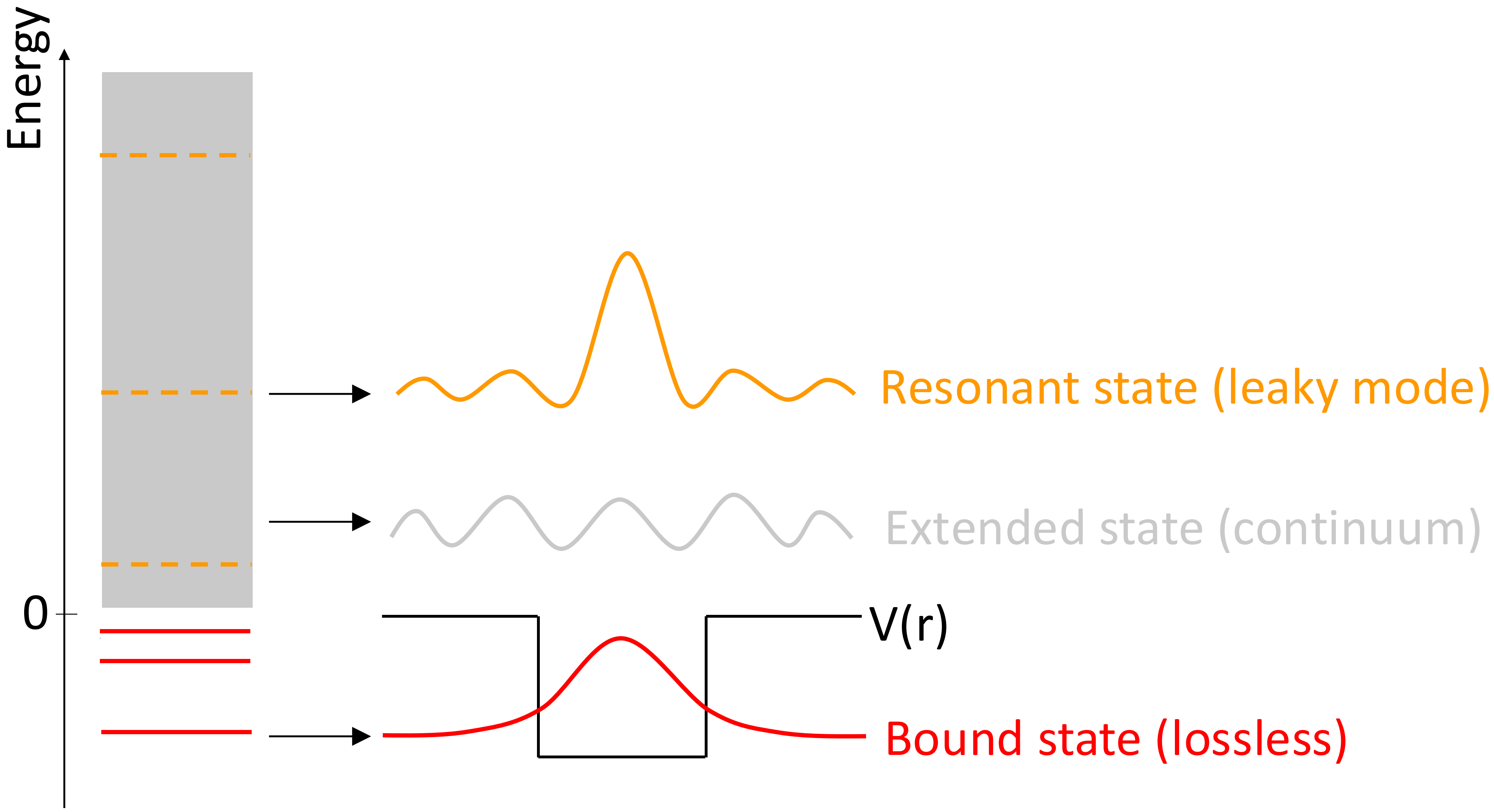
Non-Hermitian Topological Singularities :
Photonic Analog of Quantum Subradiance and Exceptional Points

Hai Son Nguyen

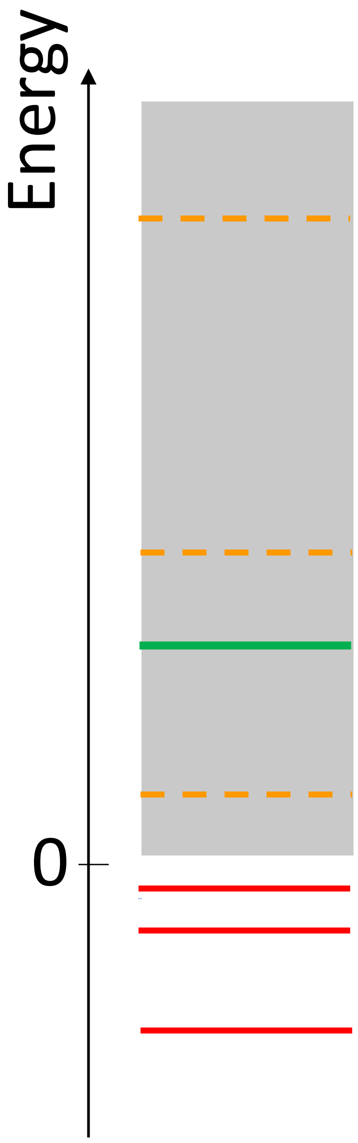
CNRS-International-NTU-Thales ResearchAlliance (CINTRA), Singapore
Institut des Nanotechnologies de Lyon (INL), Ecole Centrale de Lyon, France

100 Years of Quantum Physics
Quy Nhon 07/10/2025

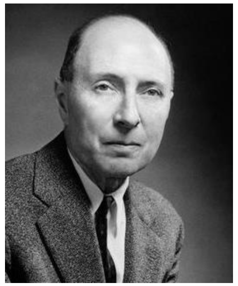
Bound states of « particle in a box »



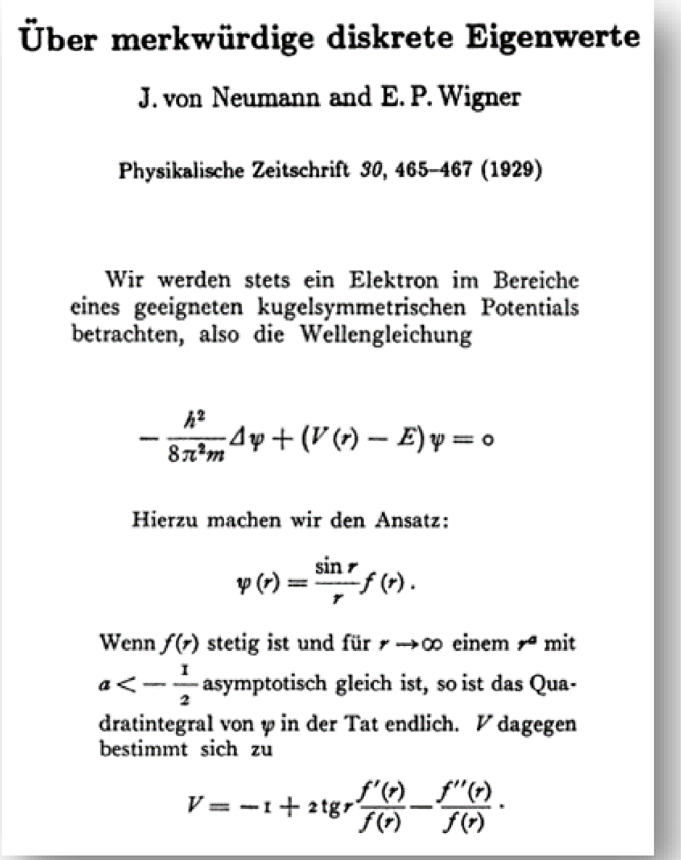
Von Neumann and Wigner's Bound states in the continuum



BICs = Confined states having infinite lifetime despite lying in the continuous spectrum of radiating/leaky states



« On curious discrete eigenvalues »



Über merkwürdige diskrete Eigenwerte

J. von Neumann and E. P. Wigner

Physikalische Zeitschrift *30*, 465–467 (1929)

Wir werden stets ein Elektron im Bereiche eines geeigneten kugelsymmetrischen Potentials betrachten, also die Wellengleichung

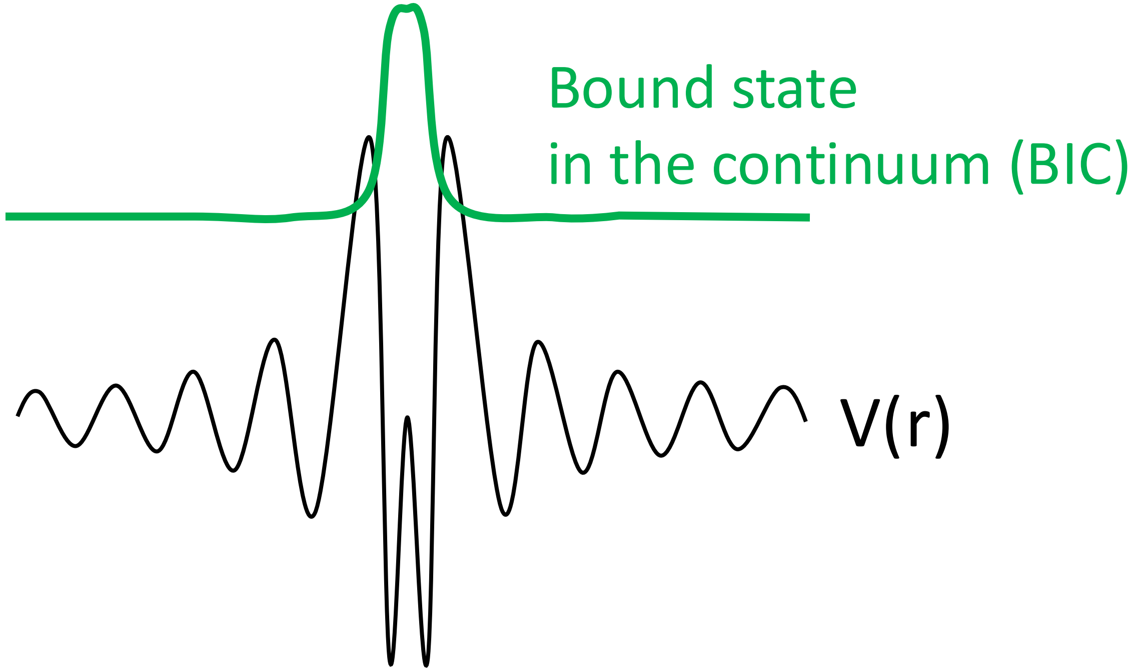
$$-\frac{h^2}{8\pi^2 m}\Delta\psi + (V(r) - E)\psi = 0$$

Hierzu machen wir den Ansatz:

$$\psi(r) = \frac{\sin r}{r} f(r).$$

Wenn $f(r)$ stetig ist und für $r \to \infty$ einem r^a mit $a < -\frac{1}{2}$ asymptotisch gleich ist, so ist das Quadratintegral von ψ in der Tat endlich. V dagegen bestimmt sich zu

$$V = -1 + 2\,\mathrm{tg}\,r \frac{f'(r)}{f(r)} - \frac{f''(r)}{f(r)}.$$



J. Von Neumann and E. Wigner, **Z. Phys** 30, 465 (1929)

Origin of BICs

PHYSICAL REVIEW A VOLUME 32, NUMBER 6 DECEMBER 1985

Interfering resonances and bound states in the continuum

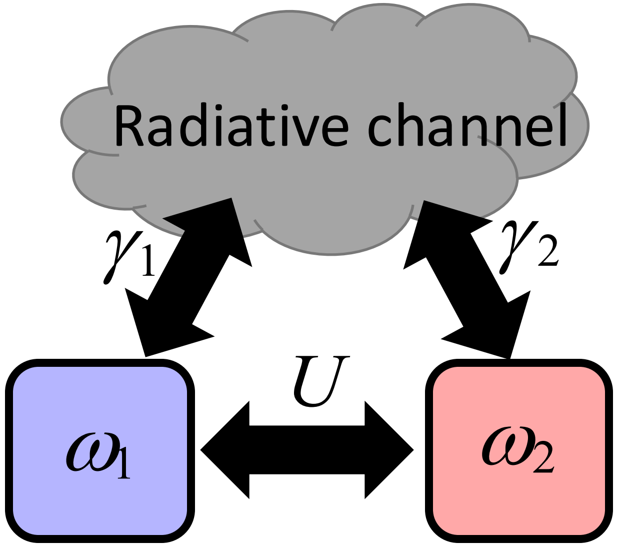
H. Friedrich and D. Wintgen
Physik Department, Technische Universität München, D-8046 Garching, West Germany
(Received 24 June 1985)

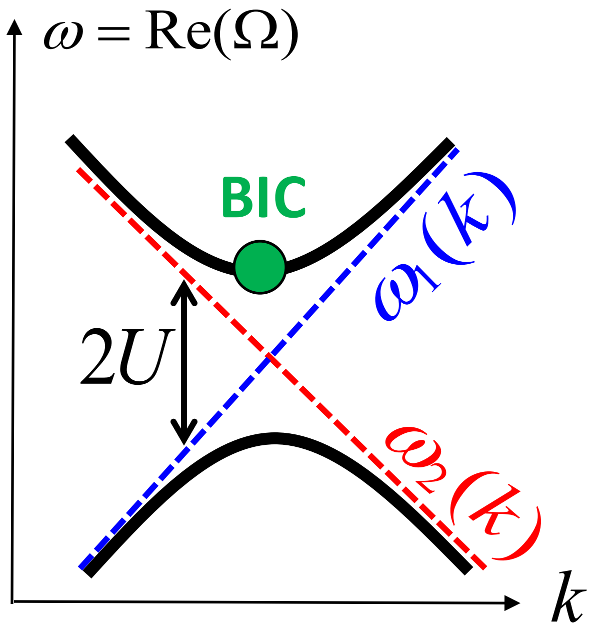
$$H = \begin{pmatrix} \omega_1 & U \\ U & \omega_2 \end{pmatrix} + i \begin{pmatrix} \gamma_1 & e^{i\phi}\sqrt{\gamma_1\gamma_2} \\ e^{i\phi}\sqrt{\gamma_1\gamma_2} & \gamma_2 \end{pmatrix}$$

Interference of losses

*« …**bound states in the continuum (BIC's) can occur due to the interference of resonances** belonging to different channels. »*

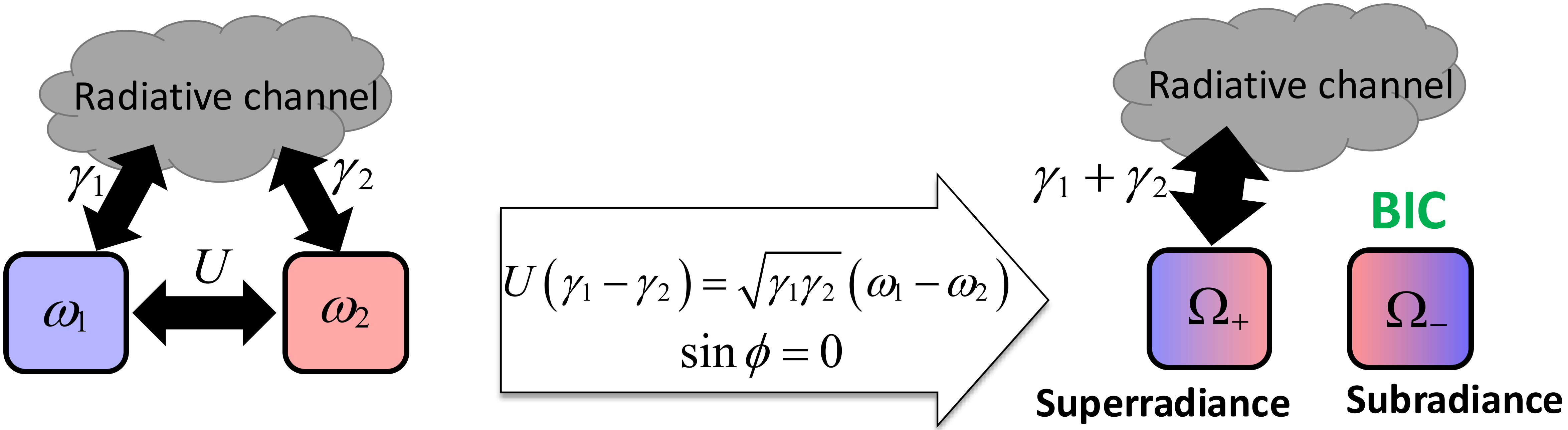
*« If two resonances pass each other as a function of a continuous parameter, then **interference causes an avoided crossing of the resonance positions** and for a given value of the continuous parameter **one resonance has exactly vanishing width and hence becomes a BIC** ».*





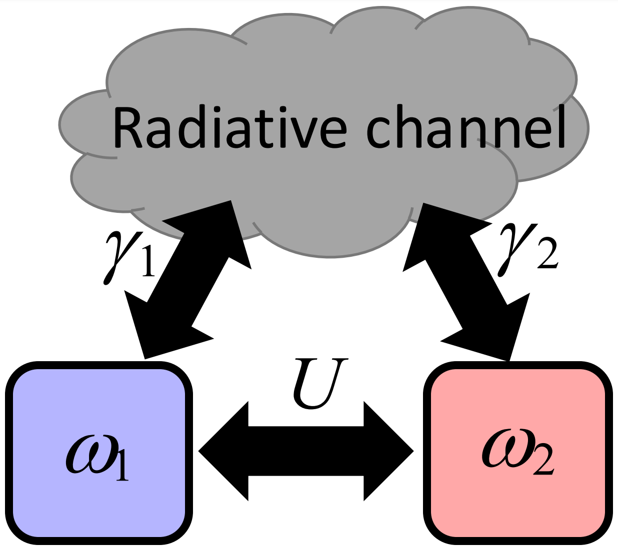
Friedrich– Wintgen interefence: super- and sub-radiance

$$H = \begin{pmatrix} \omega_1 & U \\ U & \omega_2 \end{pmatrix} + i \begin{pmatrix} \gamma_1 & e^{i\phi}\sqrt{\gamma_1\gamma_2} \\ e^{i\phi}\sqrt{\gamma_1\gamma_2} & \gamma_2 \end{pmatrix}$$



How to make this subradiance feature in photonic analog ?

Non-Hermitian degeneracy



$$\omega_1 - \omega_2 = \pm 2\sqrt{\gamma_1 \gamma_2}\cos\varphi$$

$$U = \pm \frac{\gamma_1 - \gamma_2}{2} + \sqrt{\gamma_1 \gamma_2}\sin\varphi$$

Exceptional Points

$$\boxed{\Omega_+ = \Omega_- = \frac{\omega_1 + \omega_2}{2} + i\frac{\gamma_1 + \gamma_2}{2}}$$

Complex degeneracy

$$\Omega_\pm = \frac{\omega_1 + \omega_2}{2} + i\frac{\gamma_1 + \gamma_2}{2} \pm \underbrace{\sqrt{\left(\frac{\omega_1 - \omega_2}{2} + i\frac{\gamma_1 - \gamma_2}{2}\right)^2 + \left(U + i\sqrt{\gamma_1 \gamma_2}e^{i\phi}\right)^2}}_{0}$$

$$\left|\Omega_+\right\rangle = \left|\Omega_-\right\rangle = \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$

Eigenstates Coalesce !!!

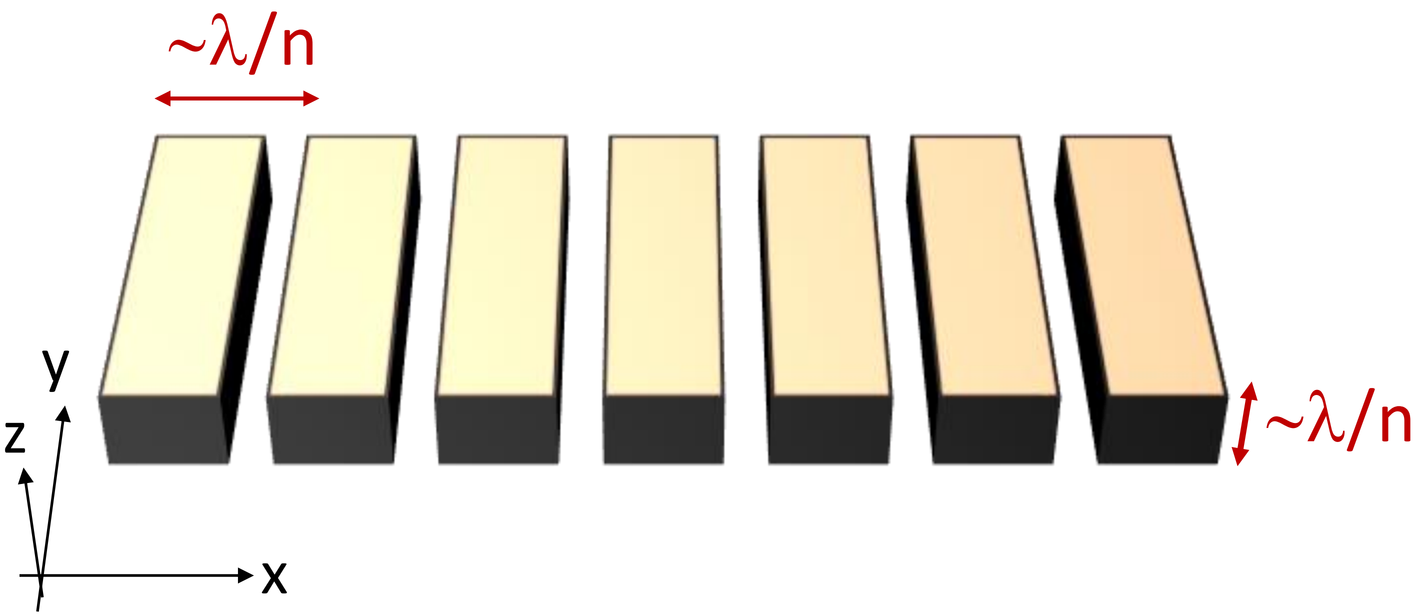
$$\left\langle \Omega_+ \middle| \Omega_+ \right\rangle = \left\langle \Omega_- \middle| \Omega_- \right\rangle = \begin{pmatrix} 1 & \mp i \end{pmatrix} . \begin{pmatrix} 1 \\ \pm i \end{pmatrix} = 0$$

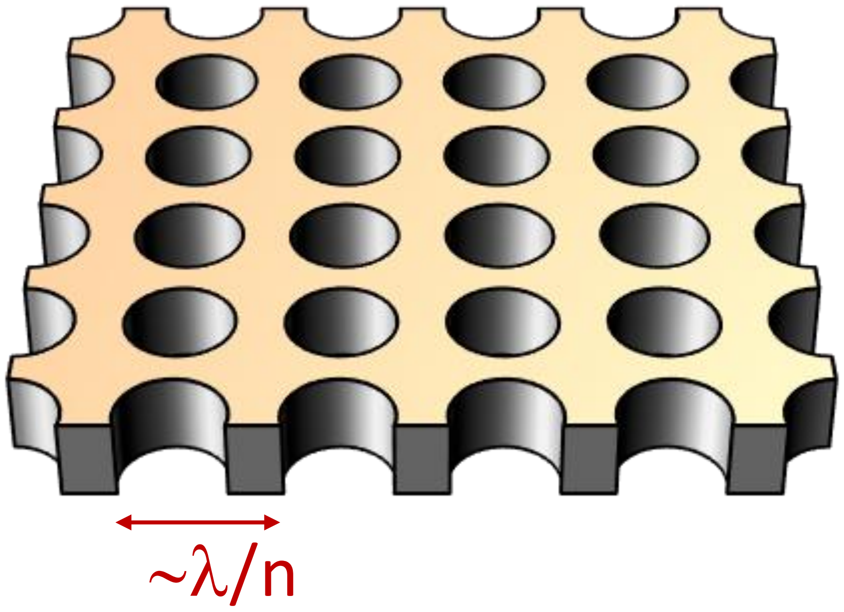
Eigenstates are self-orthogonal !!!

How to make exceptionbal point in photonic analog ?

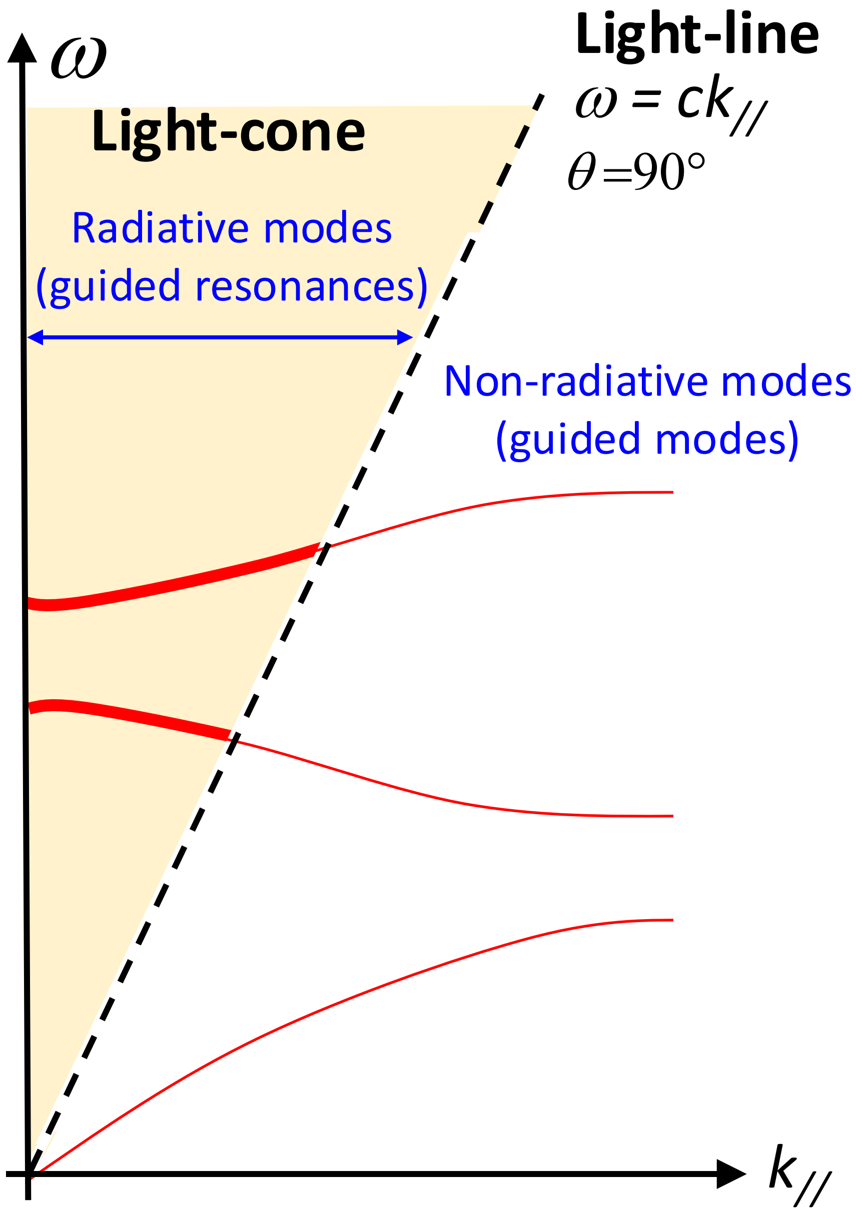
Photonic crystal slab

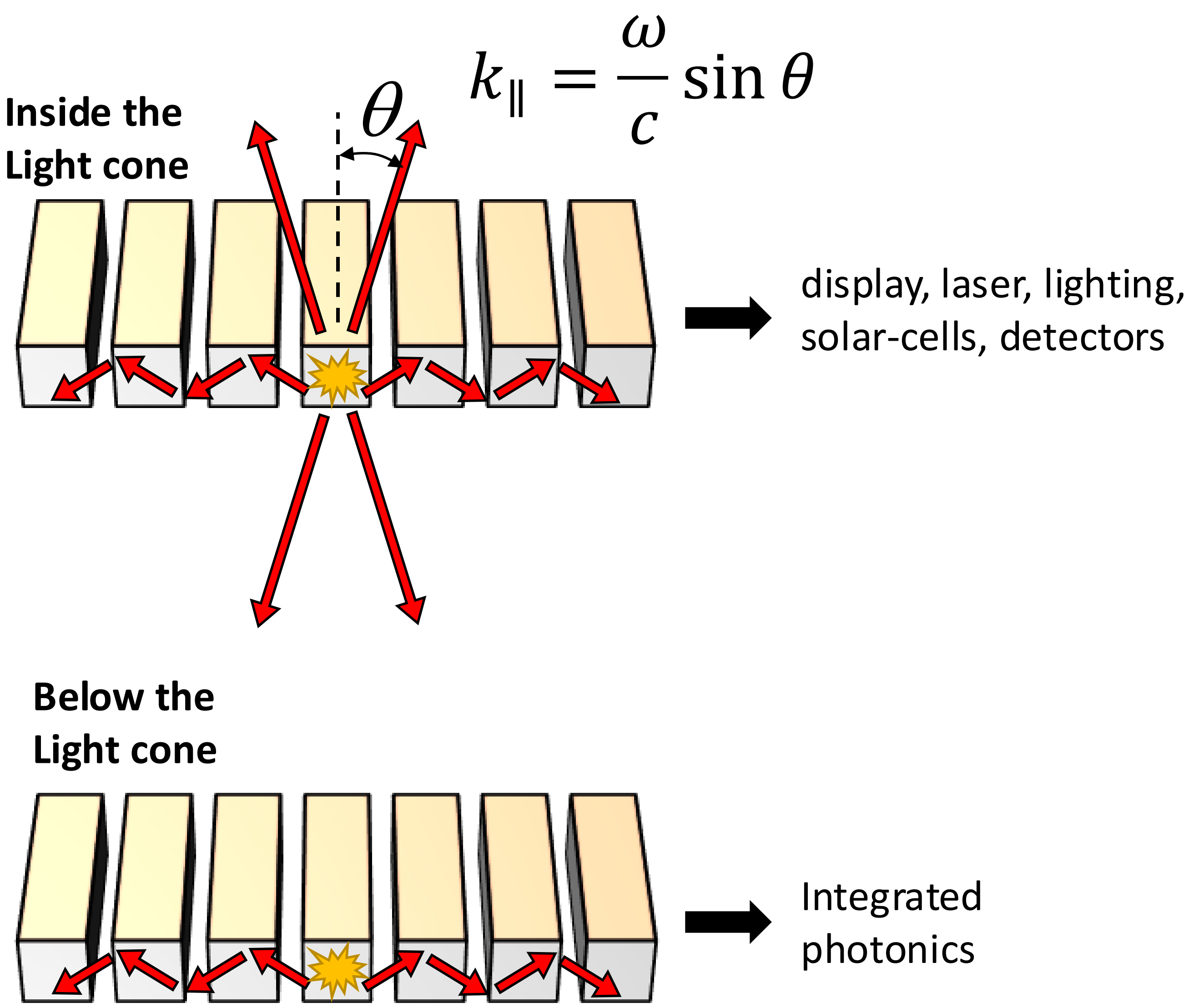
Photonic crystal slab = **Thin dielectric layer patterned with in-plane periodic structure**



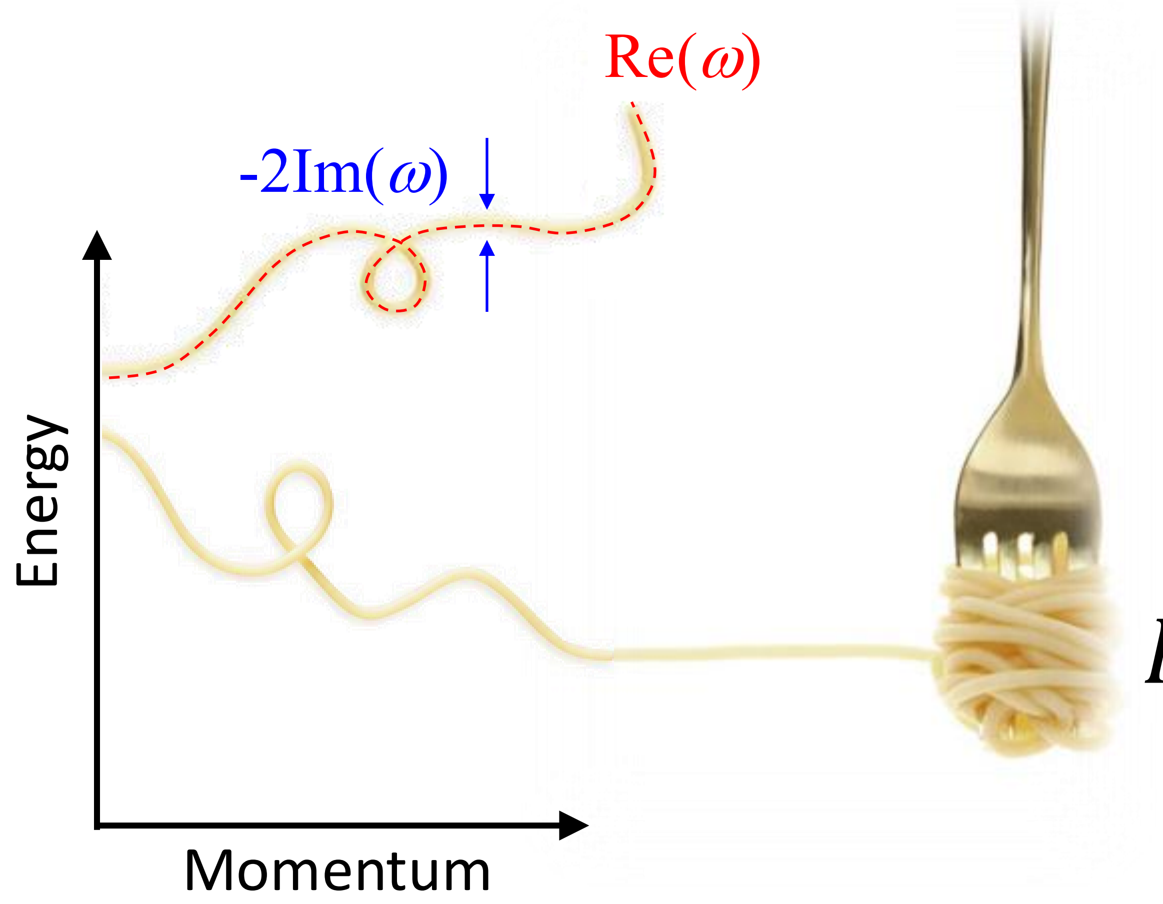


Lossy vs Lossless band structure



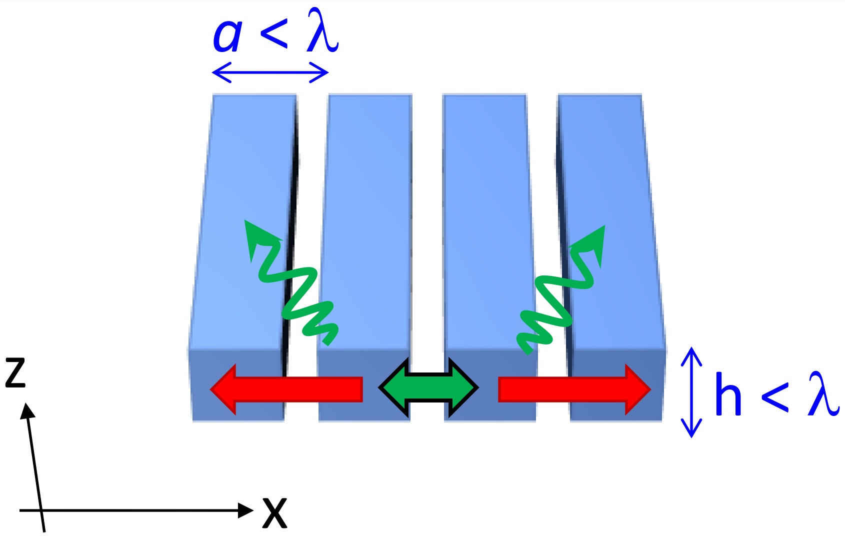


effective non-Hermitian Hamiltonian



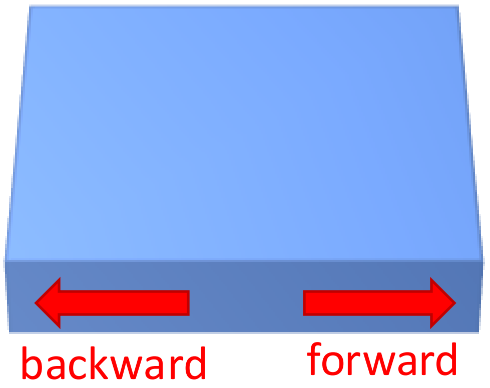
$$H|\Psi\rangle = \omega|\Psi\rangle$$

From waveguide to photonic grating



=

Planar waveguide



+ **Periodic perturbation**

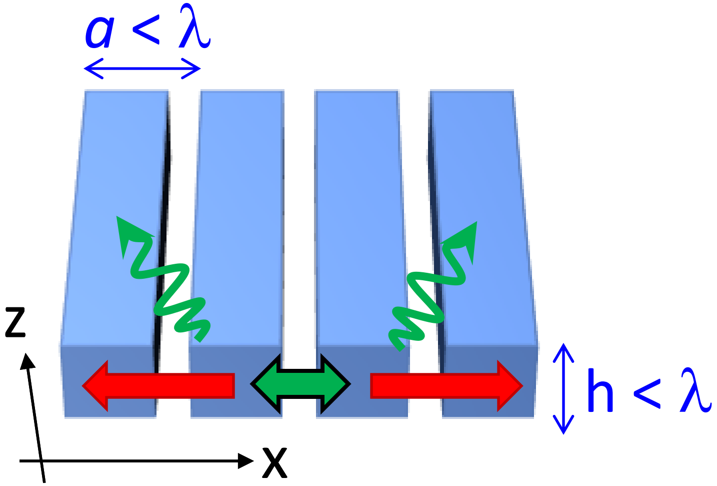
$$\varepsilon(x,z) = \varepsilon_{eff}(z) + \sum_{m\neq 0} \varepsilon_m(z).\exp\left(mKx\right)$$

« Creating »
guided modes

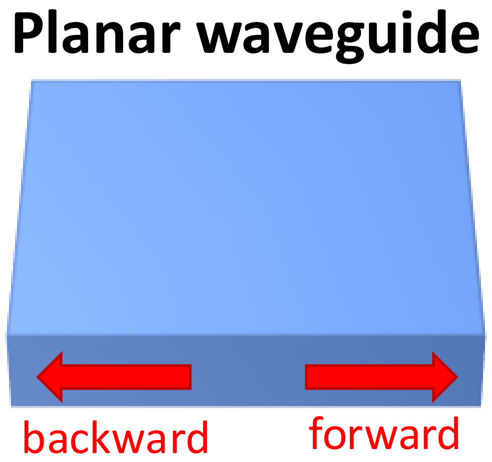
« Coupling» between guided modes → **gaps**

« Leaking » to the radiative continuum → **losses**

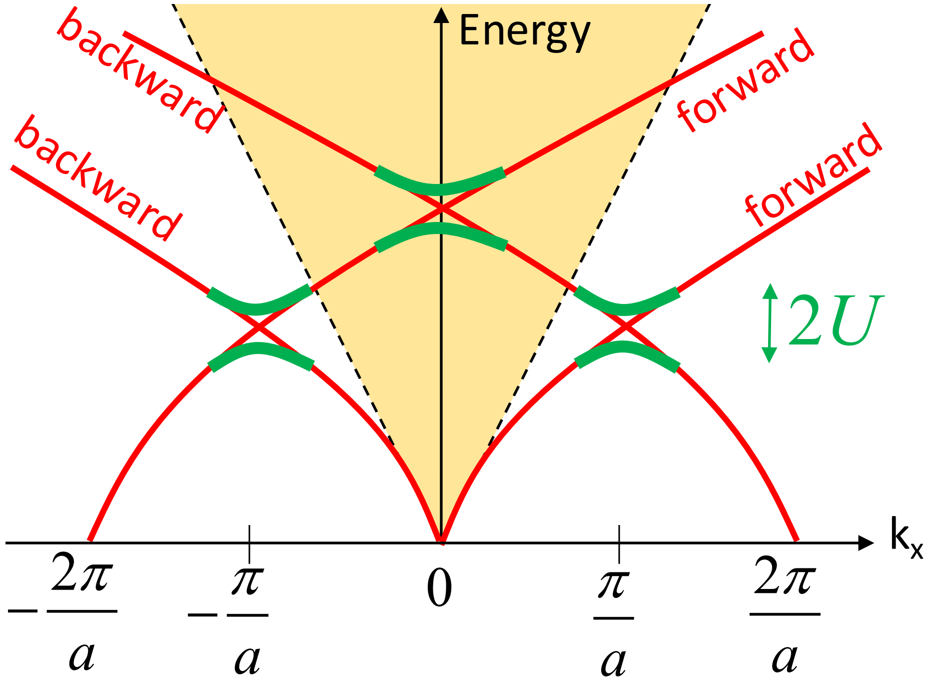
Band structure of 1D photonic crystal

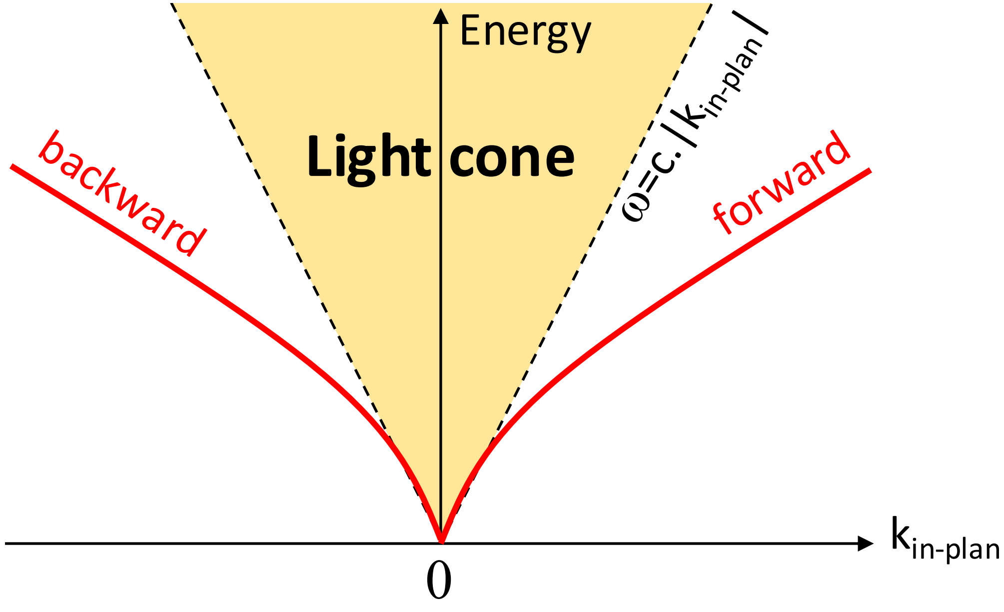


=

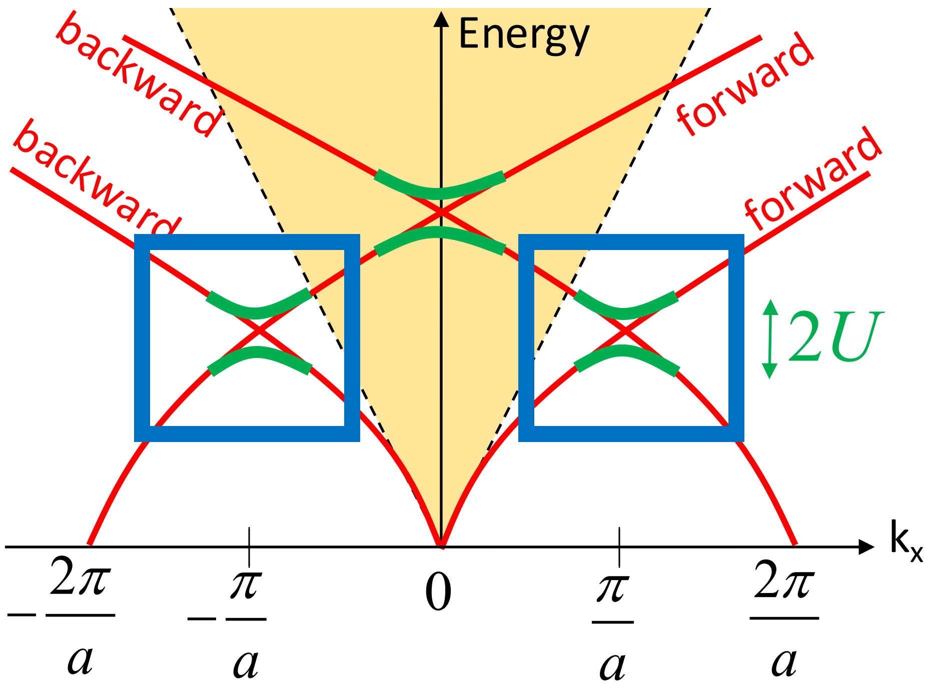


+ Periodic perturbation



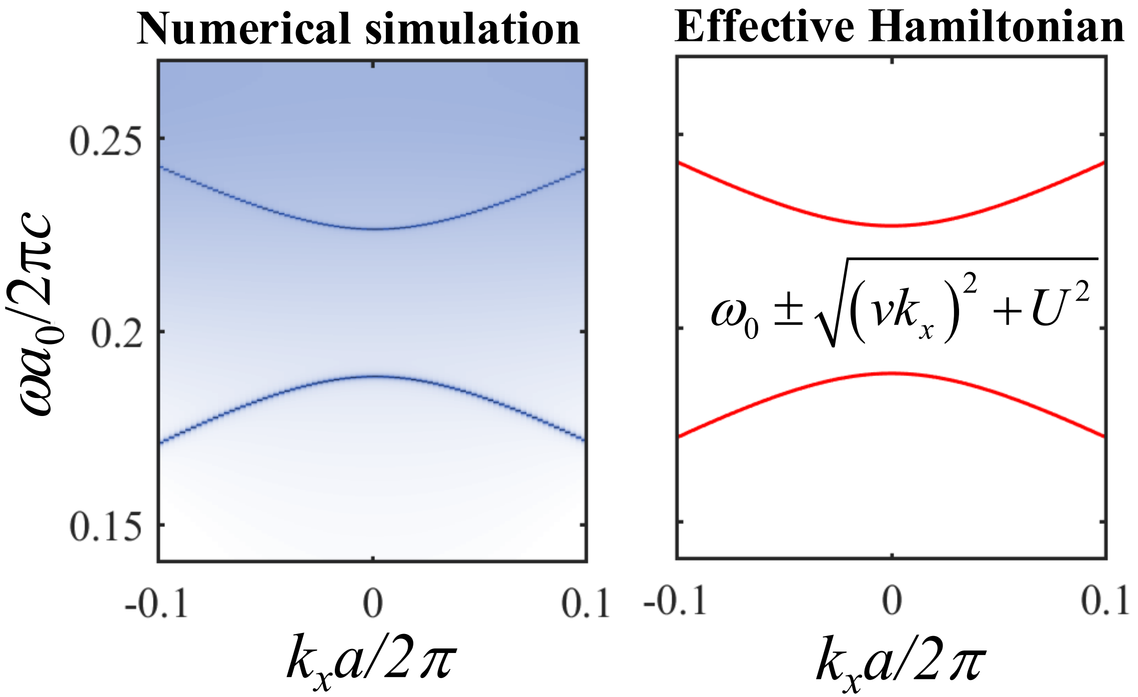


Effective model at the first band gap: Hermitian physics

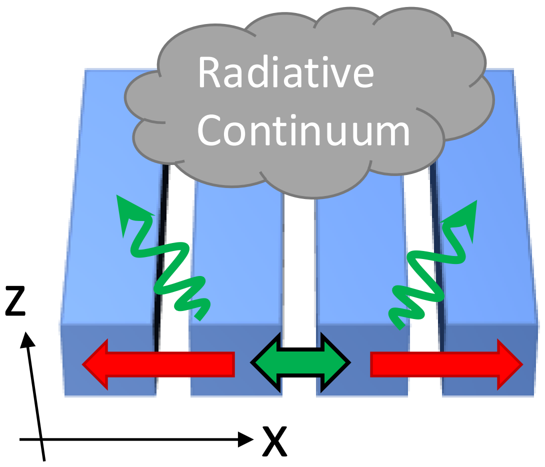


$$H = \begin{pmatrix} \omega_0 + v.k_x & U \\ U & \omega_0 - v.k_x \end{pmatrix}$$

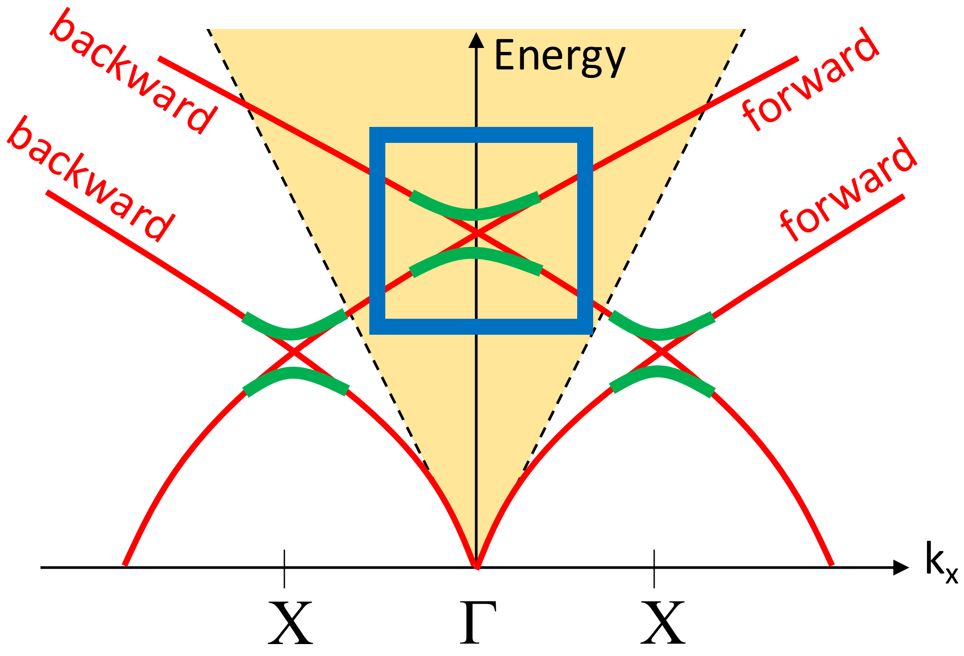
- Below the light cones → No radiative losses
- Photonic modes is given by a **Hermitian** Hamiltonian



Second band gap: non-Hermitian physics

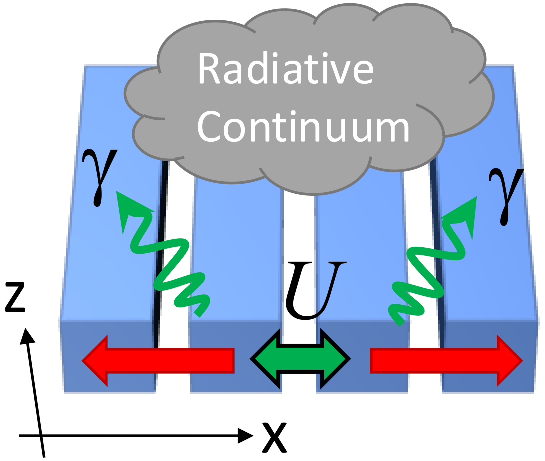


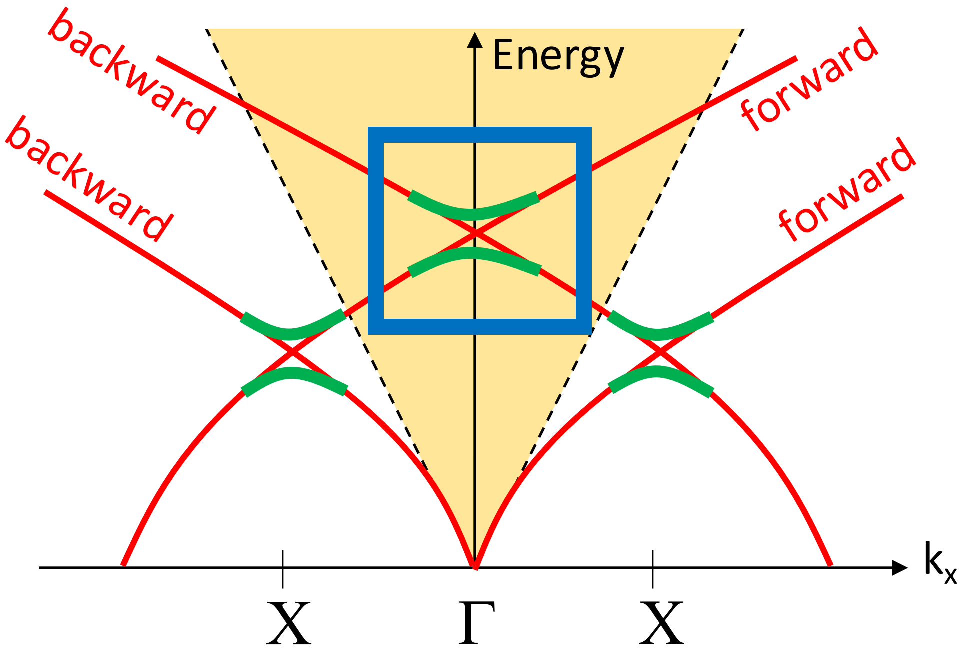
- Above the light cones → Can radiate to free space
- Photonic modes is given by a **<u>non-Hermitian</u>** Hamiltonian



Friedrich-Wintgen inteference via loss channels

Second band gap: non-Hermitian Hamiltonian





$$H = \begin{pmatrix} \omega_1 & U \\ U & \omega_2 \end{pmatrix} + i \begin{pmatrix} \gamma_1 & e^{i\phi}\sqrt{\gamma_1\gamma_2} \\ e^{i\phi}\sqrt{\gamma_1\gamma_2} & \gamma_2 \end{pmatrix}$$

$$\boxed{\begin{aligned} \gamma_1 &= \gamma_2 = \gamma \\ \phi &= \pi \end{aligned}}$$

$$\boxed{H = \begin{pmatrix} -v.k_x & U \\ U & v.k_x \end{pmatrix} + i \begin{pmatrix} \gamma & -\gamma \\ -\gamma & \gamma \end{pmatrix}}$$

Non-Hermitian band inversion

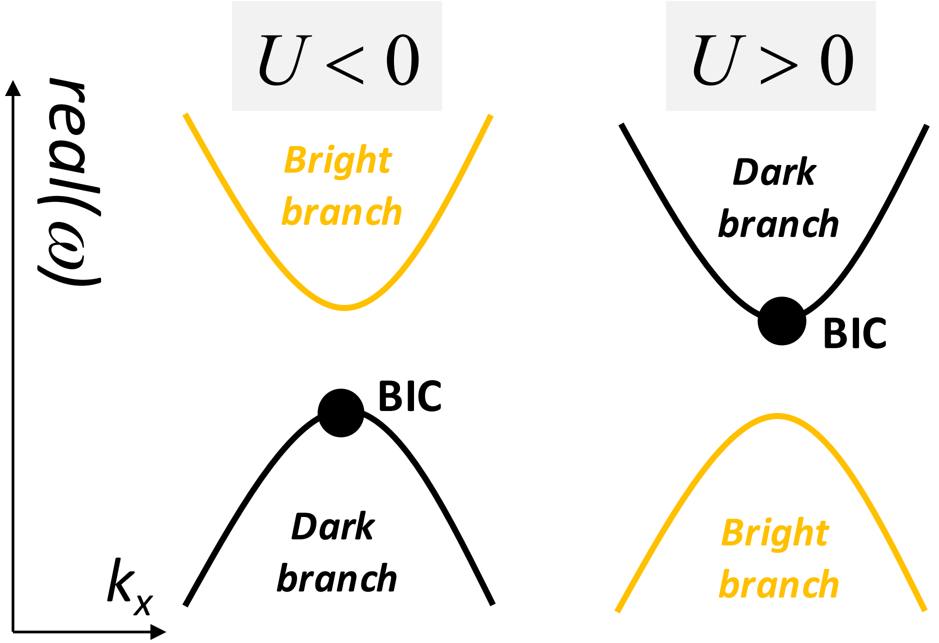
$$\boxed{H = \begin{pmatrix} \omega_0 + v.k_x & U \\ U & \omega_0 - v.k_x \end{pmatrix} + i \begin{pmatrix} \gamma & -\gamma \\ -\gamma & \gamma \end{pmatrix}}$$

$$\Omega_\pm(k_x) = \omega_0 + i\gamma \pm \sqrt{(v.k_x)^2 + (U - i\gamma)^2}$$

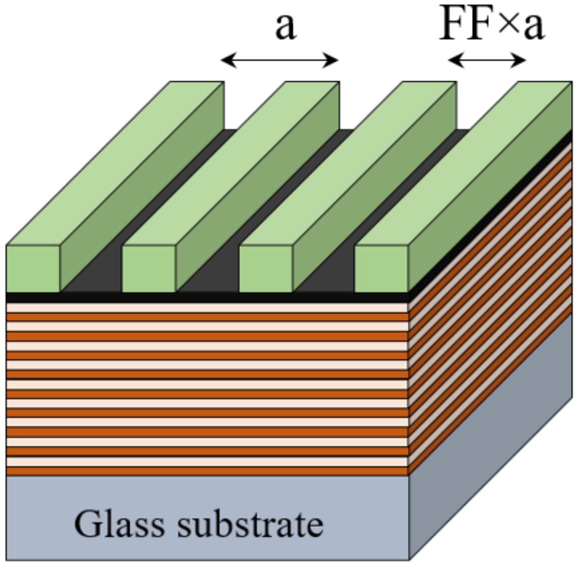
Eigenvalues at k_x=0:

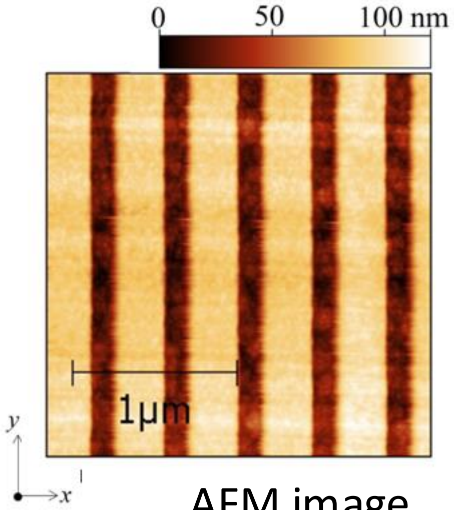
$$\begin{cases} \boxed{\Omega_+ = \omega_0 + U} \\ \Omega_- = \omega_0 - U + 2i\gamma \end{cases}$$

Bound state in the continuum

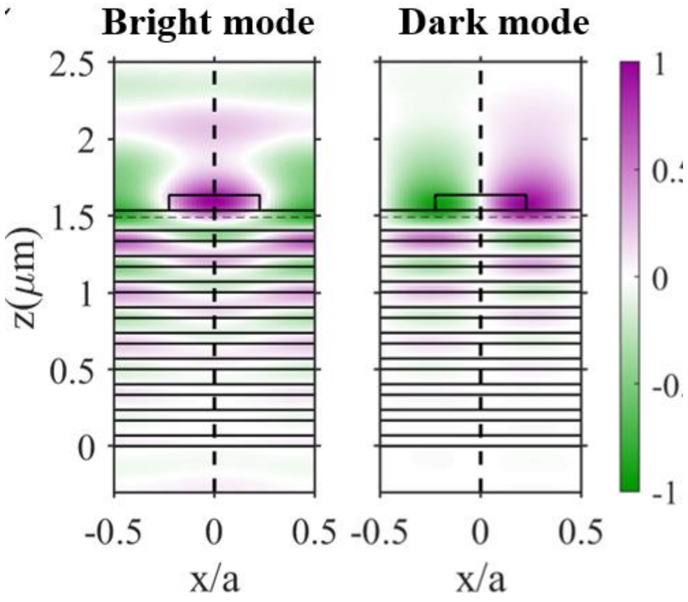


Real structures



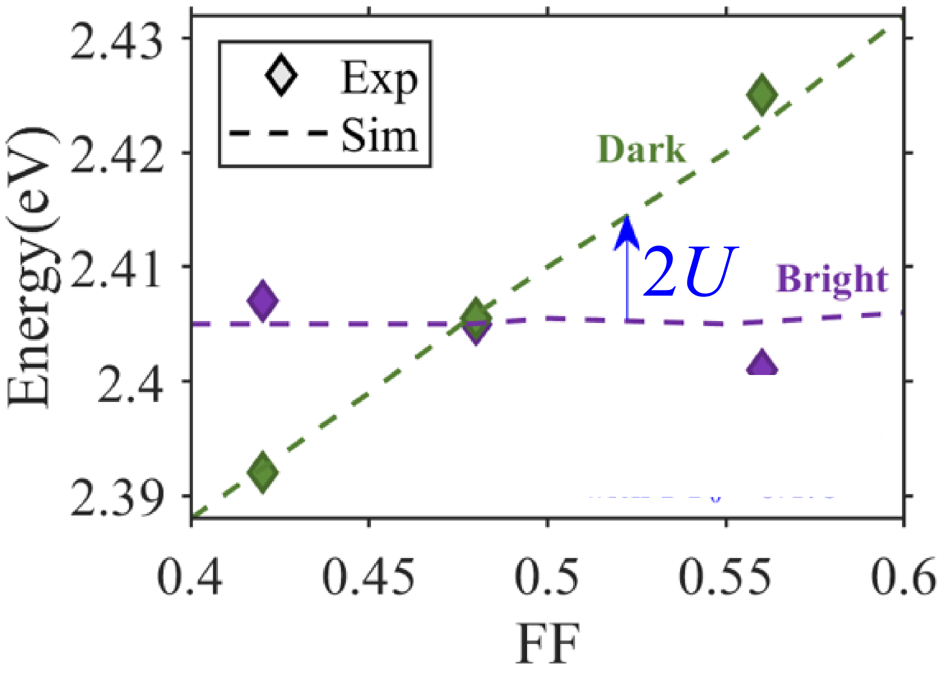


AFM image



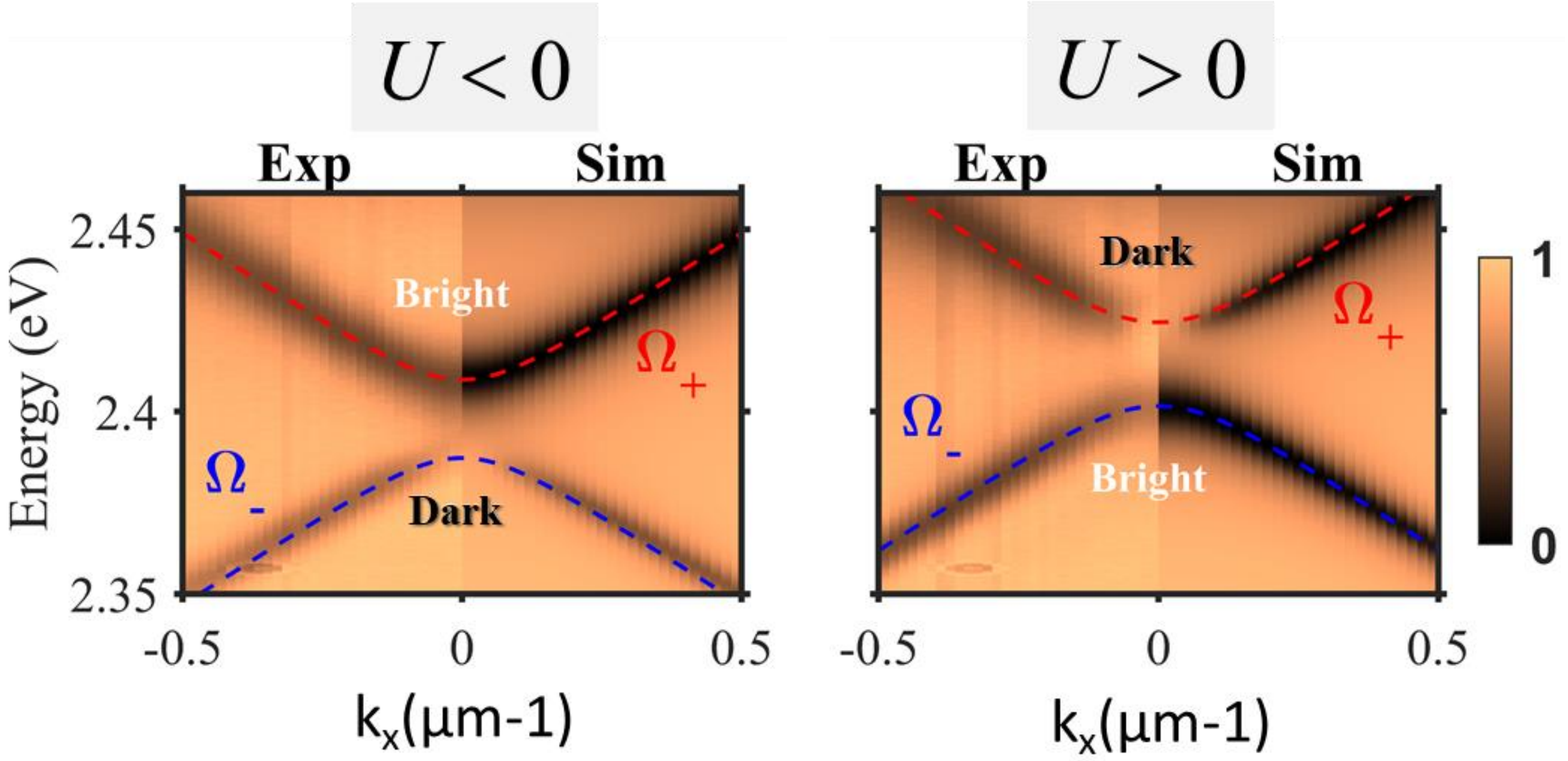
Symetric Antisymetric

At k_x=0



Physical review letters 129, 083602 (2022)

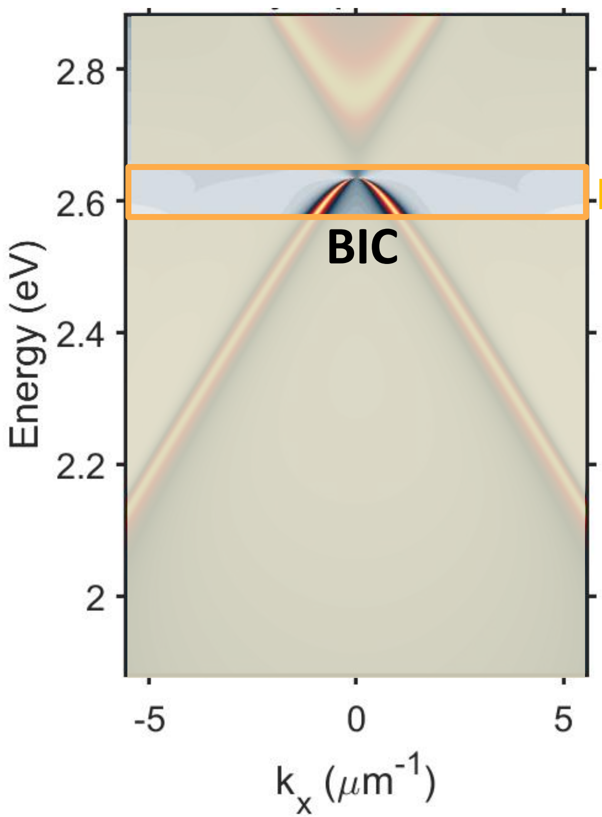
Demonstration of non-Hermitian bandstructures



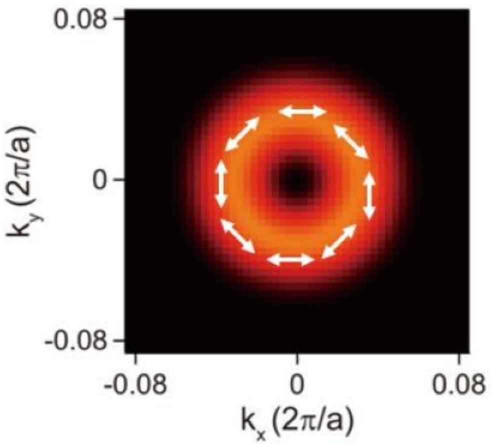
Observation of BIC = local vanishing of the resonance !

Physical review letters 129, 083602 (2022)

BIC = Polarization vortex in momentum space

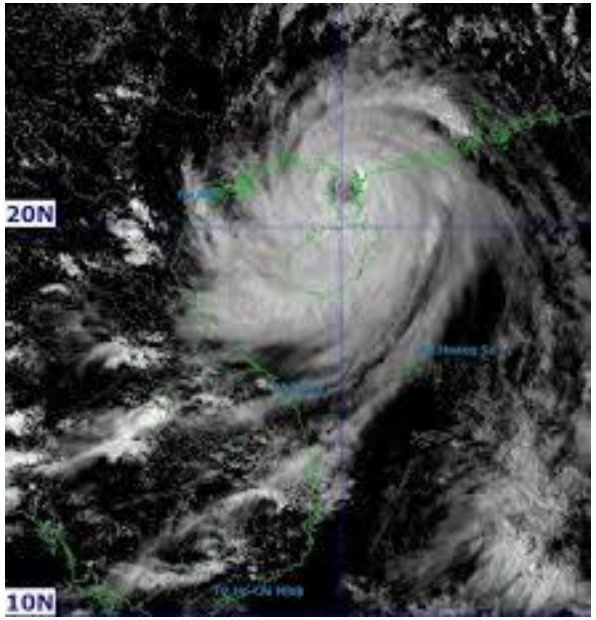


Farfield radiation



Polarization singularity in (k_x,k_y) plane

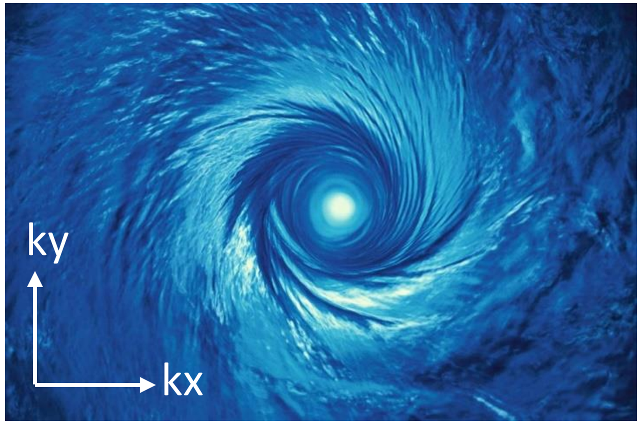
Typhoon Matmo (Vietnam 10/2025)

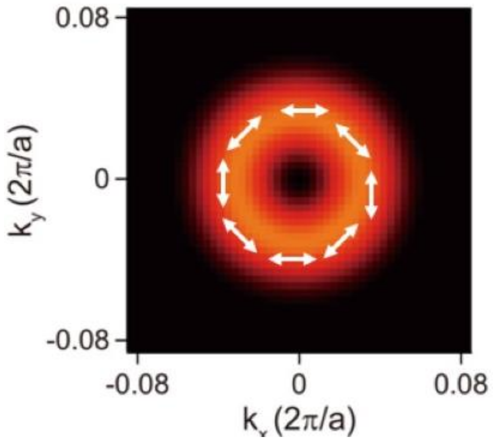


Velocity singularity in (x,y) plane

Topological nature of photonic BIC

BICs in photonic crystal = **Polarization singularity in of radiation pattern in the farfield**





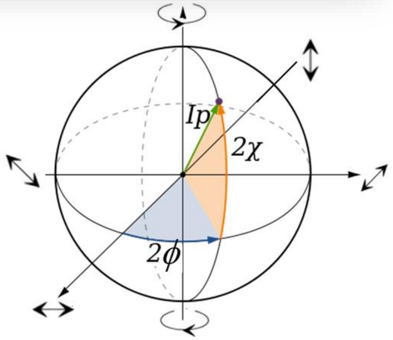
Topological charge = winding number

PRL **113,** 257401 (2014) PHYSICAL REVIEW LETTERS week ending 19 DECEMBER 2014

Topological Nature of Optical Bound States in the Continuum

Bo Zhen,[1,*] Chia Wei Hsu,[1,2] Ling Lu,[1] A. Douglas Stone,[3] and Marin Soljačić[1]

$$q = \frac{1}{2\pi}\oint_C d\mathbf{k}\cdot\nabla_{\mathbf{k}}\phi(\mathbf{k})$$

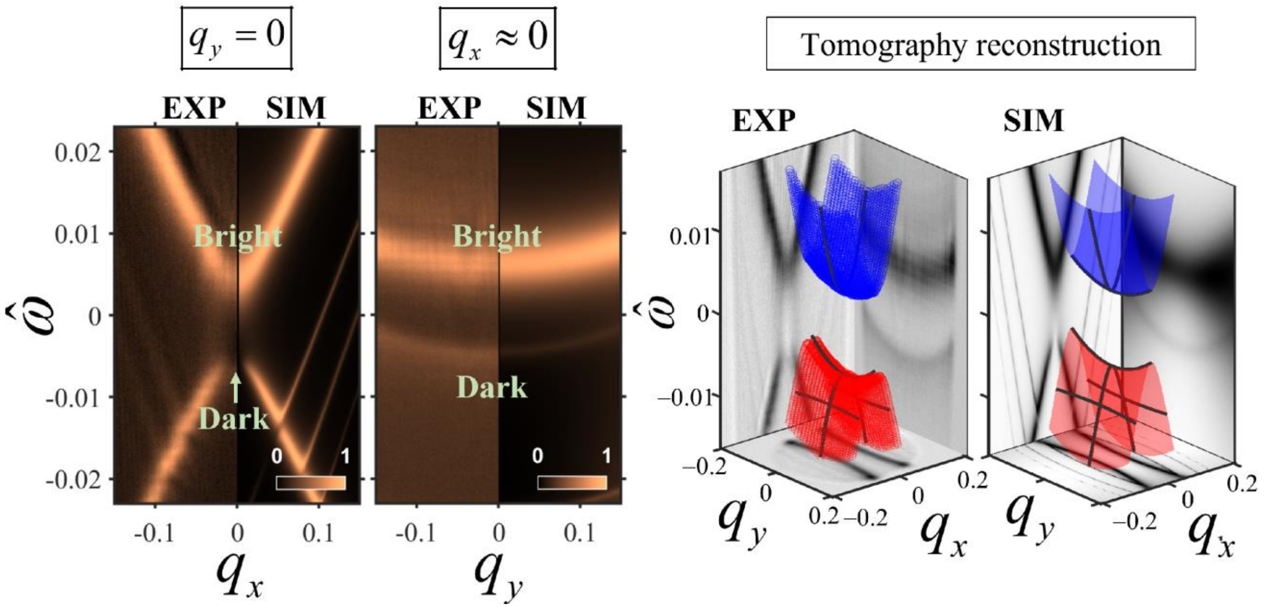


ϕ: orientation angle

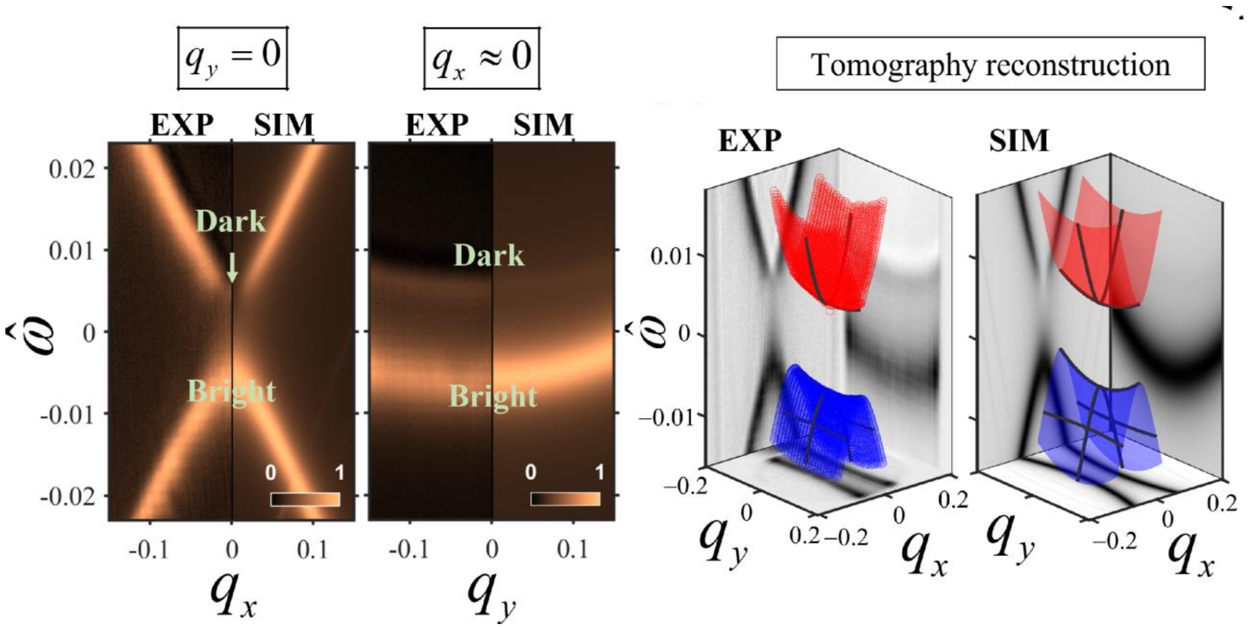
of the polarization

Experimental demonstration of band inversion: Mode tomography

$U<0$

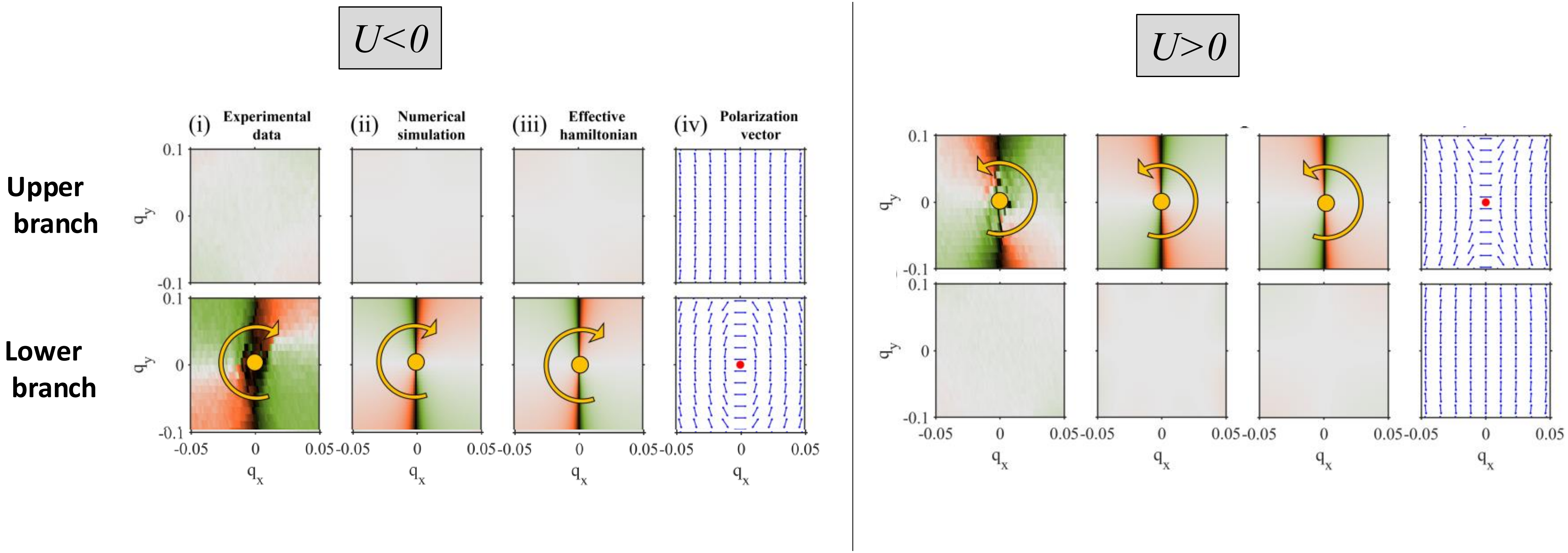


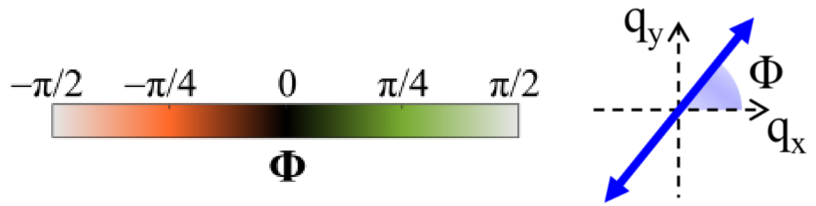
$U>0$



arXiv:2211.09884

Experimental demonstration of band inversion: Winding numbers



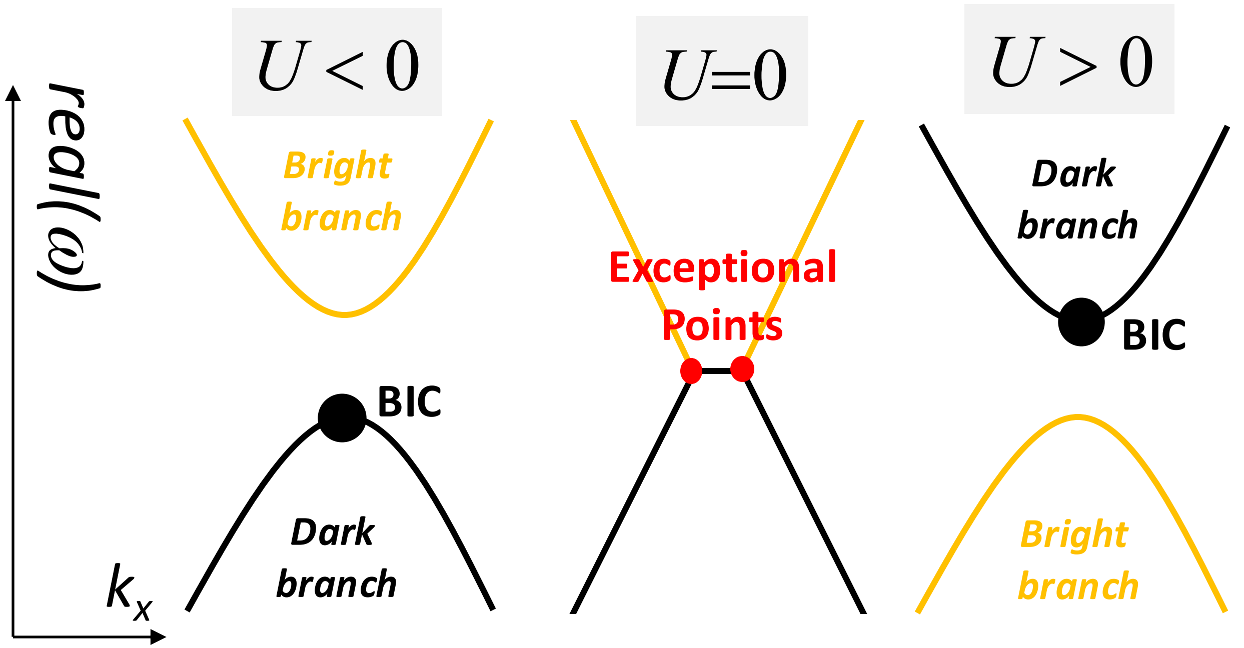


arXiv:2211.09884

Closing non-Hermitian gap

$$\boxed{H = \begin{pmatrix} \omega_0 + v.k_x & U \\ U & \omega_0 - v.k_x \end{pmatrix} + i \begin{pmatrix} \gamma & -\gamma \\ -\gamma & \gamma \end{pmatrix}}$$

$$\Omega_\pm(k_x) = \omega_0 + i\gamma \pm \sqrt{(v.k_x)^2 + (U - i\gamma)^2}$$



Eigenvalues at k_x=0:

$$\begin{cases} \boxed{\Omega_+ = \omega_0 + U} \\ \Omega_- = \omega_0 - U + 2i\gamma \end{cases}$$

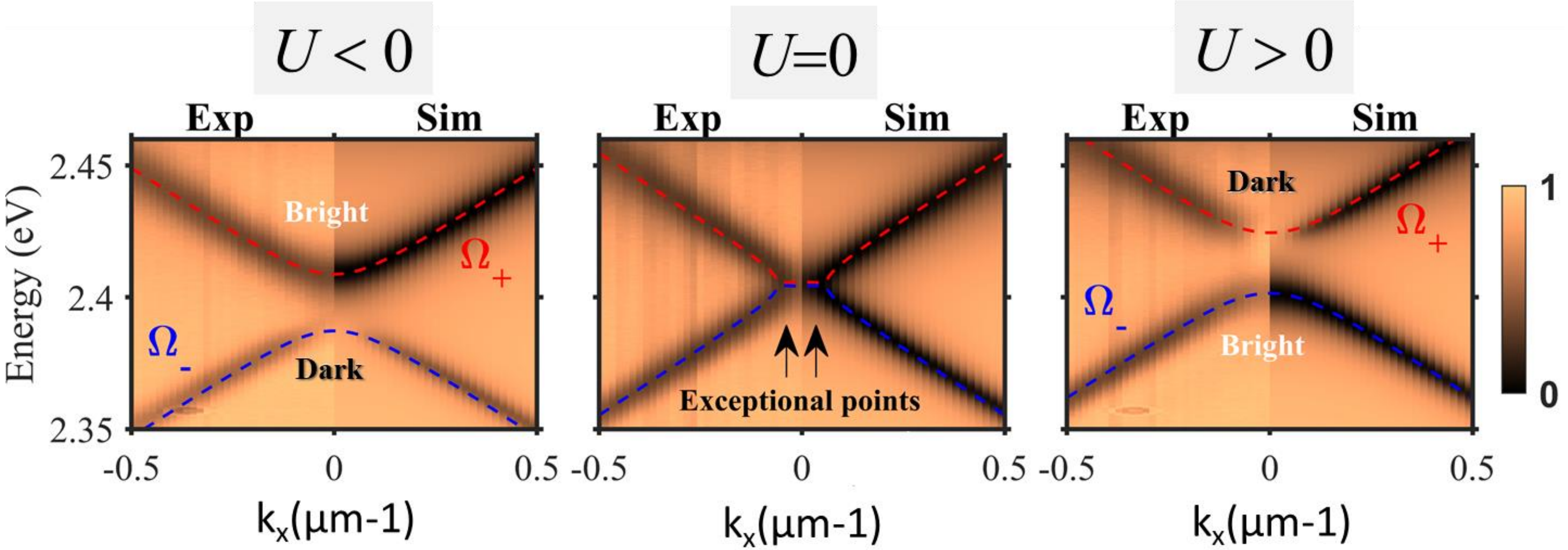
Bound state In the Continuum

Eigenvalues at $k_x = \pm \gamma/v$

$$\boxed{\Omega_+ = \Omega_- = \omega_0 + i\gamma}$$

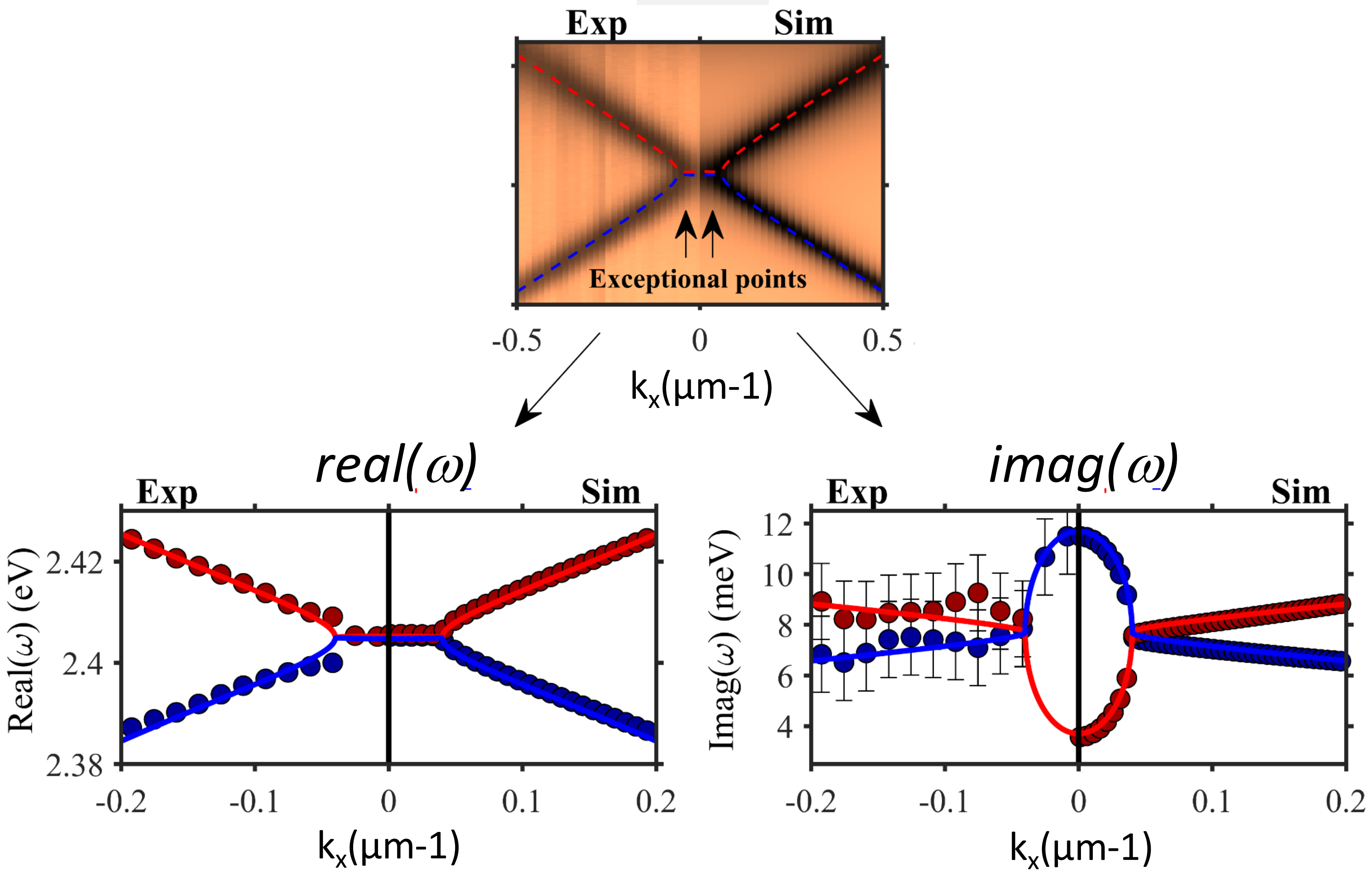
Exceptional points

Observation of exceptional points



Physical review letters 129, 083602 (2022)

Observation of exceptional points



Physical review letters 129, 083602 (2022)

Enhancement of LDOS at Exceptional points: 1 + 1 > 2

Research Article
Vol. 25, No. 11 | 29 May 2017 | OPTICS EXPRESS 12325
Optics EXPRESS

General theory of spontaneous emission near exceptional points

ADI PICK,[1,2,*] BO ZHEN,[1,3,4] OWEN D. MILLER,[5] CHIA W. HSU,[5] FELIPE HERNANDEZ,[6] ALEJANDRO W. RODRIGUEZ,[7] MARIN SOLJAČIĆ,[8] AND STEVEN G. JOHNSON[6]

PRL 117, 107402 (2016) PHYSICAL REVIEW LETTERS week ending 2 SEPTEMBER 2016

Enhanced Spontaneous Emission at Third-Order Dirac Exceptional Points in Inverse-Designed Photonic Crystals

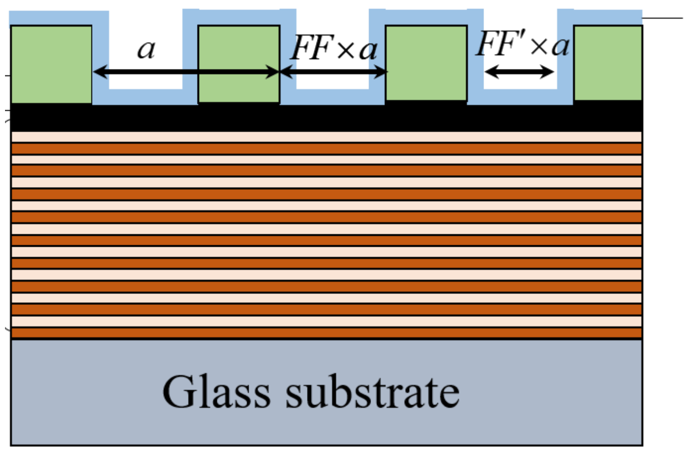
Zin Lin,[1,*] Adi Pick,[2,3] Marko Lončar,[1] and Alejandro W. Rodriguez[4]
[1]*John A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, Massachusetts 02138, USA*
[2]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[3]*Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[4]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*
(Received 19 May 2016; published 30 August 2016)

LDOS at Exceptional point $>$ $\sum$LDOS of non-degenerated states

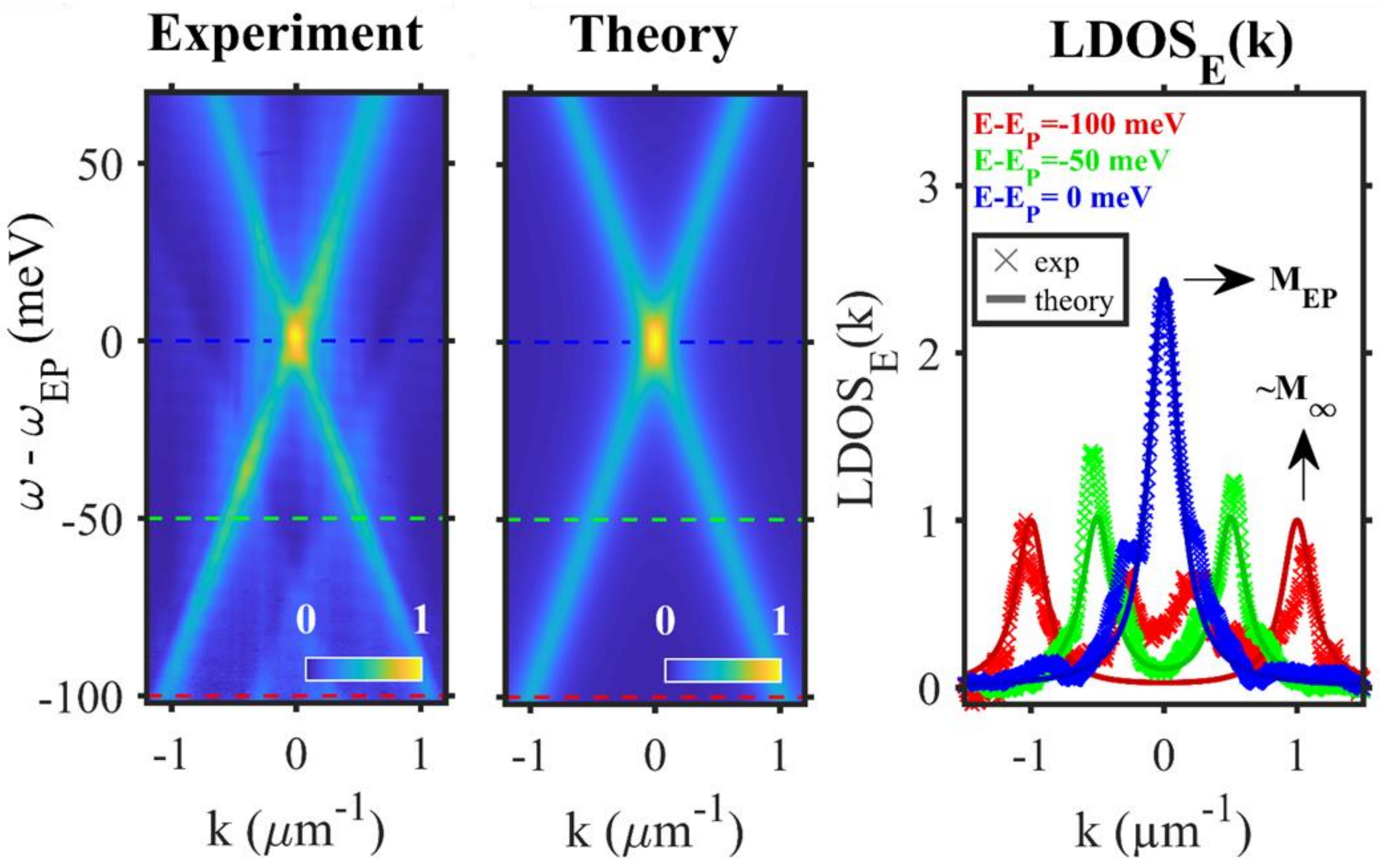
Squared Lorentzian

Lorentzian

Demonstration of LDOS enhancement at Exceptional Points



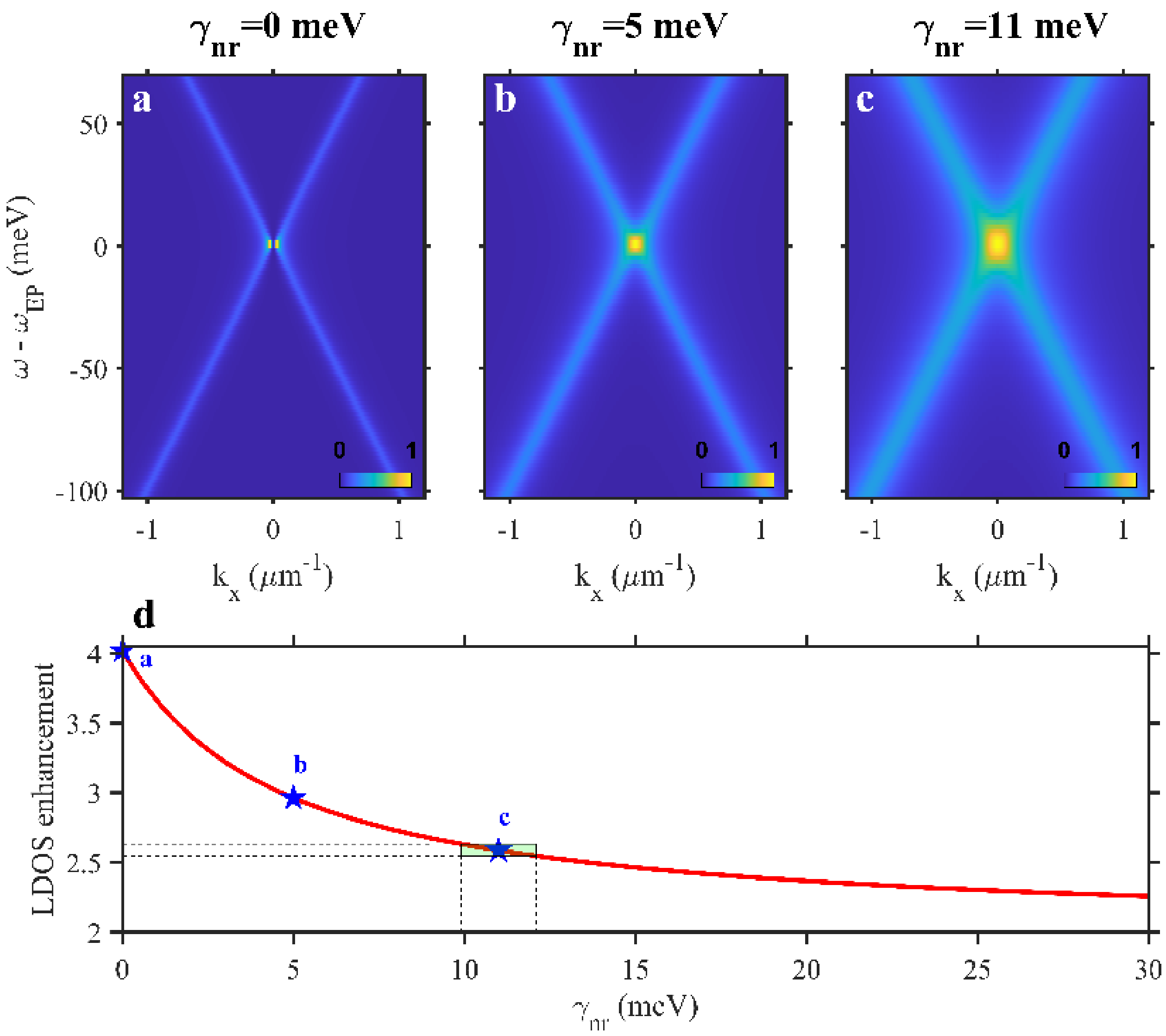
Extraction of the experimental LDOS from the quantum dots emission



Enhancement factor = 2.56

Physical review letters 129, 083602 (2022)

Limitation due to non-radiative losses



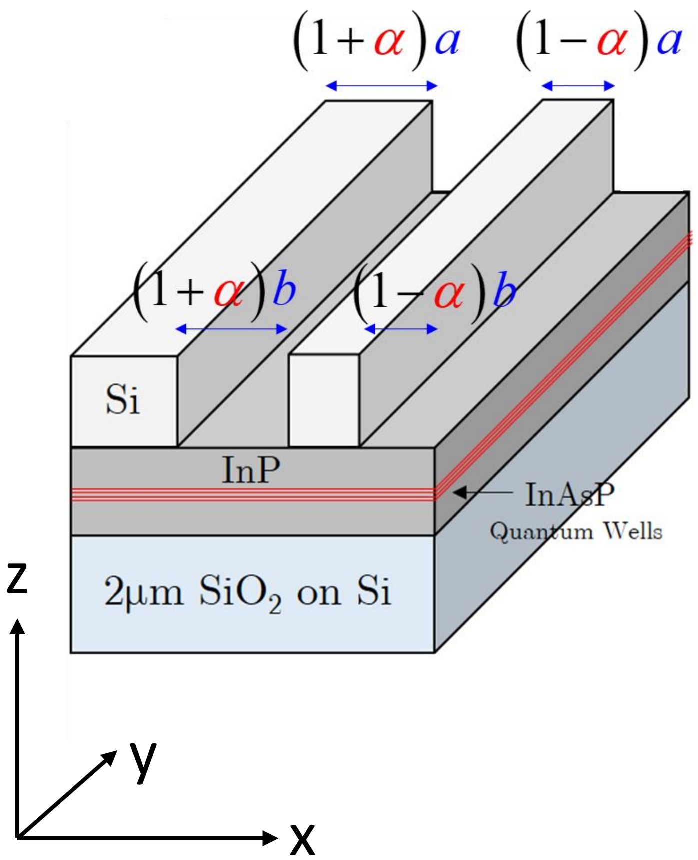
$$H = \begin{pmatrix} -v.k_x & U \\ U & v.k_x \end{pmatrix} - i\begin{pmatrix} \gamma & -\gamma \\ -\gamma & \gamma \end{pmatrix}$$

$$- i\begin{pmatrix} \gamma_{nr} & 0 \\ 0 & \gamma_{nr} \end{pmatrix}$$

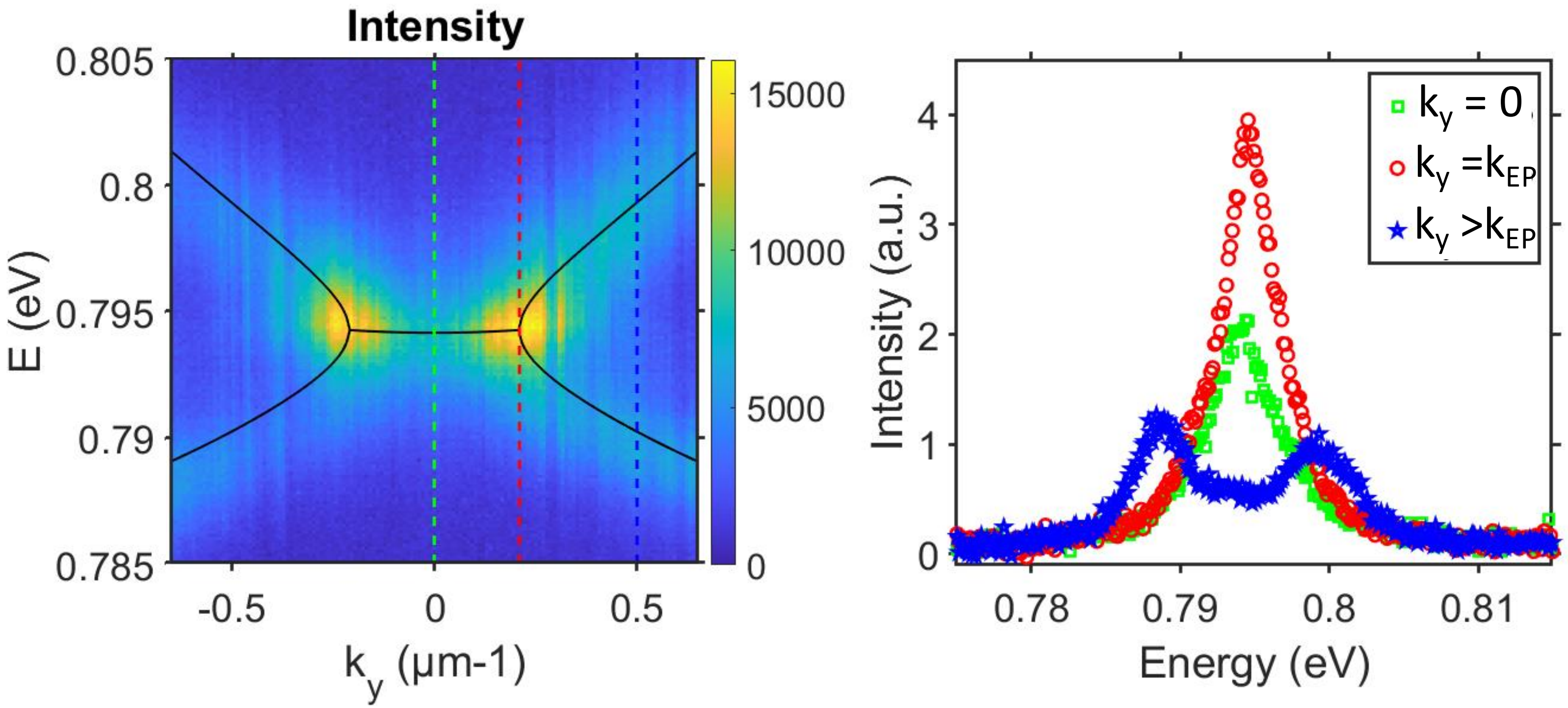
Non-radiative losses

Physical review letters 129, 083602 (2022)

Reaching the theoretical limit: 1+1=4



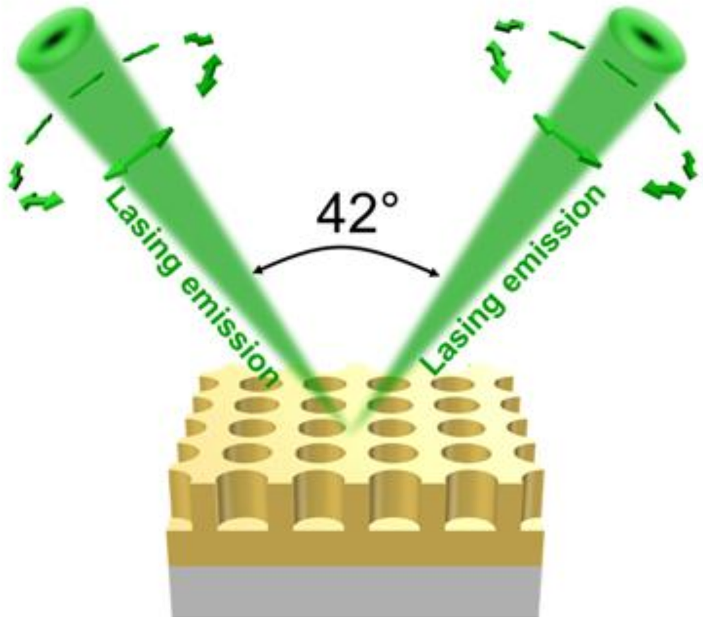
$k_x = k_0$

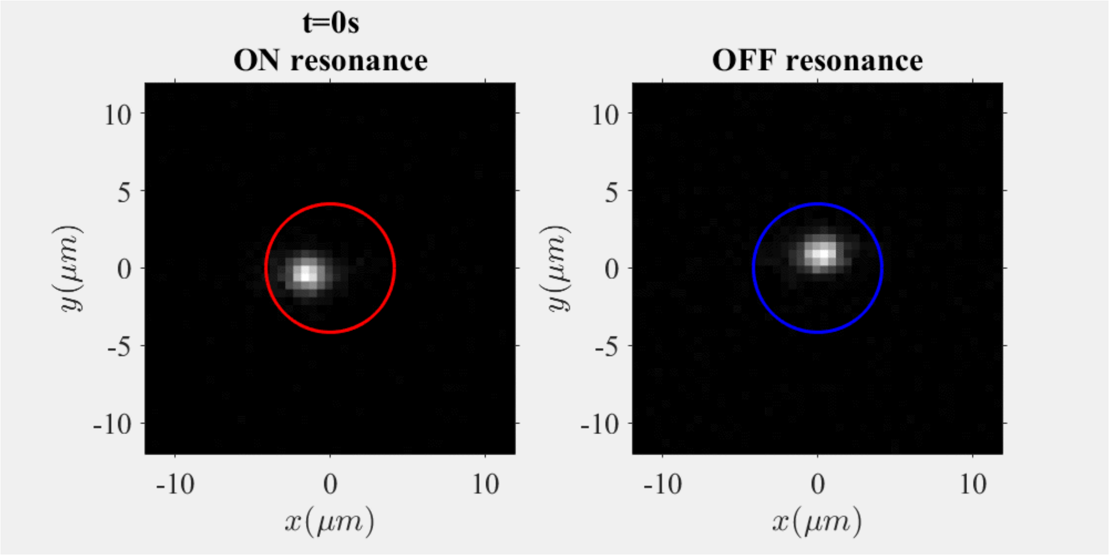


Enhancement ~ 4 = theoretical limit

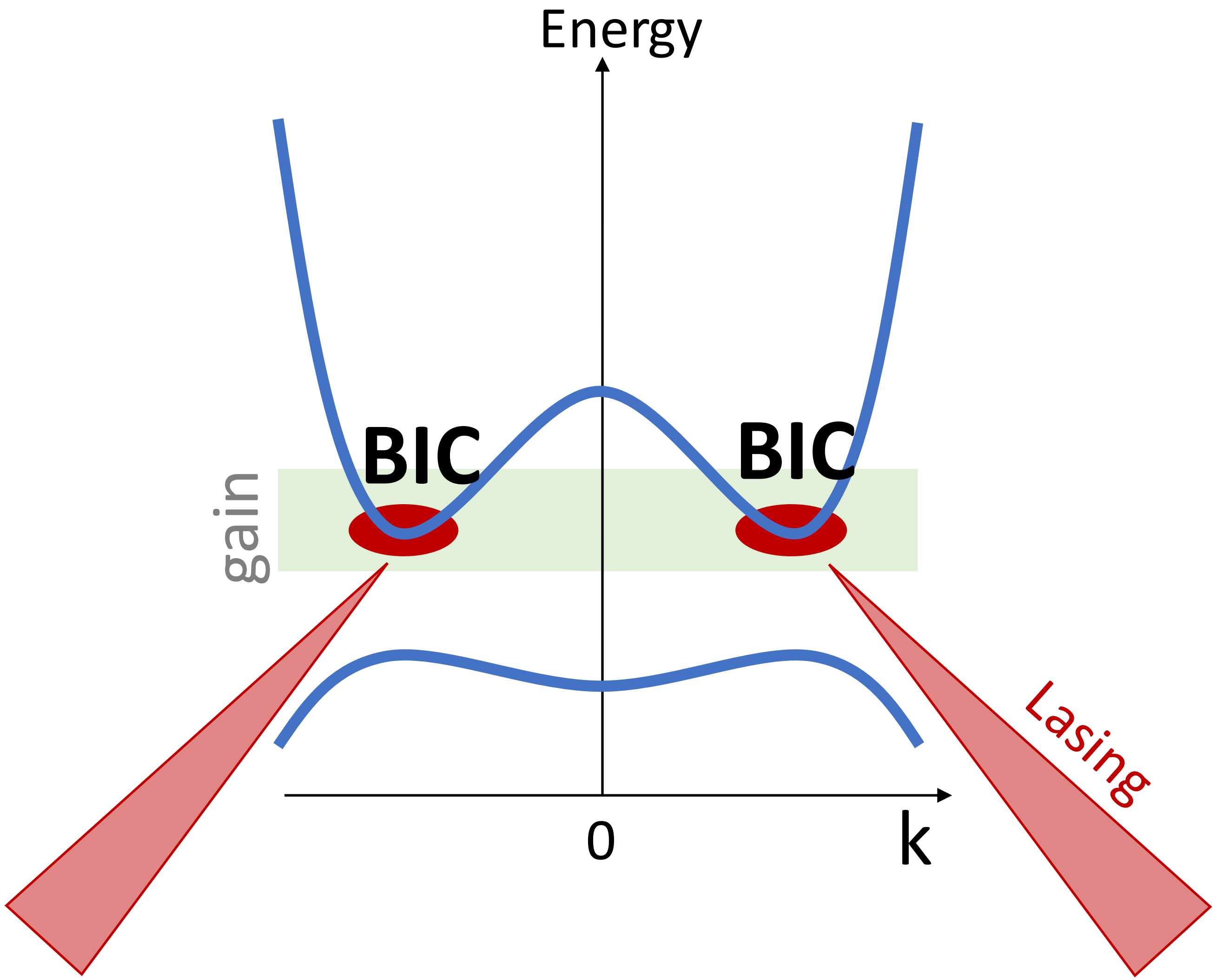
(unpublished data)

Applications of Radiation Topology





Lasing emission in high oblique angle ?

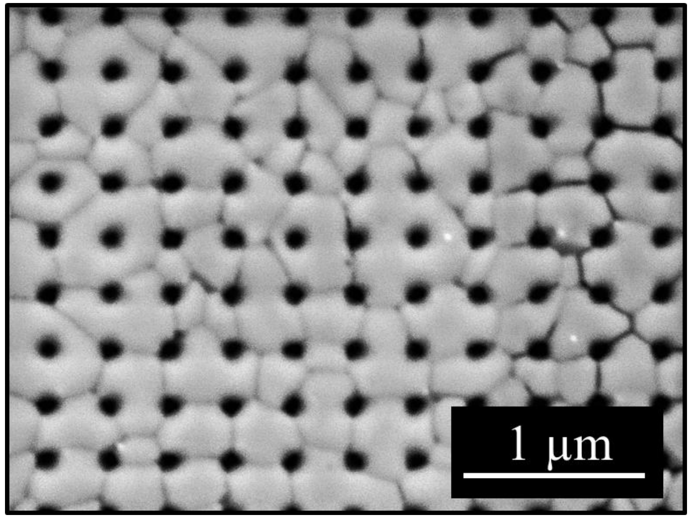


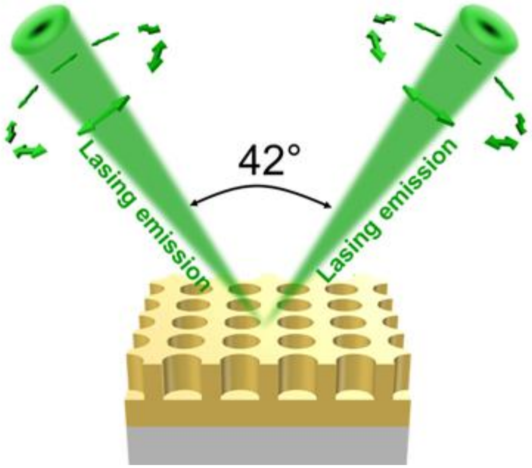
- Slow light at high oblique angle
- High quality factor at valley points

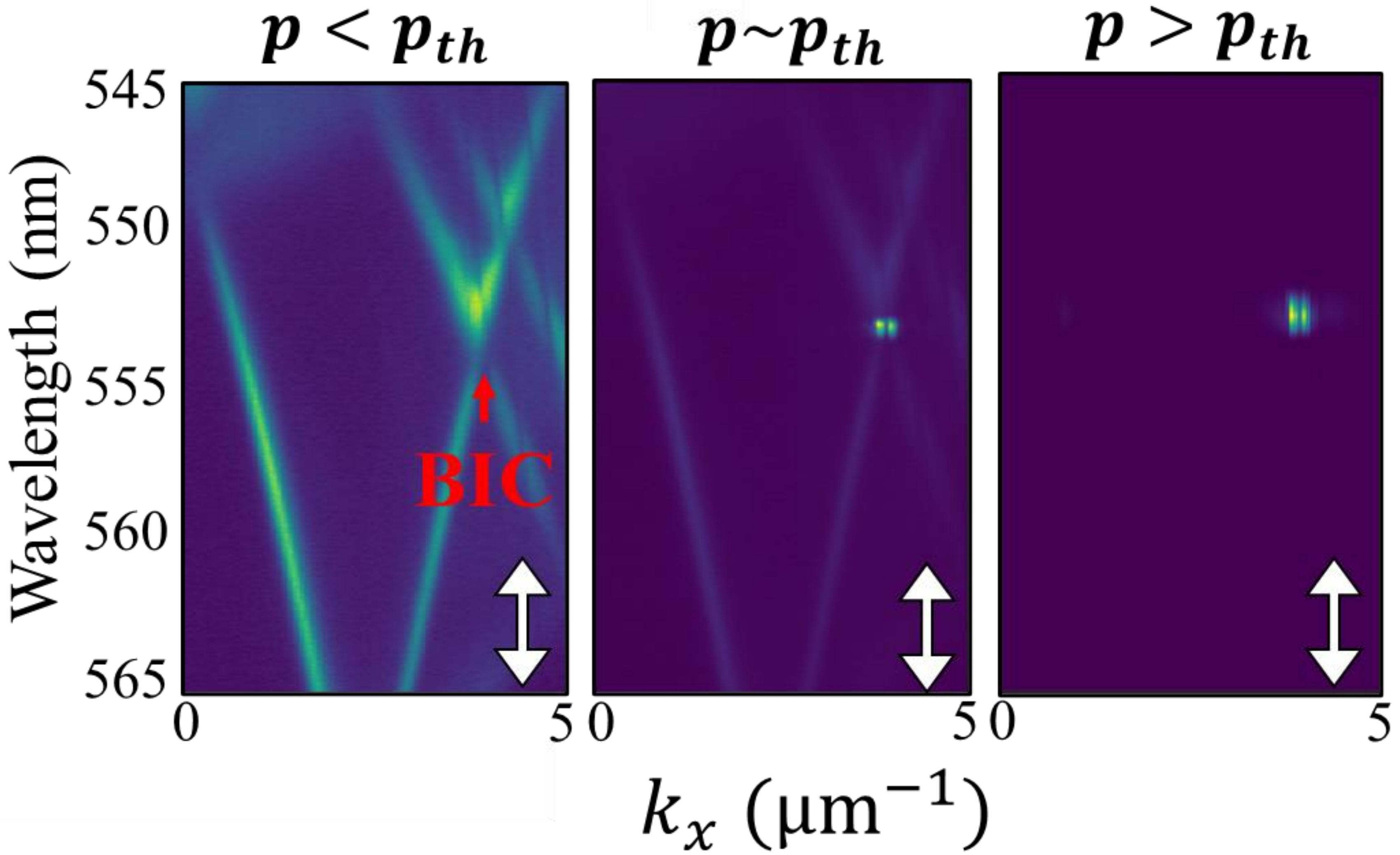
Directional Lasing at high Oblique angle

Oblique BIC: Lasing emitting at high angle

Perovskite metasurface

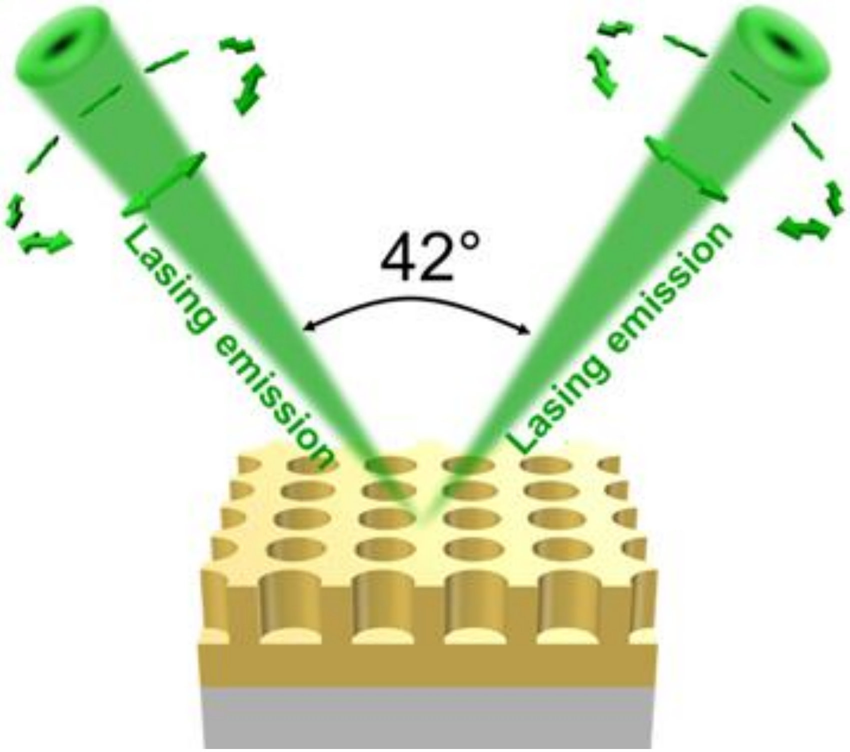


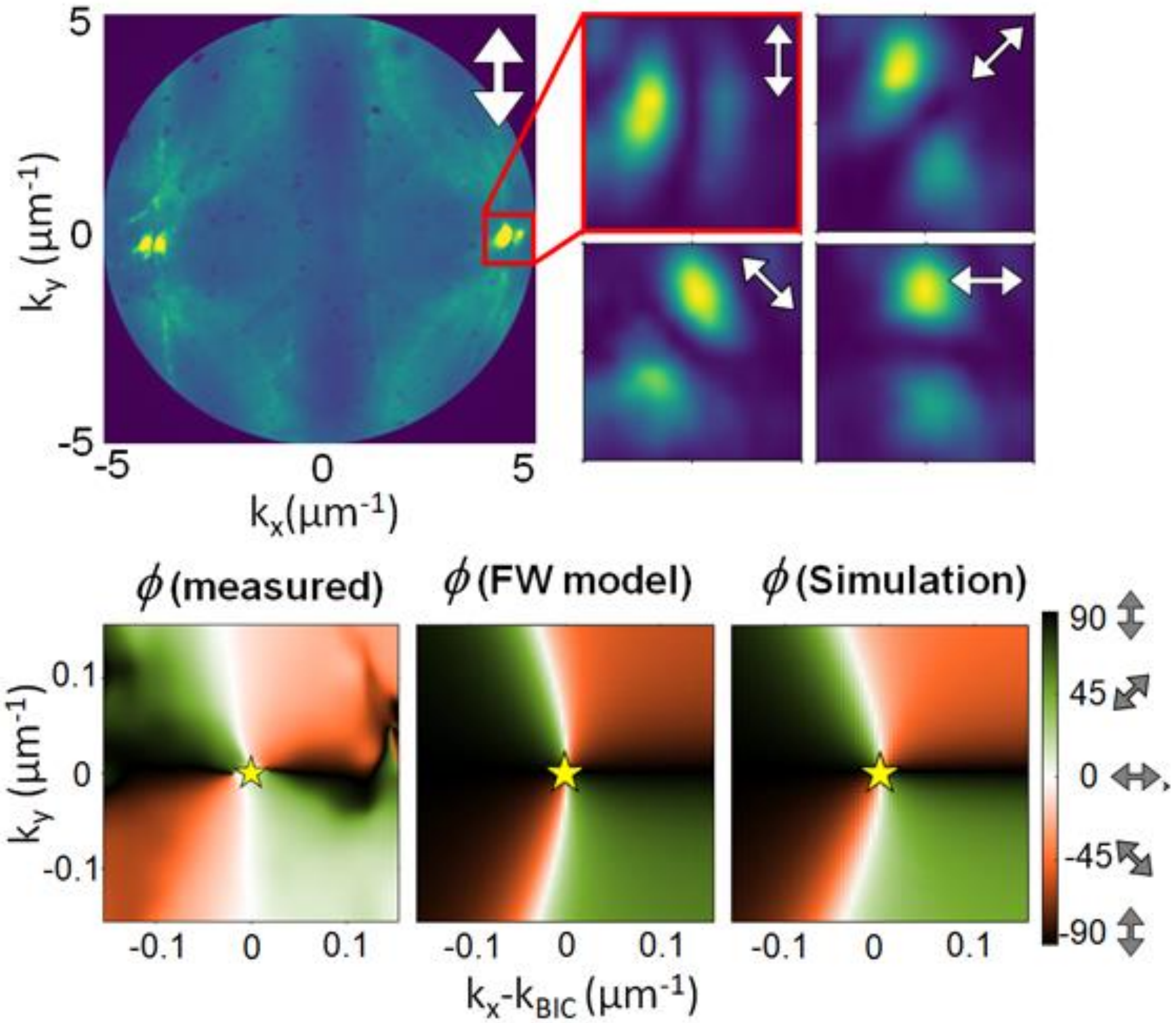




Nano Letters 23, 10, 4152–4159 (2023)

Vortex lasing emitting at high angle

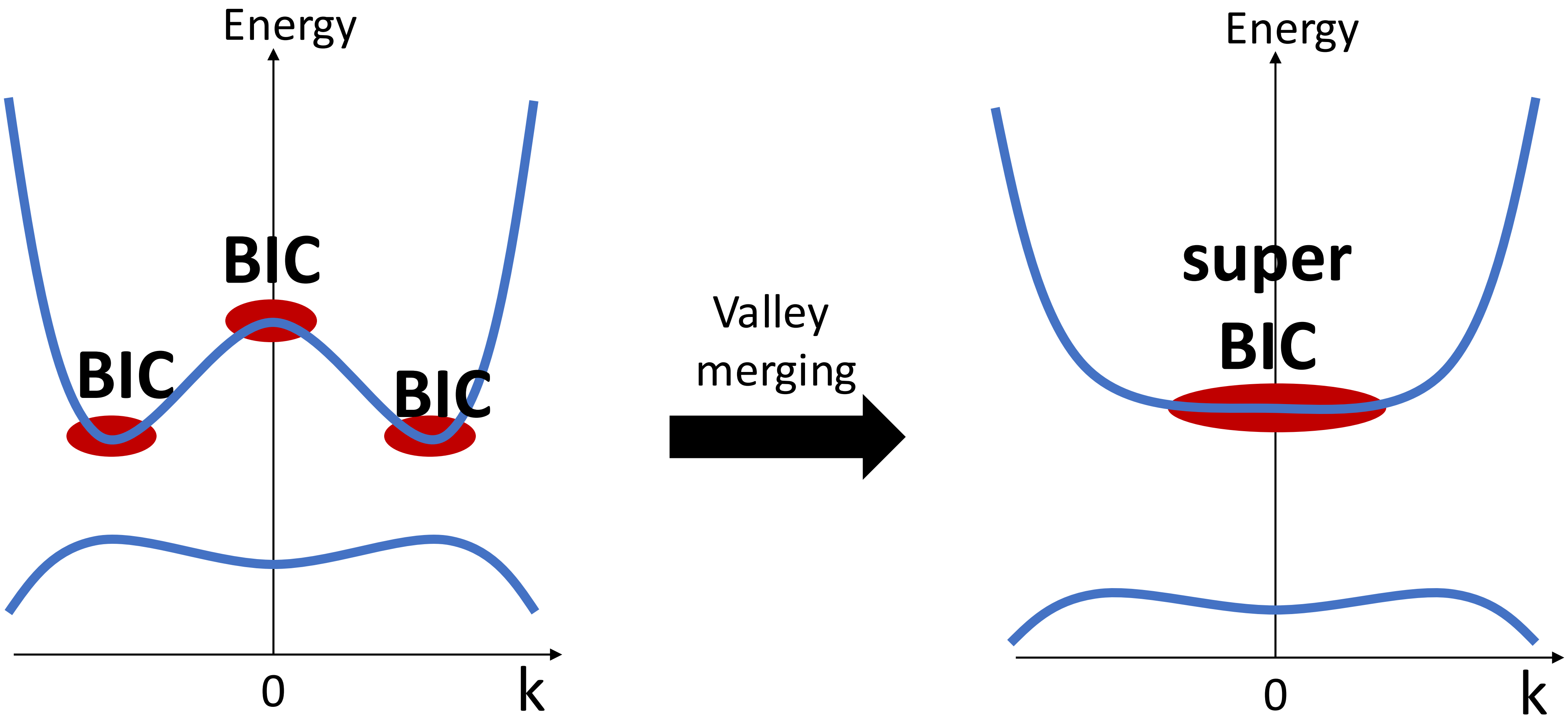




Nano Letters 23, 10, 4152–4159 (2023)

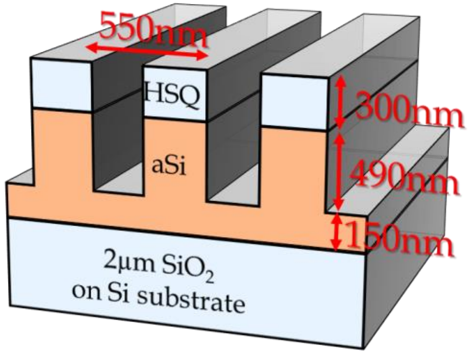
Super BIC (Merging BIC) on a flatband

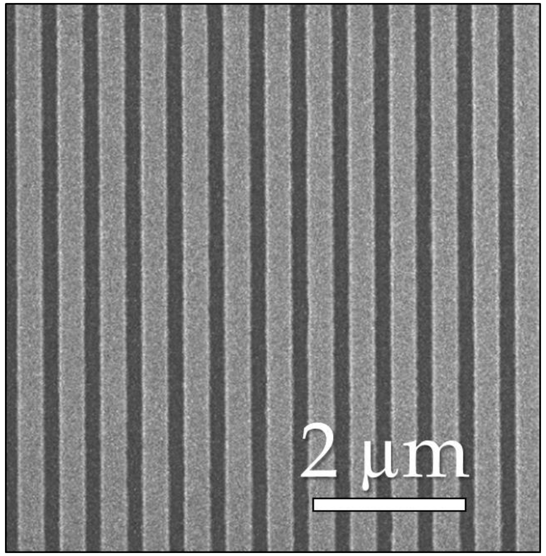
Physical Review Letters 132, 173802 (2024)

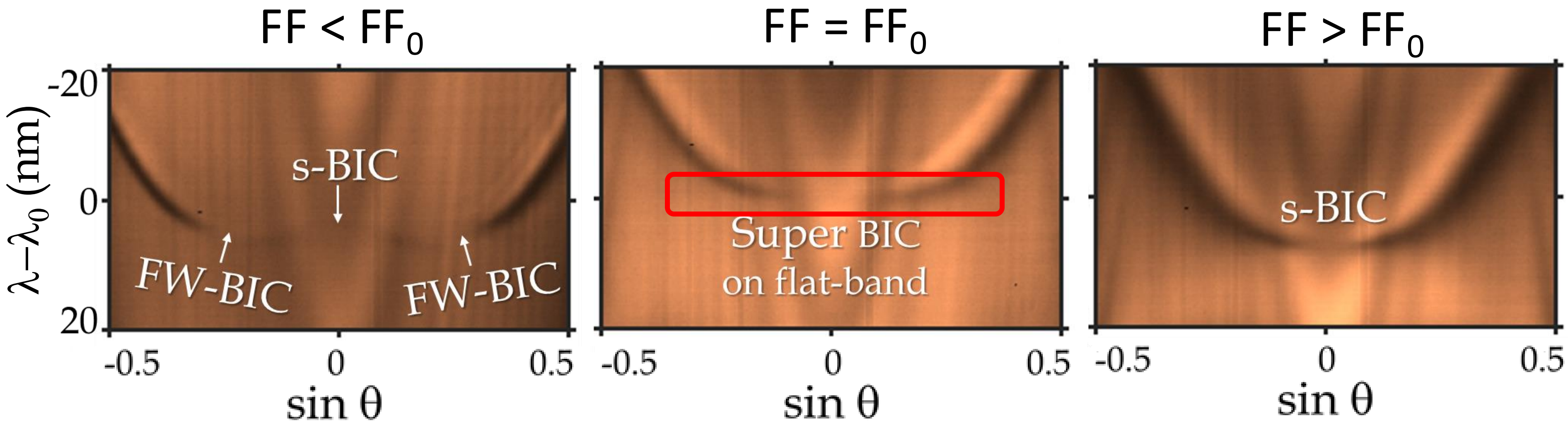


Super BIC on a flatband : experimental demonstration

Physical Review Letters 132, 173802 (2024)







Ultra-high quality factor

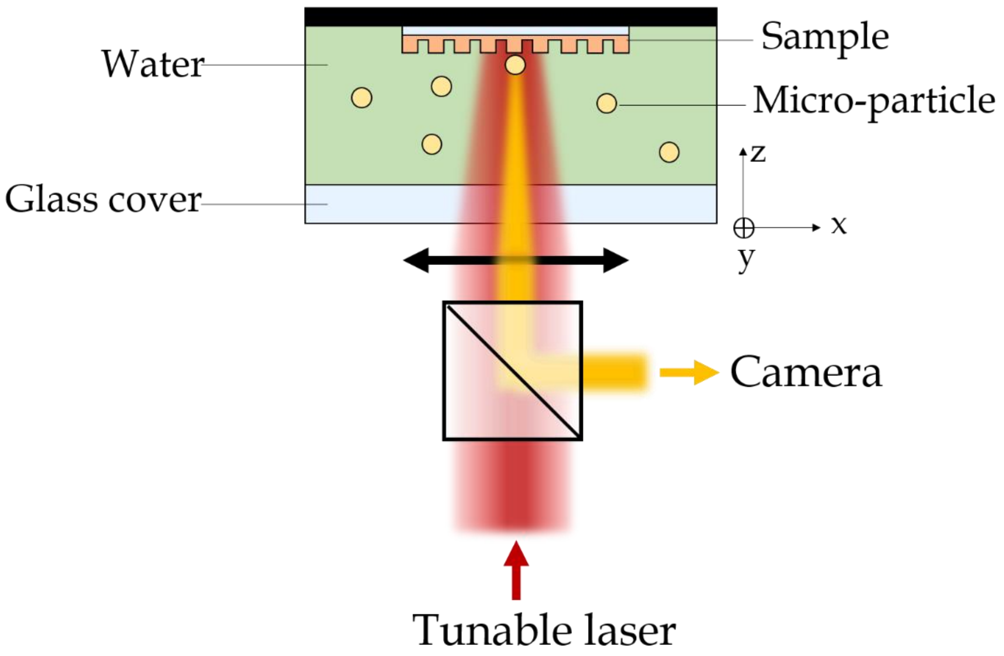
+

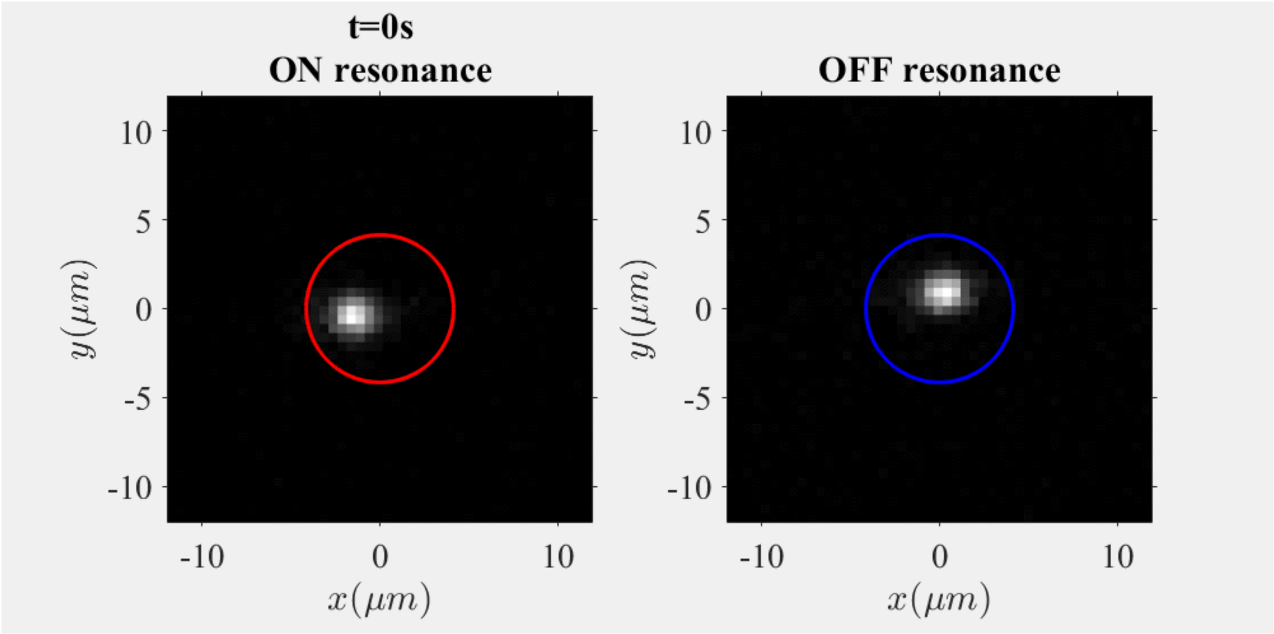
Ultra-high density of states over a broad-range of angles

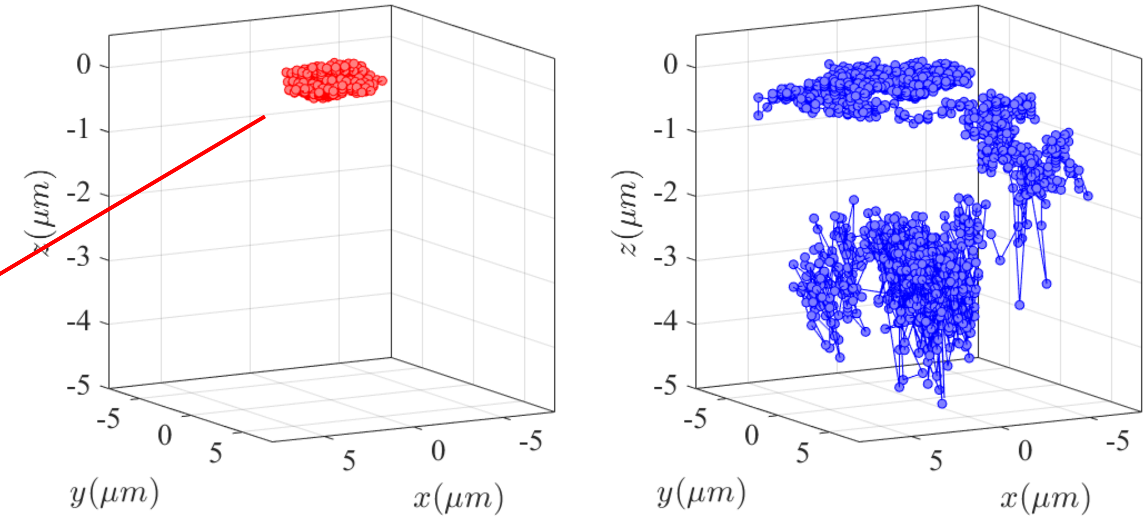
Ideal for optical trapping

Super BIC on a flatband: Optical trapping application

Physical Review Letters 132, 173802 (2024)





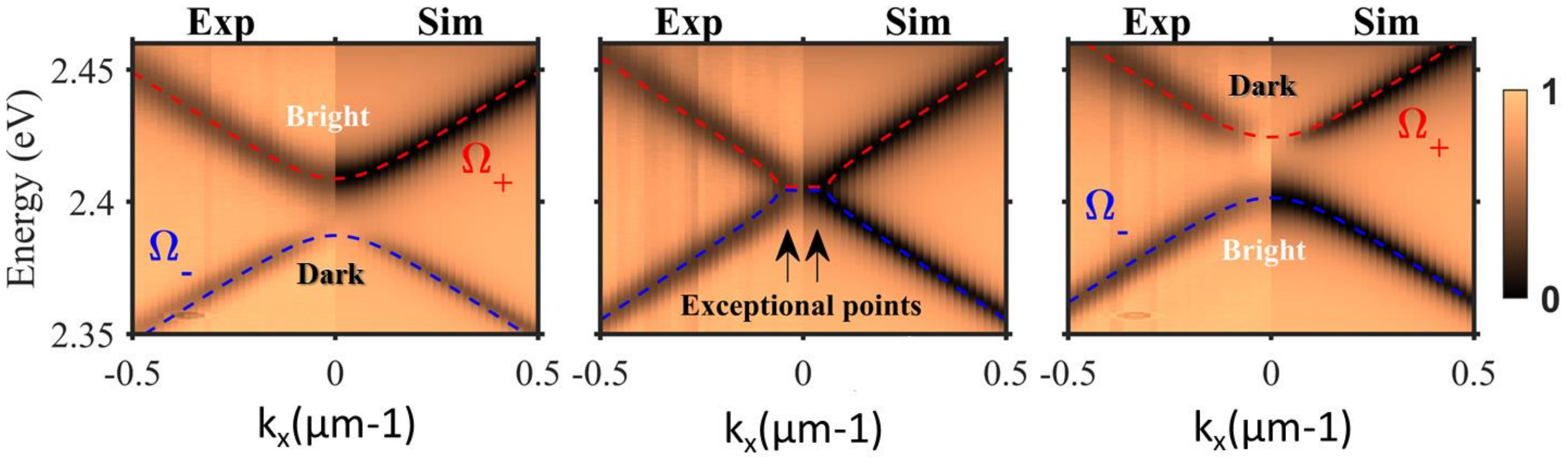


Three-dimensional trapping
of individual particles

Conclusion

- Complex band structure is dictated by Friedrich-Wintegen inteference

- Subwavelength-scale photonic crystal = Platform to study non-Hermitian physics
  → Non-hermitian objects: Bound states In the Continuum, Exceptional points



Physical review letters 129, 083602 (2022)

PHYSICAL REVIEW LETTERS **129**, 083602 (2022)

Unveiling the Enhancement of Spontaneous Emission at Exceptional Points

L. Ferrier,[1,*] P. Bouteyre,[1,†] A. Pick,[2] S. Cueff,[1] N. H. M Dang,[1] C. Diederichs,[3] A. Belarouci,[1] T. Benyattou,[1] J. X. Zhao,[4] R. Su,[4] J. Xing,[5] Qihua Xiong,[6,7,8] and H. S. Nguyen[1,8,‡]

[1]*Université Lyon, Ecole Centrale de Lyon, CNRS, INSA Lyon, Université Claude Bernard Lyon 1, CPE Lyon, CNRS, INL, UMR5270, 69130 Ecully, France*
[2]*Applied Physics Department, Hebrew University of Jerusalem, Israel*
[3]*Laboratoire de Physique de l'École normale supérieure, ENS, Université PSL, CNRS, Sorbonne Université, Université de Paris, F-75005 Paris, France*
[4]*Division of Physics and Applied Physics, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore 637371, Singapore*
[5]*College of Chemistry and Molecular Engineering, Qingdao University of Science and Technology, Qingdao 266042, China*
[6]*State Key Laboratory of Low-Dimensional Quantum Physics and Department of Physics, Tsinghua University, Beijing 100084, People's Republic of China*
[7]*Beijing Academy of Quantum Information Sciences, Beijing 100193, People's Republic of China*
[8]*Institut Universitaire de France (IUF), F-75231 Paris, France*


L. Ferrier
P. Bouteyre
S. Cueff
T. Benyattou
N.H.M.Dang

inl

J. Xing
C. Diederichs
J.X.Zhao
R.Su
Qihua Xiong

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

TSINGHUA UNIVERSITY 1911

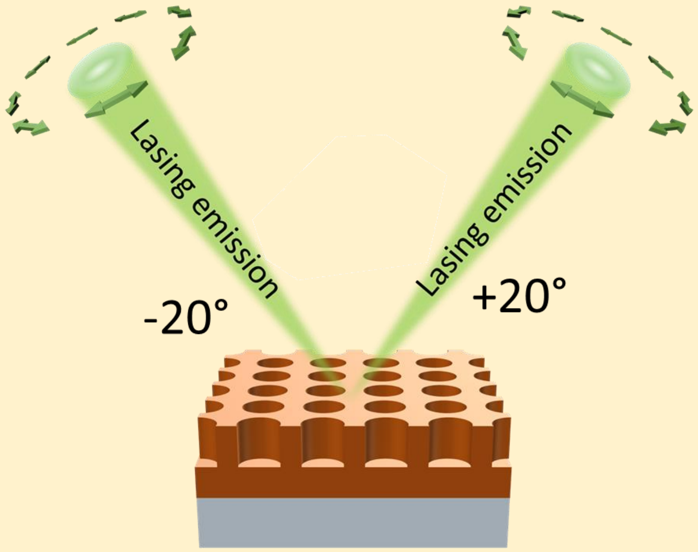
Adi Pick

THE HEBREW UNIVERSITY OF JERUSALEM

Physical review letters 129, 083602 (2022)

Applications

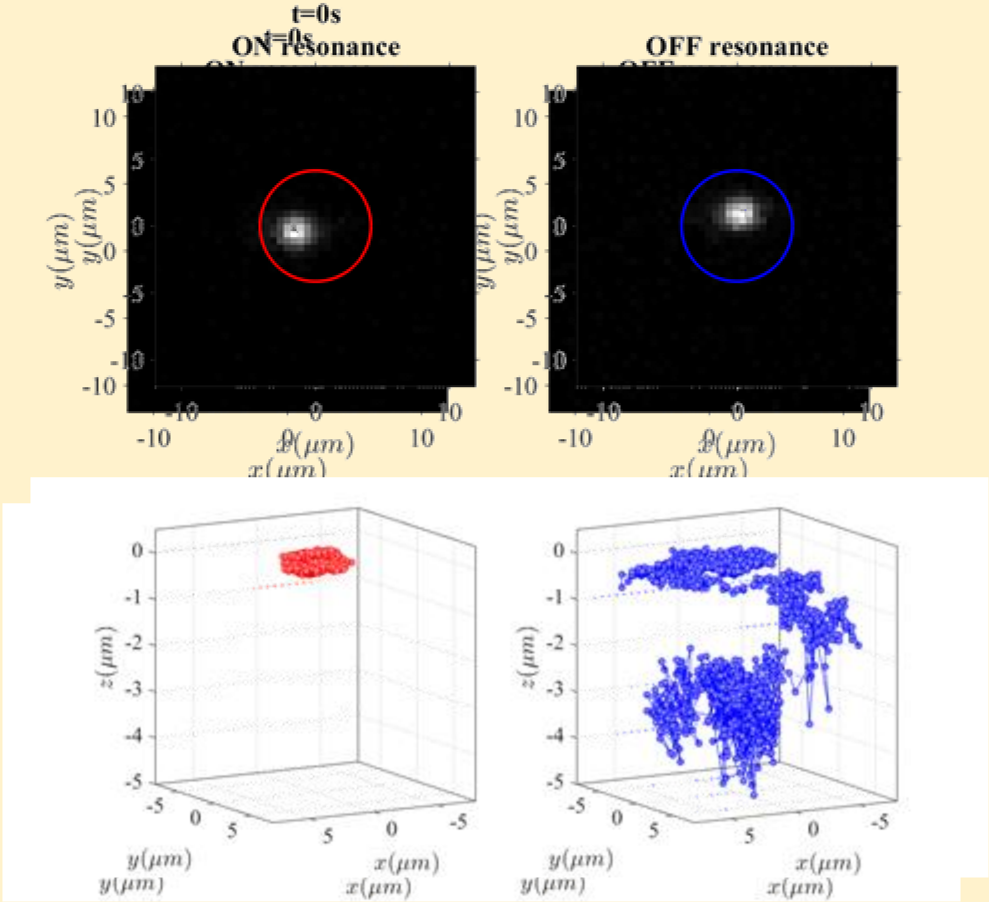
Micro lasers



Nano Letters 23, 10, 4152–4159 (2023)

- **Lasing emission in oblique emission**

Optical trapping



Physical Review Letters 132, 173802 (2024)

- **Three-dimensional trapping without cavity**

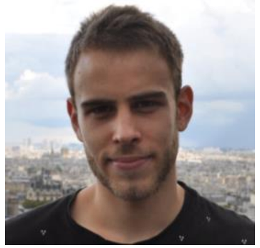
NANO LETTERS

pubs.acs.org/NanoLett Letter

Taming Friedrich–Wintgen Interference in a Resonant Metasurface: Vortex Laser Emitting at an On-Demand Tilted Angle

Raphael Mermet-Lyaudoz, Clémentine Symonds, Florian Berry, Emmanuel Drouard, Céline Chevalier, Gaëlle Trippé-Allard, Emmanuelle Deleporte, Joel Bellessa, Christian Seassal, and Hai Son Nguyen*

Nano Letters 23, 10, 4152–4159 (2023)



R.Mermet-Lyaudoz
N.Duc. Le
P. Bouteyre
F. Dubois
S. Cueff
L. Berguiga
X. Letartre
P. Viktorovitch
T. Benyattou
E. Drouard
C. Chevalier
C.Seassal

PHYSICAL REVIEW LETTERS **132,** 173802 (2024)

Super Bound States in the Continuum on a Photonic Flatband: Concept, Experimental Realization, and Optical Trapping Demonstration

Ngoc Duc Le,[1,2] Paul Bouteyre,[1,3,*] Ali Kheir-Aldine,[1] Florian Dubois,[1,4] Sébastien Cueff,[1] Lotfi Berguiga,[1] Xavier Letartre,[1] Pierre Viktorovitch,[1] Taha Benyattou,[1] and Hai Son Nguyen[1,5,†]

[1]*Univ Lyon, ECL, INSA Lyon, CNRS, UCBL, CPE Lyon, INL UMR 5270, 69130 Écully, France*
[2]*Université Paris-Saclay, CNRS, CEA, Institut de Physique Théorique, 91191 Gif-sur-Yvette, France*
[3]*Department of Physics and Astronomy, University of Sheffield, S3 7RH, Sheffield, United Kingdom*
[4]*Silicon Austria Labs GmbH (SAL), 9524 Villach, Austria*
[5]*Institut Universitaire de France (IUF), 75231 Paris, France*

Physical Review Letters 132, 173802 (2024)

G. Trippe-Allard
E. Deleporte

LuMIn
Lumière, Matière et Interfaces

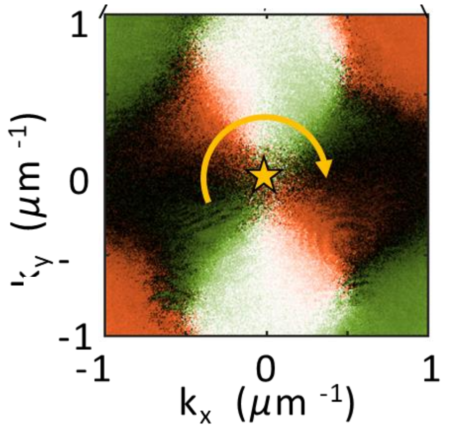
C. Symonds
J. Bellessa

iLM
INSTITUT LUMIÈRE MATIÈRE

Perspective: the strong coupling regime

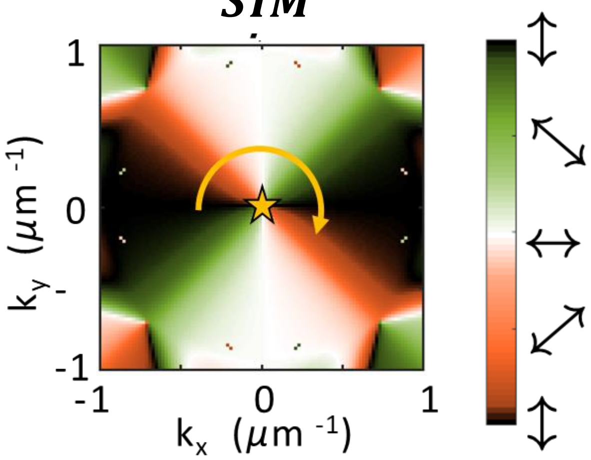
Polaritonic BIC

EXP



Photonic BIC

SIM



Dang *et al.* **Advanced Optical Materials** 2102386 (2022)

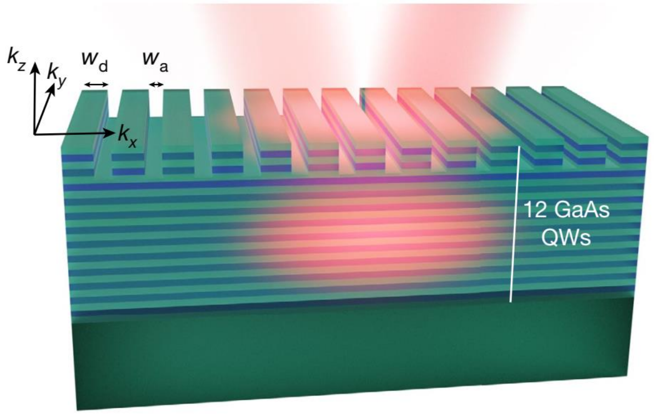
RESEARCH ARTICLE

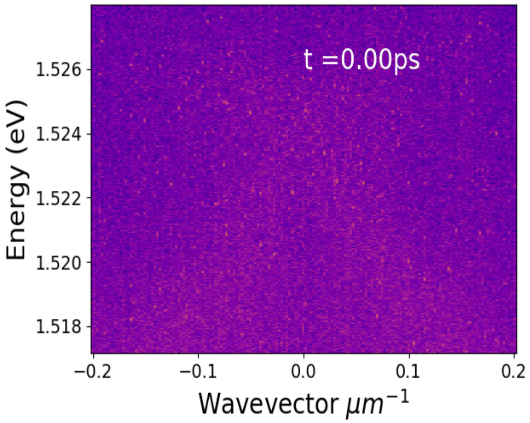
ADVANCED OPTICAL MATERIALS

www.advopticalmat.de

Realization of Polaritonic Topological Charge at Room Temperature Using Polariton Bound States in the Continuum from Perovskite Metasurface

*Nguyen Ha My Dang, Simone Zanotti, Emmanuel Drouard, Céline Chevalier, Gaëlle Trippé-Allard, Mohamed Amara, Emmanuelle Deleporte, Vincenzo Ardizzone, Daniele Sanvitto, Lucio Claudio Andreani, Christian Seassal, Dario Gerace, and Hai Son Nguyen**





Ardizzone *et al*. **Nature** 605, 447–452 (2022)

Article

Polariton Bose–Einstein condensate from a bound state in the continuum

https://doi.org/10.1038/s41586-022-04583-7

Received: 7 May 2021

Accepted: 25 February 2022

V. Ardizzone[1,2], F. Riminucci[1,2,3], S. Zanotti[4], A. Gianfrate[1], M. Efthymiou-Tsironi[1,2], D. G. Suàrez-Forero[1,2], F. Todisco[1], M. De Giorgi[1], D. Trypogeorgos[1], G. Gigli[1,2], K. Baldwin[5], L. Pfeiffer[5], D. Ballarini[1], H. S. Nguyen[6,7], D. Gerace[4] & D. Sanvitto[1]