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ICISE at Quy Nhon in Vietnam

arXiv: 2311.01199 (PRR2024)

arXiv: 2408.01706 (PRR2025)

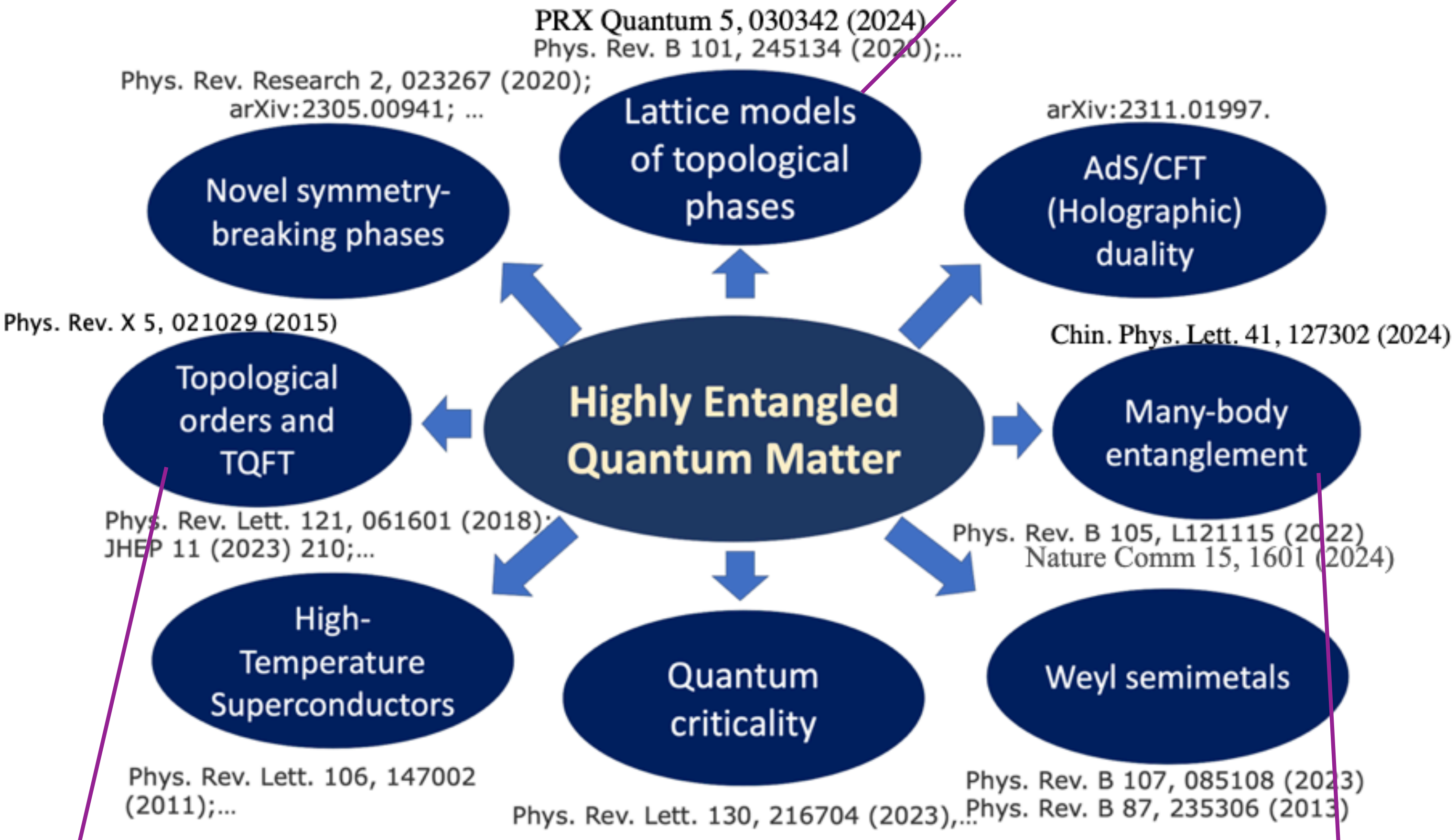
arXiv:2312.08632(NC2024)

arXiv:2205.01654 (PRB2023)

About my research group @ Guangzhou

Research field: **Condensed Matter Theory**
(**Quantum many-body theory**)

first version of talk title comes from here.
Too computer-science



2nd version of talk title
Too high-energy and mathematical physics

The present title
(sounds quantum and more Schrodinger!)

Our work about entanglement

Entanglement

Hermitian System

- **arXiv: 2311.01199 (PRR2024)**
- **arXiv: 2408.01706 (PRR2025)**
- arXiv:2312.08632(NatCom2024)
- arXiv:2205.01654 (PRB2023)
- arXiv:2211.14136 (PRB2023)
- arXiv:2401.00505 (PRX Quantum 2024)
- arXiv:1410.8670 (PRB2015)
- arXiv:1403.1039 (J. Stat. Mech. (2014))

Non-Hermitian System

- arXiv:2112.13411 (PRB2022)
- arXiv:2408.11652 (CPL2024, review)
- arXiv:2009.00546 (SciPost Phys. 2021)

Quantum entanglement v.s. superposition

Consider two spins (qubits), labeled by numbers 1 and 2:

Trivial product state (unentangled) $|\uparrow\rangle_1|\downarrow\rangle_2$

Entangled states: $\frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2)$

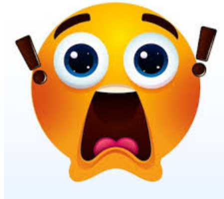
more superposed, more entangled?

Let us consider the following states of N qubits:

$$\sum_{\sigma\in\{\text{all Ising basis}\}} |\sigma\rangle$$

Entanglement entropy: A measure of the strength of entanglement

However, this highly superposed state is a trivial product state!



$$
\begin{aligned}
&\sum_{\sigma\in\{\text{all Ising basis}\}} |\sigma\rangle \\
=&(|\uparrow\rangle_1+\downarrow\rangle_1)\otimes(|\uparrow\rangle_2+\downarrow\rangle_2)\otimes(|\uparrow\rangle_3+\downarrow\rangle_3)\otimes\cdots\otimes(|\uparrow\rangle_N+\downarrow\rangle_N) \\
=&|\rightarrow\rangle_1|\otimes|\rightarrow\rangle_2\otimes|\rightarrow\rangle_3\otimes\cdots\otimes|\rightarrow\rangle_N
\end{aligned}
$$

We need von Neumann entropy (entanglement entropy, EE) to quantitatively measure entanglement!

$$S = -\mathrm{Tr}\rho_A \log \rho_A$$

ρ_A is the reduced density matrix of subsystem A

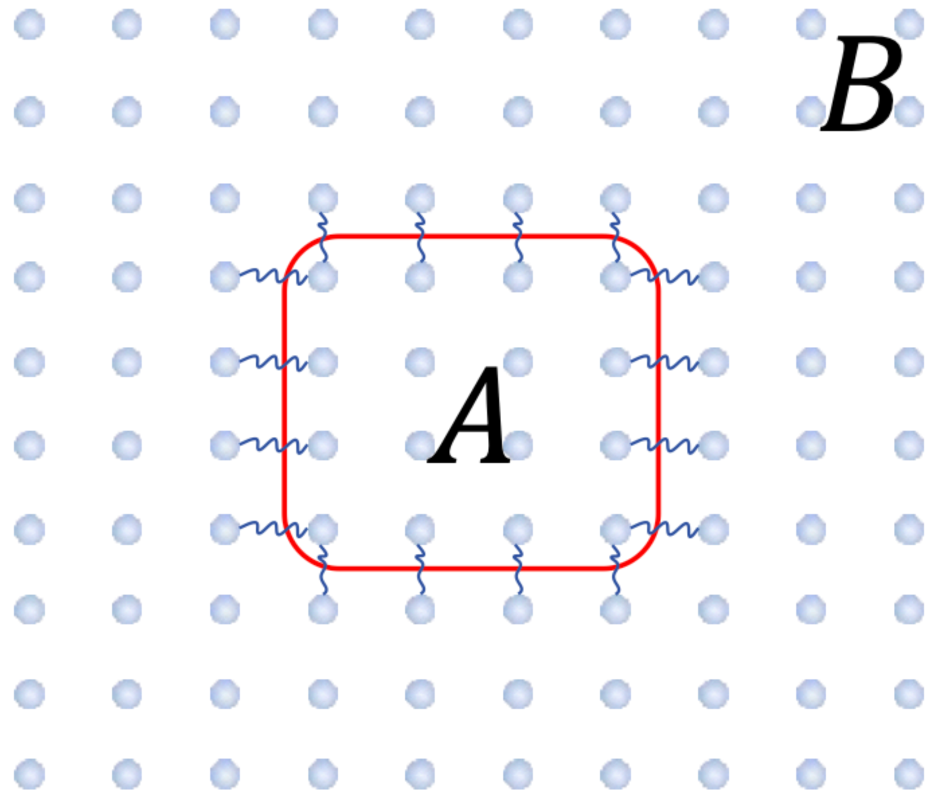
S: characterize # of maximally entangled pairs between two subsystems; many applications in quantum-information science. PRA 53, 2046 (1996)

$$S = -\mathrm{Tr}\rho_A \log \rho_A$$

Consider a condensed-matter problem (in the thermodynamical limit) or a quantum field theory problem:

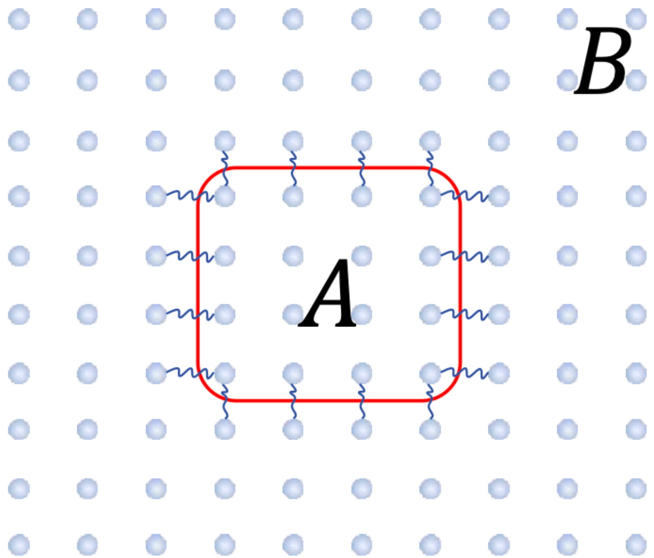
$$N \to \infty$$

utilize quantum information to characterize and even classify quantum materials?



Area law due to **locality** of many-body Hamiltonians

$$S = -\mathrm{Tr}\rho_A \log \rho_A$$



Suppose A's linear size is L

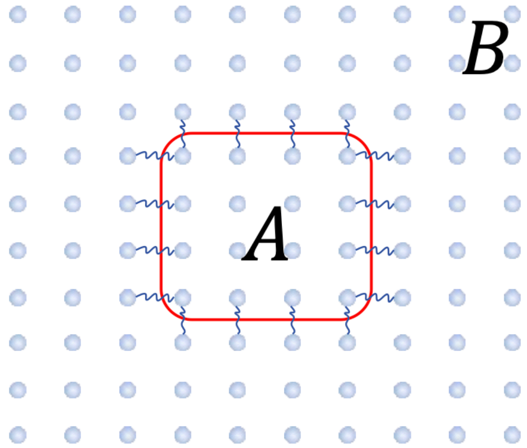
Ask: how EE scales with L when L is sufficiently large (note: thermodynamical limit is taken first)

Growing no slower than "Area law" scaling term (*area means the "area" of boundary of subsystem A*) for ground states of local Hamiltonians

$$S \sim L^{d-1}$$

d is the spatial dimension of the whole system

Super-area law: fermionic statistics is a giant entanglement reservoir!

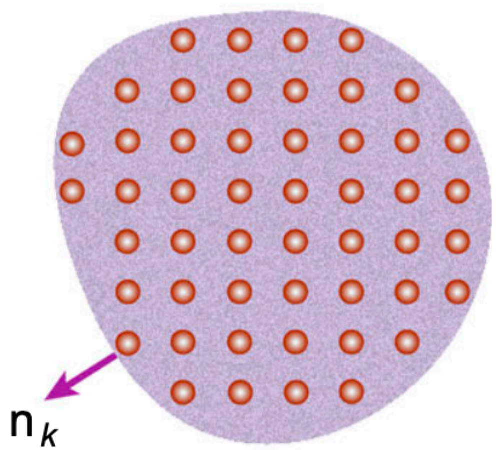


Consider a tight-binding model (or fermi gas) with a finite Fermi surface (codimension-1)

EE scales with a **super-area law** with an additional logarithmic divergence

(Also called: Gioev-Klich-Widom scaling law)

Fermi surface



$$S \sim L^{d-1} \log L$$

Necessary conditions: **Translation invariance with Fermi surface of codim-1** (such that the Widom conjecture of Toeplitz matrices is applicable)

D. Gioev and I. Klich, Phys. Rev. Lett. 96, 100503 (2006).

Consider tight-binding models on a lattice that breaks translation symmetry?

$$S = -\mathrm{Tr}\rho_A \log \rho_A$$

Difficulties:

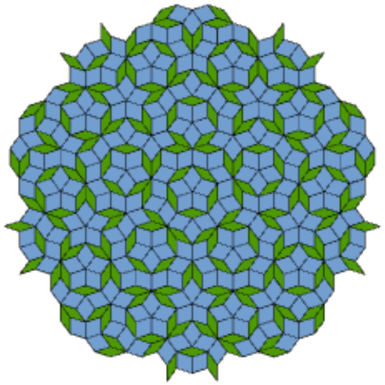
1. Fermi surface is no longer meaningful. Instead, "finite DoS at the chemical potential" is meaningful.

2. The Widom conjecture of Toeplitz matrices is no longer applicable.

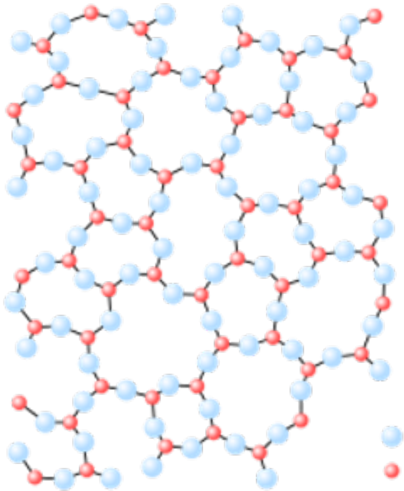
Ask: how EE scales with L.

Non-crystals

Quasicrystal

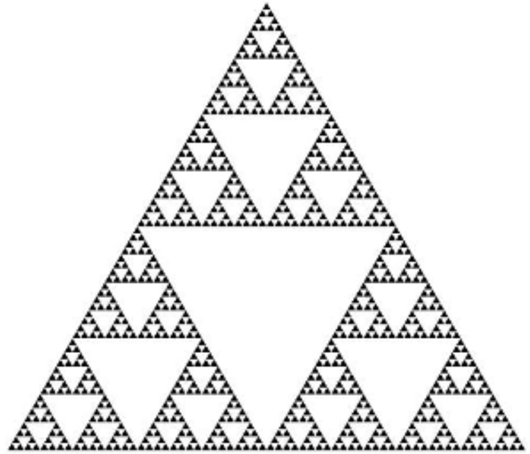


amorphous material (glass)

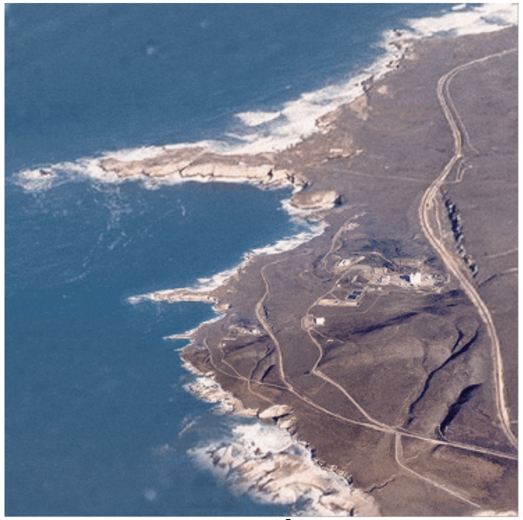


arXiv: 2311.01199

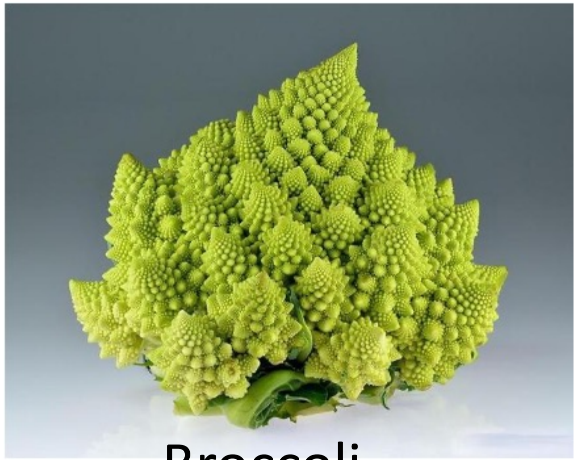
Exact self-similarity: identical at all scales.



Quasi self-similarity: approximates the same pattern at different scales.



coastline



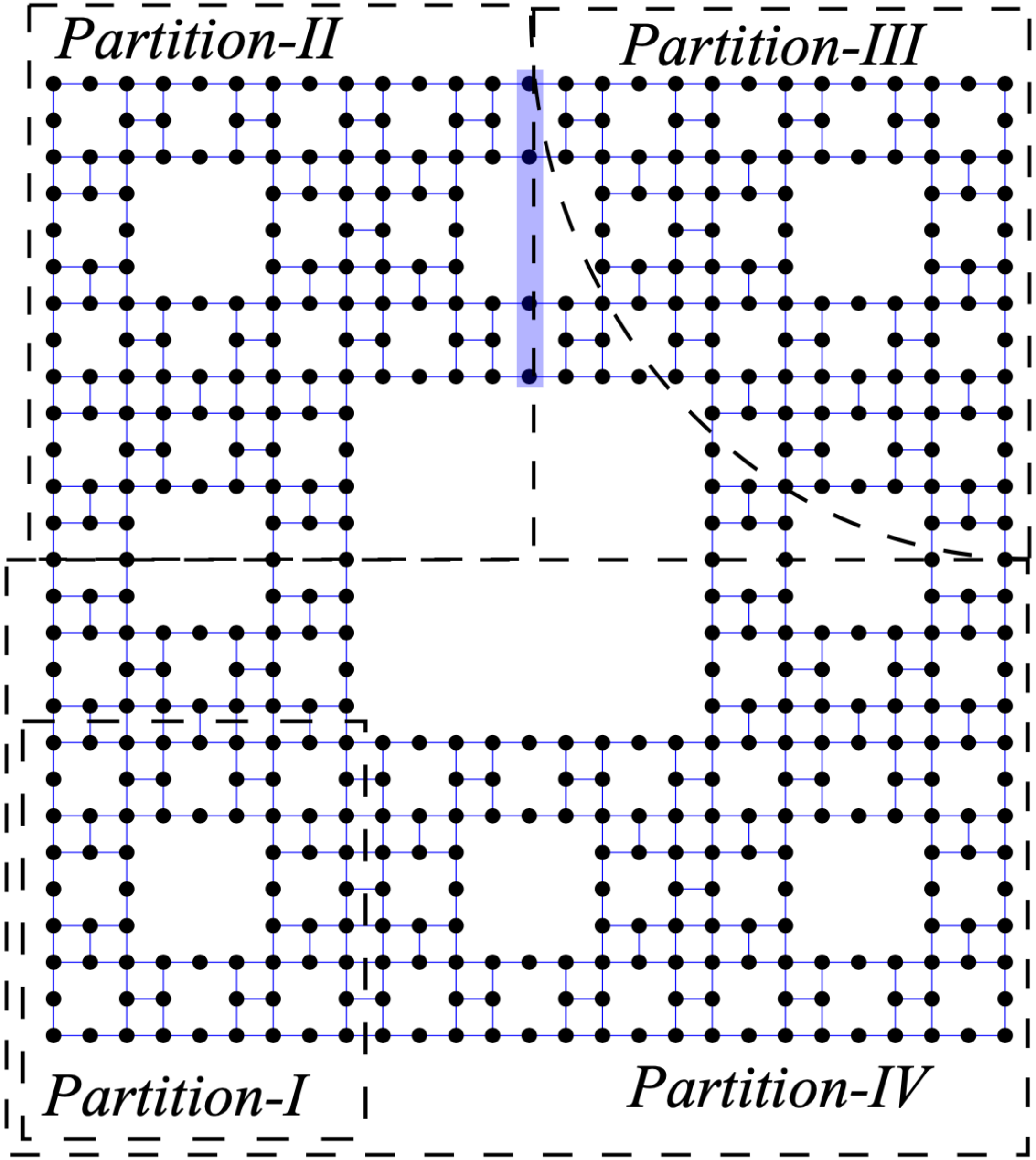
Broccoli

snowflakes

Entanglement on fractal lattices

arXiv: 2311.01199

Sierpinski carpet



SC(3,1)

One-orbital

$$H_1 = -t \sum_{<ij>} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

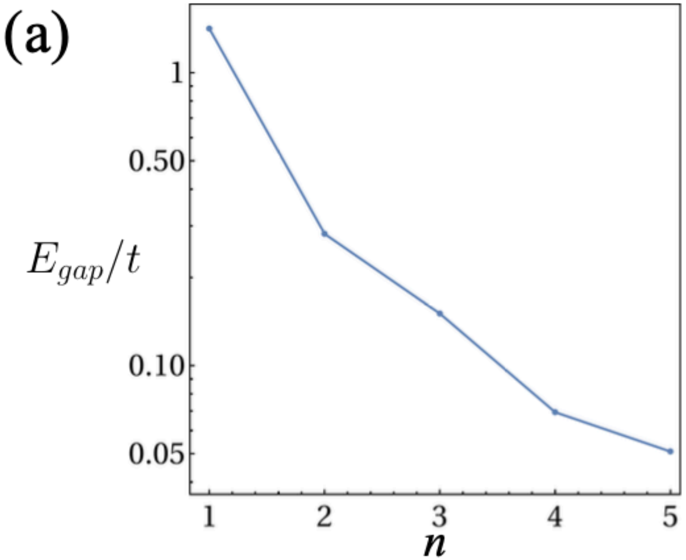
Two-orbitals

$$H_2 = \sum_i (c_{s,i}^\dagger c_{p,i}^\dagger) t_1 \sigma_x \begin{pmatrix} c_{s,i} \\ c_{p,i} \end{pmatrix} + \sum_{<i,j>} (c_{s,i}^\dagger c_{p,i}^\dagger) t \sigma_z \begin{pmatrix} c_{s,j} \\ c_{p,j} \end{pmatrix}$$

arXiv: 2311.01199

Confirmation of Density-of-States (DOS)

(a)

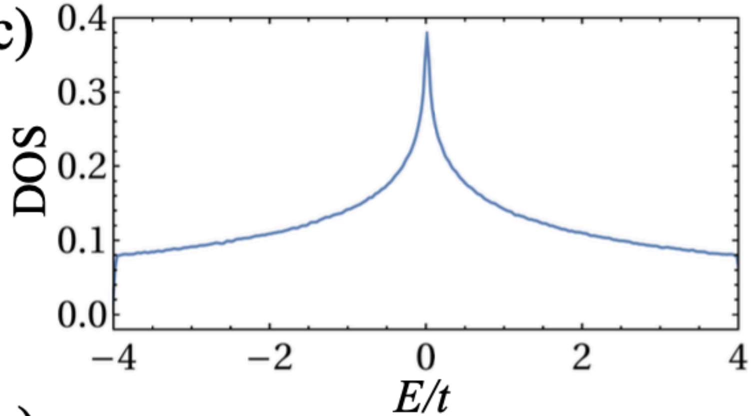


$$H_1 = -t \sum_{<ij>} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

scaling of gap w.r.t. iteration number n.

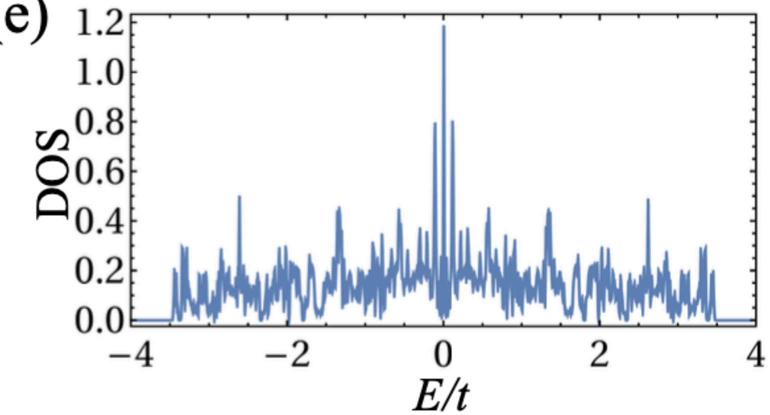
Gapless at large n.

(c)



DOS on square lattice (benchmark)

(e)

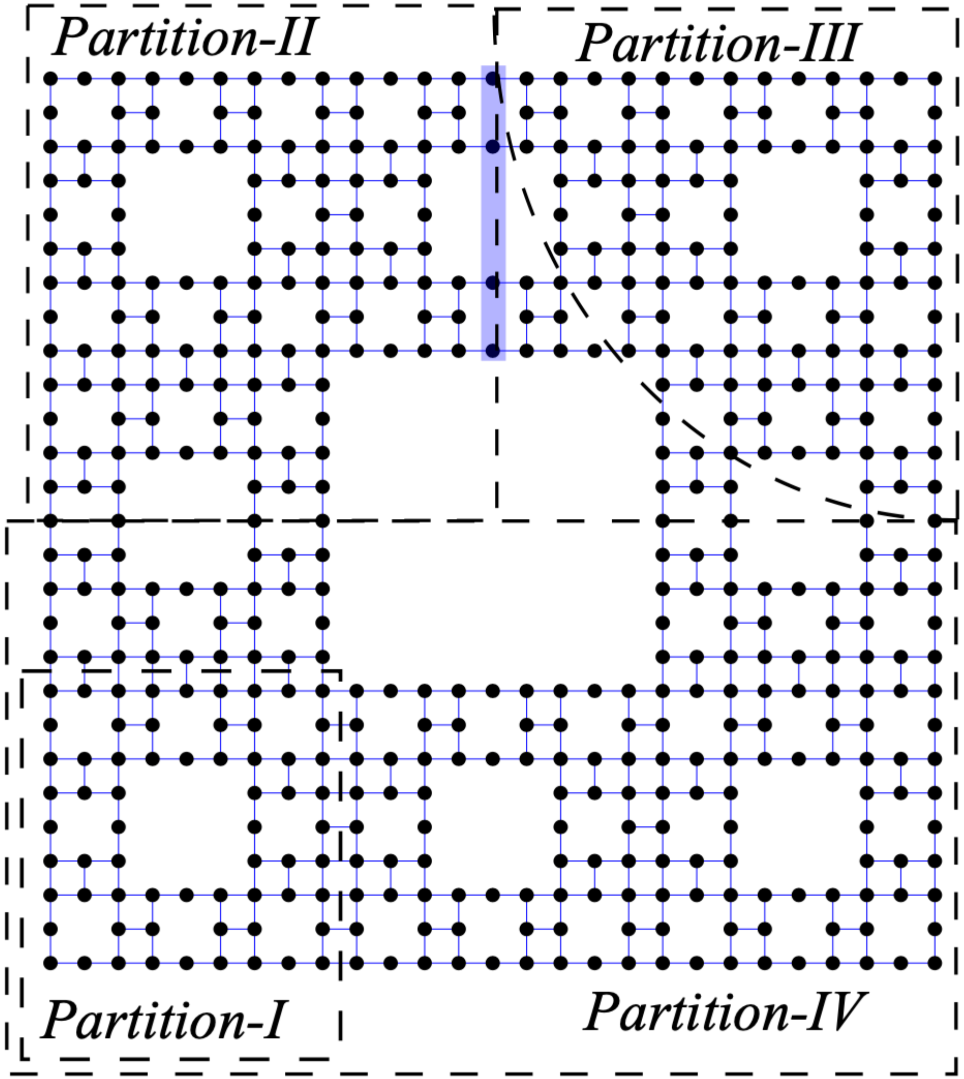


DOS on Sierpinski carpet SC(6,1).

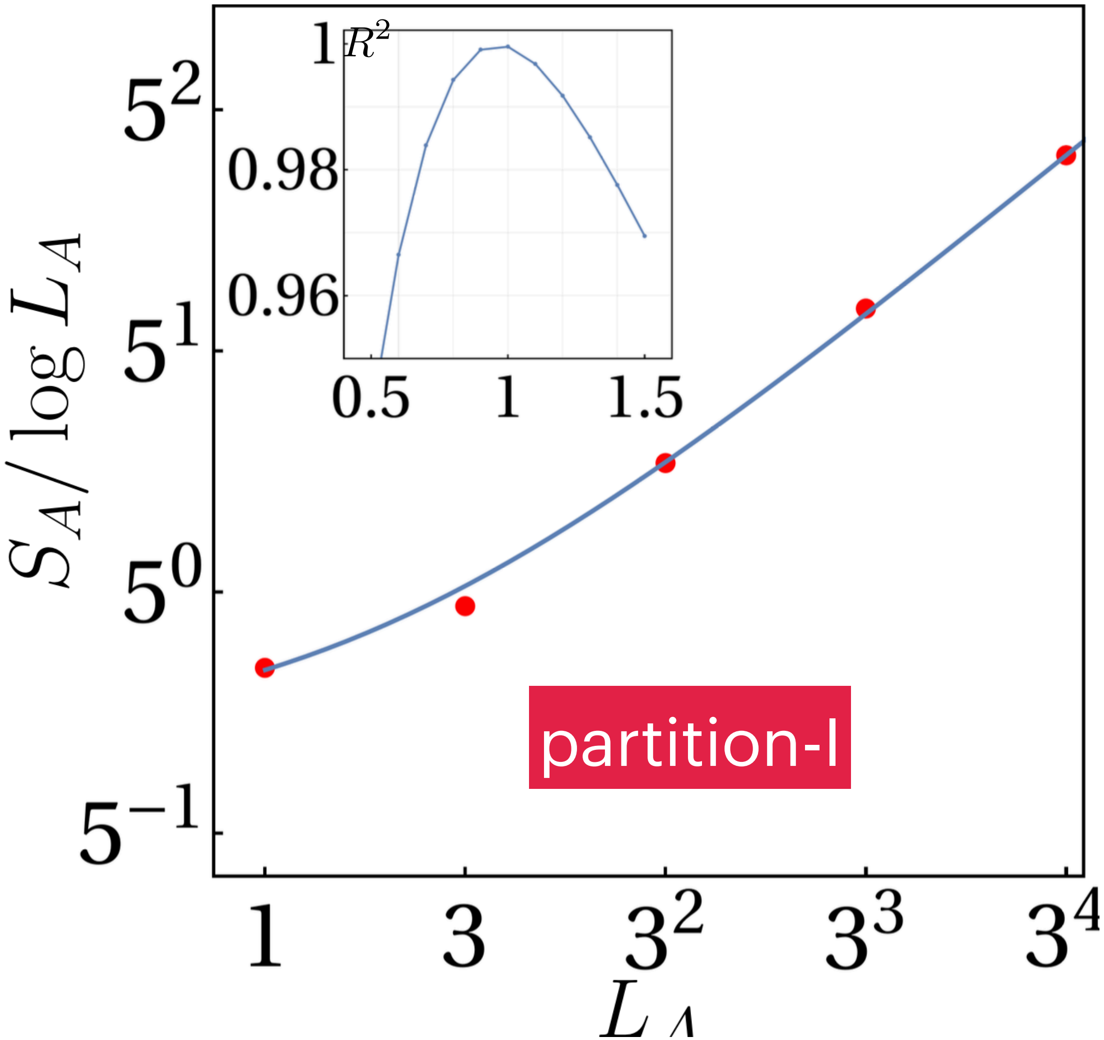
Finite DOS at zero chemical potential.

EE scaling behavior when **DOS is finite** (Partition-I)

arXiv: 2311.01199



SC(3,1)



$$S_A = aL_A^{\alpha}\log L_A + \cdots$$

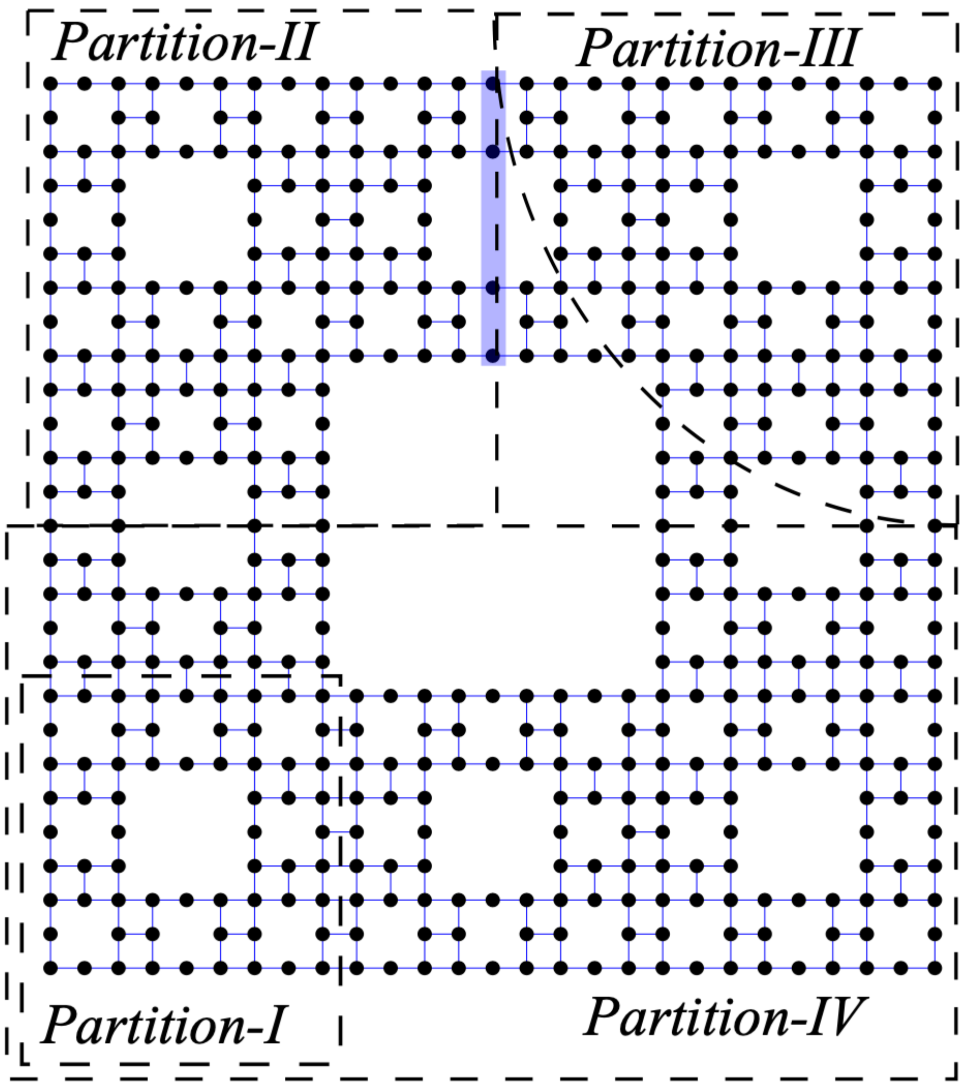
We observe that $\alpha \approx 1$ yields the best fit

Partition-I respects all symmetry

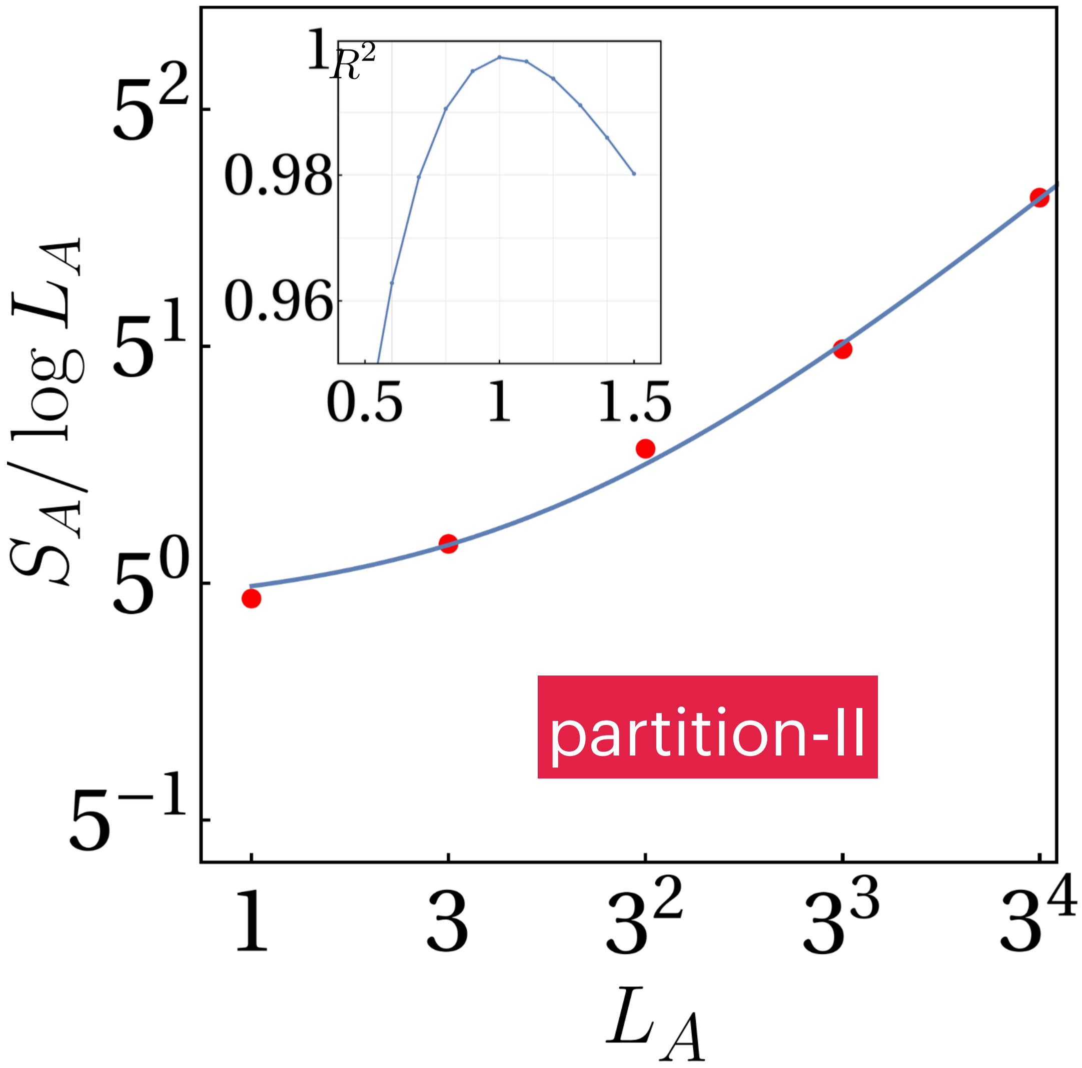
R^2 is coefficient of determination

EE scaling behavior when **DOS is finite** (Partition-II)

arXiv: 2311.01199



SC(3,1)



Partition-II has fractal boundary (Cantor set)

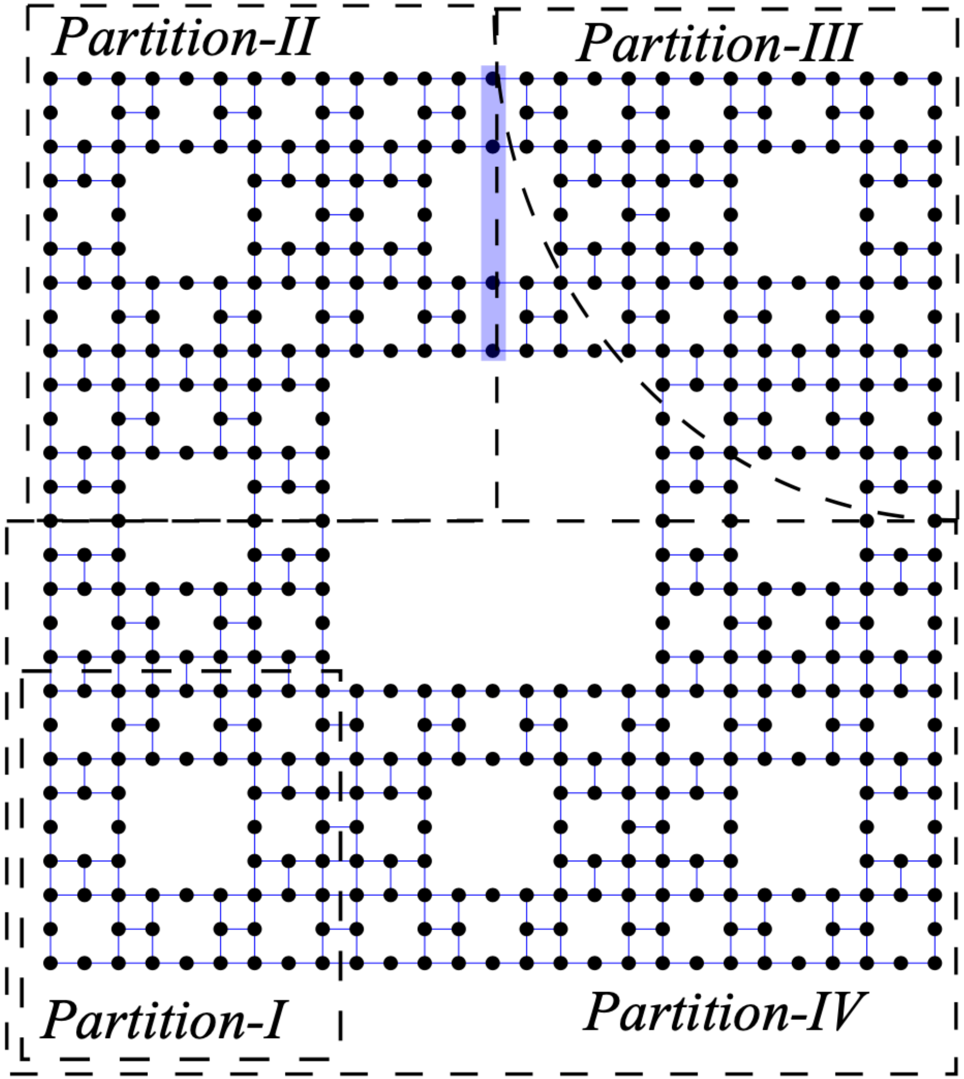
$$S_A = aL_A^{\alpha} \log L_A + \cdots$$

We observe that $\alpha \approx 1$ yields the best fit

R^2 is coefficient of determination

EE scaling behavior when **DOS is finite** (Super-area law wins!)

arXiv: 2311.01199



SC(3,1)

Partition-I respects all symmetry

Partition-II has fractal boundary

We claim that for any partition, EE scales as:

$$\boxed{S_A = aL_A^{d_s-1}\log L_A + \cdots}$$

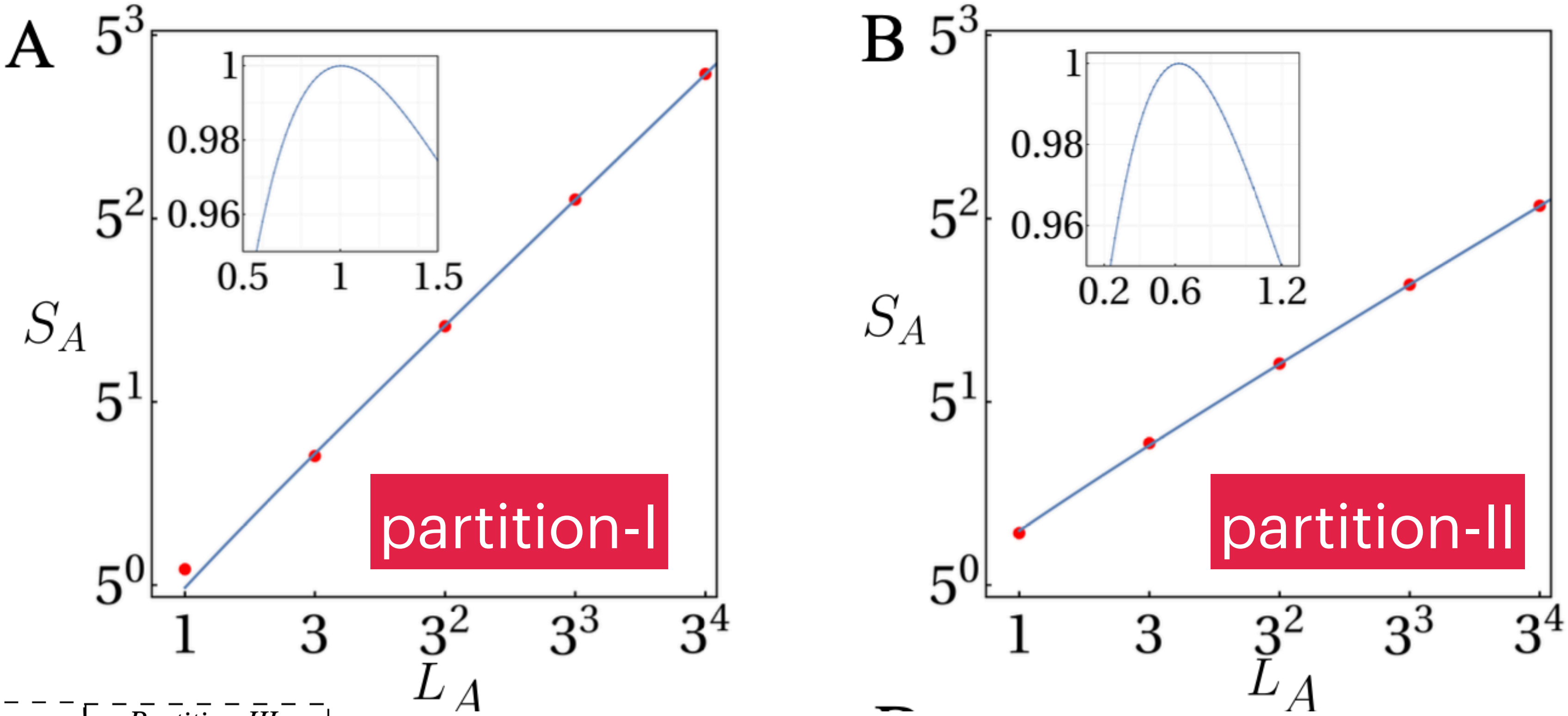
d_s is the dimension of space where fractal lattice is embedded.

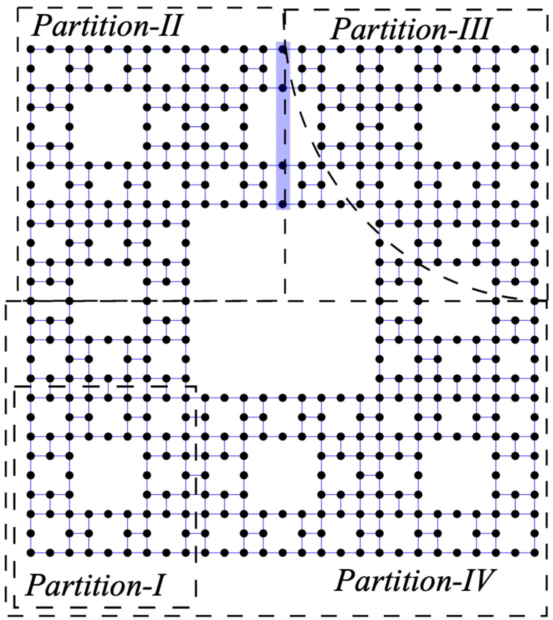
In our case, $d_s = 2$

Significantly generalize the Gioev-Klich-Widom scaling law

EE scaling behavior **when gap is present**.

arXiv: 2311.01199



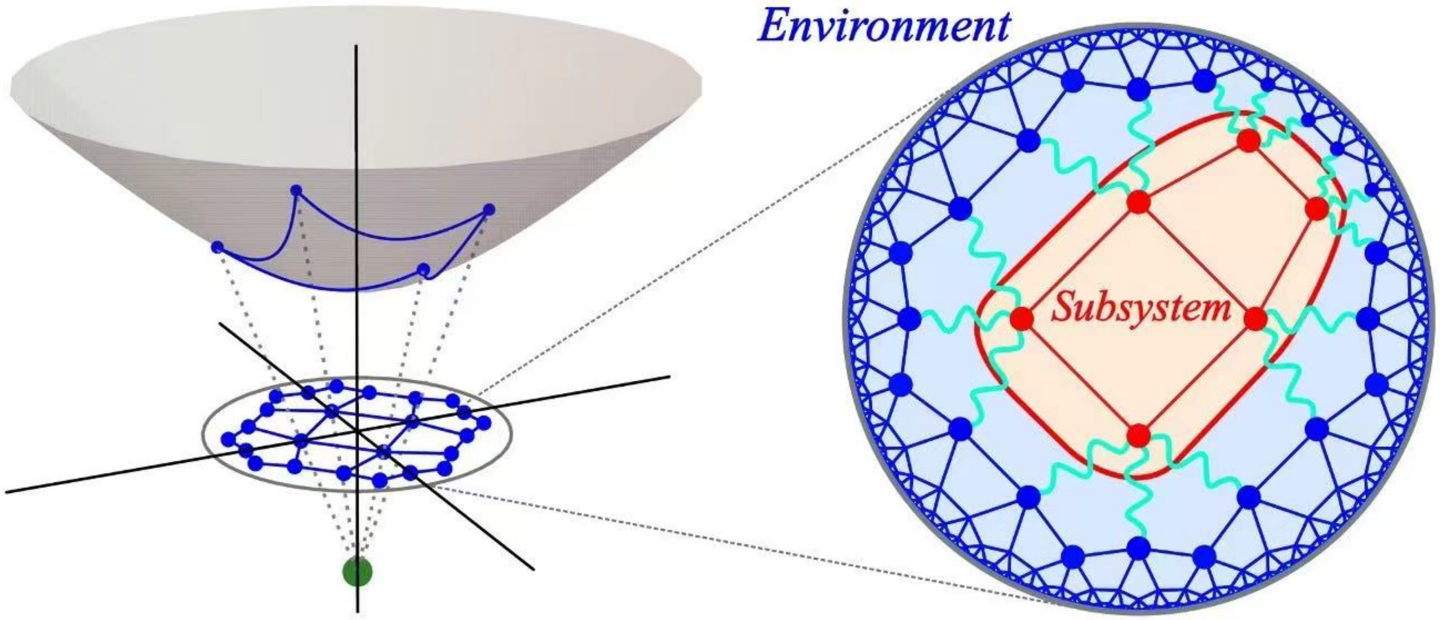


We claim that for *any* partition, EE scales as:

$$S_A = aL_A^{d_{\mathrm{bf}}} + \cdots$$

d_{bf} is the Hausdorff dimension of boundary of *A*.

generalize the area law of gapped ground states.

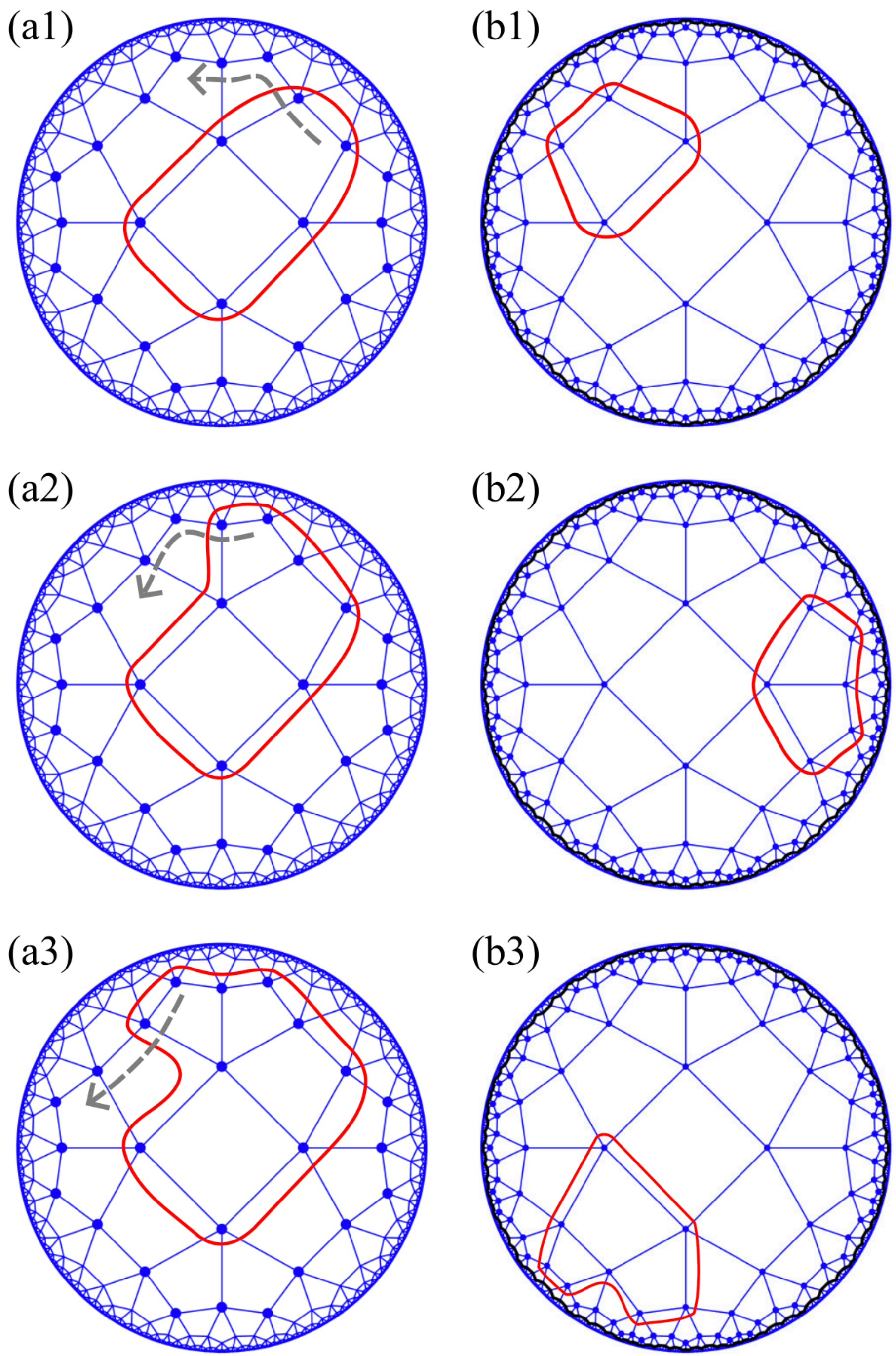


Curved Geometry Suppresses Quantum Correlations in Hyperbolic Lattices

arXiv: 2408.01706

Two ways of entanglement partitions (cuts)

arXiv: 2408.01706

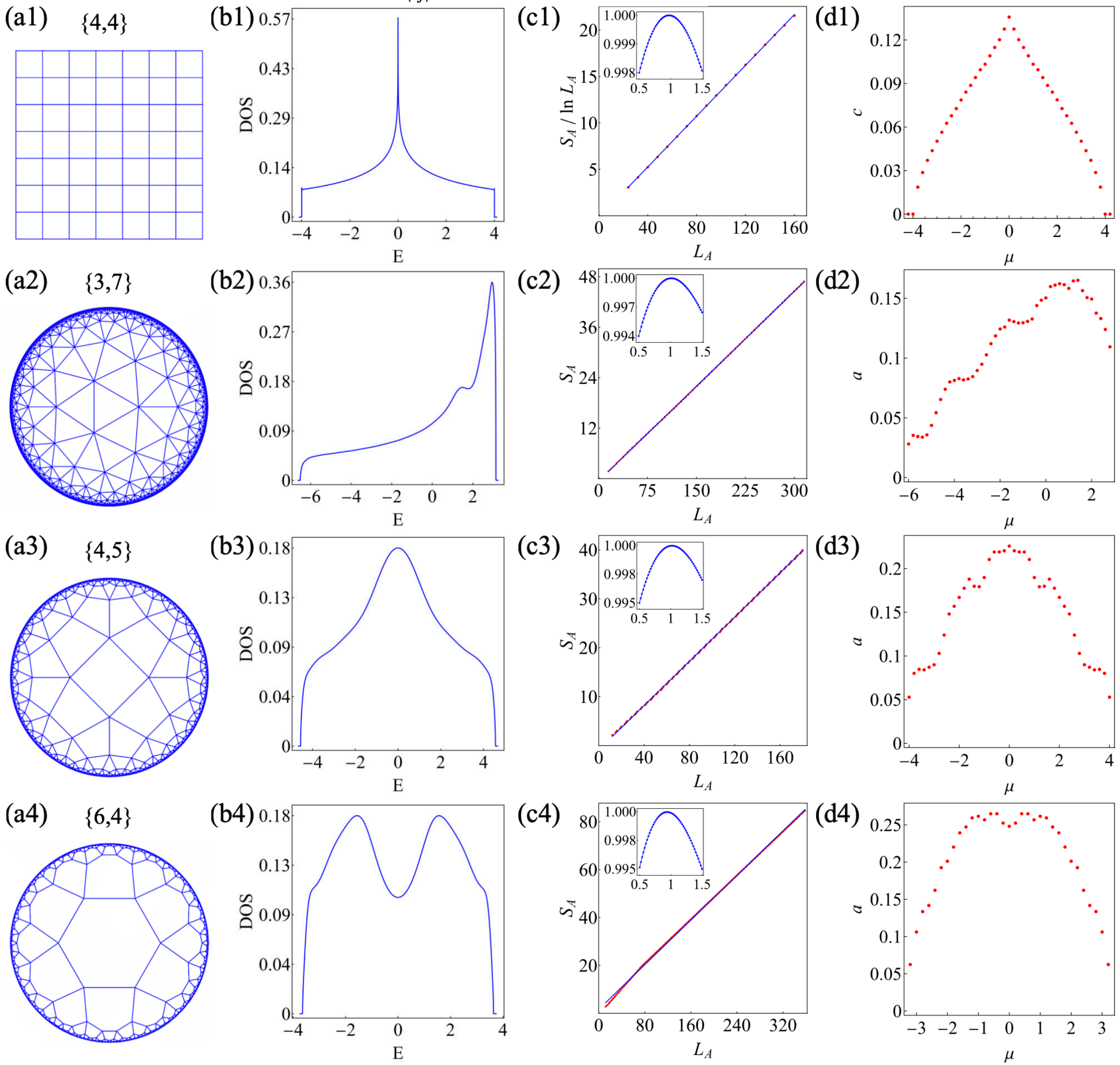


(a1,2,3): step-by-step enlarge size

(b1,2,3): random choose (keep a minimal distance from boundary) arXiv: 2408.01706

Entanglement of gapless models with finite DOS (hyperbolic metals)

$$H_1 = -t\sum_{\langle ij\rangle}(c_i^\dagger c_j + \mathrm{H.c.}) - \mu\sum_i c_i^\dagger c_i$$



DOS: Haydock recursion method

area law

$$S_A = aL_A + \cdots$$

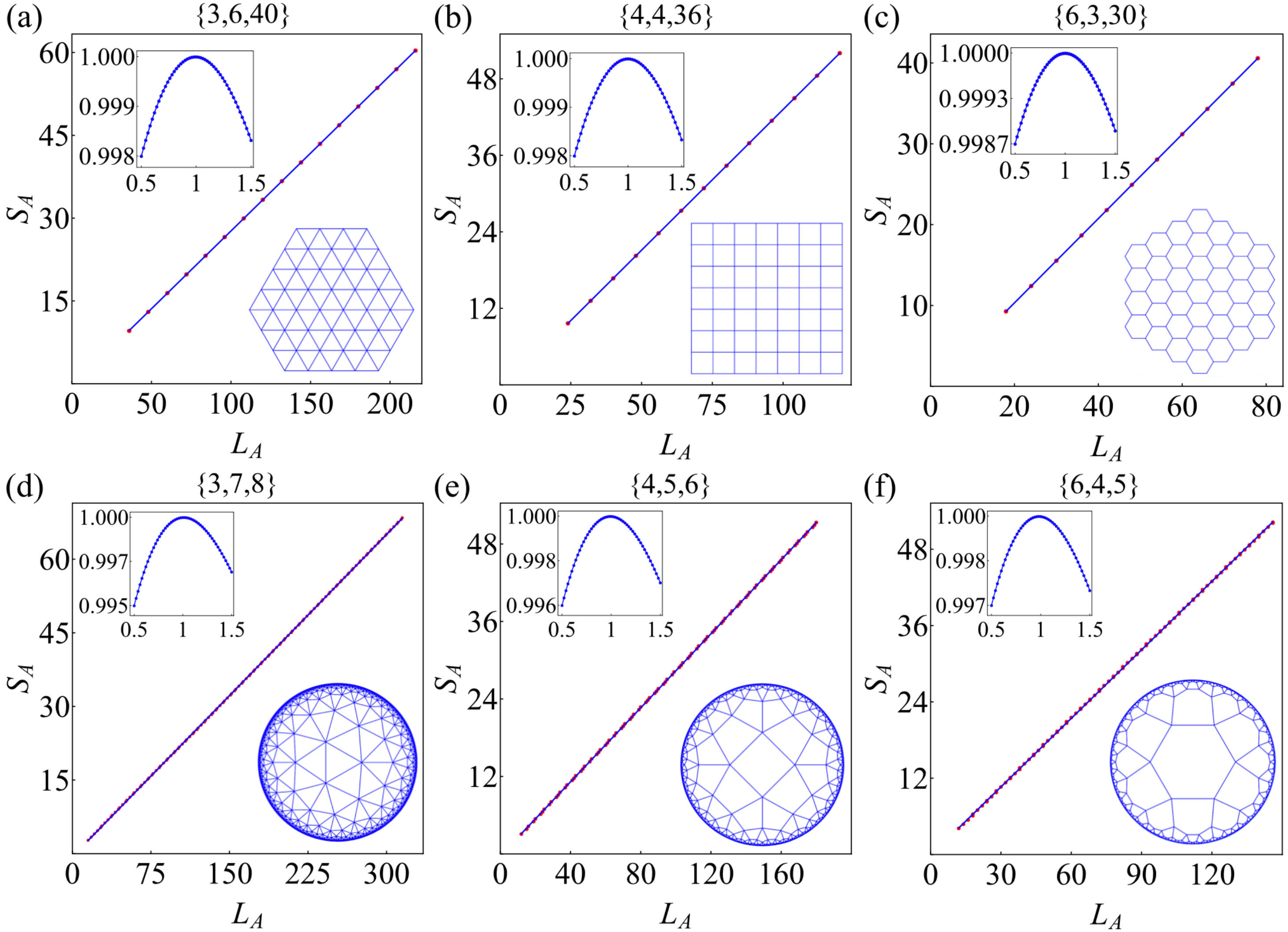
Linear fit is best fit due to the coefficient of determination

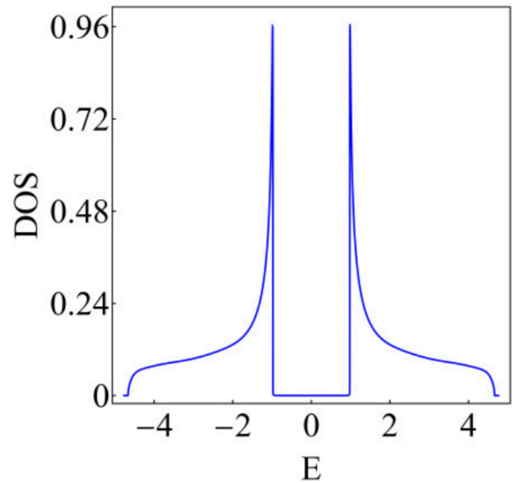
Logarithmic divergence is suppressed by AdS geometry!

Entanglement of gapped models (hyperbolic insulators)

arXiv: 2408.01706

$$H_2 = -\sum_i t_1(c^\dagger_{s,i}c_{p,i} + \text{H.c.}) - \sum_{\langle ij\rangle} t_2(c^\dagger_{s,i}c_{s,j} - c^\dagger_{p,i}c_{p,j})$$





{4, 6}

Summary and Outlook

TABLE I. Scaling behavior of the EE of ground states of free-fermionic systems on a *d-dimensional* Euclidean lattice with translation invariance, fractal lattice with self-similarity, and *two-dimensional* hyperbolic lattice. In the fractal case, d_{bf} denotes the *Hausdorff dimension* of the boundary of subsystem A, while the EE of gapless systems exhibits a fractal-like distribution [15].

Lattice	Phase	S_A
Euclidean lattice [2,3]	Gapped, DOS $= 0$	$\sim L_A^{d-1}$
	Gapless, DOS > 0	$\sim L_A^{d-1} \ln L_A$
Fractal lattice [15]	Gapped, DOS $= 0$	$\sim L_A^{d_{\mathrm{bf}}}$
	Gapless, DOS > 0	$\sim L_A^{d-1} \ln L_A$
Hyperbolic lattice	Gapped, DOS $= 0$	$\sim L_A$
	Gapless, DOS > 0	$\sim L_A$

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arXiv:2205.01654 (PRB2023)

1. Analytic study (underlying mechanism?)
2. How interaction affects EE?

Thanks for your attentions

Cảm ơn