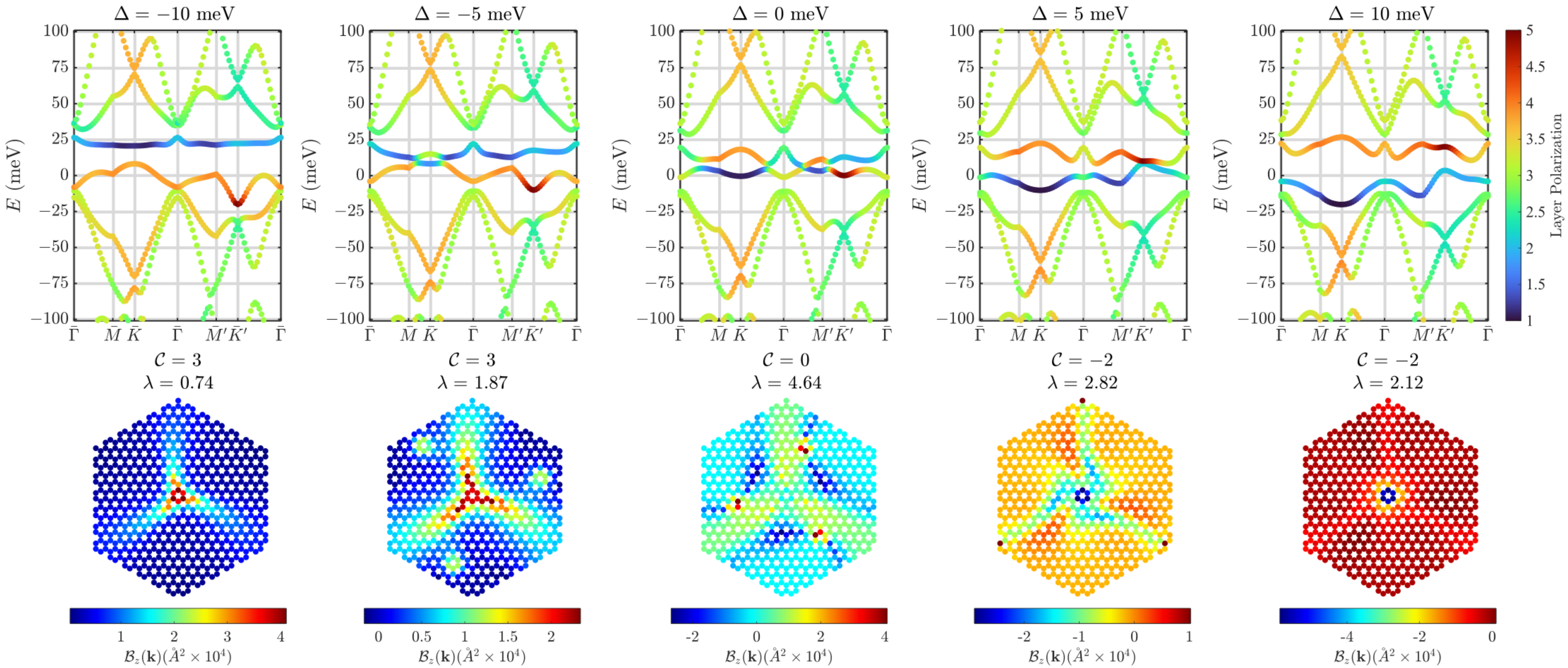
There exists a region in phase space which simultaneously has:

(1) large gaps, (2) small bandwidths, (3) Chern number = 3, and

(4) the trace condition is quite small ~ 0.74

Non-Interacting Phase Diagram

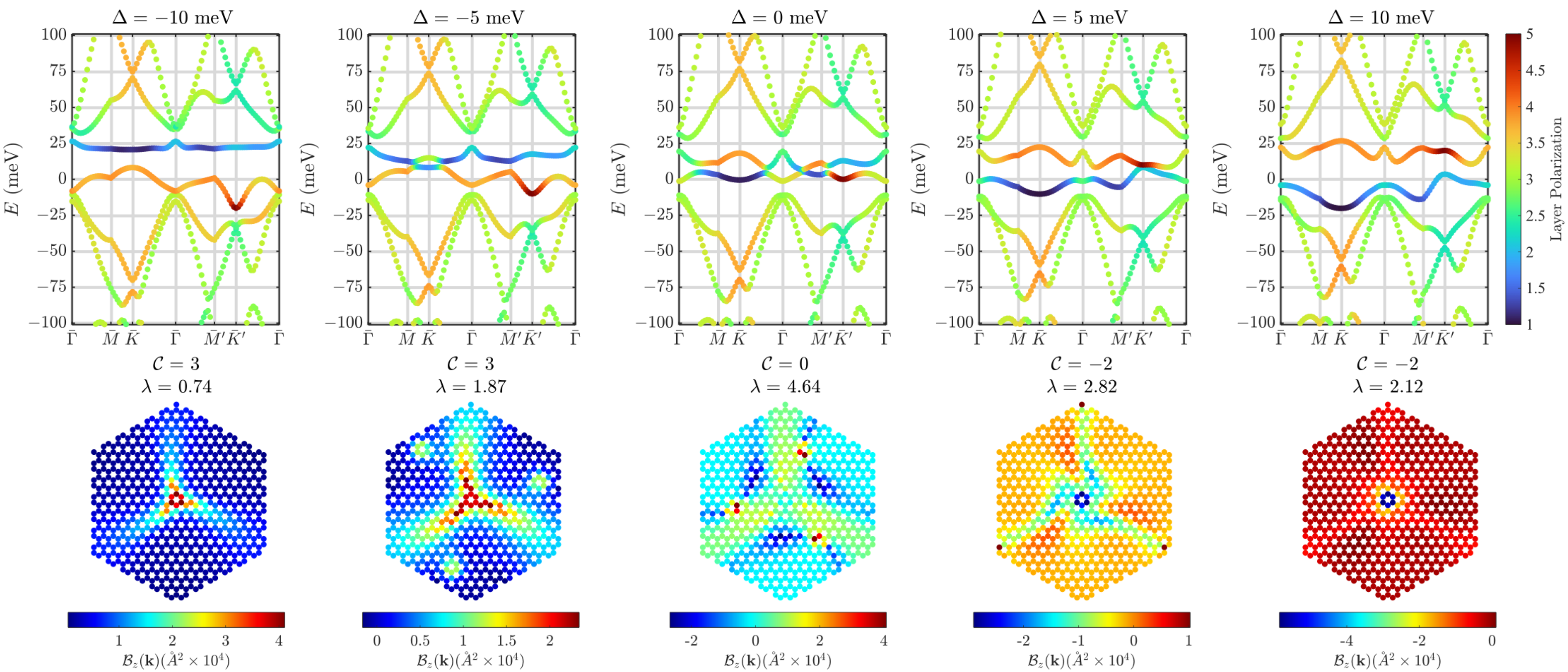
Parallel Stacking Order



- As Δ increases in the **negative** direction, a Chern-three band flattens. Charge density is localized on the trilayer substack.

Non-Interacting Phase Diagram

Parallel Stacking Order



- As Δ increases in the **negative** direction, a Chern-three band flattens. Charge density is localized on the trilayer substack.
- As Δ increases in the **positive** direction, a Chern-two band develops. Charge density is localized on the bilayer substack.

Self-Consistent Mean-Field Theory

$$\hat{\mathcal{H}} = \sum_{\substack{\mathbf{k}\in\mathrm{mBZ}\\ n,n',\xi,\xi'}} \hat{\Psi}^{\dagger}_{n,\xi,\mathbf{k}} \left[\mathbb{K}^{n,n'}_{\xi,\xi'}(\mathbf{k}) + \mathbb{H}^{n,n'}_{\xi,\xi'}(\mathbf{k}) + \mathbb{F}^{n,n'}_{\xi,\xi'}(\mathbf{k})\right] \hat{\Psi}_{n',\xi',\mathbf{k}},$$

$$\mathbb{K}^{n,n'}_{\xi,\xi'}(\mathbf{k}) = \delta_{\xi,\xi'}\delta_{n,n'}E_{n,\xi}(\mathbf{k}),$$

$$\mathbb{H}^{n,n'}_{\xi,\xi'}(\mathbf{k}) = +\delta_{\xi,\xi'} \sum_{\substack{\mathbf{p}\in\mathrm{mBZ}\\ n_1,n_3,\xi_1}} \mathbb{V}^{n_1,n,n_3,n'}_{\xi_1,\xi}(\mathbf{p},\mathbf{k},\mathbf{0})\mathbb{D}^{n_1,n_3}_{\xi_1,\xi_1}(\mathbf{p})$$

$$\mathbb{F}^{n,n'}_{\xi,\xi'}(\mathbf{k}) = -\sum_{\substack{\mathbf{p}\in\mathrm{mBZ}\\ n_1,n_4}} \mathbb{V}^{n_1,n,n',n_4}_{\xi',\xi}(\mathbf{k},\mathbf{p},\mathbf{p}-\mathbf{k})\mathbb{D}^{n_1,n_4}_{\xi',\xi}(\mathbf{p}).$$

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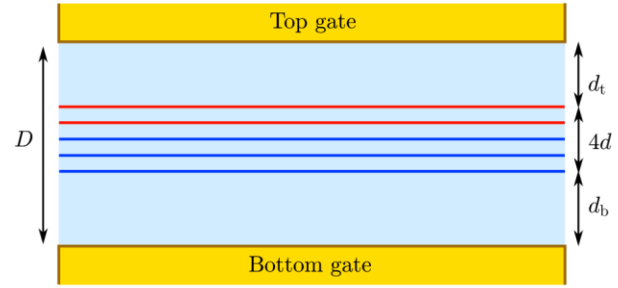
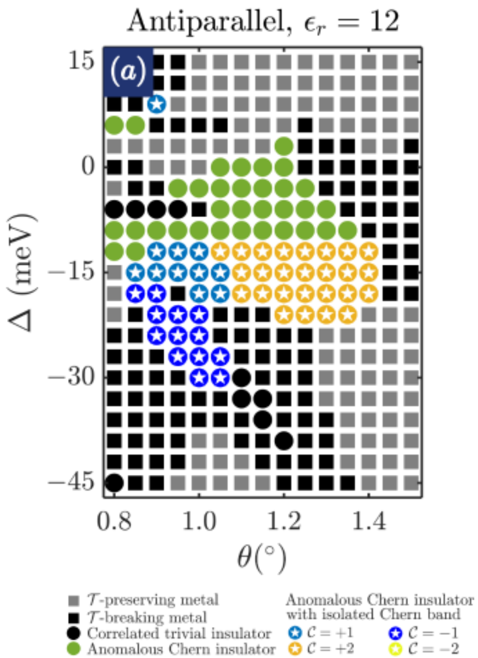
$$\mathcal{V}(\mathbf{k},z,z') = \frac{e^2\,\mathrm{csch}(kD)}{2\epsilon_0\epsilon_r k}\left[\cosh(k\left[D-|z-z'|\right]) - \cosh(k\left[D-|z+z'|\right])\right]$$

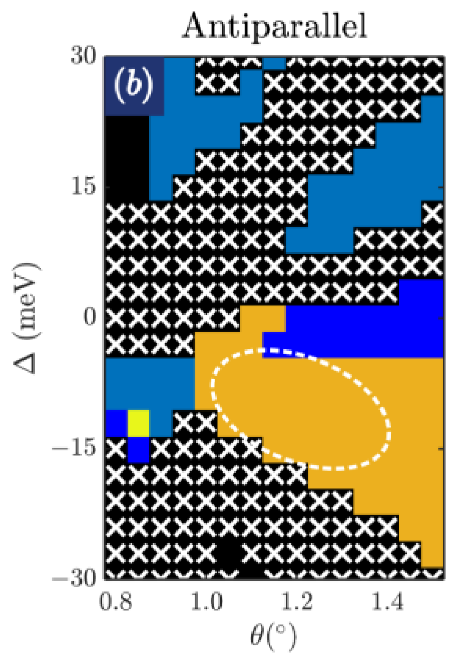
- Use layer-dependent Coulomb potential to account for uneven charge distribution
- Search for T-breaking isospin-polarized ground states



Hartree-Fock Results

Antiparallel Stacking Order





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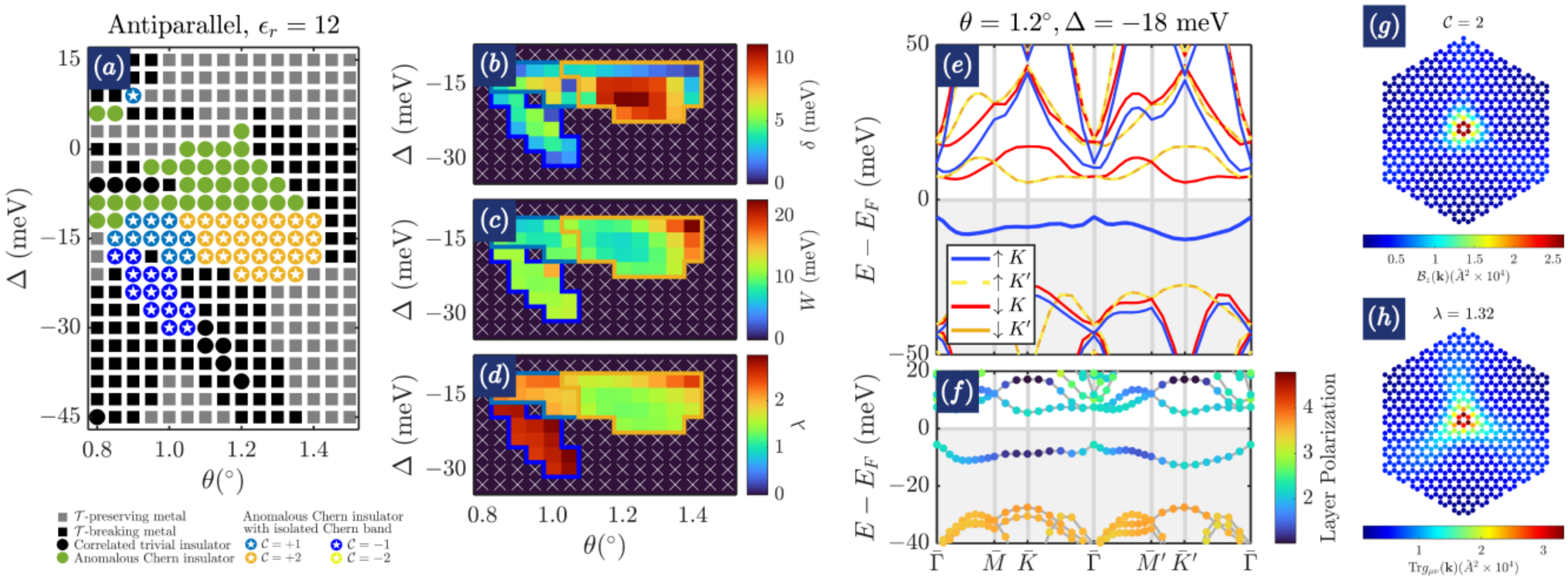

- The Chern-two phase is present even when interactions are included at the mean-field level
- Bands remain relatively narrow and well-isolated

Hartree-Fock Results

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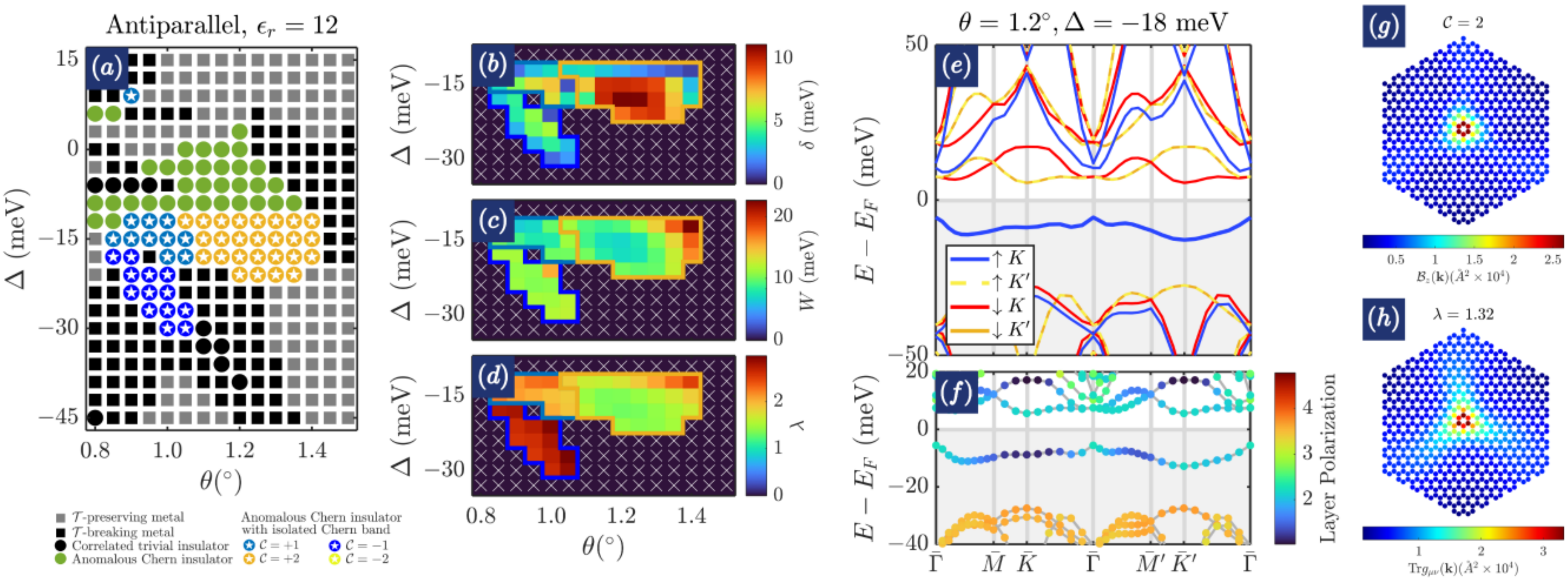

- The Chern-two phase is present even when interactions are included at the mean-field level
- Bands remain relatively narrow and well-isolated
- The trace violation is reduced by interactions: 1.7 → 1.4

Hartree-Fock Results

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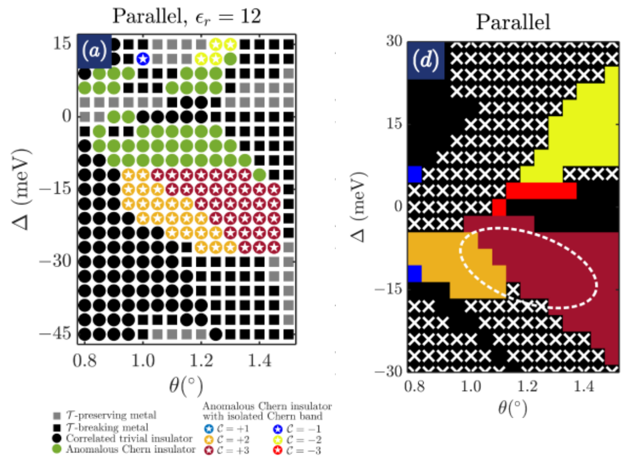
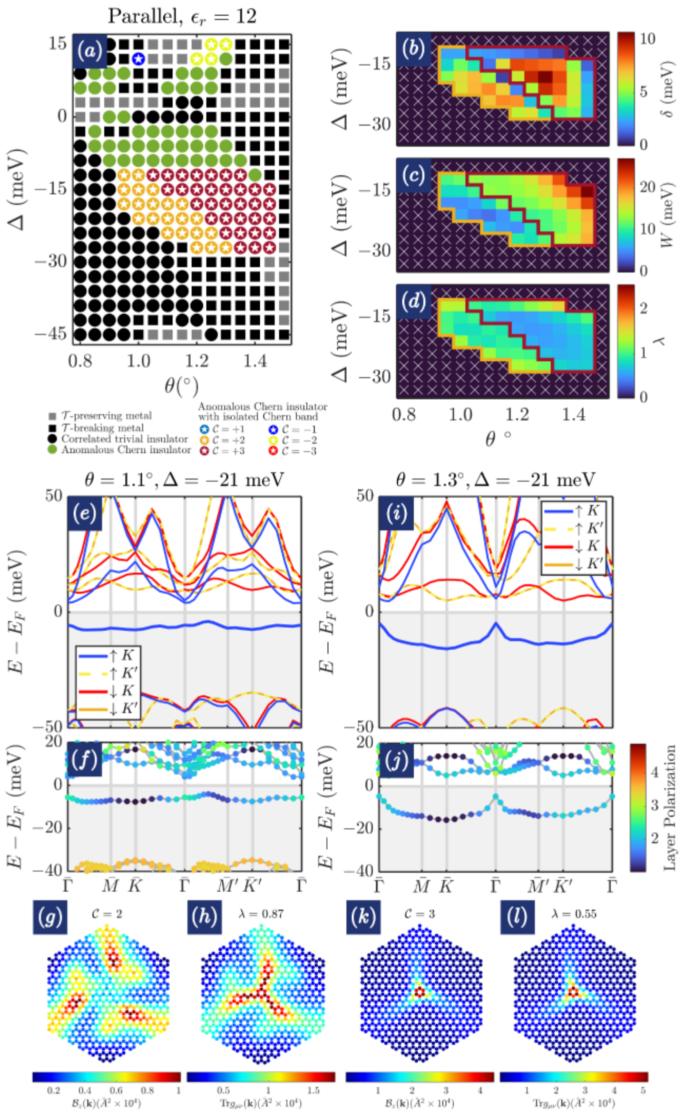


Hartree-Fock Results

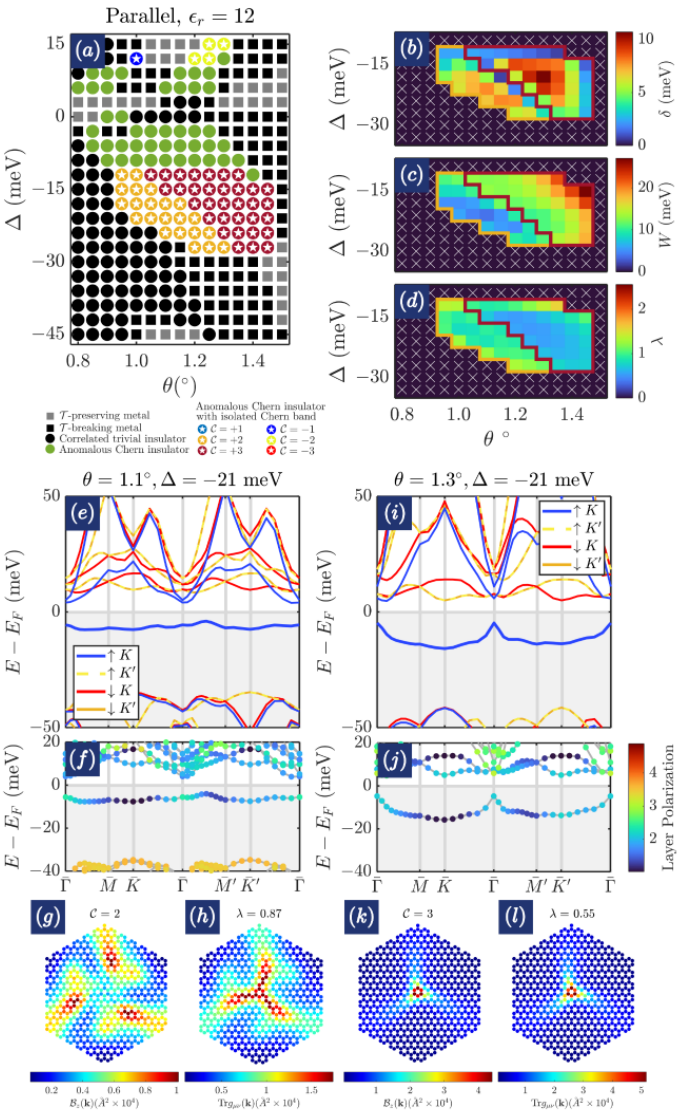
Parallel Stacking Order



- The Chern-three phase is intertwined with the Chern-two phase when interactions are included at the mean-field level

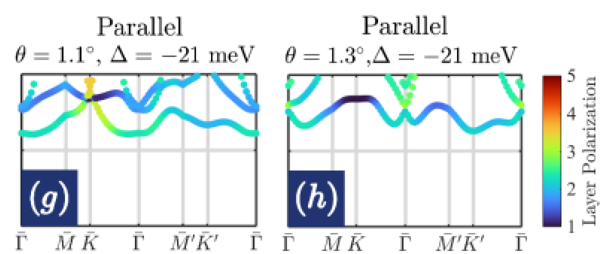
Hartree-Fock Results

Parallel Stacking Order



- The Chern-three phase is intertwined with the Chern-two phase when interactions are included at the mean-field level

- The Chern-two phase exists in a parameter regime where the non-interacting bands are not even isolated or topological



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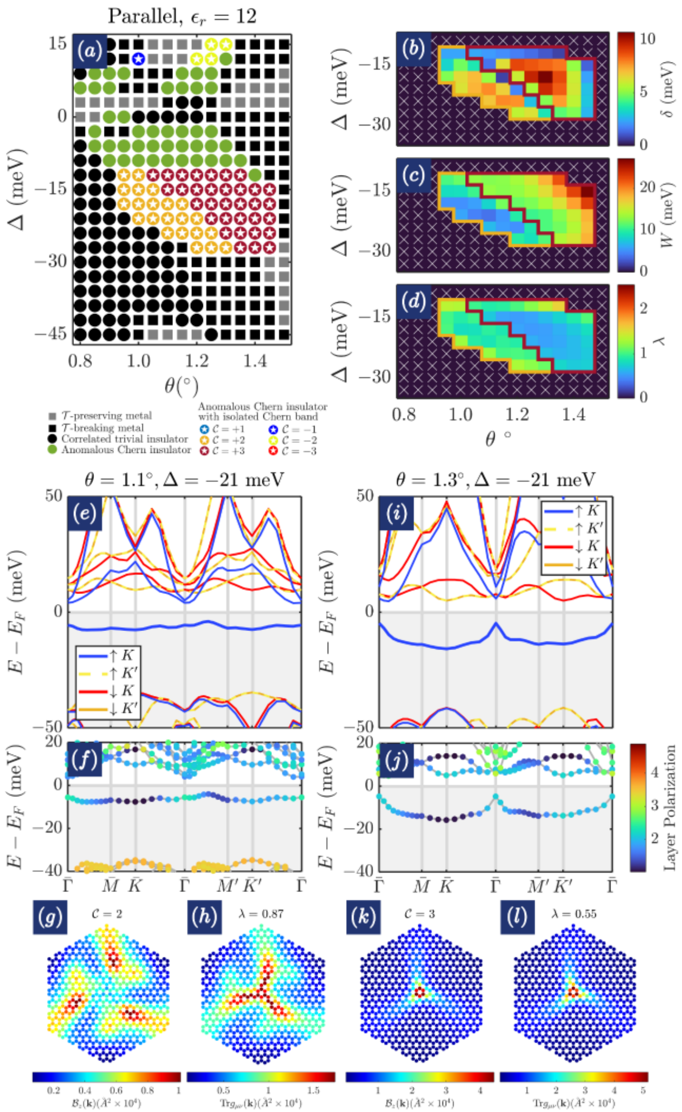


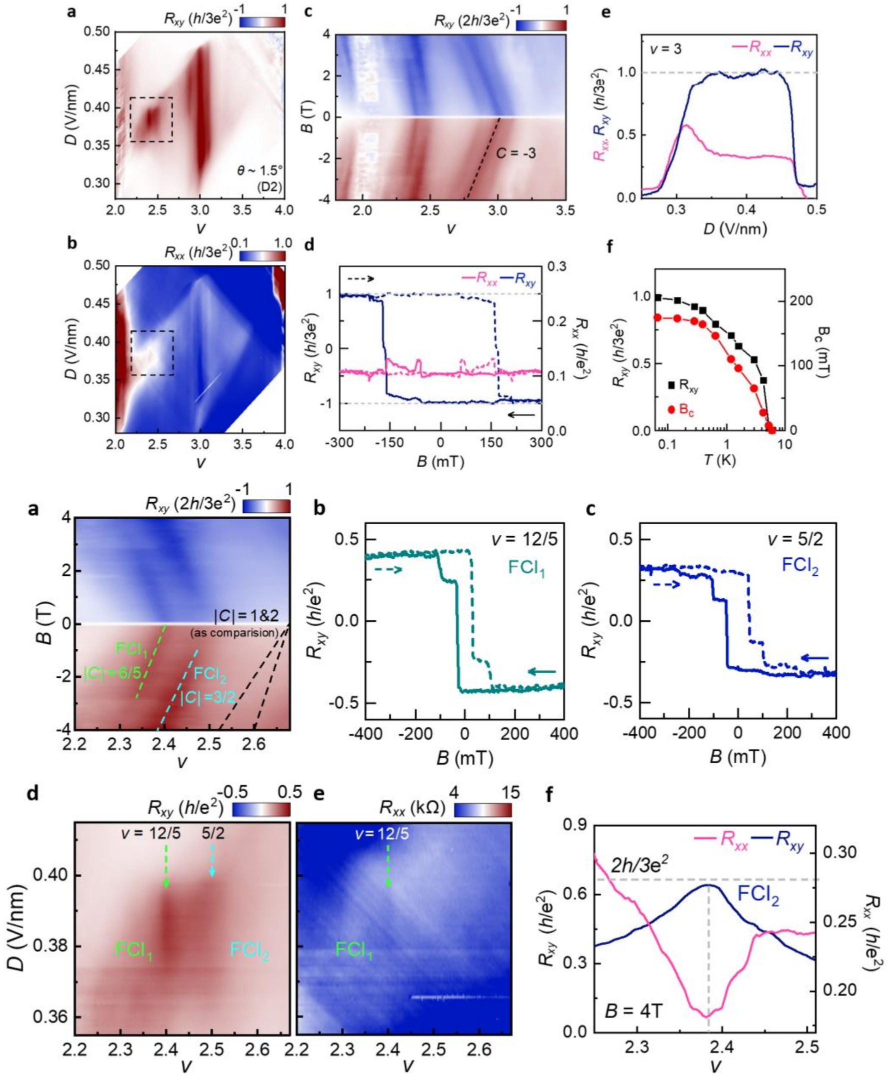
- The Chern-three phase is intertwined with the Chern-two phase when interactions are included at the mean-field level
- The Chern-two phase exists in a parameter regime where the non-interacting bands are not even isolated or topological
- The trace violation is reduced by interactions: 0.74 → 0.5

Trace condition for pentalayer: 0.6-1.45

Trace condition for MoTe2: 0.1-0.7

Experimental Confirmation

Dong, Jingwei, et al. "Observation of Integer and Fractional Chern insulators in high Chern number flatbands." *arXiv preprint arXiv:2507.09908* (2025).

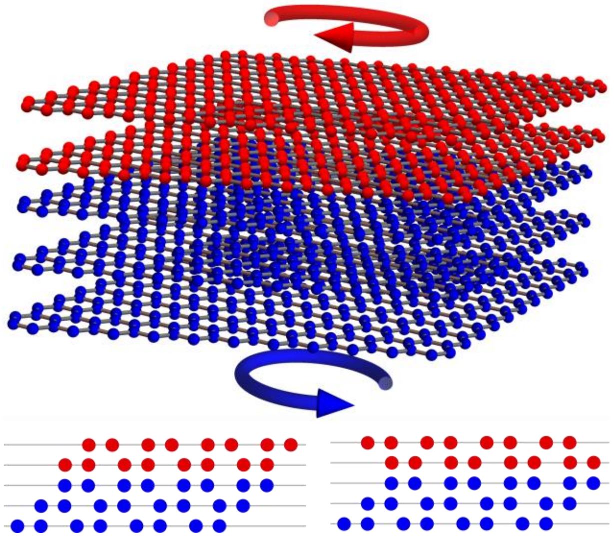


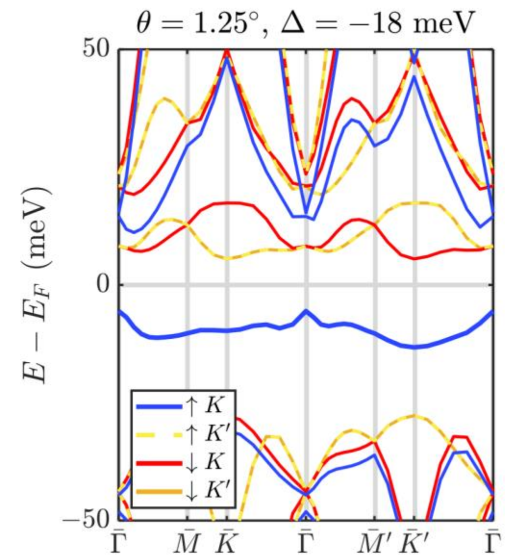
Transport experiment on twisted 3+2 rhombohedral graphene:

- Experimental evidence using Hall conductivity and Streda's slope
- Quantized C = 3 Chern insulator
- Fractional Chern insulators at $\nu \sim 12/5$ and $\nu \sim 5/2$
- Anomalous Hall crystals near $\nu \sim 1$

Conclusions

➢ Twisted $M + N$ multilayer graphene features narrow topological bands with $C > 1$.





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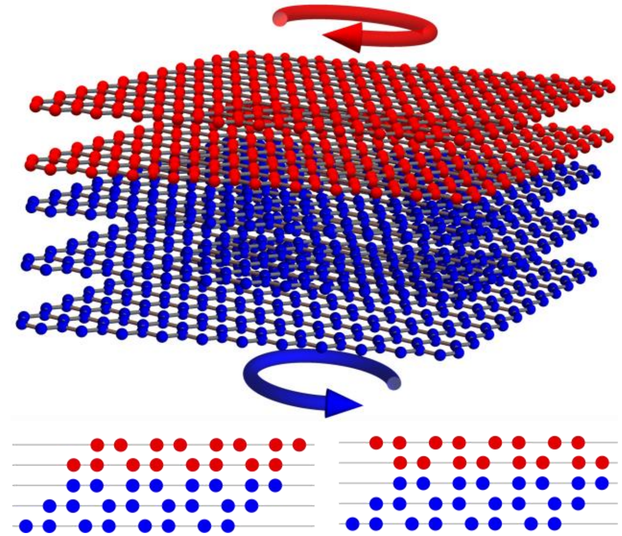

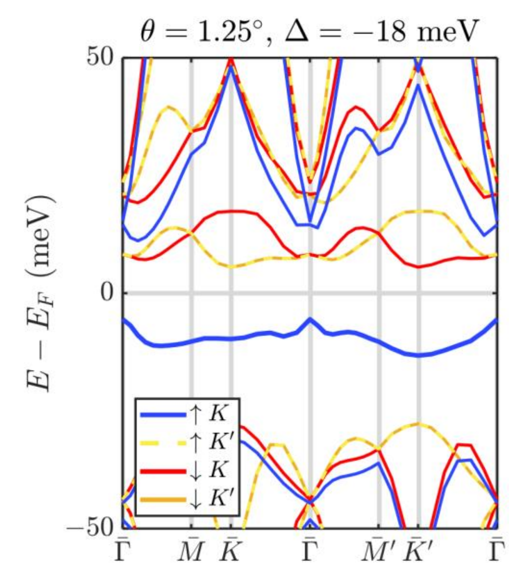

Conclusions

- Twisted $M + N$ multilayer graphene features narrow topological bands with $C > 1$.
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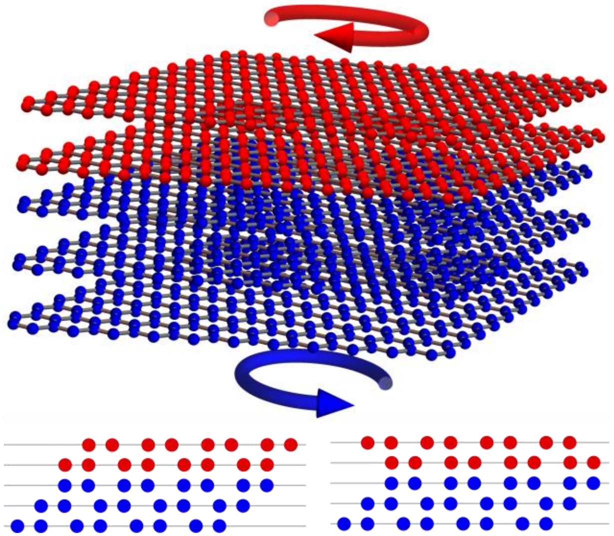

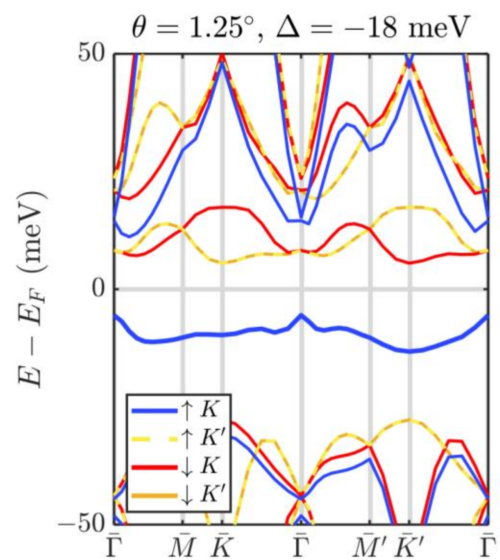

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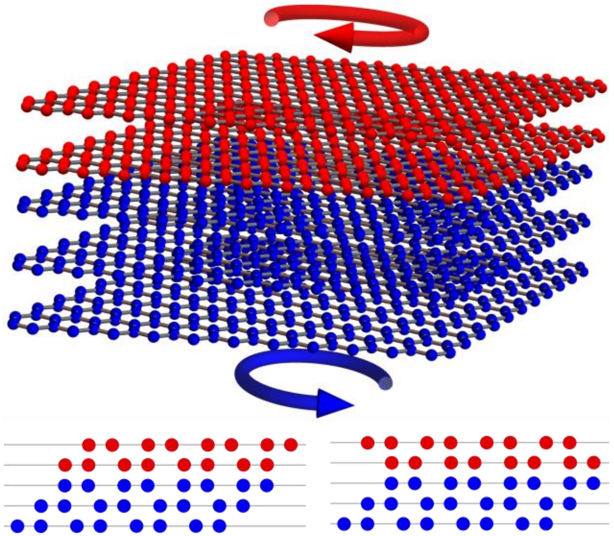
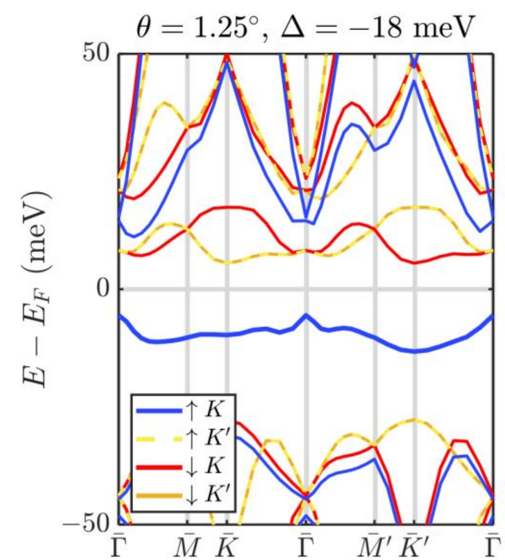
- ➢ The trace condition suggests that these bands are favorable for fractionalized phases.





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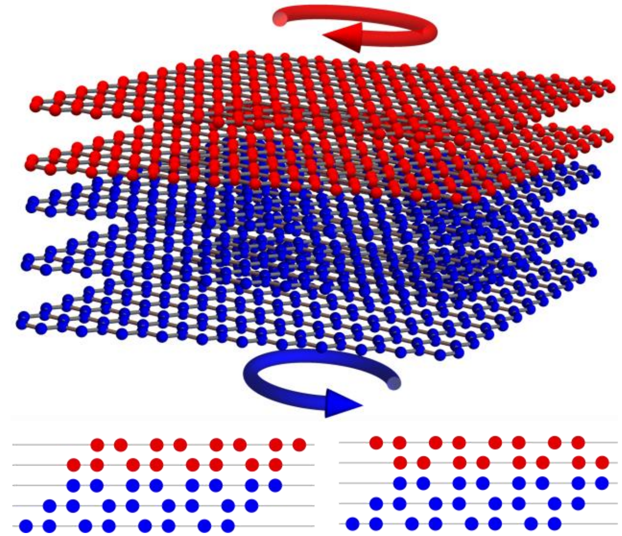
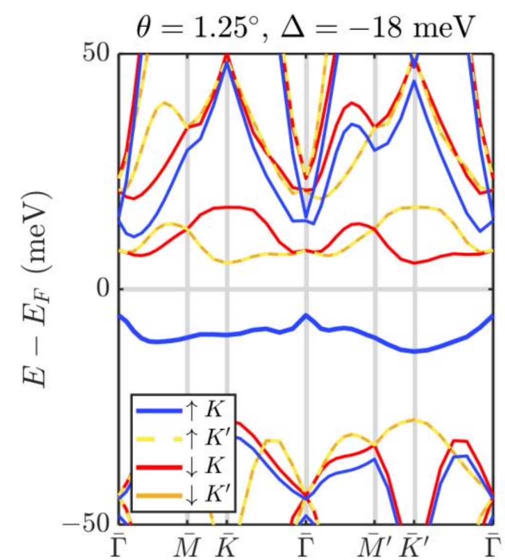
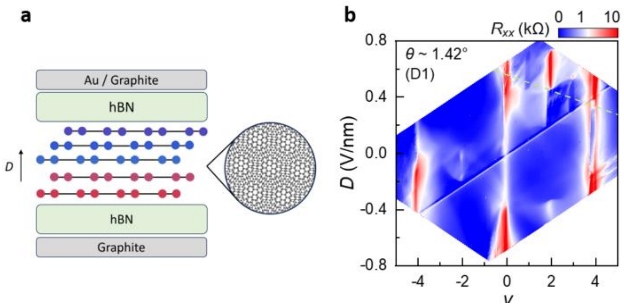




Thank you!

Topological Chern Insulators in D1 and D3



- D1 at $\theta = 1.42°$ at $T = 1$ K

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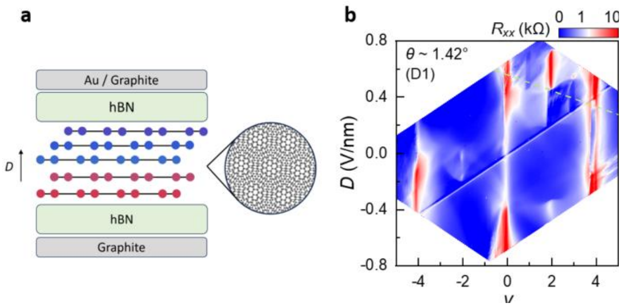
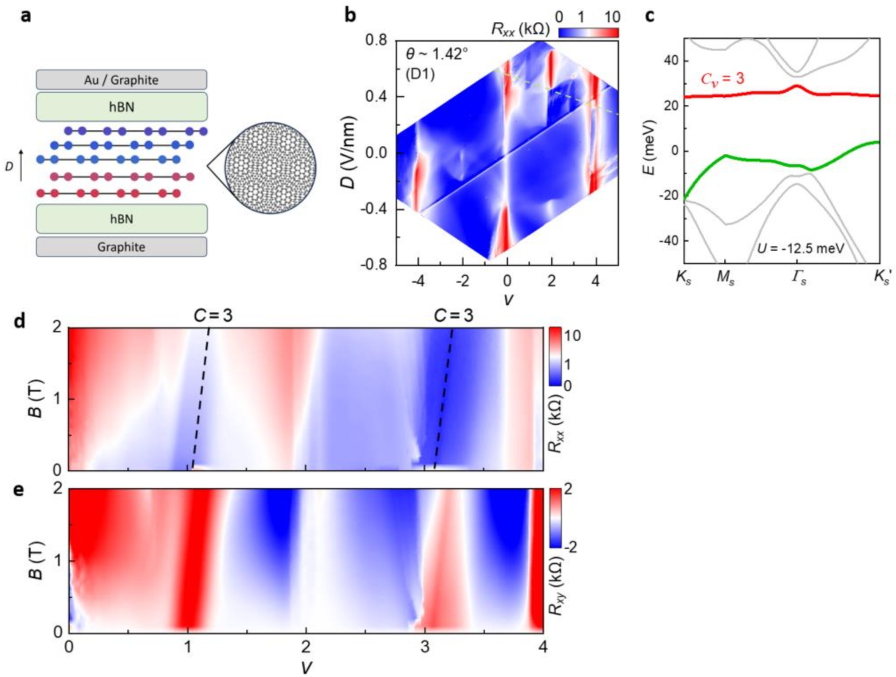


Positive D pushes electrons to the trilayer side

- D1 at $\theta = 1.42°$ at $T = 1$ K
- Correlated insulators for ν = 1, 2, and 3 at positive D

Topological Chern Insulators in D1 and D3



- D1 at $\theta = 1.42°$ at $T = 1$ K
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- $\nu = 1$ with $D = 0.517$ V/nm according to Streda's formula but no quantized Hall conductivity.
- $\nu = 3$ with $D = 0.376$ V/nm according to Streda's formula but no quantized Hall conductivity.

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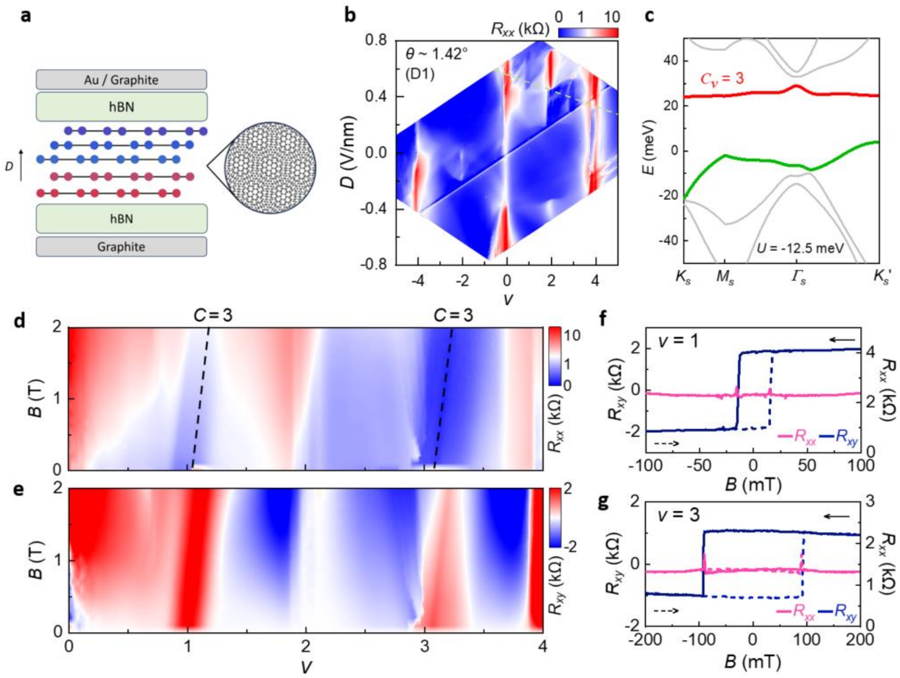

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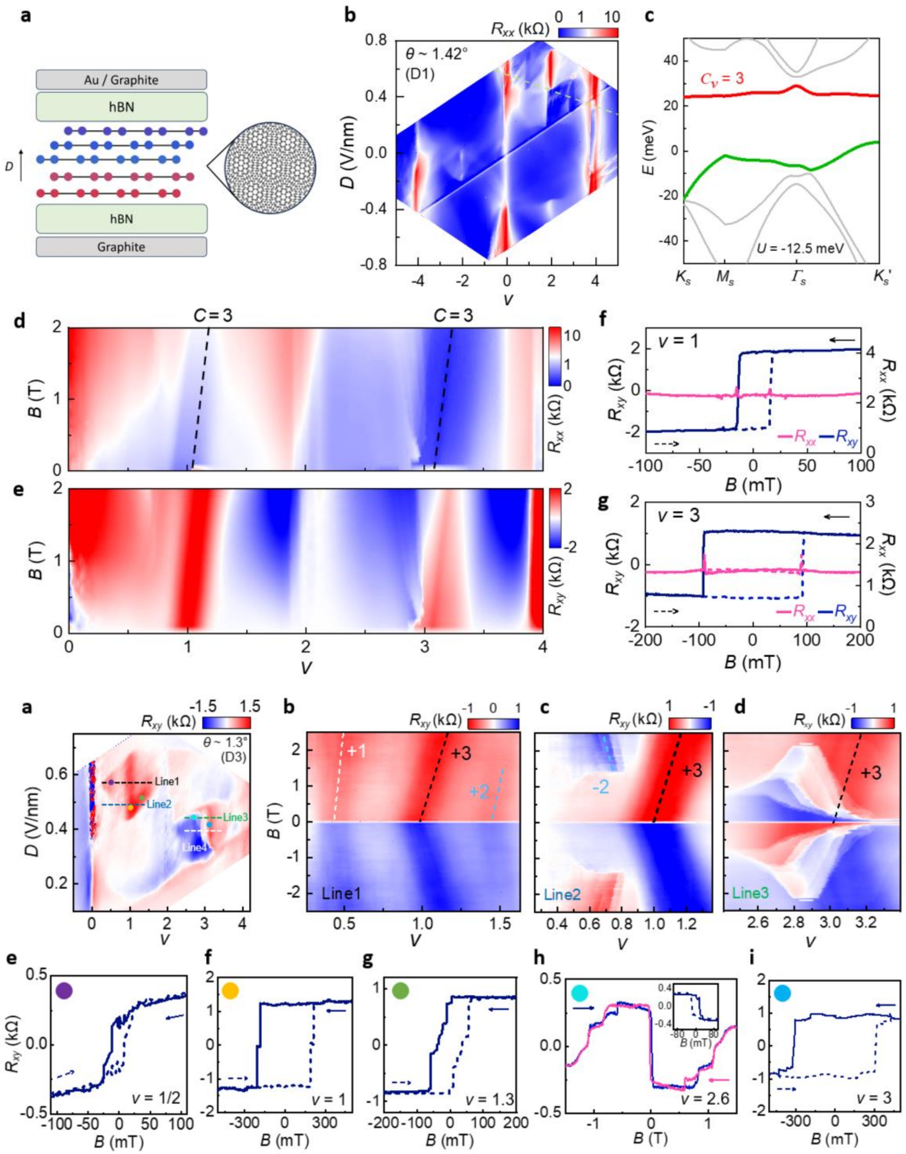

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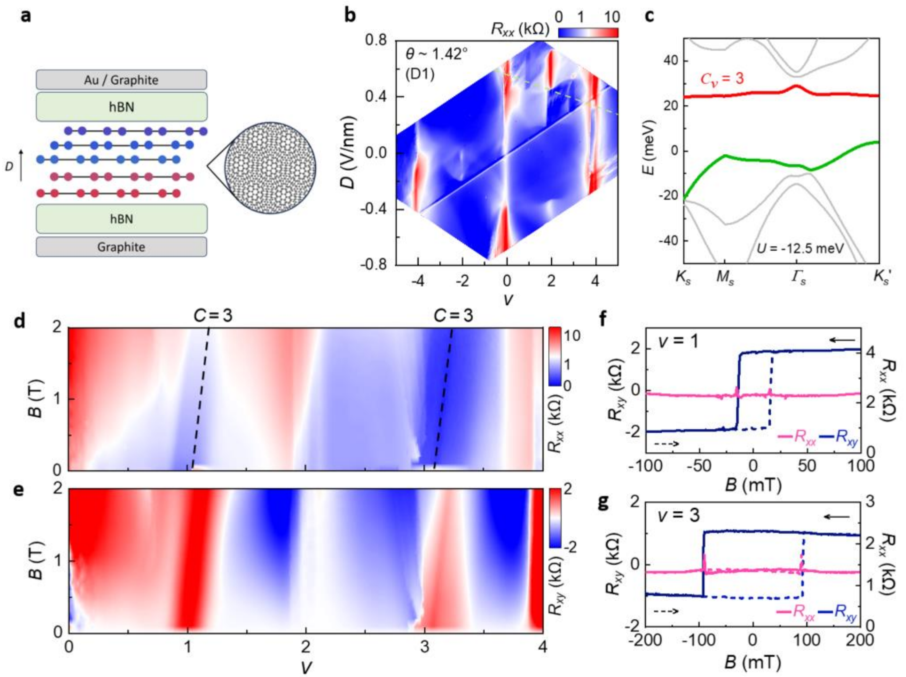
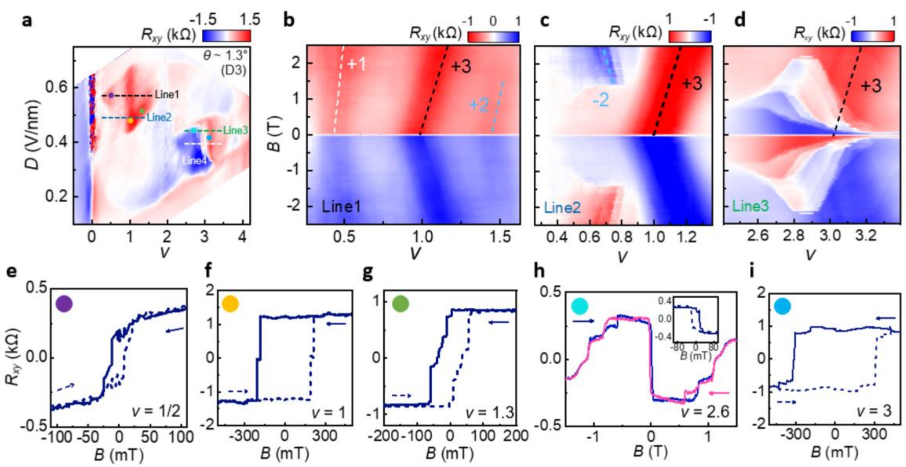
- D3 at $\theta = 1.30°$ at $B = 0.5$ T and $T = 1.1$ K

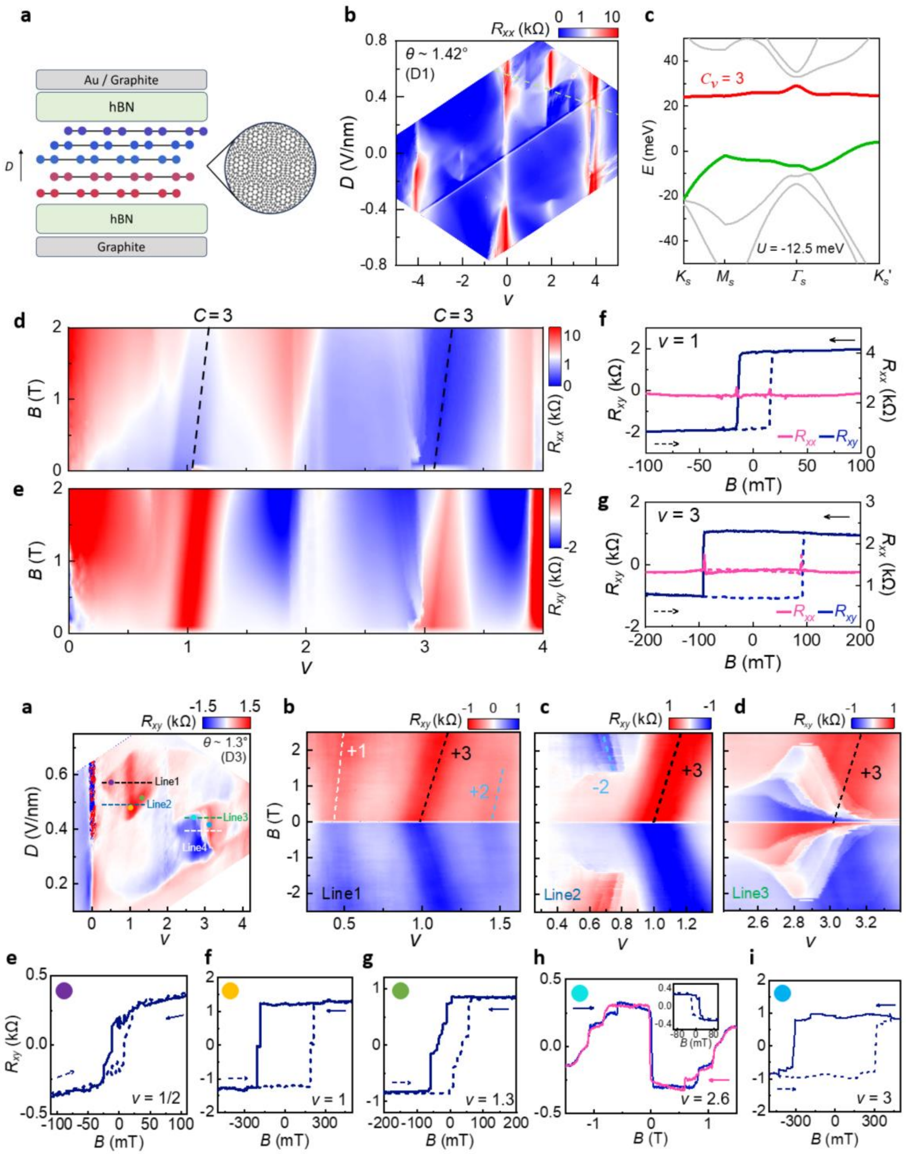
Topological Chern Insulators in D1 and D3





Why no quantization of Hall conductivity?

Topological Chern Insulators in D1 and D3



Why no quantization of Hall conductivity?

- Degradation due to cooling down? Second cooling down has worse quantization

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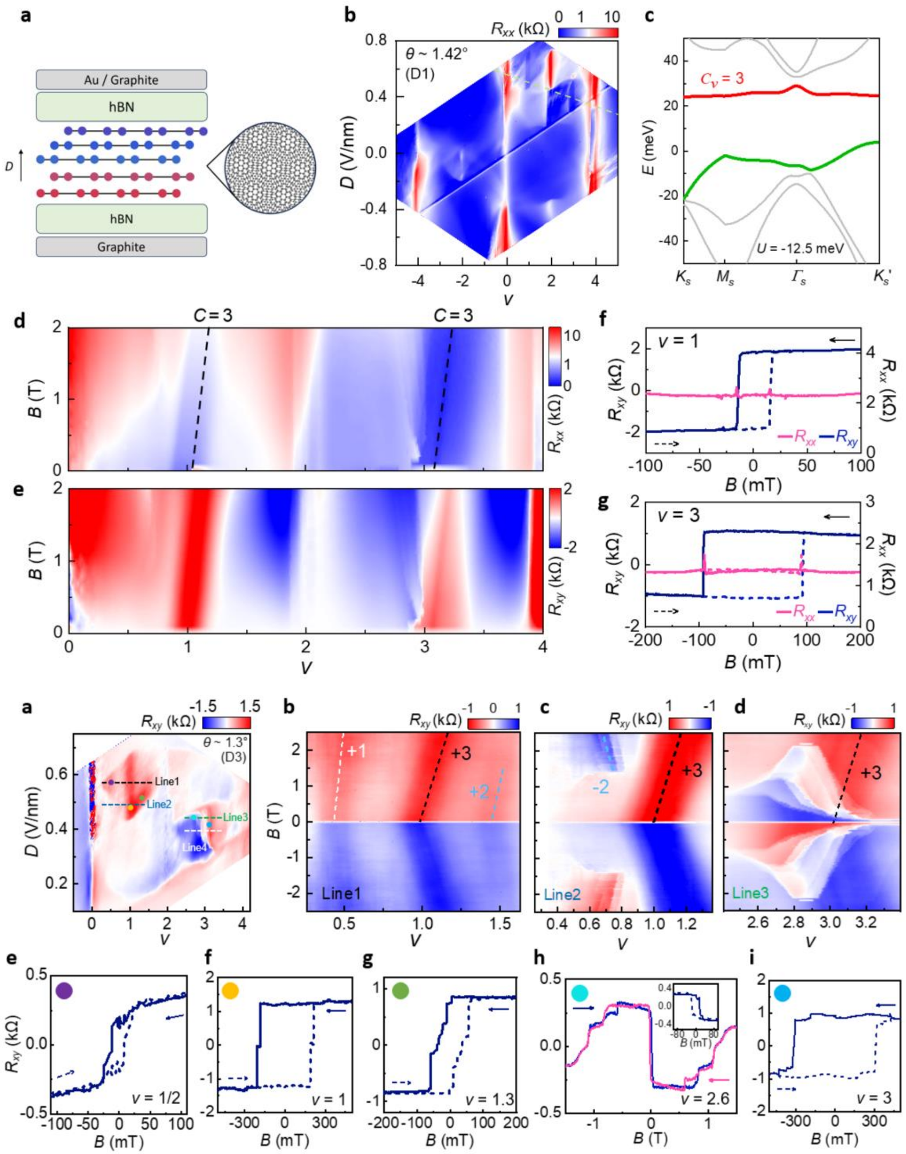


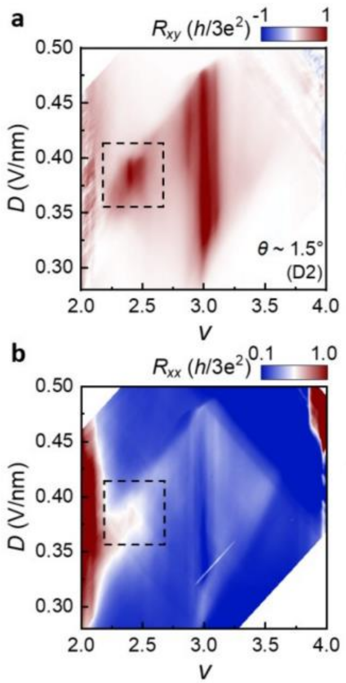
Why no quantization of Hall conductivity?

- Degradation due to cooling down? Second cooling down has worse quantization
- Thermal absorption due to evaporated metal particles? Changing gates can help

Quantized $C = 3$ Chern Insulator in D2

- D2 contains thin graphite gates at $\theta = 1.50°$

Quantized $C = 3$ Chern Insulator in D2



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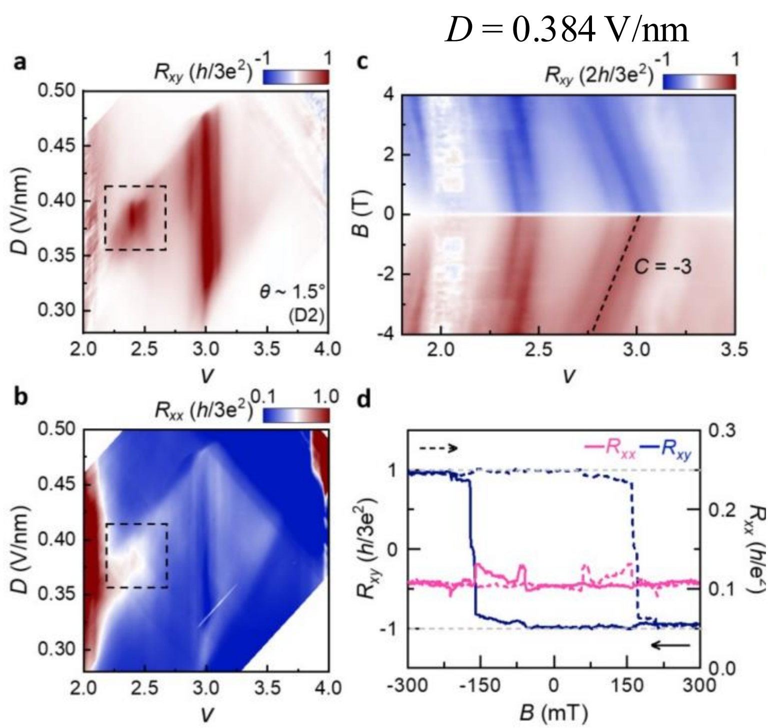

- D2 contains thin graphite gates at $\theta = 1.50°$
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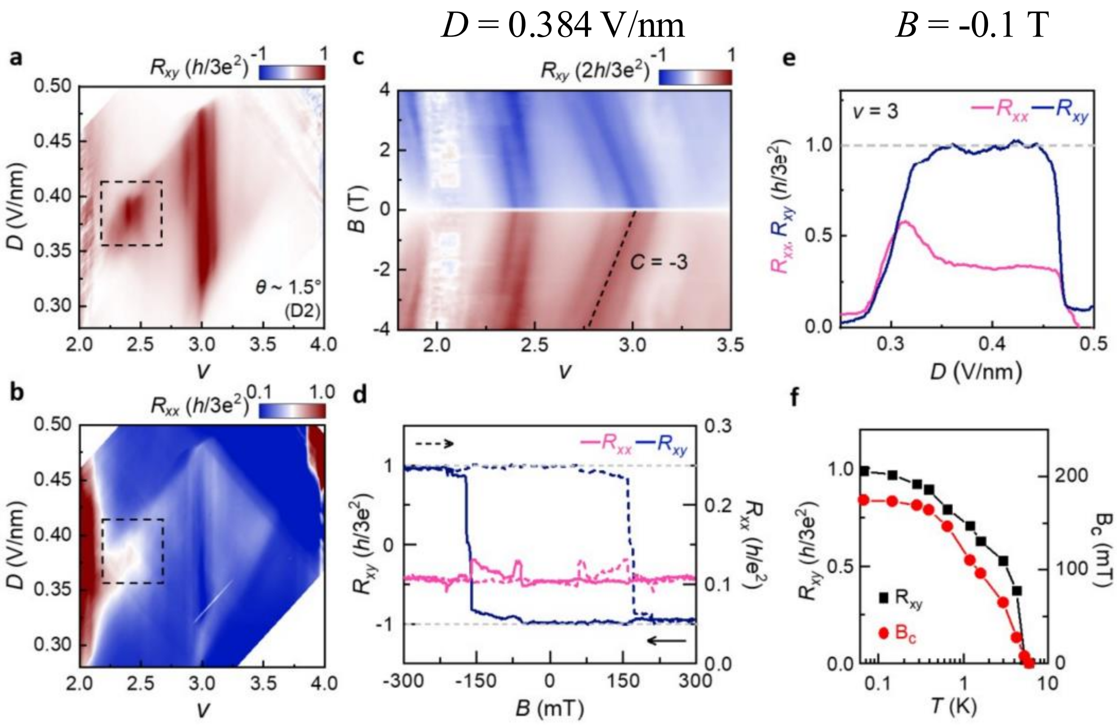

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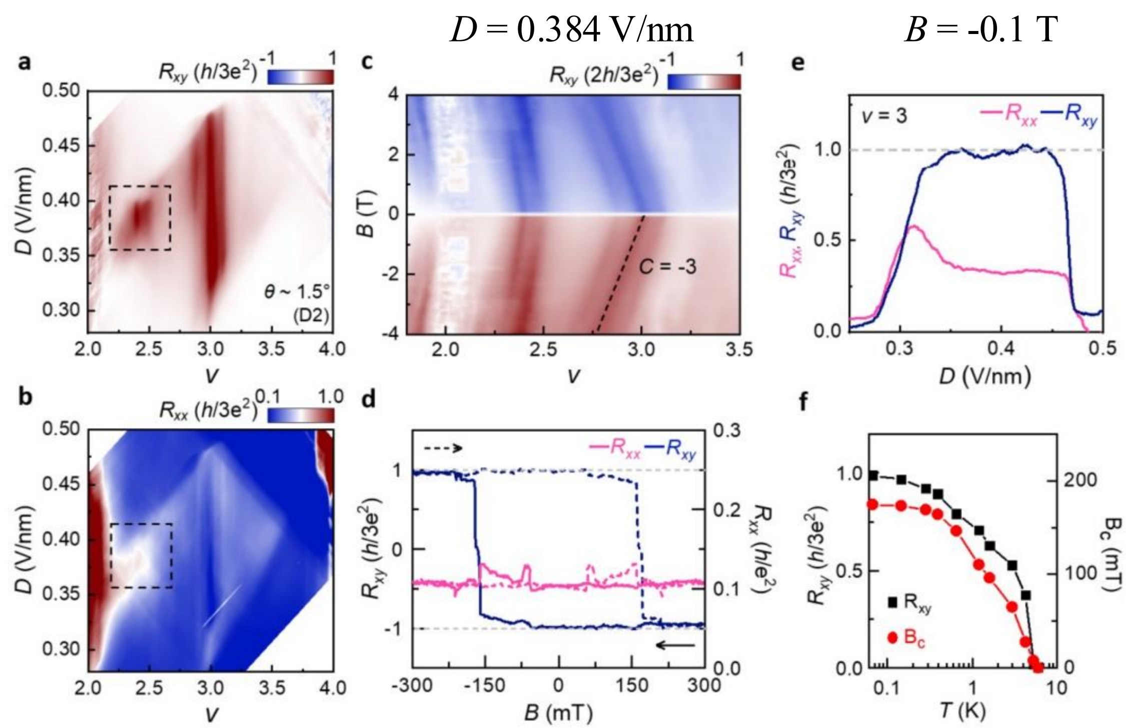
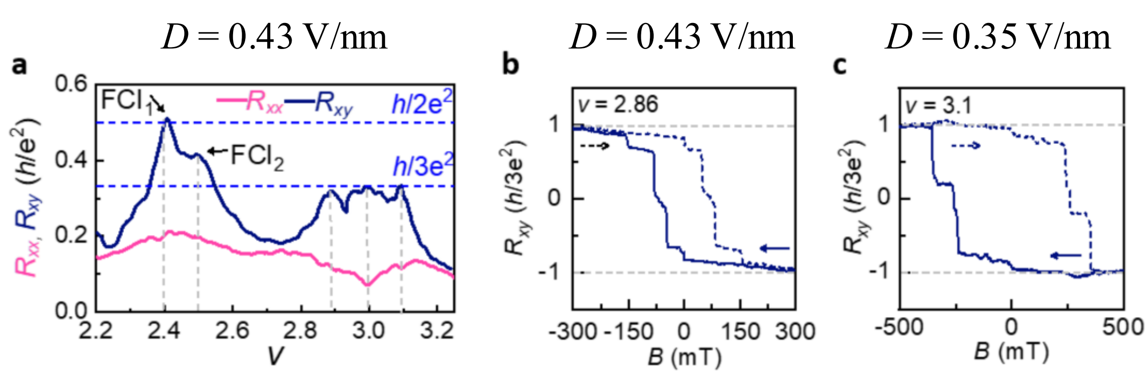
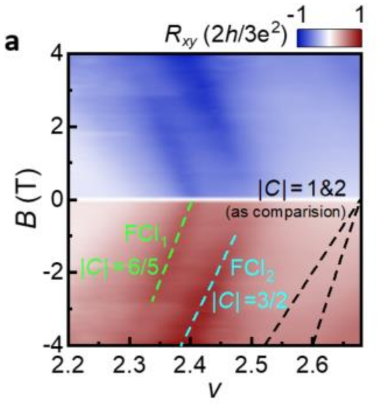




- D2 contains thin graphite gates at $\theta = 1.50°$
- Quantized Hall conductivity with accuracy > 98.2%
- Topological states at incommensurate fillings $\nu = 2.86$ and 3.1? Anomalous Hall crystal?

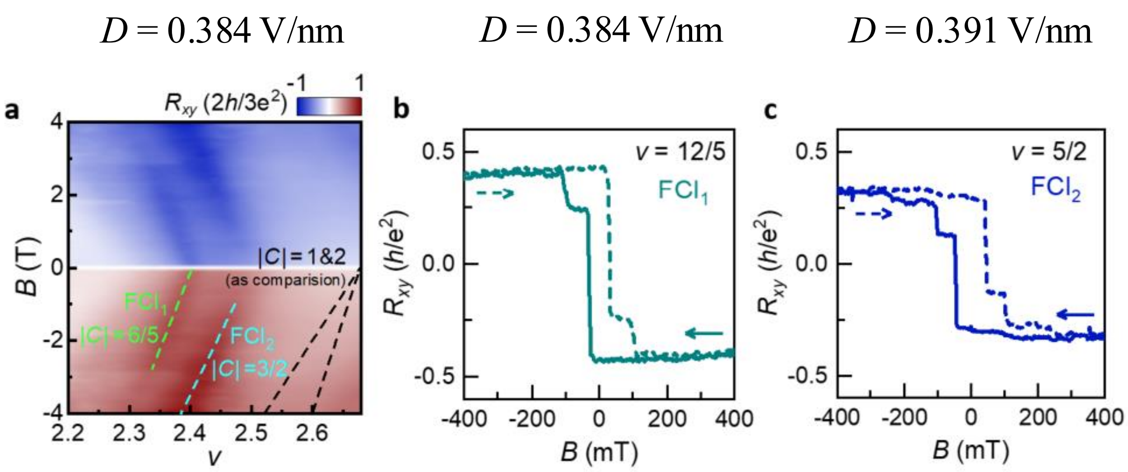
Fractional Chern Insulators in D2

$D = 0.384$ V/nm



- FC1
  - $\nu \sim 12/5$ ($\nu^* \sim 2/5$)
  - $C = 6/5$ odd denominator
- FC2
  - $\nu \sim 5/2$ ($\nu^* \sim 1/2$)
  - $C = 3/2$ even denominator

Fractional Chern Insulators in D2



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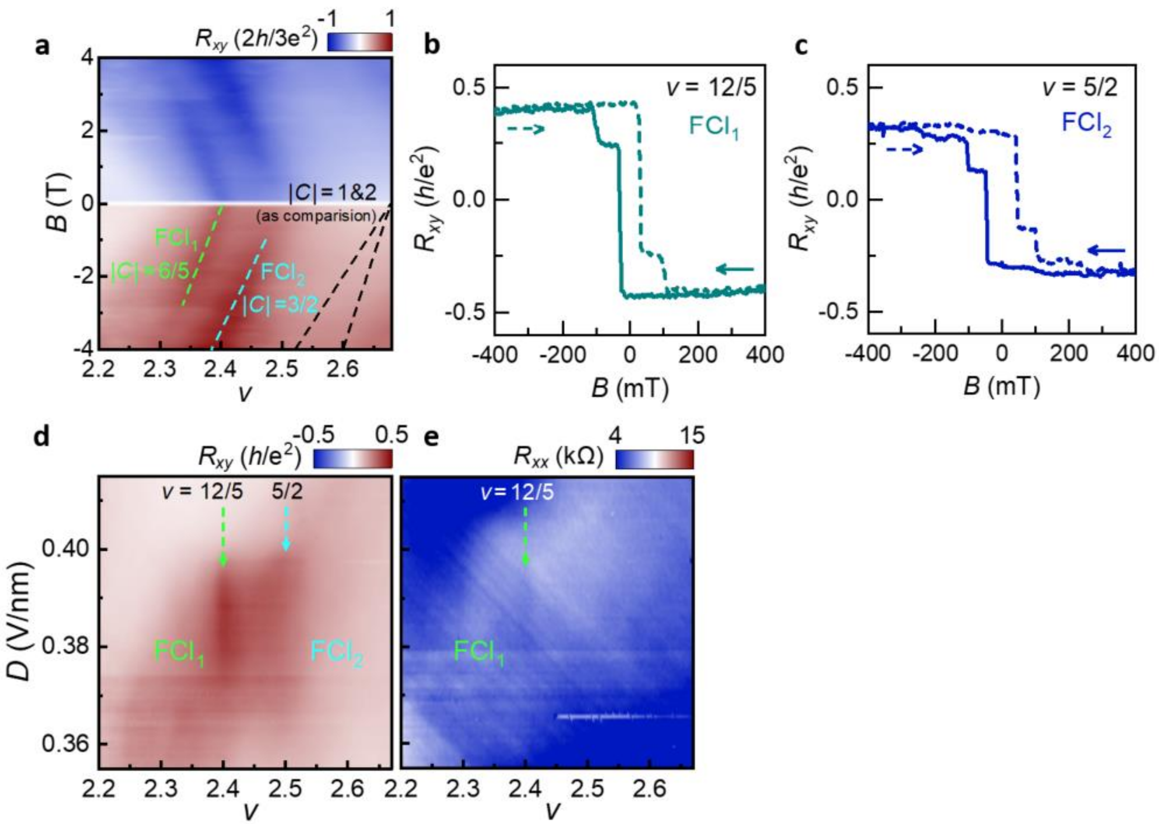
Fractional Chern Insulators in D2

$D = 0.384$ V/nm $D = 0.384$ V/nm $D = 0.391$ V/nm



$B = \pm 0.1$ T

- FC1
  - $v \sim 12/5$ ($v^* \sim 2/5$)
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  - Features resistance minimum at zero field
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Fractional Chern Insulators in D2

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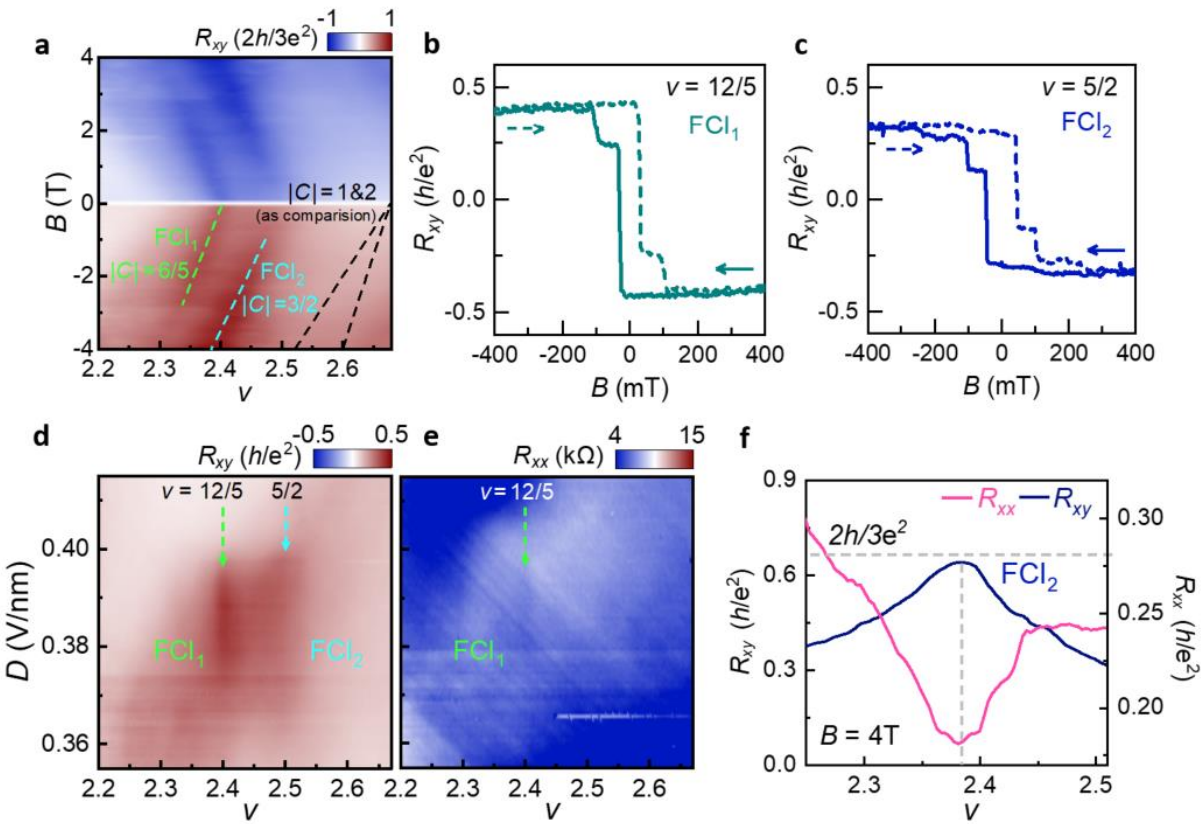

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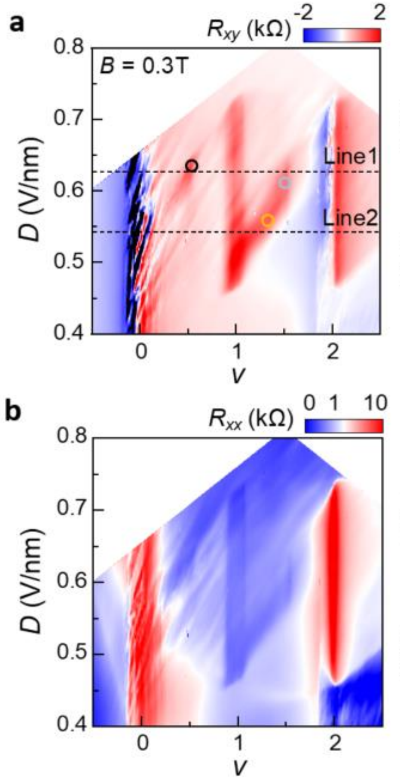
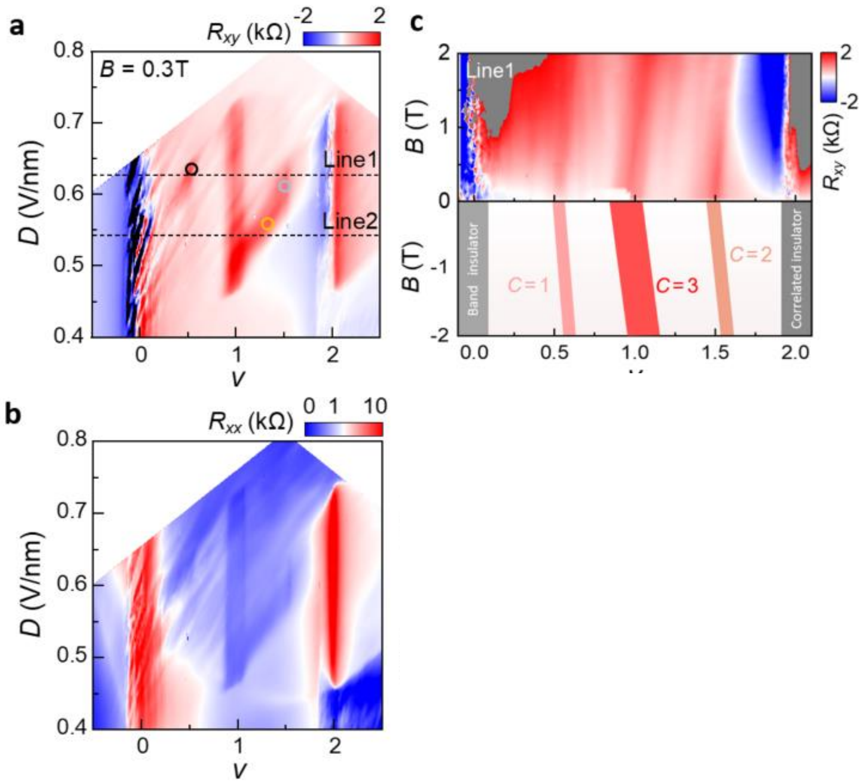
  - $v \sim 5/2$ ($v^* \sim 1/2$)
  - $C = 3/2$ even denominator
  - Can be stablized by a magnetic field of 4 T

Anomalous Hall Crystals in D1



- Integer Chern insulators at fractional fillings

Anomalous Hall Crystals in D1



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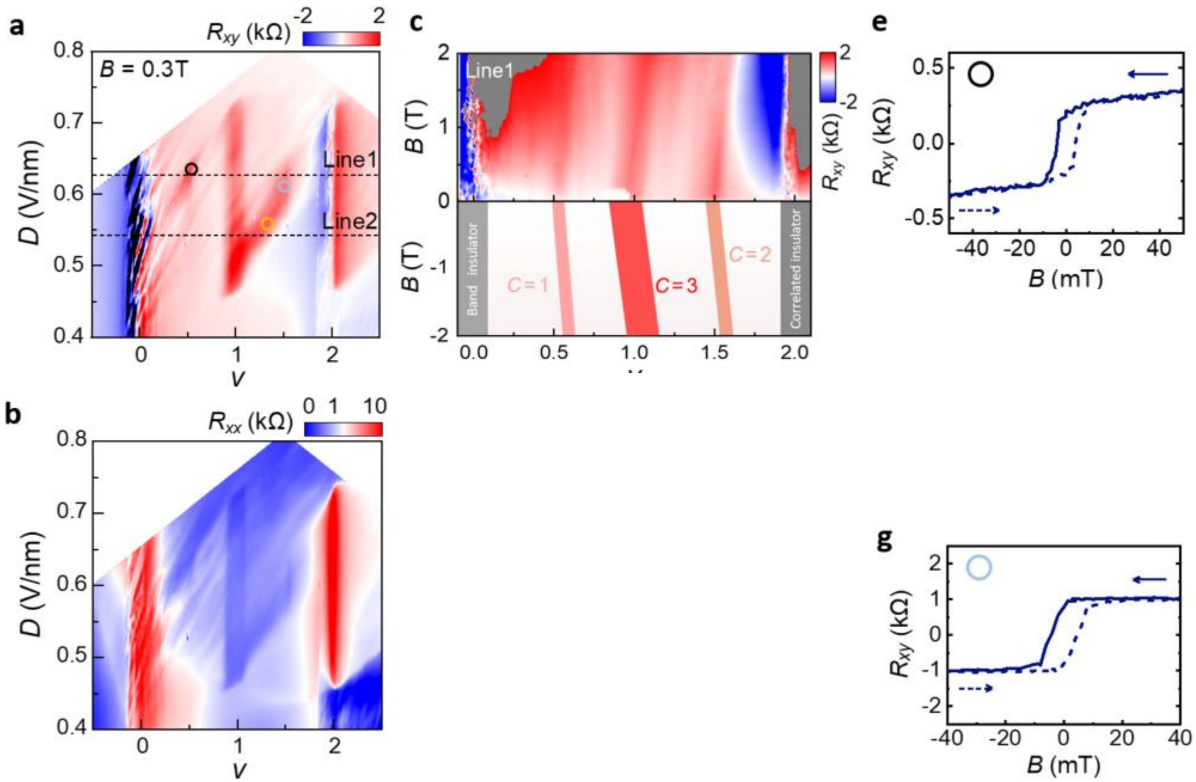

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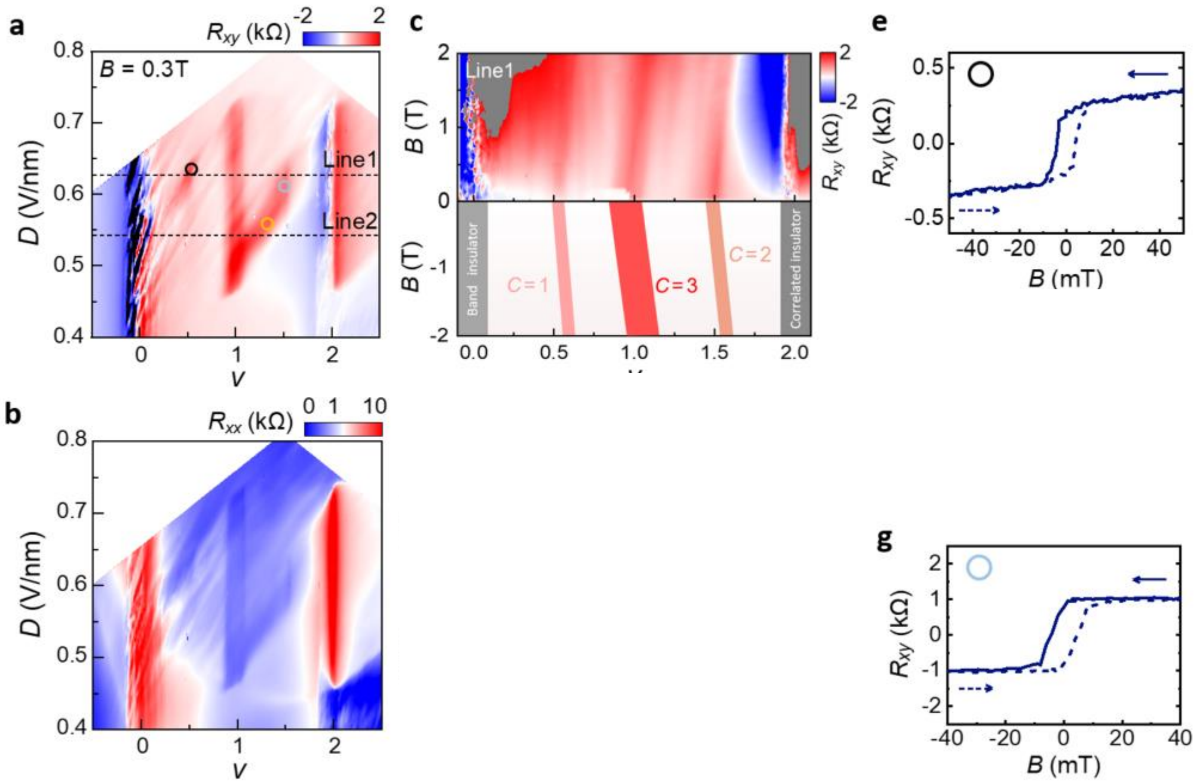

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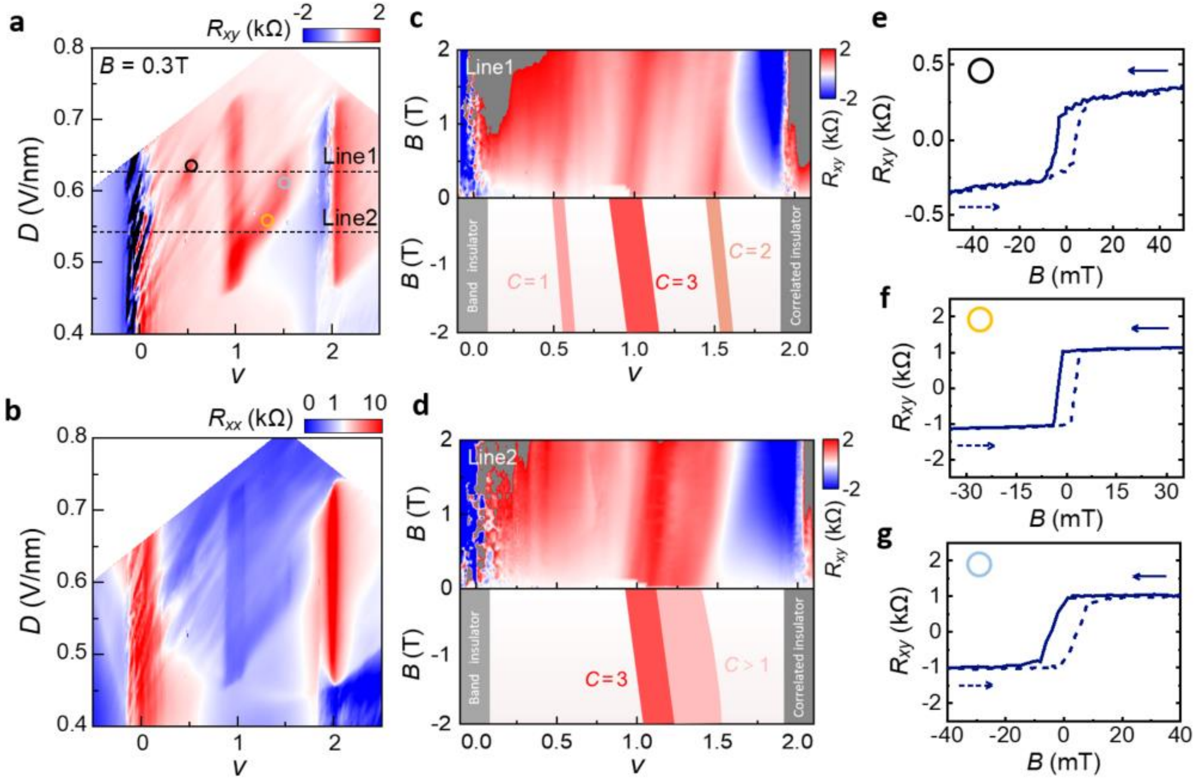

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