Fractional Quantum Hall Gravitons: From theories to experiments

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100 years of Quantum Physics

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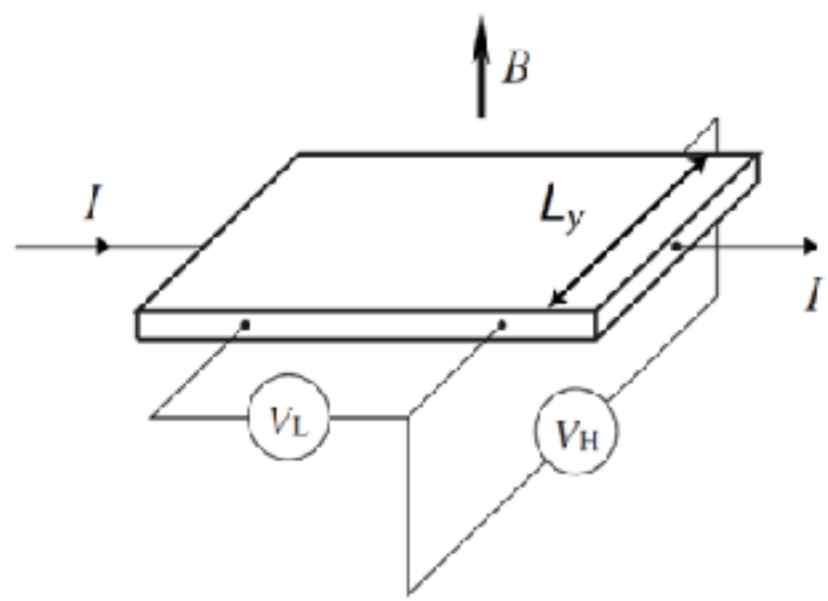
Plan

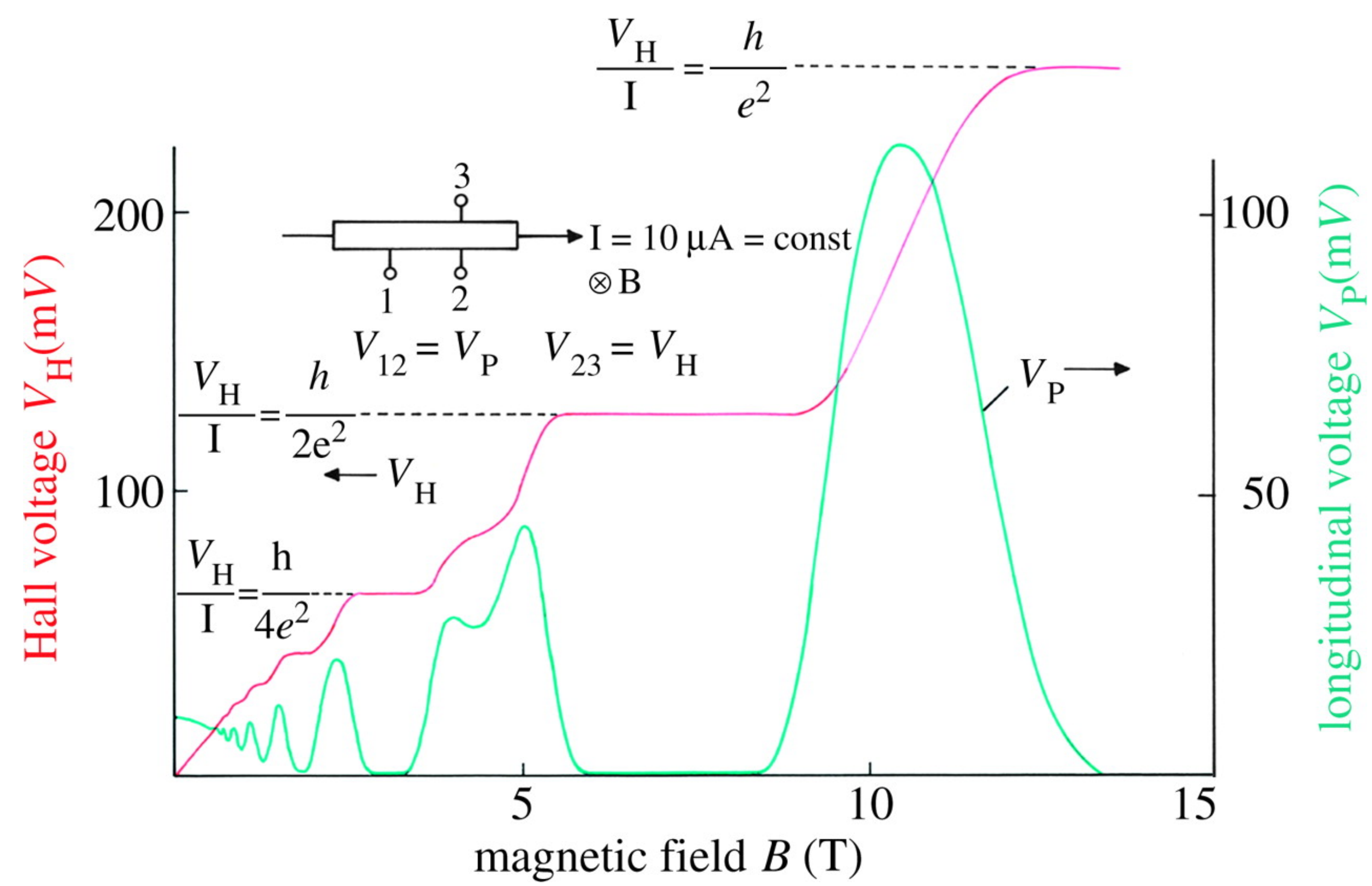
- (Brief) Introduction to Fractional Quantum Hall effect
- Magneto-roton is a massive graviton-like excitation
- Graviton(s) in Dirac CF models
- Polarized Raman scattering experiment

Fractional Quantum Hall effect

Integer quantum Hall

- 2D electrons gas in an applied magnetic field
- In a high magnetic field, we see "Integer plateaux" in the measurement of Hall resistance (conductance)





On plateaux

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

ν is an iteger

- Energy of a single electron is quantised to Landau levels

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c, \quad \omega_c = \frac{Be}{mc}$$

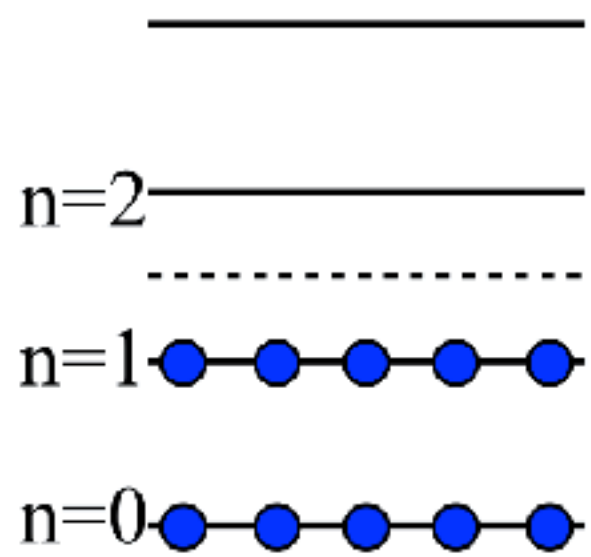
(m is electron's mass)

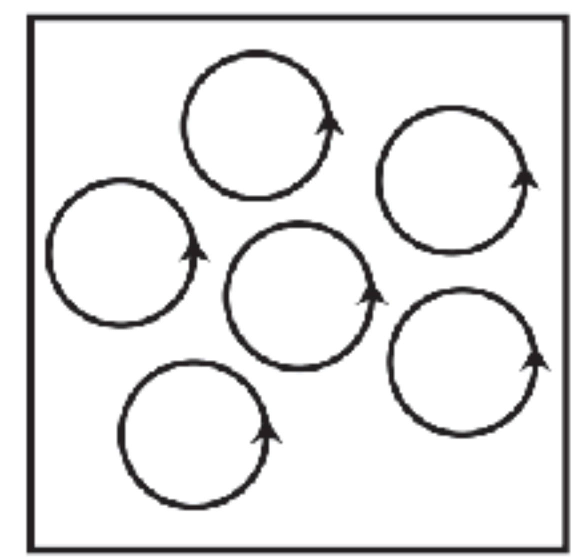
- In semi-classical picture: electrons in cyclotron orbits (orbitals)

Number of orbitals on each Landau level:

$$N_\phi = \frac{B \times \text{Area}}{\phi_0}, \quad \phi_0 = \frac{h}{e}$$

In applied electric field $\mathbf{E}$, each orbital has drift velocity $\mathbf{v} = \dfrac{\mathbf{E} \times \mathbf{B}}{B^2}$





$$\ell_B \sim \sqrt{1/B}$$

- Hall conductance if we have fully filled Landau levels

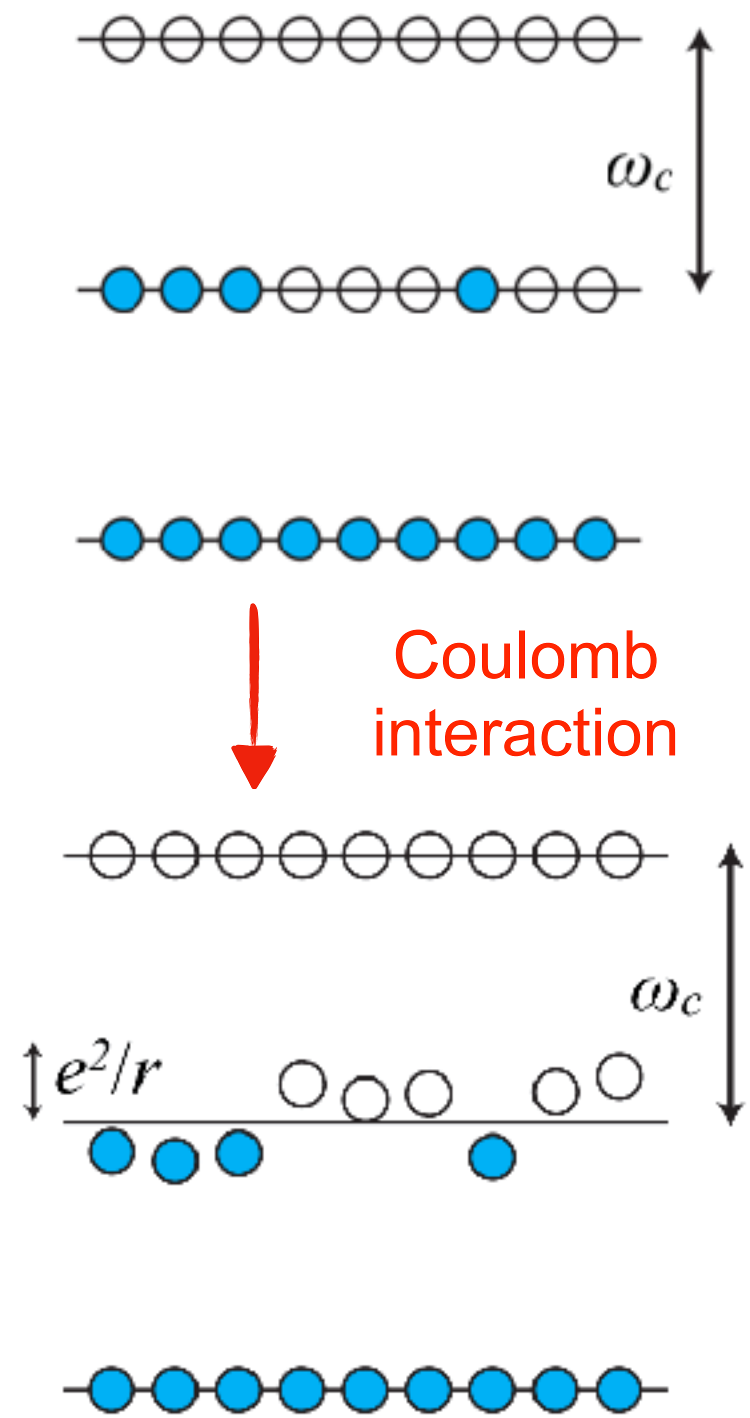
$$\sigma_{xy} = \nu \frac{e^2}{h} \qquad \nu = \frac{N_e}{N_\phi}$$

Filling fraction

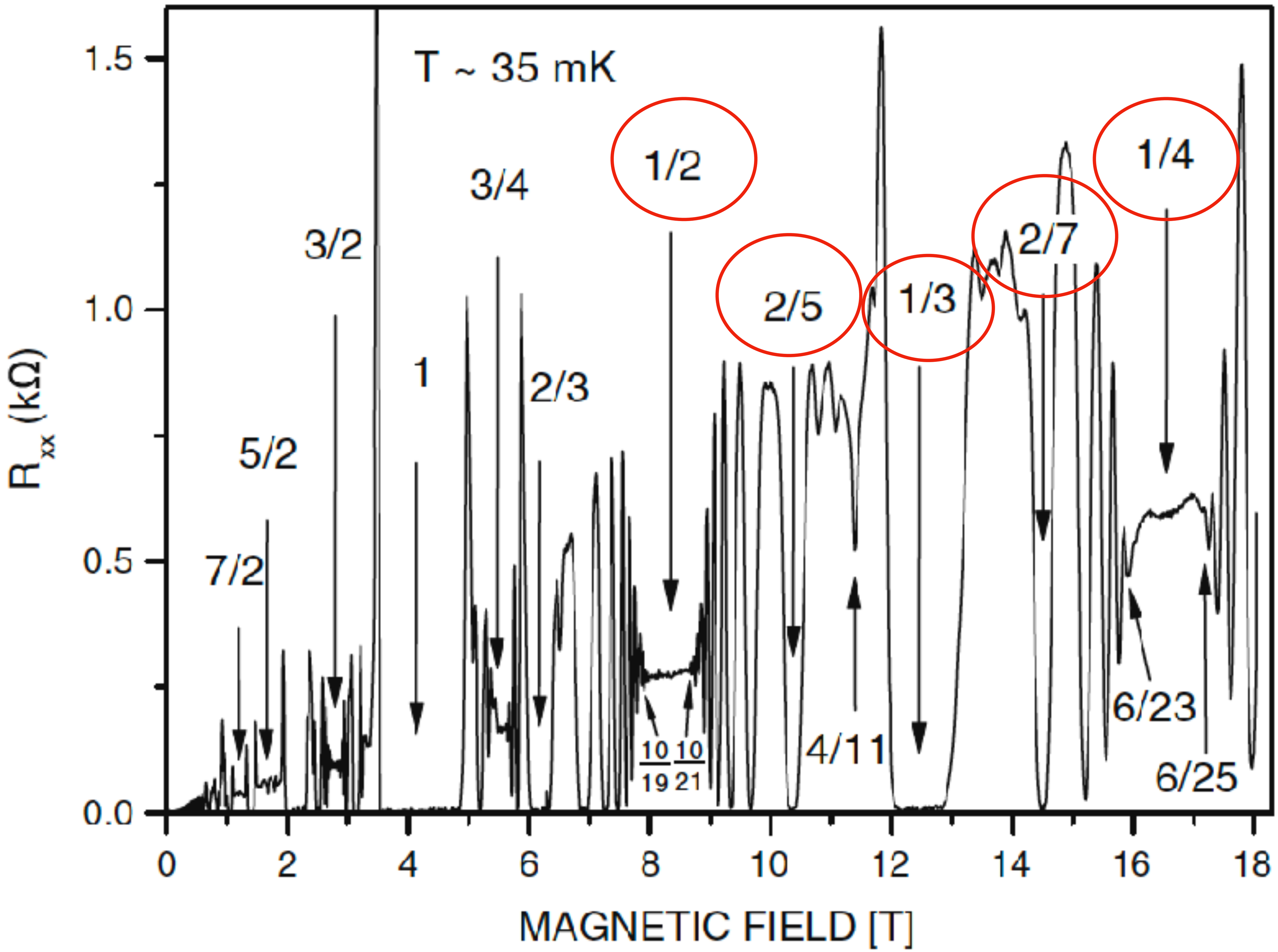
Fractional quantum Hall

What happen if we only partially fill a Landau level?

- Without interaction, a partially filled Landau level is gapless with the huge degeneracy
- Coulomb interaction splits the degeneracy to have the unique gapped ground state
- The measurement of Hall resistance shows small plateaux with fractionally quantised σ_{xy}
- In the lowest Landau level limit $\omega_c \to \infty$ $(m \to 0)$, one can ignore higher Landau levels
- In this limit, the kinetic energy effectively vanishes
- We have to deal with a strongly interacting problem with **no direct solution**



Fractional quantum Hall in experiment

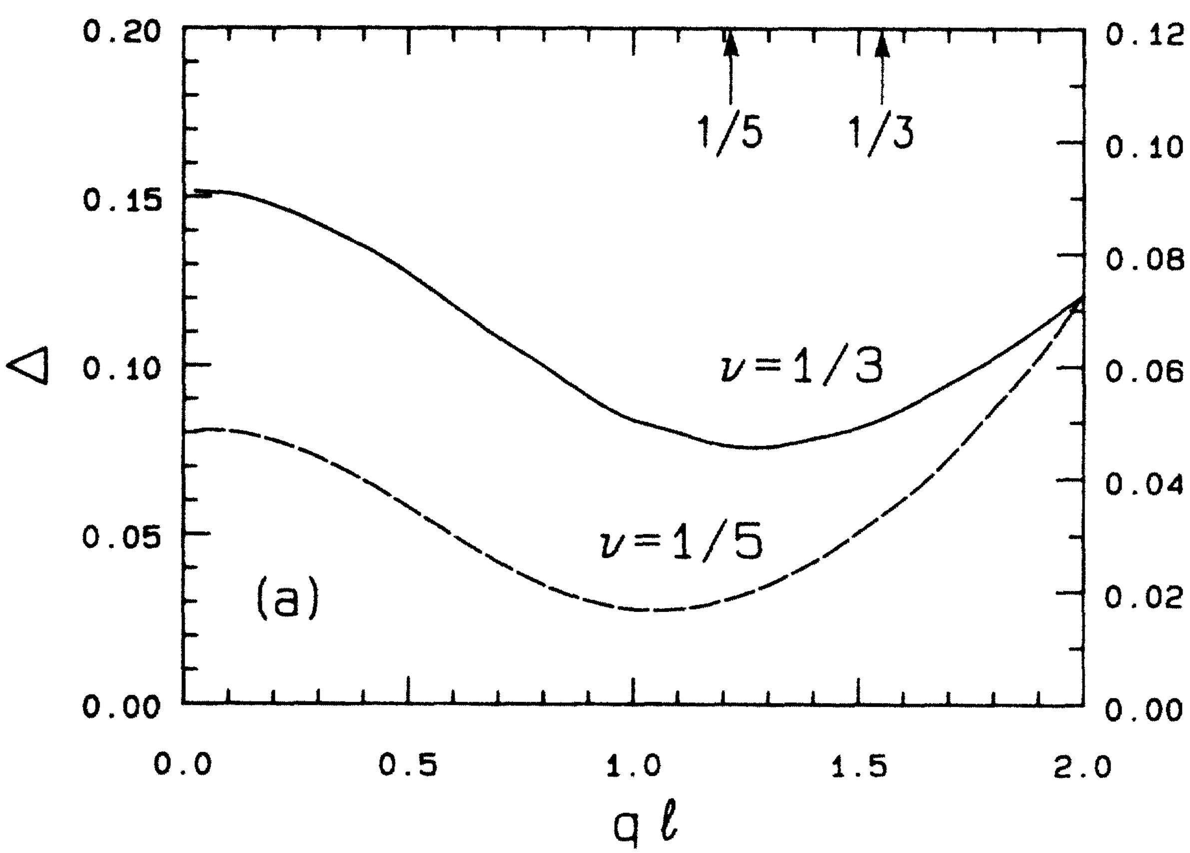


In this talk, we concentrate on states near $\nu = 1/2$ and $1/4$

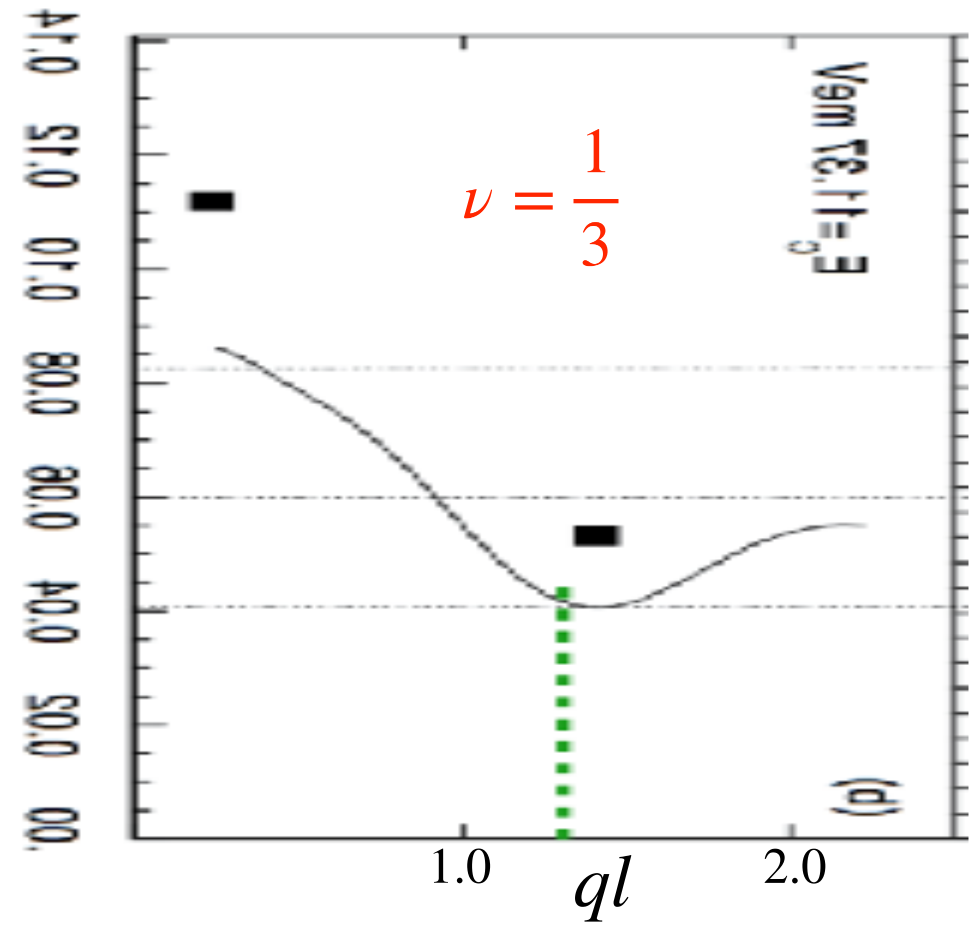
Magneto-roton as a graviton

Magneto-roton excitations

- Neutral excitation appears in Gapped FQH states
- Proposed by Girvin, MacDonald, Platzman (GMP 1986) as the charge density wave $\rho(\mathbf{q})|0\rangle$ in the lowest Landau level (LLL)
- The energy spectrum of magneto-roton (in GMP paper) was derived using the single mode approximation (similar as Feymann's model of roton in superfluid ^4He).



GMP 1986



Observed in Pinczuk et al. 1993

Lowest Landau level conservation laws and magneto-roton spin

$$\omega_c = \frac{B}{m}$$

- Symmetries lead to the Ward identities

Charge conservation

$$\partial_t \rho + \nabla \cdot \mathbf{j} = 0$$

Momentum Conservation

$$\partial_t(\cancel{m j_i}) + \partial_k T_{ki} = (\mathbf{j} \times \mathbf{B})_i$$

LLL limit

$$\mathbf{j} \sim \frac{1}{B}\partial T \qquad \dot{\rho} + \frac{1}{B}\partial^2 T = 0$$

$$\langle \rho\rho \rangle \sim \frac{q^4}{\omega^2}\langle TT \rangle$$

(Fracton)

- $$\langle \rho\rho \rangle = \sum_n \langle 0 | \rho | n \rangle \langle n | \rho | 0 \rangle$$

- Gapped excitations at $q \approx 0$ can be classified by spin

- For a spin-s excitation (rotational symmetry)
  - $\langle q, s | \rho | 0 \rangle \sim q^s$ (leading order)
  - $\langle \rho\rho \rangle \sim q^4 \Rightarrow s = 2$

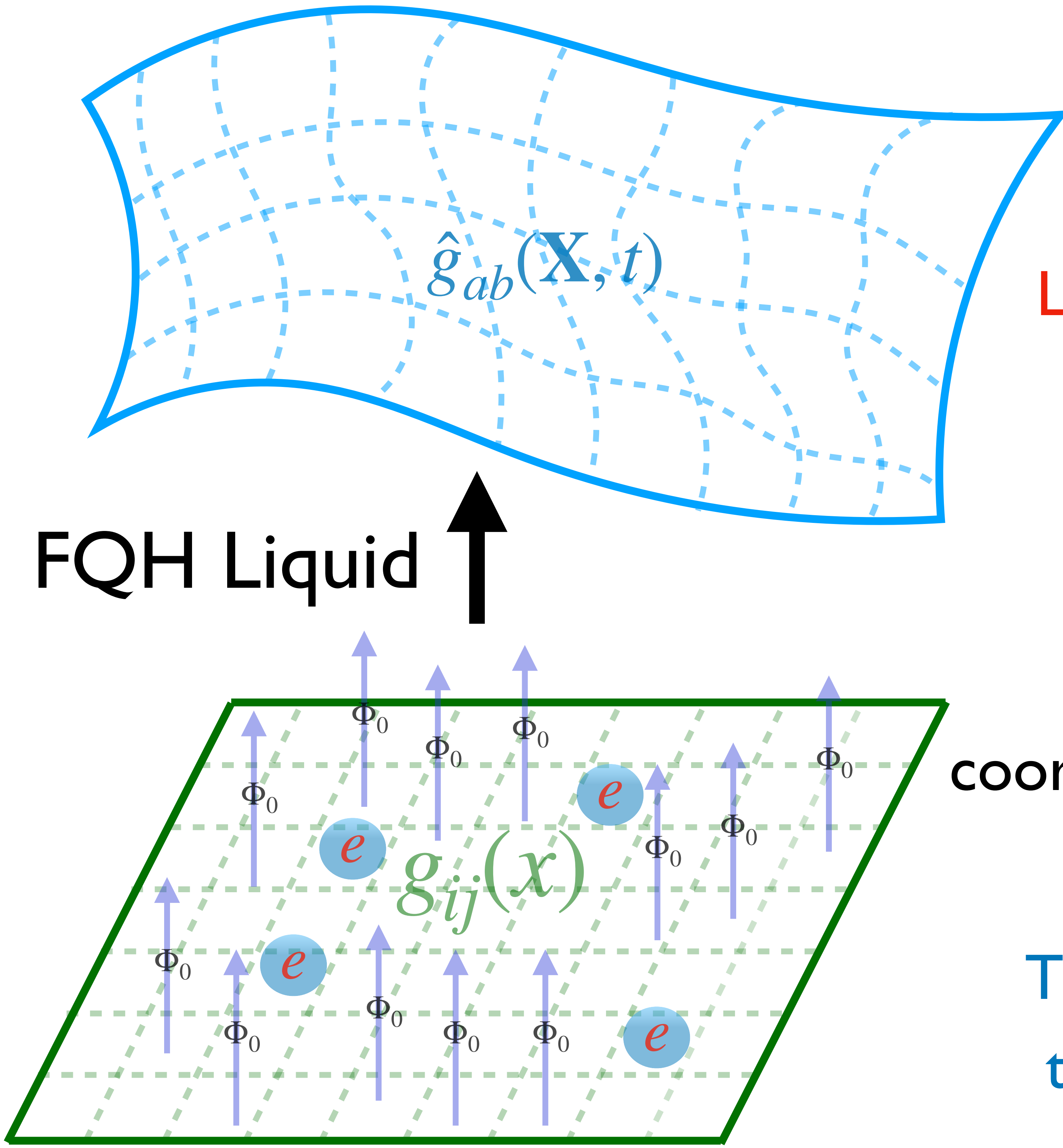
$\langle q = 0, s | T | 0 \rangle \neq 0$

Magnero-roton at $q = 0$ is created by T

Magneto-roton as graviton excitations

- Due to its spin-2, magneto-roton can be considered as the ***graviton-like*** mode in FQH.
- The idea of quantum Hall "internal metric" was initiated by **Haldane** in arXiv:0906.1854 (2009)
- Magneto-roton in Laughlin states is a dynamical spin-2 (graviton-like) excitation generated by $T_{zz}(\mathbf{q})|0\rangle$ (Golkar, DXN, Son, 2013) $(z = x + iy)$
- Gromov and Son (2017) proposed the bi-metric theory of FQH, in which the "*intrinsic metric*" is identified with a massive graviton excitation.

Emergent metric comes from the "area-reserving diffeomorphism" of the internal space of the Fractional quantum Hall fluid



$\mathbf{X}$ is the (non-commutative) Lagrange coordinate of FQH fluid (Guiding center coordinate)

x is the background Euler coordinate of 2D electrons in $\mathbf{B}$ field

The emergent graviton $\hat{g}_{ij}(x, t)$ is the pull back of $\hat{g}_{ab}(X, t)$ to the background coordinate

Dirac Composite Fermion model of general Jain's states

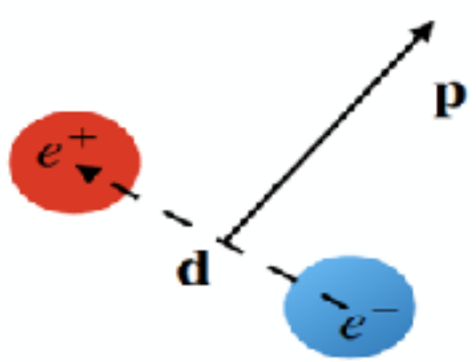
DXN, and Son PRR **3**, 033217 (2021)

Dirac composite fermion models near $\nu = 1/2$

$$\mathscr{L}_{CF} = \frac{i}{2}\left(\psi^\dagger \overleftrightarrow{D}_t \psi + \psi^\dagger \sigma^i \overleftrightarrow{D}_i \psi + v^i \psi^\dagger \overleftrightarrow{D}_i \psi\right) - \frac{1}{4\pi}\tilde{A} da + \frac{1}{8\pi}\tilde{A} d\tilde{A},$$

- ψ is a neutral the Dirac field
- a_μ is an emergent gauge field
- $D_\mu = \partial_\mu - ia_\mu$ and $v^i = \epsilon^{ij} E_j / B$ is the **drift velocity**
- Field equation of a_μ gives constraints

$$\rho_{\rm CF} = \psi^\dagger \psi = \frac{B}{4\pi} \qquad J^i_{\rm CF} = \psi^\dagger \sigma^i \psi \sim \epsilon^{ij} E_j / B$$



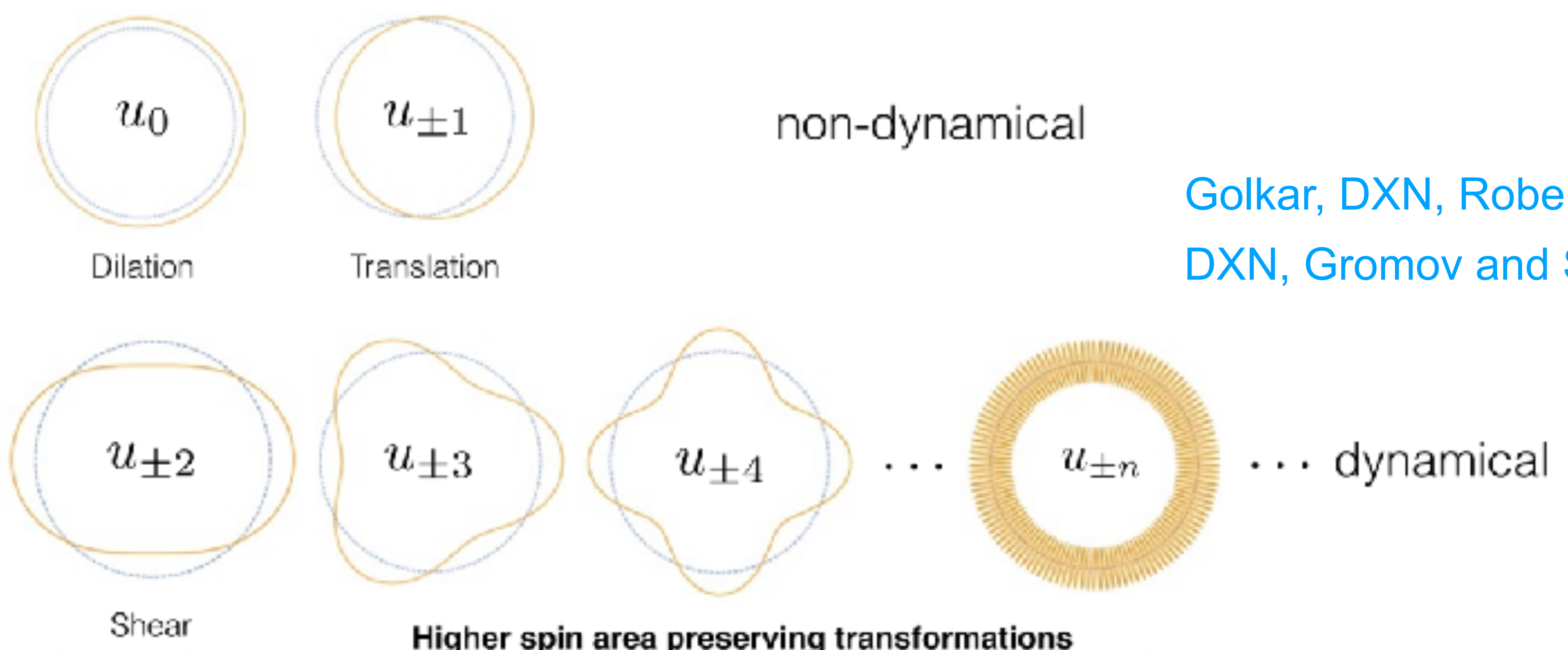
- Composite fermions have a finite density and form a Fermi-surface

Son - Dirac Composite Fermion (2015)

Graviton (Magneto-roton) as deformation of DCF Fermi surface

- Low energy degrees of freedom are multipolar deformations of CF Fermi surface

$$\rho_{\mathrm{CF}} = \psi^\dagger \psi = \frac{B}{4\pi} \qquad J^i_{\mathrm{CF}} = \psi^\dagger \sigma^i \psi \sim \epsilon^{ij} E_j / B$$



Golkar, DXN, Roberts, Son (2016)

DXN, Gromov and Son (2018)

- The shear mode is spin-2, and it is the massive graviton excitation.
- The chirality of the graviton is determined by the topological quantum number of the FQH states

Dirac composite fermion models near $\nu = 1/2n$

$$\mathscr{L}_{CF} = \frac{i}{2}\left(\psi^\dagger \overleftrightarrow{D}_t \psi + \psi^\dagger \sigma^i \overleftrightarrow{D}_i \psi + v^i \psi^\dagger \overleftrightarrow{D}_i \psi\right)$$

$$-\frac{1}{8\pi}\left(1 - \frac{1}{n}\right) ada + \frac{1}{4\pi n}\tilde{A}da + \frac{1}{8\pi n}\tilde{A}d\tilde{A},$$

Goldman, Fradkin (2019)

Jie Wang (2019)

- The general Jain's states (p is large)

$$\nu_\pm = \frac{p}{2np \pm 1}$$

- **Problems:** Some **topological properties** of FQH states are not satisfied by the Dirac composite fermion model (Wen-Zee shift and Haldane's bound are violated)

DXN, and Son PRR **3**, 033217 (2021)

Extra graviton(s) (spin-2)

- The problems of the Dirac model can be fixed by adding an addition spin-2 mode.
- We suggest that the new mode comes from the dynamics of the internal structure of composite fermions (confirmed latter by numerics and model wavefunctions)
- We add a spin-2 mode by introducing an additional emergent metric $\hat{\tilde{g}}_{ij}$ as the new dynamical degree of freedom and name it **Haldane mode**.
- The effective theory of the extra mode follows the bi-metric theory by Gromov and Son 2017.
- The extra graviton is **universal** for **all** FQH states near $\nu = 1/4$

DXN, and Son PRR **3**, 033217 (2021)

Numerical results

DXN, Haldane, Rezayi, Son, Yang, PRL 128 (24), 246402 (2022)

- All the conclusions for the second graviton mode for FQH states near filling fraction 1/4 were confirmed numerically.
- I only show the selective results

Chiral Graviton spectral functions

$(z = x + iy)$

- We define the chiral graviton's spectral functions (a $\mathbf{q} = 0$)

$$I_{-}(\omega) = \frac{1}{N_e} \sum_n |\langle n| \int d\mathbf{x}\, T_{zz} |0\rangle|^2 \delta(\omega - \omega_n),$$

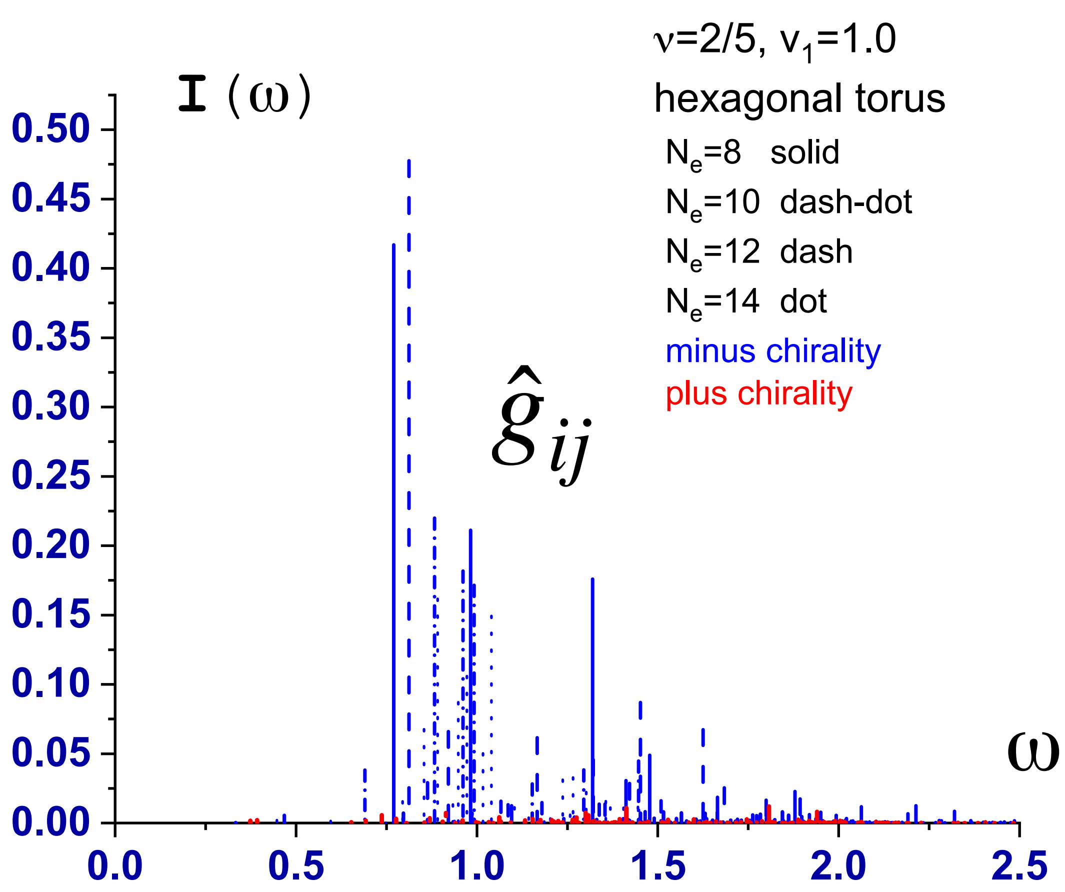
$$I_{+}(\omega) = \frac{1}{N_e} \sum_n |\langle n| \int d\mathbf{x}\, T_{\bar{z}\bar{z}} |0\rangle|^2 \delta(\omega - \omega_n),$$

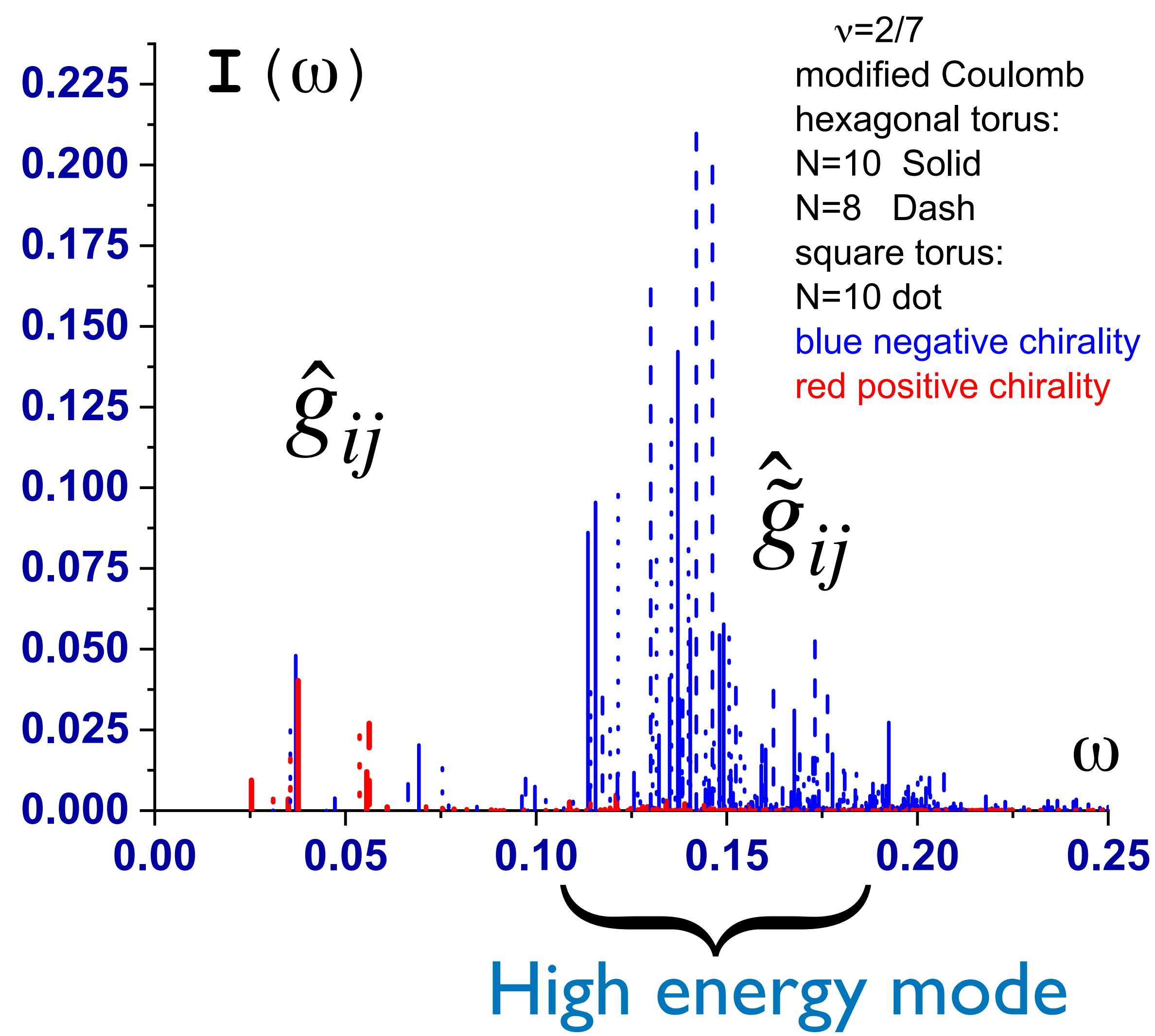
N_e : Total number of electrons

ω_n : Energy of n^{th} state

- The chirality of graviton modes can be determined in the field theory model (DXN, Son 2021)
- The spectral function also can be determined through the topological quantum numbers of FQH states.

The broaden of the spectral peaks dues to both finite size effect and the life time of the graviton modes





- State $\nu = 2/5$ (similar to $\nu = 1/3$).
- Graviton has negative chirality
- No high-energy mode.

- State $\nu = 2/7$.
- Low energy graviton has positive chirality
- High energy graviton has negative chirality

DXN, Haldane, Rezayi, Son, Yang (PRL 2022)

Probe gravitons in the bulk

DXN, Son Phys. Rev. Research **3**, 023040 (2021)

Raman scattering experiment

Energy momentum conservation:

- Momentum of incident and scattered lights $\mathbf{k}_I, \mathbf{k}_S$.
- Frequency of incident and scattered lights ω_I, ω_S
- Magneto-roton's momentum and energy (dispersion)

$$\mathbf{q} = \mathbf{k}_I - \mathbf{k}_S, \quad \omega = \omega_I - \omega_S$$

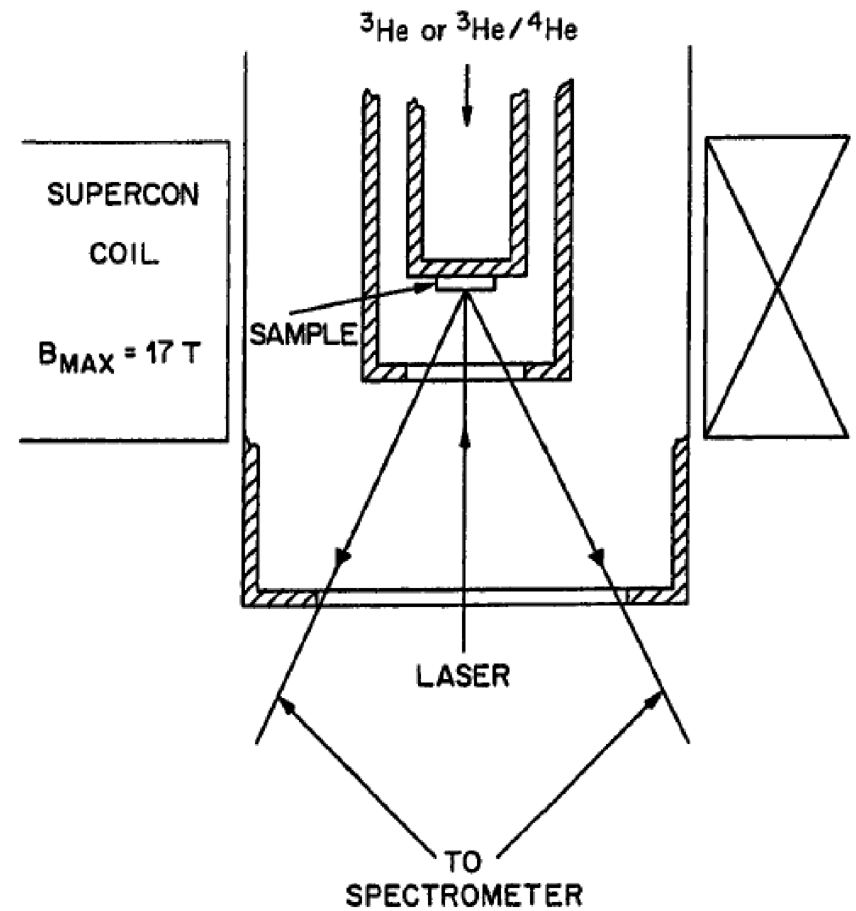


Figure 8.3. Schematic representation of the low-temperature inelastic light-scattering experiment. The magnetic field is perpendicular to the two-dimensional electron layer. In this configuration the in-plane component of the light-scattering wavevector is $k < 10^4\ \mathrm{cm}^{-1}$.

- In previous theoretical model (Platzman, He 1994), the scattered light intensity I_S measures the ***dynamic*** structure factor $S = \langle \rho\rho \rangle$

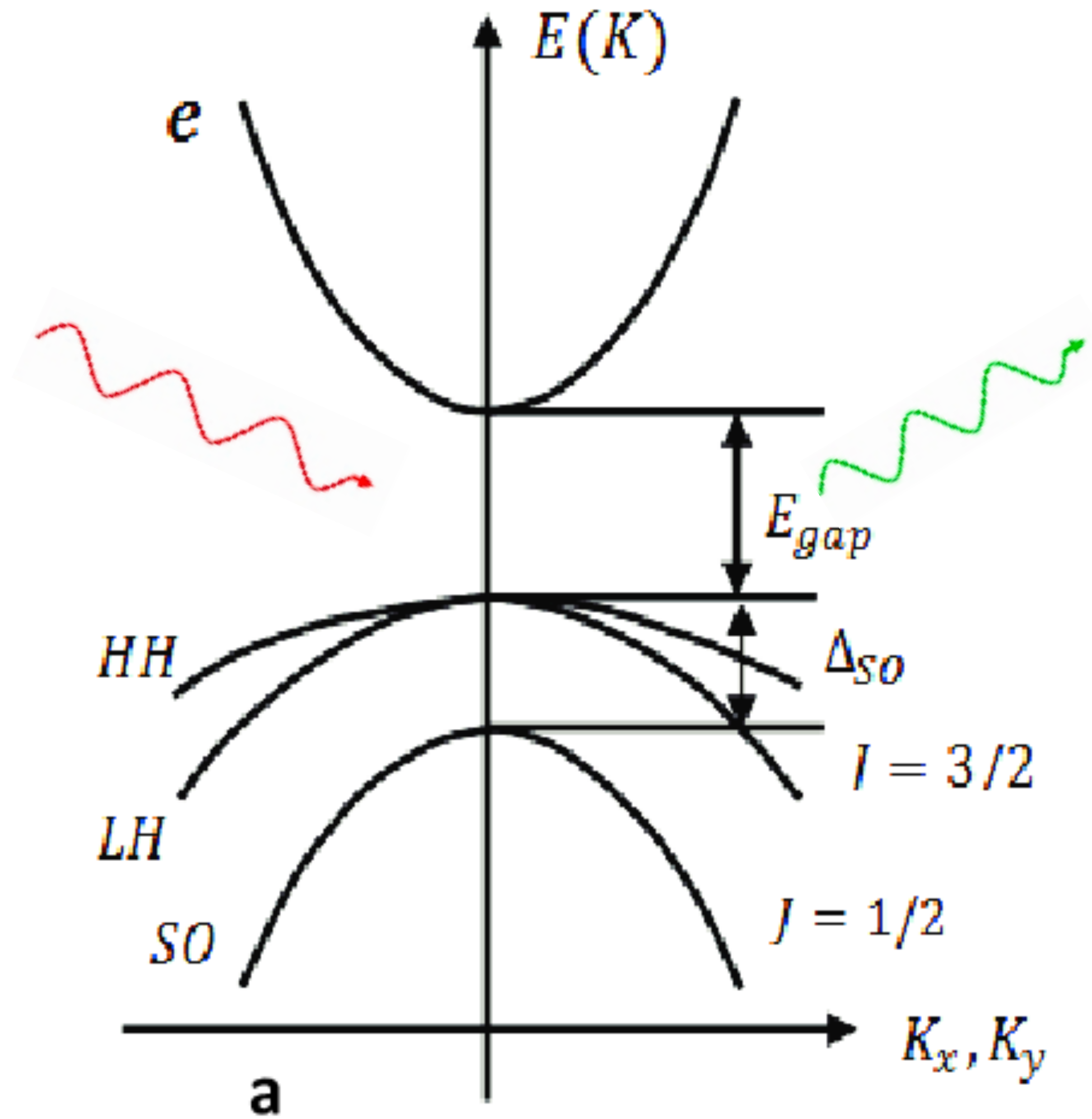
$$I_S(\omega, q) \sim S(\omega, q) \sim q^4 \qquad \text{(GMP 1986)}$$

$\rightarrow$ Can not explain the peak at zero momentum $q \sim 0$

Resonant Raman scattering

- $E_\gamma \approx E_{\text{gap}}$ (resonance)
- $\gamma_I \to$ particle-hole pair
- $\to \gamma_S$+excitation

$$\omega_I, \omega_S \gg \omega$$



Luttinger model of GaAs

DXN, Son Phys. Rev. Research **3**, 023040 (2021)

Effective coupling

- After performing second-order perturbation theory (integrating out hole-bands): coupling of EM field to electron in the conductance band

- The effective coupling for circular polarised Raman scattering

$$\mathscr{L}_{\gamma\psi} \sim E^z E^z (T_{zz} + 0.16\, T_{\bar{z}\bar{z}}) + \text{h.c.} \qquad (z = x + iy)$$

  - predominantly spin-conserving, not changing by 4 (dues to slightly broken rotational symmetry of the Luttinger model)

- The scattered light intensity measures the stress tensor correlation function

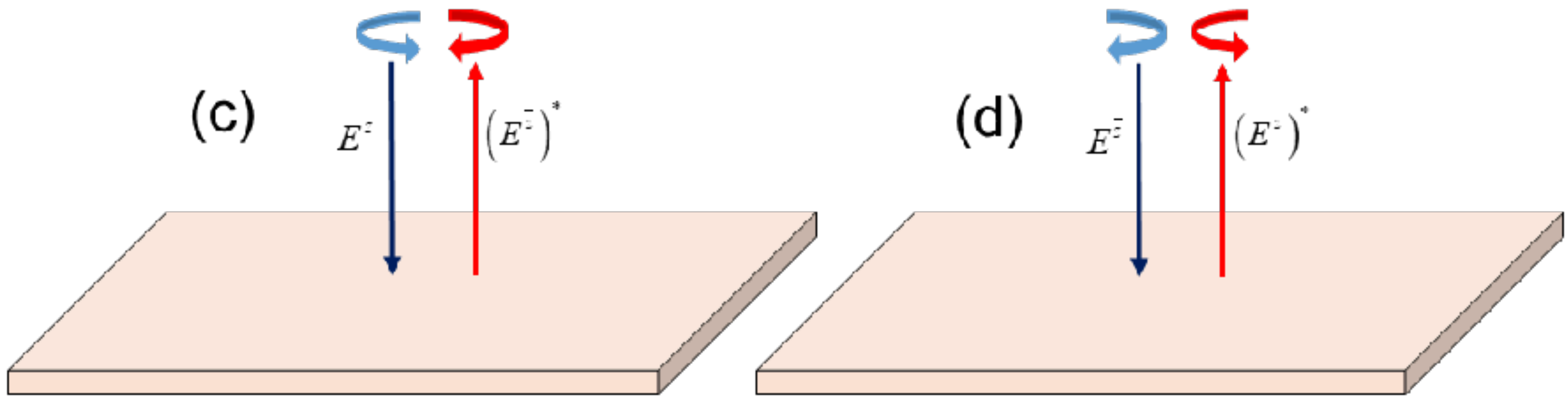
$$I_S(q) \sim \langle TT \rangle \sim q^0$$

Explains the peak in Raman scattering near $q = 0$

DXN, Son Phys. Rev. Research **3**, 023040 (2021)

Determining the spin structure of the magnetoroton

- Use circular polarized Raman scattering (at $\mathbf{q} = 0$)



$I_S(\omega,0)$ measures $I_-(\omega)$

Large peak if magnero-roton has spin -2

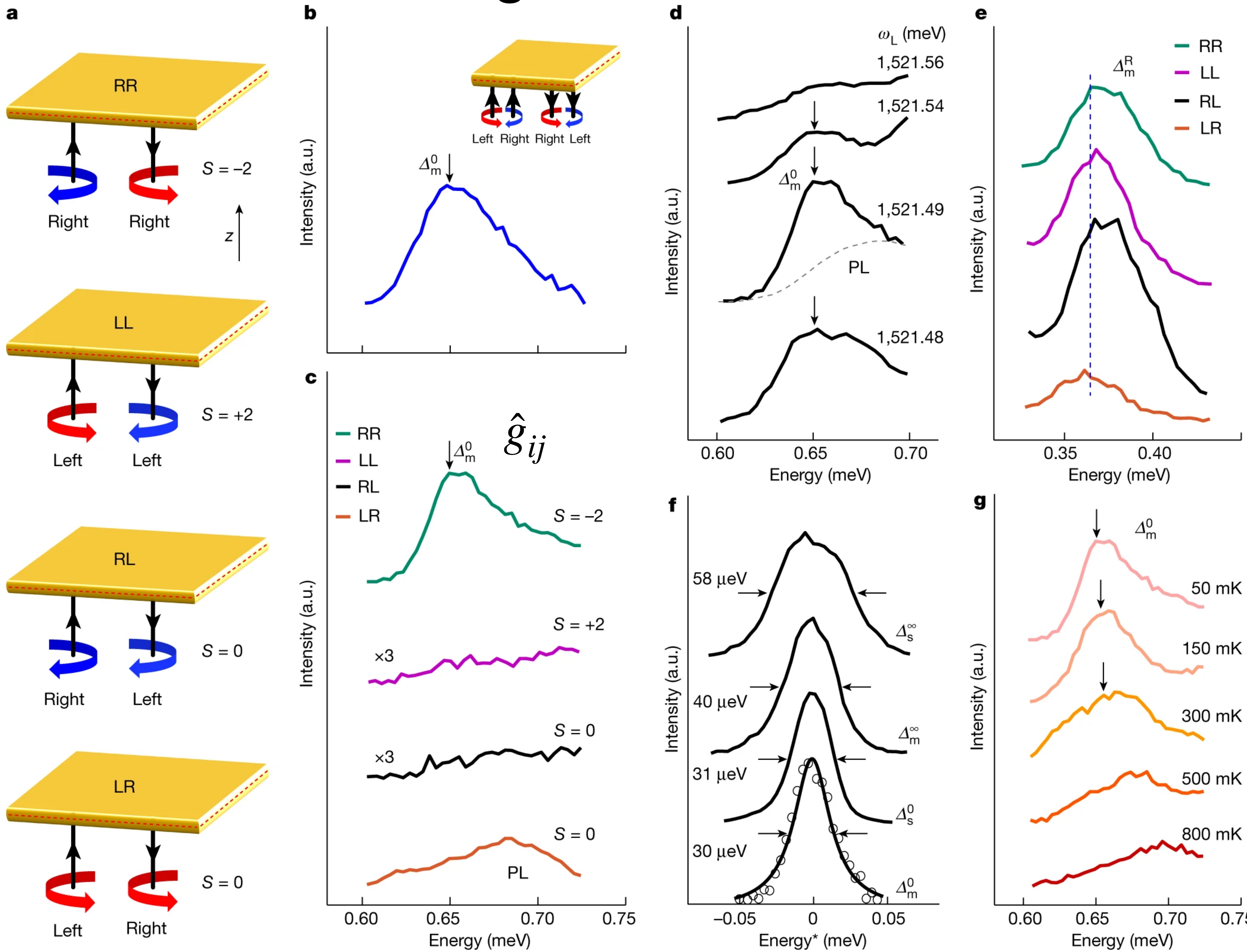
$I_S(\omega,0)$ measures $I_+(\omega)$

Large peak if magnero-roton has spin +2

Prediction: dominance by one sign of photon spin flip for chiral sates

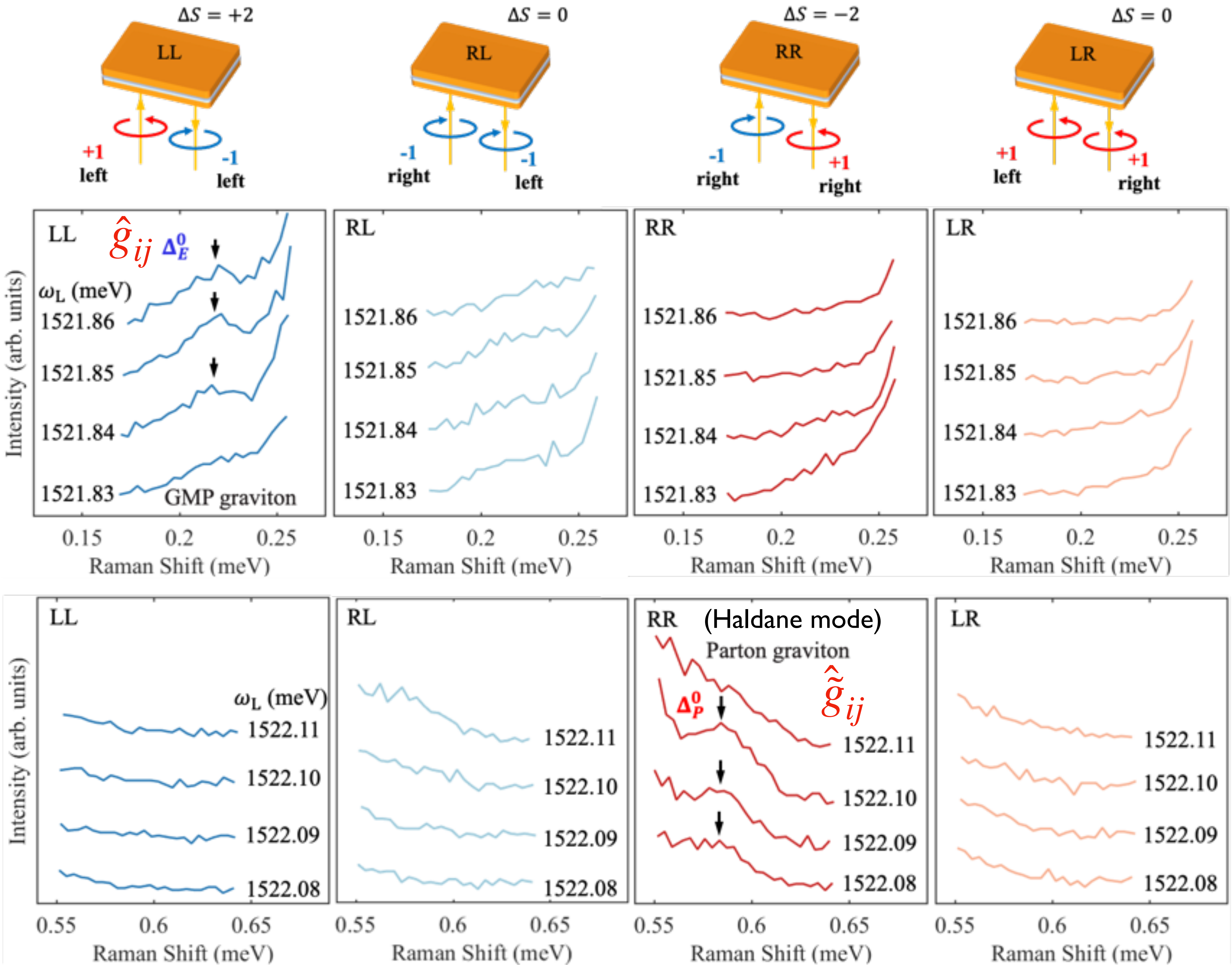
DXN, Son Phys. Rev. Research **3**, 023040 (2021)

Chiral graviton at $\nu = 1/3$ (Near $\nu = 1/2$)



[Nature 628, 78–83 (2024)]

Two chiral gravitons at $\nu = 2/7$ (Near $\nu = 1/4$)



[New experimental results from Nanjing U + Princeton group (2025)]

Conclusions:

- Magneto-roton in a gapped FQH is a graviton-like excitation
- The **newly proposed** high energy graviton is universal for FQH states near $\nu = 1/4$.
- Experiments confirmed our theoretical model of FQH graviton using polarised Raman scattering [Nature 628, 78–83 (2024) + More exp results]
- Experiments **confirmed** of the **new high energy graviton-like mode** (**The excitation that people missed for nearly 40 years**).

Outlook:

- **Related work:**
  - There are two neutral excitations of Haldane-Rezayi state with spin-2 and spin-3/2 (seen in numerics), we proposed the **supergravity** model that captures both **graviton and gravitino.**

(DXN, K Prabhu, AC Balram, A Gromov PRB 107 (12), 125119 (2023))

  - Graviton in Fractional Chern Insulators

Y Wang, J Huxford, DXN, G Ji, YB Kim, B Yang (2025)

- **Future directions:**
  - Experimental proposal to verify the supergravity theory?
  - Higher-spin $(s \geq 2)$ theory of FQH and connection to string theory?