Skyrmions: from hadrons to magnetic textures

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Based on arXiv:2509.17403
"Dynamics of CP(N-1) skyrmions"

21st Recontres du Vietnam

We are celebrating 100 years of quantum physics

Discuss a theme that is both ancient, and active

Skyrmions

Skyrmion: definition & history

- Smooth topological texture in *d* space dimension
- Characterized by $\pi_d(S^d) = \mathbb{Z}_N$ homotopy (winding numbers)
- Skyrme (1958,1961,1962) envisioned *nonlinear* field theory with *topological* current

$$J_\alpha = (2\pi)^{-1}\epsilon_{\alpha\beta}\epsilon_{ab}n_a\partial_\beta n_b \text{ (1d)}$$

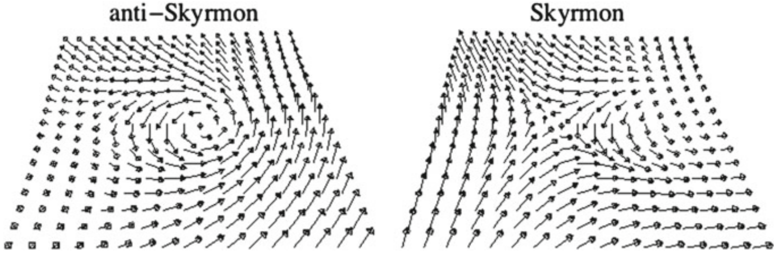
$$J_\alpha = (8\pi)^{-1}\epsilon_{\alpha\beta\gamma}\epsilon_{abc}n_a\partial_\beta n_b\partial_\gamma n_c \text{ (2d)}$$

$$J_\alpha = (12\pi^2)^{-1}\epsilon_{\alpha\beta\gamma\delta}\epsilon_{abcd}n_a\partial_\beta n_b\partial_\gamma n_c\partial_\delta n_d \text{ (3d)}$$

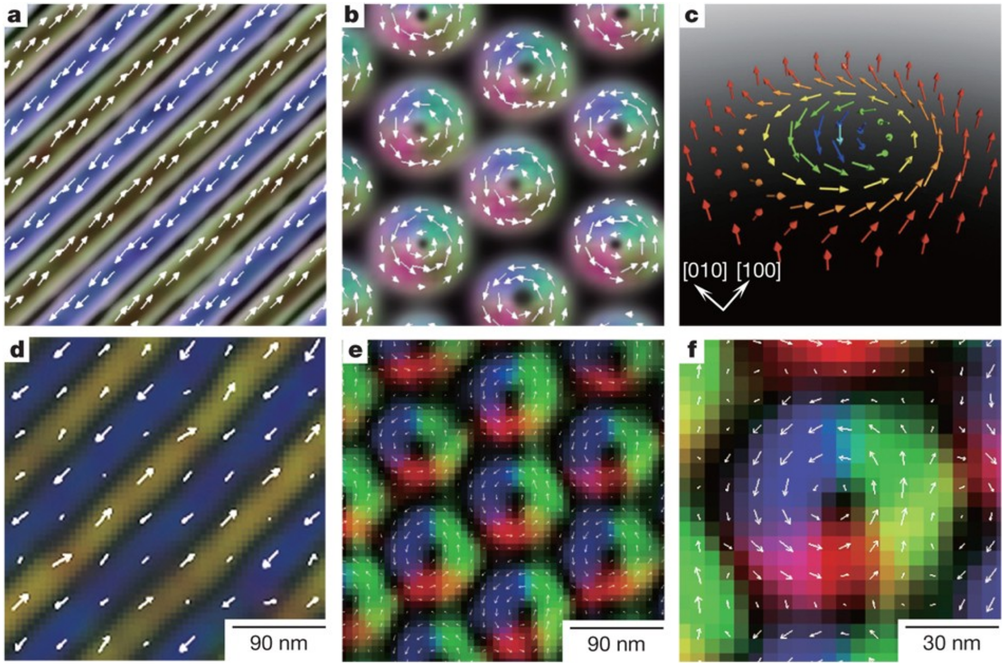
Unit vector field $\sum_{n=1}^{d} n_a n_a = 1$

- Traditionally a realm of particle and nuclear physics

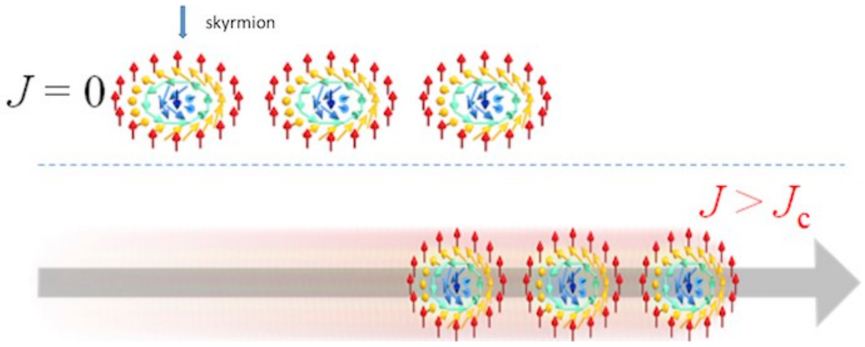
"Baby" Skyrmions in 2d and magnets

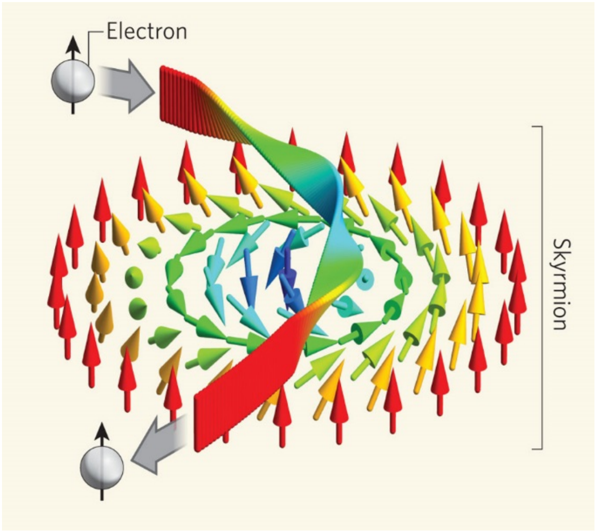


- Skyrme glossed over 2d
- 2d texture is called "baby" skyrmion with O(3) vector
- Classically, magnets are described by O(3) unit vector
- Experimental discovery of baby skyrmions in chiral magnets (2009,2010)
- Extensive search for materials with higher Tc and maneuverability



Skyrmions and electrons





- Chiral magnets are often metals
- Skyrmion motion by electric current
- Skyrmion racetrack memory - Skyrmionics
- Electrons see skyrmions as source of emergent magnetic field - topological Hall effect

Skyrmionics

- Without electric current: Thiele's equation:

$$\mathbf{F} + G(\hat{z} \times \dot{\mathbf{R}} - \alpha \dot{\mathbf{R}}) = 0$$

- G : Gyrotropic constant
- α : Gilbert damping, skyrmion Hall angle

- With electric current: modified Thiele's equation:
  $$\mathbf{F} + \hat{z} \times \left([G + h\rho^e]\dot{\mathbf{R}} - h\mathbf{J}^e\right) - \alpha G \dot{\mathbf{R}} = 0$$

- When $\mathbf{F} = 0$, $\mathbf{J}^e \neq 0$:
  $$\dot{\mathbf{R}} \simeq \frac{h\mathbf{J}^e}{G} - \alpha \hat{z} \times \frac{h\mathbf{J}^e}{G}$$

- Current-driven skyrmion motion -> skyrmionics

CP(N-1) skyrmions

- CP(N-1) topological current:
  $$J_\mu = \frac{1}{8\pi}\epsilon_{\mu\nu\lambda} f_{abc} n^a \partial_\nu n^b \partial_\lambda n^c$$
  (f_{abc} : SU(N) structure constant)
  (n^a : $N^2 - 1$ component unit vector)

- Previously, only N=2 considered
- N>2 realizable in multipolar magnets (dipole, quadrupole)

- Continuity equation: $\partial_\mu J_\mu = \partial_t J_t + \partial_j J_j = 0$
- Conserves $Q = \int_{\mathbf{r}} J_t$

CP(N-1) skyrmions

- Skyrmions are “fractons” - Papanicolaou-Tomaras (91)
- Fractonic continuity equation:
  $\partial_t J_t + \partial_i \partial_j J_{ij} = 0$
- $J_{ij} = \frac{\epsilon_{ik}}{2\pi} \sigma_{kj}$, $\sigma_{kj} = \mathscr{H} \delta_{kj} - \frac{\partial \mathscr{H}}{\partial(\partial_j n^a)} \partial_k n^a$
- Conserves $Q = \int_{\mathbf{r}} J_t$ and $\mathbf{D} = \int_{\mathbf{r}} \mathbf{r} J_t$
- We prove it for arbitrary CP(N-1)

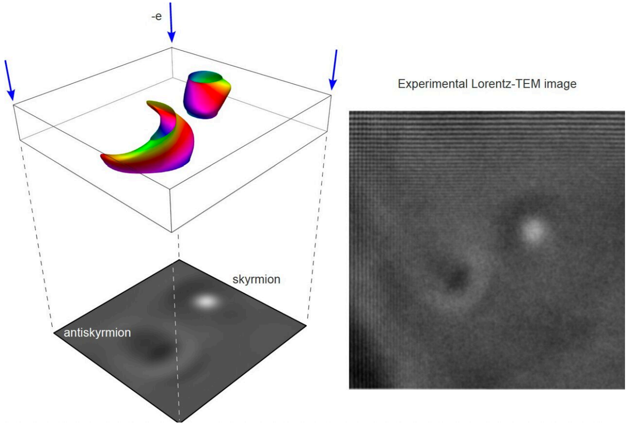
Effect of damping

- Dipole conservation forbids motion of single skyrmion
- Gilbert damping “relieves” dipole constraint:

$$\partial_t J_t + \partial_i \partial_j J_{ij} = \frac{\alpha}{8\pi} \epsilon_{ij} \partial_i [\partial_t n^a \partial_j n^a] \neq 0$$

(α : Gilbert damping)

- Single skyrmion motion possible due to friction
- Skyrmion-antiskyrmion creation possible due to friction



Girvin-MacDonald-Platzman (GMP) algebra

- GMP algebra for density operators in LLL
- Consequence of strong magnetic field

- GMP algebra for CP(1) topological density [Garst, 24]
- No magnetic field!

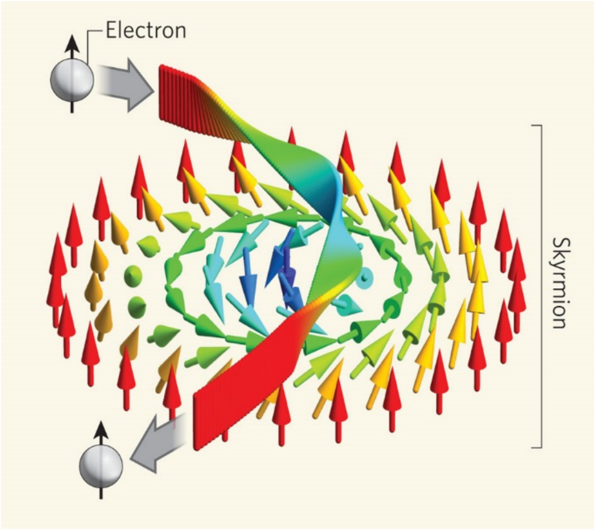
- Generalization to CP(N-1):

$$\{J_t(\mathbf{r}, J_t(\mathbf{r}')\} = \frac{\epsilon_{ij}}{2\pi}\partial_i J_t(\mathbf{r})\partial_j \delta(\mathbf{r} - \mathbf{r}')$$

- Importance of GMP algebra in fractonic field theory emphasized by DX Nguyen, DT Son [SciPost 22]
- GMP algebra related to fractons more than magnetic field

Topological Hall effect of CP(N-1) skyrmions

- CP(2) skyrmions develop texture in dipolar ($\mathbf{S}$) and quadrupolar ($Q_{\alpha\beta}$) sectors
- Only the dipolar sector couples to electron's magnetic moment
- Hund's coupling ~ $J(\psi^\dagger \sigma \psi) \cdot \mathbf{S}$
- Some CP(2) skyrmion texture may develop significant dipolar texture and leads to THE, but not the quadrupole skyrmion



Summary

- CP(N-1) skyrmion dynamics exhibit fractonic dynamics, which is destroyed by damping

- GMP algebra (same as LLL) holds

- Skyrmions as description of subatomic particles (60s) -> magnetic solitons ('09) -> realization in multipolar magnets?