Perfect Vortex Beams for photon entanglement
Orbital Angular Momentum for quantum interplaying

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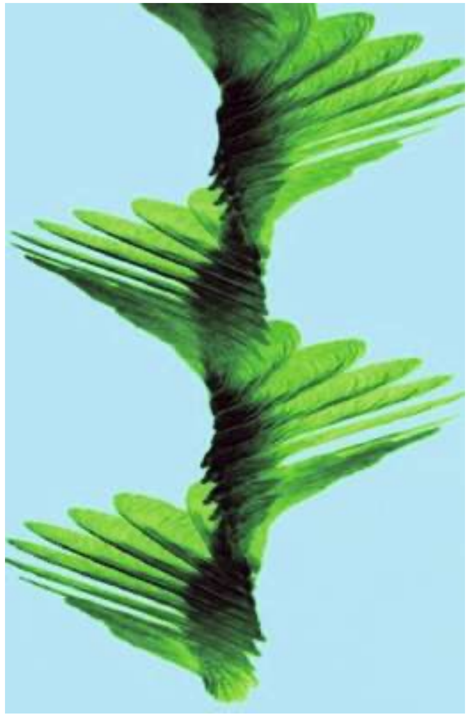
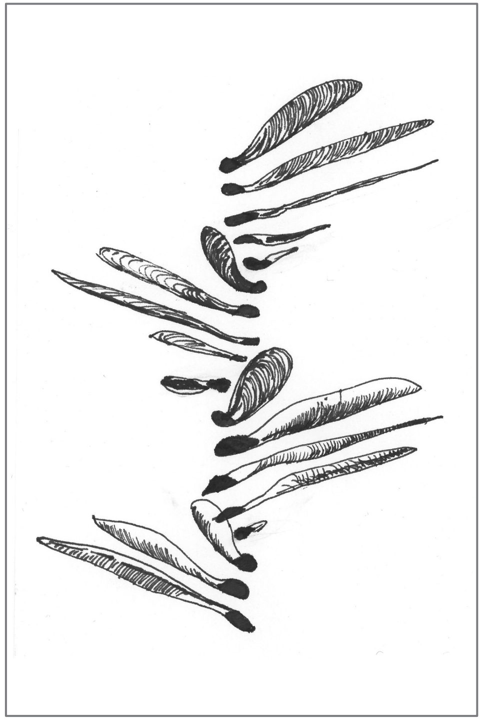
100 Years of Quantum Physics– october 2025





Falling maple fruit; photo of David Lentink & Illustration by Angèle Brune, from the chronophotography.

Introduction

Discrete quantum variables of photon

- **SAM** : Spin Angular Momentum (or polarisation)
  two values ➤ qubit
- **OAM** : Orbital Angular Momentum (phase twist)
  $\ell \in \mathbb{Z}$ ➤ qudit

Processes to entangle, store, …

Atom-light interaction

Non-linear processes

Outline

1. Vortex beams and OAM

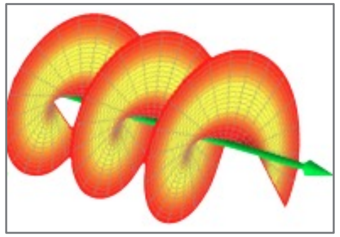
2- Photon pairs

3- Exp : Conversion of Laguerre-Gaussian beams

4- Using Perfect Vortex beams

5- Conclusion

Photonic orbital angular momentum (OAM)



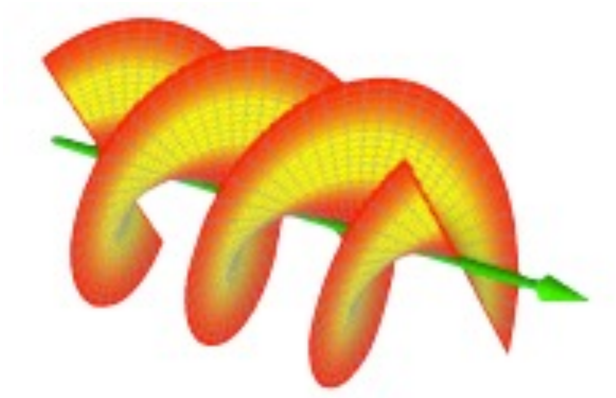
Optical Vortex – Orbital Angular Momentum

A vortex beam carries a helical phase (Coullet 1989)

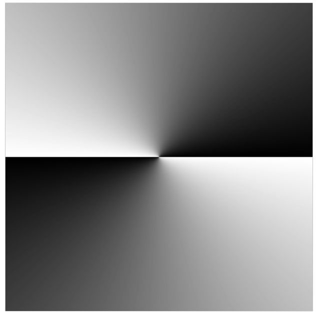
$$\varphi = \ell\theta \text{ with } \ell \in \mathbb{Z}$$

An OAM is associated to the singularity (Allen 1992)

Quantized as $\hbar\boldsymbol{\ell}/\text{photon}$



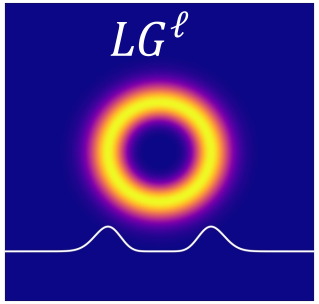
$\ell = 2$



Phase at z

Ex: Laguerre-Gauss

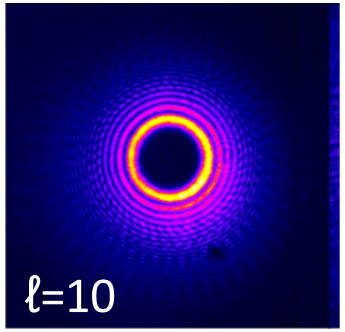
$$e^{ikz}\, e^{i\ell\theta} . \left(\frac{2r^2}{w^2}\right)^{\ell/2} . e^{-\frac{r^2}{w^2}}$$



Experimentally

High OAM vortex beams are currently generated

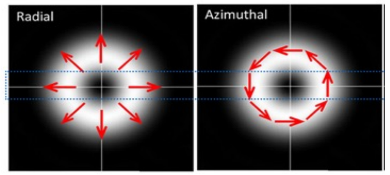
But ring radius and thickness depend on ℓ



Vortex beams - **Orbital Angular Momentum**

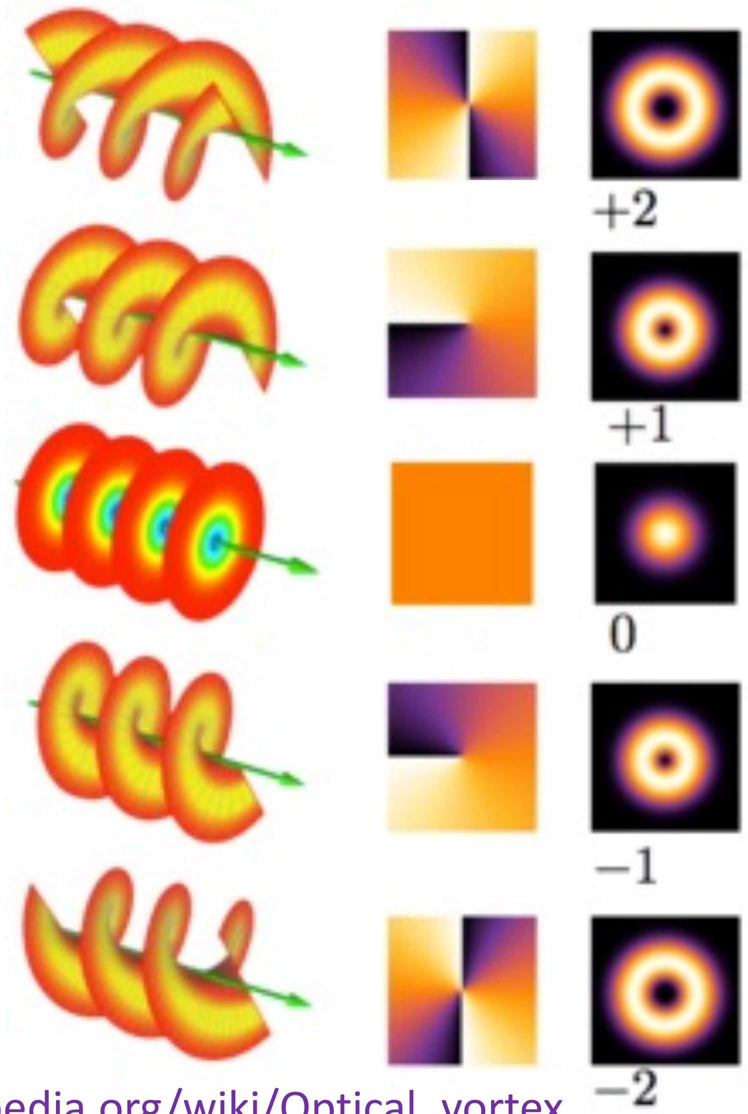
Properties

- Handedness
- A large space for encoding → **qudit**
- Large compare to SAM (polarisation)
- Carried by waves -> transmission over long distances
- Entangled by non linear processes (Zeilinger 2001)
- Multi-entanglement as $|\ell\rangle\ \ell'\rangle$
- OAM & SAM coupling (vector beams)



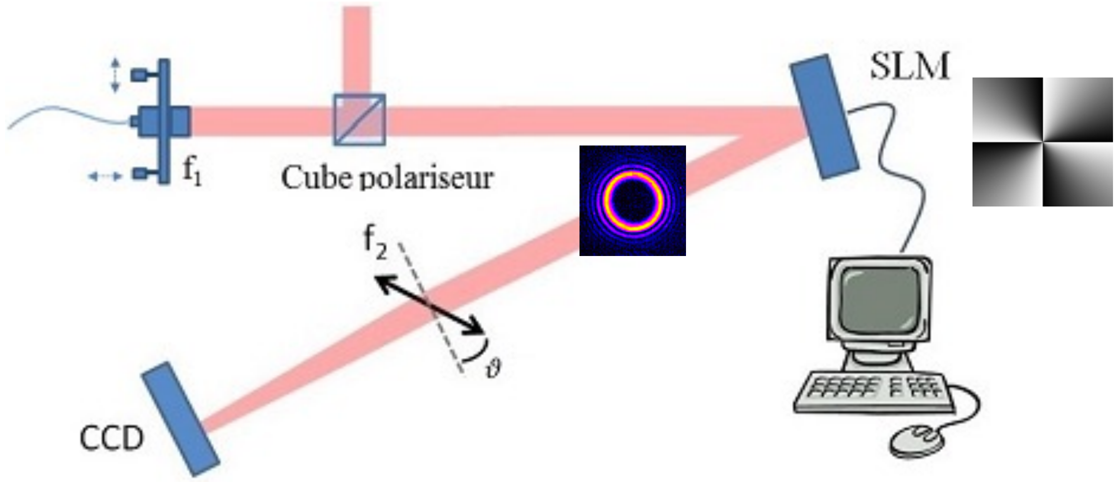
Several classes of vortex beams

Several preparation techniques



wikipedia.org/wiki/Optical_vortex

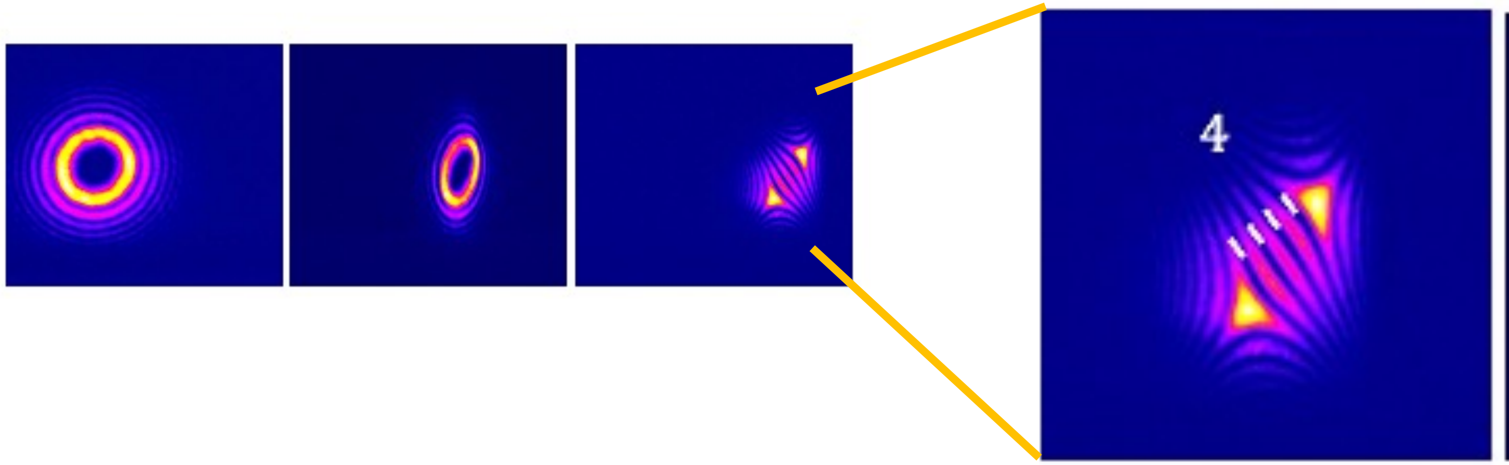
Generation and detection of Laguerre-Gaussian modes



SLM : Spatial light modulator
Liquid crystal pixels

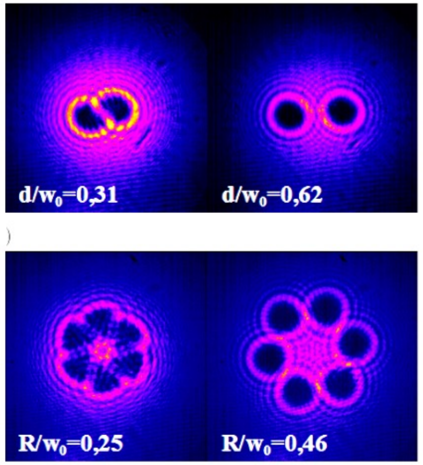
Tilted Lens Method

- Self interferences
- $|\ell|$ = number of dark lines
- Sign = inclination

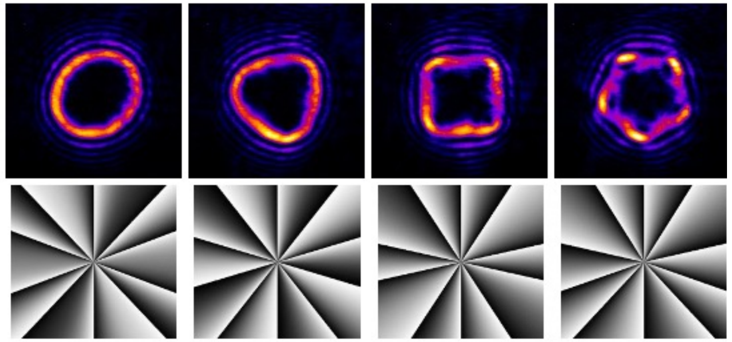


Vaity et al. Phys. Lett. A 377, 1154, 2013
C. Cabrera, PhD- thesis, LAC, Orsay 2014

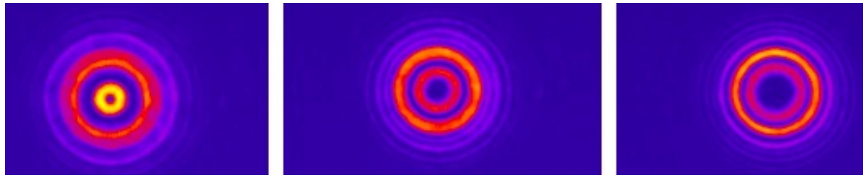
Combination of vortex beams



Vortex with different centres

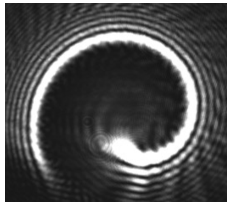


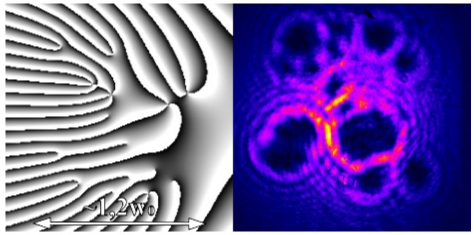
Combinations of 3 OAMS



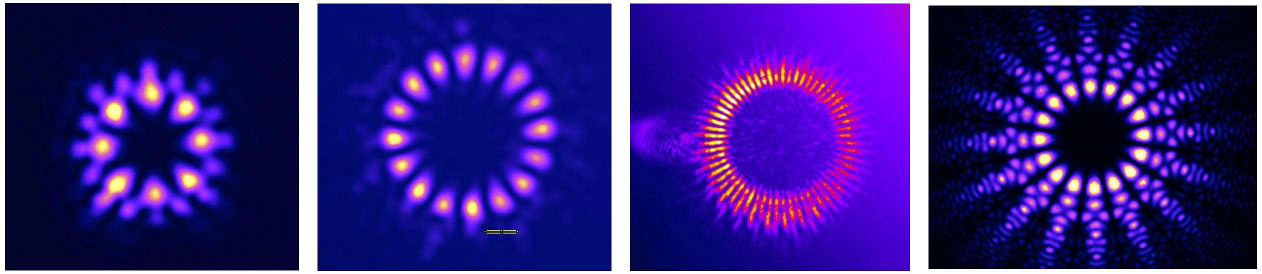
Multi-ring Vortices

Combination of 10 OAM



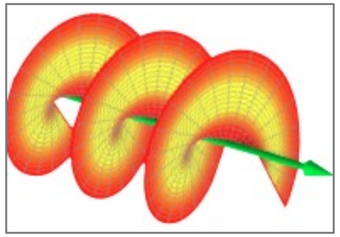


Random vortices



Phase-shaping, for example for atom multi-traps

Photon pairs



Processes to generate photon pairs

- SPDC in a crystal (Spontaneous Down Conversion)
  - Easy to implement
  - Can be fibered
  - One input : No flexibility for the input
- **FWM in a vapor (Four Wave Mixing, up-conversion)**
  - With hot or cold atoms
  - Resonant or not
  - A variety of outputs due to its two inputs
- Requirements
  - Energy conservation
  - Respect of polarizations
  - Phase-matching

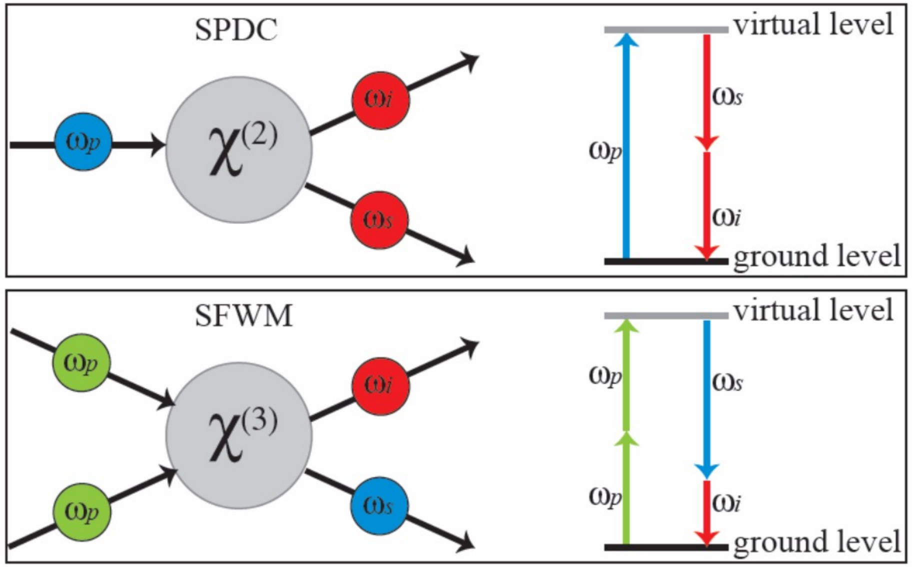
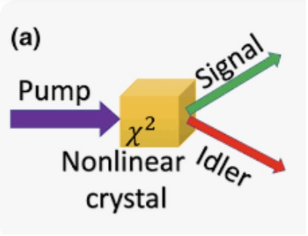
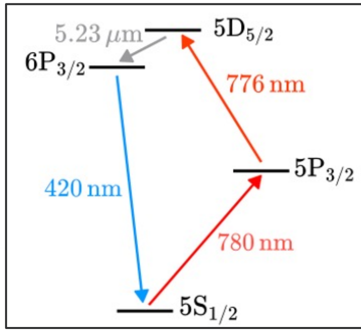


Figure of J Moreno Thesis, 2012





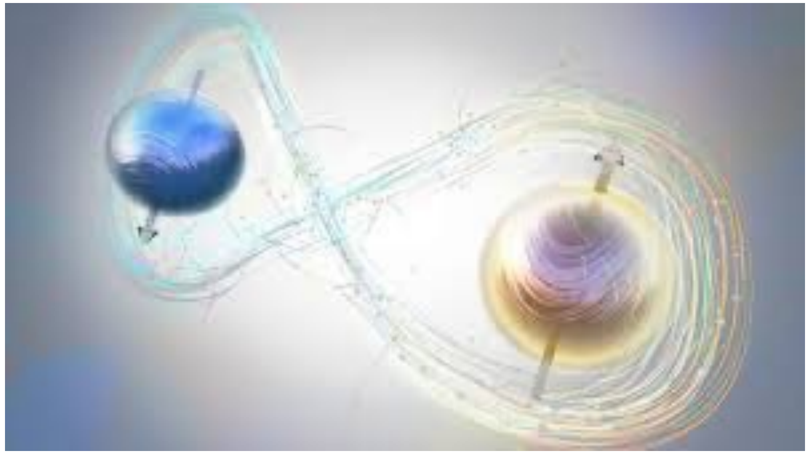
Photonic entanglement

- **Polarisation entanglement**

  very often used, robust, Easy to detect and analyse

  Only 2 polarization states per photon (2 qubits)

  a play limited to ↑⟩↑⟩; ↑⟩↓⟩; ↓⟩↓⟩; ↓⟩↓⟩



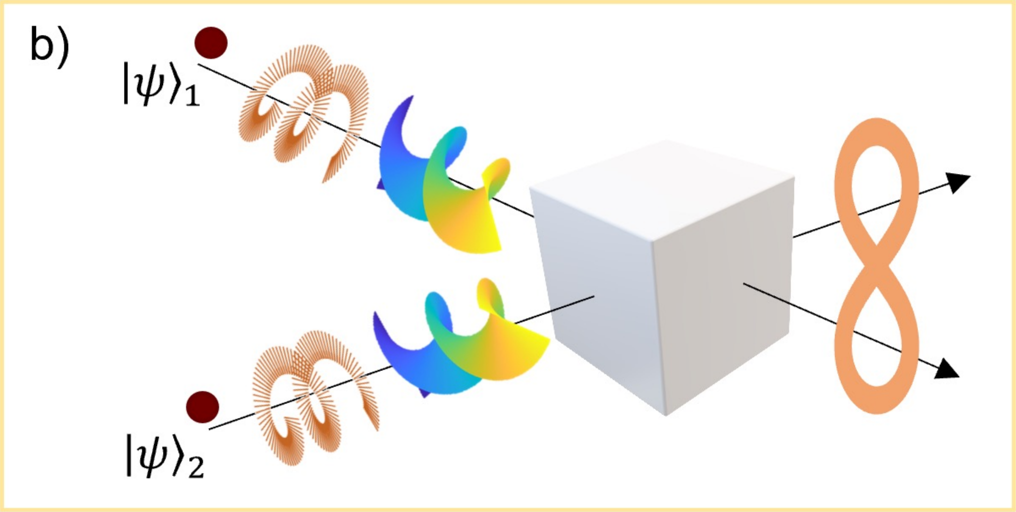
From Science News 2022

- **Orbital angular momentum (OAM)**

  qudits

  multi-space entanglement $|\ell\rangle\ \ell'\rangle$

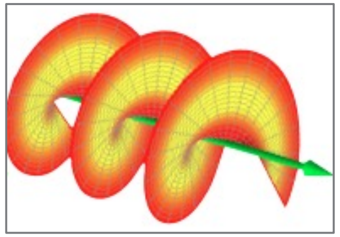
  but vulnerable (cf. phase)



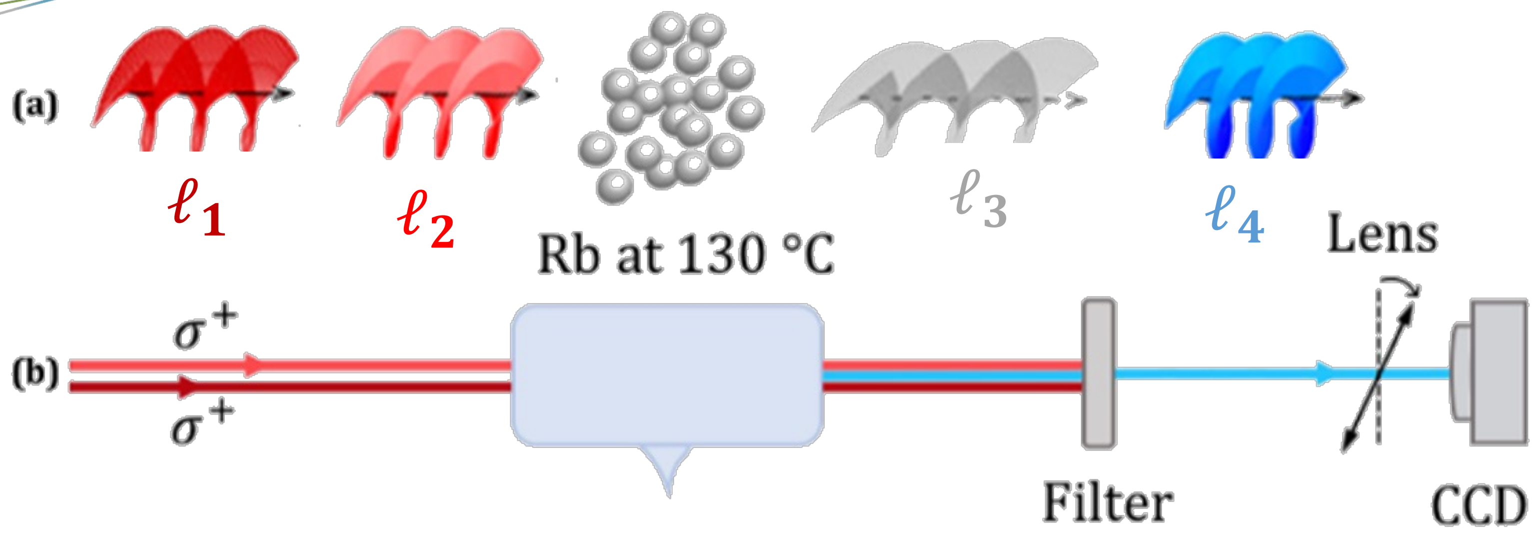
From F Sciarrino website

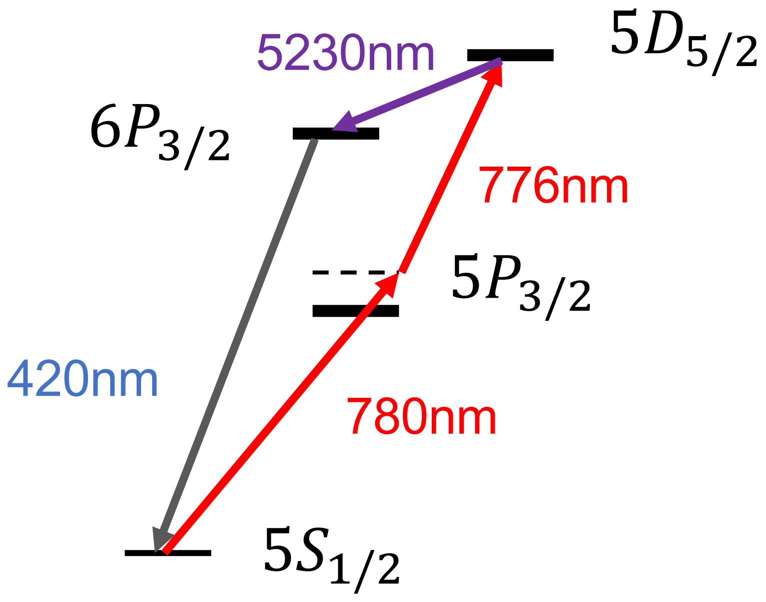
Mair, Zeilinger, Nature, 412, 313, 2001
Leach … Boyd, Phys. Rev. A , 85, 060304,2012
Krenn et al., Phil. Trans. R. Soc. A 375: 20150442, 2017
Offer ... Franke-Arnold, Com. Phys. 1, 184, 2018

Experiment : Conversion of Laguerre-Gaussian beams by FWM in a rubidium vapour



Four Wave Mixing with Laguerre-Gauss beams





A Rb vapour addressed by two vortex beams (780 & 776 nm) creates IR and blue (5230 & 420 nm)

Conditions :

- Energy conservation
- Phase matching and momentum conservations

$$k_3 + k_4 = k_1 + k_2$$

$$\ell_3 + \ell_4 = \ell_1 + \ell_2 = L$$

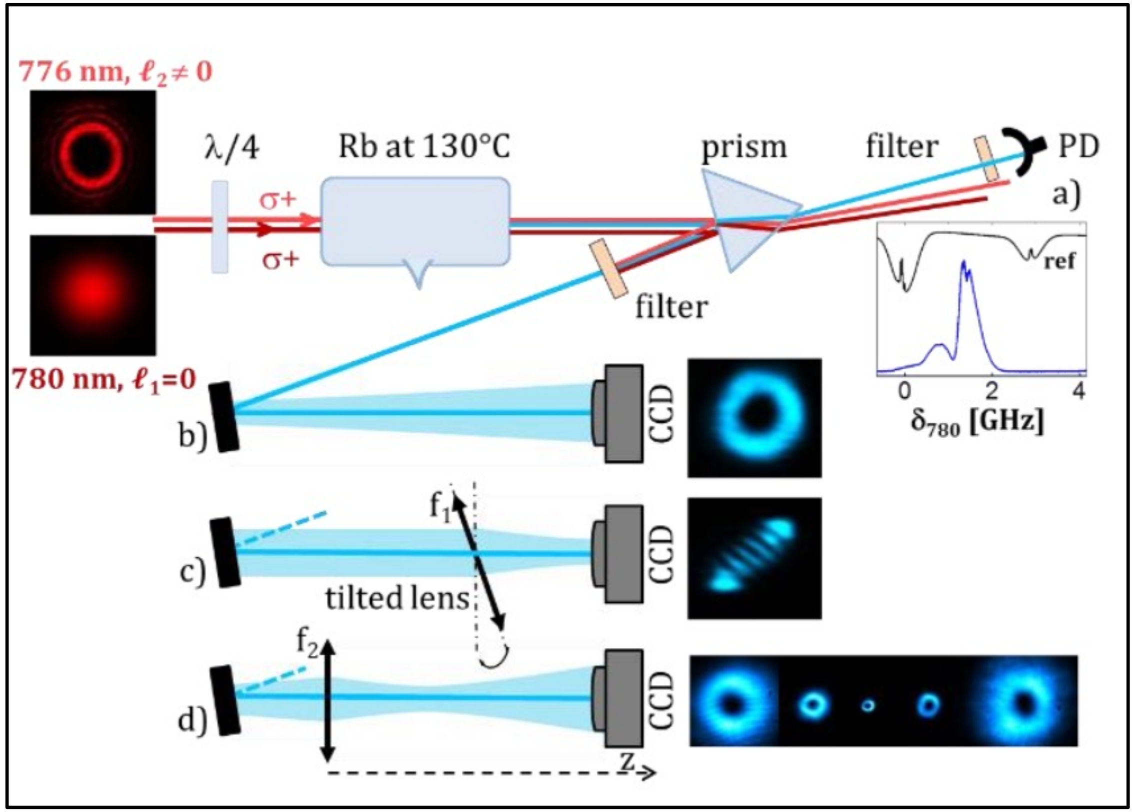
➤ determines the OAM-pairs (ℓ_3, ℓ_4)

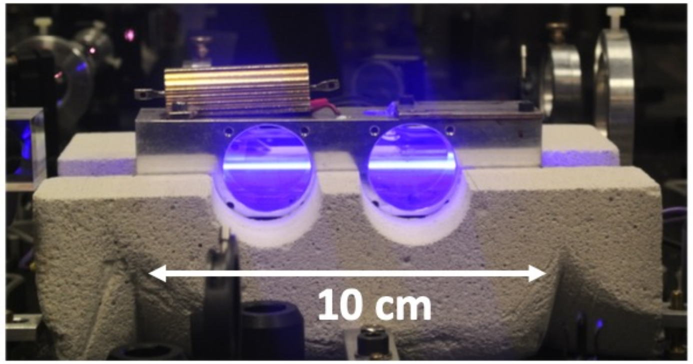
Gouy phase matching limits the number of pairs

For $\ell_1 . \ell_2 \geq 0$ only $\ell_3 . \ell_4 \geq 0$ are allowed ➤ only $L + 1$ pairs

Experiment

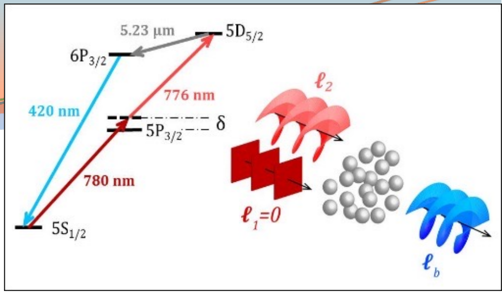
- **Input vortices are Laguerre-Gaussian modes**
  ℓ_1 is fixed, ℓ_2 will be varied
- Input : long Rayleigh range (z_R>> L_{cell})
- **Analysis of the blue output versus ℓ**
  spectrum,
  efficiency,
  intensity shape,
  OAM,
  propagation





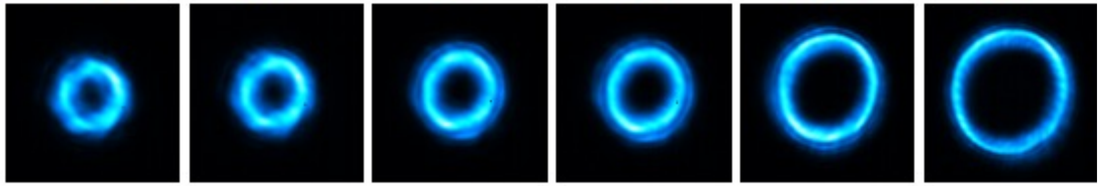
Details in Chopinaud, Jacquey, Viaris, Pruvost, PRA **97**, 063806, 2018.

Efficiency of vortex conversion

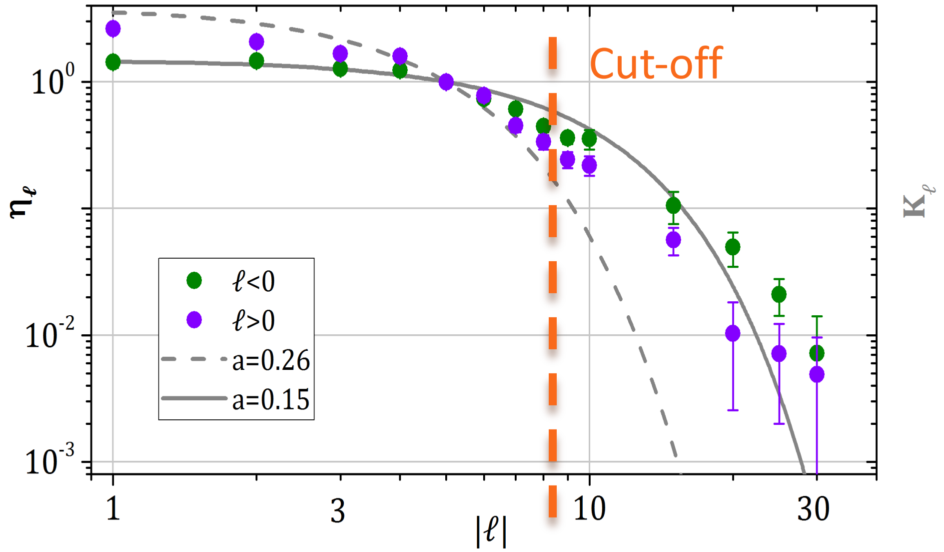


- $|\ell|$ up to 30

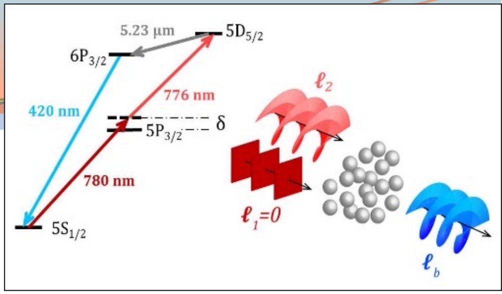
$\ell = +4$ 6 8 10 20 30



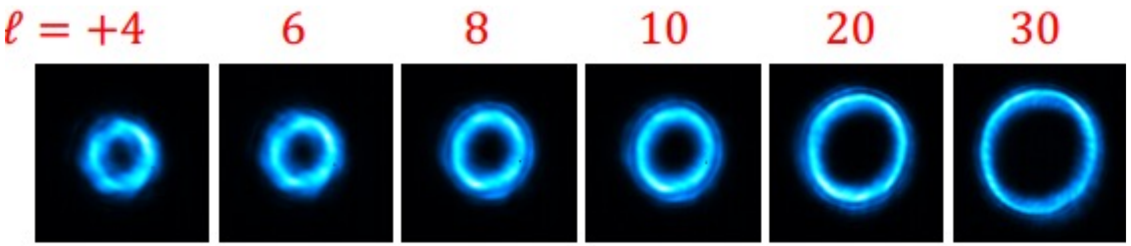
- A cut-off $\ell_{\text{cut-off}} \sim 8$



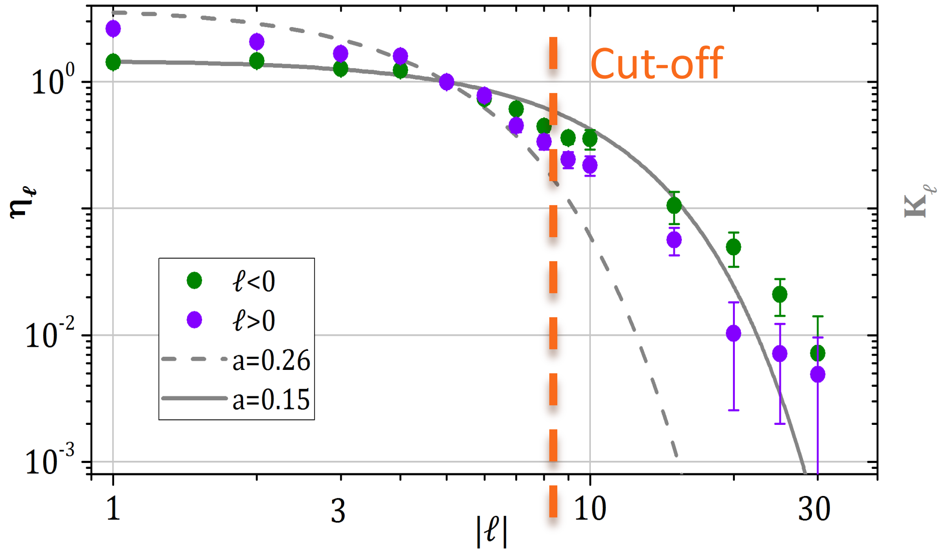
Efficiency of vortex conversion



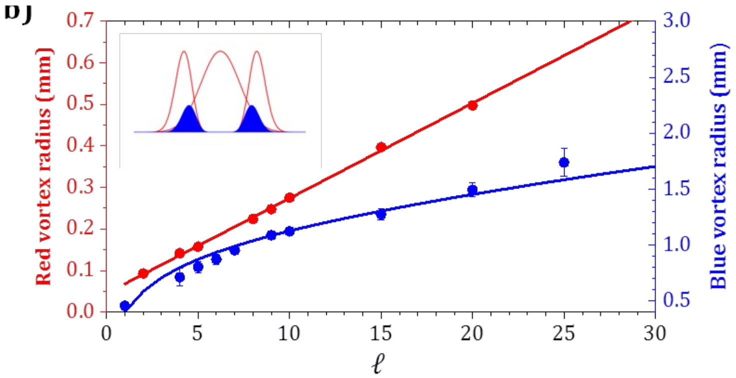
- $|\ell|$ up to 30



- A cut-off $\ell_{\text{cut-off}}$ ~8

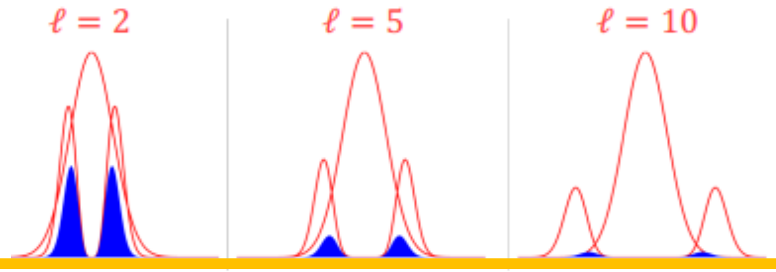


Input LG radius strongly varies as ℓ

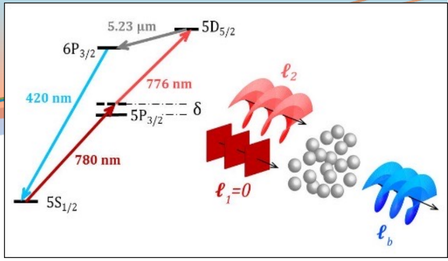


Efficiency = overlap of input beams

The spatial overlap rapidly drops down with ℓ



FWM and OAM output



$\ell_1 = 0$ **and** $\ell_2 = \ell$

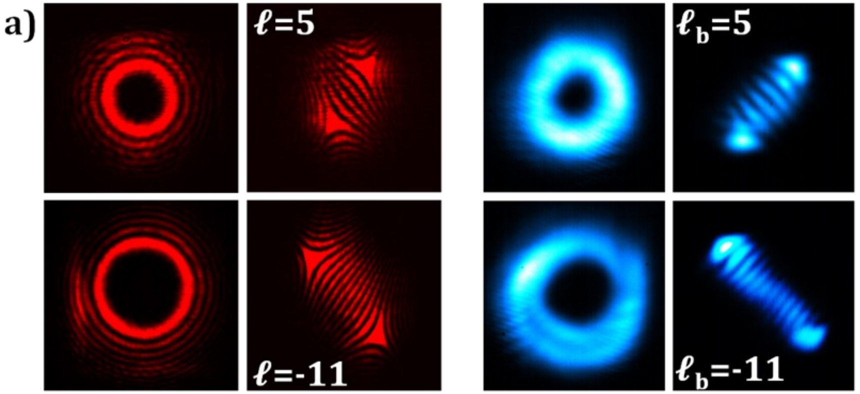
- up to $\ell \sim 10$ the output contains one output

$$|\ell_3\rangle\, |\ell_4\rangle = |0\rangle\, |\ell\rangle$$

- For $\ell = 11$ the data show **2 outputs** :

$$|0\rangle\, |\ell\rangle \;\; \& \;\; |1\rangle\, |\ell - 1\rangle$$

Input / output



The OAM goes preferentially to the blue wave

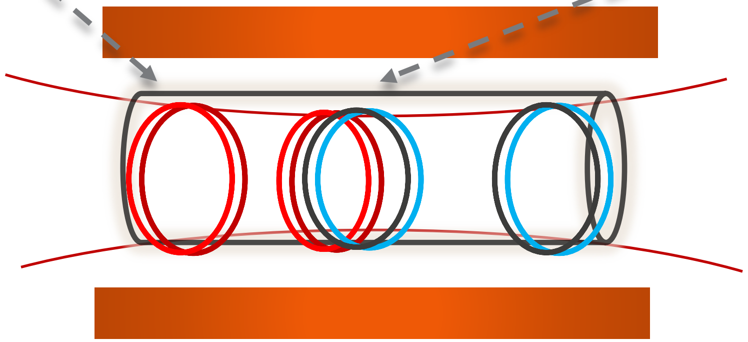
Explained by the wavelength ratio $\frac{\lambda_3}{\lambda_4} = 12$

Two key-parameters

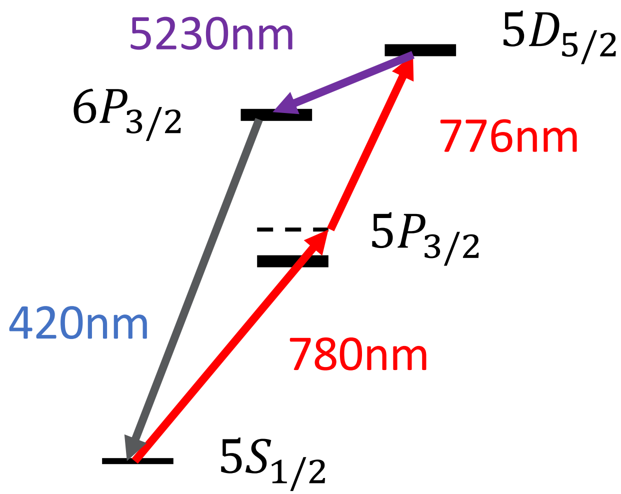
- Efficiency = the **overlap of the input laser beams**

- Pair probability = **Overlap of the four fields**

$$\int A_1(r)A_2(r)A_3(r)A_4(r)2\pi\, rdr$$



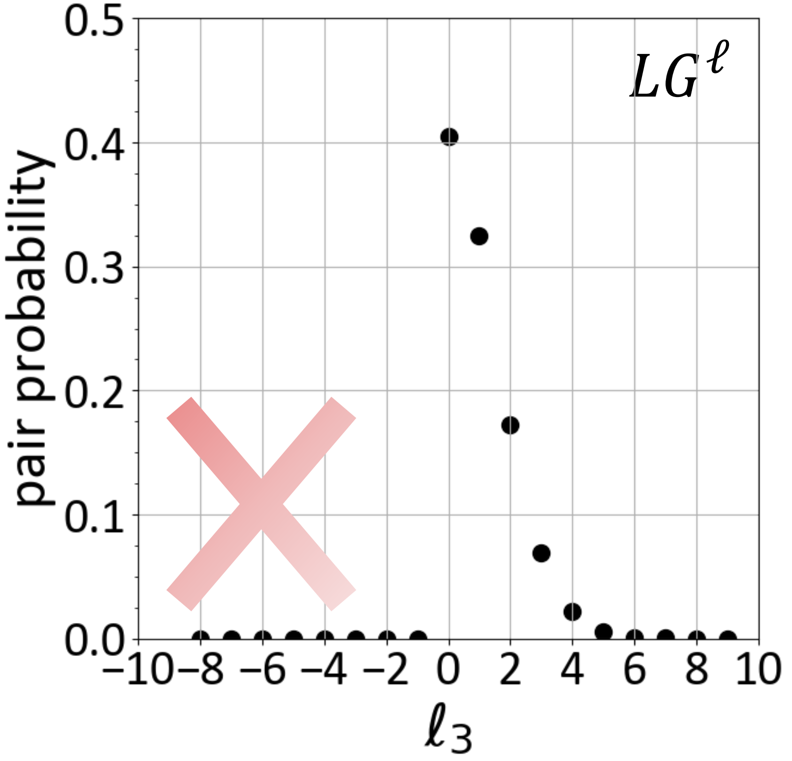
Usual NL-optics assumption : same Rayleigh ranges (**Boyd's criterion**)



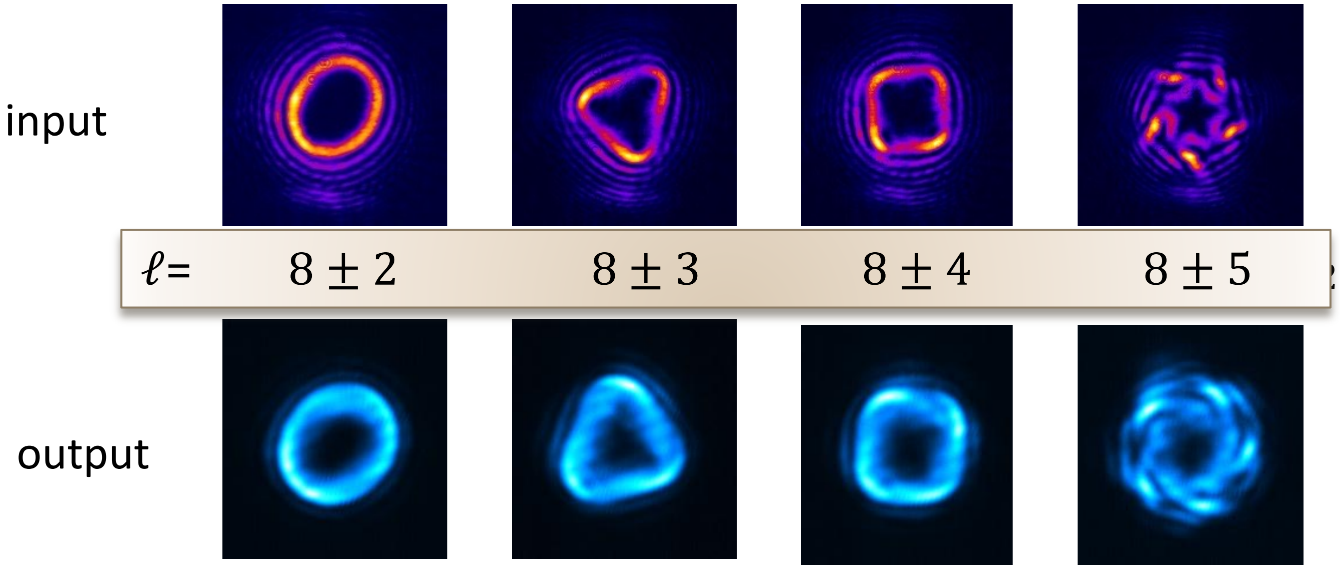
Because $\frac{\lambda_3}{\lambda_4} = 12$

IR is wider than blue beam

Low ℓ_3 is favorable



FWM with an input carrying many OAMs



The inputs are superpositions of 3 LG modes, created by a modulation of the helical phase

Experiments

Early ones and with low OAM values

S. Franke-Arnold (Glasgow, UK) : Phys. Rev. Lett. 2012

I. Novikova (Williamsburg, US) : Optics Lett. 2015

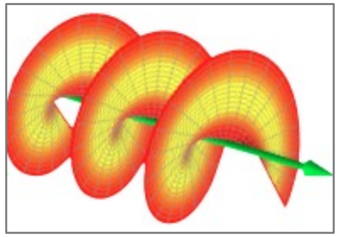
L. Wang (Shanxi and Taiyuan, CHN) : Laser Phys. Lett. 2023

With LG beams, the number of outputs $|\ell_3\rangle\,|\ell_4\rangle$ is limited

Large OAM values

L. Pruvost (Paris, F) : PRA 2018 (ℓ up to 50)

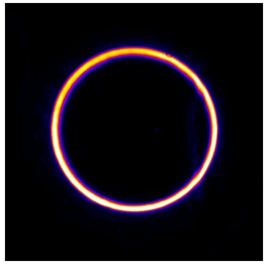
L. Wang : Optics Com. 2024 (ℓ up to 13)

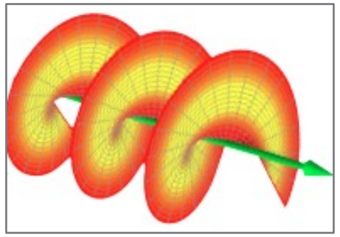


Increase the number of OAM-pairs by using Perfect Vortex Beams

Defined by $e^{i\ell\theta}\,\delta(r-R)$ it has a ring radius independent of the OAM

Realistic form $e^{i\ell\theta}\,e^{-(r-R)^2/w^2}$ with the finesse $F=\frac{R}{w}$

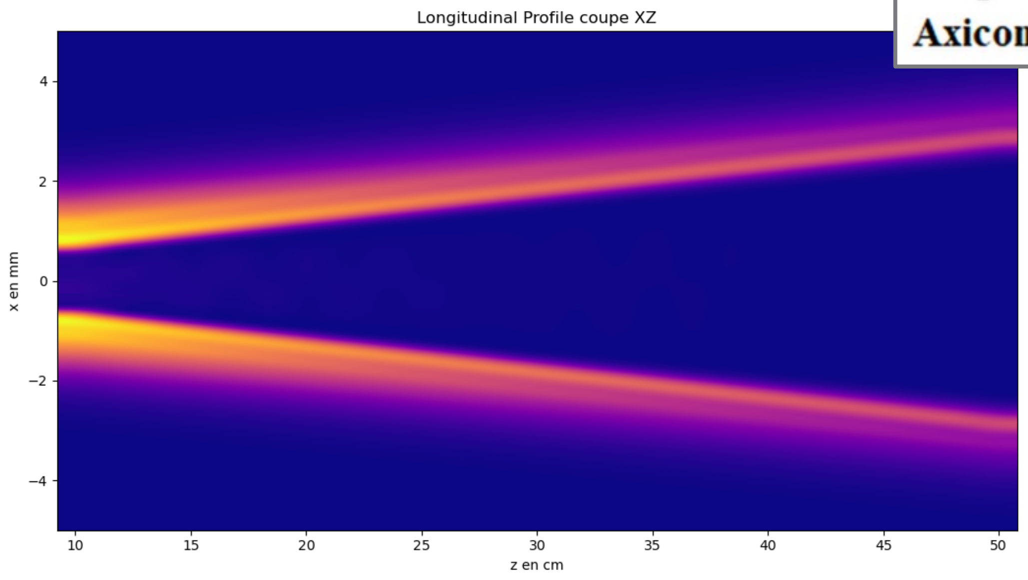


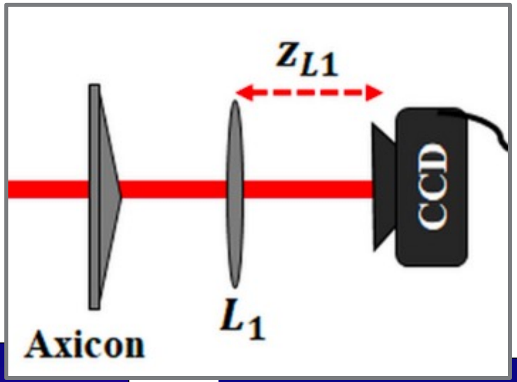


The perfect vortex beam : FT of a Bessel-Gaussian beam

The conical phase

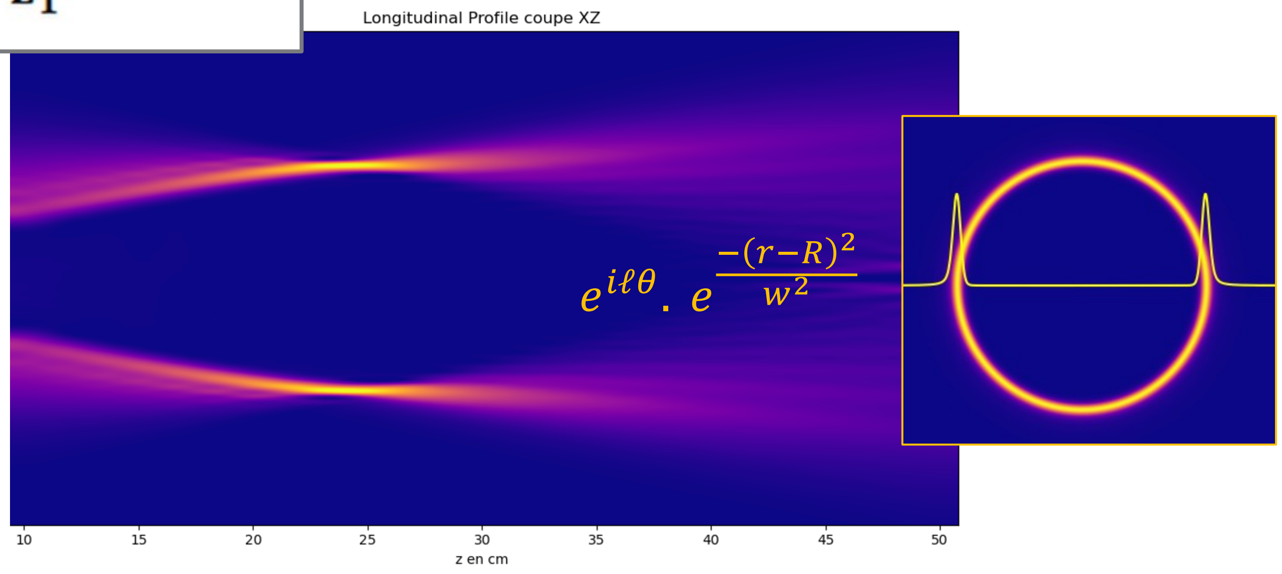
produces a divergent BG beam





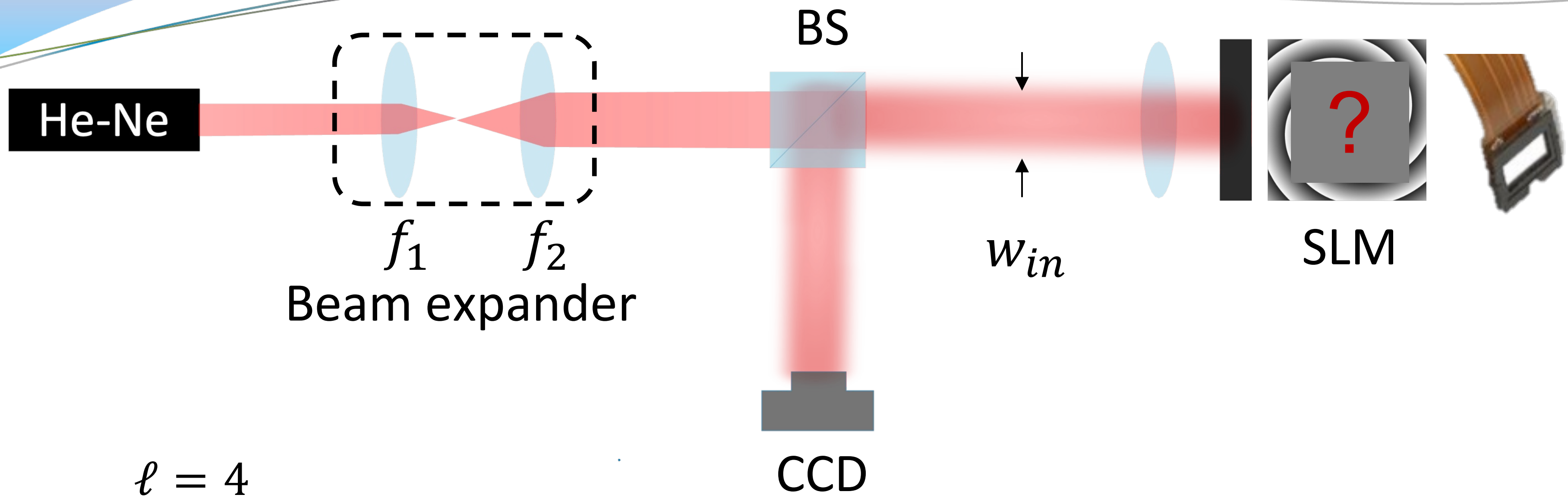
A lens compensates the divergence

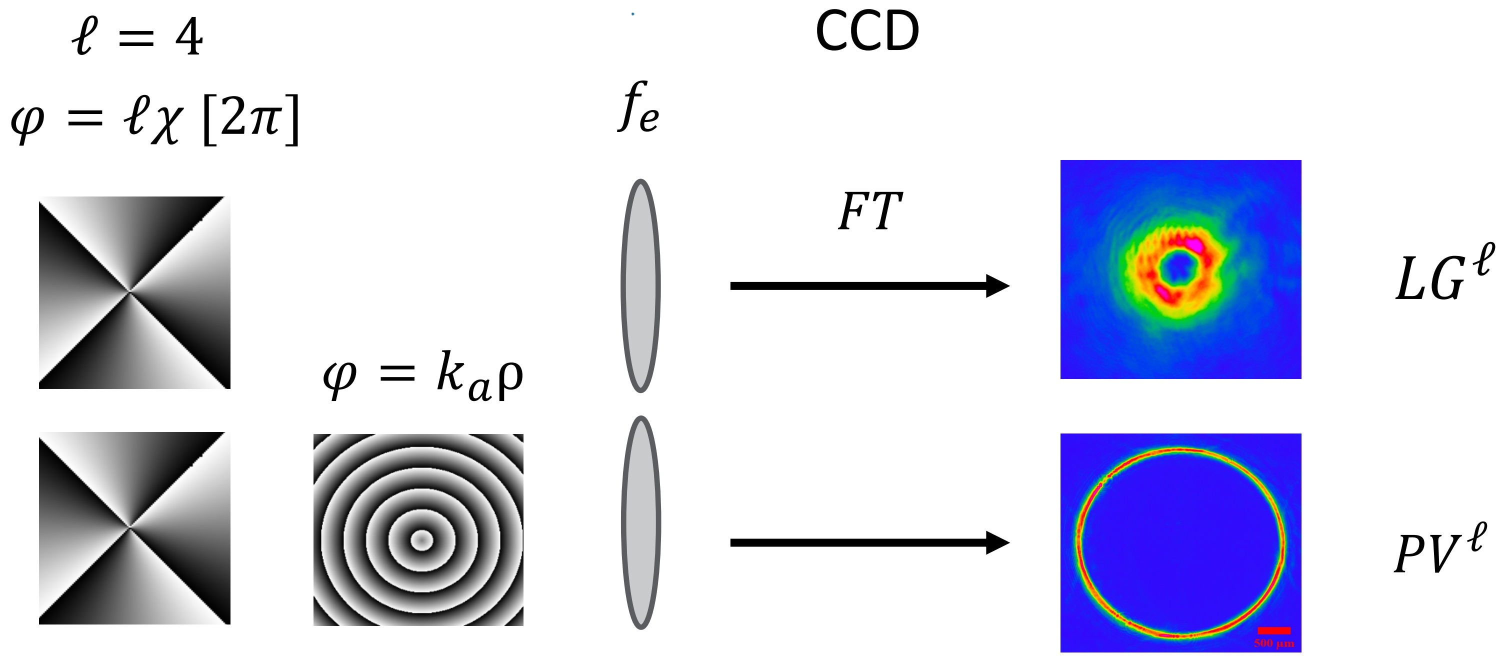
- PV beam obtained at the focus point
- Radius and thickness adjustable
- Radius nearly independent of OAM



Computation and experiments done by Paul Seguigne (PhD)

Preparation of a Perfect Vortex





Settings:

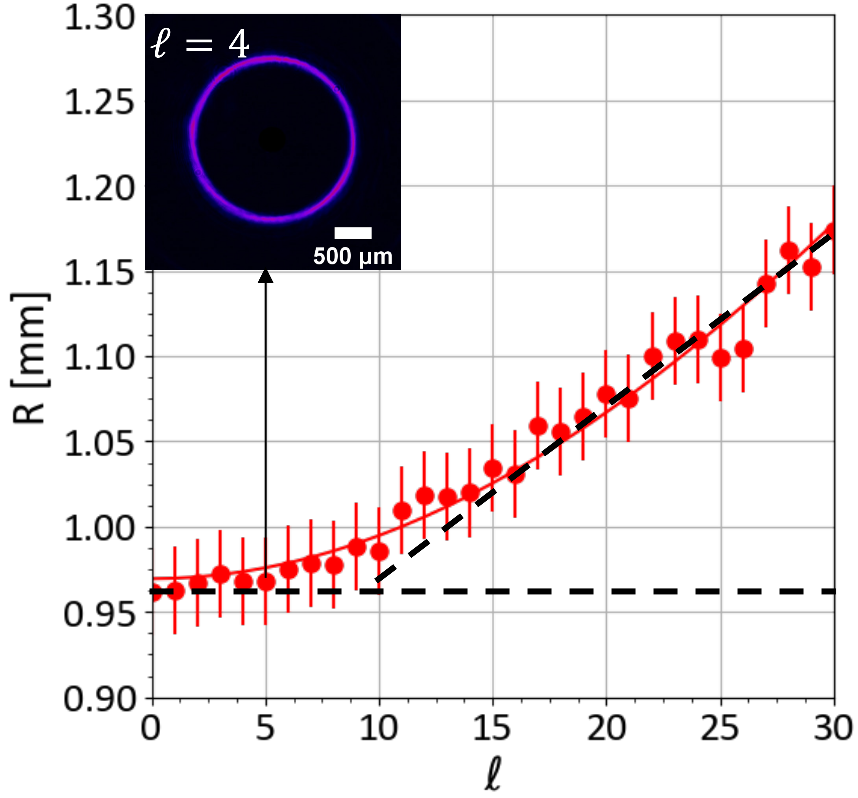
- ℓ
- $\boldsymbol{k_a}$ is conical momentum
- Input waist w_{in}
- Effective lens f_e

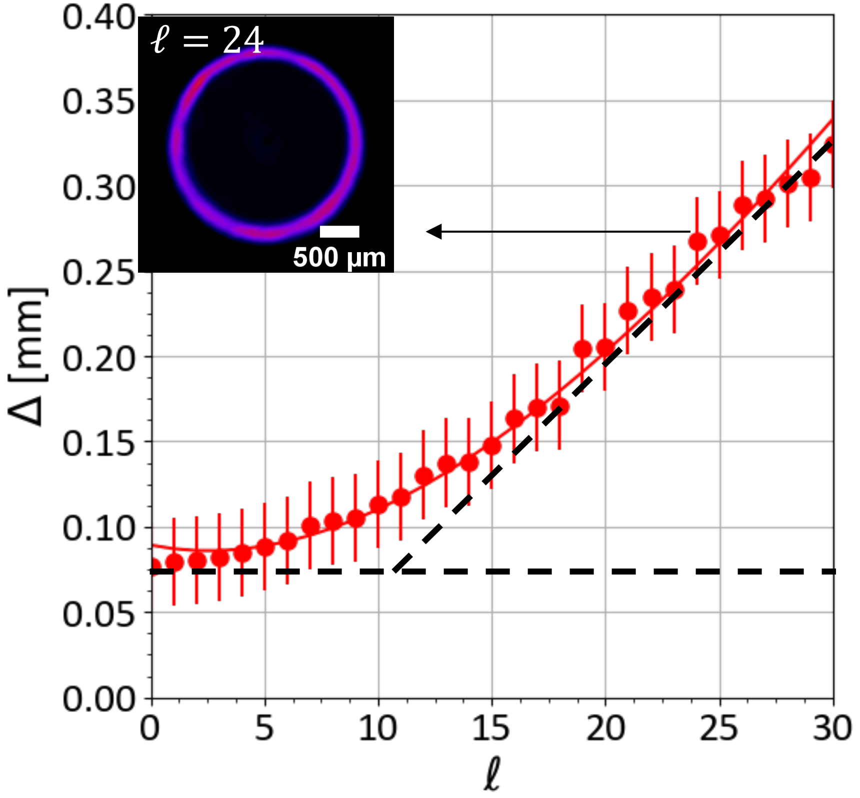
Modelable (first two settings)

Fixed (last two settings)

Radius and thickness of an experimental Perfect Vortex

At the **focal point, we measure** the ring radius and and its FWHM versus ℓ ($k_a = 35\, mm^{-1}$)





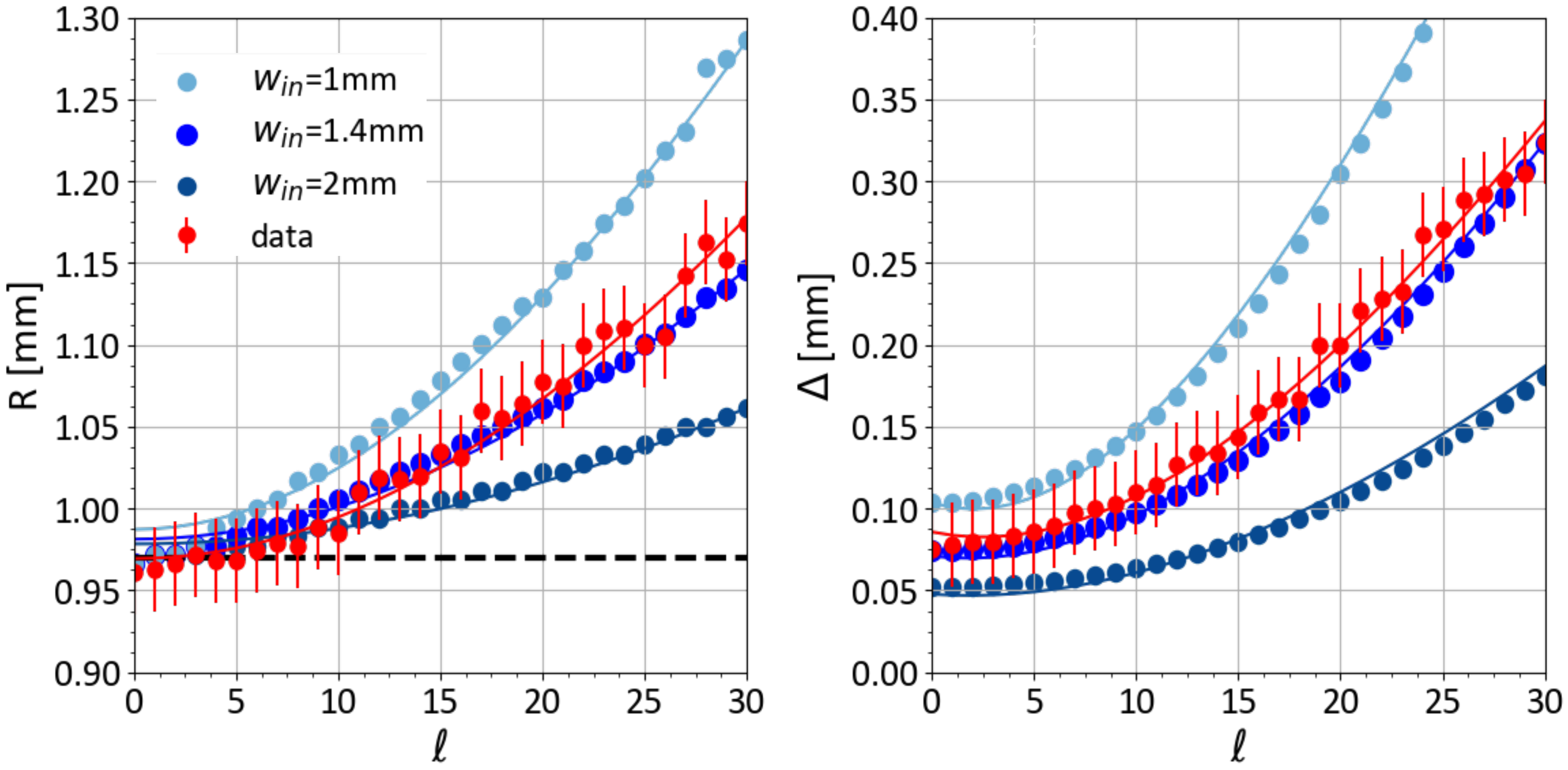
Radius (R) and thickness (Δ)

- Constant up to $\ell \sim 10$
- Then linear

→ Finesse $F \sim 12$

Comparison with numerical approach

Hughens-Fresnel : $E(r,\theta,z) = \frac{e^{ikz}}{i\lambda z} \iint E_{in} \, . \, e^{i\varphi(\rho,\chi)} \; . \, e^{i\frac{k(\rho^2+r^2-2\rho r \cos(\chi-\theta))}{2z}} \rho d\rho d\chi$



Hyperbolic-like variation

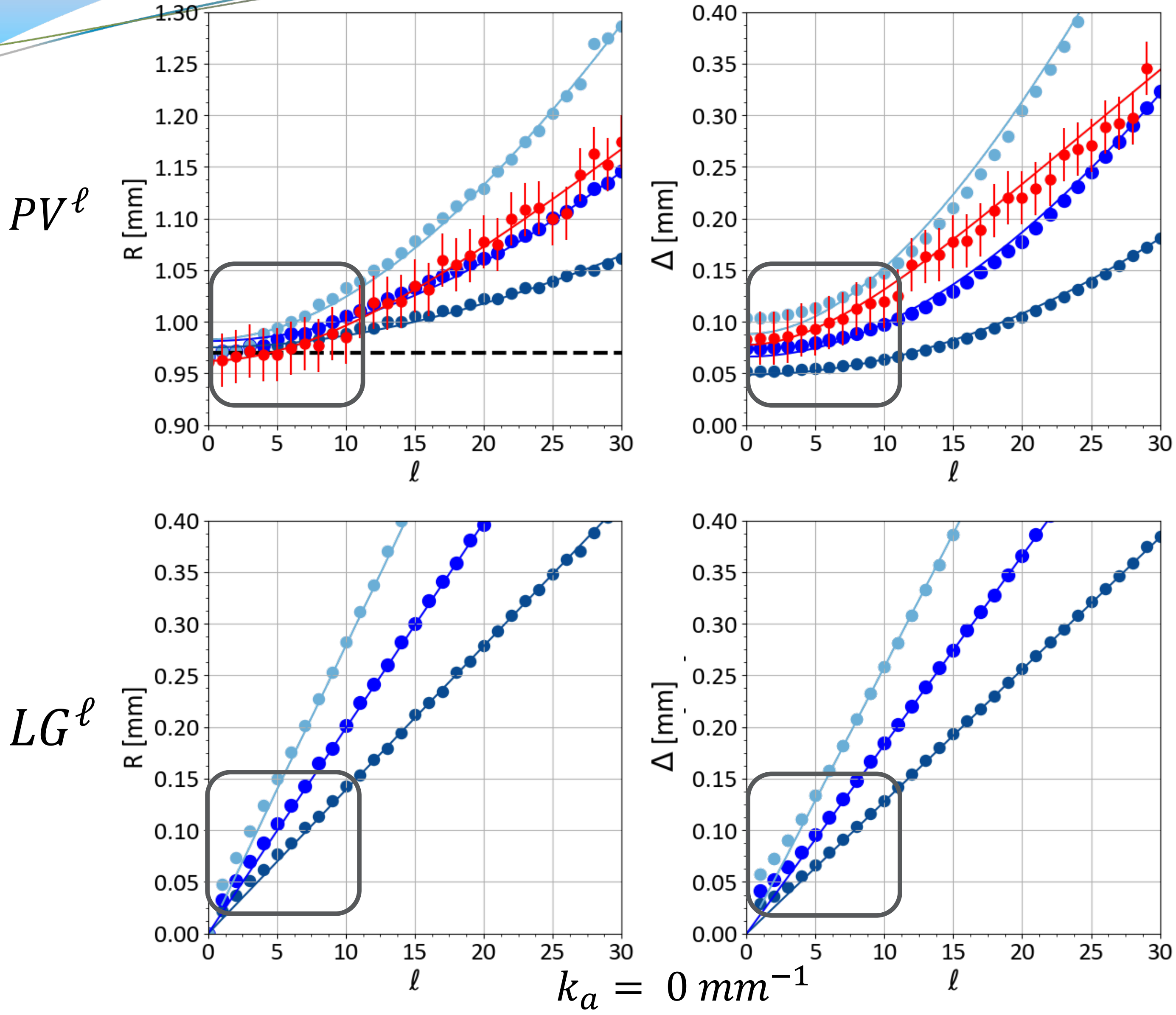
Radius, thickness related to
- input waist,
- conical momentum and
- lens focal

$$R_0 = \frac{k_a}{k} f_e . ; \Delta_0 = \frac{2}{k w_{in}} f_e$$

Then linear , the slopes being

$$\alpha = \frac{1}{k w_{in}} f_e ; \beta = \frac{0.93}{k w_{in}} f_e$$

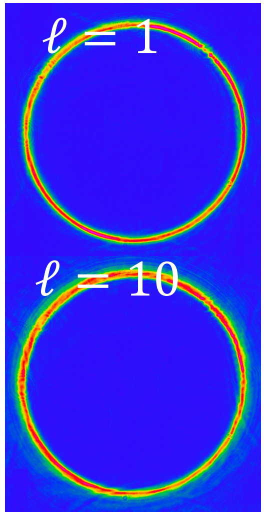
Perfect Vortex compared to exp. LG beams



Radius and thickness constant

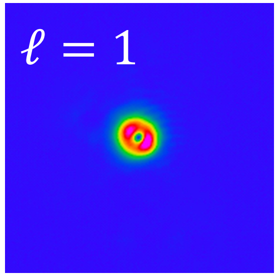
Finesse about 10

→ **Good for input overlap**

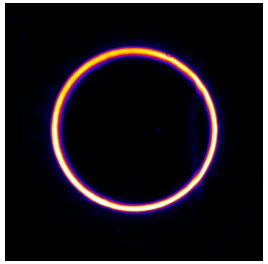


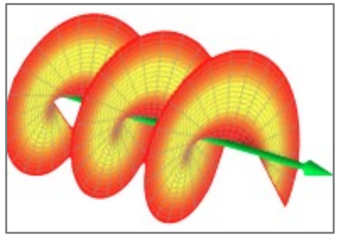
Radius and thickness are linear

- Low Finesse $F = \frac{\alpha}{\beta} \sim 1$



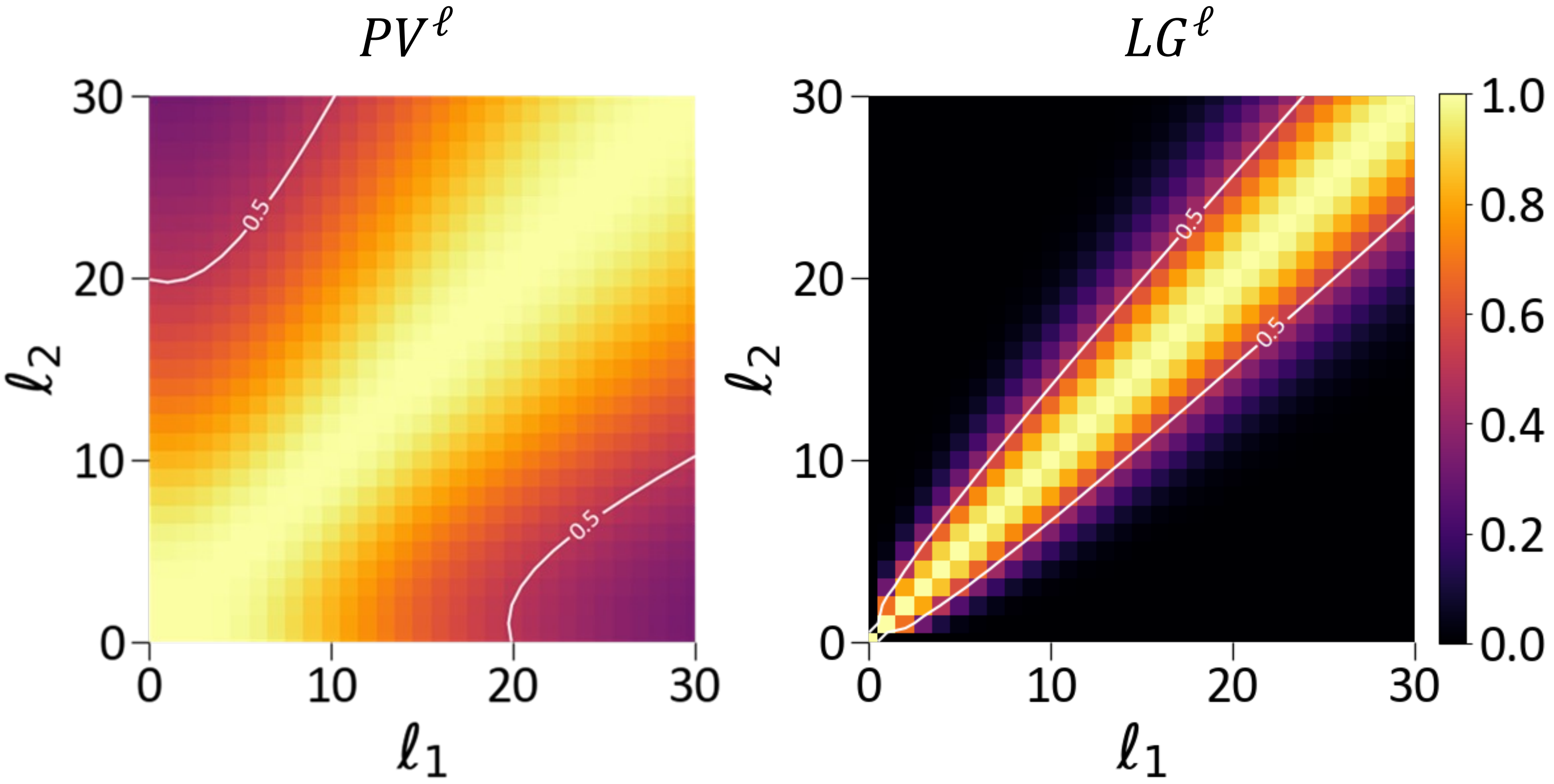
Prediction about OAM-pairs with PVB





Input space

Overlap of input vortex beams : $\int A_1(r).A_2\,(r)\;2\pi\;rdr$



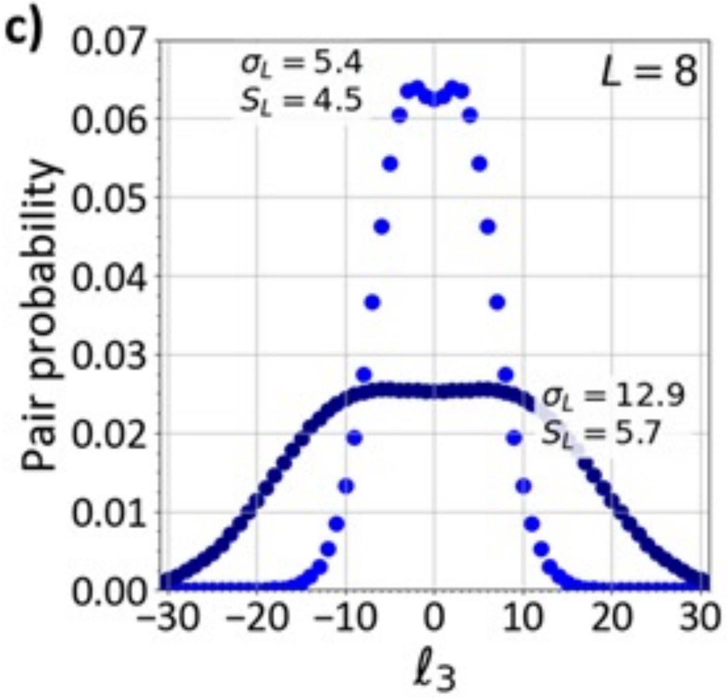
Computed for PV of finesse 5

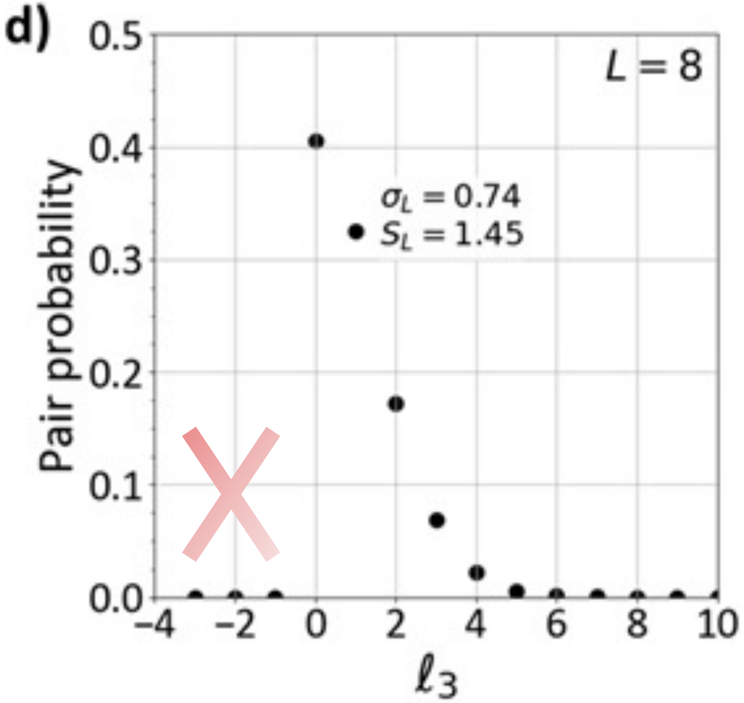
The input space is larger for PV beams than LG (by a factor 5)

Output space (number of pairs)

Phase matching and **Overlap of the four fields :** $\int A_1(r)A_2(r)A_3(r)A_4(r)2\pi\, rdr$

PV^{ℓ} LG^{ℓ}





Computed

- For $\ell_1 = \ell_2 = 4$ ($L = 8$)
- Unfocused beams
- Finesse =5 (light bleu)
- Finesse =10 (dark blue)

No Gouy phase condition
The variance (σ_L) and the Shannon entanglement entropy (S_L) increase for PV.

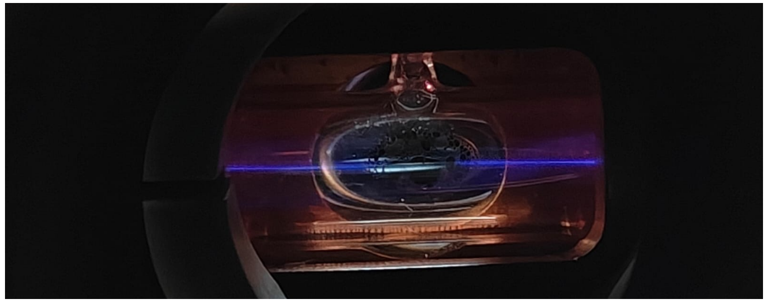
➤ possible increase of the number of OAM-pairs and the entanglement

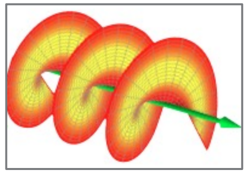
$$S(\ell_{\rm T}) = -\sum_{\ell_{\rm B}=-\infty}^{\infty} P_{\ell_{\rm B},\ell_{\rm IR}} \log_2 P_{\ell_{\rm B},\ell_{\rm IR}},$$

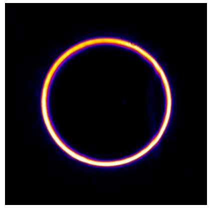
Conclusion

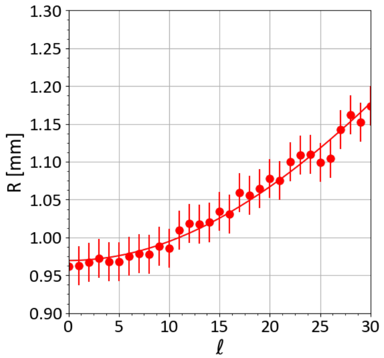
- OAM = qudits
- Perfect Vortex Beams (PVB) adjustable in size and finesse
- PVB to increase the output space of OAM-pairs

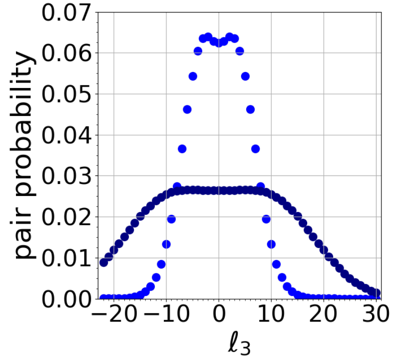
- Experiment in progress using 5S->5D transition at 778 nm





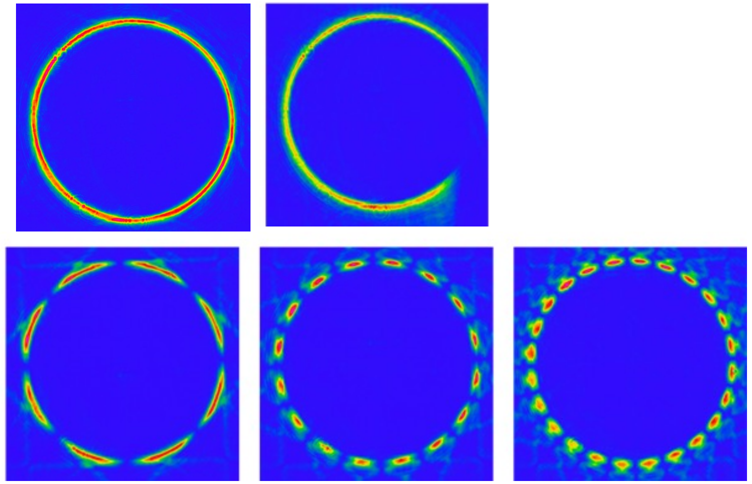






Opening for PVB

- PVB for deeper blue-detuned traps
- PVB for red-detuned ring traps
- Opened PVB for linear red-detuned traps
- Modulated PVB for multi-traps





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"Phase shaping for new atomic lattice clocks"

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