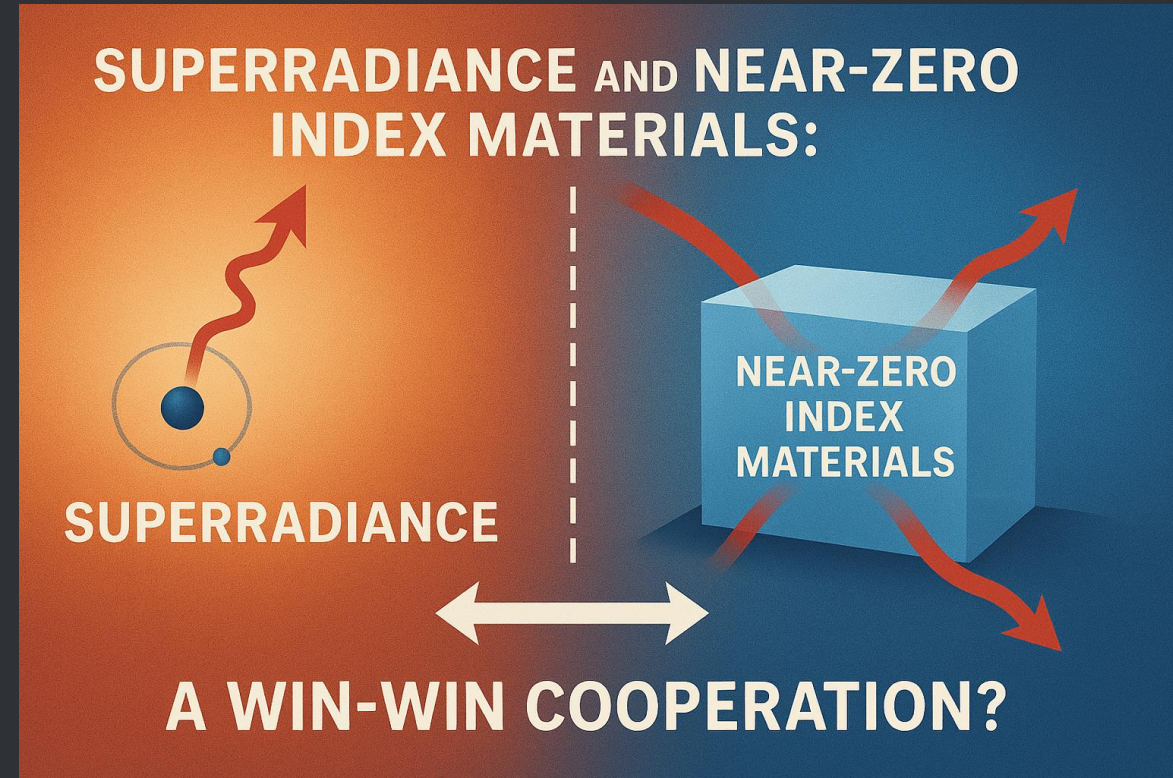


# ***Superradiance and near-zero refractive index materials: a win-win cooperation?***

21<sup>st</sup> Rencontres du Vietnam, Quy  
Nhon  
October 8<sup>th</sup> 2025

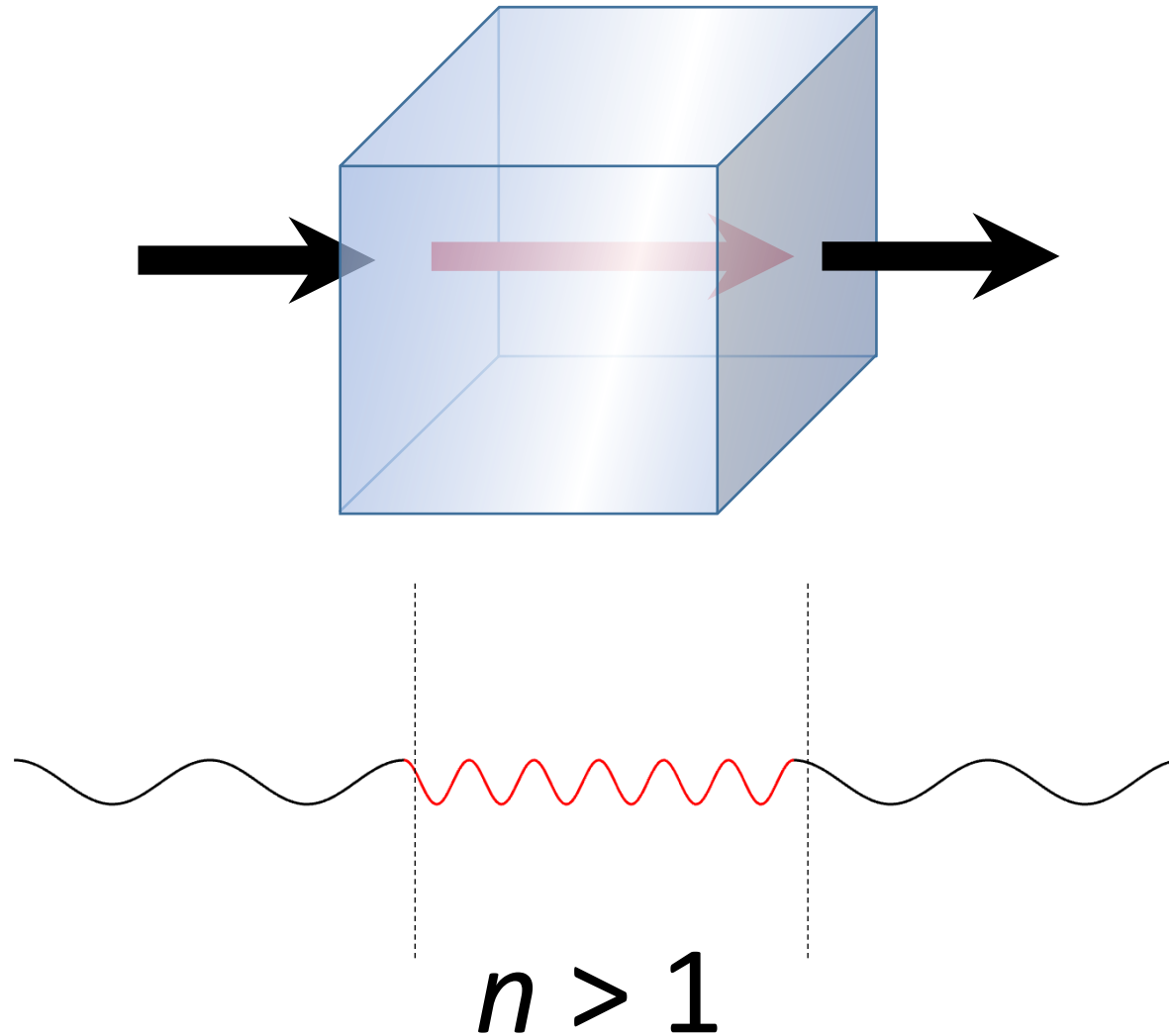
*Slides available on simple request*



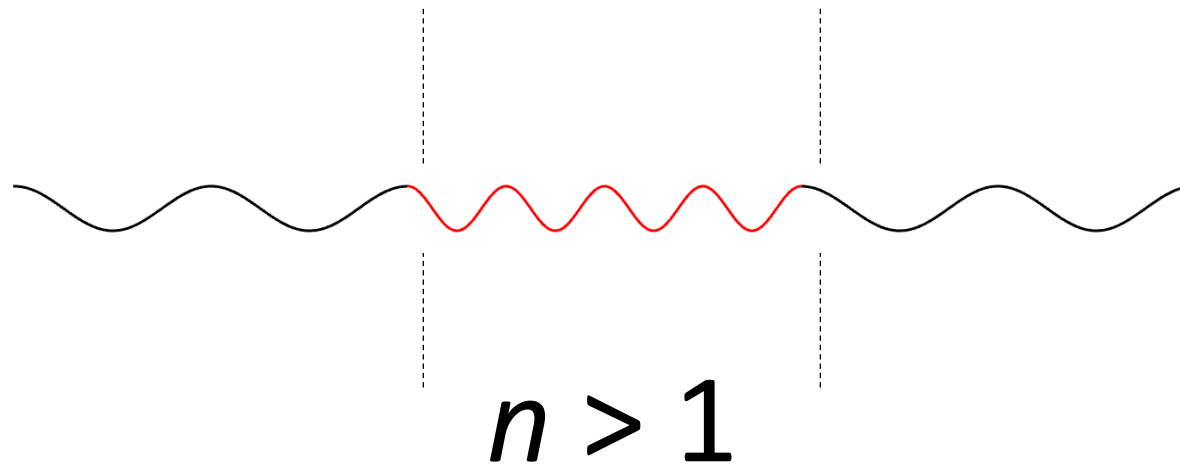
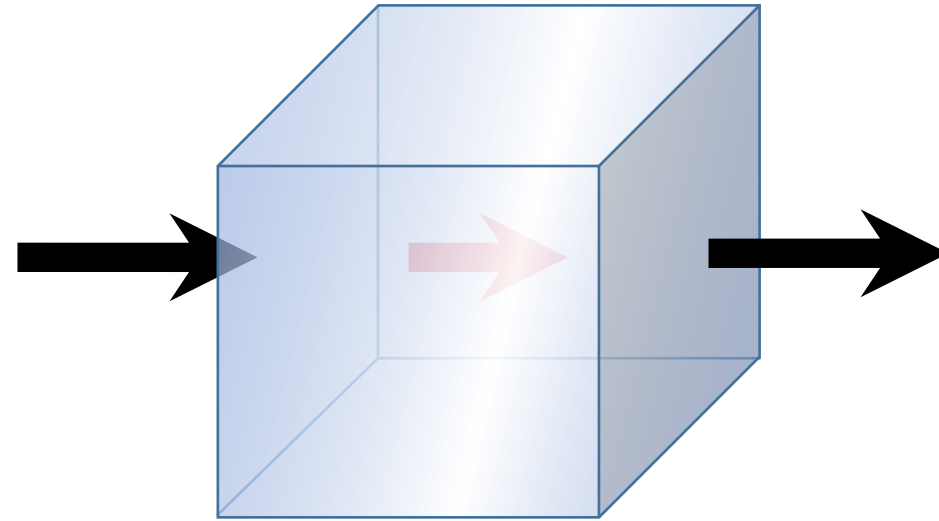
**M. Lobet,**  
O. Melo, A. Debacq, L. Vertchenko, S.  
Nelson, D. Guney, E. Mazur



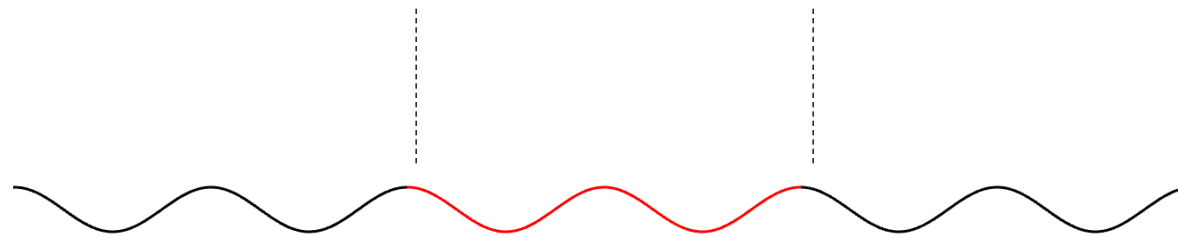
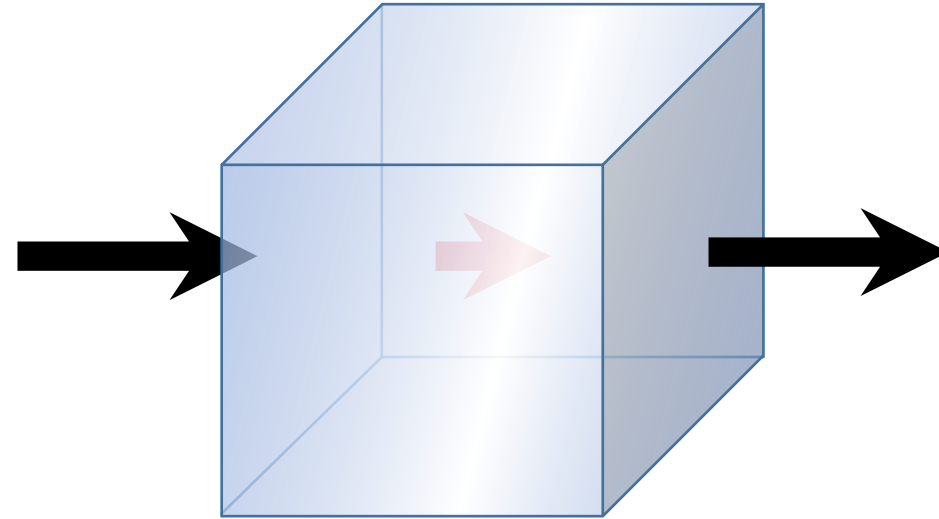
## What are near-zero index media?



As the index **decreases**, the effective wavelength **increases**.

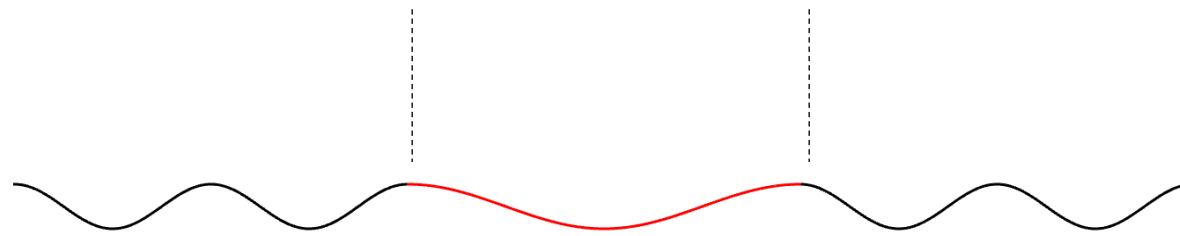
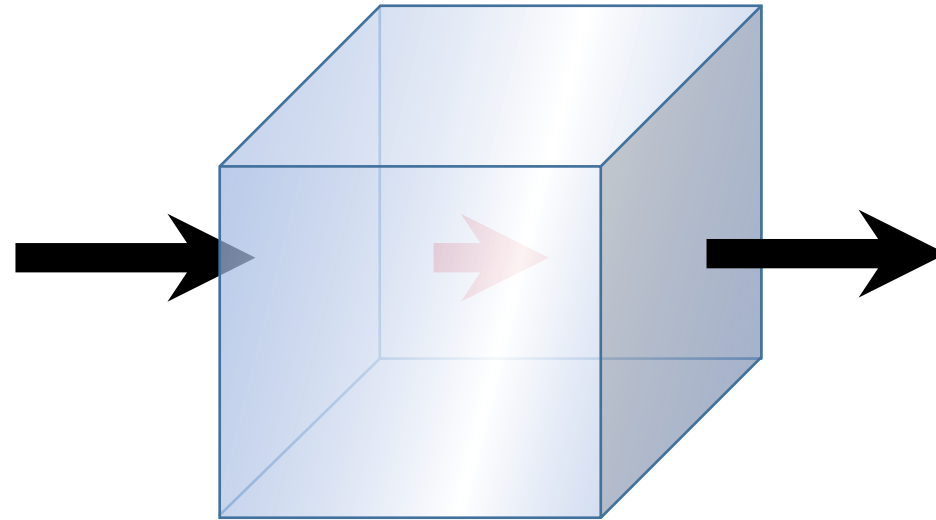


As the index **decreases**, the effective wavelength **increases**.



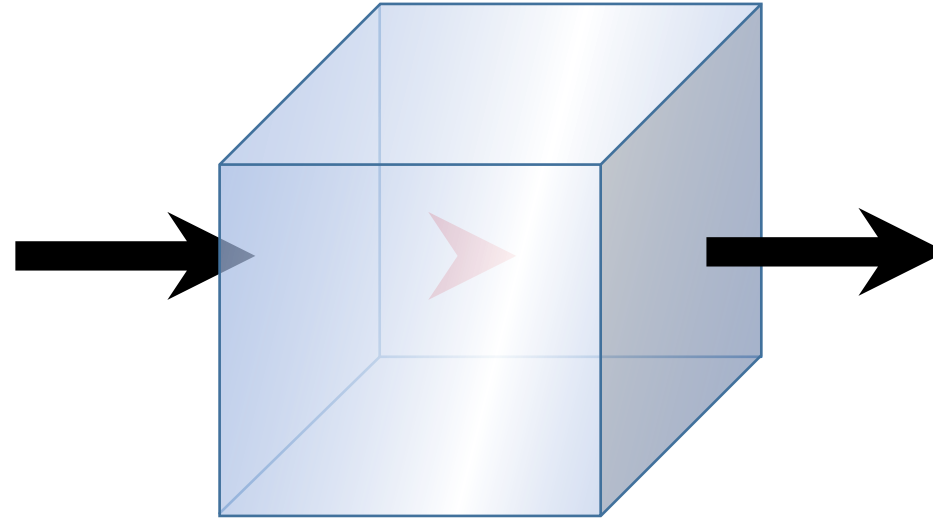
$$n = 1$$

As the index **decreases**, the effective wavelength **increases**.



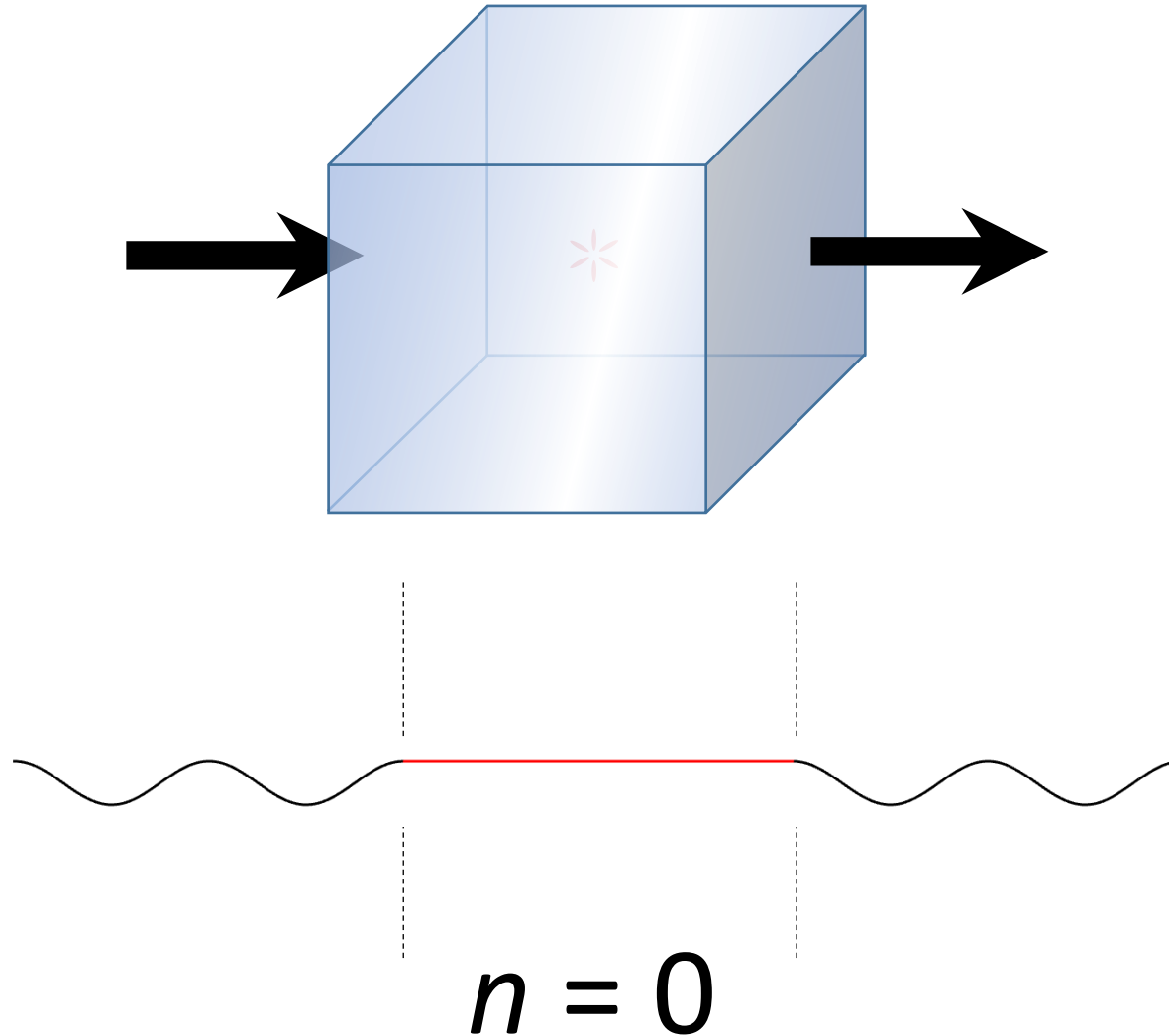
$$n < 1$$

As the index **decreases**, the effective wavelength **increases**.



$$n \ll 1$$

As the index **decreases**, the effective wavelength **increases**.





$$\lambda = \frac{\lambda_0}{n}$$

Infinite wavelength

$$k = \frac{\omega n}{c}$$

Zero  $k$  vector

Lobet, M.. *et al.* Momentum considerations inside near-zero index materials. *Light Sci Appl* **11**, 110 (2022).

$$\cancel{v} = f \cancel{\lambda}$$

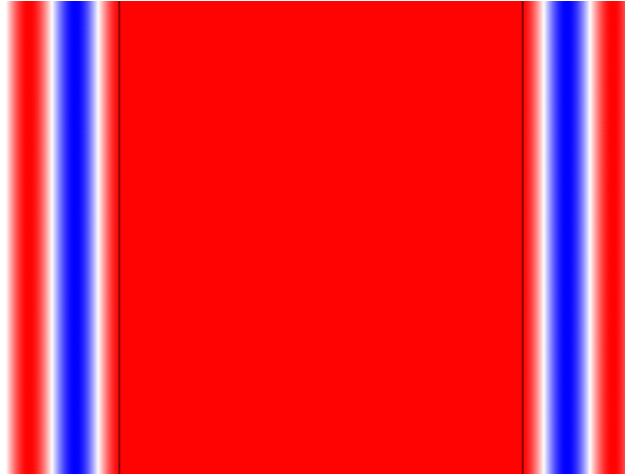
Infinite phase velocity

$$\phi = ikx$$

Zero phase change

Interestingly: no diffraction and no Doppler effect occur in NZI  $\vec{E} = \vec{E}_0 e^{-i\omega t}$

Air ( $n = 1$ )   NZIM ( $n = 0$ )   Air ( $n = 1$ )



Zero  $k$  vector

Infinite wavelength

Infinite phase velocity

Zero phase change

Introducing 3 classes

$$n = \sqrt{\epsilon\mu}$$

$$\epsilon \rightarrow 0$$

**ENZ**

$$\epsilon \text{ and } \mu \rightarrow 0$$

**EMNZ**

$$\mu \rightarrow 0$$

**MNZ**

Lobet, Michaël, et al. "Fundamental radiative processes in near-zero-index media of various dimensionalities." *ACS Photonics* 7.8 (2020): 1965-1970.

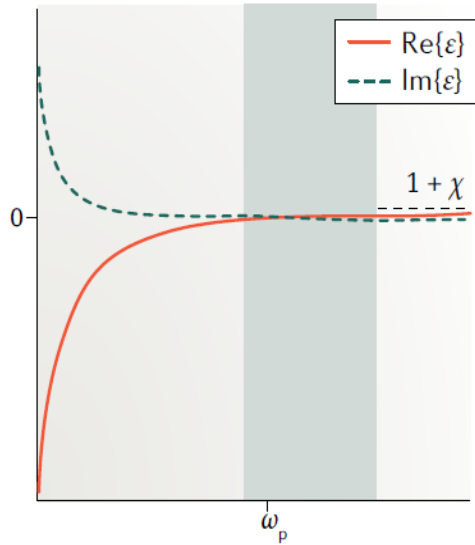
Lobet, Michaël, et al. "New horizons in near-zero refractive index photonics and hyperbolic metamaterials." *ACS photonics* 10.11 (2023): 3805-3820.

# How can we make NZI media?

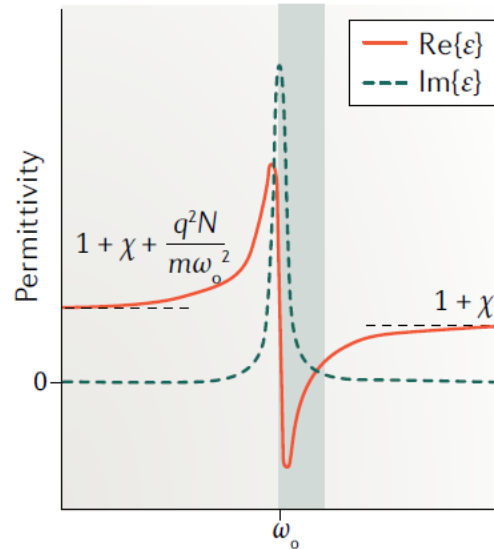
$$\epsilon_{Drude}(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - i\gamma\omega} \quad \text{At } \omega_p, \epsilon \rightarrow 0$$

$$\epsilon_{Lorentz}(\omega) = \epsilon_\infty + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

**b** Drude model



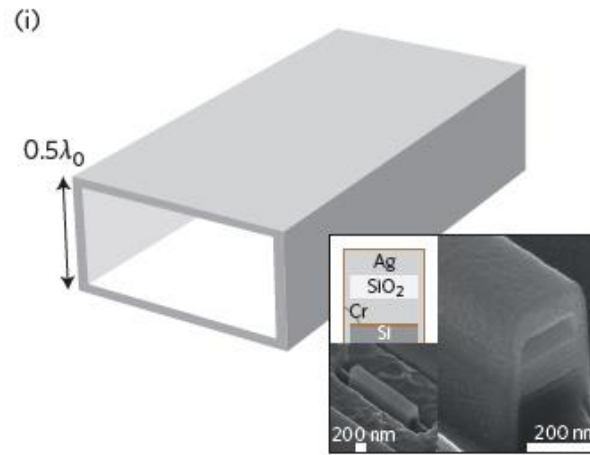
**a** Lorentz model



## Drude/Lorentz materials

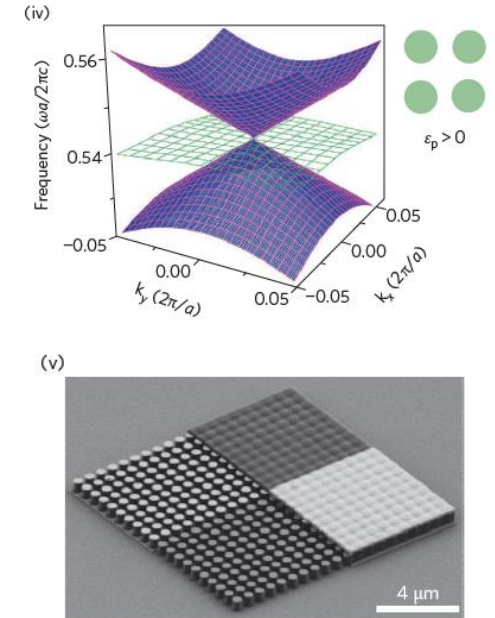
See Kinsey or Liberal/Engheta reviews

**b**



## Waveguides at cut-off

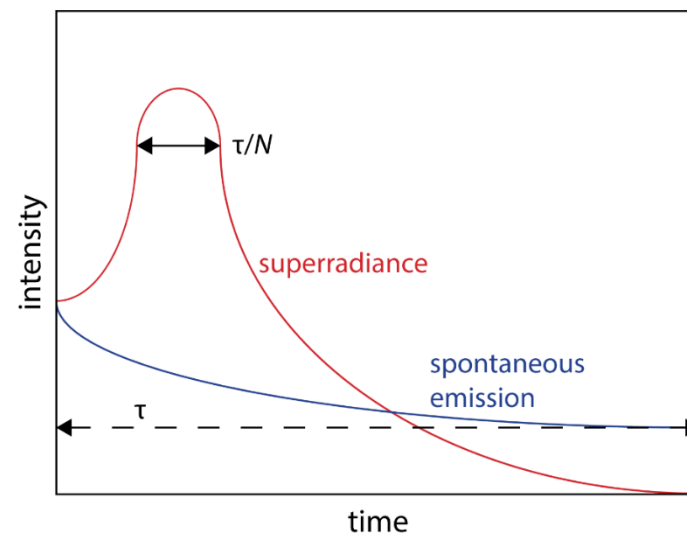
See Engheta/Polman works



## Photonic crystals (Dirac cone MM)

See Mazur/CT Chan works

# How can it be useful for superradiance?



M. Gross and S. Haroche, *Phys. Rep.* (1982)

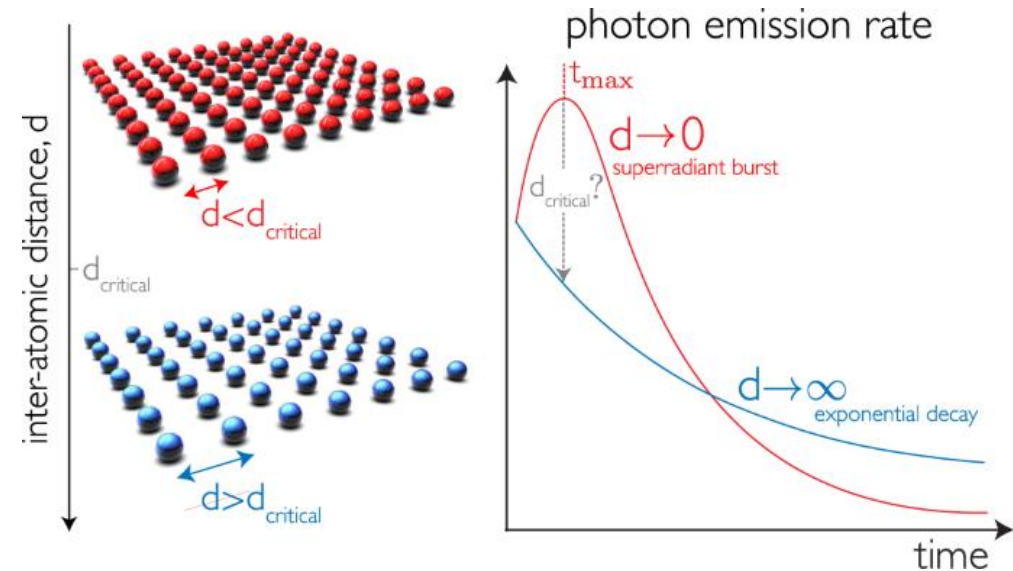
✓ *It's a near-field interaction*



All emitters need to be within  
 $1 \lambda$  from each other

$$\Leftrightarrow kr < \lambda$$

Need to adjust precisely  $r$ , the  
inter-emitter spacing




S. Masson and A. Asenjo-Garcia, *Nat. Com.* (2022)

## **Limitations**

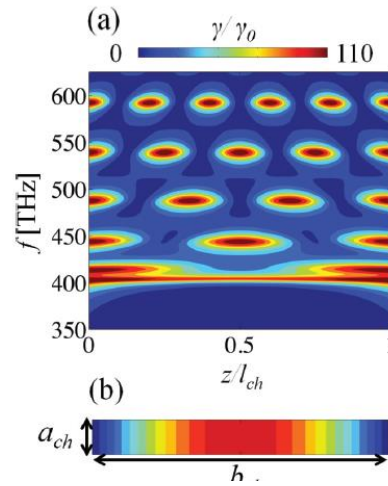
- Small number of atoms/emitters
- Small spatial extent

$$0 \leftarrow n \frac{2\pi}{\lambda_0} r = kr < \lambda = \frac{\lambda_0}{n} \rightarrow \infty$$

*As small as possible    As big as possible*

✓ Use metamaterials?  Use Near-Zero Refractive Index (NZI) materials!  
(No phase change  $\Delta\varphi \rightarrow 0$ )





PHYSICAL REVIEW B **87**, 201101(R) (2013)

## Enhanced superradiance in epsilon-near-zero plasmonic channels

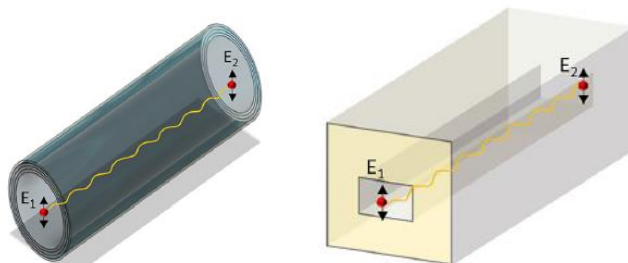
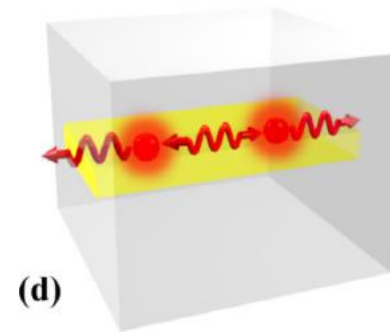
Romain Fleury and Andrea Alù\*

**1D ENZ: *Enhanced* spontaneous emission**



Vol. 24, No. 23 | 14 Nov 2016 | OPTICS EXPRESS 26696

## Controlling collective spontaneous emission with plasmonic waveguides

YING LI AND CHRISTOS ARGYROPOULOS\*

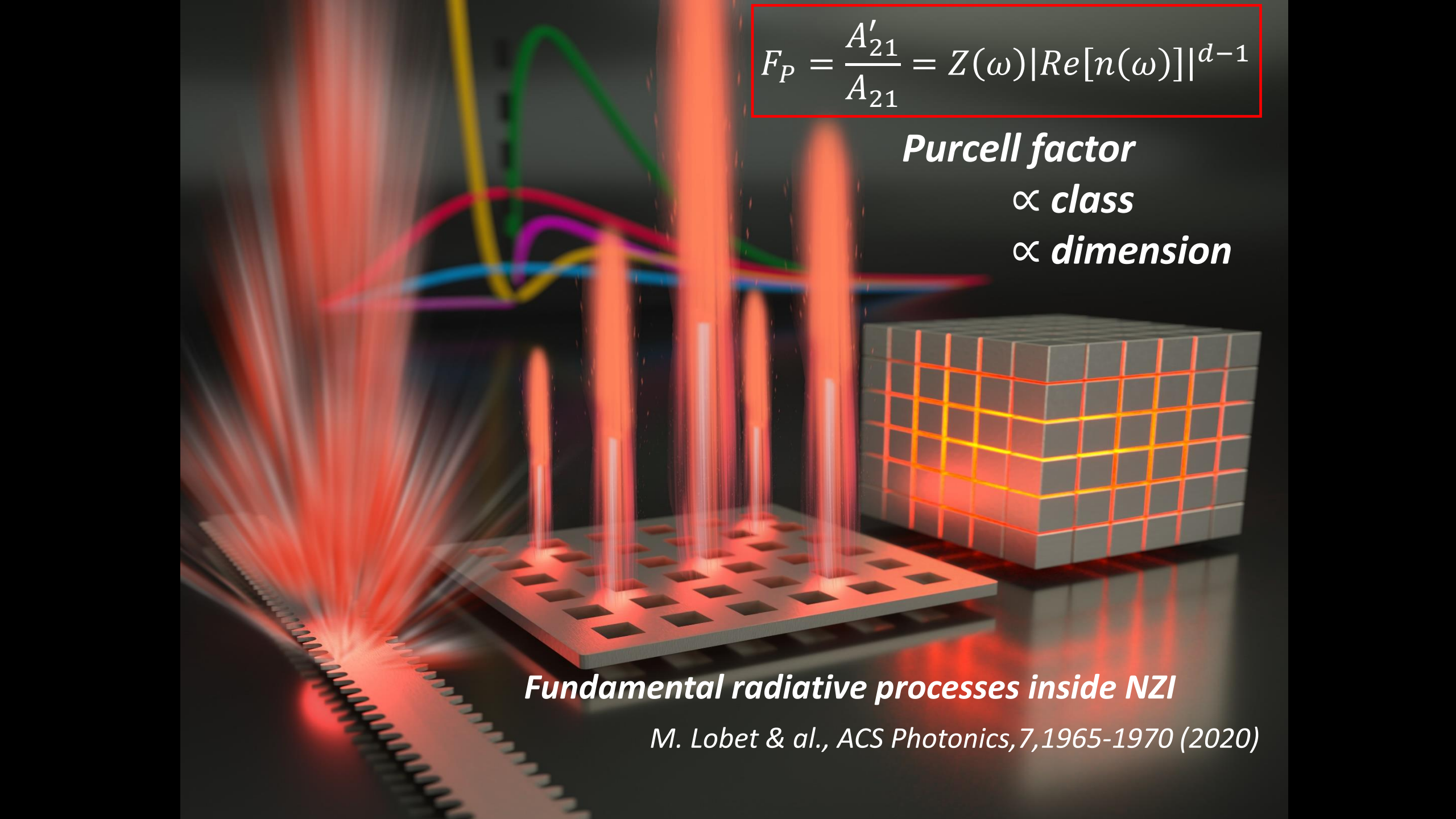


## Qubit-qubit entanglement mediated by epsilon-near-zero waveguide reservoirs EP

Ibrahim Issah  and Humeysra Caglayan<sup>a)</sup> 

Appl. Phys. Lett. **119**, 221103 (2021)




$$F_P = \frac{A'_{21}}{A_{21}} = Z(\omega) |Re[n(\omega)]|^{d-1}$$

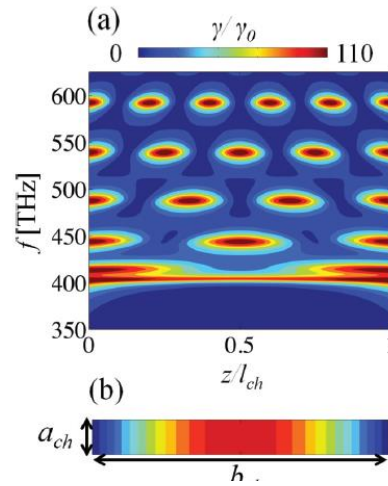
**Purcell factor**

$\propto$  **class**

$\propto$  **dimension**

**Fundamental radiative processes inside NRI**

M. Lobet & al., ACS Photonics, 7, 1965-1970 (2020)



PHYSICAL REVIEW B **87**, 201101(R) (2013)

## Enhanced superradiance in epsilon-near-zero plasmonic channels

Romain Fleury and Andrea Alù\*

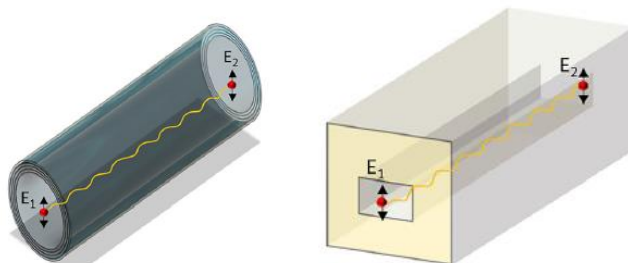
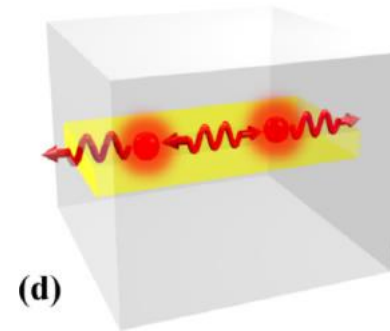
**1D ENZ: Enhanced spontaneous emission**

Vol. 24, No. 23 | 14 Nov 2016 | OPTICS


## Controlling collective spontaneous emission with plasmonic waveguides

YING LI AND CHRISTOS ARGYROPOULOS\*

**! Losses !**



## Qubit-qubit entanglement mediated by epsilon-near-zero waveguide reservoirs

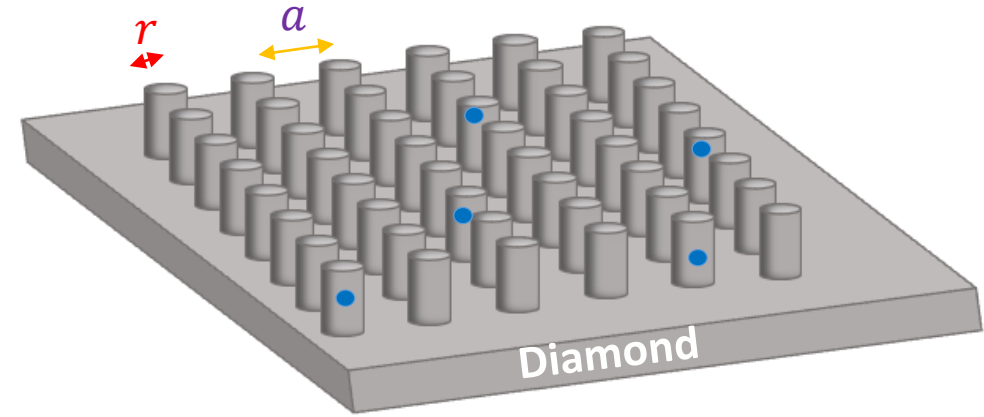
Ibrahim Issah  and Humeysra Caglayan<sup>a)</sup> 

Appl. Phys. Lett. **119**, 221103 (2021)

# Design of NZI platforms

- All-dielectric
- Visible
- Suitable for quantum optics

*2D ENZ diamond  
platform with SiV  
centers*



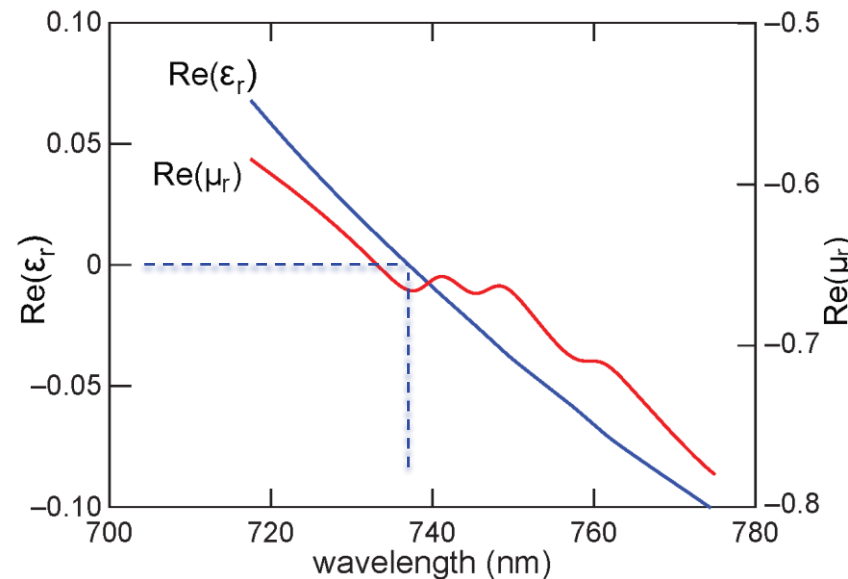
*(emitters operating at 737 nm)*

Tuning  $a$  and  $r$  such that:

$$\varepsilon \rightarrow 0 \text{ at } 737 \text{ nm}$$

$$\mu \rightarrow 0 \text{ at } 737 \text{ nm}$$

*Near-Zero crossing*



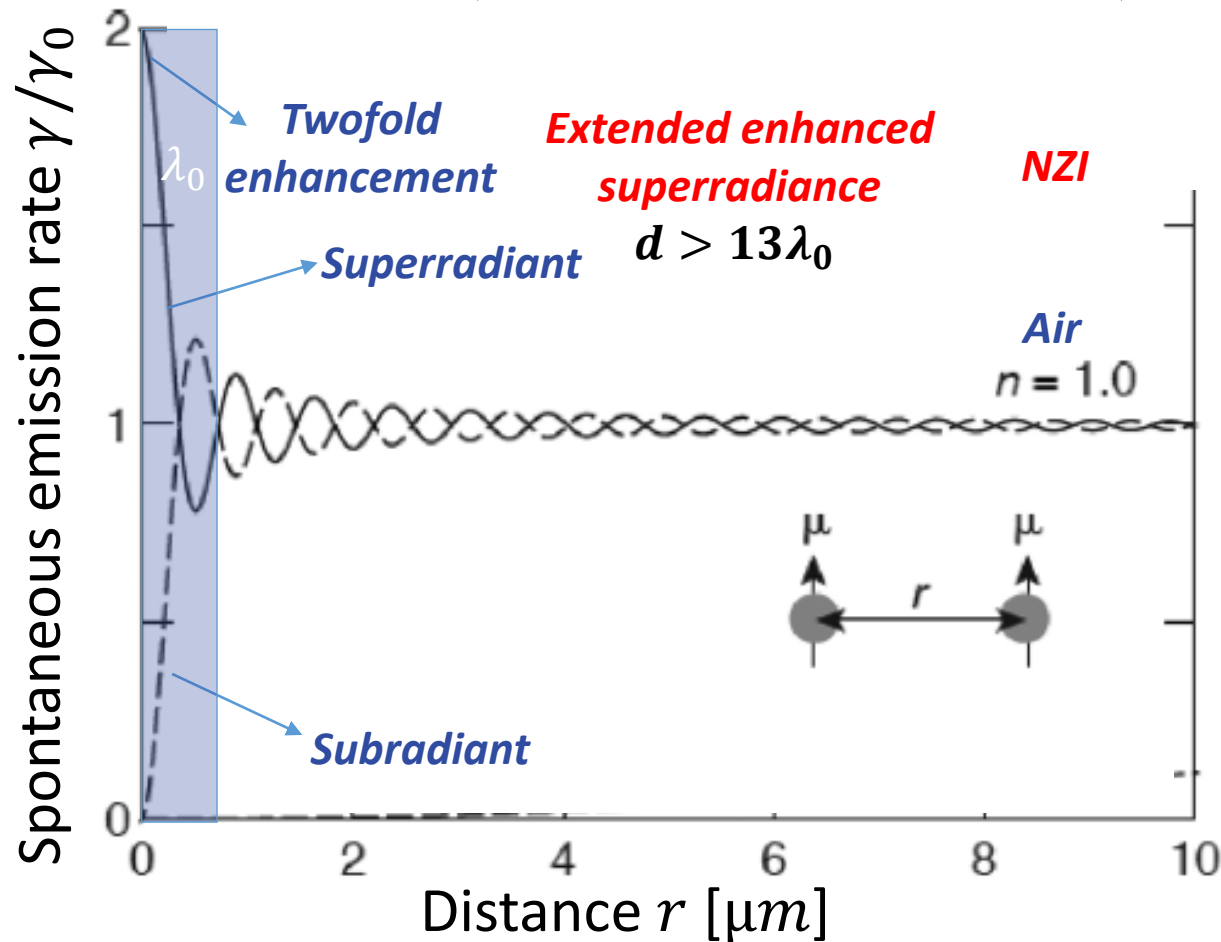
✓ O. Mello et al., Appl. Phys. Lett. 120, 061105 (2022)

✓ O. Mello et al., Light: Sci & App. 14.1, 300 (2025).

# How does it help for superradiance?

## Dicke superradiance for two two-level system in NZI

Spontaneous emission rate  $\gamma_{\pm} = \gamma_0 \left[ 1 \pm \frac{3}{2} \left( \frac{\sin(kr)}{kr} + \frac{\cos(kr)}{(kr)^2} - \frac{\sin(kr)}{(kr)^3} \right) \right]$



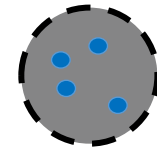
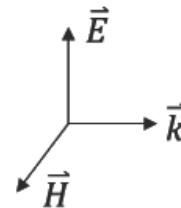
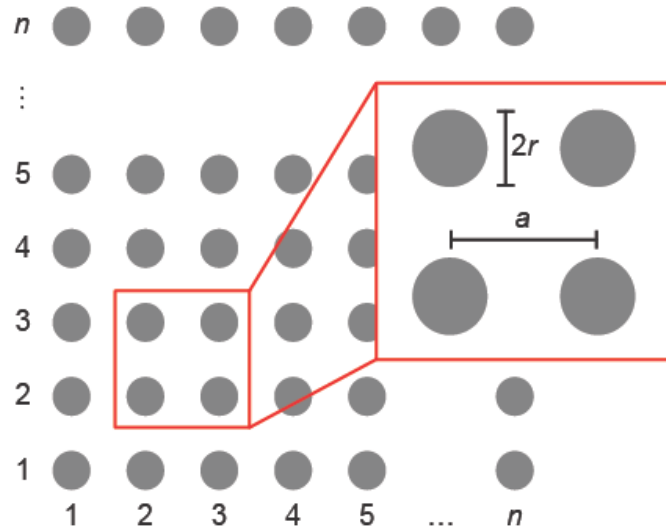
✓ O. Mello et al., Appl. Phys. Lett. 120, 061105 (2022)

✓ O. Mello et al., *Light: Sci & App.* 14.1, 300 (2025).

*What if we increase the number of emitters?*

Coherent emission from  $N$  dipoles?

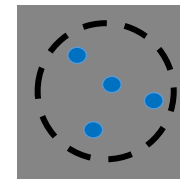
3 situations



**Emitters** randomly distributed inside diamond *pillars*



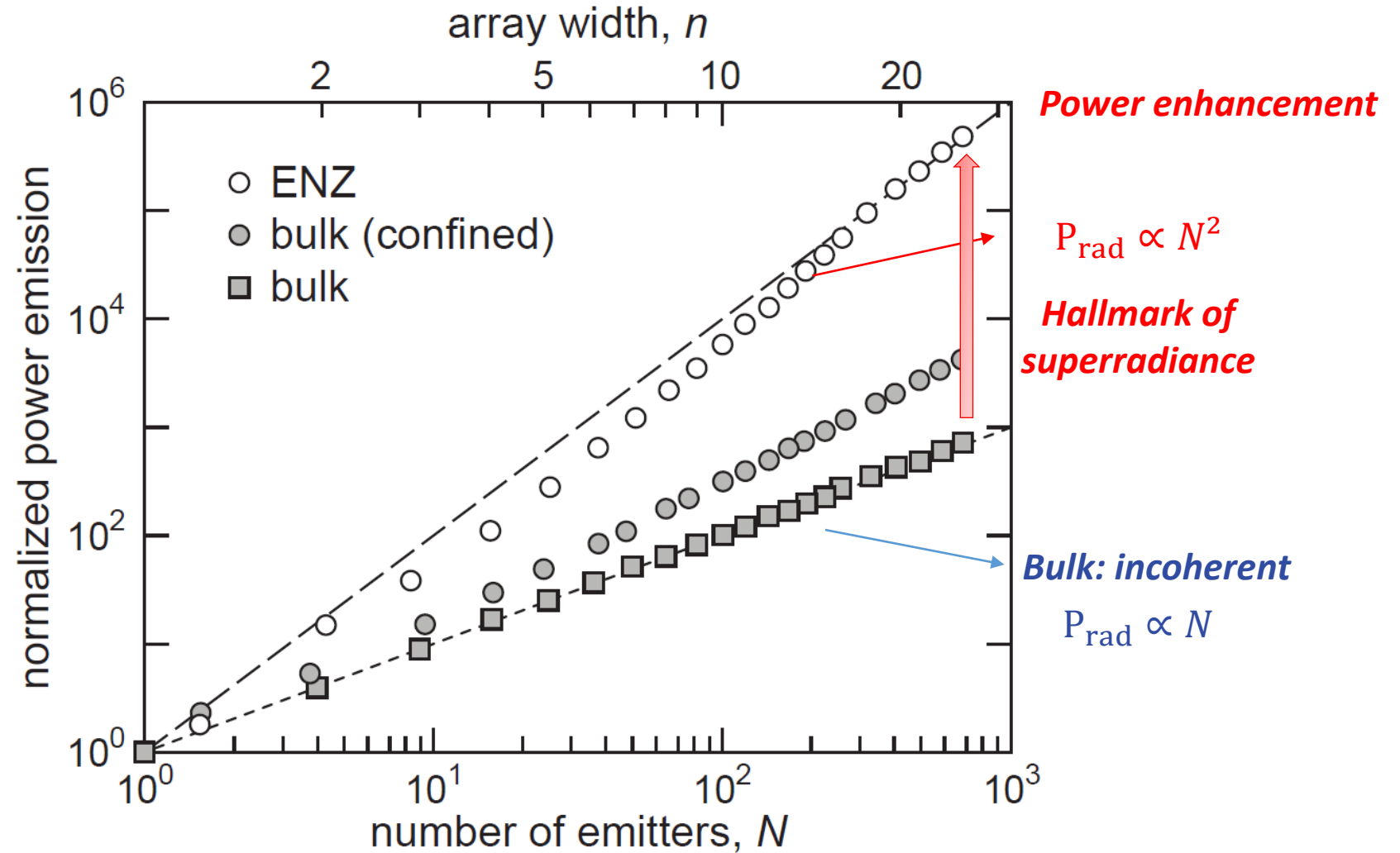
**Emitters** randomly distributed inside *bulk* diamond



**Emitters** randomly distributed inside *confined bulk* diamond

✓ O. Mello et al., Appl. Phys. Lett. 120, 061105 (2022)

# Emission from randomly positioned N dipoles

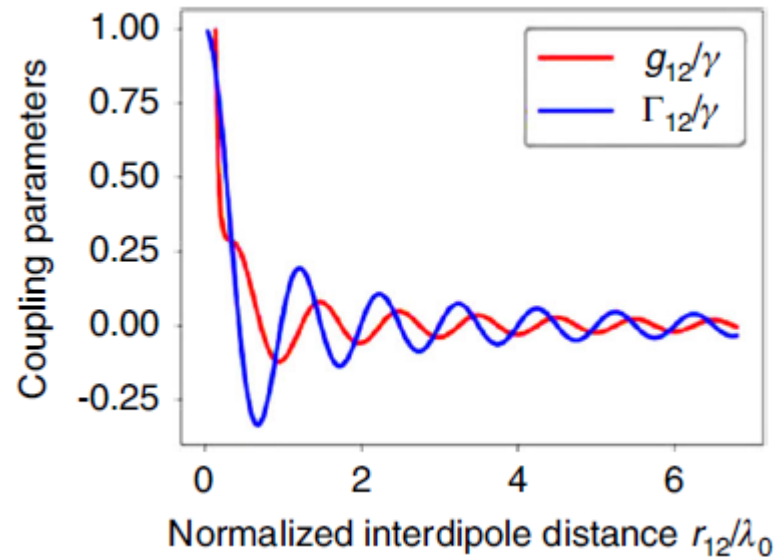


✓ O. Mello et al., Appl. Phys. Lett. 120, 061105 (2022)

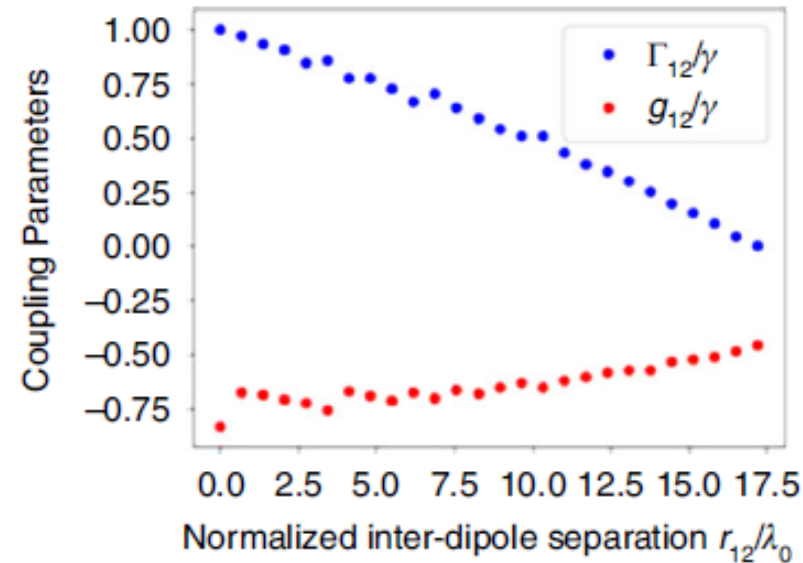


*Dipole-dipole coupling term*

$$g_{ij} = \frac{2\omega_j^2}{\hbar\epsilon_0 c^2} d_i \text{Re} \left[ \vec{G}(r_i, r_j, \omega_j) \right] d_j^*$$



*Free space*



*MNZ metamaterial*

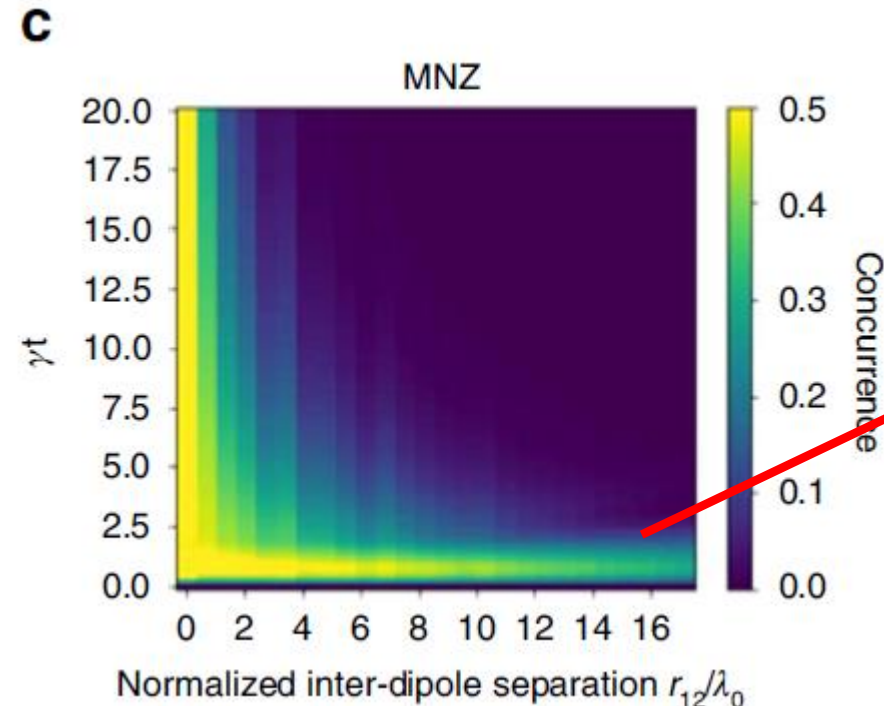
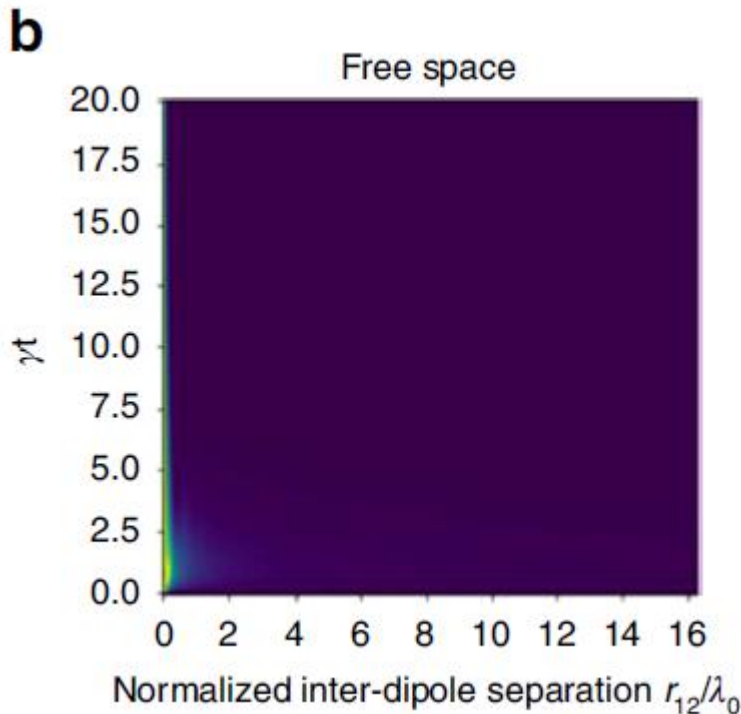
**Having a  $\Gamma_{12} \gg g_{12}$  is ideal for attaining a large transient concurrence**

✓ O. Mello et al., *Light: Sci & App.* 14.1, 300 (2025).

**Concurrence** [Wouters PRL 1998]

$$C = \frac{1}{2} \sqrt{(e^{-(\gamma+T_{12})t} - e^{-(\gamma-T_{12})t})^2 + 4e^{-2\gamma t} \sin^2(2g_{12}t)}$$

**Transient  
concurrence**



$C = 0.35$   
at  $r_{12} = 17\lambda_0$  i.e.  
 $12.5\mu\text{m}$

→ 10x ENZ  
plasmonic  
waveguide

**Robust entanglement**

✓ O. Mello et al., *Light: Sci & App.* 14.1, 300 (2025).

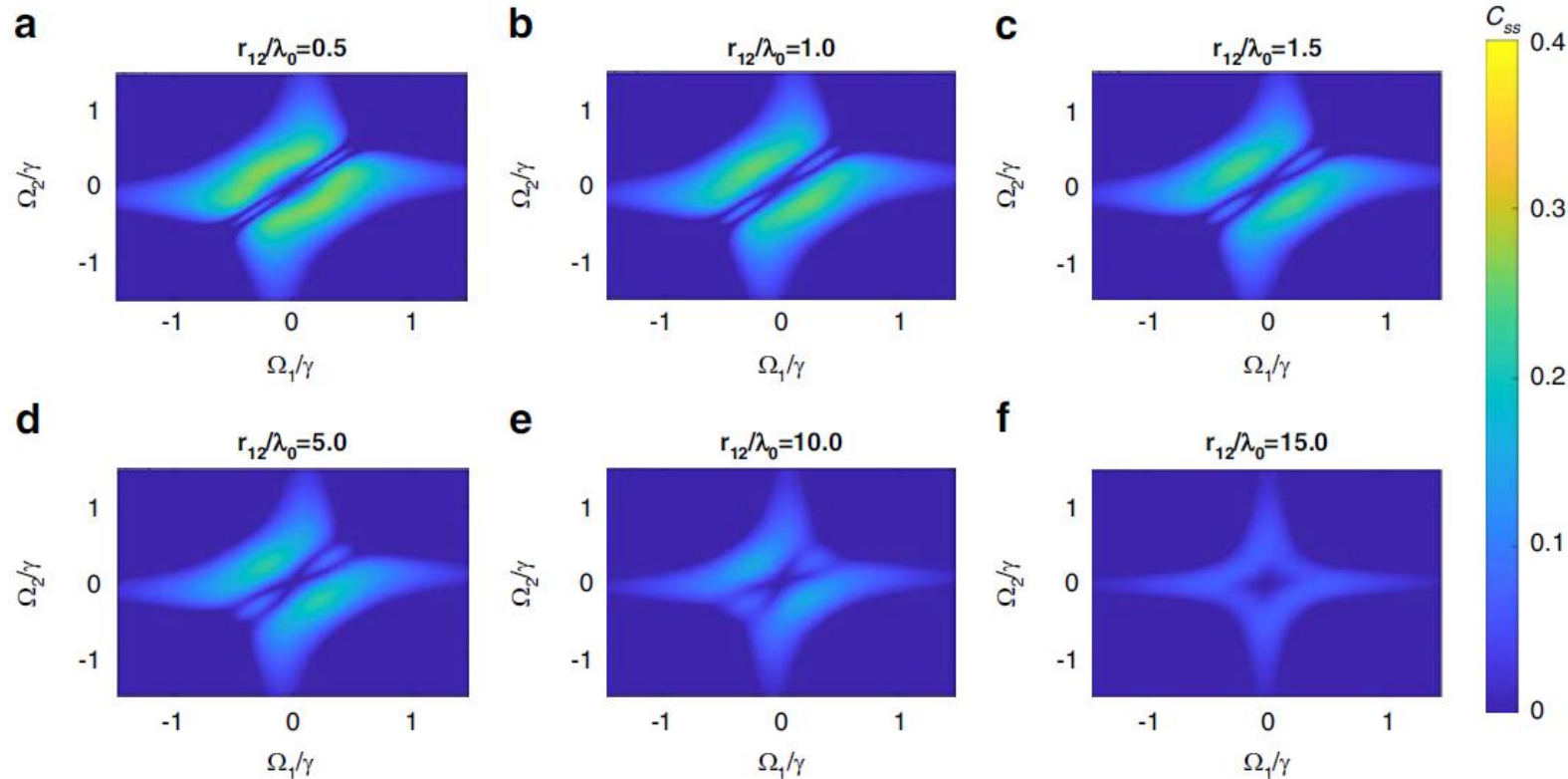


## Steady-state concurrence

Add pumping

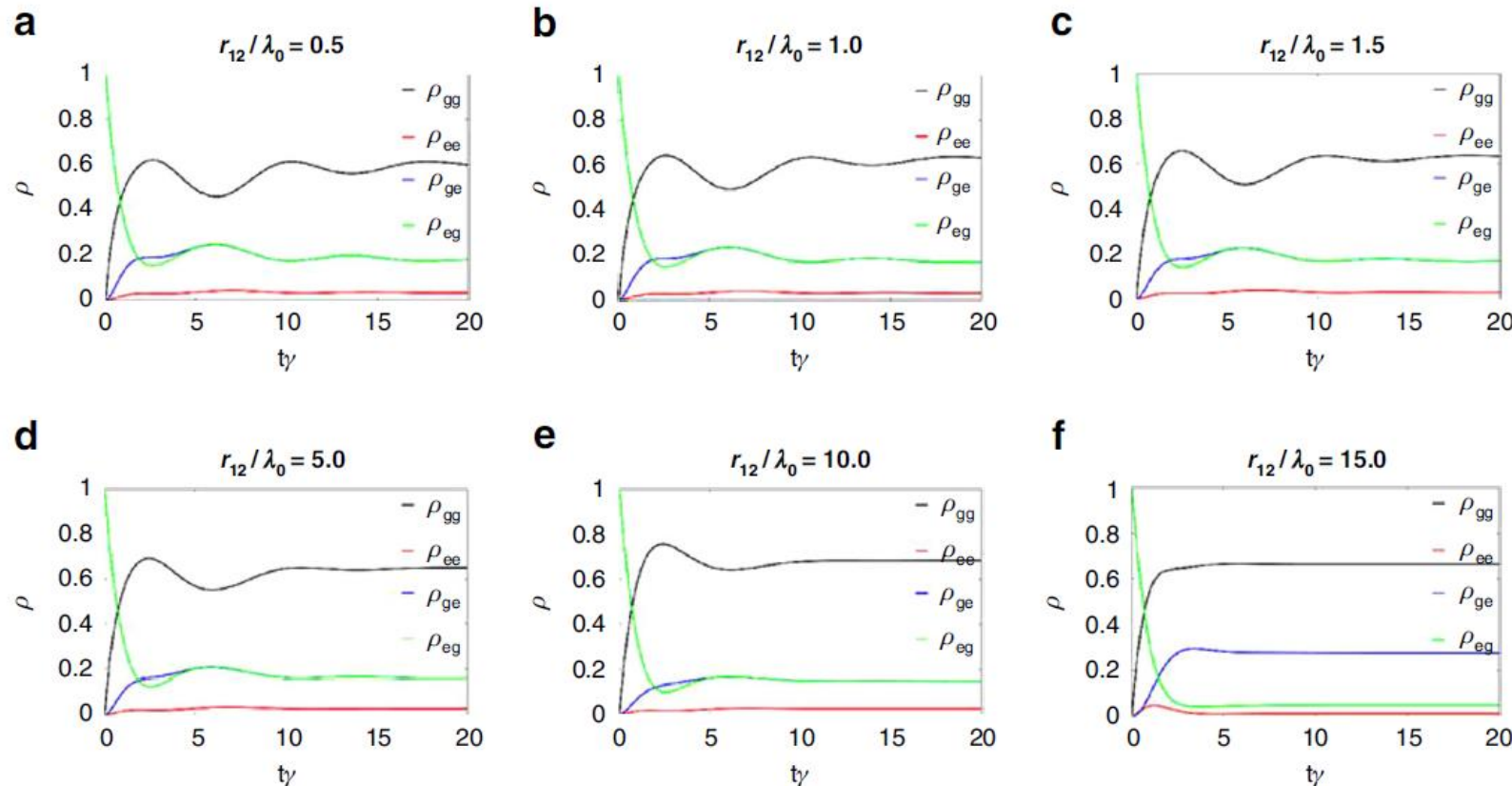
$$V = -\sum_{i=1}^2 \hbar \left( \Omega_i e^{-i\Delta_i t} \sigma_i^\dagger + \Omega_i^* e^{i\Delta_i t} \sigma_i \right)$$

+ solve time evolution of density matrix



**One order of magnitude enhancement in the entanglement range compared to ENZ plasmonic waveguide**

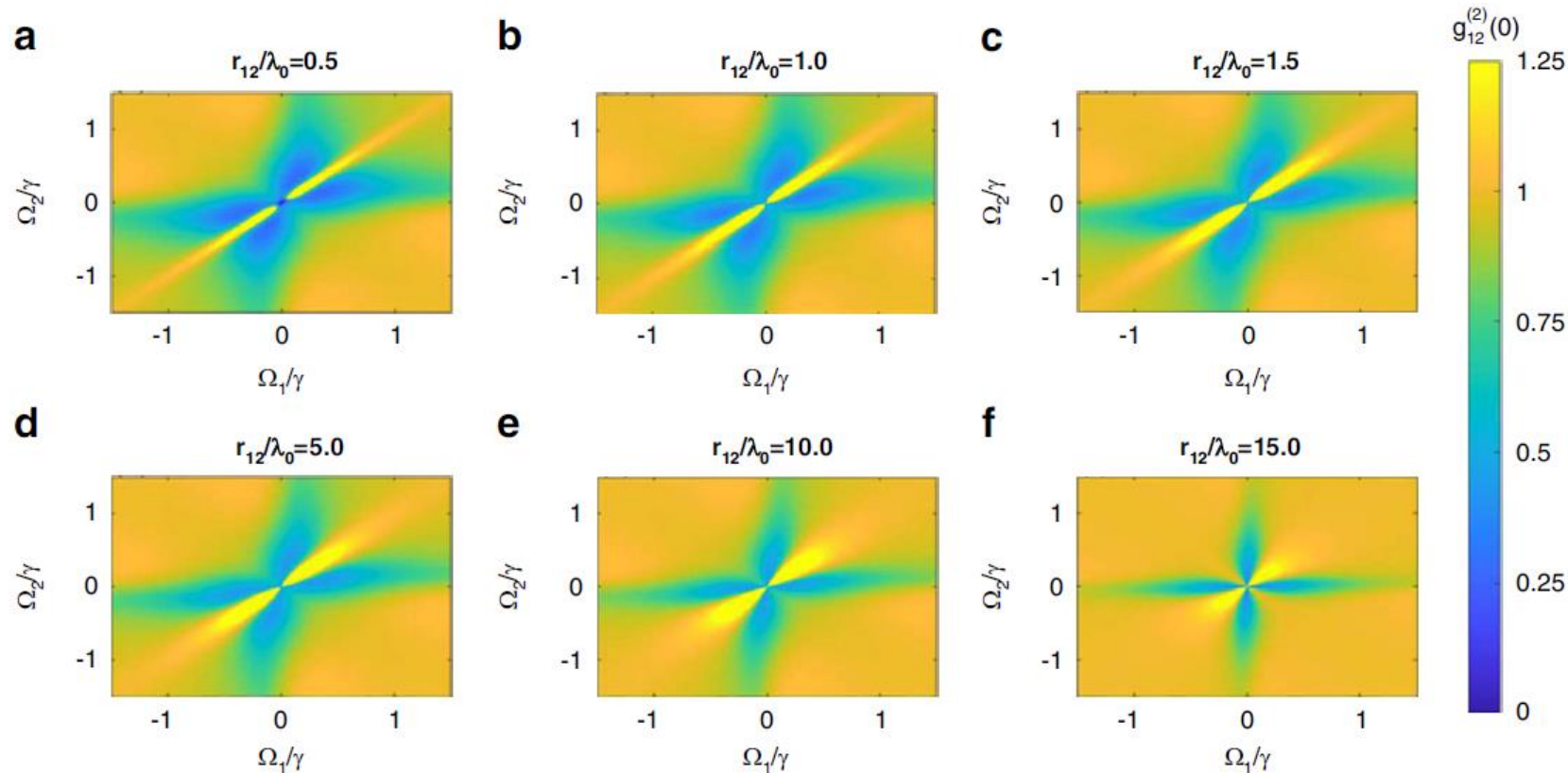
## Time evolution of the probabilities for the basis states



**Steady state at  $yt = 20$**

## Photon–photon intensity correlations at zero-time delay

$$g_{12}^{(2)}(0) = \frac{\langle \sigma_1^\dagger \sigma_2^\dagger \sigma_2 \sigma_1 \rangle}{\langle \sigma_1^\dagger \sigma_1 \rangle \langle \sigma_2^\dagger \sigma_2 \rangle} = \frac{\rho_{ee}}{(\rho_{eg} + \rho_{ee})(\rho_{ge} + \rho_{ee})}$$



High  $C$ ,  
small  $g_{12}^{(2)}(0)$

With the increasing separation between the quantum emitters, the parameter space for antibunching shrinks as expected.

Transitions from bunching to antibunching ( $g_{12}^{(2)}(0) = 0$ , correlated with entanglement) depending on the values of Rabi frequencies  $\Omega_1, \Omega_2$

## Superradiance and near-zero refractive index materials: a win-win cooperation?

Yes!

*That would be hype*

Yes, maybe, if we  
can prove it  
experimentally

*That's about trust*



# Acknowledgments



O. Melo



L. Vertchenko



S. Nelson



D. Guney



E. Mazur

Thank you for your attention





# CONGRATULATIONS NOBEL PRIZE SERGE HAROCHE



Made with Copilot October 7th



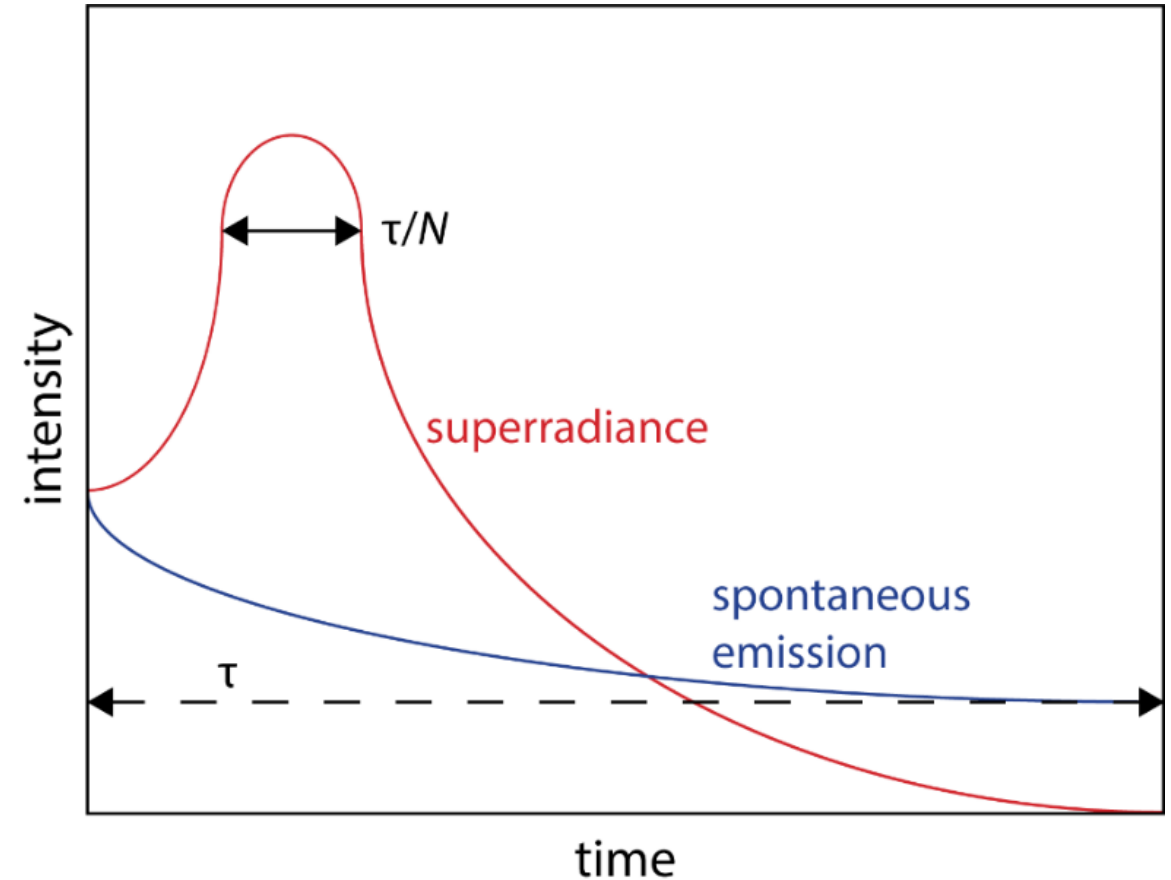
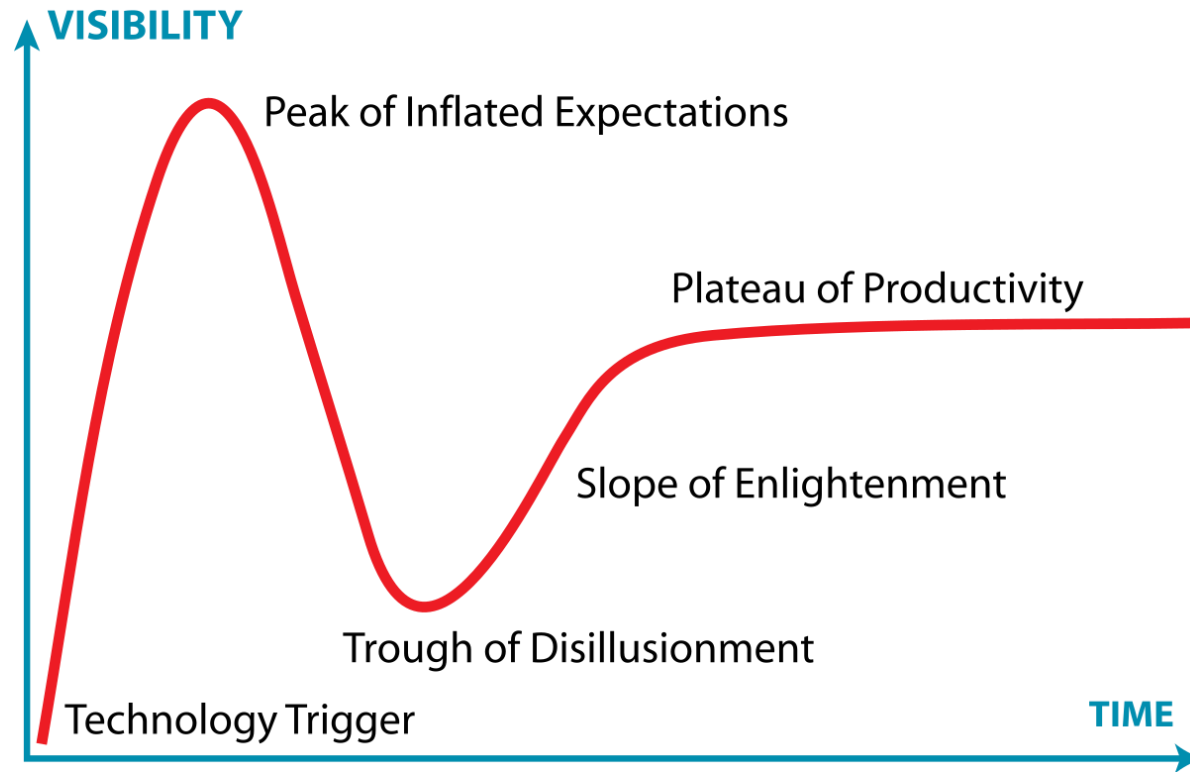
# Conclusions

---

- ✓ 2D ENZ low loss metamaterial platform compatible for quantum optics
- ✓ High degree of superradiance for distance greater than  $13\lambda_0$
- ✓ Power enhancement over 3 orders of magnitude higher than bulk for large arrays with  $N^2$  scaling



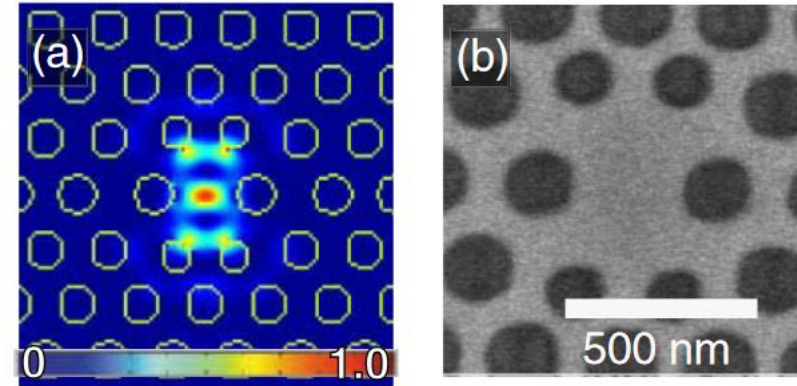
# Conclusions



# Literature review

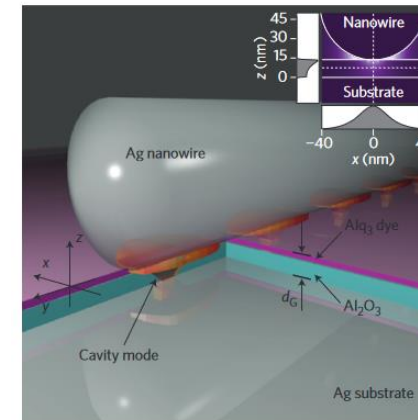
## Strategies

✓ Use photonic crystals

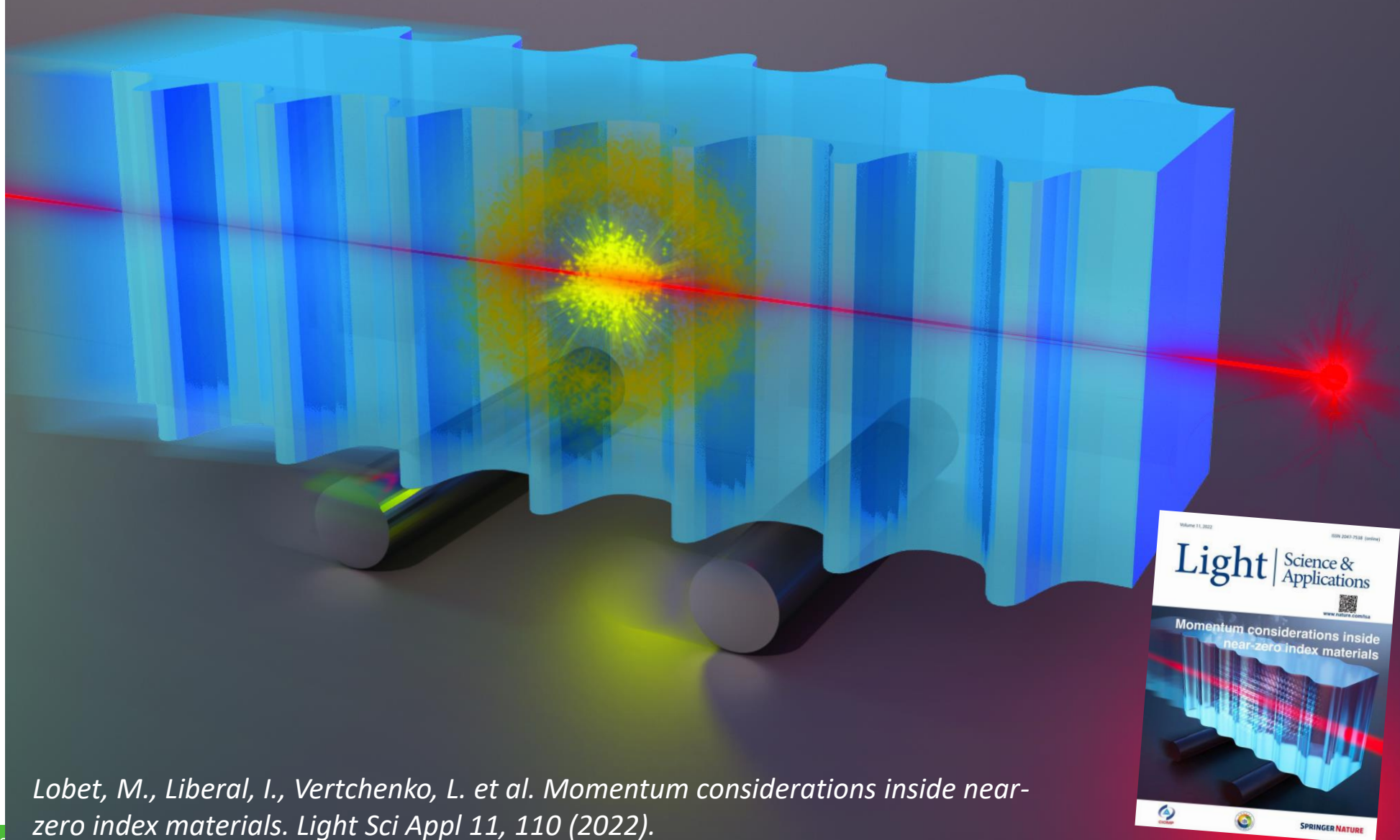


D. Englund, ..., J. Vuckovic, *PRL*. (2005)

✓ Use (plasmonic) cavities

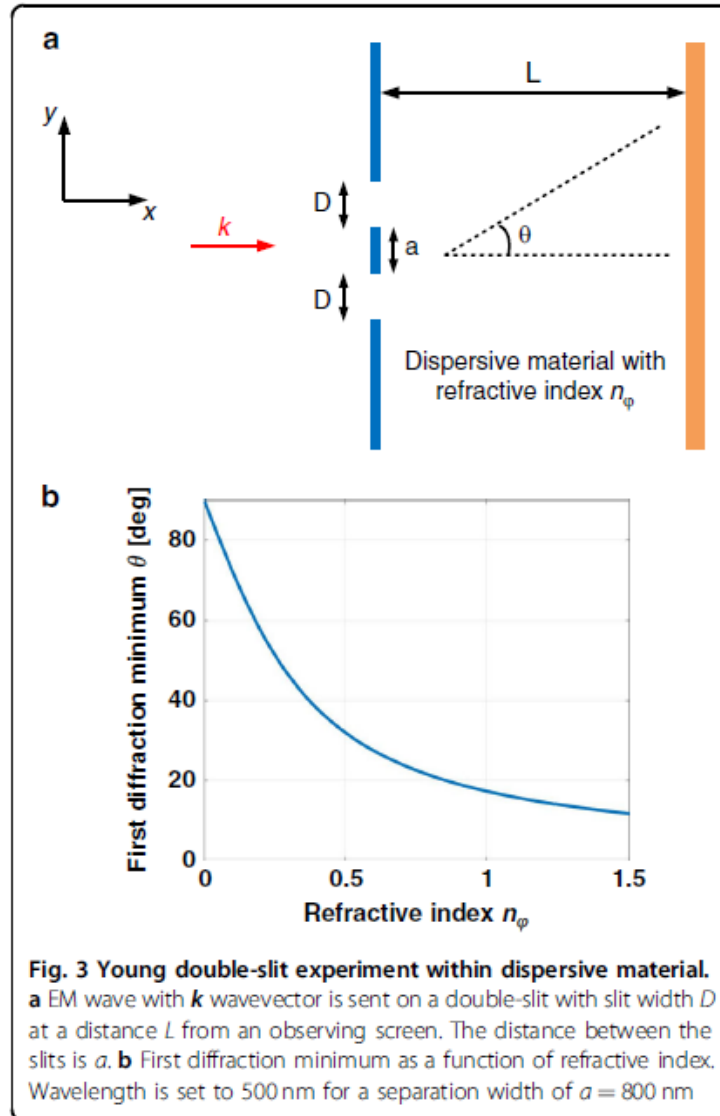


K. Russel, ... E. Hu, *Nat. Phot.* (2012)



Lobet, M., Liberal, I., Vertchenko, L. et al. Momentum considerations inside near-zero index materials. *Light Sci Appl* 11, 110 (2022).

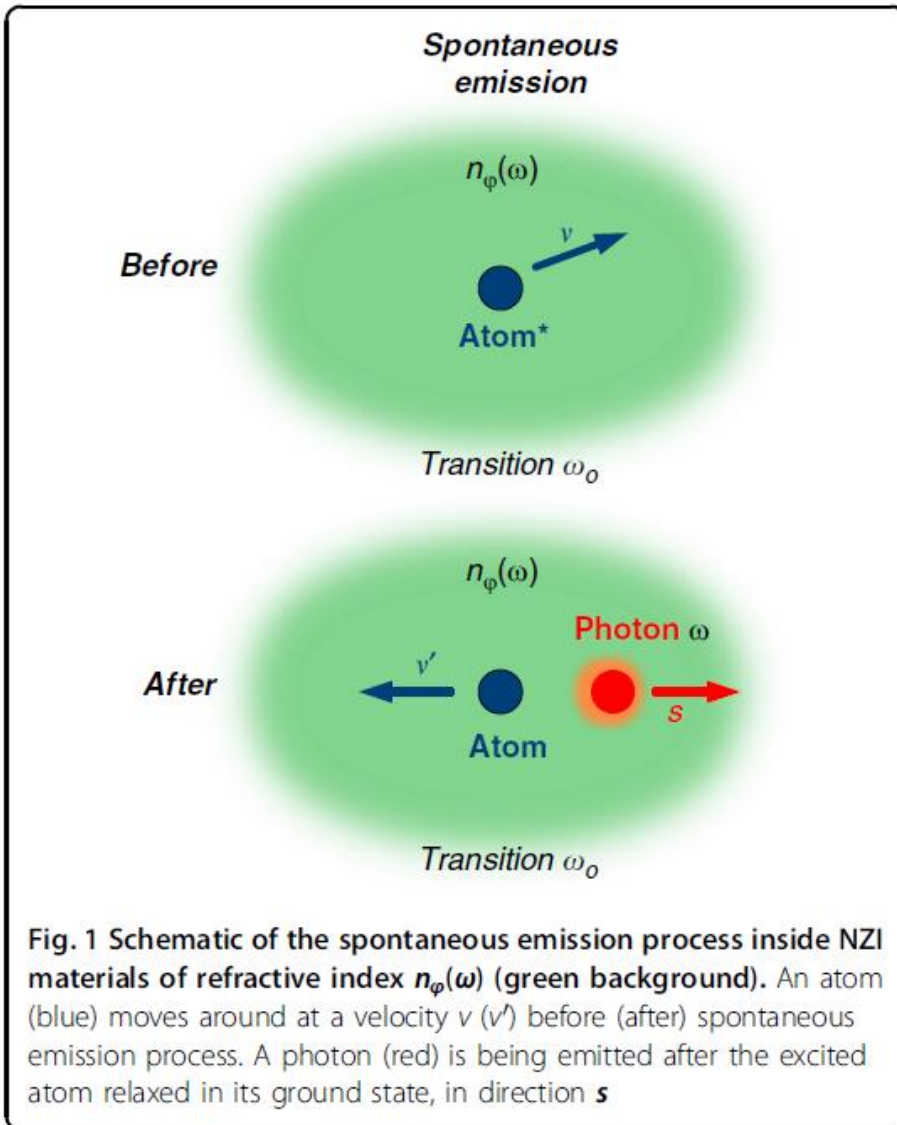
# Applications of near-zero index materials



$$\tan(\theta) = \frac{\lambda_0}{2a|n_\varphi|}$$

$$I(y) = \frac{I_0}{2} \frac{\sin^2\left(\frac{\pi Dy}{\lambda L}\right)}{\left(\frac{\pi Dy}{\lambda L}\right)} \left[ 1 + \cos\left(\frac{2\pi ay}{\lambda L}\right) \right]$$

# Applications of near-zero index materials



In the non-relativistic approximation<sup>41</sup>, conservation of energy for the spontaneous emission process implies

$$\frac{mv^2}{2} + \hbar\omega_0 = \frac{mv'^2}{2} + \hbar\omega \quad (7)$$

while the conservation of linear momentum can be expressed as

$$m\mathbf{v} = m\mathbf{v}' + \hbar\mathbf{k} \quad (8)$$

with  $\mathbf{k} = [n_{\varphi}(\omega) \frac{\omega}{c}] \mathbf{s}$ ,  $\mathbf{s}$  being a unit vector pointing in the direction of the emitted photon and  $-\hbar\mathbf{k}$  is the recoil momentum of the atom. As is well known from classical physics, the frequency of the emitted light, as it appears to the moving atom, is increased due to the Doppler shift. The Doppler shift formula can be deduced as<sup>42,43</sup>

$$\omega = \omega_0 \left[ 1 + \frac{n_{\varphi}(\omega)}{c} v \cos\theta \right] \quad (9)$$

where  $\theta$  is the angle between  $\mathbf{v}$  and  $\mathbf{s}$ . In general,  $\hbar\mathbf{k}$  is not solely the momentum of the emitted photon, but corresponds to the total momentum transferred from the atom to both the emitted photon and the medium<sup>6</sup>.

# Applications of near-zero index materials

quantization in the dielectric<sup>9</sup>. Both approaches yield to the conclusion that the recoil momentum is the canonical momentum:

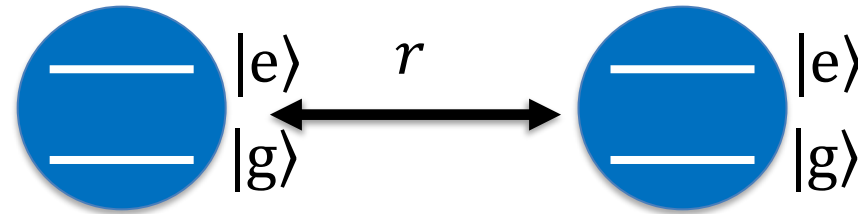
$$p_C = n_\varphi(\omega_0) \frac{\hbar\omega_0}{c} \quad (10)$$

Consequently, the recoil momentum vanishes inside NZI materials. Moreover, the Doppler shift perceived by the atom also vanishes as the phase refractive index goes to zero (Eq. (9)). This extinction of the Doppler shift can be understood as a continuous transition between inverse Doppler effect occurring in negative index materials<sup>44–46</sup>

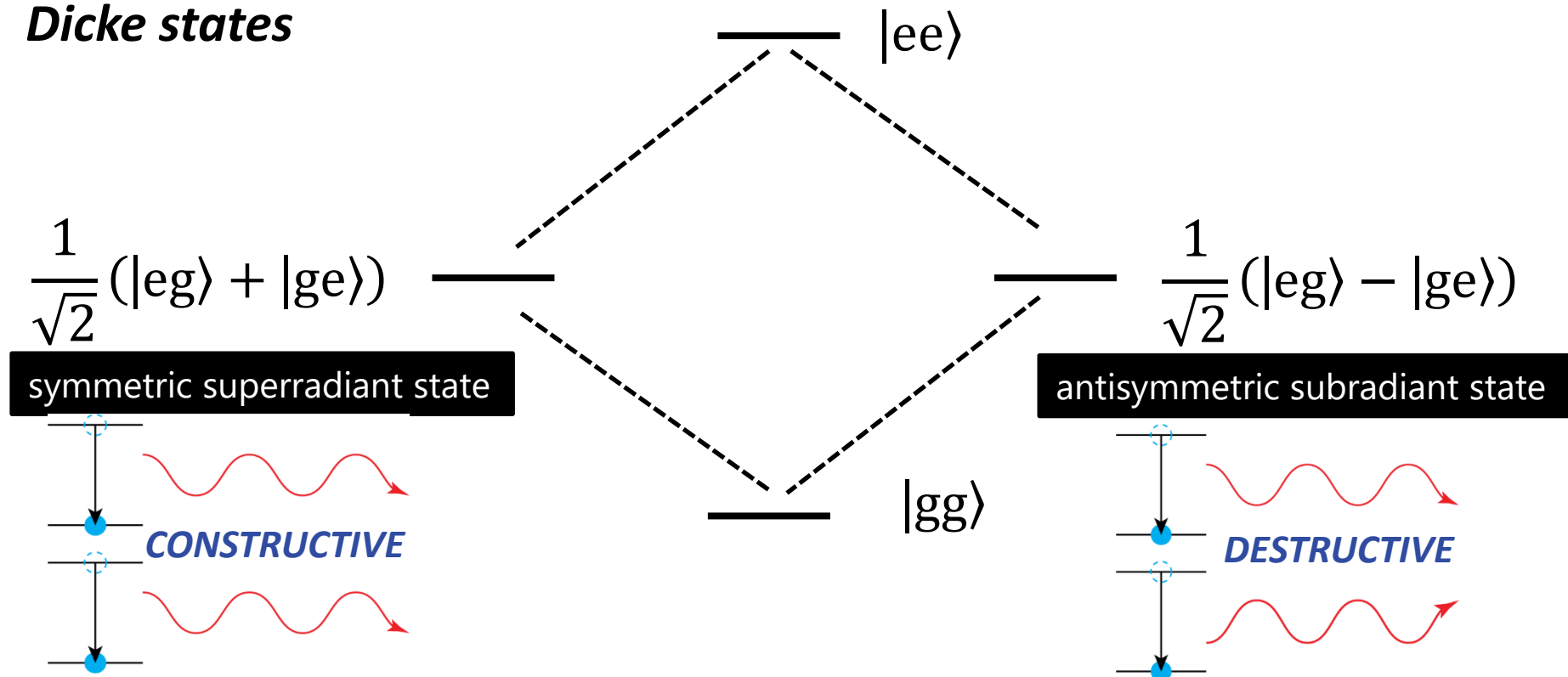


# Dicke superradiance for two two-level system

## Analytical model



## Dicke states



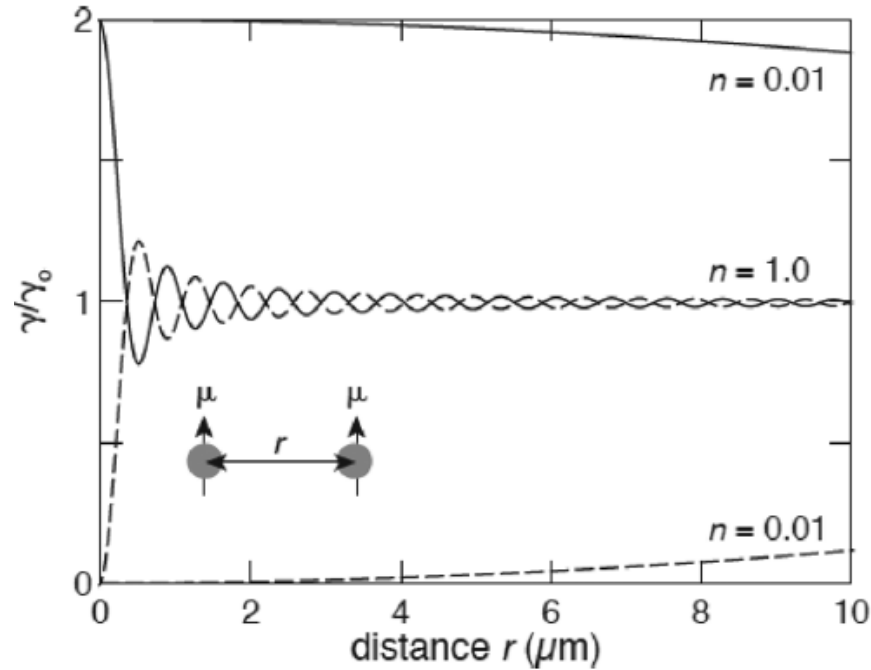
# Dicke superradiance for two two-level system in 2D ENZ

## Spontaneous emission rate

### Two-atom decay rate

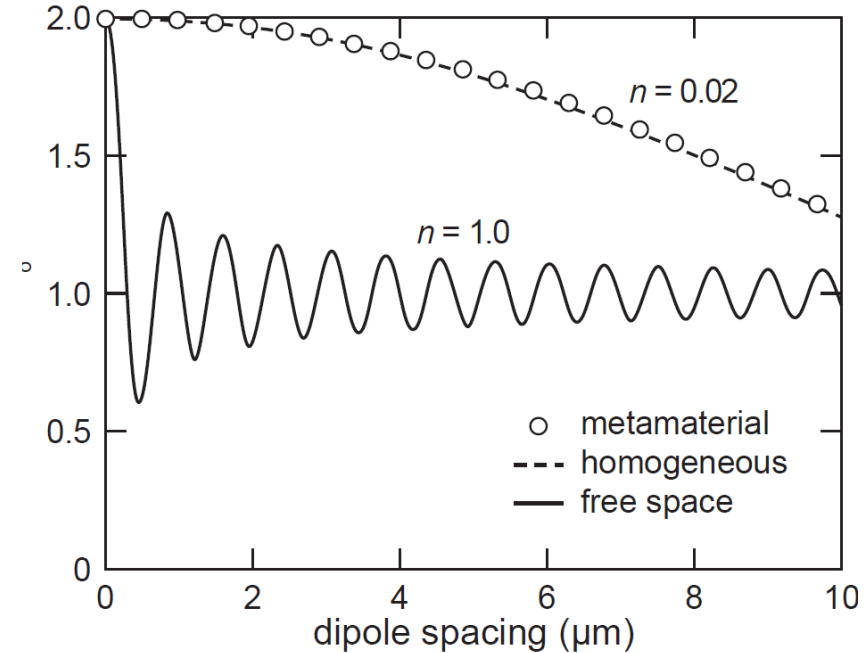
Homogeneous NZI

(Analytical model)



### Two-atom decay rate

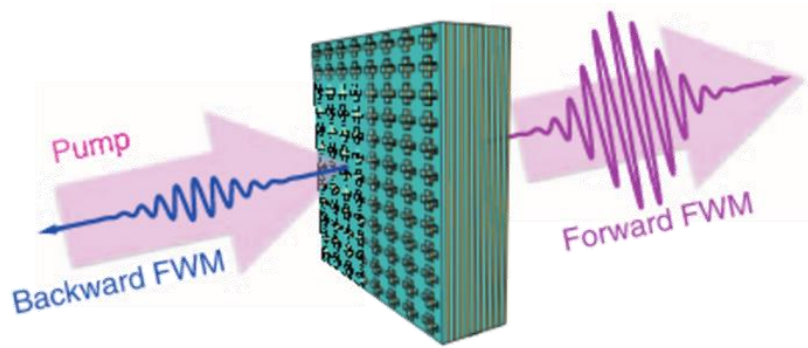
2D ENZ diamond platform SiV centers  
(COMSOL simulations)





# Applications of near-zero index materials

Nonlinear phase matching

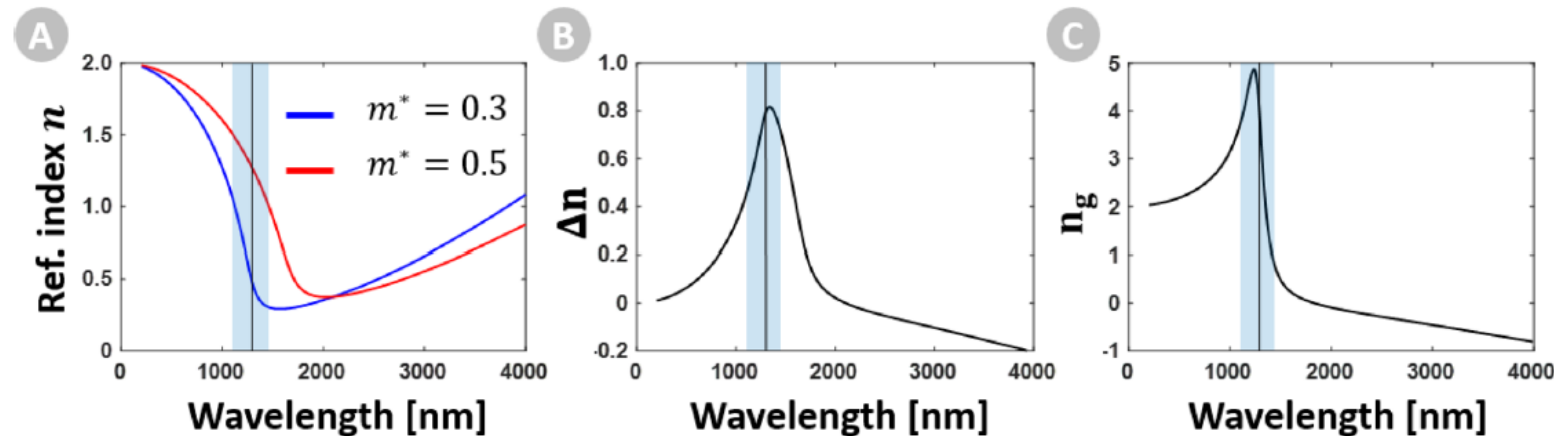


*H. Suchowski et al, Science, vol. 342, 2013*

Nonlinear optics

$$n_2 = \frac{3\chi^{(3)}}{4n_0\Re[n_0]\epsilon_0 c}$$

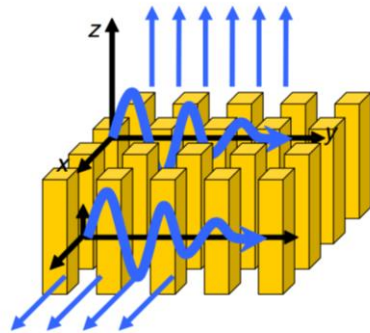
*M. Zahirul Alam et al, Science, vol. 352, 2016*



*M. Lobet et al, ACS Photonics, 10, 3805–3820 (2023).*

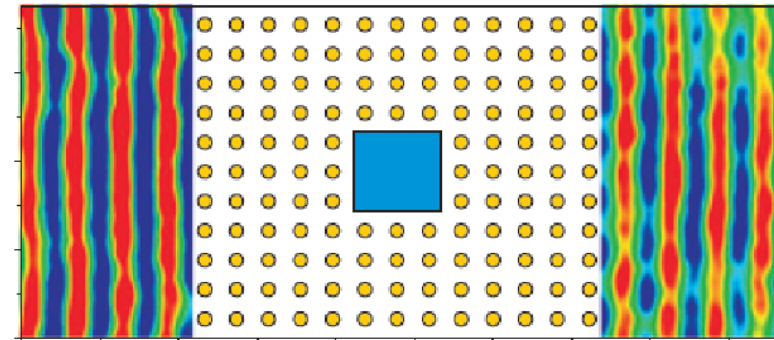
# Applications of near-zero index materials

Beam steering



*M. Memarian et al, Nature Comm., vol. 6, 2015*

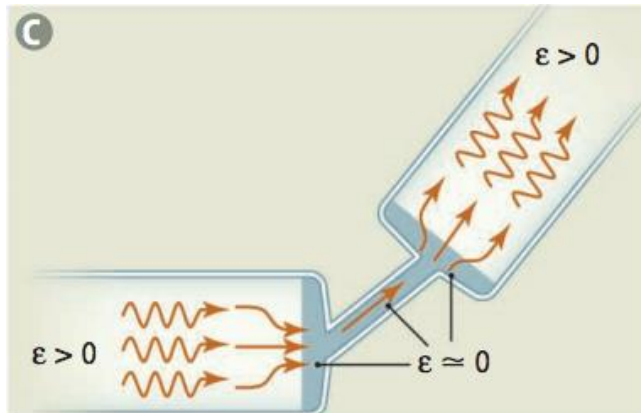
Cloaking



*X.Q. Huang et al, Nature Materials, vol. 10, 2011*

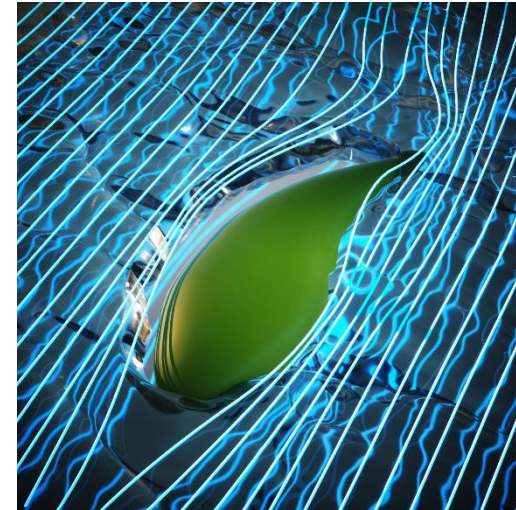
# Applications of near-zero index materials

Supercoupling



*M. Silveirinha & N. Engheta, PRL 97, 2006 / PRB 76, 2007 / N. Engheta, Science, vol. 340, 2013*

Ideal fluid



*I. Liberal, PNAS, 2020*

# Applications of near-zero index materials

$$\mathbf{S} = \mathbf{S}_R + i\mathbf{S}_I = \frac{1}{2} \mathbf{E} \times \mathbf{H}^*$$

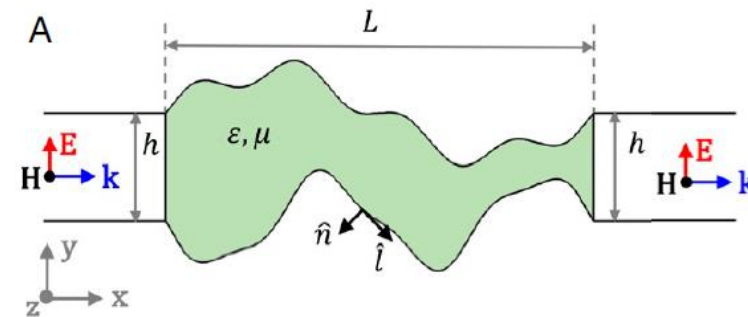
$$\nabla \cdot \mathbf{S} = -\frac{\omega}{2} [(\epsilon_0 \epsilon'' |\mathbf{E}|^2 + \mu_0 \mu'' |\mathbf{H}|^2) + i(\epsilon_0 \epsilon' |\mathbf{E}|^2 - \mu_0 \mu' |\mathbf{H}|^2)].$$

→ 0 Incompressible

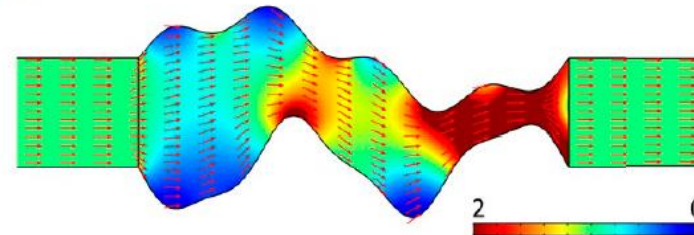
$$\nabla \times \mathbf{S} = \frac{1}{2} [(\mathbf{H}^* \cdot \nabla) \mathbf{E} - (\mathbf{E} \cdot \nabla) \mathbf{H}^* + \mathbf{E} (\nabla \cdot \mathbf{H}^*) - \mathbf{H}^* (\nabla \cdot \mathbf{E})].$$

→ 0 for TM fields in ENZ materials

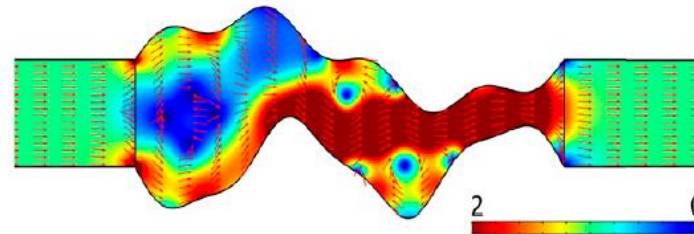
Irrotational → No vortex



B EMNZ Supercoupling ( $\epsilon, \mu \approx 0$ )

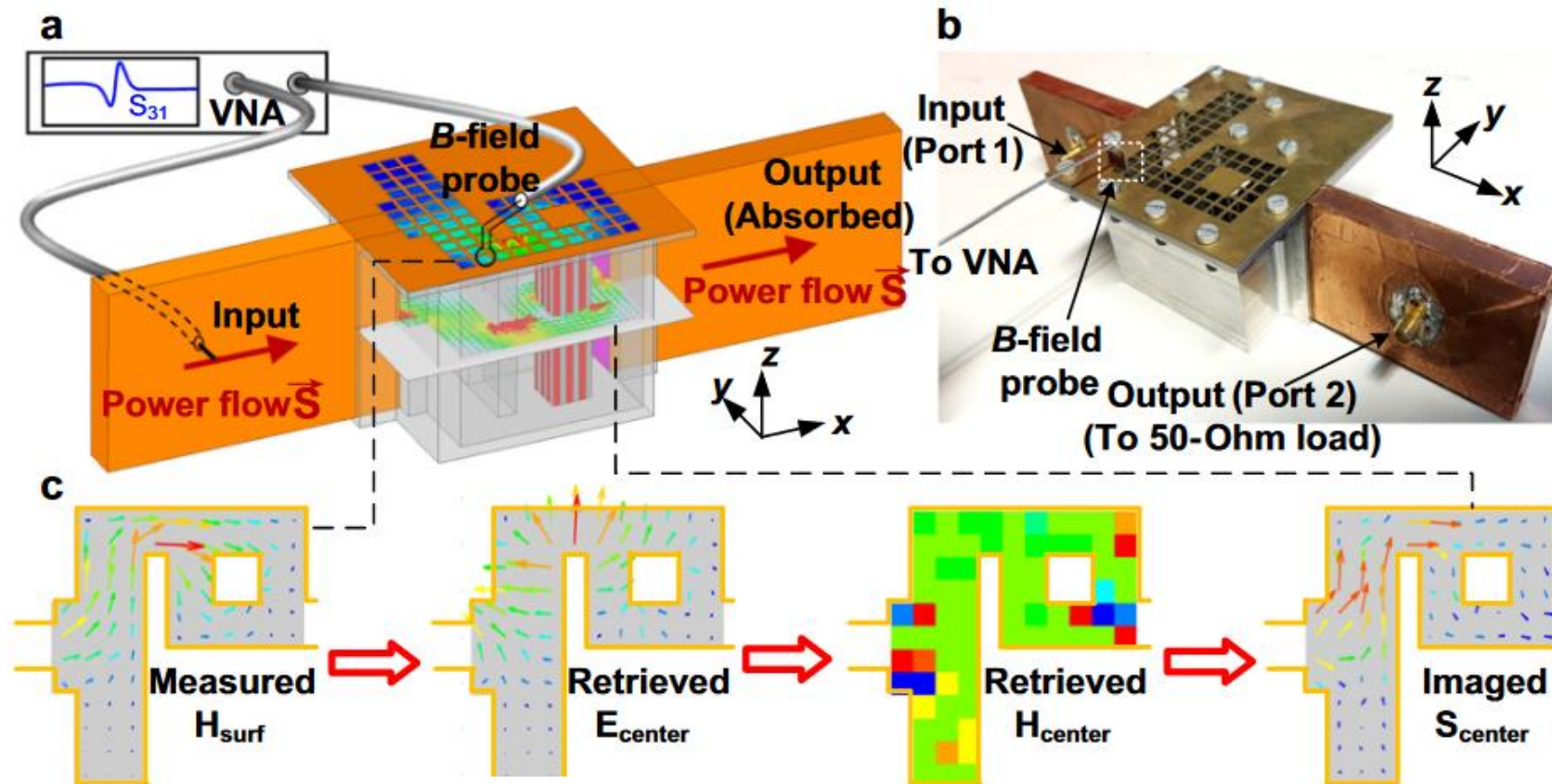


C Resonant dielectric coupling ( $\epsilon = 6.42, \mu = 1$ )



*I. Liberal, PNAS, 2020*

# Applications of near-zero index materials

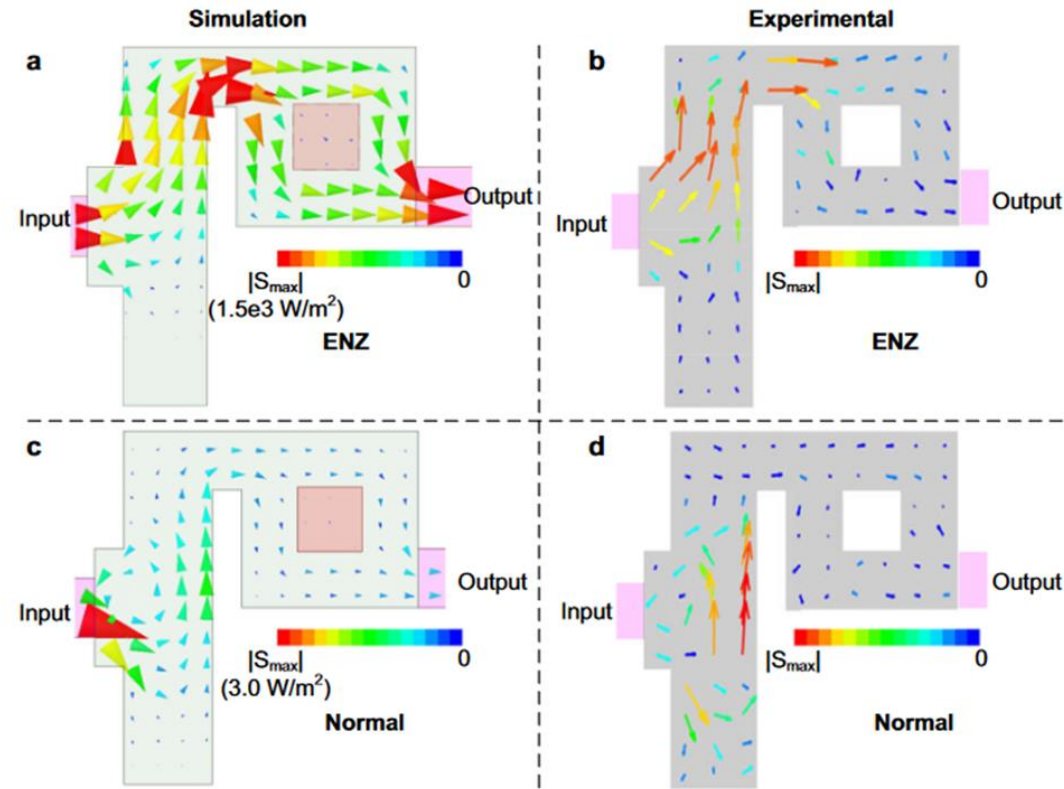


$$\mathbf{E}(x,y,0) = \frac{i\omega\mu_0 h}{\pi} \mathbf{H}(x,y,\frac{h}{2}) \times \hat{\mathbf{z}}. \quad \mathbf{H}(x,y,0) = \frac{1}{i\omega\mu_0} \nabla_{xy} \times \mathbf{E}(x,y,0). \quad \mathbf{S}_R(x,y,0) = \frac{1}{2} \text{Re}[\mathbf{E}(x,y,0) \times \mathbf{H}(x,y,0)].$$

H. Li & al. Nature Communications 13 (2022)



# Applications of near-zero index materials



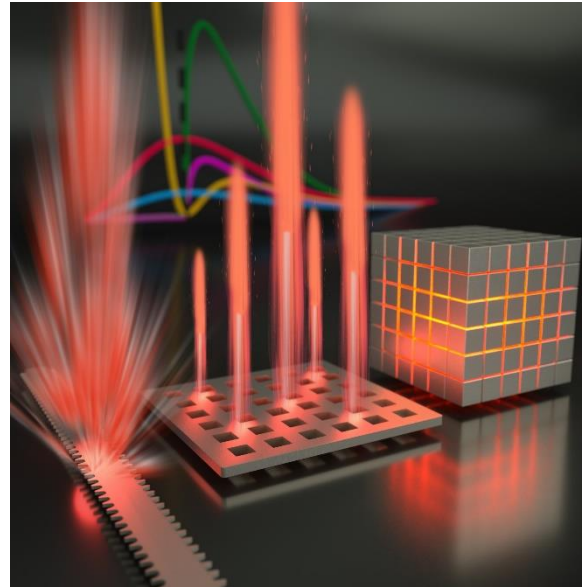
**Fig. 3 | Simulated and experimental observation of the power flow within doped ENZ and normal media. a, b** the simulated and experimentally imaged power flow of the doped ENZ medium at 3.06 GHz, **c, d** the simulated and

experimentally reconstructed power flow of the normal medium with the same dielectric particle but the operating frequency at 3.9 GHz.

*H. Li & al. Nature Communications 13 (2022)*

# Applications of near-zero index materials

Modified fundamental radiative processes



*M. Lobet , ACS Photonics, 2020*

# Applications of near-zero index materials

$$\varepsilon(\omega) = \mu(\omega) = \frac{\omega^2 - \omega_Z^2 + 2i\omega\Gamma}{\omega^2 - \omega_r^2 + 2i\omega\Gamma}$$

(No losses)

$$\omega_r = 0.1\omega_Z$$

