Secondary Gravitational wave as a new window to the early universe

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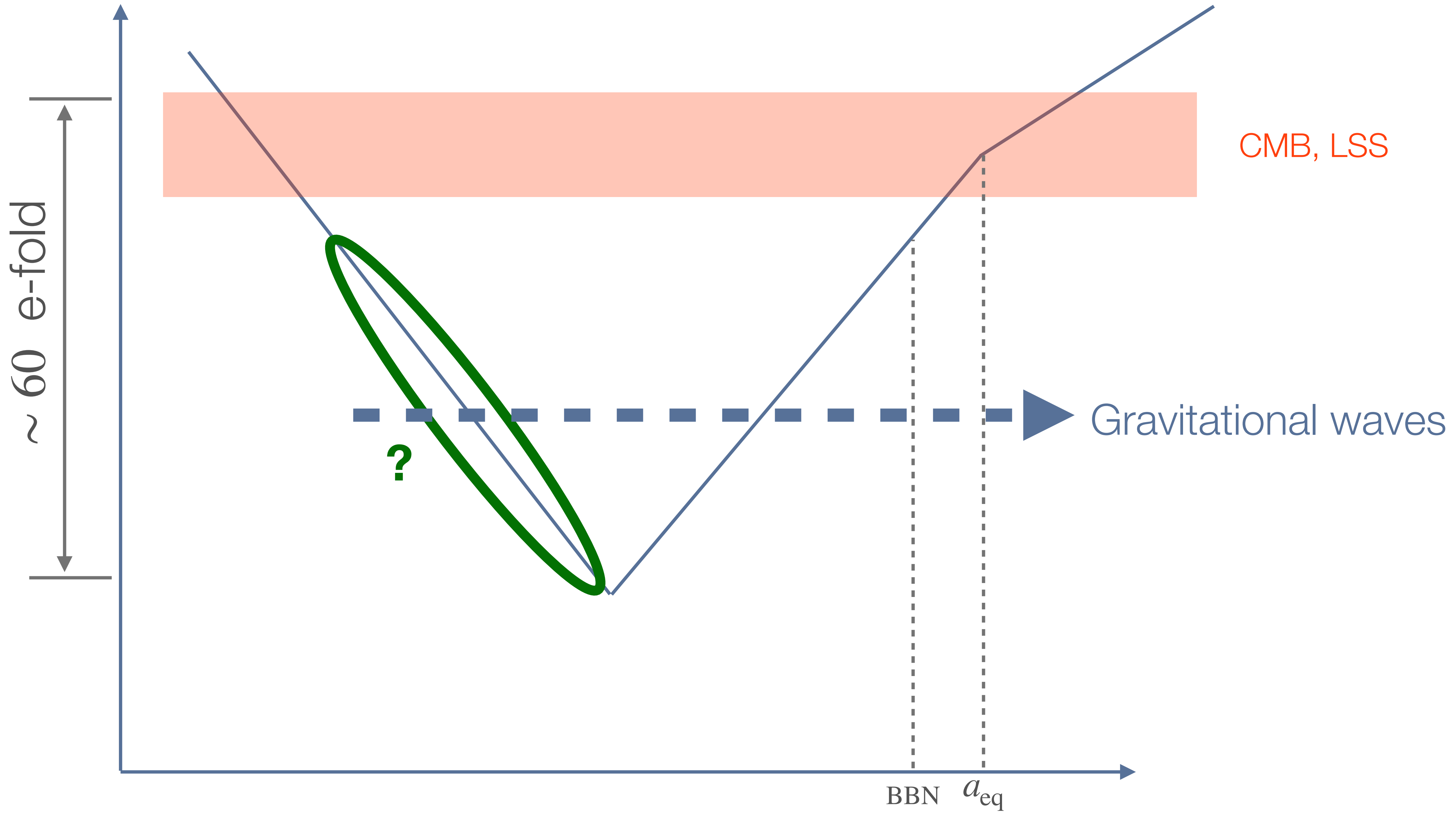
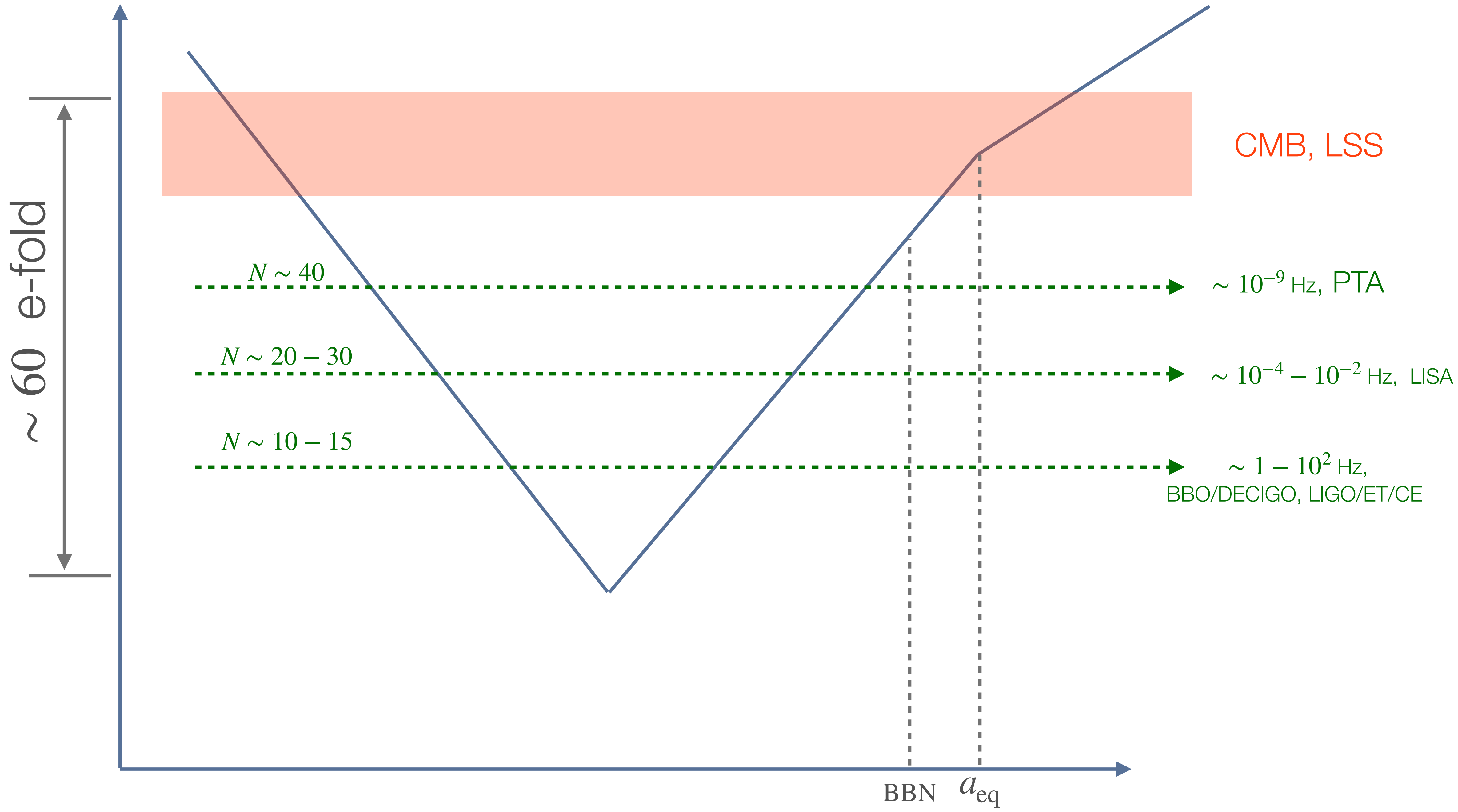
Work in collaboration with
Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, LTW, 2307.12048
Soubhik Kumar, Hanwen Tai, LTW, 2410.17291

21st Rencontres du Vietnam, Cosmology, ICISE, August 14 , 2025

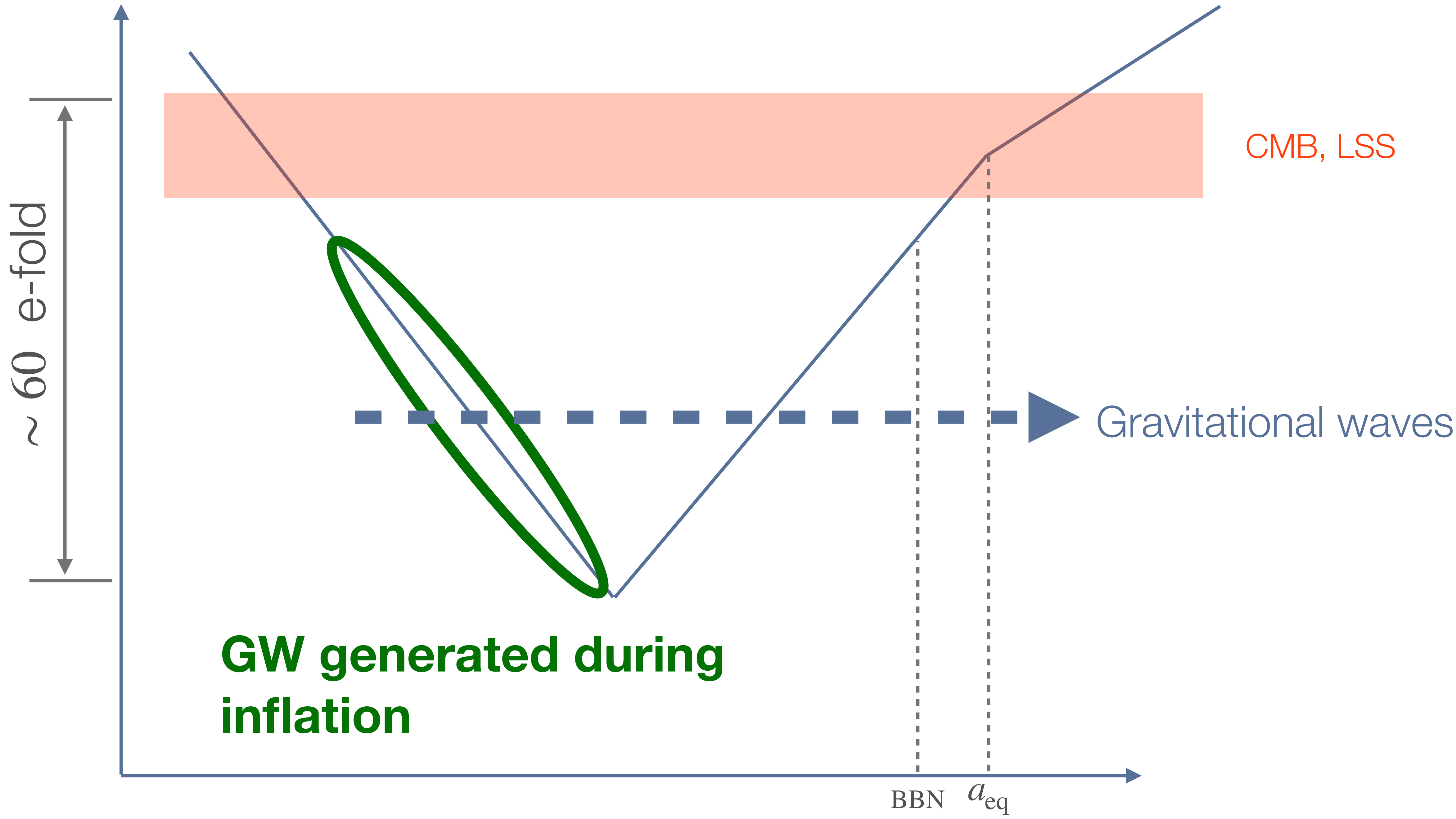
Inflationary era



Early universe

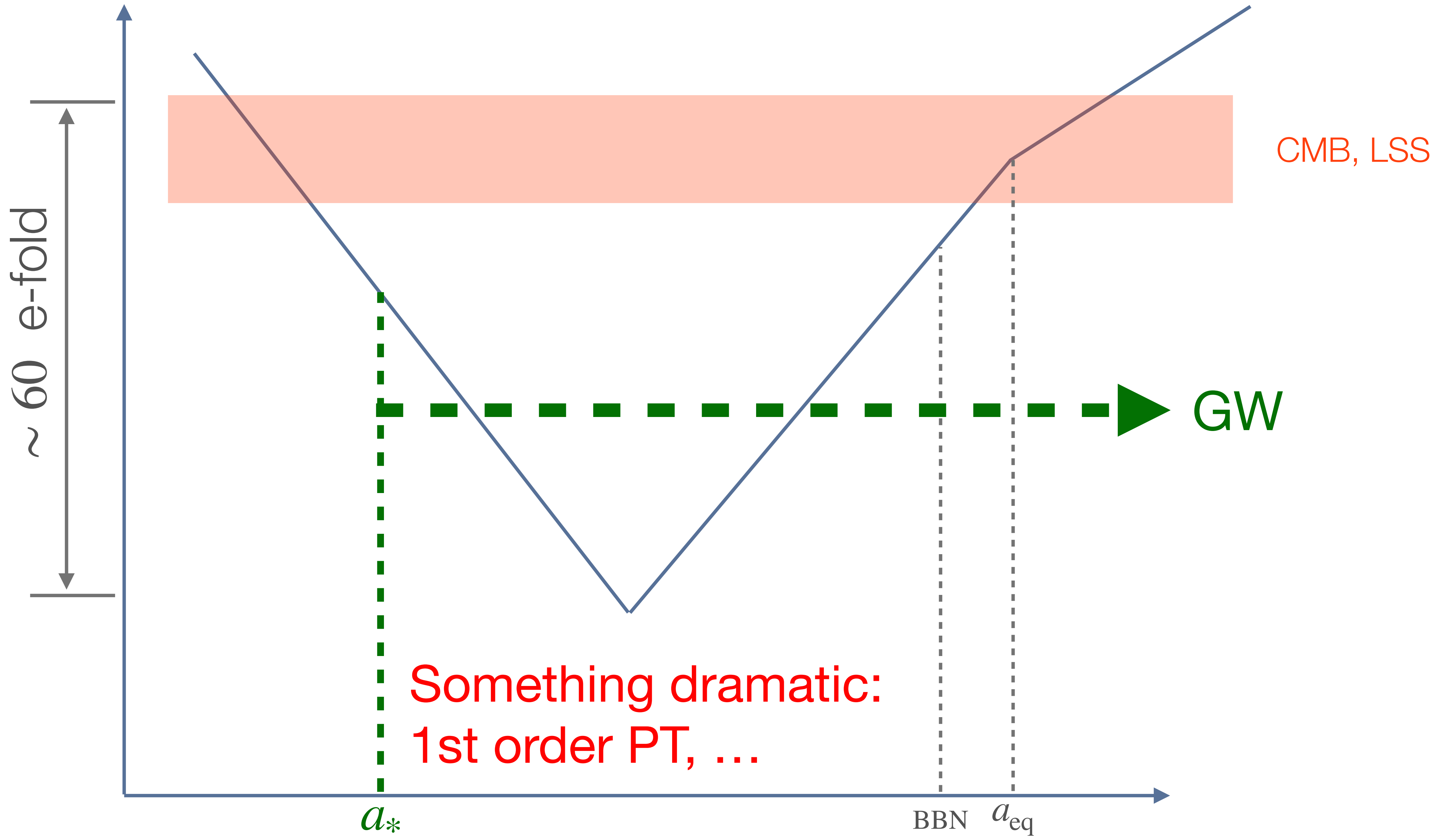


Primary GW

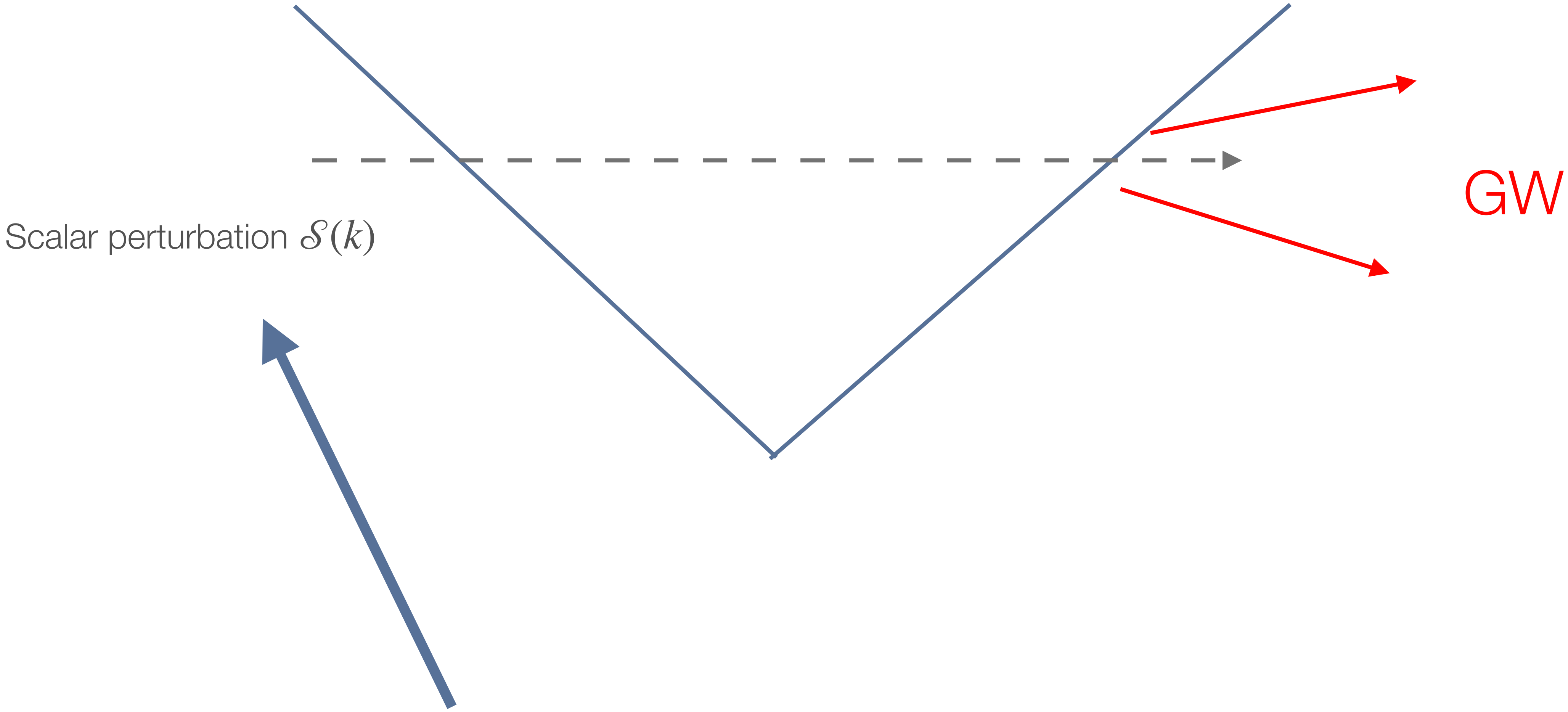


~ 60 e-fold
CMB, LSS
Gravitational waves
GW generated during inflation
BBN
a_{eq}

Early universe

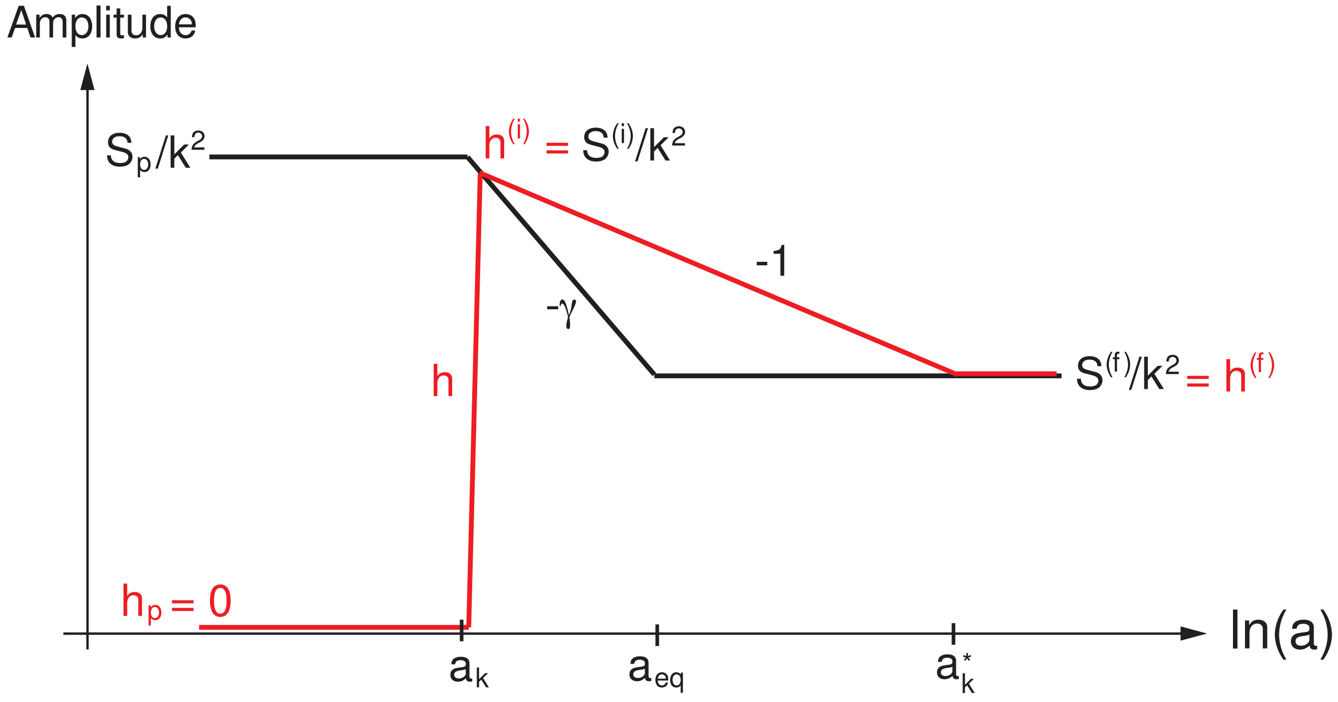


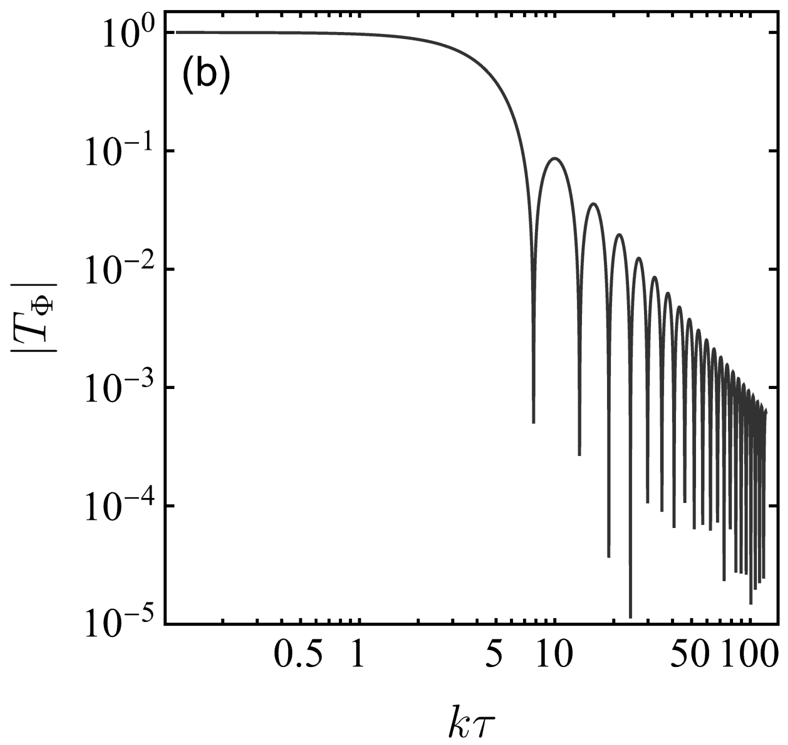
Secondary GW



In addition to the inflaton, many other fields have quantum fluctuations

Example: secondary GW





Baumann, Steinhardt, Takahashi, hep-th/0703290

Modes enter horizon during RD, starts oscillate, and generates GW

Size of the signal

Curvature perturbation $\mathbf{\Phi}$

$$\mathrm{d}s^2 = -\left(1+2\Phi\right)\mathrm{d}t^2 + a^2\left(\left(1-2\Phi\right)\delta_{ij} + \frac{1}{2}h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j$$

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Einstein equation:

$$h'' + 2Hh' + k^2 h = \boxed{\Phi\partial^2\Phi + \ldots}$$

Curvature source GW

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Gravitational wave abundance:

$$\Omega_{\rm gw} \propto (\dot{h})^2 \propto \Phi^4 \sim \Omega_{\rm rad}P_\zeta^2$$

On large (CMB, LSS) scales: $\Omega_{\rm rad} \sim 10^{-5}$, $P_\zeta \sim 10^{-9}$

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Clearly, to have observable signal, need much larger curvature perturbation on smaller scales (blue tilted).

Stories of blue spectrum

1. A spectator light scalar

R. Ebadi, S. Kumar, A. McCune, H. Tai, LTW 2023

$$\mathcal{L} = \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m^2\sigma^2 - \frac{\lambda}{4}\sigma^4$$

with $m < H$

Stochastic method

The spectrum of its fluctuation on large scales
can be studied by stochastic method

Starobinsky and Yokoyama, 1994

Fokker-Planck
$$\frac{\partial P_{\rm FP}(t,\sigma)}{\partial t} = \left(\frac{V''(\sigma)}{3H} + \frac{V'(\sigma)}{3H}\frac{\partial}{\partial \sigma} + \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial^2 \sigma} \right) P_{\rm FP}(t,\sigma)$$

$P_{\rm FP}(t,\sigma)$: 1-pt PDF for field σ

Starobinsky and Yokoyama, 1994; Markkanen, Rajantie, Stopyra, Tenkanen, 1904.11917

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Classical evolution, drift

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Classical evolution, drift   Stochastic, diffusion

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Classical evolution, drift — Stochastic, diffusion

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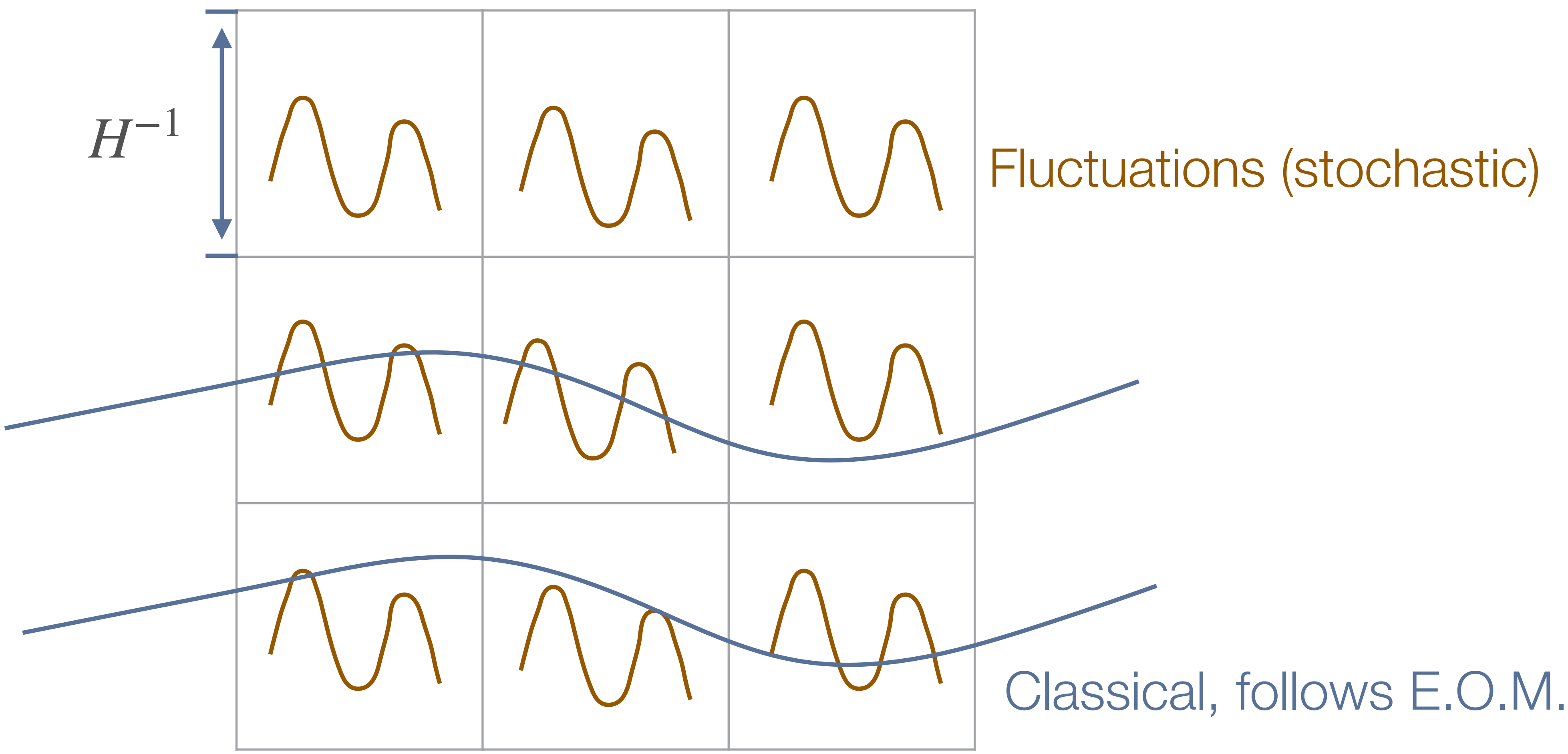
Evolution of fluctuations: small vs large scales



Light field during inflation

$$m_\sigma^2 < H^2$$

1. Massless. “Stuck” at large field value.
   - Example: misaligned axion.
2. Massive but light.

Light field during inflation

- Massive but light. (Free field for simplicity)

$$\dot{\sigma} = -\frac{m_\sigma^2 \sigma}{3H} \rightarrow \sigma = \exp\left(-m_\sigma^2 \int^t \frac{dt'}{3H(t')}\right) \cdot \sigma_i$$

Initial field value

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Initial field value

- Roughly, $-\int^t \frac{dt'}{3H(t')} \sim \frac{1}{\dot{H}}$

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Initial field value

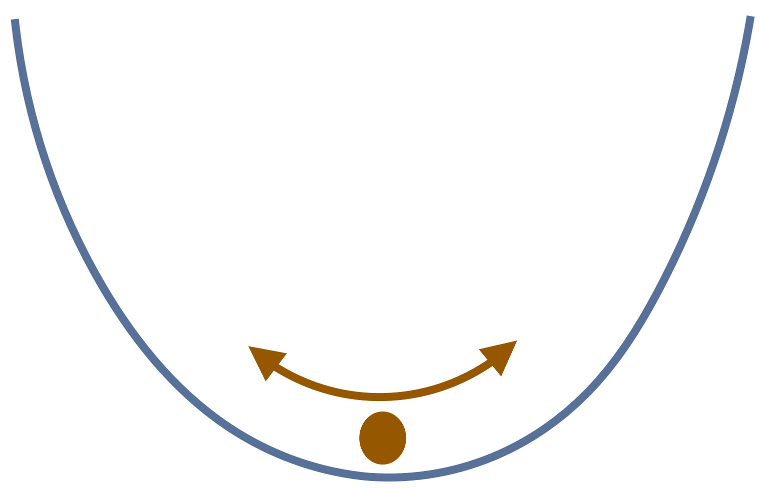
- Roughly, $-\int^t \frac{dt'}{3H(t')} \sim \frac{1}{\dot{H}}$

- If $m_\sigma^2 > \epsilon H^2$ $(\epsilon = \dot{H}/H^2)$,
  - Initial value of field does not matter. Amplitude of field dominated by stochastic fluctuation around origin

A spectator light scalar

R. Ebadi, S. Kumar, A. McCune, H. Tai, LTW 2023

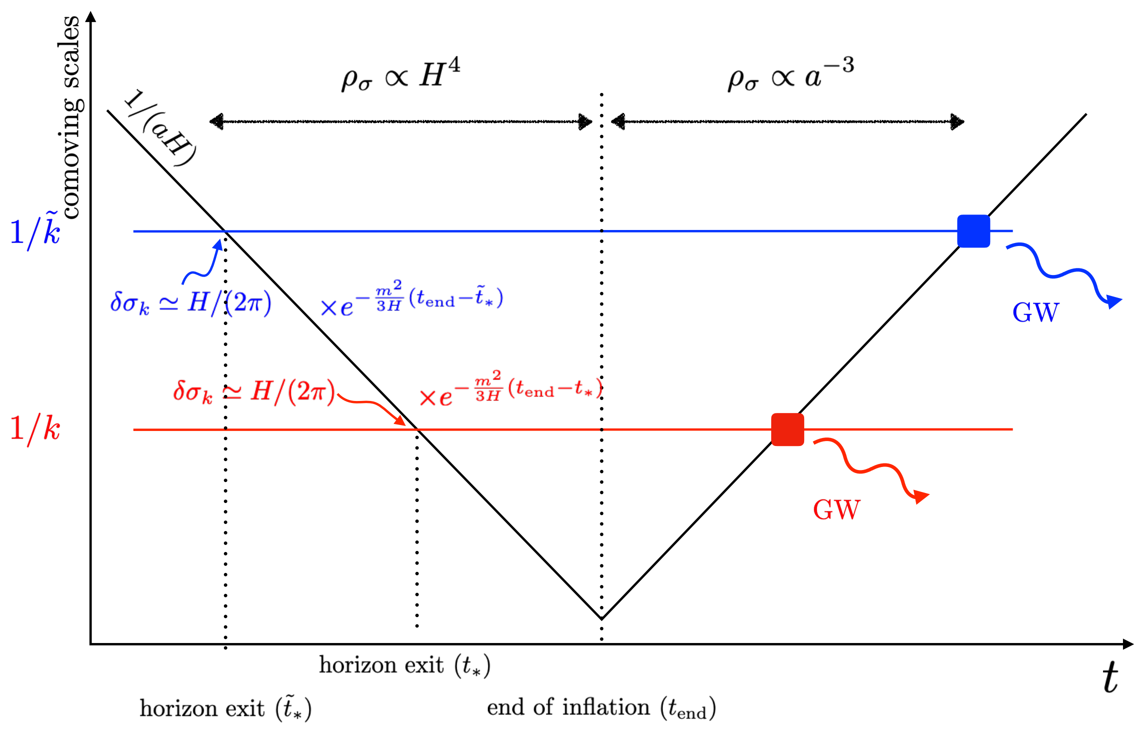
$$\mathcal{L} = \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m^2\sigma^2 - \frac{\lambda}{4}\sigma^4$$ with $m < H$



$$\mathcal{P}_f(k) = \sum_n \frac{2}{\pi} f_n^2 \Gamma\left(2 - 2\frac{\Lambda_n}{H}\right) \sin\left(\frac{\Lambda_n \pi}{H}\right)\left(\frac{k}{H}\right)^{2\Lambda_n/H} \quad \to \mathscr{A}\left(\frac{k}{H}\right)^{\frac{2\Lambda_{\text{lowest}}}{H}}$$ for $k \ll H$

Starobinsky and Yokoyama, 1994; Markkanen, Rajantie, Stopyra, Tenkanen, 1904.11917

Blue tilt

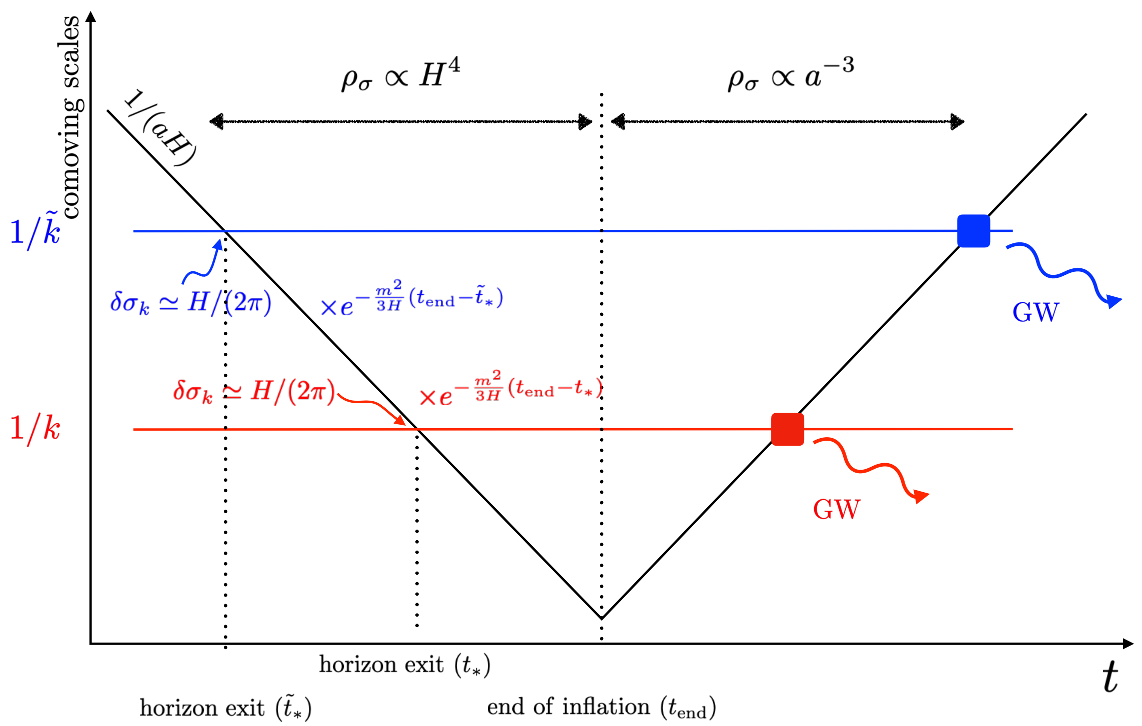


At horizon exit:
Amplitude ≈ H

After exit, damping

$$\dot{\sigma} = -\frac{m_\sigma^2 \sigma}{3H}$$

Blue tilt



At horizon exit:
Amplitude ≈ H

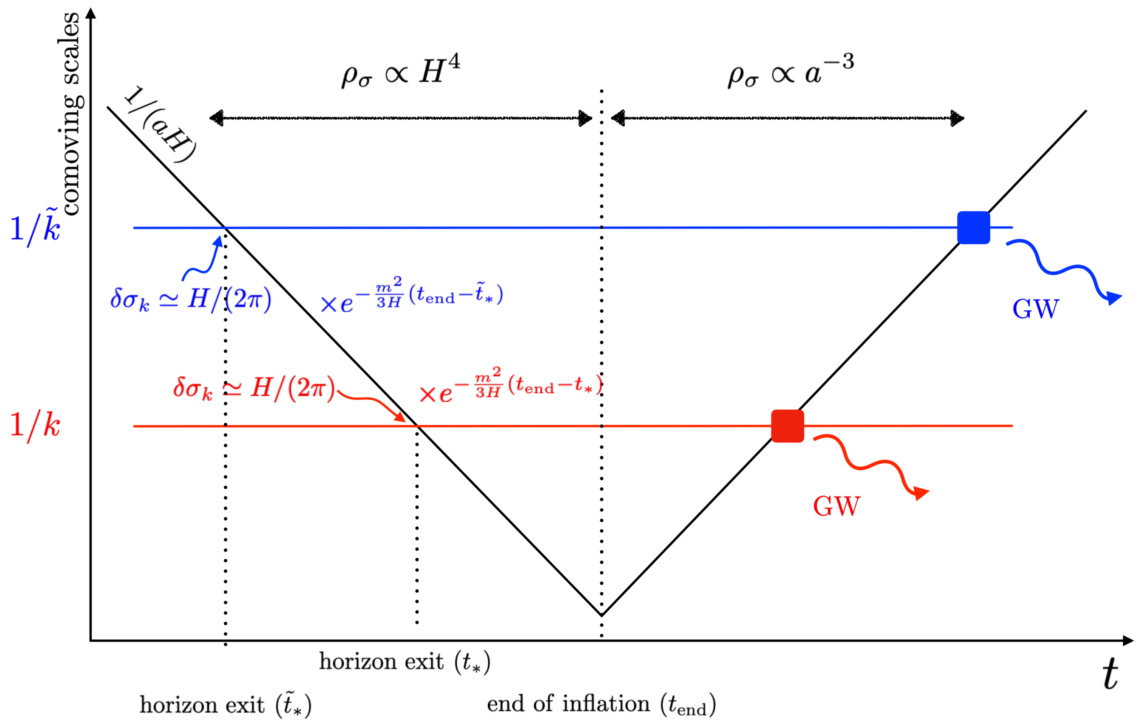
After exit, damping

$$\dot{\sigma} = -\frac{m_\sigma^2 \sigma}{3H}$$

$$\sigma_k(t) = \sigma(t_*) \exp\left(-\frac{m_\sigma^2}{3H}(t-t_*)\right) = \sigma(t_*)\left[\exp\left(-H(t-t_*)\right)\right]^{\frac{m_\sigma^2}{3H^2}} = \sigma(t_*)\left[\frac{k(t)}{H}\right]^{\frac{m_\sigma^2}{3H^2}}$$

More damping for longer wave-length (earlier exit)

Blue tilt



At horizon exit:
Amplitude ≈ H

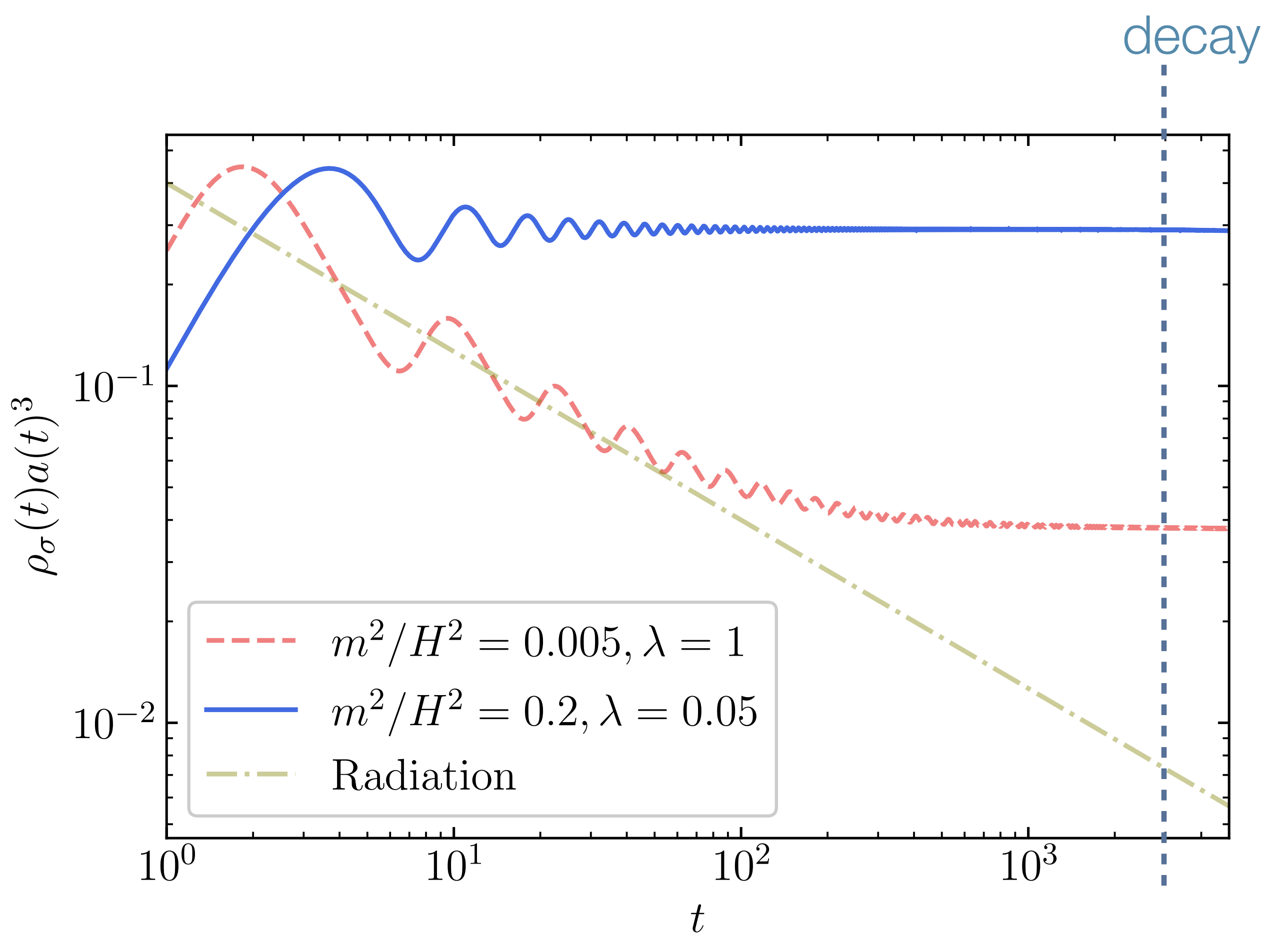
After exit, damping

$$\dot{\sigma} = -\frac{m_\sigma^2 \sigma}{3H}$$

For more general scalar theory

$$\mathcal{P}_f(k) = \sum_n \frac{2}{\pi} f_n^2 \Gamma\left(2 - 2\frac{\Lambda_n}{H}\right) \sin\left(\frac{\Lambda_n \pi}{H}\right) \left(\frac{k}{H}\right)^{2\Lambda_n/H} \quad \rightarrow \mathscr{A} \left(\frac{k}{H}\right)^{\frac{2\Lambda_{\rm lowest}}{H}} \quad \text{for } k \ll H$$

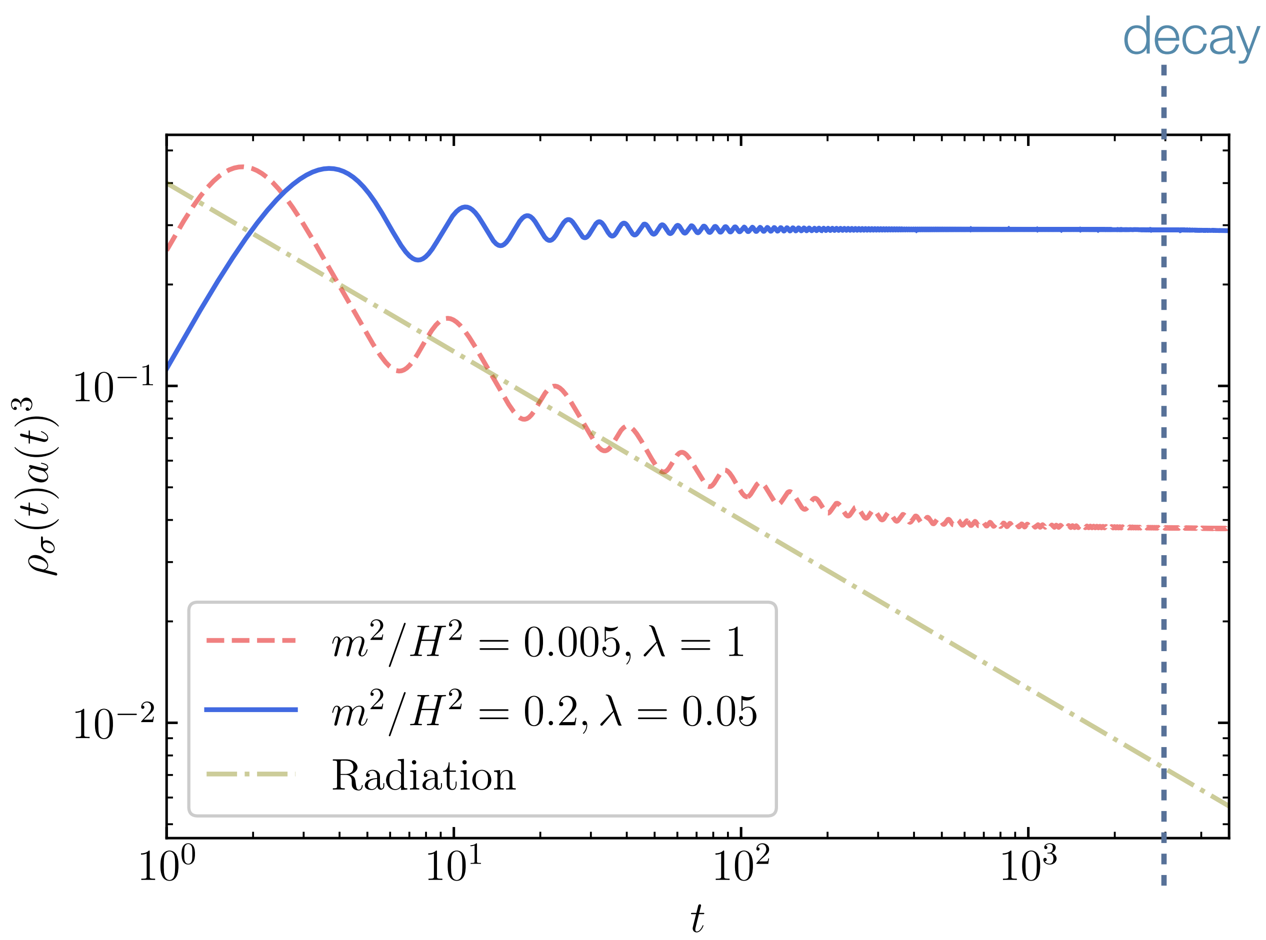
After inflation



Eventually,
evolve like matter

Can become important

After inflation



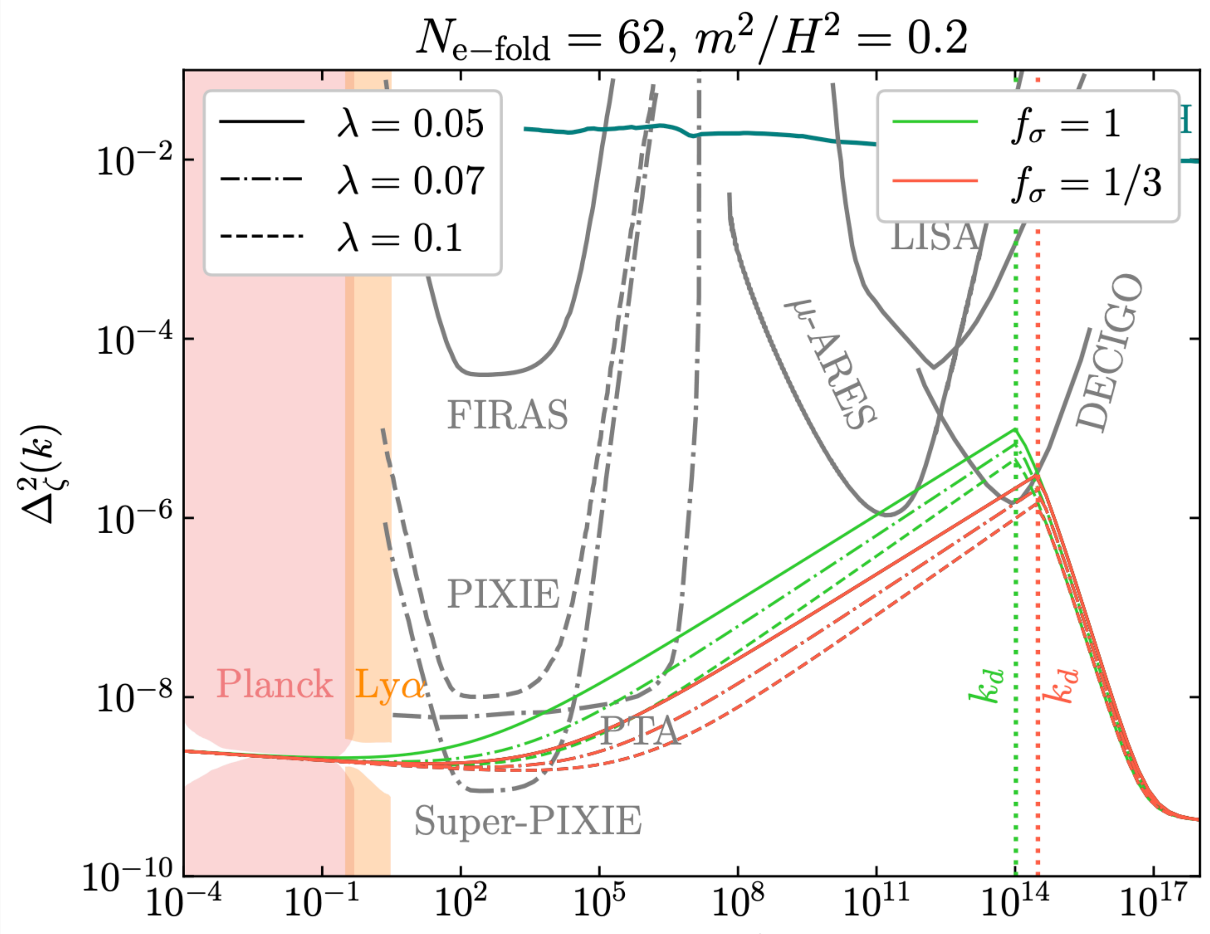
Eventually,
evolve like matter

Can become important

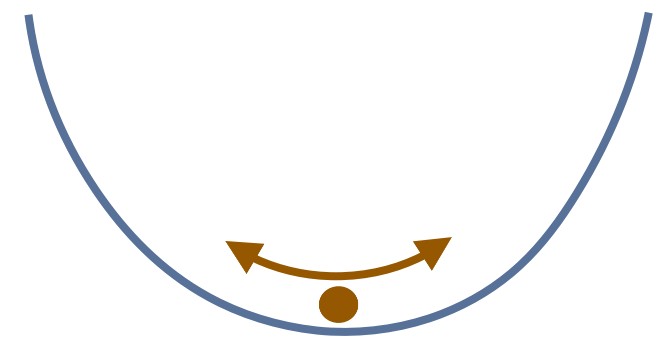
$$\Delta^2_\zeta(k) = \begin{cases} \Delta^2_{\zeta_r}(k) + \left(\frac{f_\sigma(t_d)}{4+3f_\sigma(t_d)}\right)^2 \Delta^2_{S_\sigma}(k), & k < k_d, \\ \Delta^2_{\zeta_r}(k) + \left(\frac{f_\sigma(t_d)(k_d/k)}{4+3f_\sigma(t_d)(k_d/k)}\right)^2 \Delta^2_{S_\sigma}(k), & k > k_d \end{cases}$$

Power spectrum

Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, LTW, 2307.12048



$H = 5 \times 10^{13}$ GeV

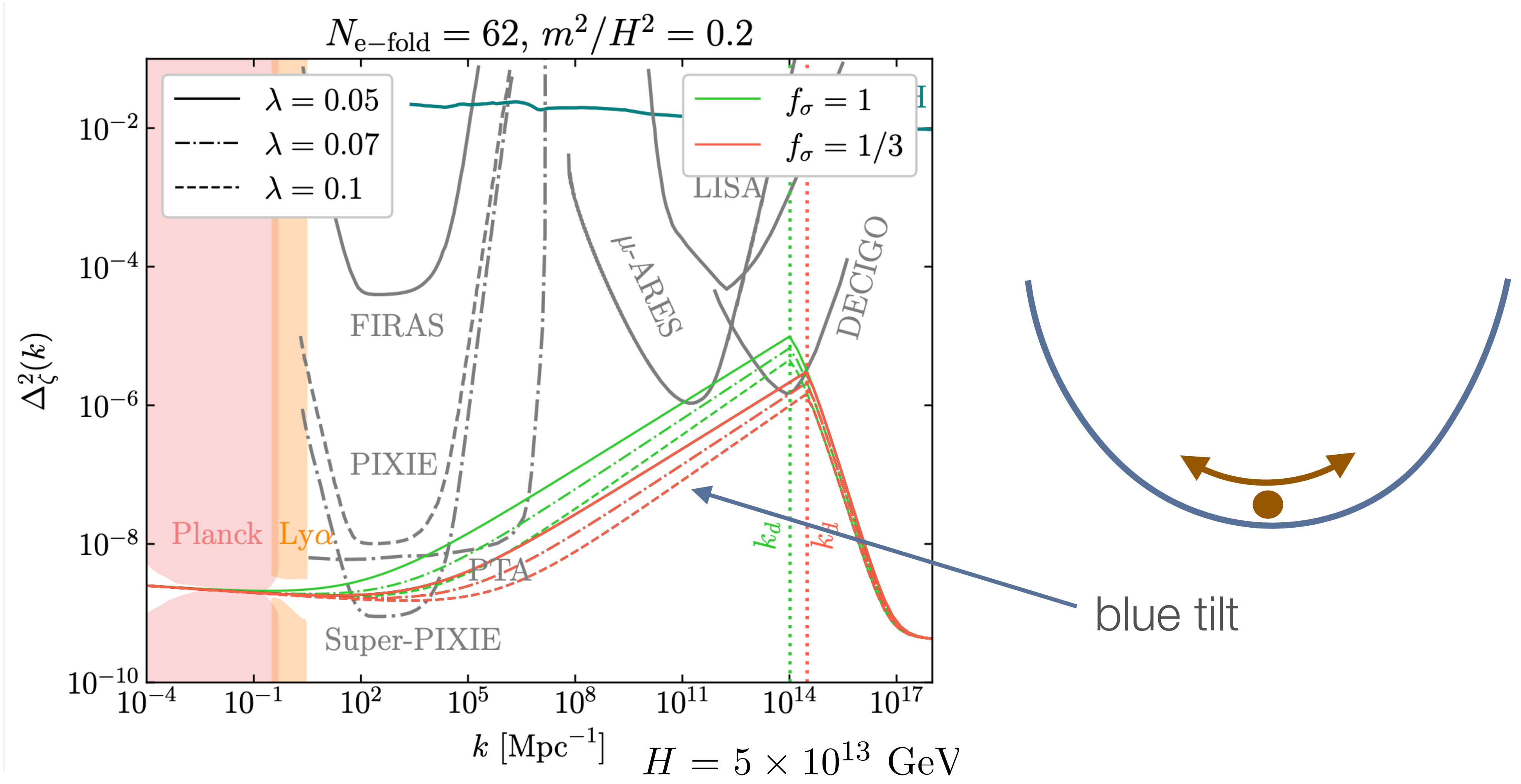


Assuming the scalar behave similar to curvaton.
Becoming important before decay.

Assumption: scalar field does not dominate (more later)

Power spectrum

Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, LTW, 2307.12048



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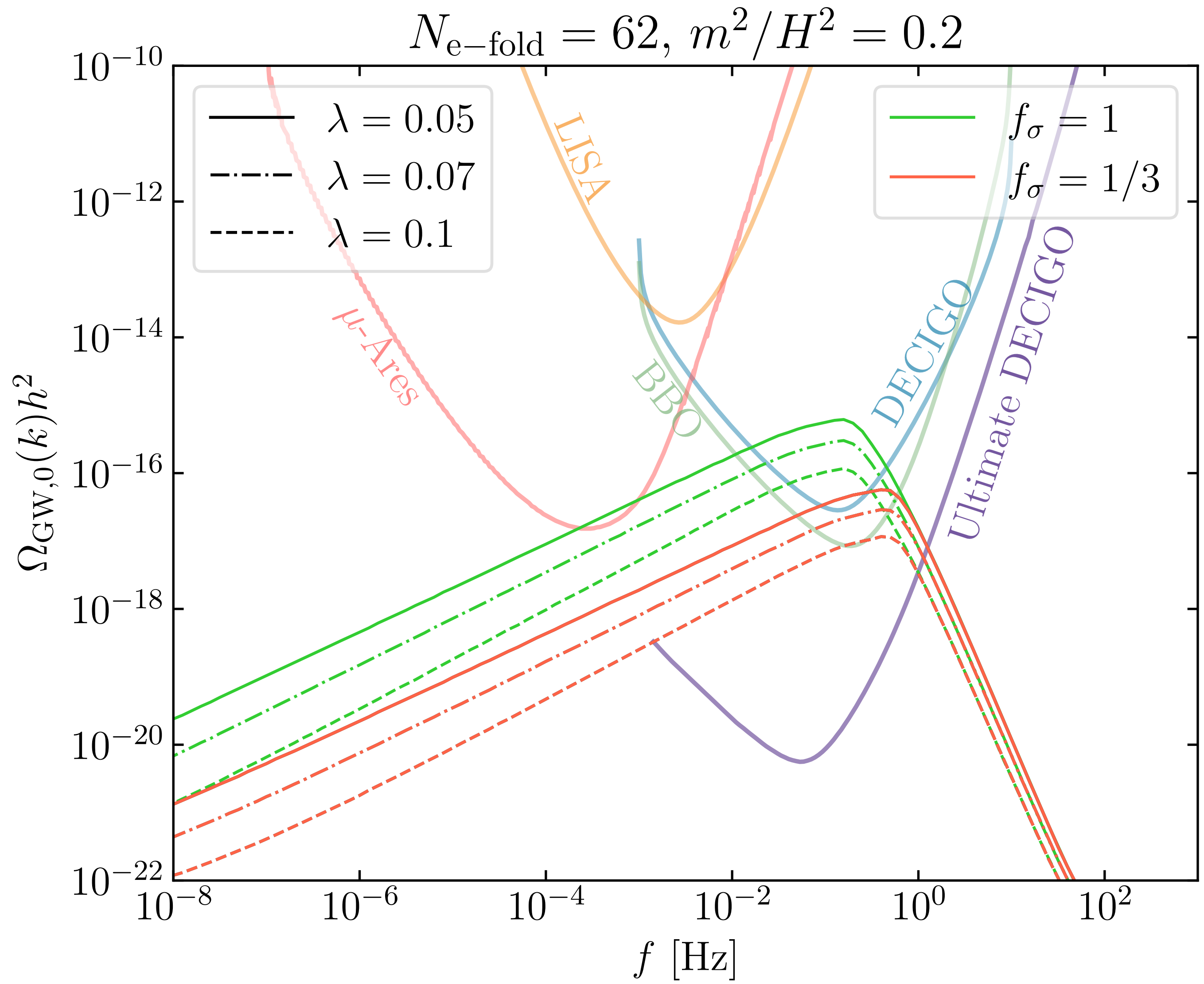
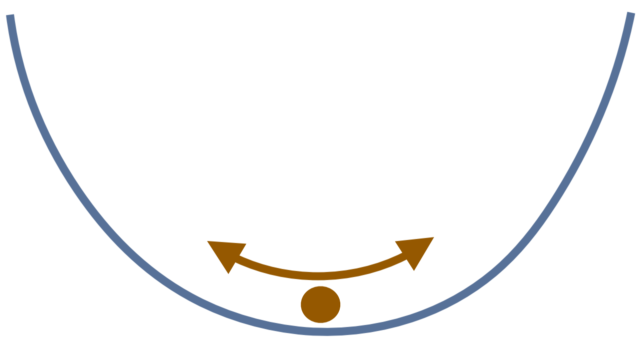
Gravitational wave

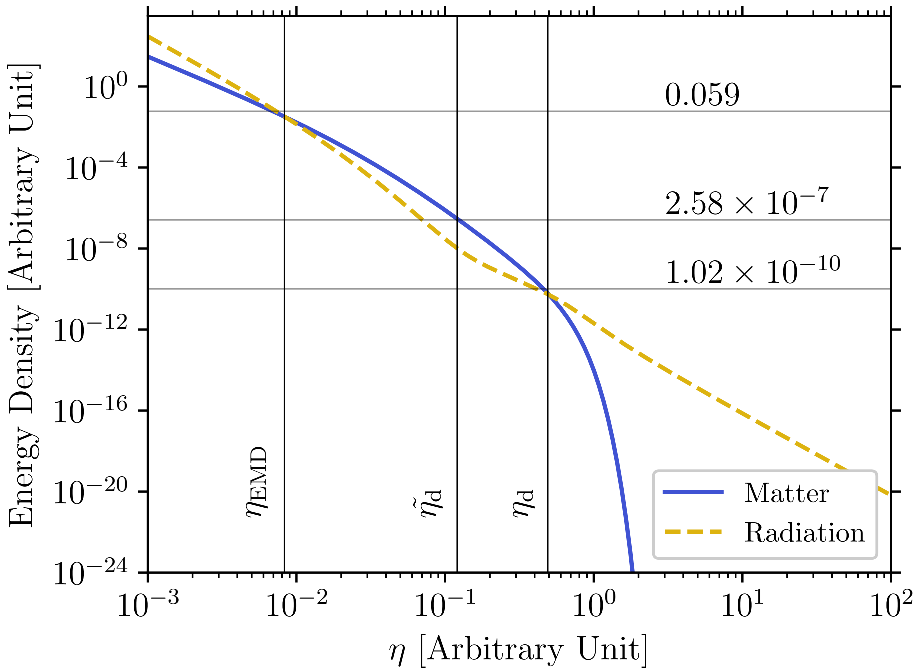
Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, LTW, 2307.12048

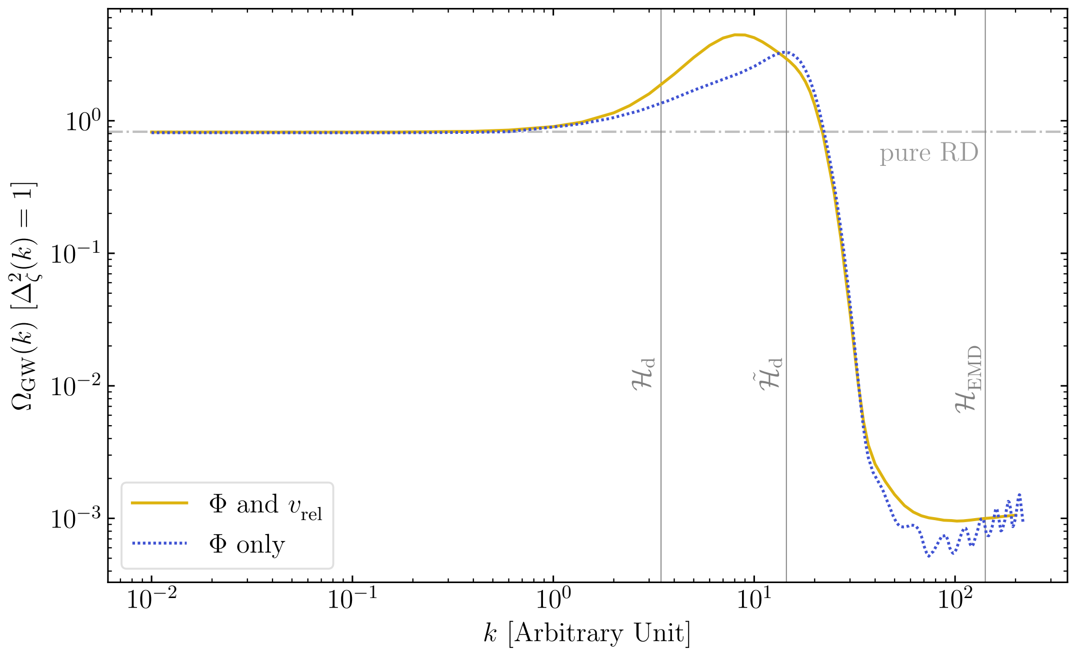




More general scenario

Soubhik Kumar, Hanwen Tai, LTW, 2410.17291



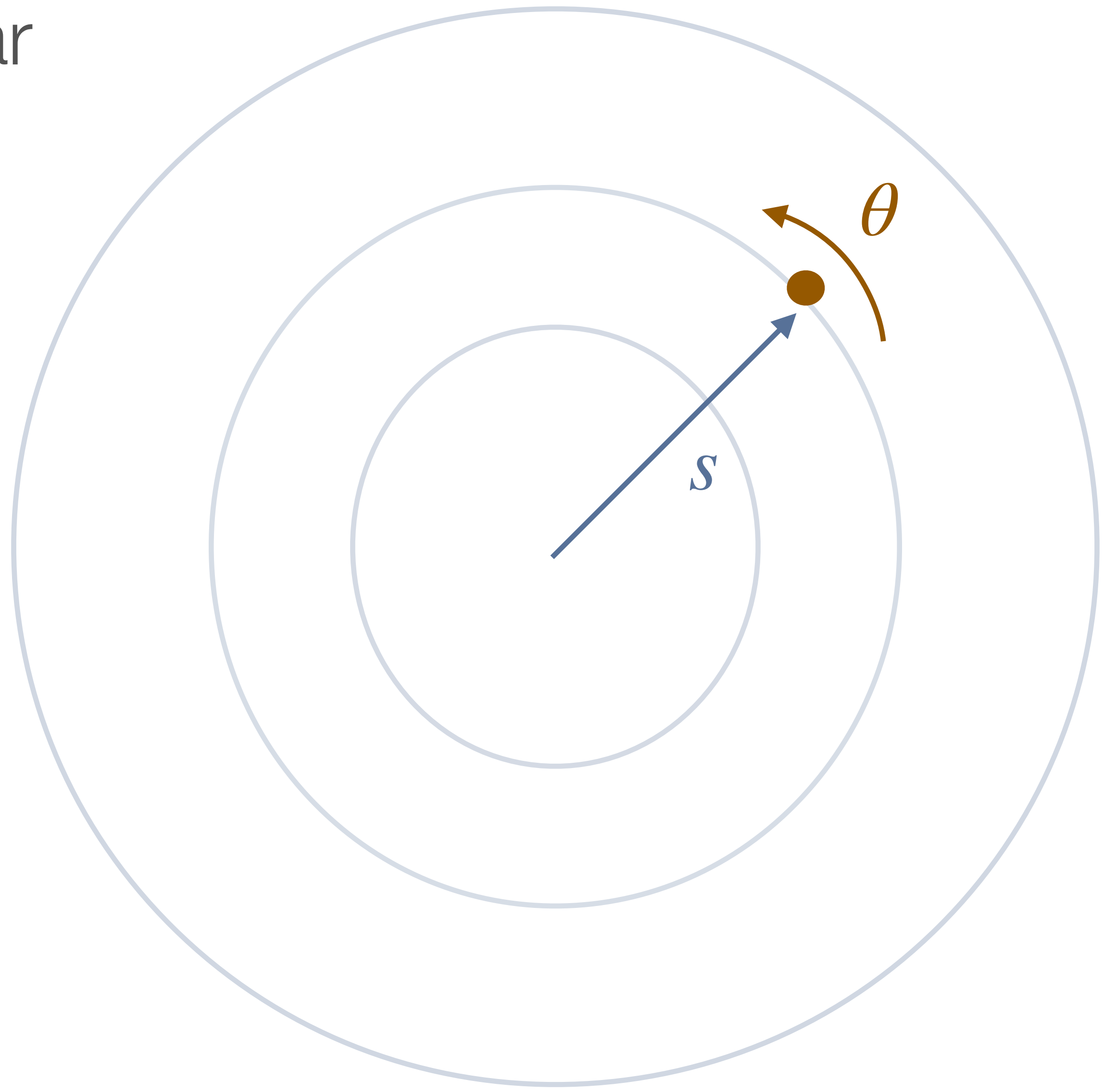


More generally, can consider the case scalar perturbation dominates (curvaton-like).
Larger signal, interesting spectral shape.
To treat this properly, much care is needed, numerically challenging.

2. Complex scalar

Soubhik Kumar, Hanwen Tai, LTW, 2410.17291

Complex scalar

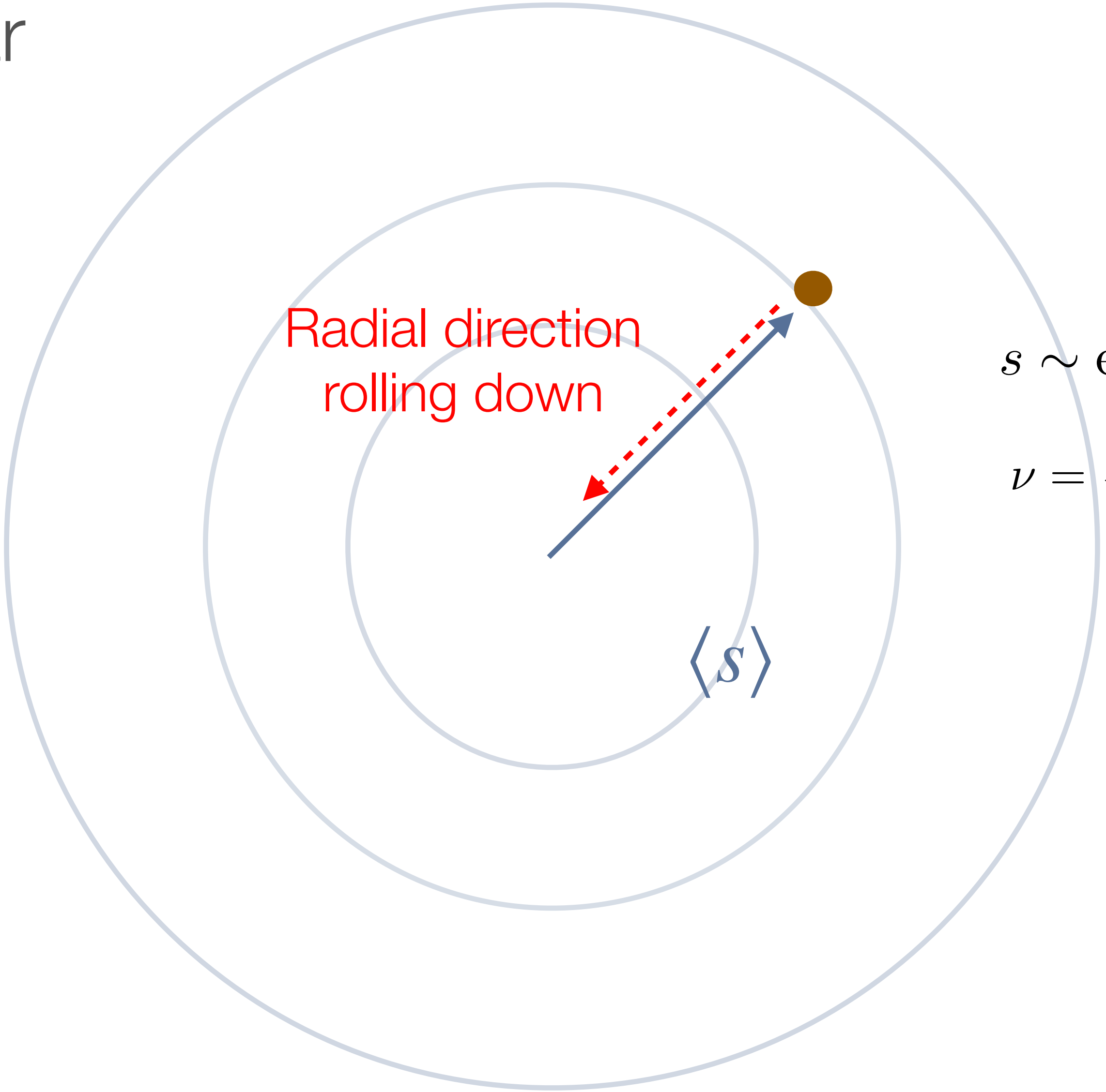


$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu s)^2 + \frac{1}{2}s^2(\partial_\mu \theta)^2 - \lambda_\Phi (s^2 - f_a^2)^2/4 + \frac{1}{2}m^2 s^2 \theta^2.$$

Rolling radial mode

Soubhik Kumar, Hanwen Tai, LTW, 2410.17291

Complex scalar



$s \sim \exp(-(3/2 - \nu)Ht)$

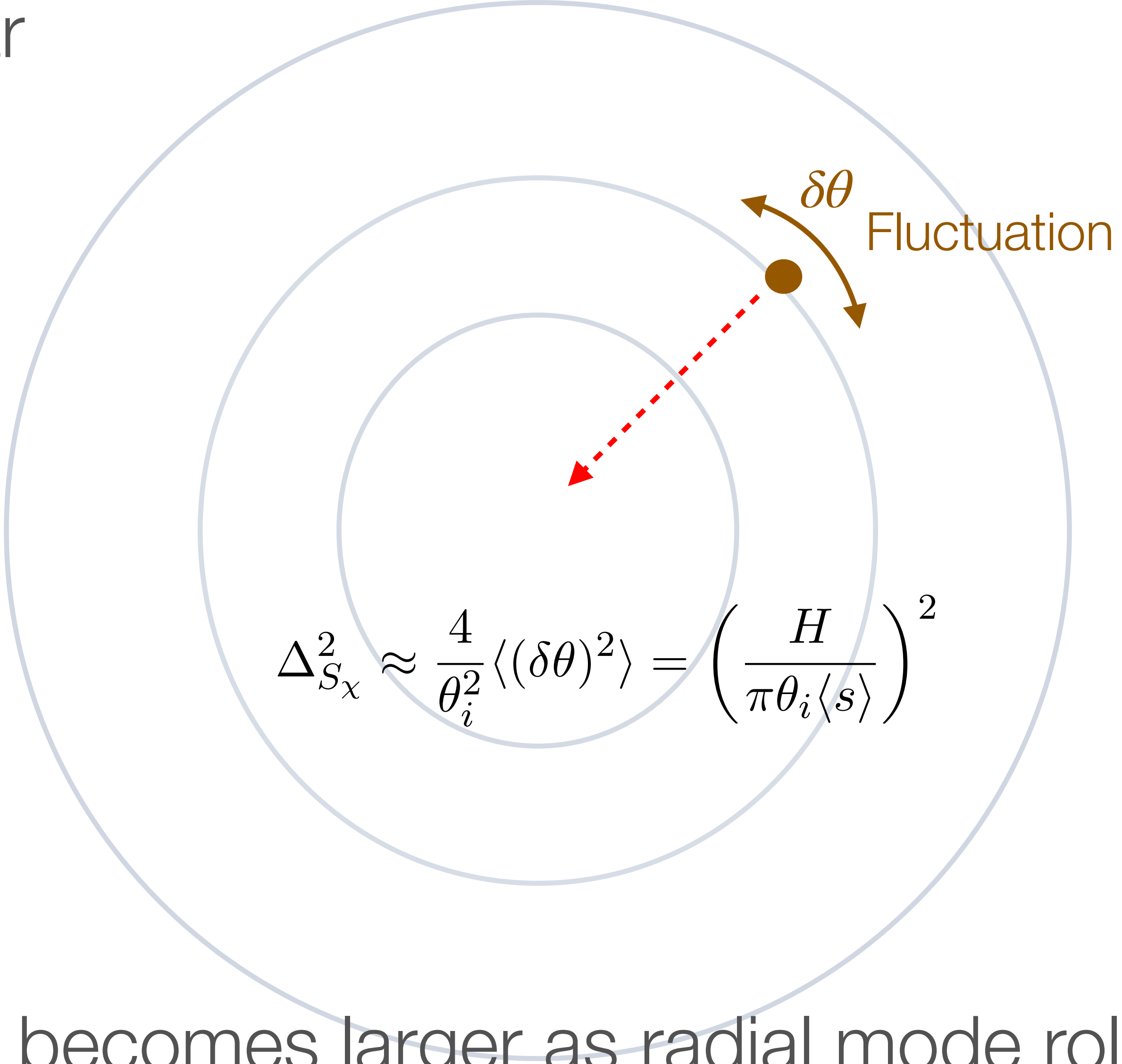
$\nu = \sqrt{9/4 - m_s^2/H^2}$

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu s)^2 + \frac{1}{2}s^2(\partial_\mu \theta)^2 - \lambda_\Phi(s^2 - f_a^2)^2/4 + \frac{1}{2}m^2 s^2 \theta^2.$$

Fluctuations

Soubhik Kumar, Hanwen Tai, LTW, 2410.17291

Complex scalar

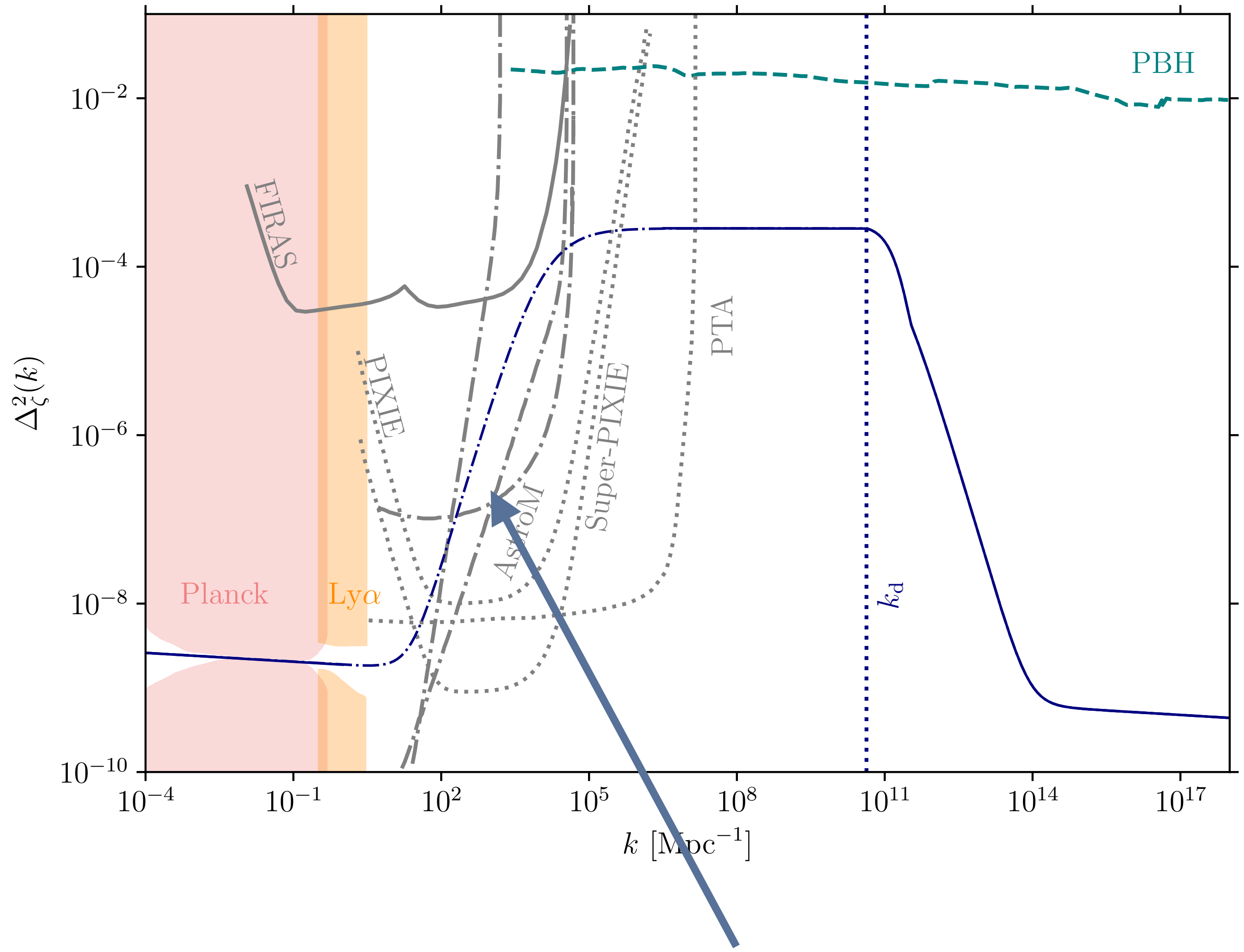


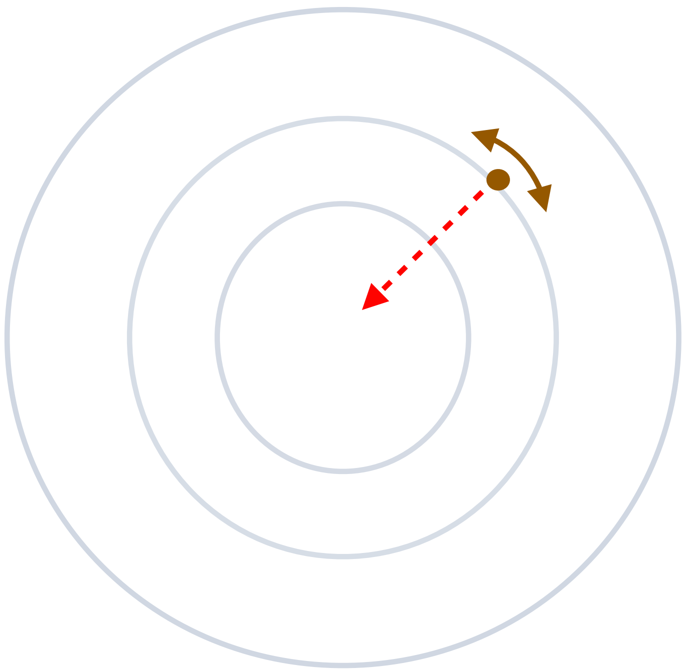
$$\Delta^2_{S_\chi} \approx \frac{4}{\theta_i^2}\langle(\delta\theta)^2\rangle = \left(\frac{H}{\pi\theta_i\langle s\rangle}\right)^2$$

Fluctuation becomes larger as radial mode rolling down

Perturbation spectrum

Complex scalar

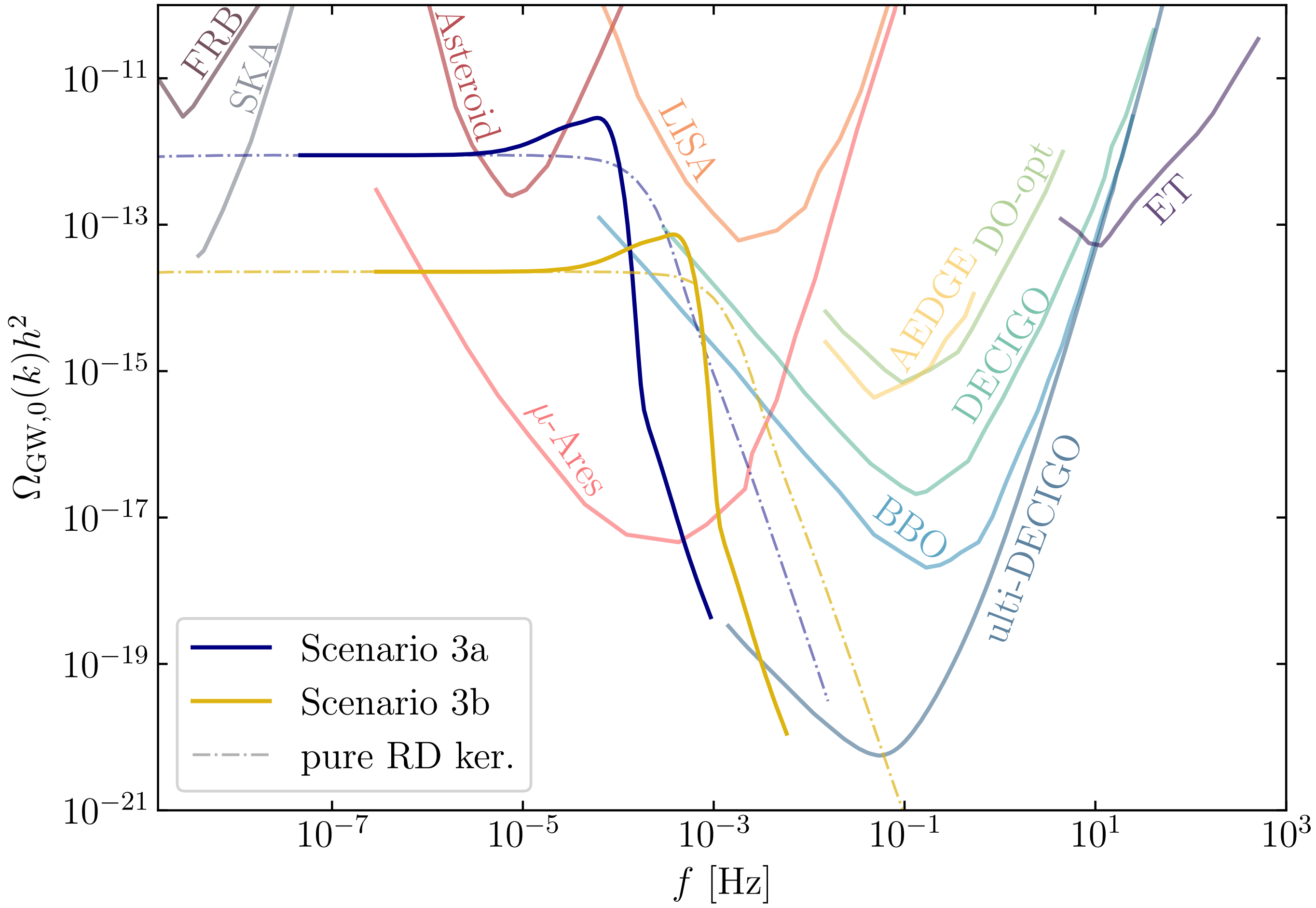


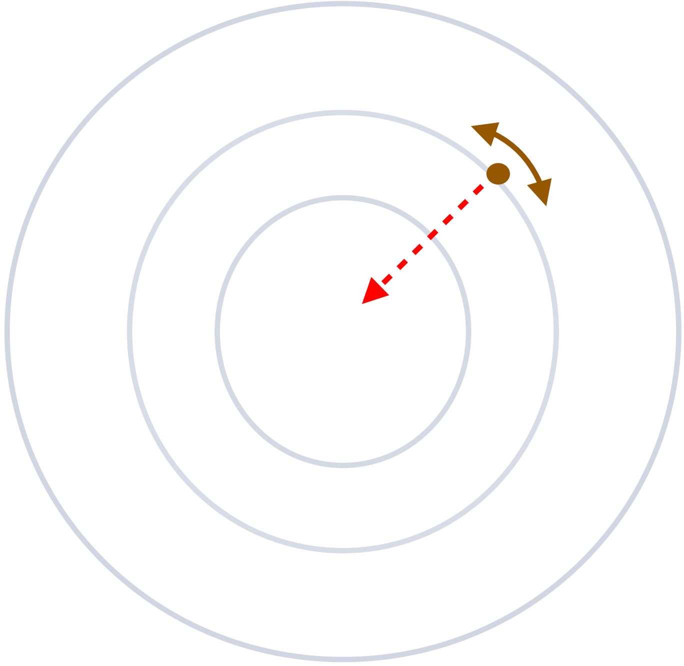


Steeper blue tilt than the previous case

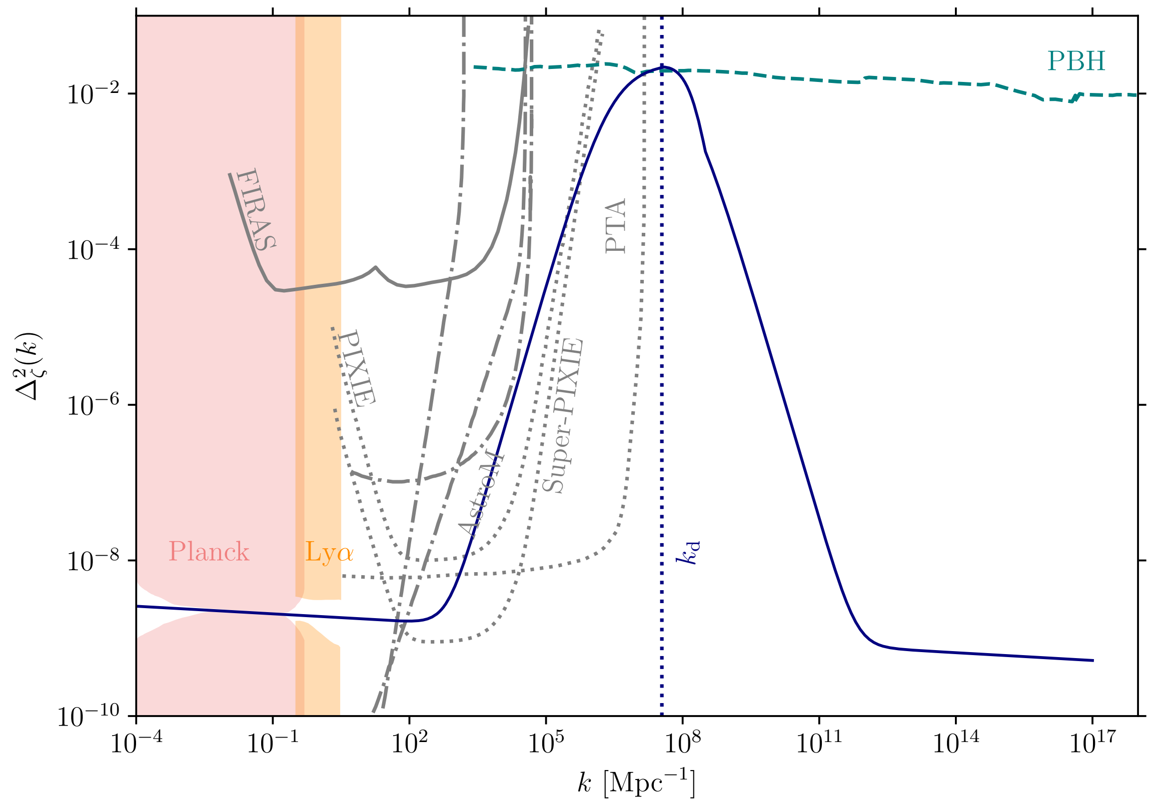
GW prediction

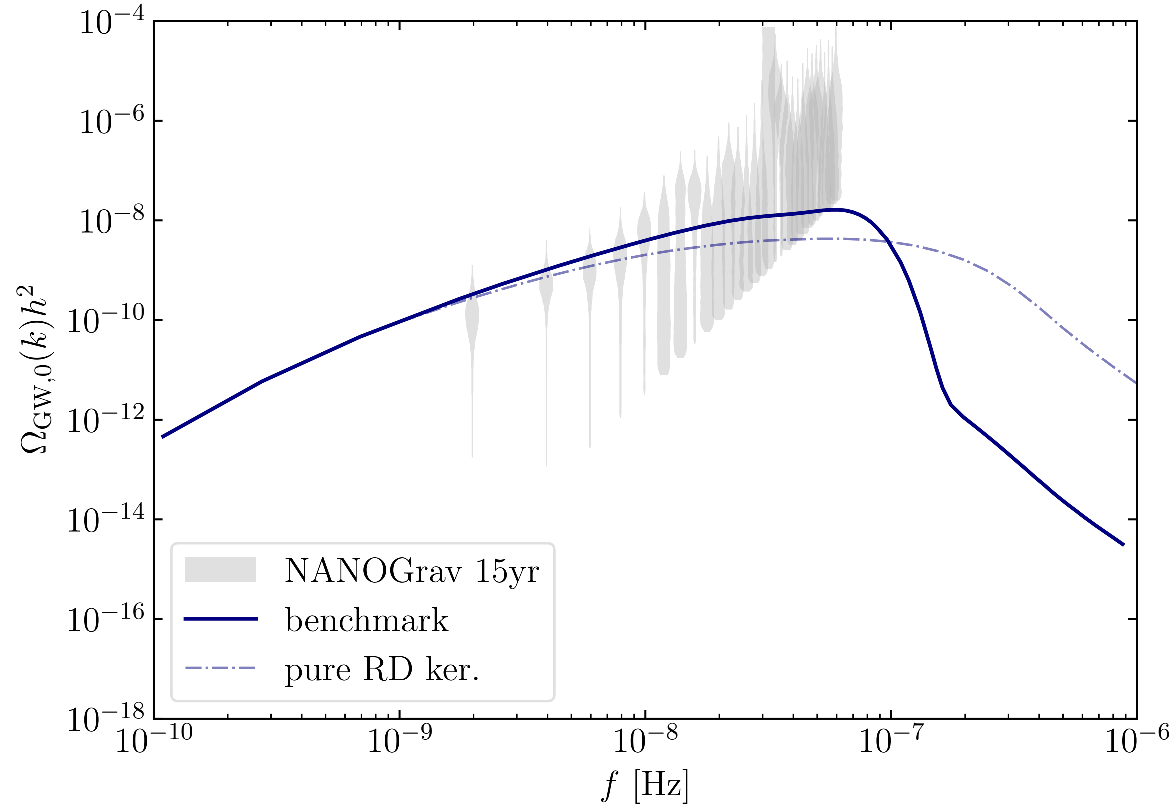
Complex scalar





Another benchmark



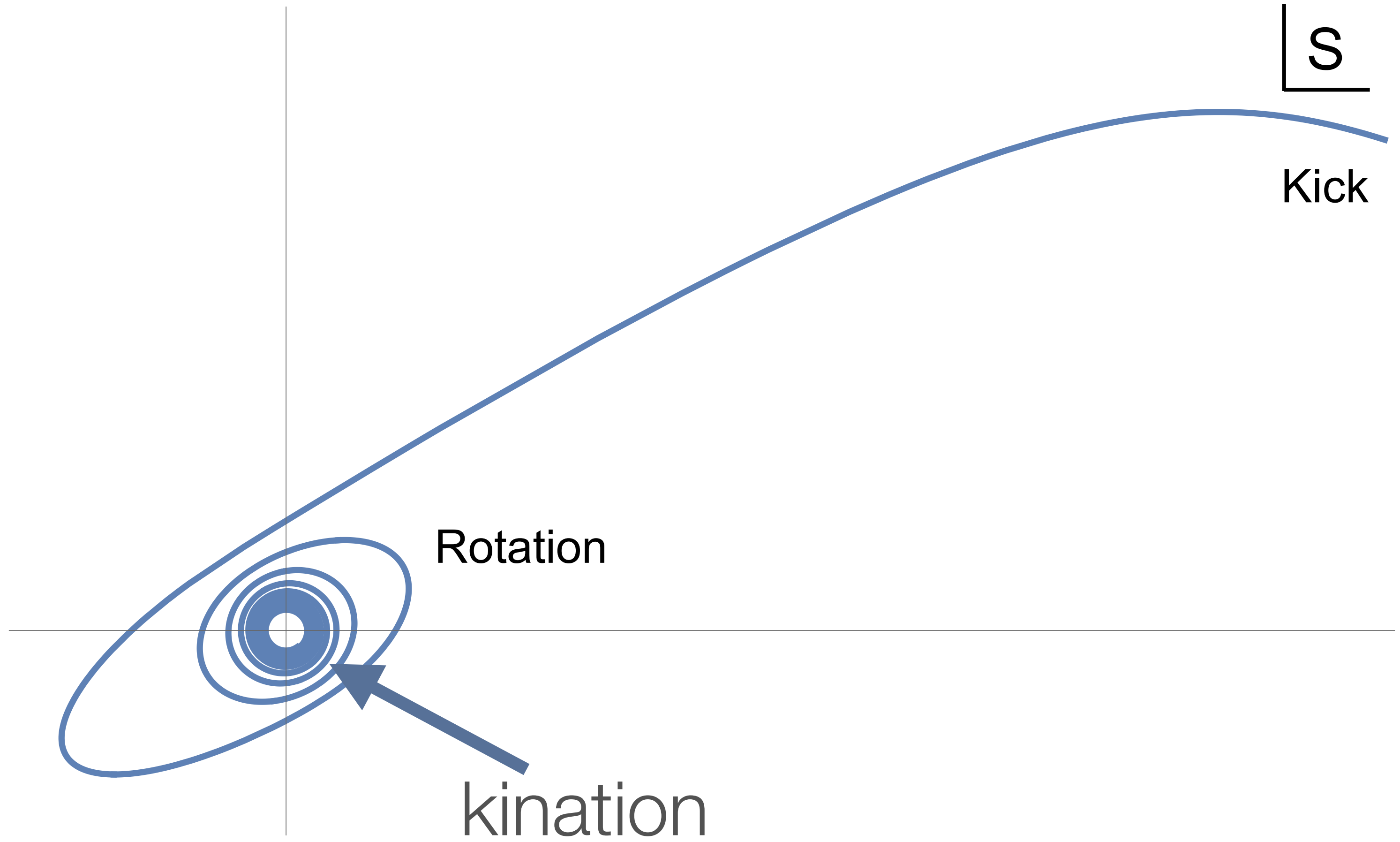


$$\chi_{0,\mathrm{end}} = f_a = 0.6H,\ H = 1.9\times10^{12}\ \mathrm{GeV},\ m = 0.05H,\ \lambda_\Phi = 0.75$$

N	$k_{\rm end}$ [Mpc^{-1}]	$k_{\rm EMD}$ [Mpc^{-1}]	$k_{\rm d}$ [Mpc^{-1}]
59.2	1.18×10^{22}	3.14×10^{8}	4.0×10^{7}

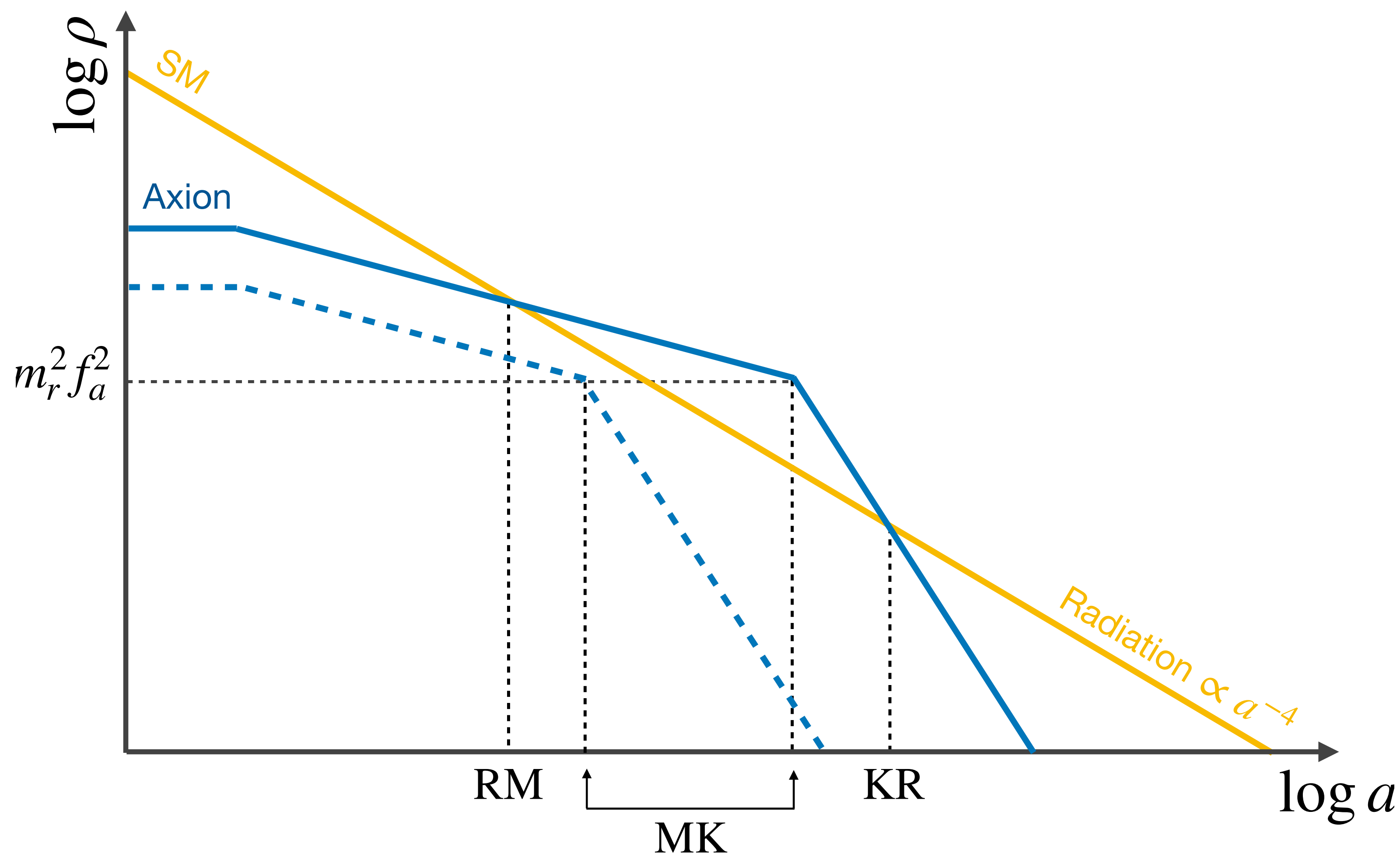
3. GW from Rotating axion

A. Bodas, K. Harigaya, K. Inomata, T. Terada, LTW 2508.08249



Evolution and GW

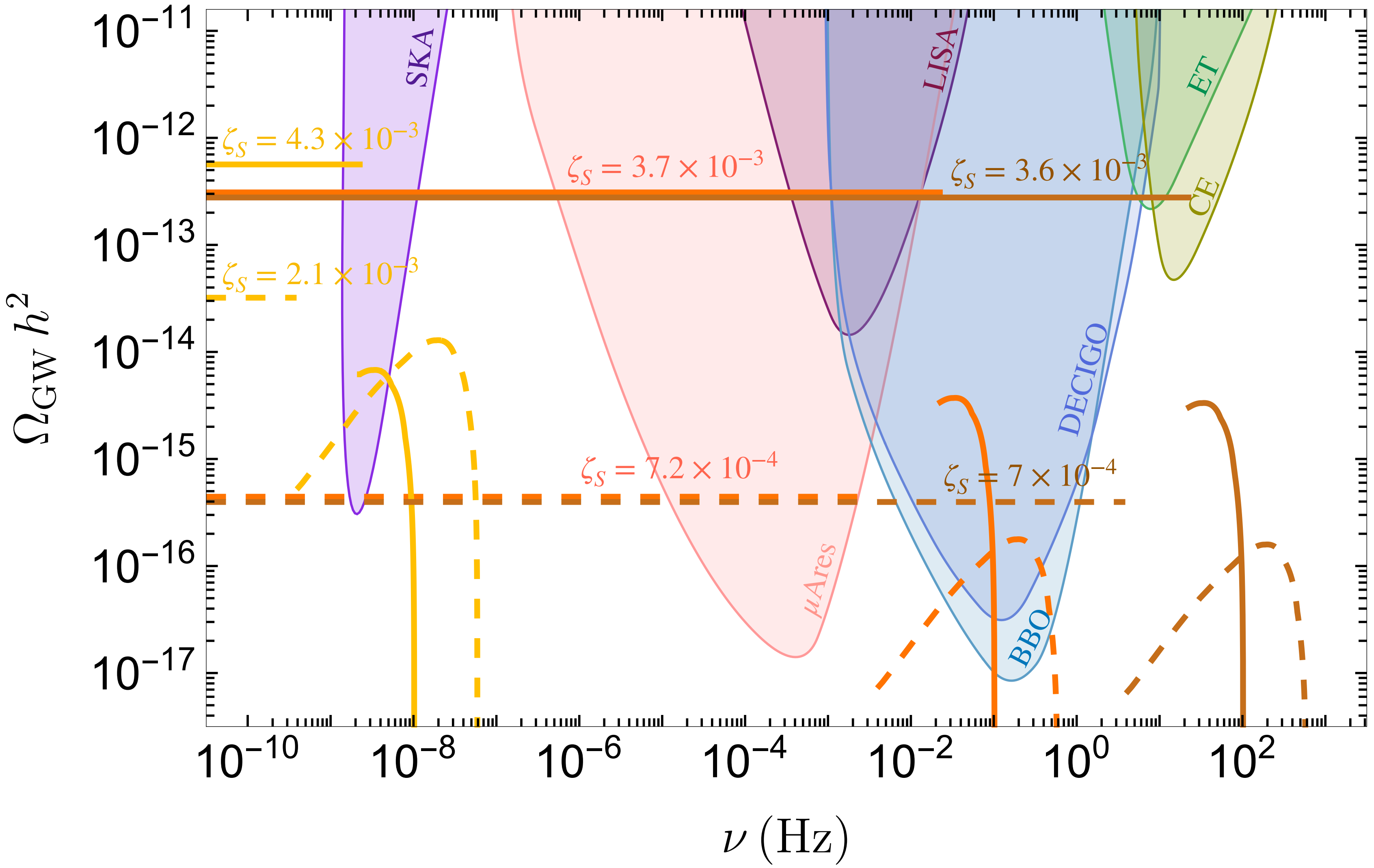
A. Bodas, K. Harigaya, K. Inomata, T. Terada, LTW 2508.08249



Fluctuation in axion field leads to secondary GW.
GW produced during eMD and MK transition (short wavelength)

GW from Rotating axion

A. Bodas, K. Harigaya, K. Inomata, T. Terada, LTW 2508.08249

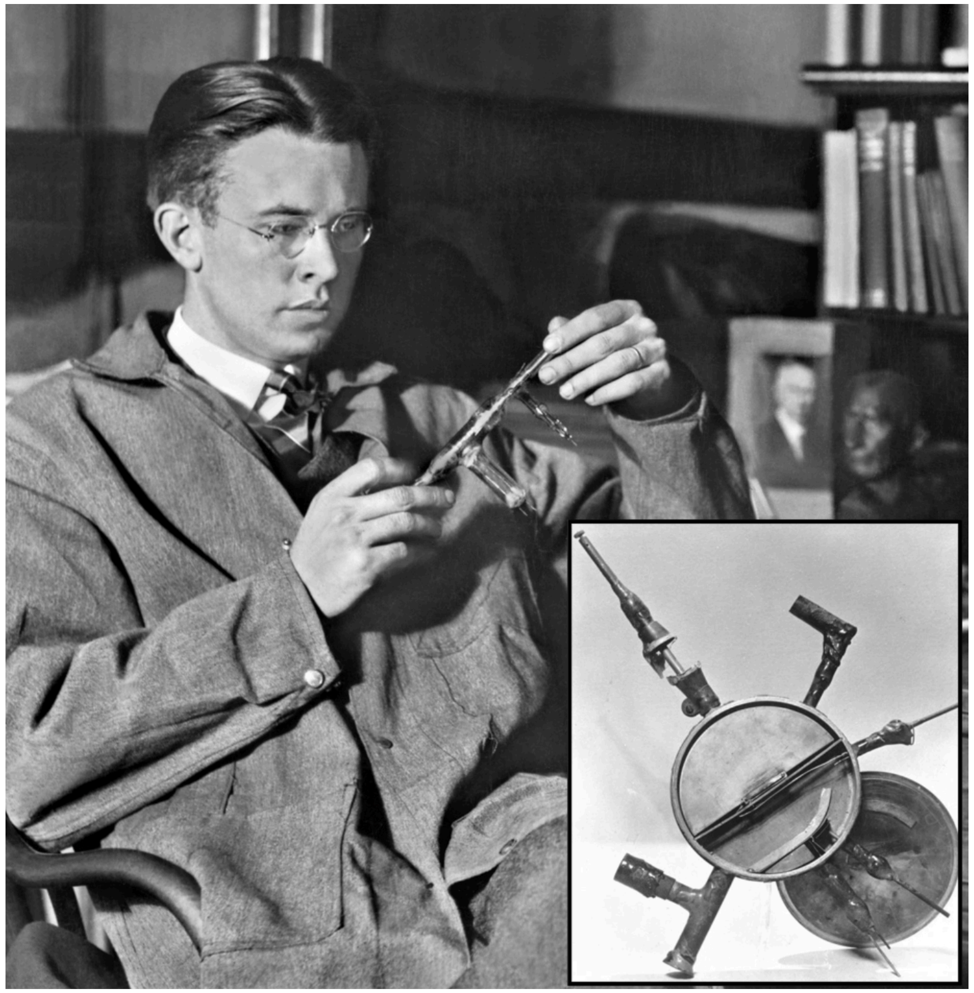


Conclusions

- We are at the beginning of a new era, gravitational wave as a new window to early universe.
- More observations of stochastic gravitational wave in the coming decades.
- Can reveal important dynamics in the early universe
- I focused on the question of new dynamics during inflation:
  - Blue spectrum of fluctuations $\rightarrow$ secondary GW
- A fast advancing field with many opportunities.

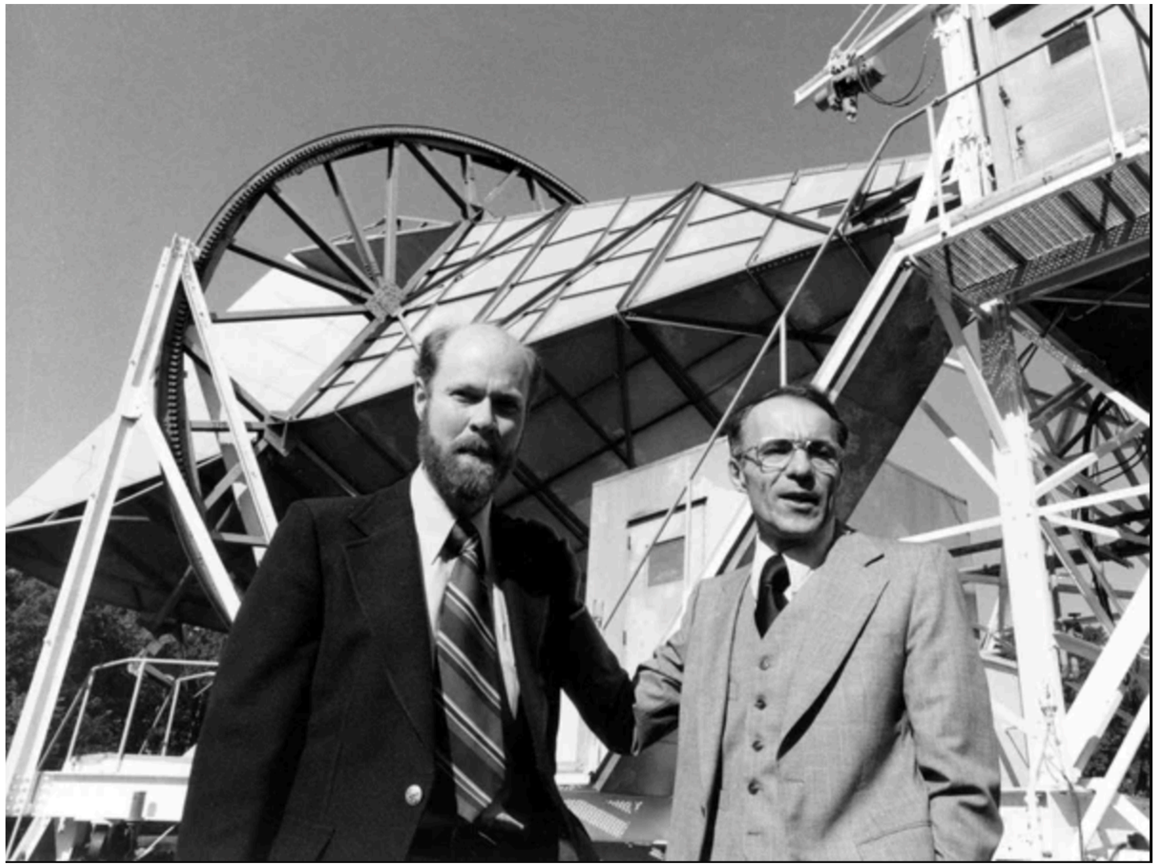
Beginnings of exciting times

E. Lawrence



LBNL

A. Penzias and R. Wilson



AP