

# Foundations of excitonic polaritons

Claude Weisbuch

*Materials Department, University of California at Santa Barbara, USA*

*Laboratoire de Physique de la Matière Condensée, CNRS, IP Paris, Ecole Polytechnique,  
Palaiseau, France*

Getting into polaritons without knowing it: **1970**

The evidence of excitonic polaritons in ultra-pure GaAs

**3D excitons in a 3D photon environment**

Measurements of 3D X-polaritons **1976-1980**: dispersion curve, resonant fluorescence, resonant electron scattering

Moving to the 2D world of the **80'ies**: quantum wells and efficient luminescence (Bell laboratories)

**2D excitons in a 3D photon environment**

2D excitons in microcavities: 2D cavity polaritons **1991 –**

**2D excitons in a 2D photon environment**



---

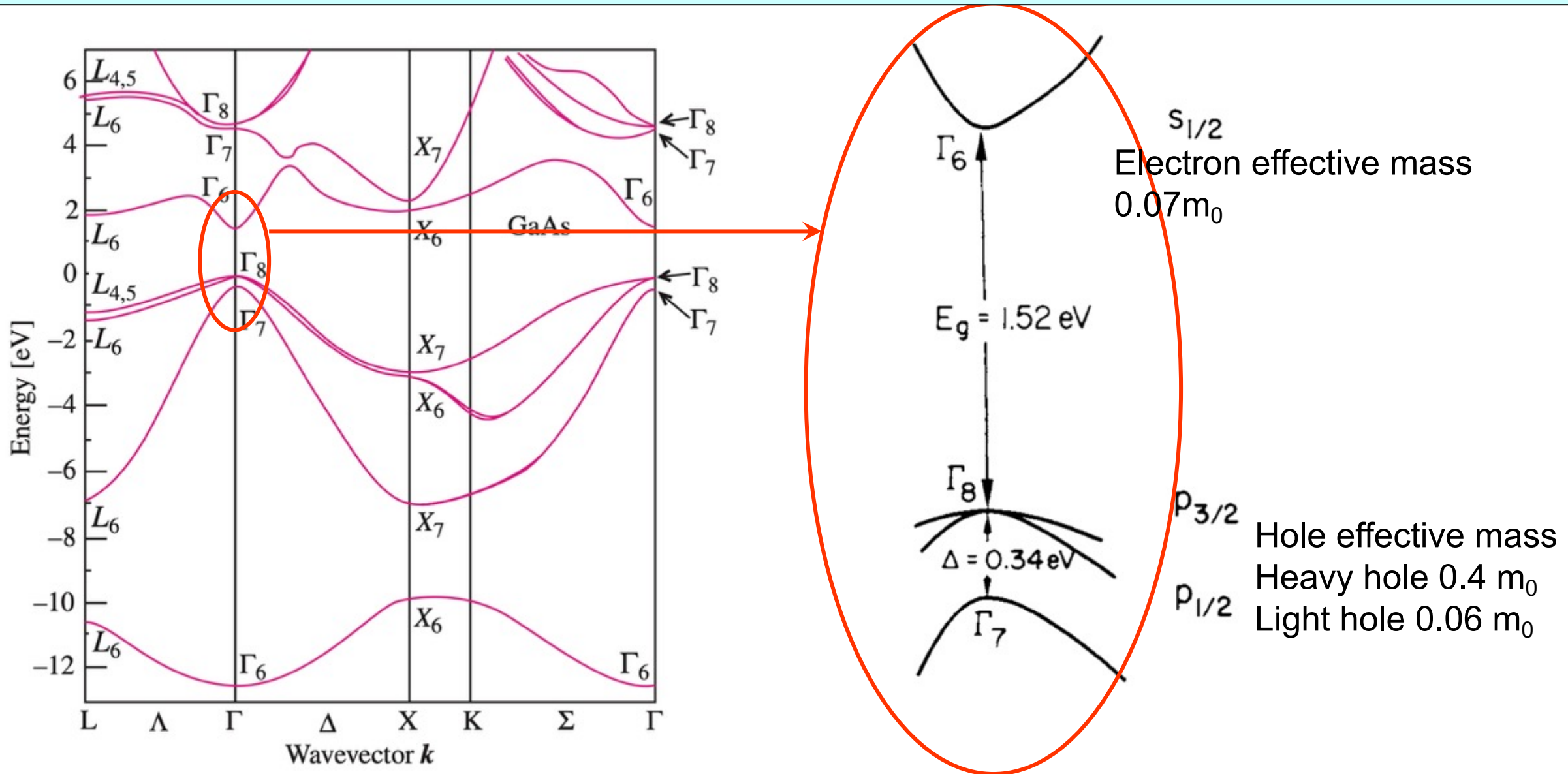
**Review**

D. N. Basov\*, Ana Asenjo-Garcia, P. James Schuck, Xiaoyang Zhu and Angel Rubio

## **Polariton panorama**

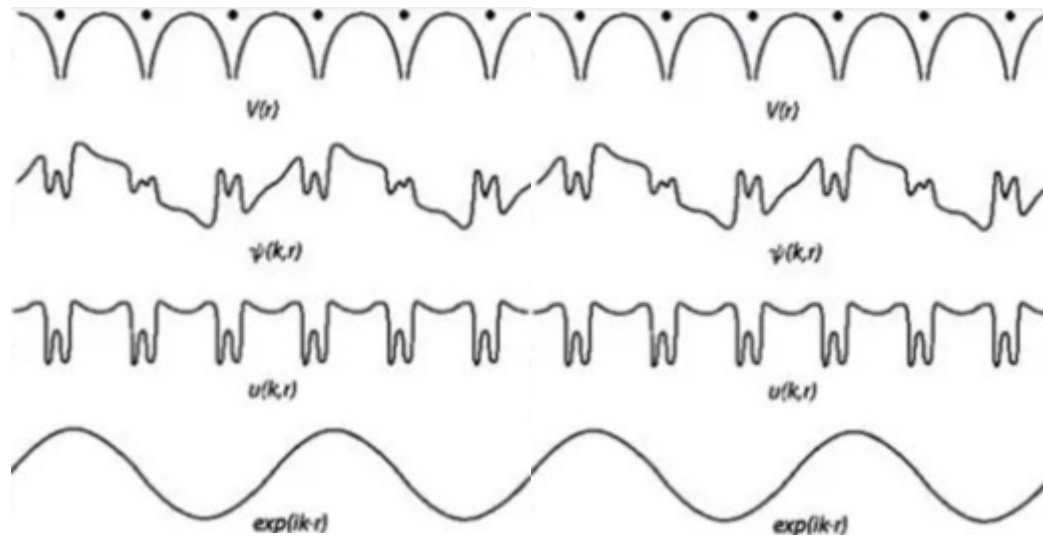
**Abstract:** In this brief review, we summarize and elaborate on some of the nomenclature of polaritonic phenomena and systems as they appear in the literature on quantum materials and quantum optics. Our summary includes at least 70 different types of polaritonic light–matter dressing effects. This summary also unravels a broad panorama of the physics and applications of polaritons.

# Band structure (electronic dispersion curves) for Gallium Arsenide (GaAs)



# Wavefunctions in an infinite perfect periodic crystal

Solution of the Schrödinger equation with a periodic potential are Bloch wavefunctions:  
**product of a periodic part and an envelope plane wave**



Lattice ion periodic potential

Bloch wavefunction

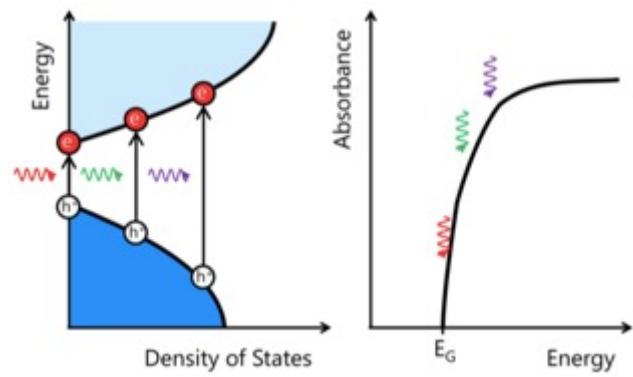
Periodic part of the Bloch wavefunction

Envelope wavefunction

This is true for electrons and holes, but ...,  
when there is Coulomb interaction, the true excitations are not separated electrons and holes,  
but **excitons** (which also have translational symmetry), **Coulomb bound electrons and holes**

# Electron-hole Coulomb interaction: exciton effects in ordered semiconductors

## no Coulomb interaction



$$\left( -\frac{\hbar^2}{2m_e^*} \Delta_{\mathbf{r}_e} - \frac{\hbar^2}{2m_h^*} \Delta_{\mathbf{r}_h} - \frac{e^2}{\epsilon_0 \epsilon_r |\mathbf{r}_e - \mathbf{r}_h|} \right) \Psi = E \Psi$$

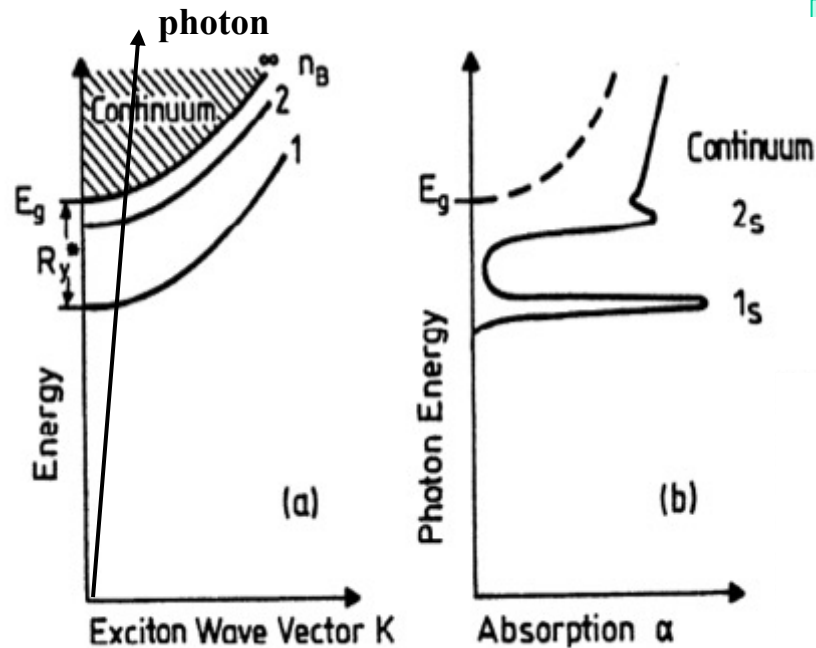
- Separation of variables:  $\mathbf{R} = \frac{m_e \mathbf{r}_e + m_h \mathbf{r}_h}{m_e + m_h}$ ,  $\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h$

$$\Psi(\mathbf{r}_e, \mathbf{r}_h) = F(\mathbf{R})g(\mathbf{r}_e - \mathbf{r}_h)$$

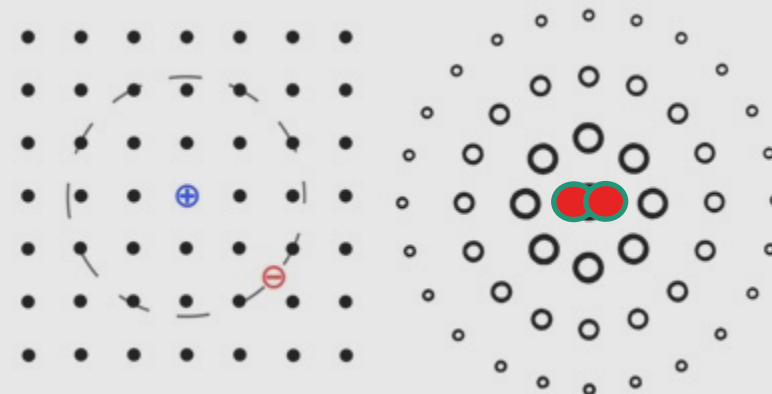
$$E_{\text{ex}}(n_B, \mathbf{K}) = E_g - Ry^* \frac{1}{n_B^2} + \frac{\hbar^2 \mathbf{K}^2}{2M}$$

$F(\mathbf{R})$  center of mass wavefunction is a plane wave  $e^{i\mathbf{K}\mathbf{R}}$

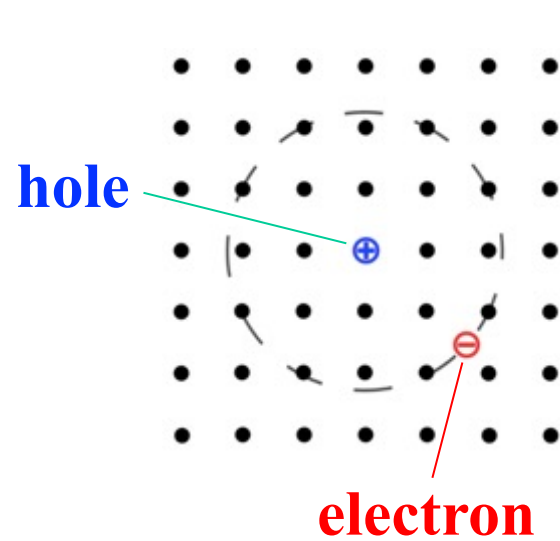
$g(\mathbf{r})$  relative electron-hole wavefunction is hydrogenoid with quantum numbers  $n=1, 2, 3, \dots$



In homogeneous semiconductors - excitons  
hydrogen-like wavefunction of electrons around hole

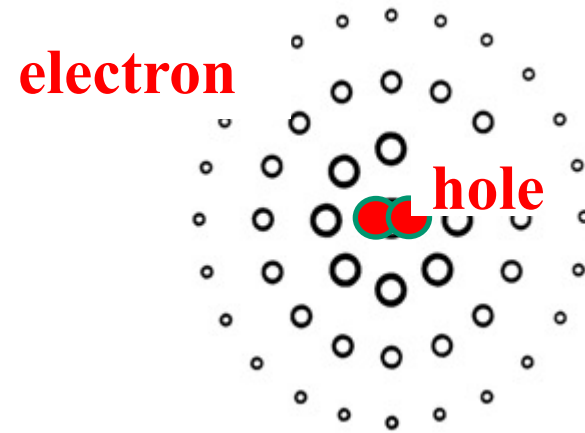


# Free excitons parameters



Inorganic semiconductors  
Large exciton radius  
Wannier excitons

$$\Psi(\mathbf{r}_e, \mathbf{r}_h) = f(\mathbf{R}) g(\mathbf{r})$$

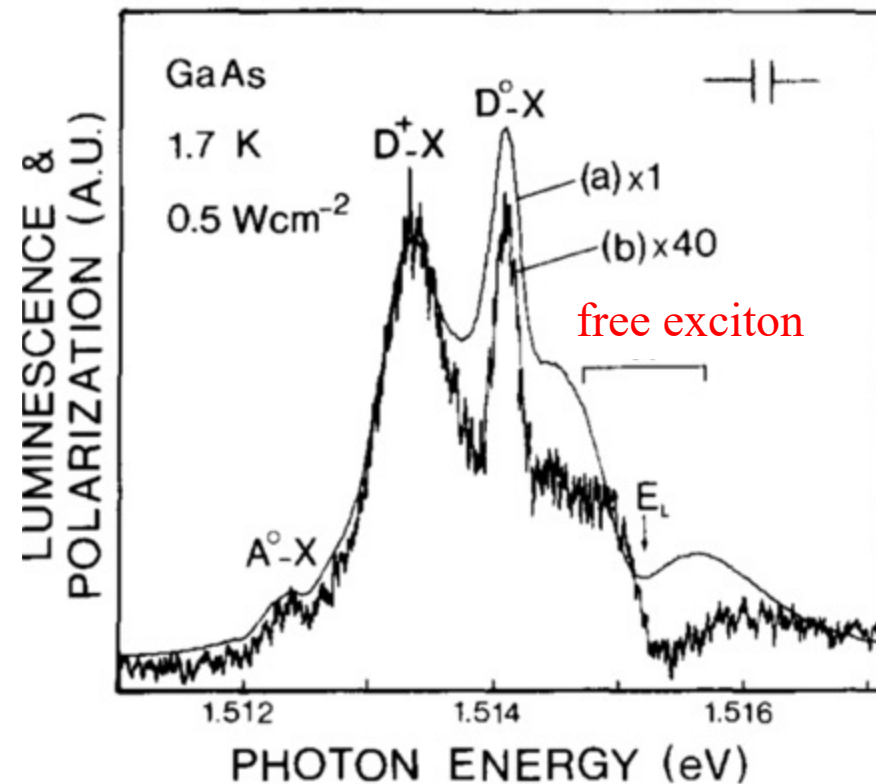
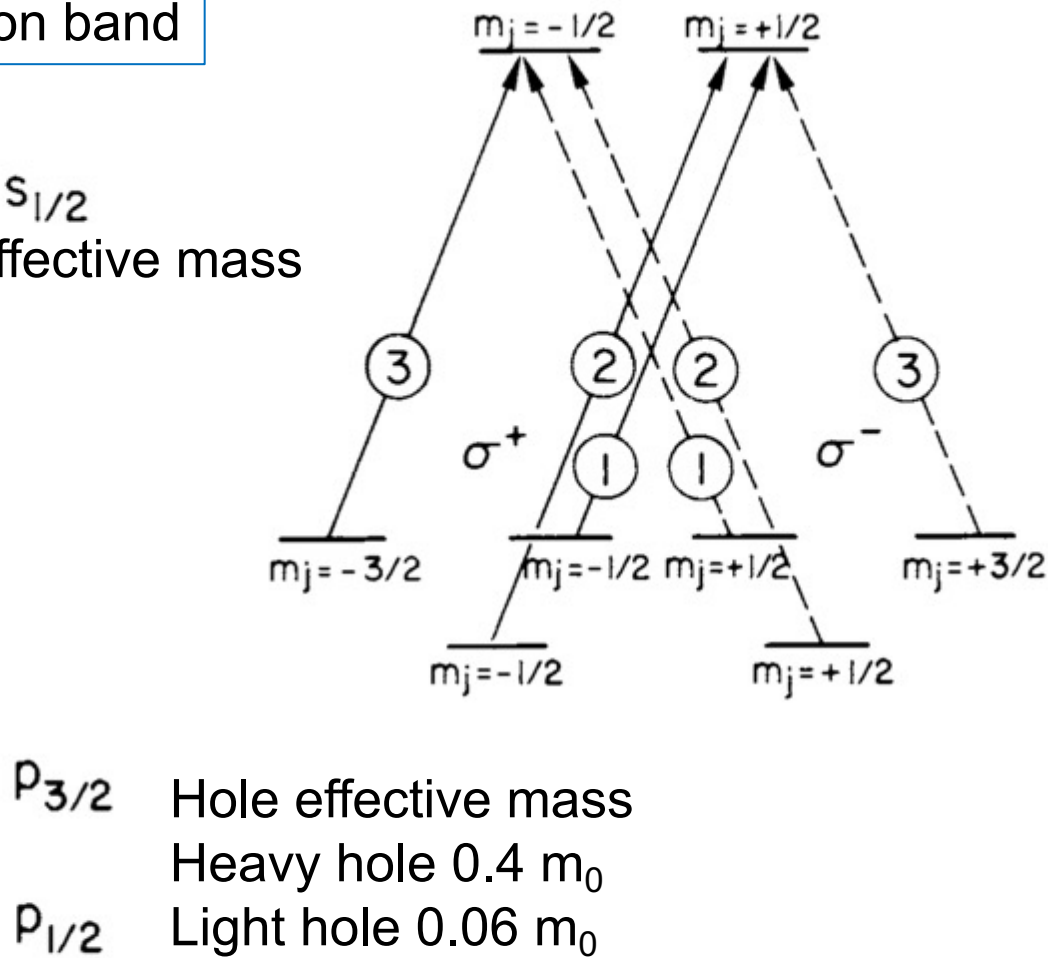
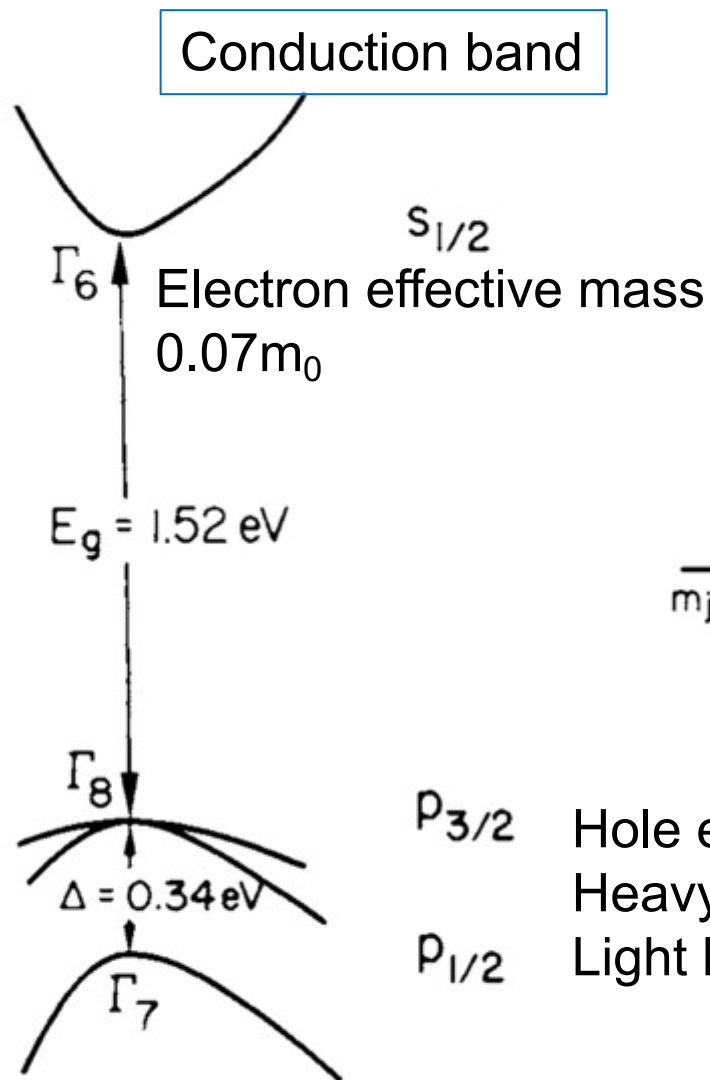


n=1 bounded pair hydrogenoid  
wavefunction of relative e-h motion

Bohr radius  $a_B^* = \epsilon_r \left( \frac{m_0}{\mu} \right) a_B$  a few nm ( $\mu$  effective mass)

Effective Rydberg  $Ry^* = 13.6 \text{ eV} \frac{\mu}{m_0} \left( \frac{1}{\epsilon^2} \right)$  a few to tens of meV  
Exciton energy

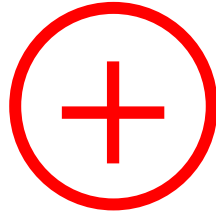
# At the beginning: search for large optical spin orientations In ultra-pure GaAs



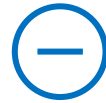
Valence band

# Bound excitons

Donor ion



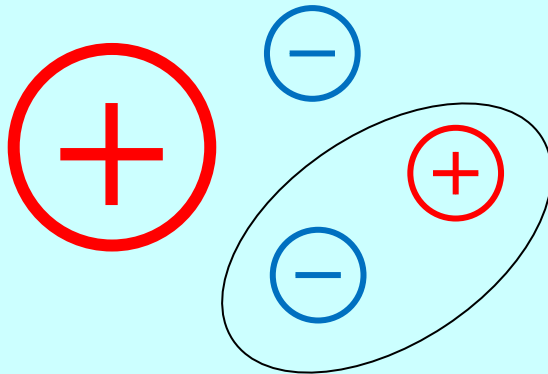
electron



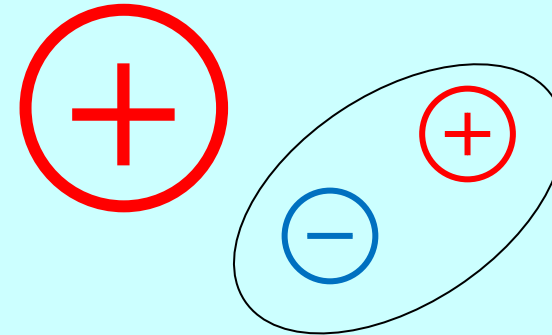
hole



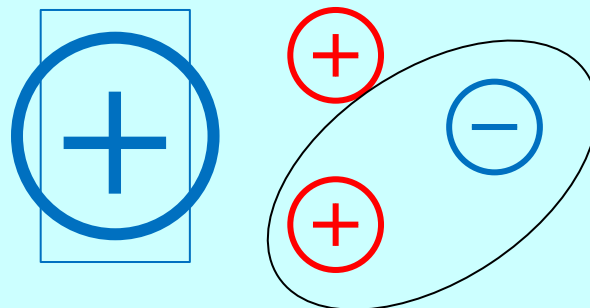
Exciton bound to a neutral donor



Exciton bound to an ionized donor



Exciton bound to a neutral acceptor





# Theory of the Contribution of Excitons to the Complex Dielectric Constant of Crystals\*†

J. J. HOPFIELD†

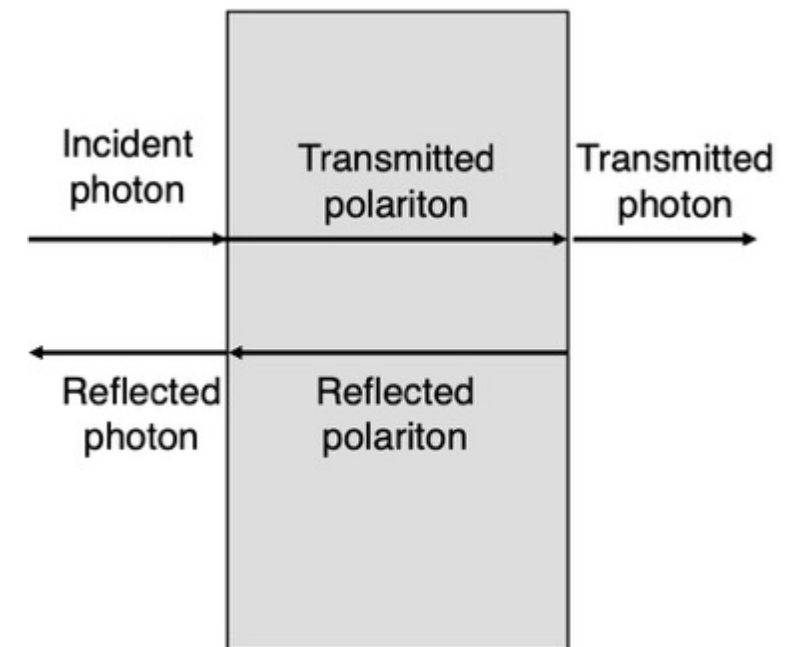
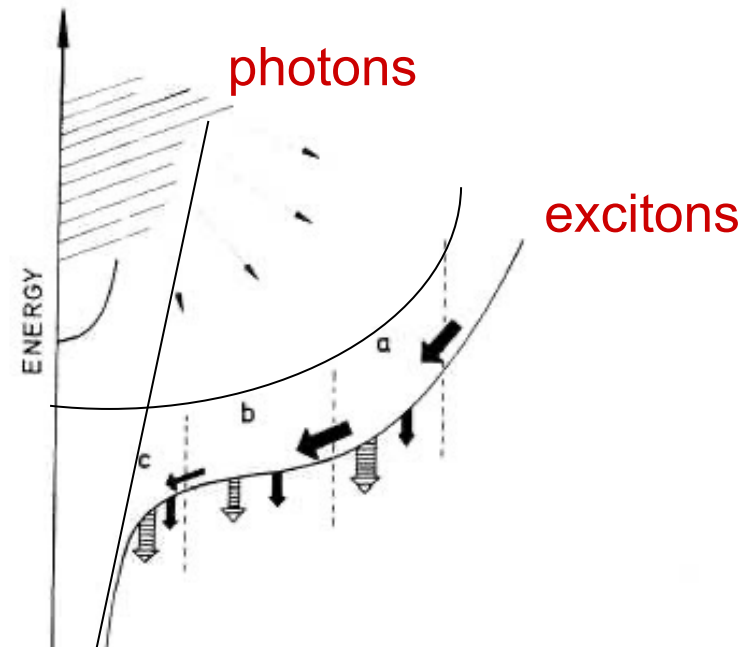
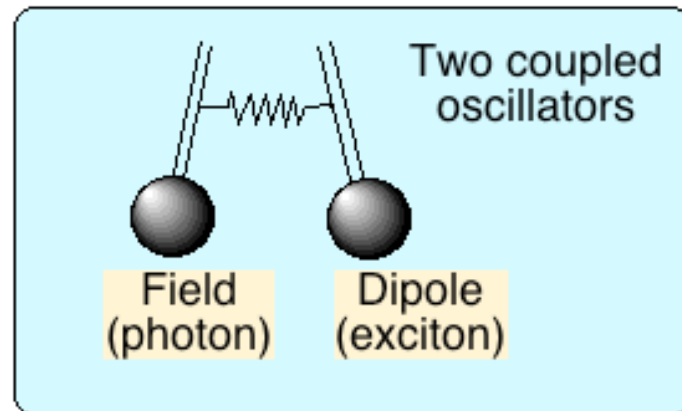
*Physics Department, Cornell University, Ithaca, New York*

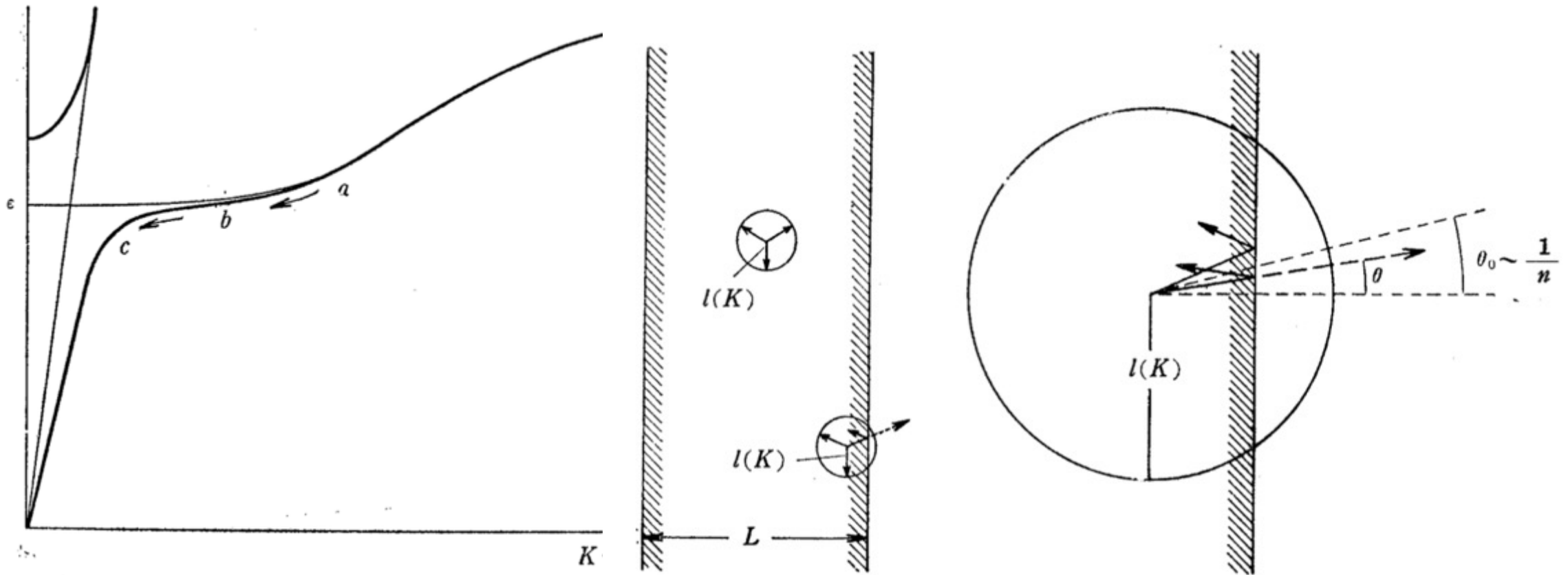
(Received July 16, 1958)

It is shown that the ordinary semiclassical theory of the absorption of light by exciton states is not completely satisfactory (in contrast to the case of absorption due to interband transitions). A more complete theory is developed. It is shown that excitons are approximate bosons, and, in interaction with the electromagnetic field, the exciton field plays the role of the classical polarization field. The eigenstates of the system of crystal and radiation field are mixtures of photons and excitons. The ordinary one-quantum optical lifetime of an excitation is infinite. Absorption occurs only when "three-body" processes are introduced. The theory includes "local field" effects, leading to the Lorentz local field correction when it is applicable. A Smakula equation for the oscillator strength in terms of the integrated absorption constant is derived.

Strongly-coupled 3D excitons and photons, excitonic polaritons are the quasi particles of the system

They retain translational invariance



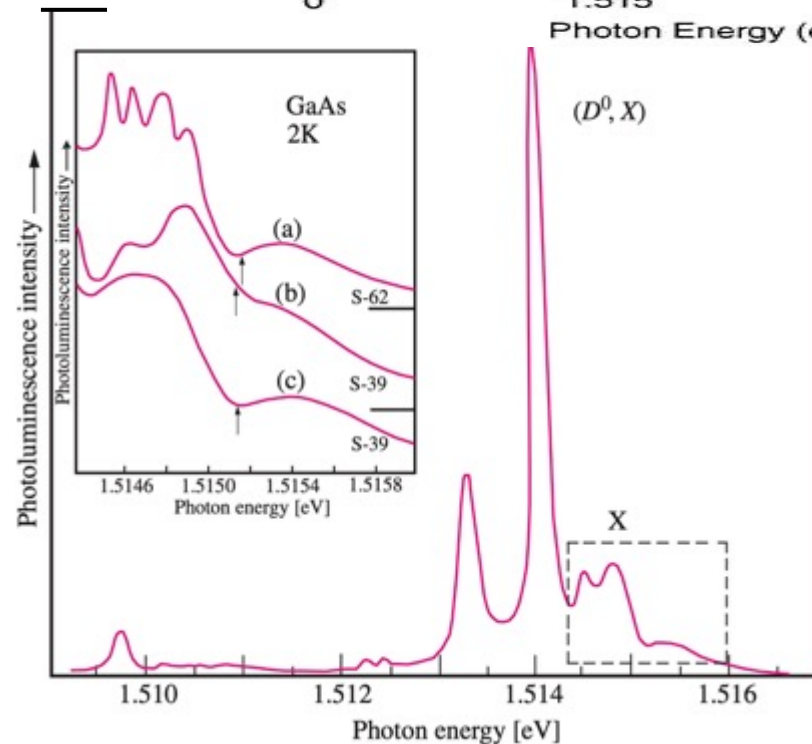
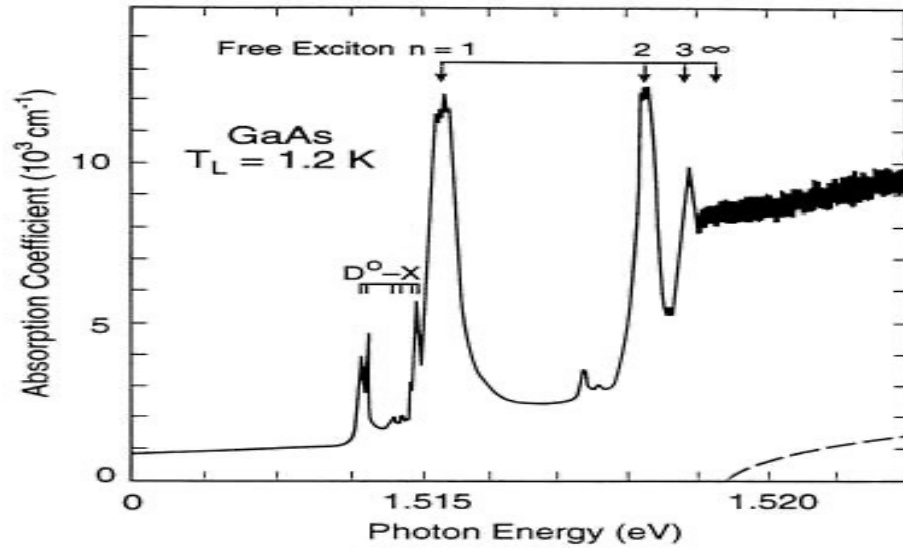


Direct radiative annihilation of an exciton  
(weak exciton-phonon coupling, and the band  
bottom at  $K=0$ )

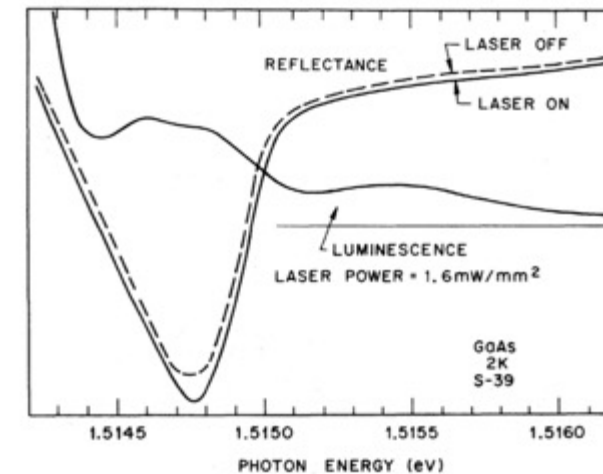
$b$   $c$  in Fig. 9 is indeed a neck because it is much longer than the thermalization time 10-12 sec. Thus we can take  $t_a$  as the life-time for radiative annihilation of an exciton. It should be borne in mind that  $t_a$  depends on the boundary condition ( $L$  in the present model) as well as on the mean depth (from the surface) of the excitons initially produced.

Yutaka Toyozawa, On the Dynamical Behavior of an Exciton Suppl. Progress Theoretical Physics, 12, 111 (1959)

# A breakthrough in 1973: identification of polaritons luminescence in high-purity GaAs

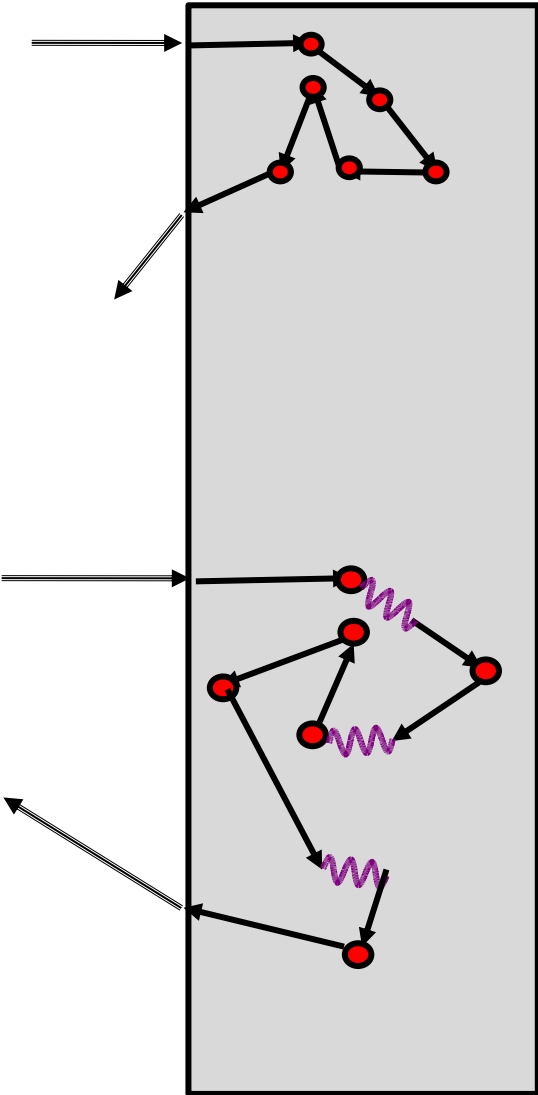


Sell et al., Polariton Reflectance and Photoluminescence in High-Purity GaAs, Phys. Rev. B7, 4568 (1973)

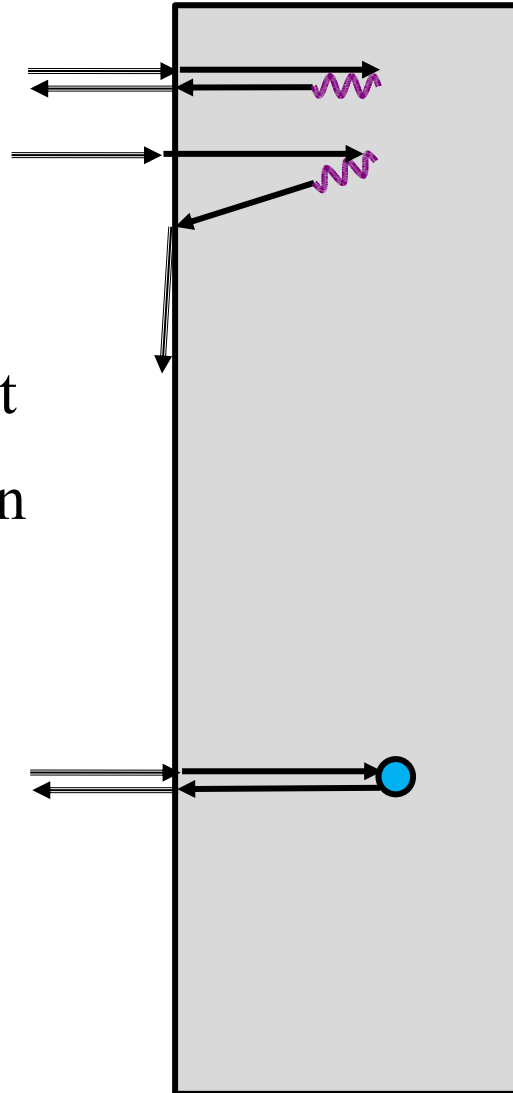


direct comparison of the luminescence and reflectance (laser on, solid; laser off, dashed curve)

# Scattering phenomena for excitonic polaritons



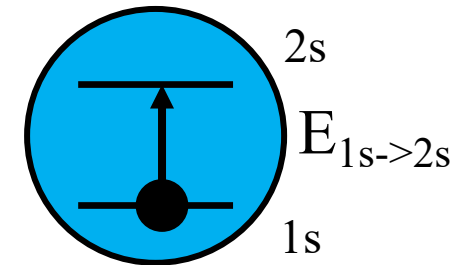
• Scattering point defect  
Lattice vibration phonon



**Single phonon scattering**  
**Energy conservation**

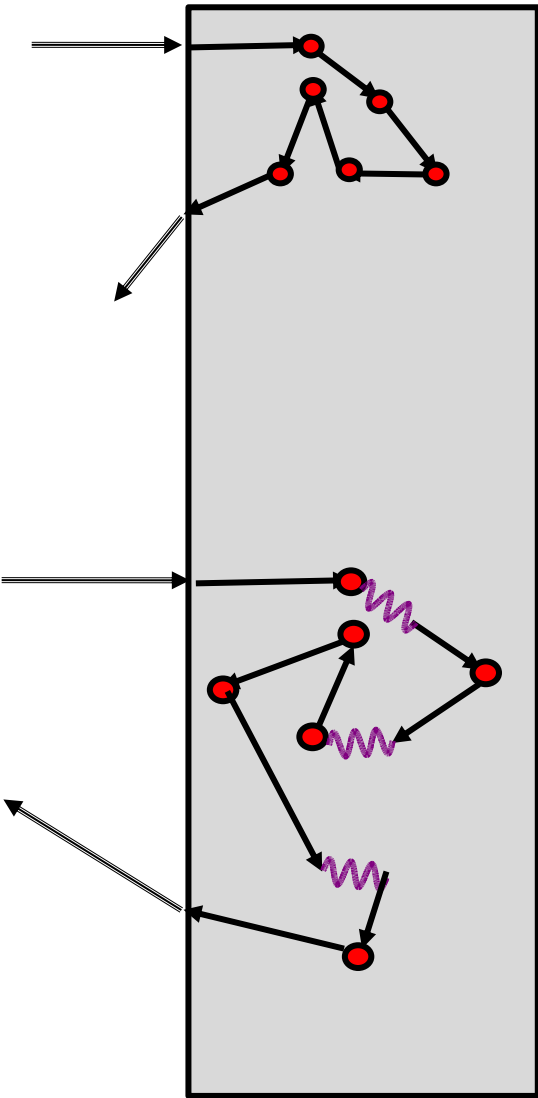
**Acoustic phonon energy**  
 $\Delta E = \hbar c_s k$

Inelastic electron scattering  
by an electron bound to a  
donor



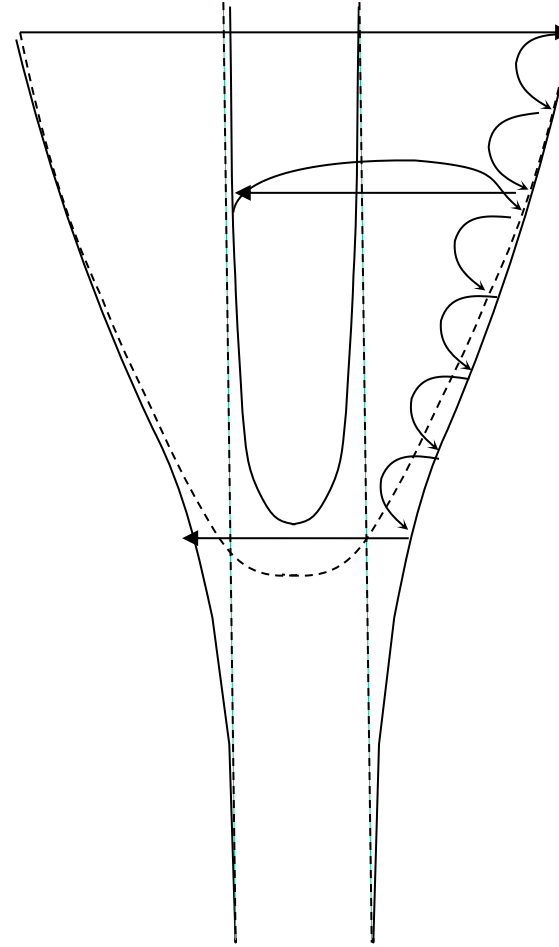
$$\Delta E = \begin{matrix} E_{2s} - E_{1s} \\ E_{3s} - E_{1s} \end{matrix}$$

# Scattering phenomena for excitonic polaritons

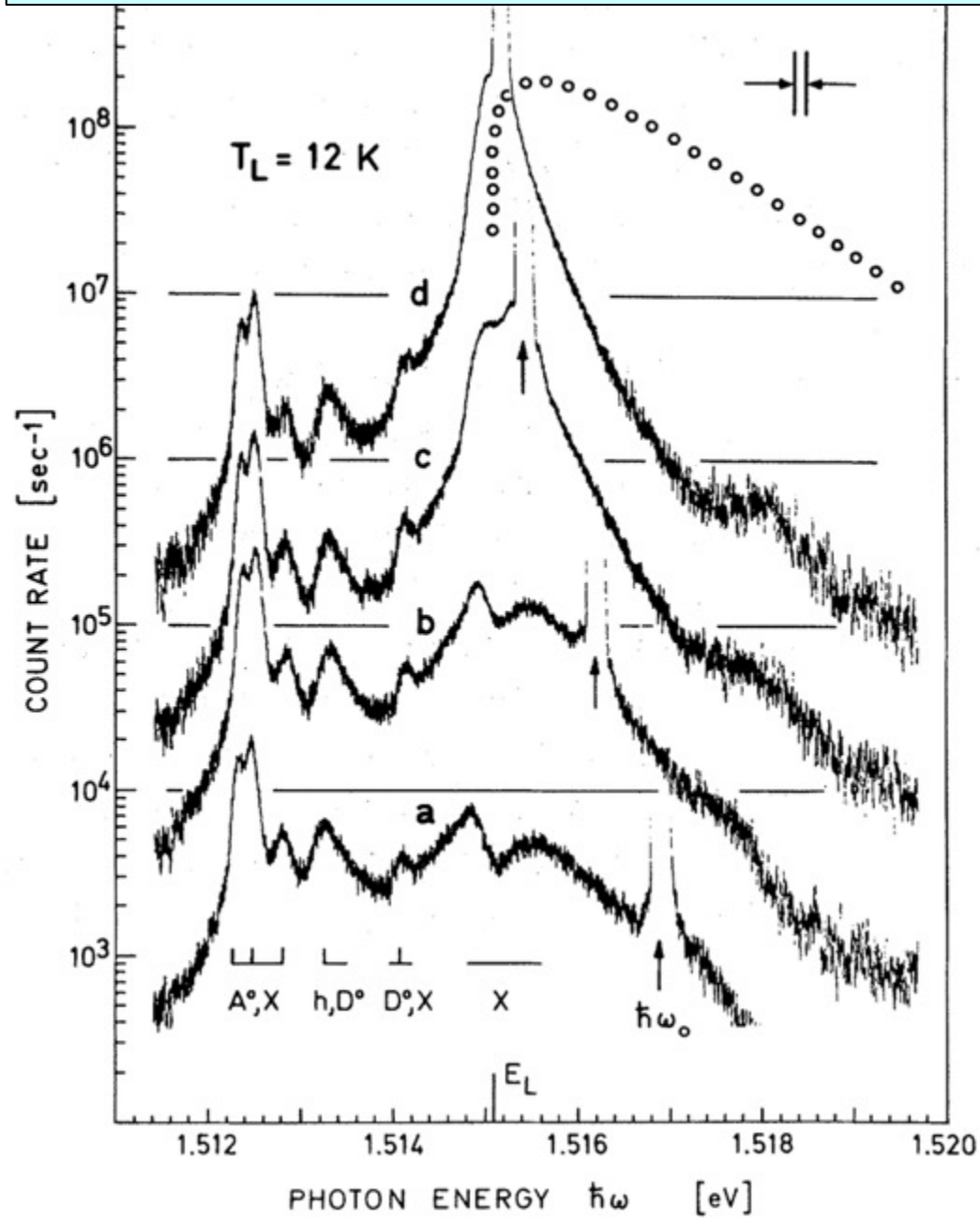


● Scattering point defect

~~~~~ Lattice vibration phonon



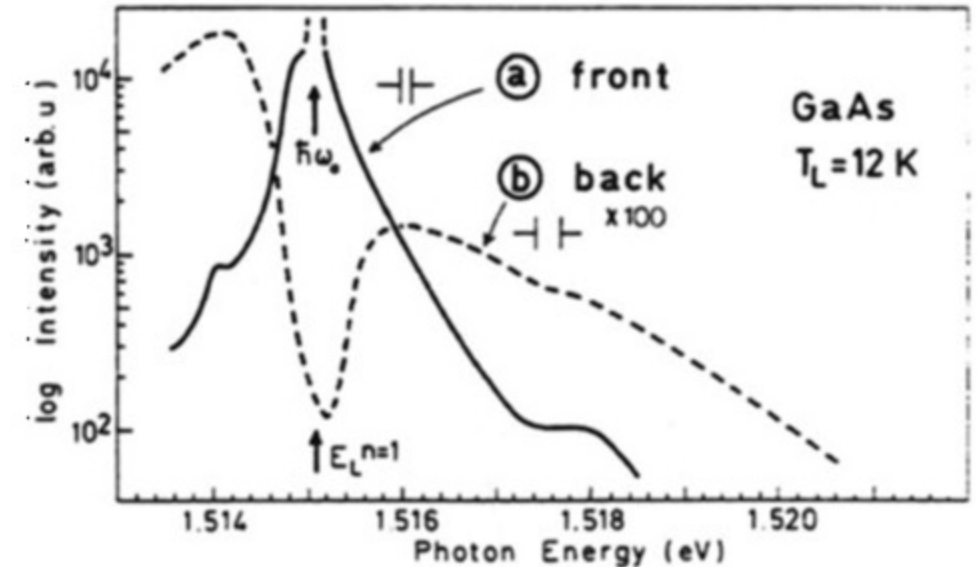
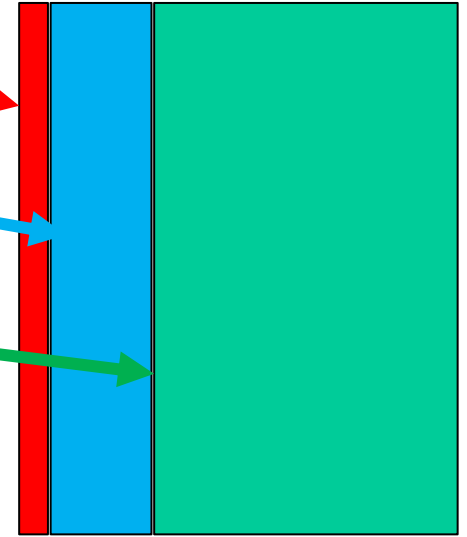
# Fluorescence of resonantly excited polaritons



Resonantly excited volume

Non-resonantly excited volume

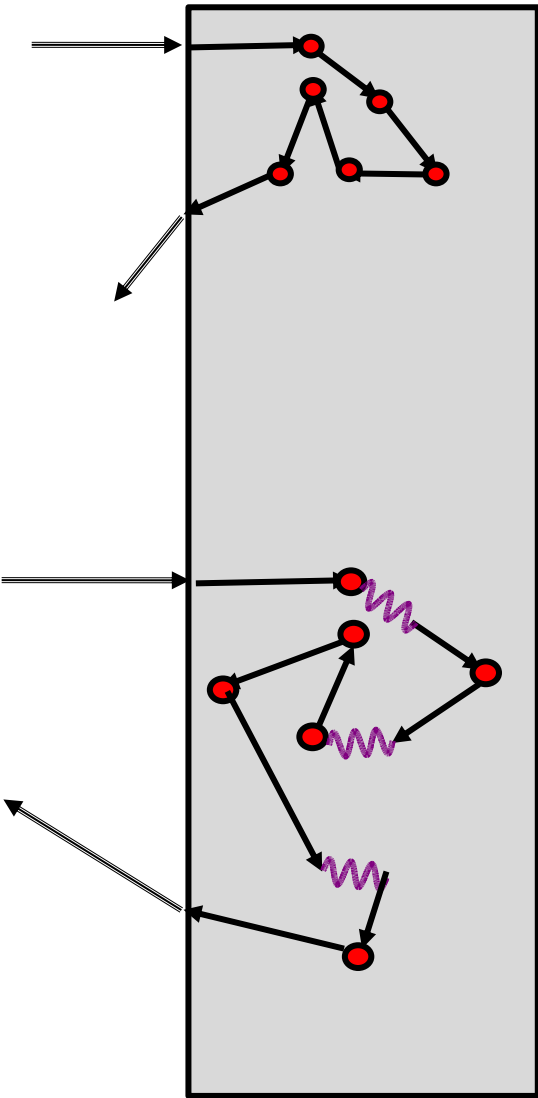
sample volume



Thick sample :  $2\mu\text{m}$

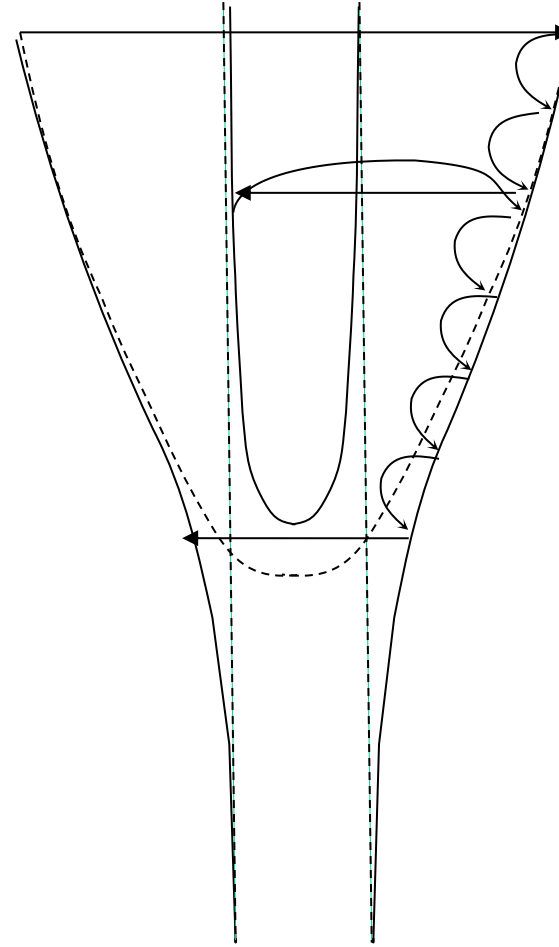
Resonant excitation depth:  $0,1\mu\text{m}$ , time of flight  $10^{-11}\text{s}$

# Scattering phenomena for excitonic polaritons



● Scattering point defect

~~~~~ Lattice vibration phonon





# Phonon Paper

$\Gamma(k, \omega)$   
 $\downarrow$   
 Total impurities?  $ABC$   
 [Diff Pot. for polaritons]

Criticism:

$$Y_n \xleftrightarrow{2} \tau_{\text{elast}}, \tau_{\text{inert}}$$

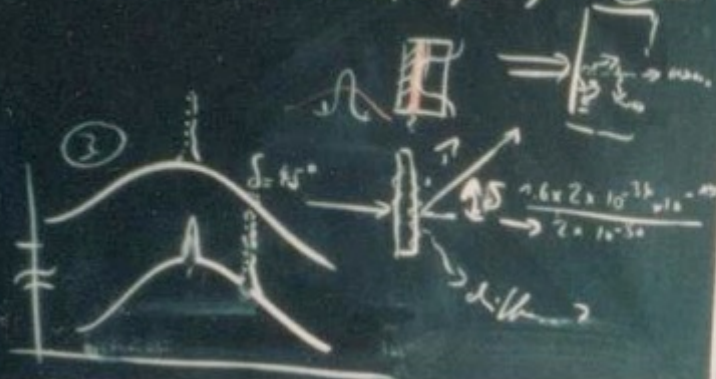
Before  $\leftrightarrow$  cryptic

[Stern  $\leftrightarrow$  not new?]

Miklós  $\leftrightarrow$  Multiple Int.  
Green's Fats!

Evangelisti  $\leftrightarrow$  surface rough  
diffuse scattering

Absorption +

$$S(\omega, \omega', k, k', z, \dots) \quad \textcircled{F} \textcircled{2}$$


## Resonant Interaction of LA phonons with Excitonic Polaritons in GaAs

## I. Introduction

Pine, Taylor/B/B

2-band model, LA phase coupling,  
penetration depth, spatial dispersion  
and ABC treated in model of BBZ

We:  $\Xi-\bar{\nu}$ , two center bands, expected 9 lines but  
found only 4. Preliminary results published. Now  
we concentrate on LA-phonon coupling, dynamical  
aspect of RBS versus RPF.

RBS<sup>+++</sup> / GaAs / CdTe\*

long version (contra/uk <sup>Carlton</sup> <sup>Wickings</sup>)

 $\bar{E}(\mathbf{k})$ 
$$T_2 = \frac{1}{2}, \frac{1}{2}^*)$$

Polarization

Lincoln

ABC's (?)

Oscillator strength  $\stackrel{?}{\Rightarrow} E_{LT}$  too small  
dipole matrix element  $\mu_{ij}$

## II. Experiments

Book sitting config. photon counting

high resolution with F.Y. spectrometer ( $\frac{1}{2}$ )

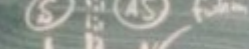

  
 $\text{width} \leq 40 \text{ nm}$   
 $\leq \frac{1}{12} \Delta E$   
 $z_0 \leq 0.8 \mu\text{m}$   
 from Hall's  
 $z_0 \geq 0.5 \mu\text{m}$   
 Phenon Phenon

Fig. A RBS spectrum - GaAs.  
Note the asymmetry

RPF<sub>2</sub> + + + +

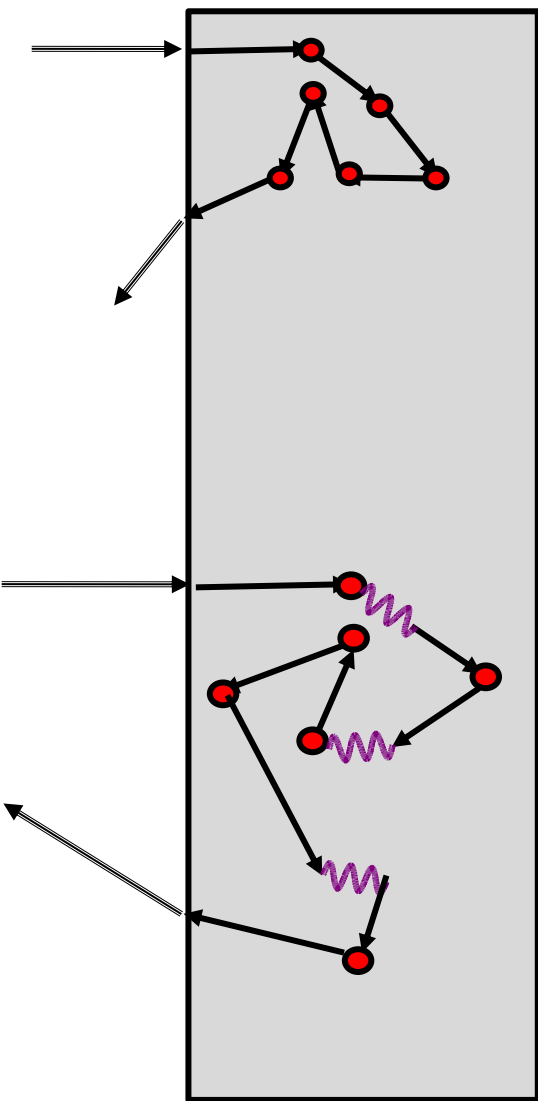
long version

f)  $ABC \cong \triangle$

\_\_\_\_\_

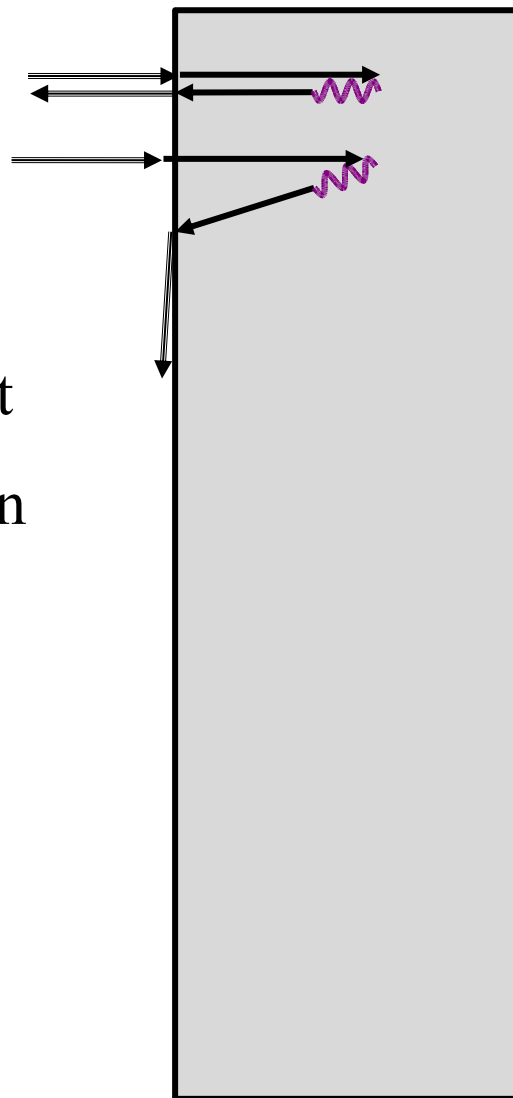


# Scattering phenomena for excitonic polaritons



● Scattering point defect

~~~~~ Lattice vibration  
phonon



**Single phonon scattering**

**Energy conservation**

**Acoustic phonon energy**

$$\Delta E = \hbar c_s k$$

# Light Scattering in the Polariton Framework

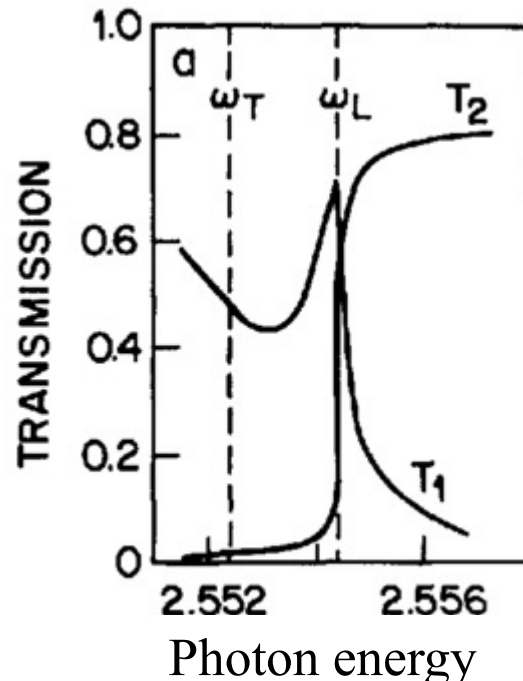
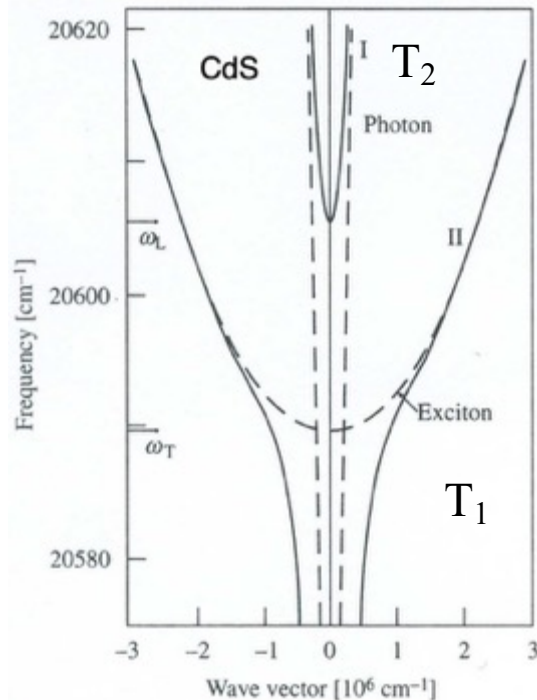
A scattering event is the *succession of three steps*:

- 1) transmission of an incoming photon at the interface as a polariton inside the crystal;
- 2) scattering from one polariton state to another inside the crystal;
- 3) propagation and transmission of the scattered polariton outside the crystal as a photon.

the scattering probability  $P_{\text{out}}$  (or efficiency) can be factorized as

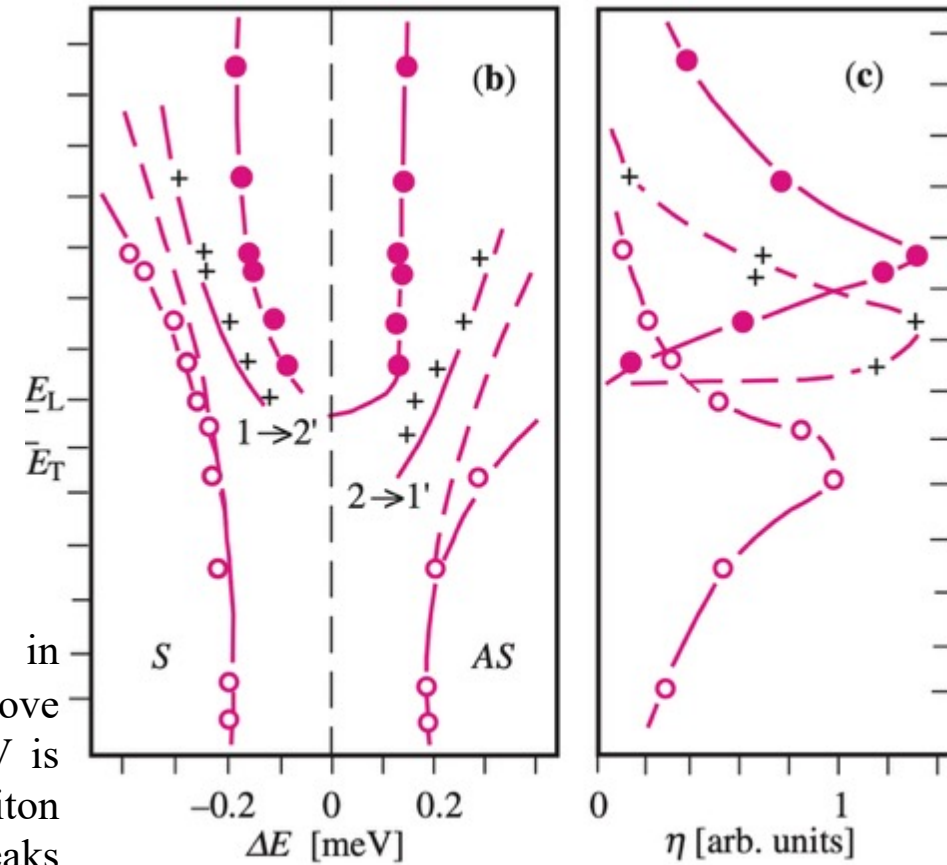
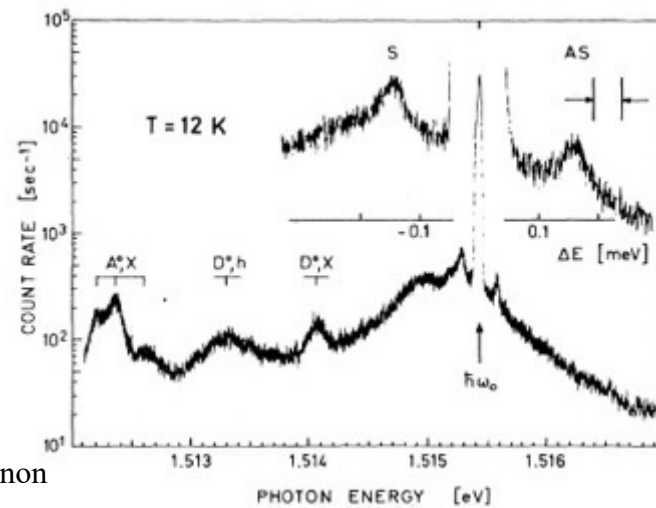
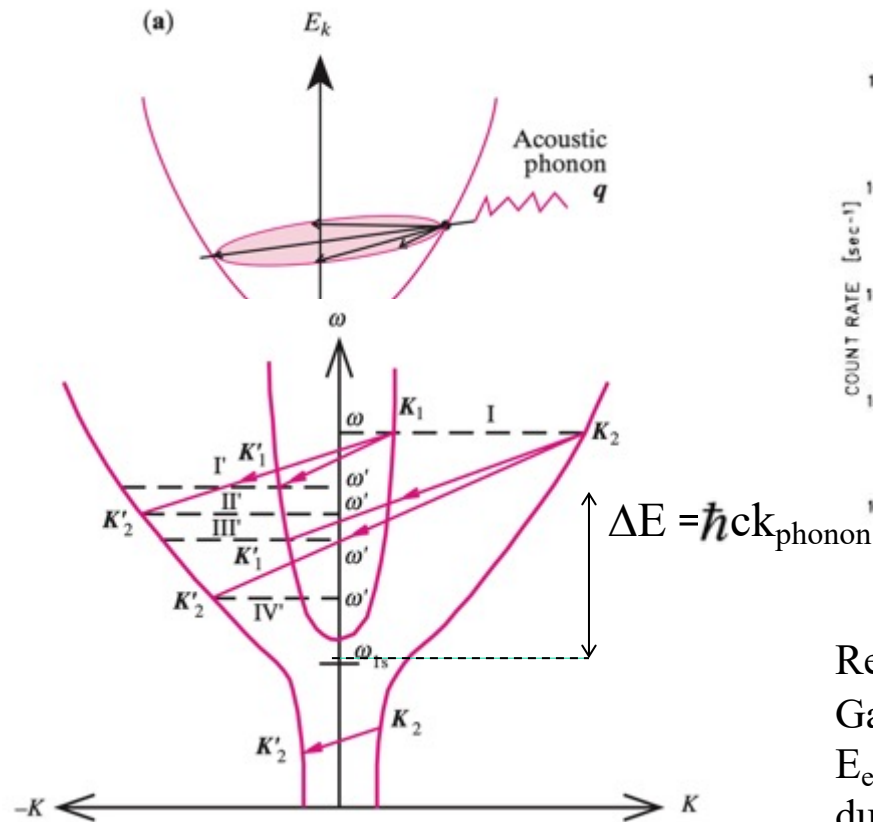
$$P_{\text{out}} = T(\omega_i) P_{\text{scatt, in}} T(\omega_s),$$

$T(\omega_i)$  and  $T(\omega_s)$  transmission coefficients of ingoing and outgoing polaritons at the crystal interface and  $P_{\text{scatt, in}}$  scattering probability of polaritons inside the crystal.



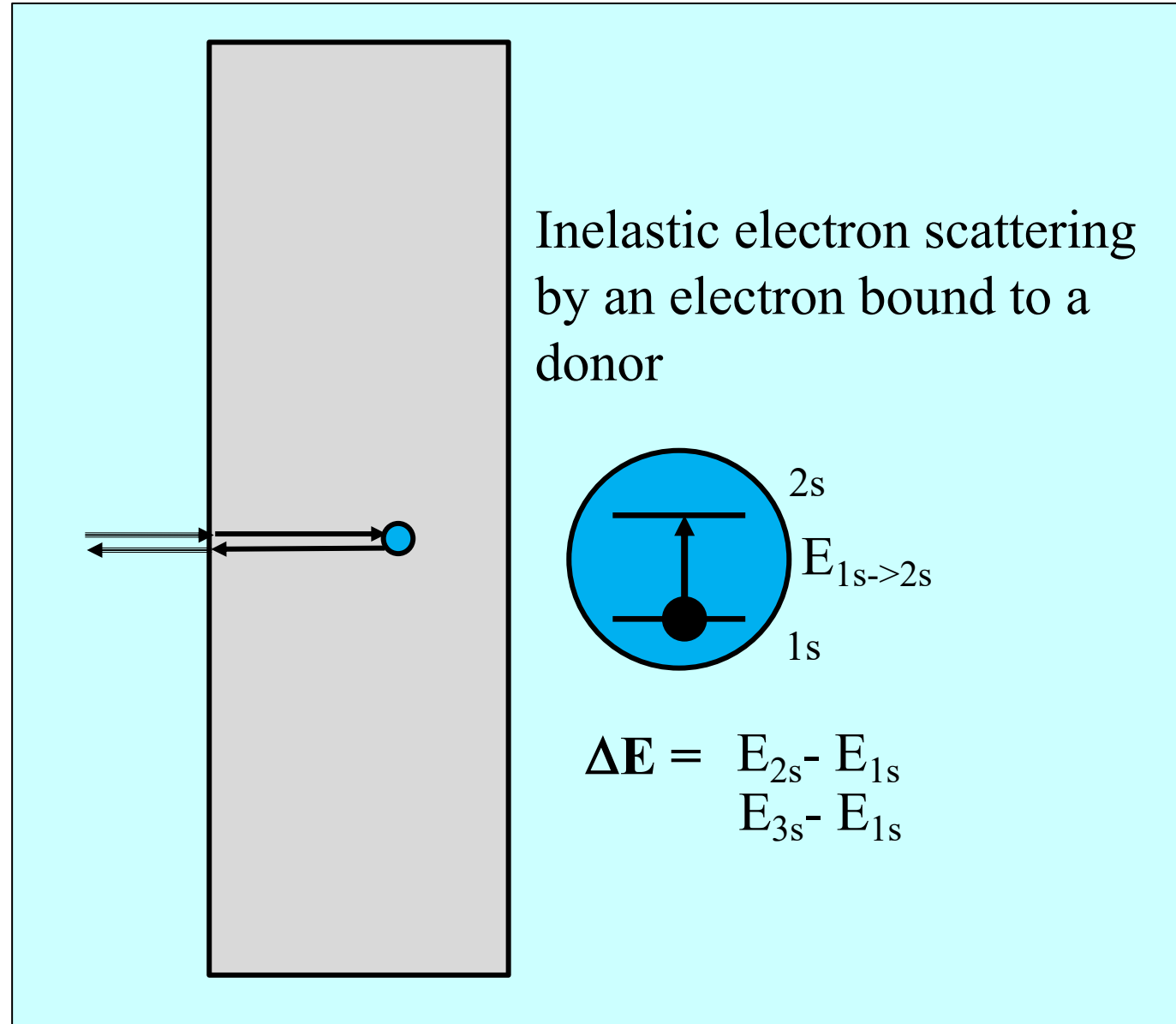
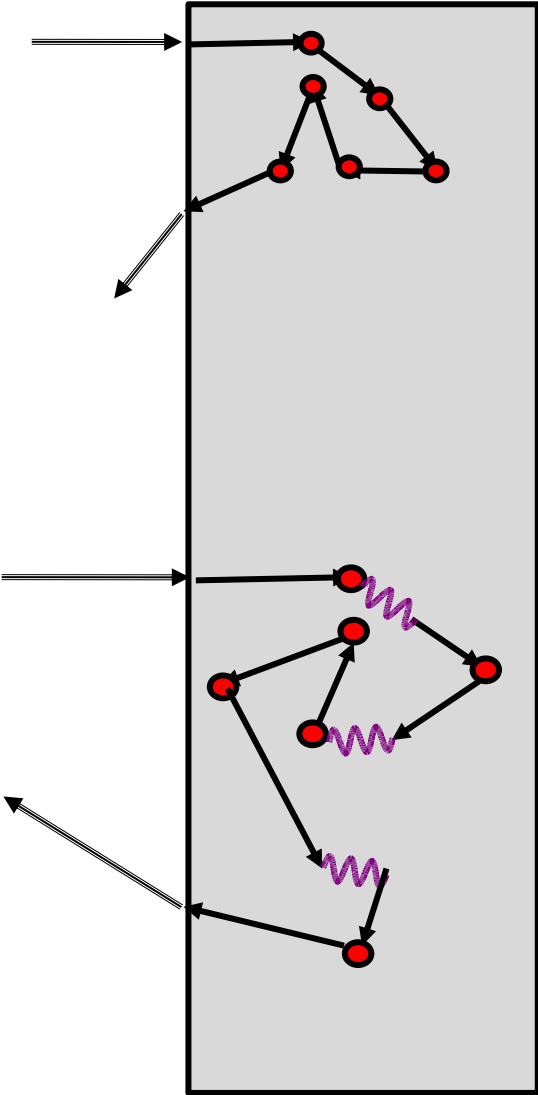
Transmission coefficients of outside photons as excitonic polaritons as a function of incoming photon energy in CdS (normal incidence, E, K1c). T<sub>1</sub> and T<sub>2</sub> are the transmission coefficients of photons as LPB or UPB polaritons  $\Gamma=0.05\text{meV}$

# Resonant Brillouin scattering by bulk excitonic polaritons: single acoustic phonon (sound waves) scattering



Resonant Brillouin scattering (RBS) in GaAs near-resonant  $\hbar\omega$ , slightly above  $E_{\text{exc}}$ . The broad band around 1.515 eV is due to  $n = 1$  exciton-polariton luminescence. Other luminescence peaks according to the corresponding electronic transitions.

# Inelastic electron scattering phenomena for excitonic polaritons



# Resonant Electronic Raman Scattering in Semiconductors

Rainer G. Ulbrich

*Institut für Physik der Universität Dortmund, D-4600 Dortmund, Germany*

and

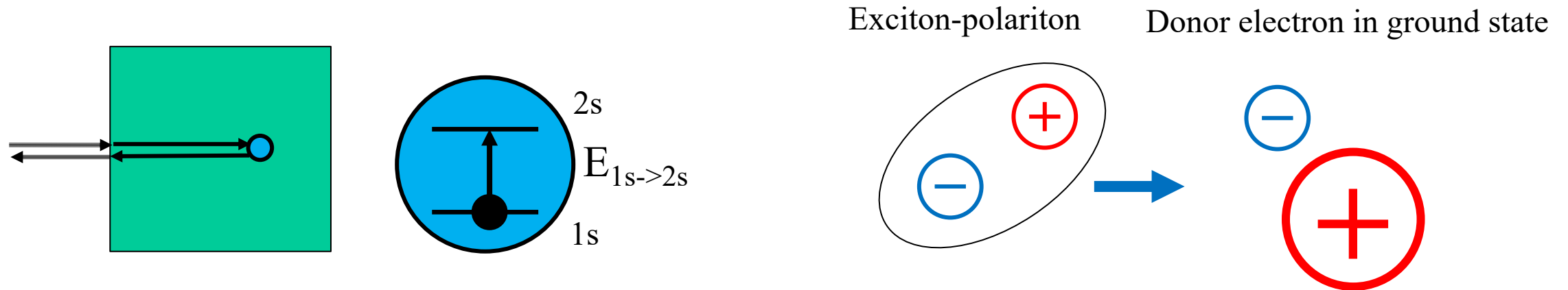
Nguyen Van Hieu

*Institute for Physics, Nghia Do, Tu Liem, Hanoi, Vietnam*

and

Claude Weisbuch

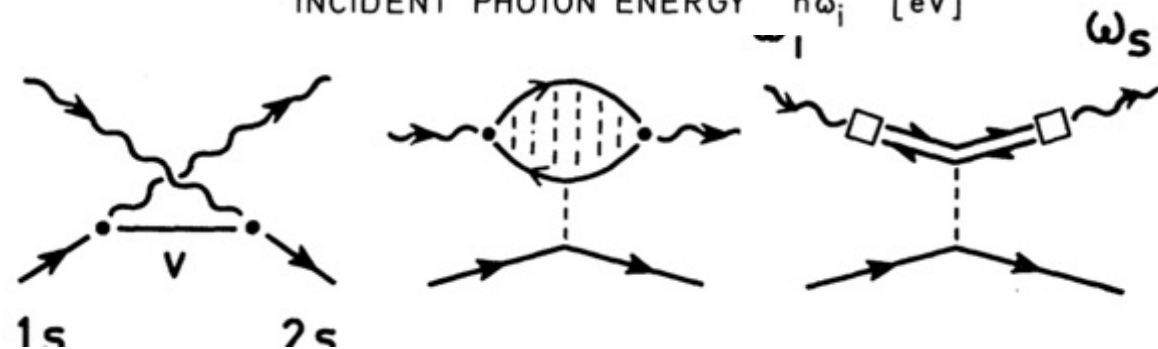
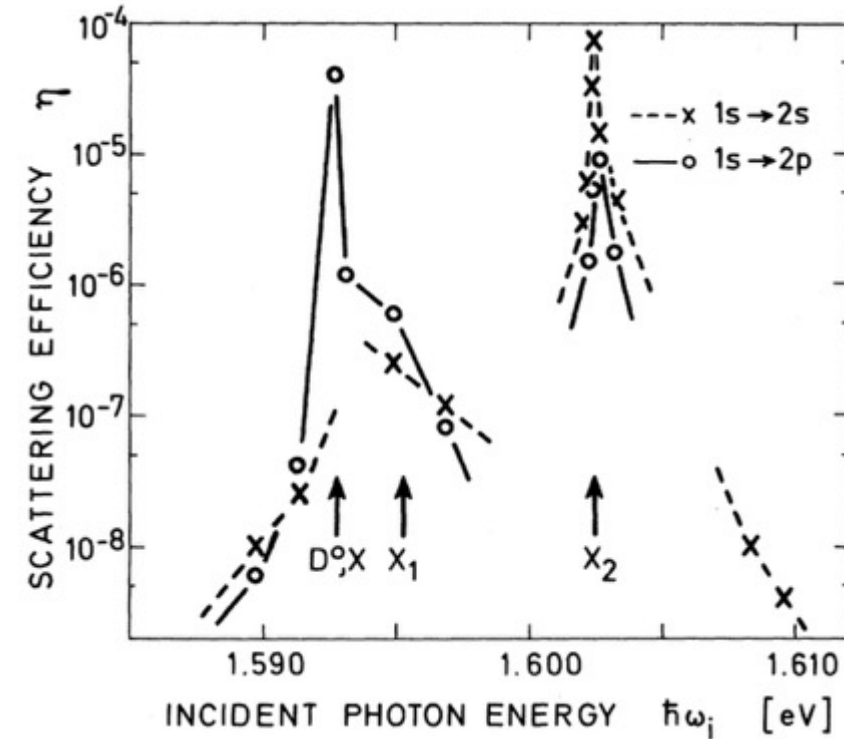
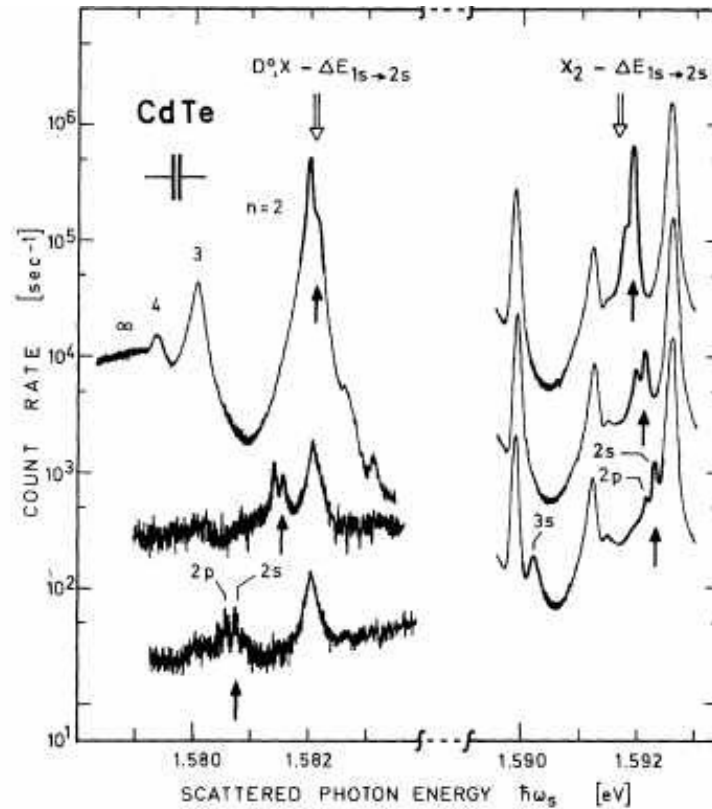
*Laboratoire de Physique de la Matière Condensée, Ecole Polytechnique, F-91128 Palaiseau, France*



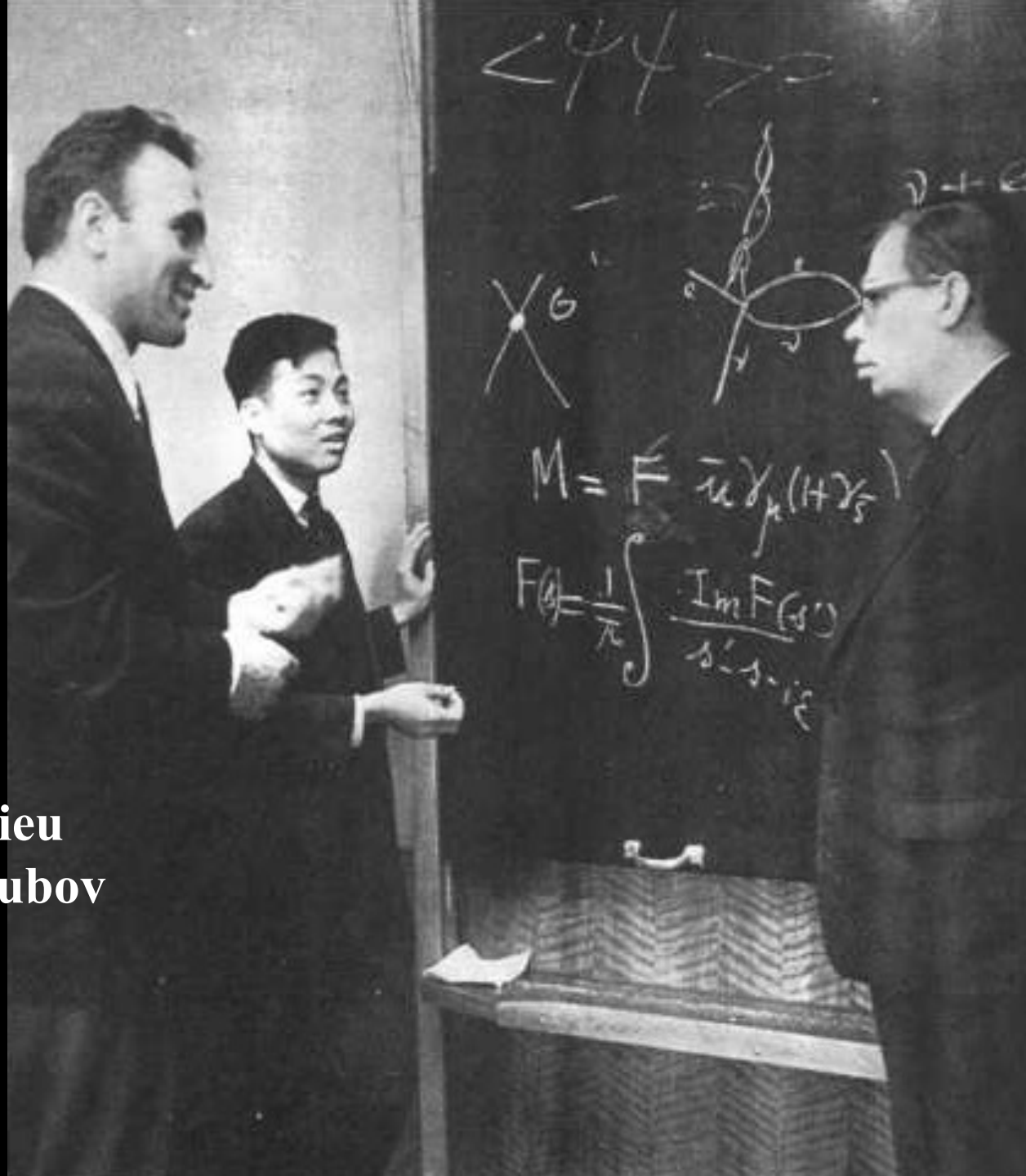
Electronic Raman scattering on shallow donors is enormously enhanced when the incident photon energy approaches the fundamental exciton region. **This excitonic resonance was observed for the first time** and it was found that  $\sigma_{\text{peak}} \approx 5 \times 10^{-14} \text{ cm}^2$  for CdTe and GaAs in the  $n=2$  free-exciton resonance. The **enhancement of  $\gtrsim 10^{10}$**  over off-resonance excitation

R. Ulbrich, Nguyen van Hieu & cw, Phys. Rev. Lett. 46, 531(981)

# Resonant Electronic Raman scattering



**Center**  
**Nguyen Van Hieu**  
**Right: Bogoliubov**  
**Dubna 1962**





$$T_{ex\gamma} \approx e \frac{\pi}{m} \frac{a_0^{3/2}}{\sqrt{2\epsilon E_\gamma}} \cdot I_{ex\gamma}, \quad a_0^{3/2} = a_0^2 \cdot e \sqrt{\frac{m_e^*}{\epsilon}},$$

$$I_{ex\gamma} = \int \tilde{F}_\gamma(\pi) \tilde{\Phi}_{1s}(\pi) \tilde{\Phi}_{2s}^*(\pi) \frac{d^3\pi}{(2\pi)^3}$$

$$T_{ex\gamma} = e a_0^2 \frac{\pi}{m} \frac{1}{\epsilon} \sqrt{\frac{m_e^*}{2\epsilon E_\gamma}} I_{ex\gamma}$$

$$ex + e \rightarrow ex + e$$

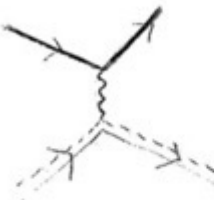


$$\gamma+(1s) \rightarrow \gamma'+(2s)$$

$$\Delta Q = Q' - Q$$

$$\frac{e^2}{\epsilon(Q-Q')^2} \iint F_\gamma(p) \Phi_{1s}(q) F_{\gamma'}^*(p + \frac{\Delta Q}{2}) \Phi_{2s}^*(q - \Delta Q)$$

$$\frac{d^3p}{(2\pi)^3} \cdot \frac{d^3q}{(2\pi)^3}$$



$$-\frac{e^2}{\epsilon(Q-Q')^2} \iint F_\gamma(p) \Phi_{1s}(q) F_{\gamma'}^*(p - \frac{\Delta Q}{2}) \Phi_{2s}^*(q + \Delta Q) \frac{d^3p}{(2\pi)^3} \cdot \frac{d^3q}{(2\pi)^3}$$



$$\frac{e^2}{\epsilon} \iint \frac{F_\gamma(p) \Phi_{1s}(q) F_{\gamma'}^*(p + \frac{\Delta Q}{2}) \Phi_{2s}^*(q - \Delta Q)}{(q - p + \frac{Q - Q'}{2})^2} \frac{d^3p}{(2\pi)^3} \frac{d^3q}{(2\pi)^3}$$

$$\left\{ \approx \frac{e^2}{\epsilon} \iint \frac{F_\gamma(p) \Phi_{1s}(q) F_{\gamma'}^*(p) \Phi_{2s}^*(q)}{(p - q)^2} \frac{d^3p}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \right.$$

c)



$$-\frac{e^2}{\epsilon} \iint \frac{F_\gamma(p) \Phi_{1s}(p + Q' - \frac{Q}{2}) F_{\gamma'}^*(p') \Phi_{2s}^*(p + \frac{Q}{2})}{(p - p' + \frac{\Delta Q}{2})^2} \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3}$$

$$\approx \left[ \frac{e^2}{\epsilon} \iint \frac{F_\gamma(p) \Phi_{1s}(p) \Phi_{2s}^*(p) F_{\gamma'}^*(p')}{(p - p')^2} \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \right.$$



$$-\frac{e^2}{\epsilon} \iint \frac{F_\gamma(p) \Phi_{1s}(p + \frac{Q}{2}) F_{\gamma'}^*(p') \Phi_{2s}^*(p' - \frac{Q'}{2})}{(p - p' + \frac{\Delta Q}{2})^2} \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3}$$

$$\approx \left[ \frac{e^2}{\epsilon} \iint \frac{F_\gamma(p) \Phi_{1s}(p) \Phi_{2s}^*(p) F_{\gamma'}^*(p')}{(p - p')^2} \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \right.$$

$$\overline{\Pi}_{exen} = \frac{e^2 a_0^2}{\epsilon} I_{exen}$$

$$I_{exen} = \iint \frac{\tilde{F}_\gamma(\pi) \tilde{\Phi}_{1s}(\pi') \tilde{F}_{\gamma'}^*(\pi') \tilde{\Phi}_{2s}^*(\pi')}{(\pi - \pi')^2} \frac{d^3\pi}{(2\pi)^3} \frac{d^3\pi'}{(2\pi)^3}$$

$$- \iint \frac{\tilde{F}_\gamma(\pi) \tilde{\Phi}_{1s}(\pi) \tilde{F}_{\gamma'}^*(\pi') \tilde{\Phi}_{2s}^*(\pi')}{(\pi - \pi')^2} \frac{d^3\pi}{(2\pi)^3} \frac{d^3\pi'}{(2\pi)^3}$$

$$- \iint \frac{\tilde{F}_\gamma(\pi) \tilde{\Phi}_{1s}(\pi') \tilde{F}_{\gamma'}^*(\pi') \tilde{\Phi}_{2s}^*(\pi')}{(\pi - \pi')^2} \frac{d^3\pi}{(2\pi)^3} \frac{d^3\pi'}{(2\pi)^3}$$



# Free Exciton Recombination with the Excitation of Donor Electron

State vector of free electron-hole pair with momenta

$$\vec{p}_e = \frac{\vec{p}}{2} + \vec{p}$$

$$\vec{p}_h = \frac{\vec{p}}{2} + \vec{p}$$

$$|\vec{p}_e, \vec{p}_h\rangle \equiv |\vec{P}, \vec{p}\rangle =$$

$$e_{\frac{\vec{p}}{2} + \vec{p}}^+ h_{\frac{\vec{p}}{2} - \vec{p}}^+ |0\rangle$$

Normalization condition:

$$\langle \vec{p}_e, \vec{p}_h | \vec{p}_e, \vec{p}_h \rangle = \langle \vec{P}, \vec{p} | \vec{P}, \vec{p} \rangle =$$

$$\delta_{\vec{p}_e \vec{p}_e} \delta_{\vec{p}_h \vec{p}_h} = \delta_{\vec{P} \vec{P}} \delta_{\vec{p} \vec{p}}$$

State vector of exciton

$$|\vec{P}, v\rangle = \sum_{\vec{p}} f_v(\vec{p}) |\vec{P}, \vec{p}\rangle$$

①

$$\omega = 1 \text{ eV} \Rightarrow 1.5 \cdot 10^{15} \text{ sec}^{-1}$$

$$W_0 = \underbrace{2 \cdot 10^{-5}}_{\frac{P^2}{m}} \cdot \underbrace{2 \cdot 10^{-3}}_{\frac{d_0}{\pi}} \cdot \underbrace{4 \cdot 10^6}_{2^{22}} \cdot \underbrace{(10^{-18} \text{ cm}^3)}_{a_{ex}^3} \cdot \underbrace{1.5 \cdot 10^{15} \text{ sec}^{-1}}_{\omega} I^2$$

$$= 2.4 \cdot 10^{-9} \cdot 10^6 \cdot 10^{-18} \cdot 10^{15} \cdot I^2$$

$$= 2 \cdot 10^{-4} \text{ cm}^3 \text{ sec}^{-1} \cdot I^2$$

Probability on an unite of volume:

$$W = N W_0 \quad N = 10^{12} \text{ cm}^{-3}$$

$$W = 2 \cdot 10^8 \text{ sec}^{-1} I^2$$

Hanoi April 19, 1979

Dear Claude

I am sorry that the manuscript was in our embassy too long. I received it only recently. During this time I have finished together with my post-graduated students all calculations. Today I send to you our work to your consideration. I suggest the following.

- 1) You can include completely our works with An, Thang, Viet and write a complete article with six co-authors.
- 2) You can take any results from this theoretical work, any part you like, for improving ~~our~~<sup>my</sup> work with you and Ulbricht. In this case I would like to ask you and George to submit our work to Journal de Physique.

Classification  
Physics Abstracts  
71.35 — 71.36 — 78.20B

## Polariton theory of resonant electronic Raman scattering on neutral donor levels in semiconductors with a direct band gap

Nguyen Ba An, Nguyen Van Hieu, Nguyen Toan Thang and Nguyen Ai Viet

Institute of Physics, National Center for Scientific Research of Vietnam, Nghia Do, Tu Liem, Hanoi, Vietnam

(Reçu le 18 janvier 1980, révisé le 21 avril, accepté le 9 mai 1980)

**Résumé.** — Dans ce travail nous présentons la théorie de la diffusion de Raman sur les électrons liés aux donneurs neutres dans les semiconducteurs avec une bande interdite directe, quand la fréquence du photon incident est en résonance avec l'exciton. La section efficace de diffusion du polariton avec transition de l'électron de l'état fondamental au premier état excité a été calculée.

**Abstract.** — The theory is presented for the Raman scattering on bound electrons of neutral donors in the semiconductors with a direct band gap at the incident photon frequency in the resonance with the exciton. The cross-section of the polariton scattering with the transition of the donor electron from the ground state to the first excited state is calculated.

**1. Introduction.** — Different electronic Raman scattering processes in semiconductors were investigated both theoretically and experimentally by many authors. In the case of the scattering on a gas of mobile carriers (electrons or holes) there may be the scattering with the single particle excitation or the scattering on collective excitations in this gas. The scattering processes of this type were studied widely by Platzman, Tzoar, Wolff *et al.* [1-13]. The interband scattering processes, in which the initial and final states of electrons or holes belong to different energy bands, were studied in many works [14-17]. The initial and final states of the charge carriers may be also their bound states in the Coulomb field of the donor or acceptor ions. In this case we have the scattering on the impurity levels [18-27]. The role of different external electromagnetic fields on the electronic Raman scattering processes was also studied in details [28-31]. Recently Weisbuch and Ulbricht have investigated experimentally the resonant electronic Raman scattering on neutral donor levels at the energy of the incoming photon near that of the exciton [32]. The present work is devoted to the theoretical investigation of the resonant electronic Raman scattering on neutral donor levels in a two band semiconductor with the direct band gap and the allowed electrical dipole transition from the valence to the conduction band. In the case of the material with a very low concentration of donors the electron-hole pair can exist in their bound states — the excitons.

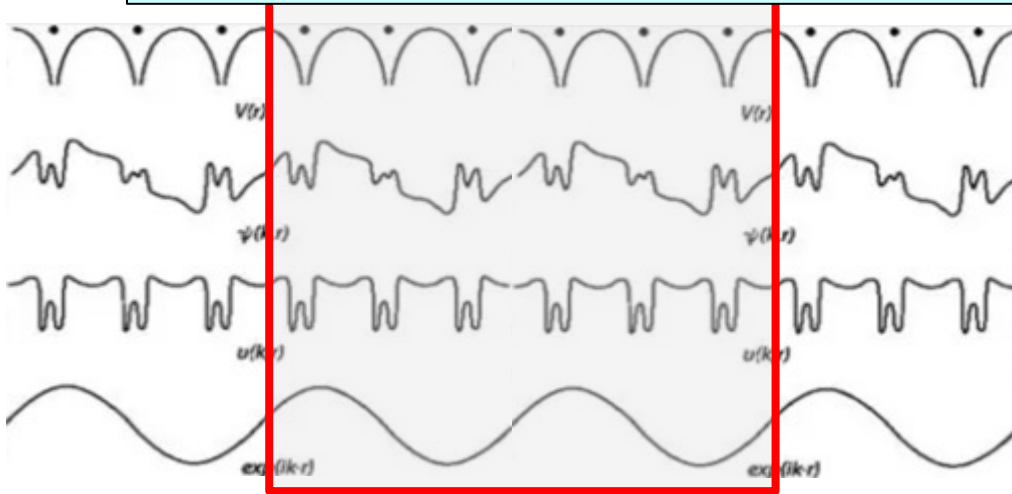
Due to the virtual transition between the photons and the excitons there arises in the semiconductor a new kind of elementary excitations — the polaritons or more precisely the excitonic polaritons discovered by Hopfield [33] and Pekar [34] and studied further in many works [35-42]. As in the case of the resonant Raman scattering, at the photon energy near that of the exciton the polariton effect becomes very important. The polariton theory of the resonant Raman scattering was developed by Mills, Burstein *et al.* [43] and also by Bendow and Birman [44].

In the present work we apply the method of Bendow and Birman [44] to the study of the resonant electronic Raman scattering on the neutral donor levels at the energy of the incoming or outgoing photon near that of an exciton. We assume that the semiconductor has a direct band gap with the allowed electrical dipole transition and consider only the case of the material with a high purity, when the donor concentration is very small. In this case there will be no screening of the Coulomb interaction between the electrons and the holes, the excitons will exist, and therefore the polariton effect must be taken into account. We shall use the unit system with  $\hbar = c = 1$ .

**2. Electromagnetic interaction and polariton scattering.** — The existence of the polariton and all the scattering processes considered in this work are the consequences of the electromagnetic interaction in the semiconductor. We begin our study by discussing the



# Two-dimensional electronic systems: quantum wells



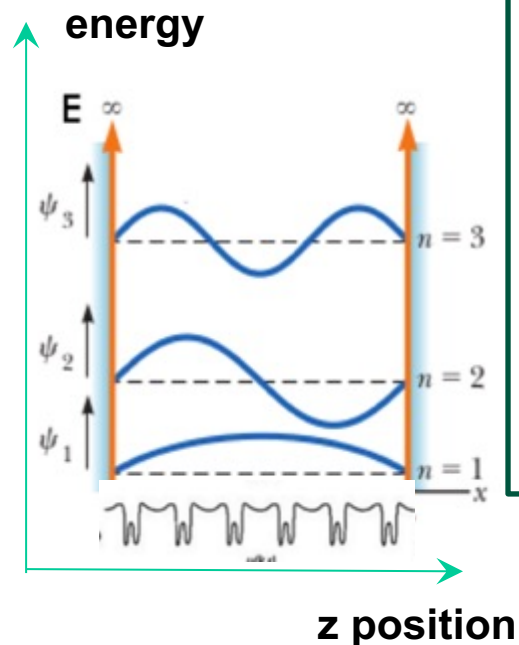
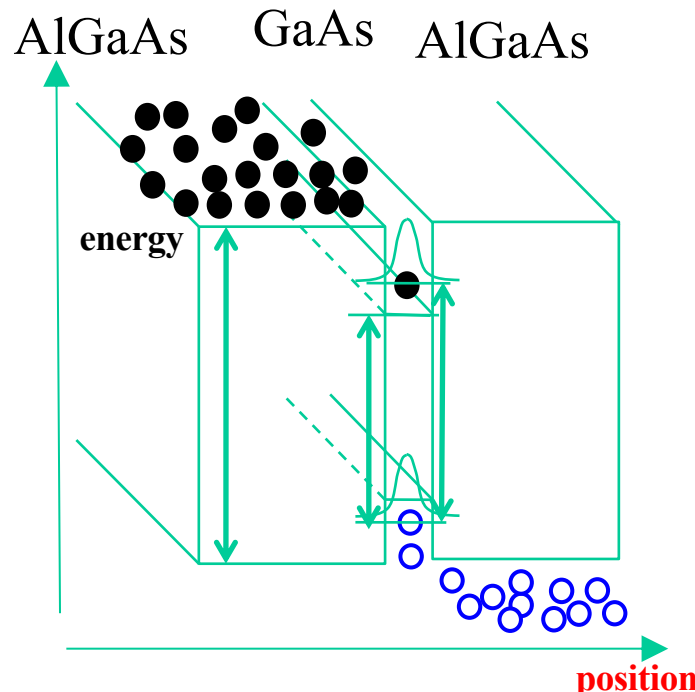
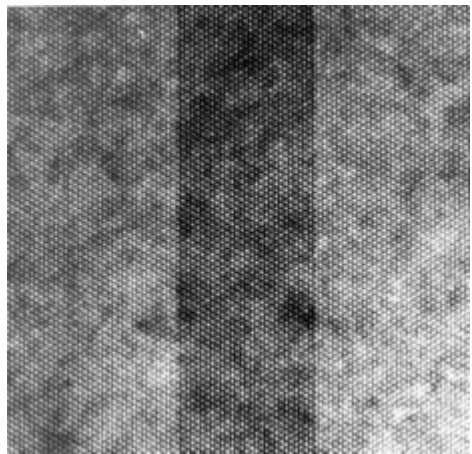
Potential ion potential

Bloch wavefunction

Periodic part of the  
Bloch wavefunction

Envelope wavefunction

22 atomic layers



Puits quantique infini:

$$\Phi(\mathbf{r}) = e^{i(k_x x + k_y y)} \Phi_n(z)$$

$$-\frac{\hbar^2}{2m} \frac{d^2 \Phi}{dx^2} = E \Phi$$

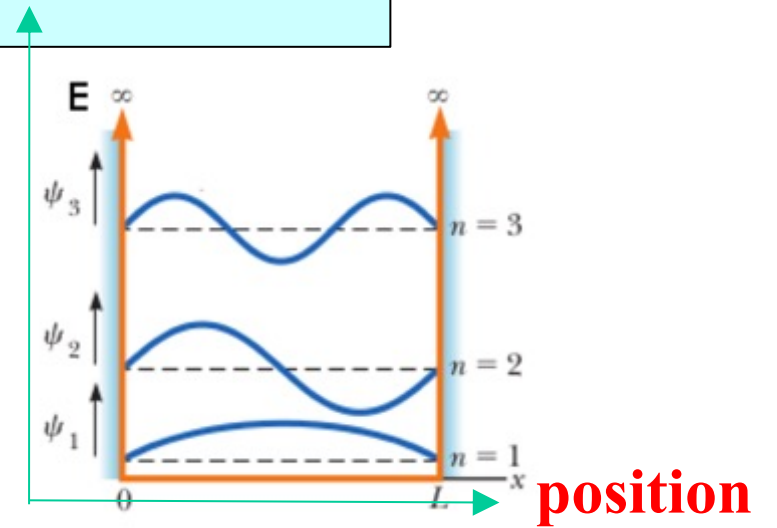
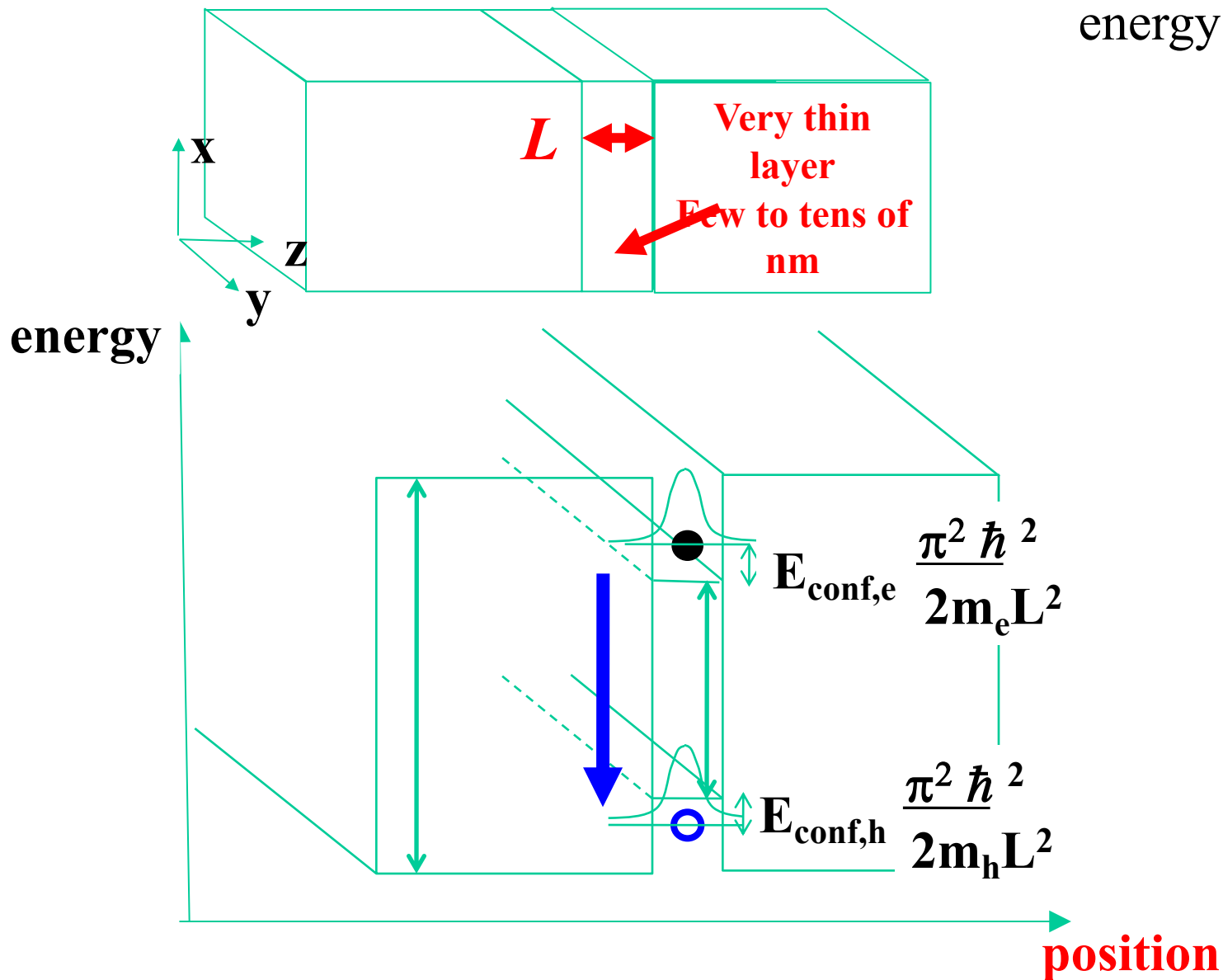
$m$  electron effective mass

$$\Phi_n(x) = \sqrt{\frac{2}{\pi}} \sin\left(\frac{n\pi x}{L}\right)$$

$$E_n = \frac{\pi^2 \hbar^2}{2mL^2} n^2$$

$$\Psi_n(\mathbf{r}) = \Phi_n(\mathbf{r}) w_0(\mathbf{r})$$

# Two-dimensional electronic systems: quantum wells



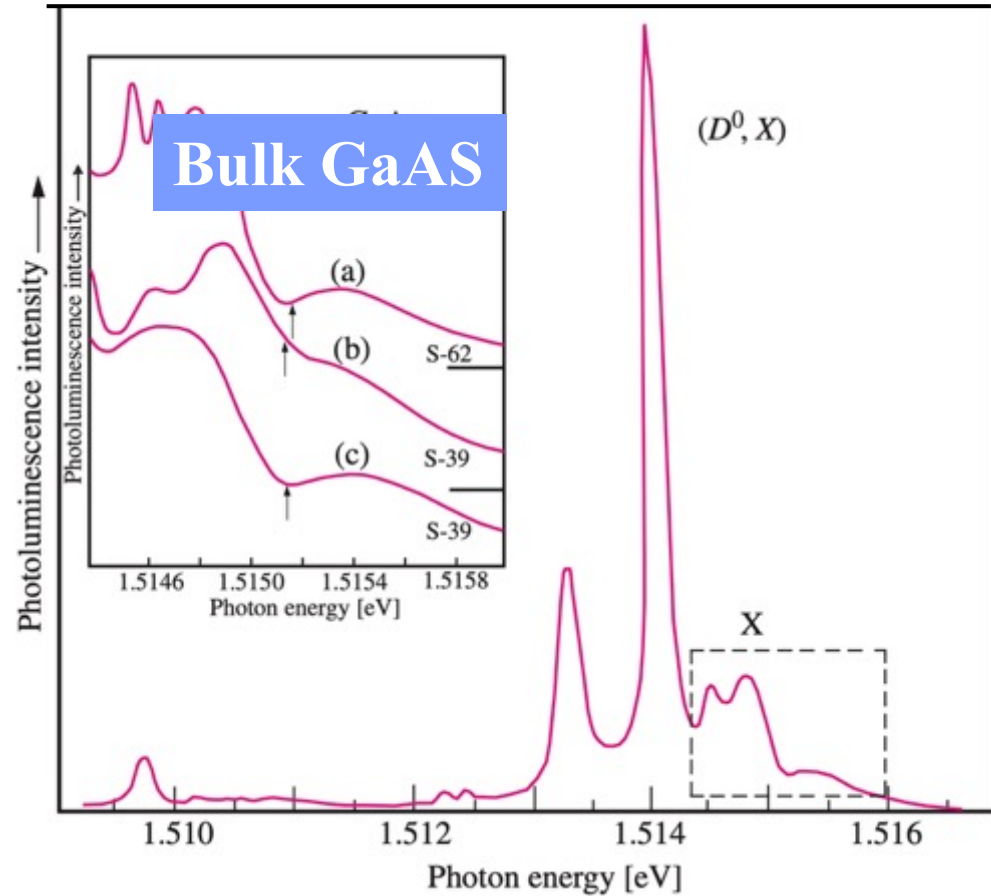
Infinite well approximation

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) = E \cdot \psi(x)$$

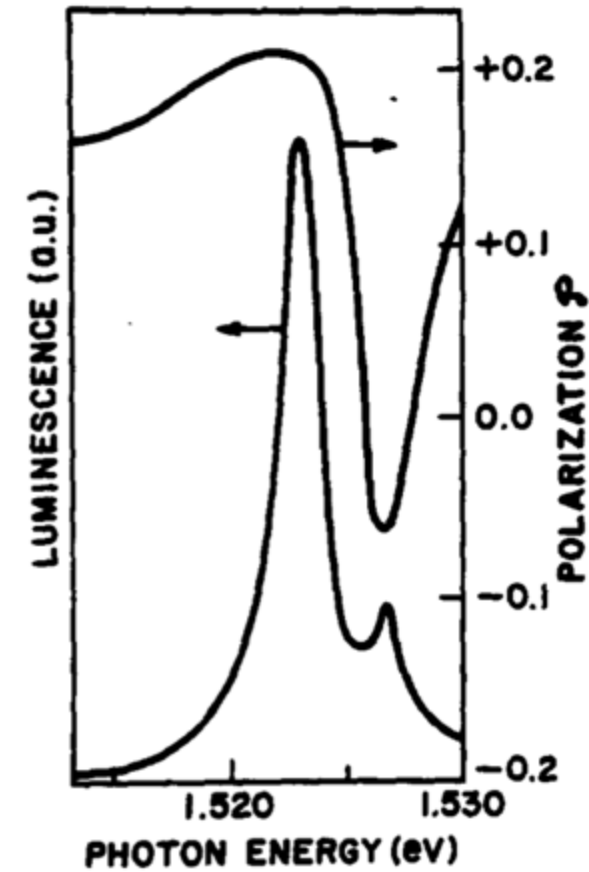
$$\psi_n(x) = \sin\left(\frac{n\pi}{L}x\right)$$

$$E(n) = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

**Surprise: in quantum wells, free excitons dominate light emission !**

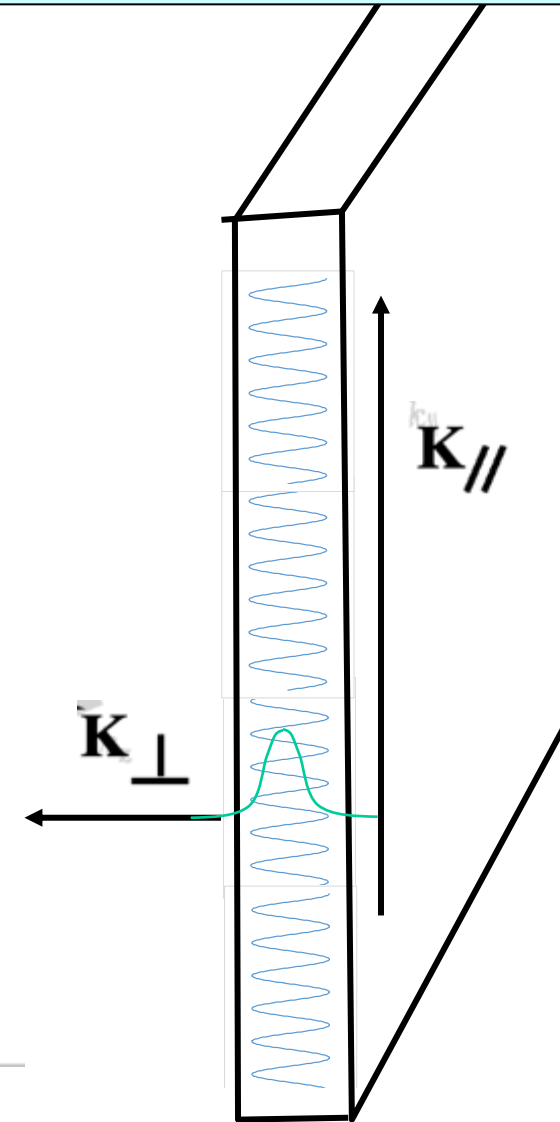
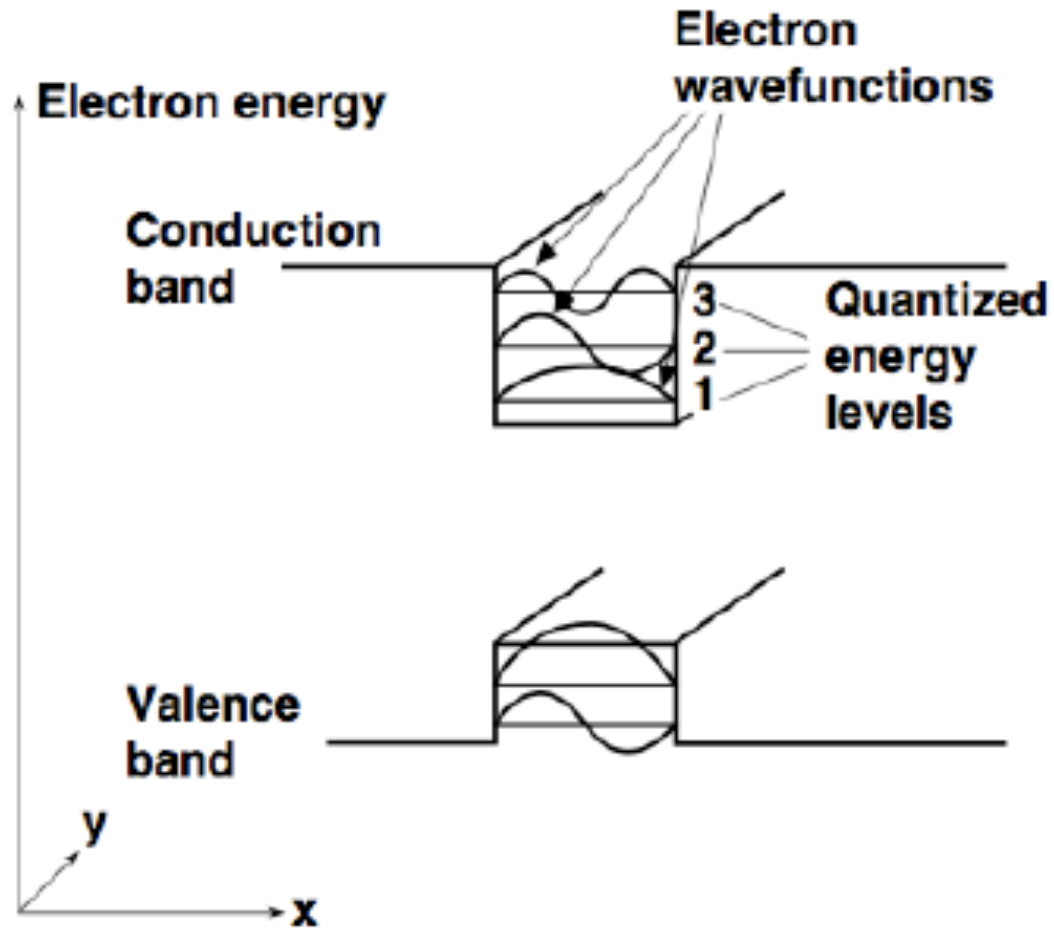


**GaAS QW**

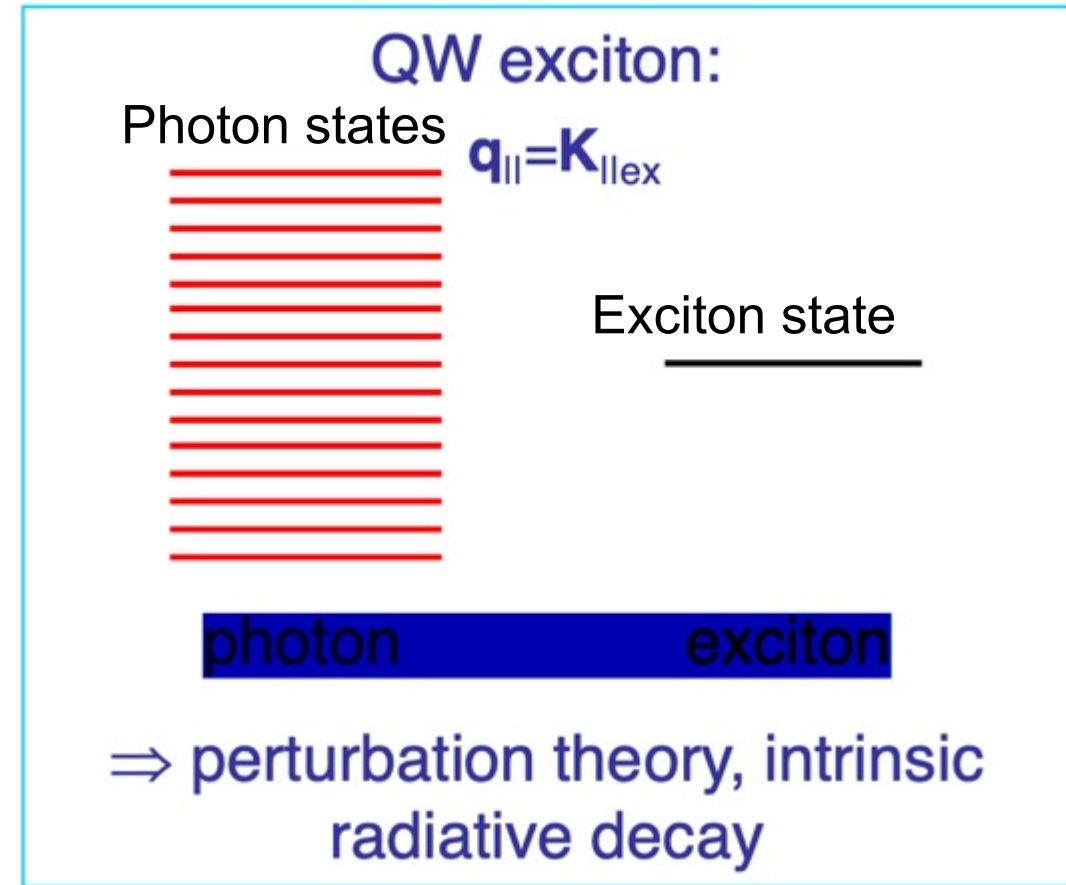
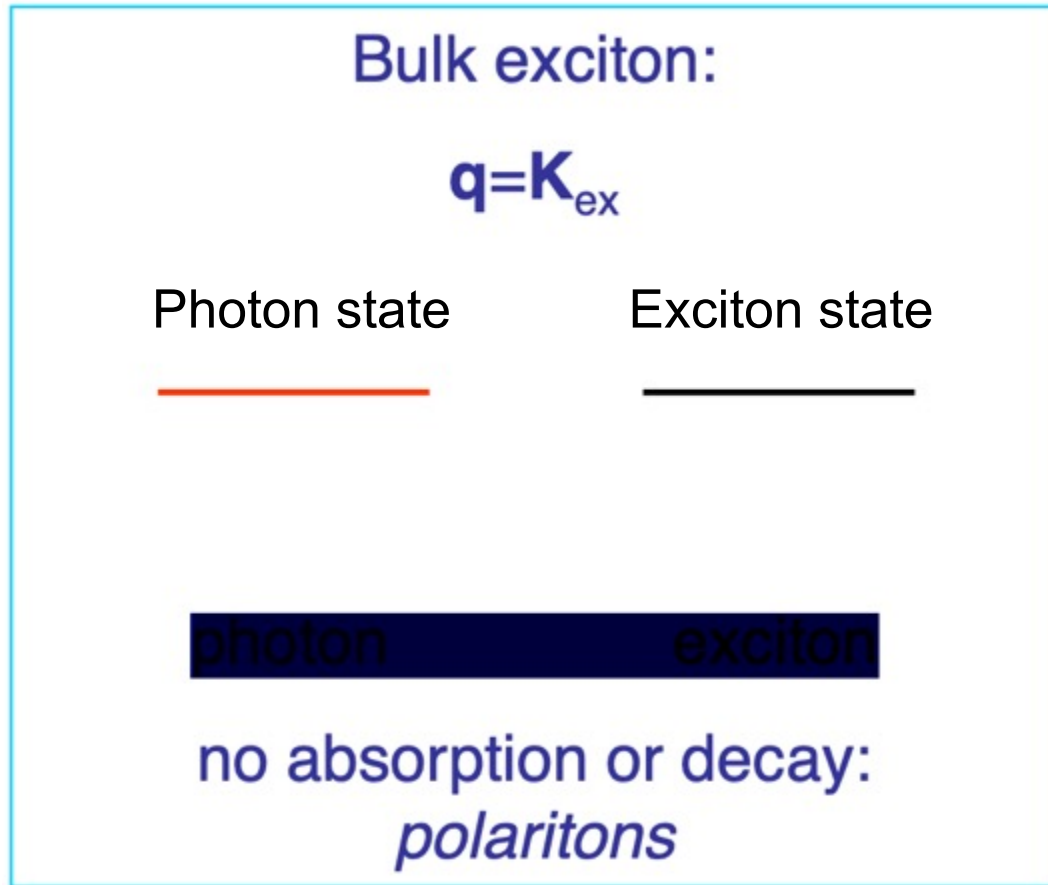


Weisbuch et al., Intrinsic Radiative Recombination From Quantum States in GaAs-AlGaAs Multi-Quantum Well Structures, Solid State Commun. 37, 219 (1981)

**In quantum wells recombination, no conservation of momentum  
Excitons are coupled to a continuum of states**

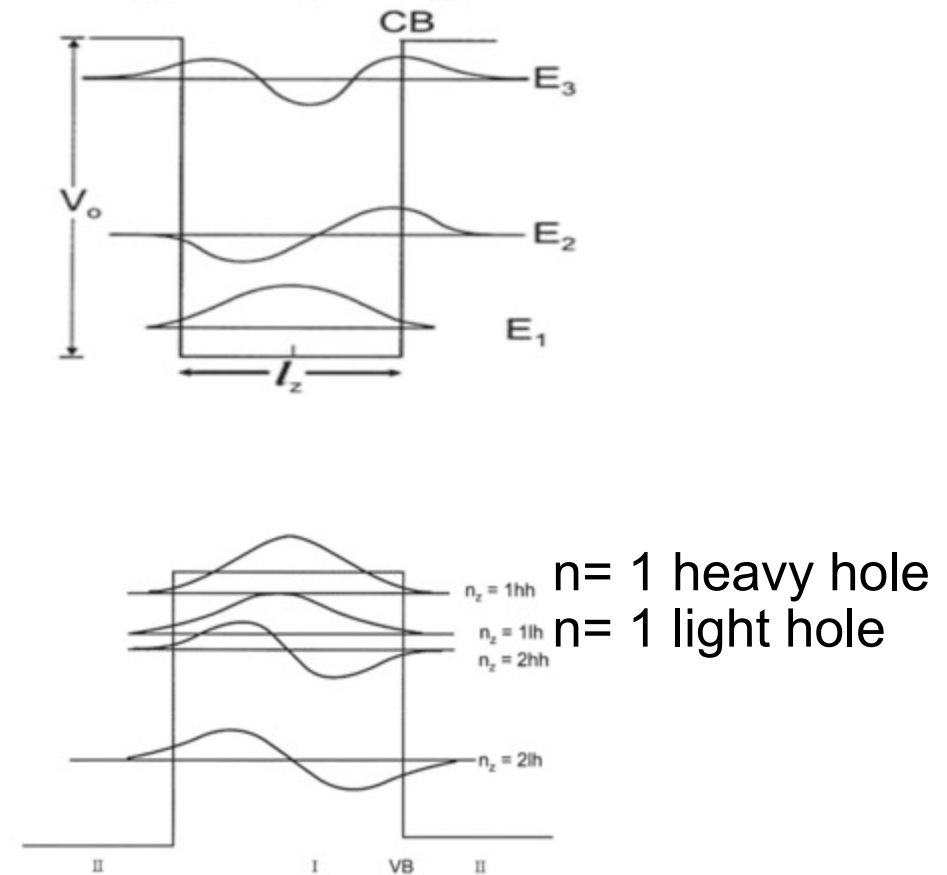
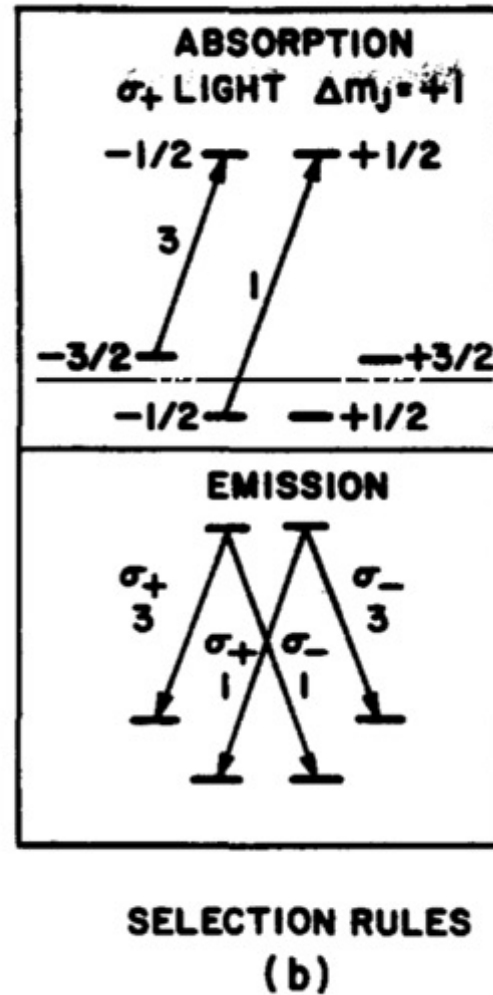
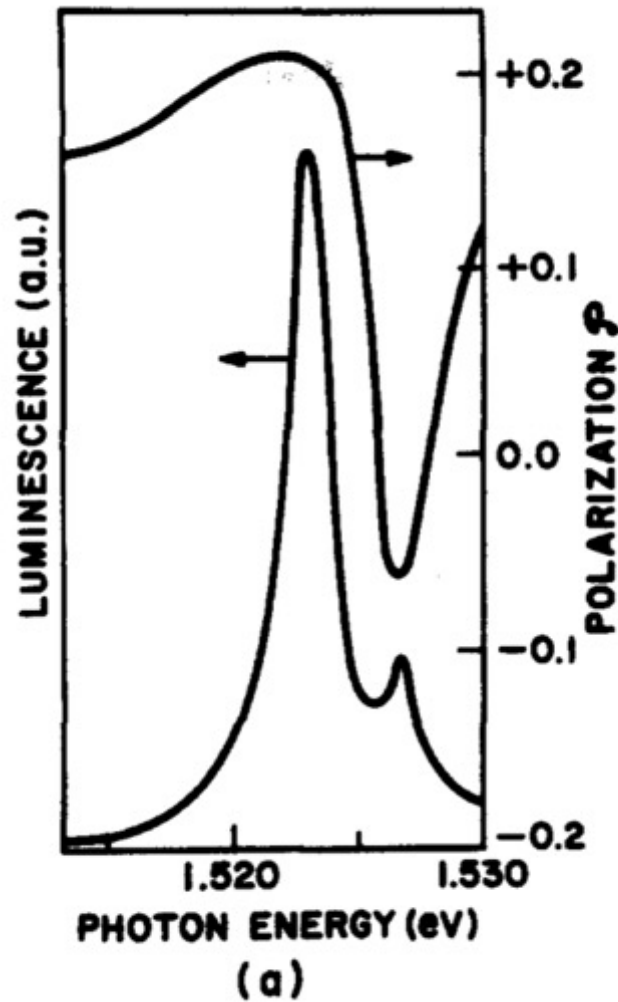


# Intrinsic radiative decay of free QW excitons



A QW exciton with given in-plane wavevector  $\mathbf{K}_{\parallel \text{ex}}$  interacts with a continuum of photons with wavevectors  $\mathbf{q} = (\mathbf{q}_{\parallel} = \mathbf{K}_{\parallel \text{ex}}, \mathbf{q}_z)$  perturbative regime with irreversible decay *intrinsic radiative recombination mechanism*.

# Intrinsic radiative decay of free QW excitons



High efficiency  
For all semiconductors, efficiency much higher for QWs than in bulk 3D material

Weisbuch et al., Intrinsic Radiative Recombination From Quantum States in GaAs-AlGaAs Multi-Quantum Well Structures, Solid State Commun. 37, 219 (1981)



# Intrinsic radiative decay of free QW excitons explained: very short radiative lifetime measured

Radiative properties of free excitons in a single GaAs quantum well are studied under resonant excitation. Enhanced radiative recombination of the excitons, caused by the breakdown of the translational symmetry of the system, is evidenced by the very short lifetime as well as by the strong intensity of the signal. Dephasing mechanisms, by transferring the excitons to nonradiative states, increase the observed lifetime. We deduce a radiative lifetime of 104 ps in the absence of dephasing mechanisms.

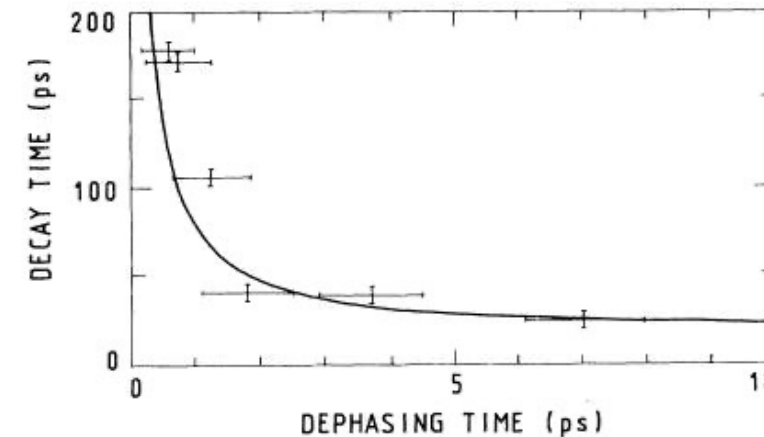
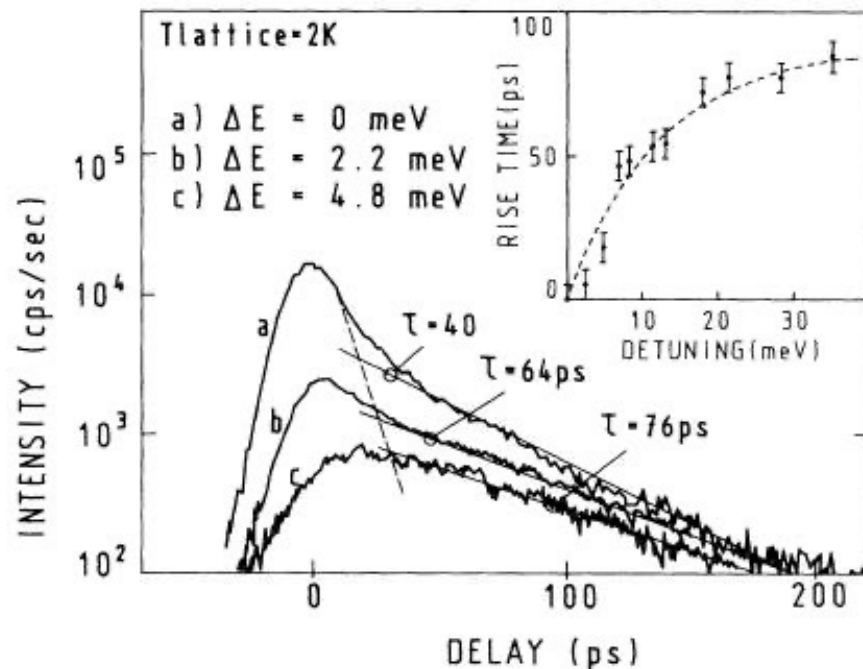
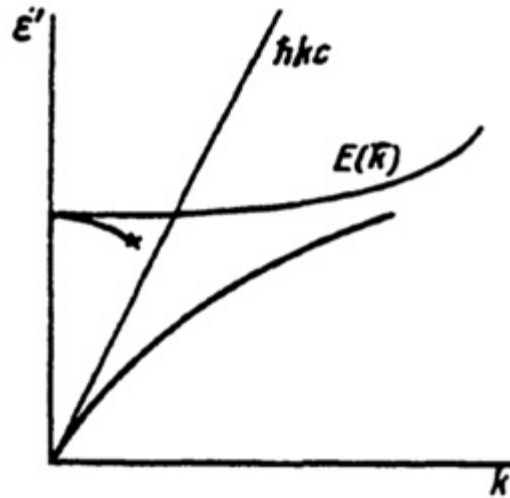


FIG. 3. Measured exciton lifetime, under resonant excitation, as a function of the dephasing time (deduced from fits similar to those of Fig. 2). The fit corresponds to Eq. (3), with  $\tau_0 = 8$  ps and  $\tau' = 8$  ps, and  $\tau_{\text{un}} = 1000$  ps.

## Intrinsic radiative decay of 2D excitons : very short radiative lifetime predicted in 1962 !

“Unlike three-dimensional crystals, allowance for retarded interaction in one- and two-dimensional periodic structures leads to several singularities in the exciton spectra ....”

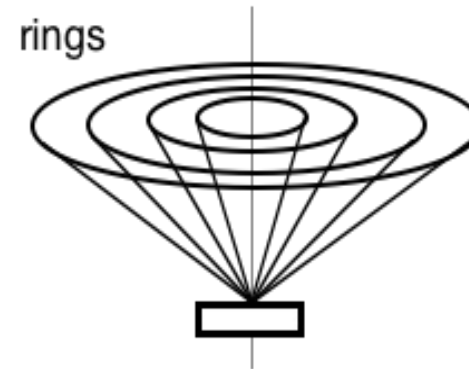
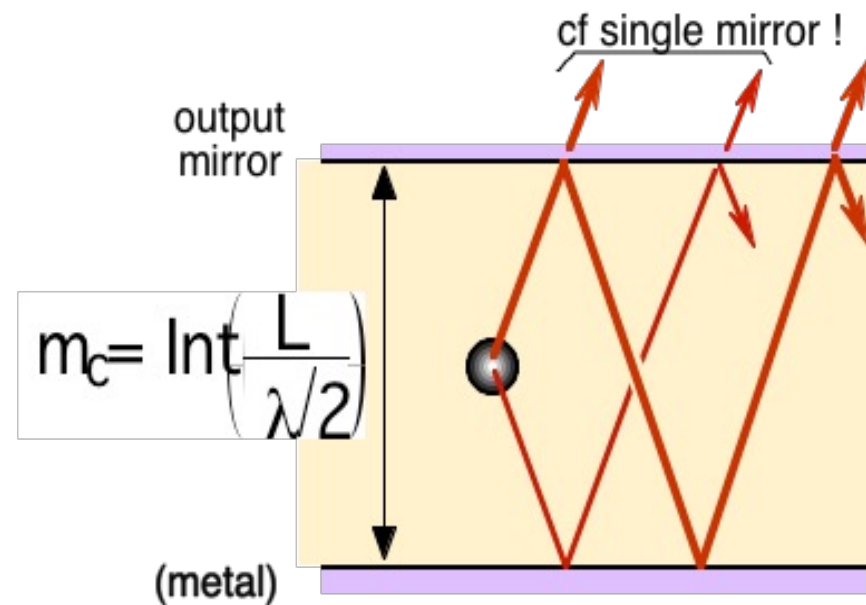


“The excitons should have in such crystals a lifetime  $\sim 10^{-13}$  -  $10^{-15}$  sec, which can be discerned from the luminescence damping time, from the absorption and emission line shapes, etc. ”

Agranovich and O. A. Dubovskii, Effect Of Retarded Interaction on the Exciton Spectrum In One-Dimensional and Two-Dimensional Crystals, JETP Lett. 3, 796, 1962

# Atomes a l'Interieur d'un Interferometre Perot-Fabry

## Microcavity emission: atoms in a Fabry-Pérot resonator



For a point source :

( Fabry-Perot interf.)  $\times$  (2-beam interf)

$$|E|^2 = |E_o|^2 \times \frac{T_1}{|1 - r_1 r_2 e^{2i\phi}|^2} \times |1 + r_2 e^{2i\phi'}|^2 = |E_o|^2 \times \frac{T_1}{|1 - r_1 r_2 e^{2i\phi}|^2} \times 2\zeta(z, \theta)$$

Exaltation or Inhibition  
due to the modal structure of the  
whole cavity

Factor from 0 to 4 depending on  
the source location with respect to  
the mode antinodes at the  
considered angles

## Microcavities : Weak Coupling Régime

Spontaneous and stimulated emissions are NOT intrinsic properties of an electronic state. They also depends of the electromagnetic field they are coupled to.

Discrete state coupled to a continuum of final states

Fermi's golden rule:

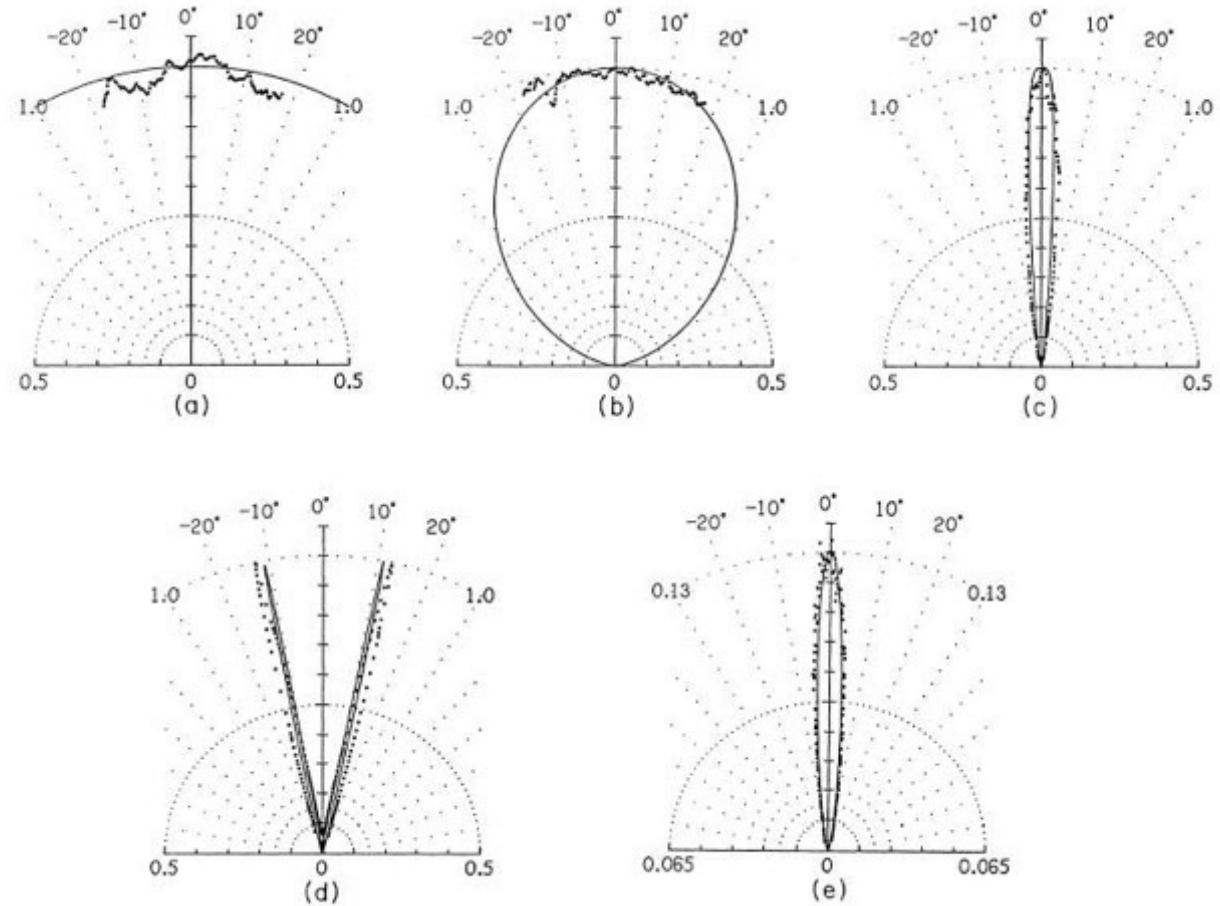
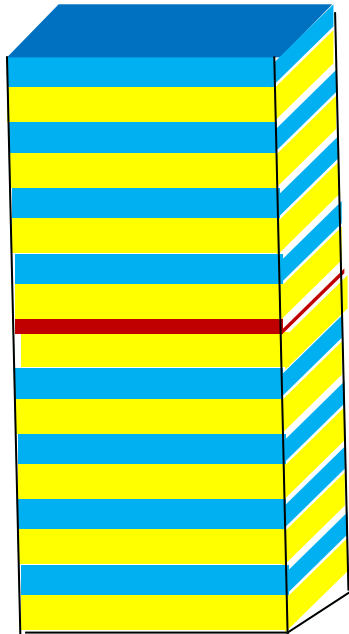
Perturbation theory, transition rate:

$$\Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} \left| \langle f | H' | i \rangle \right|^2 \rho(E_f)$$

density of photon modes

Microcavities provide the means to modify the "natural" spontaneous **emission direction**. Changing emission efficiency and **rate (Purcell effect)** is more difficult.

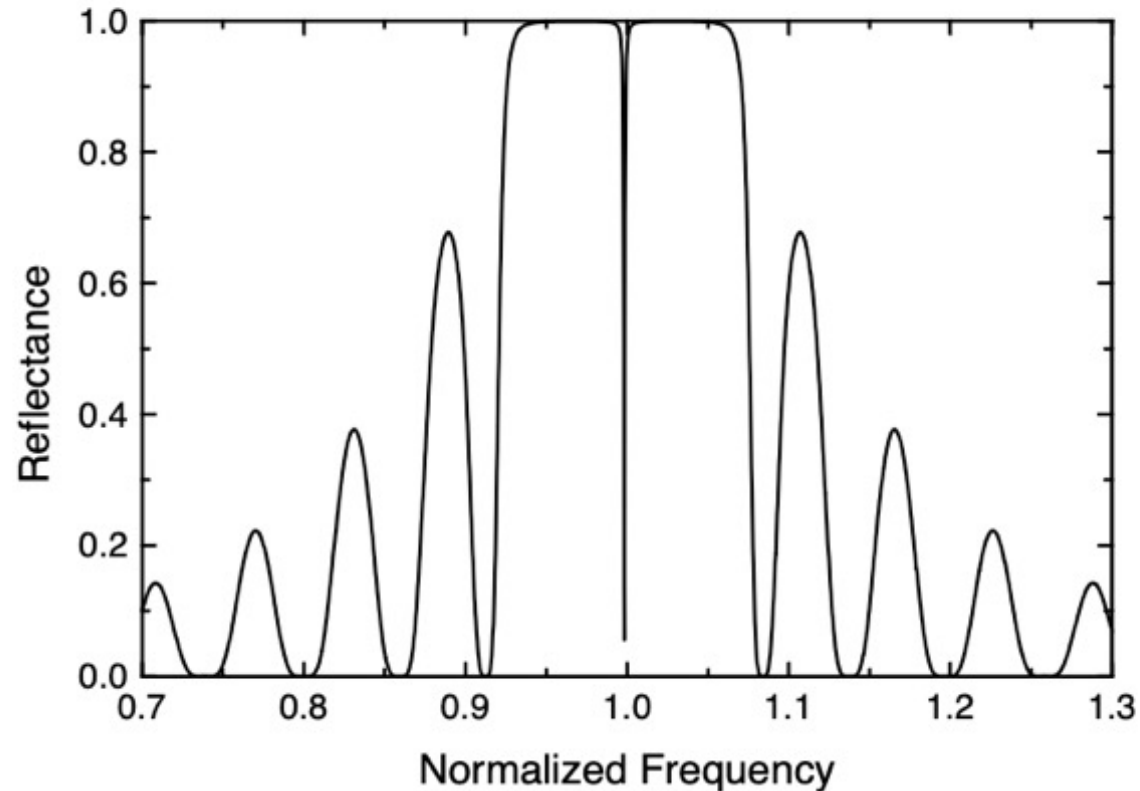
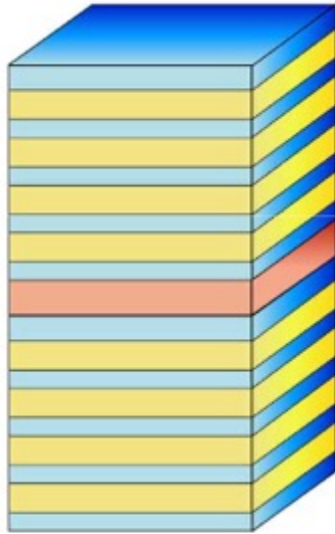
# Semiconductor microcavities in weak coupling



Experimental and theoretical radiation patterns from the GaAs quantum well. (a) a thick Al Ga 0.8 As layer (s wave); (b) a thick Al<sub>0.2</sub>Ga<sub>0.8</sub>As layer (p-wave); (c)  $\lambda$  cavity ( $\lambda_{em} = \lambda_{cav} = 800\text{nm}$ ); (d)  $\lambda$  cavity ( $\lambda_{em} = 800\text{ nm}$   $\lambda_{cav} = 815\text{nm}$ ); (e)  $\lambda$  cavity ( $\lambda_{em} = 800\text{ nm}$   $\lambda_{cav} = 790\text{nm}$ )

# Planar dielectric microcavity: Fabry-Pérot and leaky modes

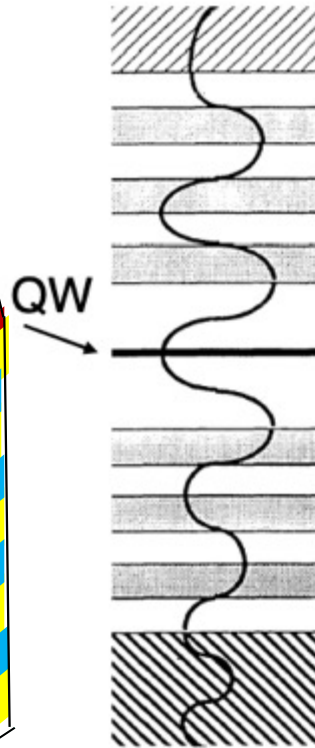
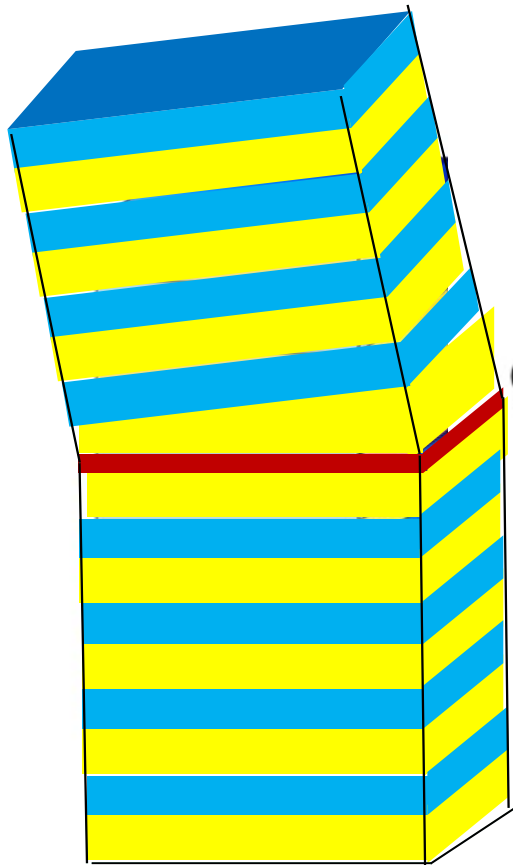
15 pairs GaAs/AlAs Distributed Bragg Reflector ( $n_1=3$ ,  $n_2=3.6$ ), normal incidence



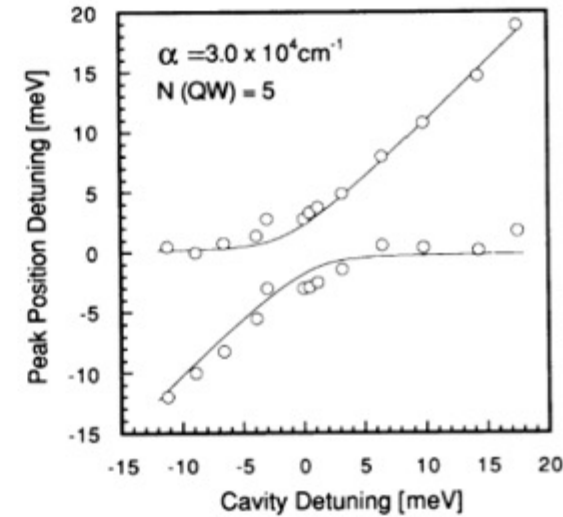
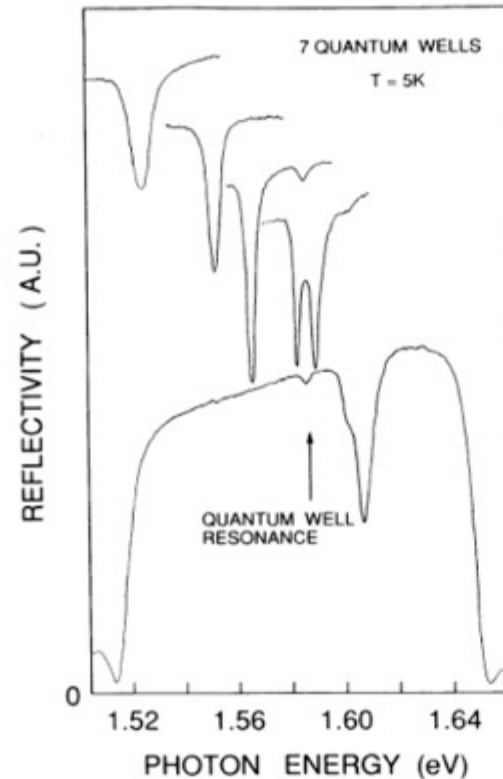
The Fabry-Pérot mode is localized within (or close to) the cavity region  
→ **quasi-2D photon states**



# Strong coupling regime in semiconductor microcavities: cavity polaritons



Cavity with 7 quantum wells:  
splitting at resonance



The coupling between cavity mode and exciton gives rise to a vacuum-field Rabi splitting ( $\sim 5$  meV) at resonance and to mixed modes:

***cavity polaritons***



Weisbuch, Arakawa et al., Observation of the Coupled Exciton-Photon Mode Splitting in a Semiconductor Quantum Microcavity, Phys. Rev. Lett. 69, 3314, 1992

“This effect can be seen as the Rabi vacuum-field splitting of the quantum-well excitons, or more classically as the normal-mode splitting of coupled oscillators, the excitons and the electromagnetic field of the microcavity. An exciton oscillator strength of  $4 \cdot 10^{12} \text{ cm}^{-2}$  is deduced for 76-A quantum wells”.

**A very disturbing paper –**

## **“Vacuum Rabi Splitting as a Feature of Linear Dispersion Theory: Analysis and Experimental Observations”**

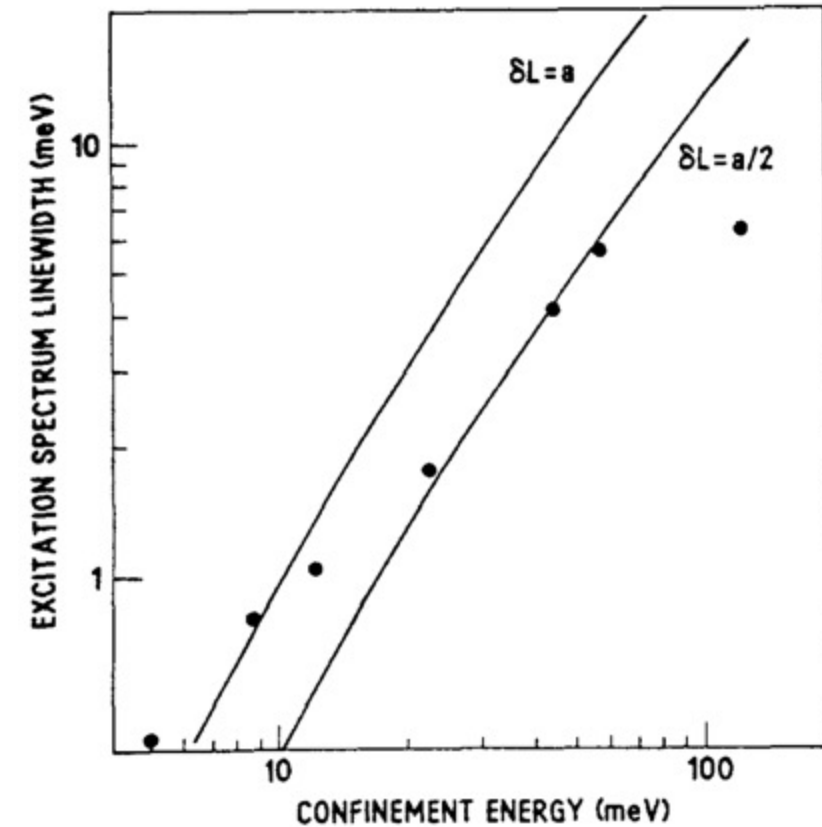
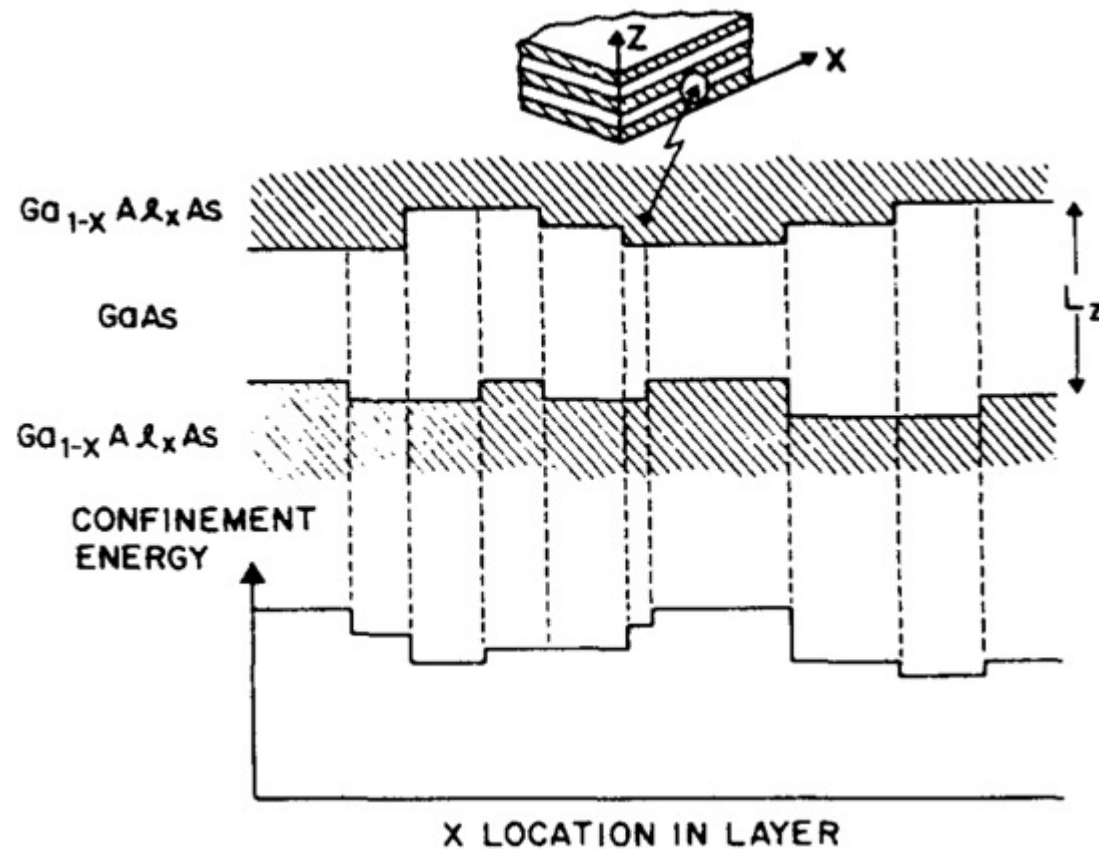
The spectral and temporal response of an optical cavity resonantly coupled to an ensemble of barium atoms has been investigated experimentally. The empty-cavity transmission resonances are found to split in the presence of the atoms and, under these conditions, the cavity's temporal response is found to be oscillatory. **These effects may be viewed as a manifestation of a vacuum-field Rabi splitting, or as simple consequence of the linear absorption and dispersion of the intracavity atom**

In summary, we have shown that the steady-state and transient transmission characteristics of a cavity coupled to a collection of atoms can be understood using an entirely classical, steady-state multibeam interference analysis of an optical cavity. In particular, it is found that the empty-cavity resonances can be split due to the effects of linear absorption and dispersion and that this mode splitting is identical to the vacuum-field Rabi splitting predicted by a fully quantum electrodynamical formalism. **This of course implies that the mere observation of oscillatory transmission and/or mode splitting is no more intrinsically quantum than the observation of linear absorption and dispersion.**

Yifu Zhu et al., Vacuum Rabi Splitting as a Feature of Linear-Dispersion Theory: Analysis and Experimental Observations, Phys. Rev. Lett. 54, 2499 (1990)

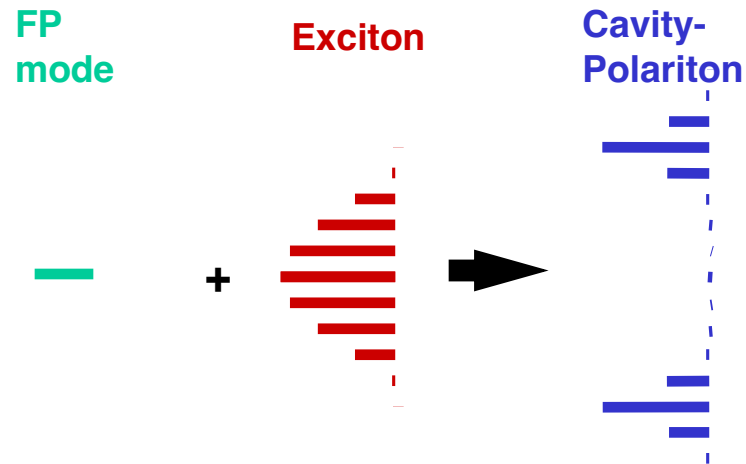


# An inhomogeneously broadened system: Interface fluctuations in quantum wells lead to position-dependent energy



C. Weisbuch et al., Optical characterization of interface disorder in  $\text{GaAs-Ga}_x\text{Al}_{1-x}\text{As}$  multi-quantum well structures, *Solid State Commun.* 38, 709-712 (1981)

# Strong coupling in an inhomogeneously broadened system

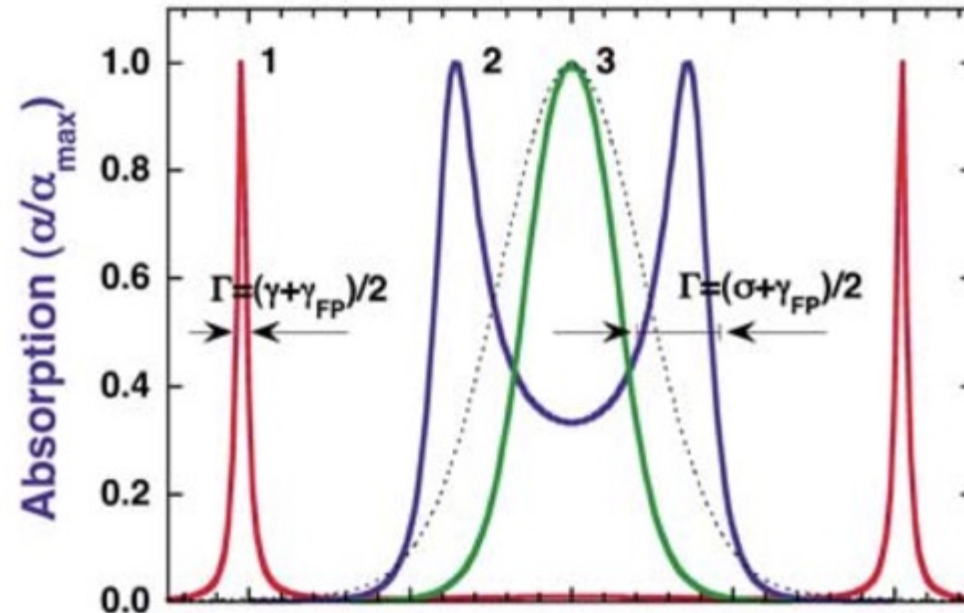


If the Rabi splitting is larger than inhomogeneous broadening

All exciton states are coupled together through **cavity photon exchange**  
Splitting is hardly perturbed

While QW inhomogeneous linewidth is  $>1$  meV, polariton linewidth can be as small as  $50 \mu\text{eV}$

**QW fluctuations are averaged** over polariton wavefunction, i.e. photon mode size,  $\approx$  tens of  $\mu\text{m}$ , vs exciton size,  $\approx$  ten nm.

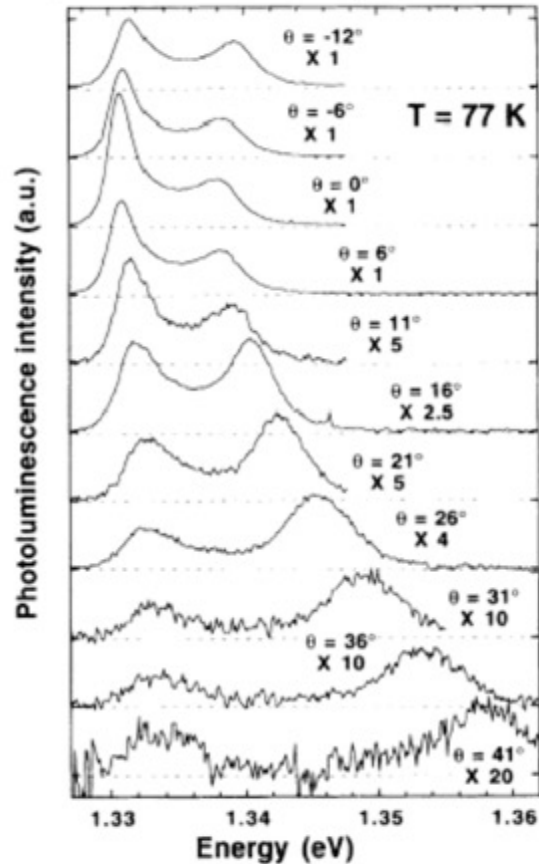


$\gamma_c/E_0 = 3 \cdot 10^{-5}$ ,  $\gamma/E_0 = 1 \cdot 10^{-4}$ ,  $\sigma/E_0 = 1 \cdot 10^{-3}$   
coupling strength are 50 (1), 5 (2) and 1 (3)

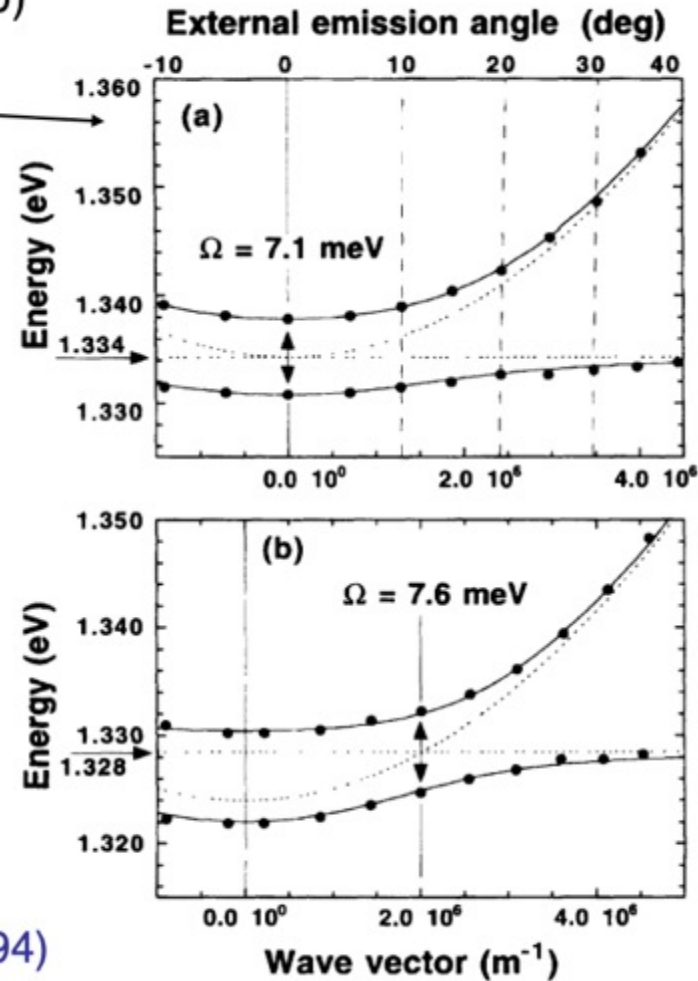
Houdré et al., Phys. Rev. A 53 (1996) 2711

# Angle-dependent photoluminescence delivers the polariton dispersion curve

Angle-resolved PL (resonance at  $\theta=0$ )



R. Houdré et al., PRL 73, 2043 (1994)



Romuald Houdré



Ursula Oesterle



Ross Stanley



# Evidence of strong coupling (vs cavity dielectric response)

## The detuning variation of emission

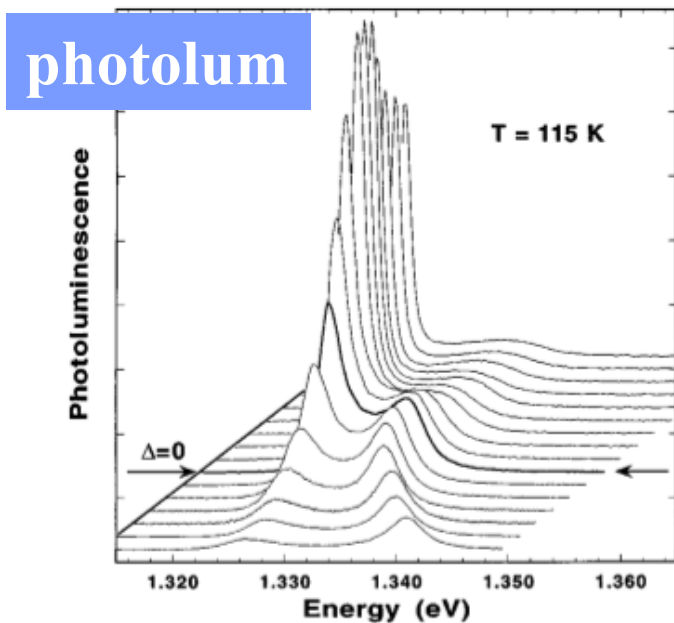


FIG. 5. Photoluminescence spectra for a range of FP-exciton detuning, ranging from  $-10 \text{ meV}$  to  $+12 \text{ meV}$ . The spectra have been shifted laterally, so that the unperturbed exciton PL (as seen in the edge PL) stays at the same energy ( $1.326 \text{ eV}$ ). The thicker line denotes the spectrum at zero detuning.

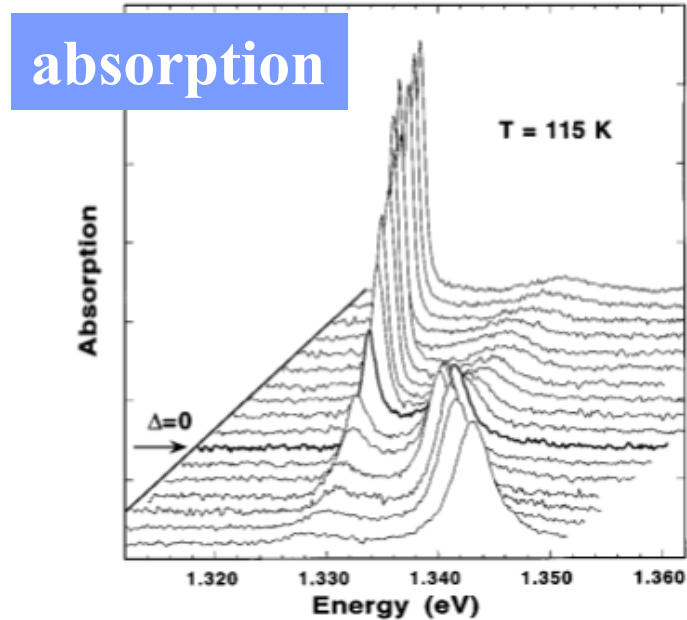


FIG. 6. Absorption spectra for a range of FP-exciton detuning, where the absorption is approximated by  $1 - \text{reflectivity}$ , ranging from  $-10 \text{ meV}$  to  $+12 \text{ meV}$ . The spectra have been shifted later-

R. Stanley et al.  
Phys. Rev. B 53, 10995 (1996)

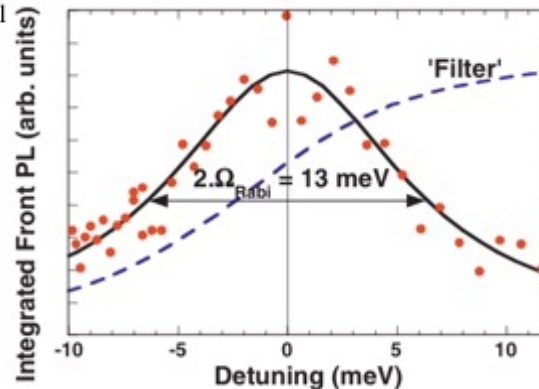


Fig. 11 (online colour at: [www.pss-b.com](http://www.pss-b.com)) Integrated front surface photoluminescence as a function of detuning (dots). The dashed line labeled 'filter' shows the asymmetry expected from the exciton in a Fabry-Pérot filter. Adapted from [38].



# Bose-Einstein condensation of cavity-polaritons

in a CdTe/CdMgTe microcavity with 16 quantum wells at 5 K / 19 K

## Parameters for BEC

| Parameter              | Atom      | Polariton                       |
|------------------------|-----------|---------------------------------|
| $m [m_e]$              | $10^4$    | $10^{-4}$                       |
| $T [K]$                | $10^{-6}$ | 20                              |
| $\lambda_{dB} [\mu m]$ | 0.7       | 3                               |
| $n_{cr} [cm^{-2}]$     | $10^8$    | $5 \cdot 10^8$                  |
| Life Time              | "long" s  | <b>"very short"</b> $10^{-12}s$ |

