Gravitational Waves from First Order Phase Transition

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- Previous work with A. Azatov and M. M. Khalil "Q-ball perturbations with more details: linear analysis vs lattice" has been published on PRD.
- E-print also available at 2412.13885

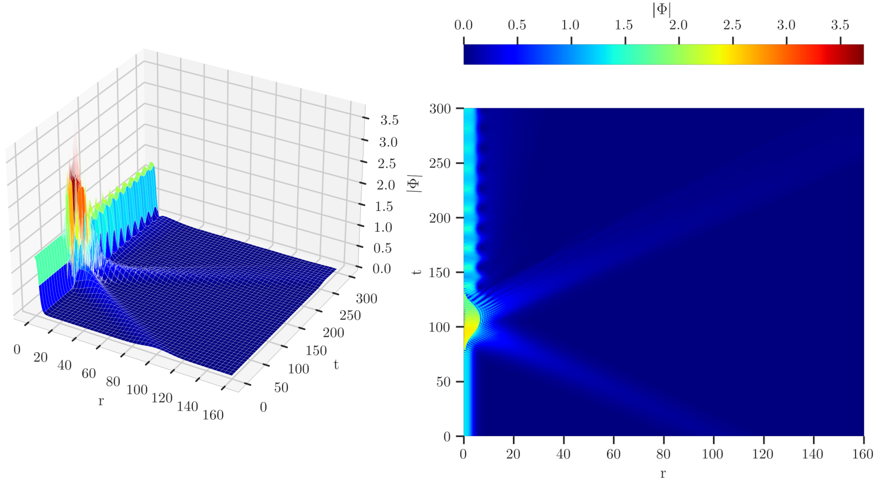


Table of Contents

Why study about Gravitational Wave Background (GWB)?

- Probes the earlier history of the Universe compared to e.g CMB.
- Uncovers exotic Astrophysical/Cosmological sources (phase transitions, cosmic strings, domain walls, ...)
- Active developments from both theory and experiments (e.g LISA, Einstein Telescope, NanoGrav, ...)
- ...

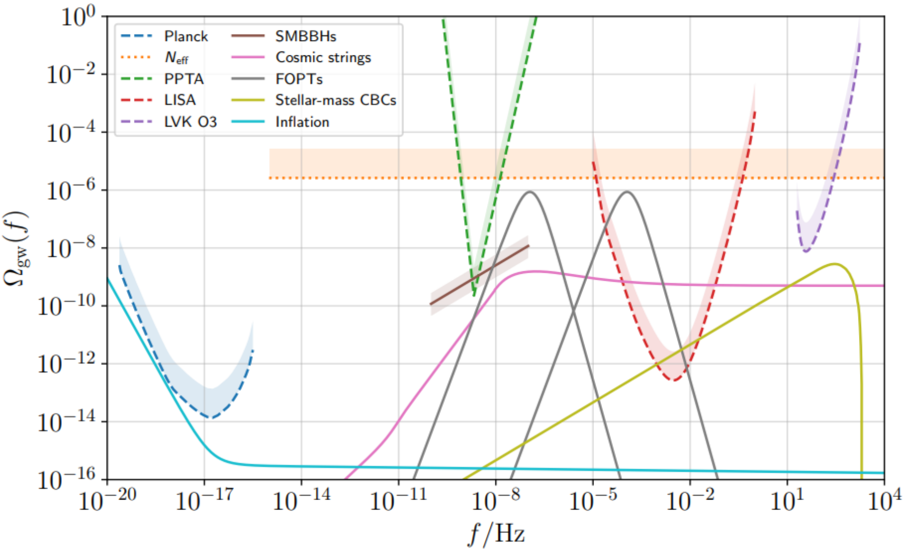
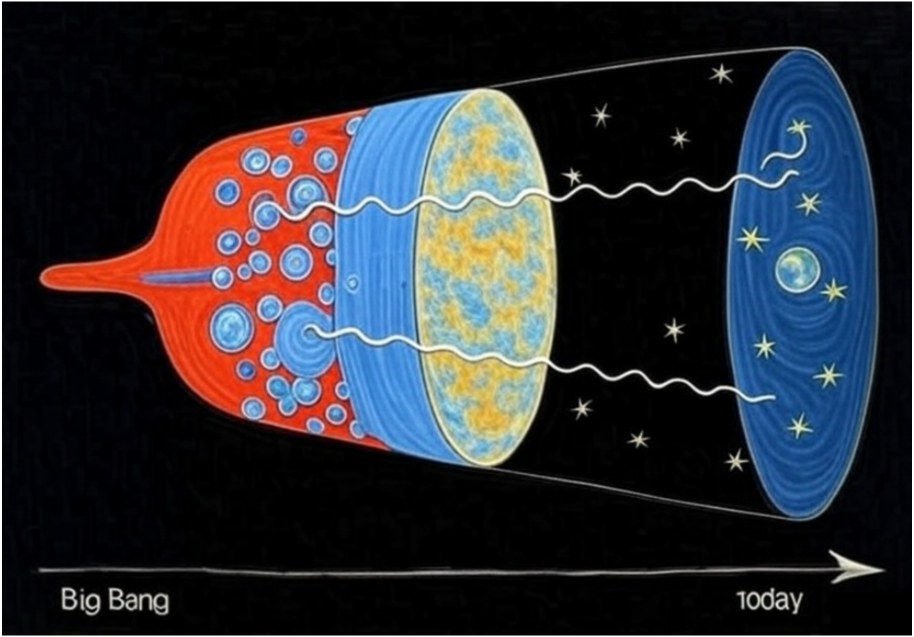


Figure 1: Overview of potential GWB signals across the frequency spectrum.
Credit: [Renzini et al., 2022]


Big Bang
Today

First Order Phase Transition as a source of GWs

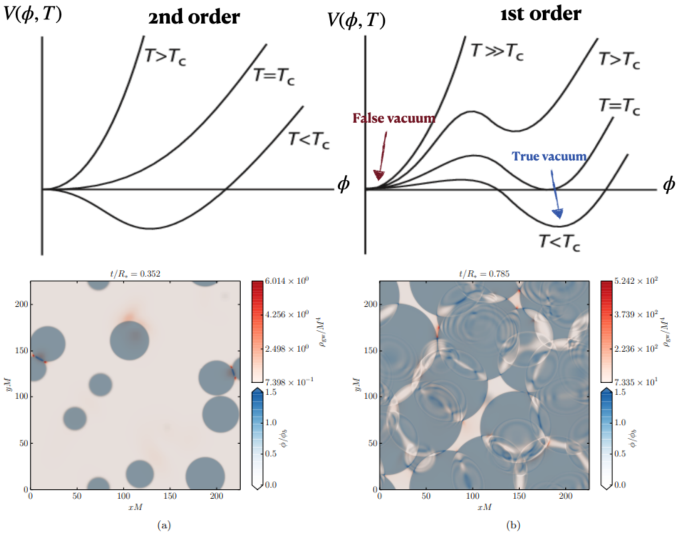


Figure 2: Credit: [Cutting, Hindmarsh, and Weir, 2018]

Approaches in computing GWs from FOPT

Lattice simulations: [Child and Giblin, 2012], [Cutting, Escartin, et al., 2021]

✔ Capture full effects of bubble dynamics.

✗ Computationally expensive.

✗ Simulations cannot capture thin-wall bubbles and high wall velocity.

✗ Limited number of bubbles in the simulation.

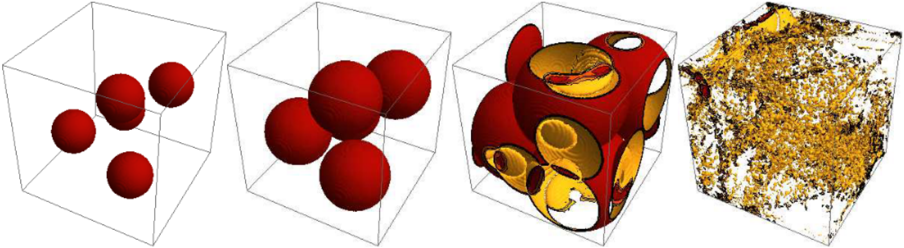


Figure 3: Lattice simulation for time evolution of 5 bubbles

2. The Envelope and Bulk-flow approximations

The amount of GW energy radiated in the direction $\hat{\mathbf{k}}$

$$\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}\omega\,\mathrm{d}\Omega} = 2G\omega^2\Lambda_{ij,lm}(\hat{\mathbf{k}})T^*_{ij}(\hat{\mathbf{k}},\omega)T_{lm}(\hat{\mathbf{k}},\omega)\,. \tag{2.1}$$

Applying this for N nucleated bubbles

$$T_{ij}(\hat{\mathbf{k}},\omega) = \frac{1}{2\pi}\int \mathrm{d}t\, e^{i\omega t}\sum_{n=1}^{N}\int \mathrm{d}\Omega\,\mathrm{d}r\, e^{-i\omega\hat{\mathbf{k}}\cdot(\mathbf{x}_n+r\hat{\mathbf{x}})}\, r^2\, T_{ij,n}(r,t). \tag{2.2}$$

Non-linearity of bubble collisions makes direct computations of T_{ij} almost impossible
$\Rightarrow$ *Approximations.*

Envelope approximation

First observation traced back to 34 years ago [Kosowsky and Turner, 1993]

$$\frac{\mathrm{d}E_{GW}}{\mathrm{d}\omega\,\mathrm{d}\Omega} = 4G\Delta V^2\omega^2(|C_+|^2 + |C_-|^2),$$

$$C_\pm(\omega) = \frac{1}{6\pi}\sum_{n=1}^{N}\int \mathrm{d}t\; e^{i\omega(t-z_n)}\, A_{n,\pm}(\omega, t),$$

$$A_{n,\pm}(\omega, t) = \int_{-1}^{1} \mathrm{d}\zeta\; e^{-i\omega(t-t_n)\zeta}\, B_{n,\pm}(\zeta, t),$$

$$B_{n,\pm}(\zeta, t) = \frac{1-\zeta^2}{2}\int_0^{2\pi} \mathrm{d}\phi\; g_\pm(\phi)\left[(t-t_n)^2\sigma(t, t_{n,c})\right].$$

Figure 4: Credit: [Konstandin, 2018]

- Before collision, the surface tension scales as: $\sigma \sim E_{\text{released}}/A \sim \mathcal{O}(R^3)/\mathcal{O}(R^2) \sim \mathcal{O}(R)$
- After collision, the surface tension vanishes $\Rightarrow \sigma(t, t_n, t_c) = \sigma_0(t - t_n)\Theta_H(t_c - t)$.

Bulk-flow approximation

Next step in improving envelope approximation, first done in [Konstandin, 2018].

- After collision, the released energy and wall width remains constant, the surface tension scales as $\sigma_{\text{collided}} \sim E_{\text{released}}/A \sim \mathcal{O}(R^{-2})$

$$\sigma(t, t_n, t_c) = \sigma_0(t - t_n)\Theta(t_c - t) + \sigma_1\Theta(t - t_c)\left(\frac{t_c - t_n}{t - t_n}\right)^2$$

- With the existence of long-lasting source, one expects both the IR and UV scaling powers are affected.

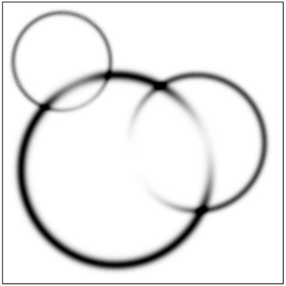


Figure 5: Credit: [Konstandin, 2018]

Envelope and Bulk-flow approximations:

- ✔ Model independent.
- ✔ Capture general features of the GW spectrum.
- ✔ Faster than simulation.
- ✔ Easy to generalize to many bubbles systems.

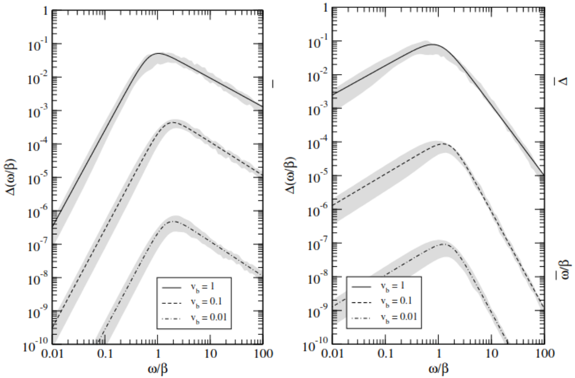


Figure 6: Envelope approximation (left) vs Bulk-flow approximation (right) for ~ 300 bubbles. Credit: [Konstandin, 2018].

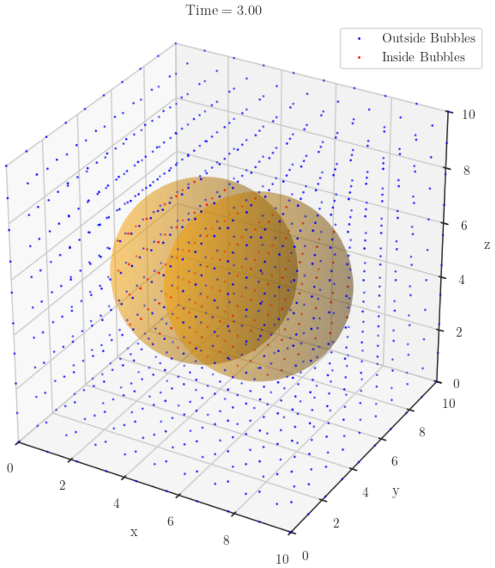
Going beyond Envelope/Bulk-flow approximations

? *What if the energy density of collided parts scale differently?* [Lewicki and Vaskonen, 2020]

- The scaling behaviors of surface tension in Envelope/Bulk-flow approximations is based on simple phenomenological argument. *How to correctly extract the scaling behaviors of* $\sigma(t, t_{n,c})$*?*

$\Rightarrow$ Two bubbles system!

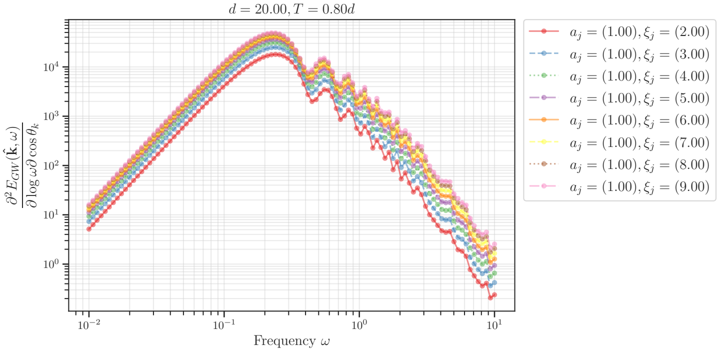
3. Two bubbles system



Why study this simple system?

- It captures general aspects of the full spectrum (e.g peak position, order of amplitude of the spectrum, ...).
- In lattice simulation, the symmetry of the system reduce the simulation from $(3+1)$D down to $(1+1)$D.
- In the approximations, it could be solved (semi)analytically, provide a good test bed for which case the approximations work.

$$\sigma(t, t_n, t_{n,c}) \approx \Theta(t_{n,c} - t)\sigma_0(t - t_n)$$
$$+\,\Theta(t - t_{n,c})\sigma_0(t_{n,c} - t_n)\left(\frac{t_{n,c} - t_n}{t - t_n}\right)^2 \sum_\xi a_\xi \left(\frac{t_{n,c} - t_n}{t - t_n}\right)^\xi \quad (3.1)$$



Field equations in hyperbolic coordinates

Given two patches $t^2 \geq x^2 + y^2$ and $t^2 \leq x^2 + y^2$ labelled as $+$ and $-$, the field equations in this coordinates become

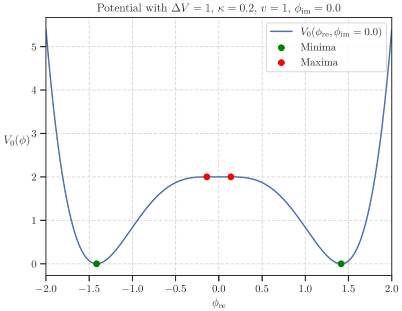
$$\pm\frac{\partial^2\Phi_\pm}{\partial s^2} \pm \frac{2}{s}\frac{\partial\Phi_\pm}{\partial s} - \frac{\partial^2\Phi_\pm}{\partial z^2} + \frac{\mathrm{d}V}{\mathrm{d}\Phi_\pm} = 0\,. \tag{3.2}$$

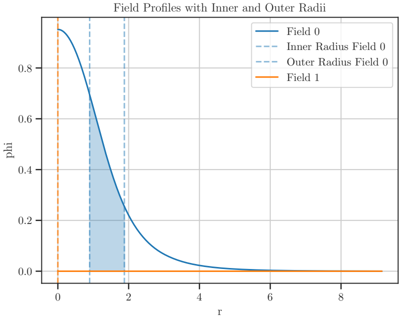
In what follows:

- Eq. (3.2) is solved on a $(1+1)D$ lattice.
- The surface tension σ is be extracted $\Rightarrow$ input of the generalized bulk-flow method to compute ***approximate GW spectrum***.
- The stress-energy tensor is computed exactly $\Rightarrow$ obtain the ***exact GW spectrum***.

Lattice simulation results

$$\textbf{Potential: } \frac{V(\Phi)}{\Delta V} = 1 + \kappa \frac{|\Phi|^2}{v^2} + \frac{|\Phi|^4}{v^4}\left[(\kappa+2)\log\left(\frac{|\Phi|^2}{v^2}\right) - (\kappa+1)\right]$$





Potential: $\dfrac{V(\Phi)}{\Delta V} = 1 + \kappa \dfrac{|\Phi|^2}{v^2} + \dfrac{|\Phi|^4}{v^4}\left[(\kappa+2)\log\left(\dfrac{|\Phi|^2}{v^2}\right) - (\kappa+1)\right]$

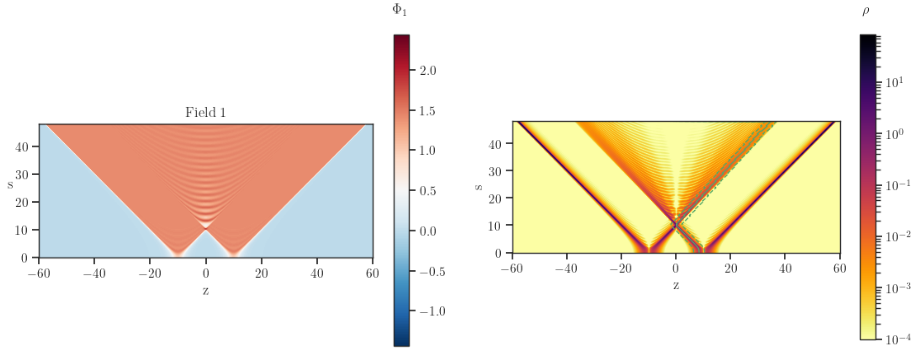


Figure 9: Field evolution (left) and its gradient energy density (right)

Fitting surface tension σ

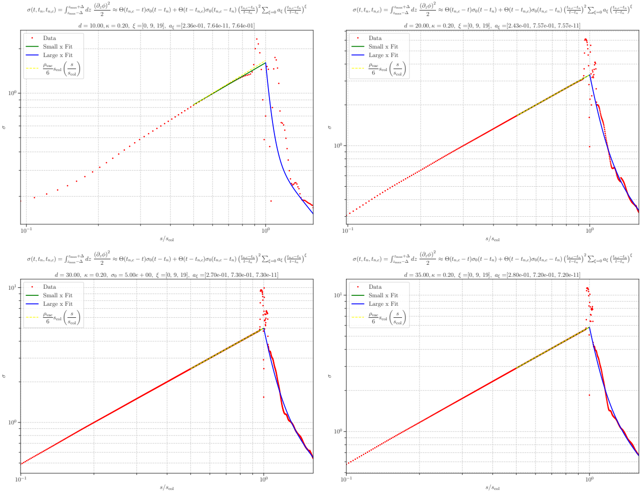
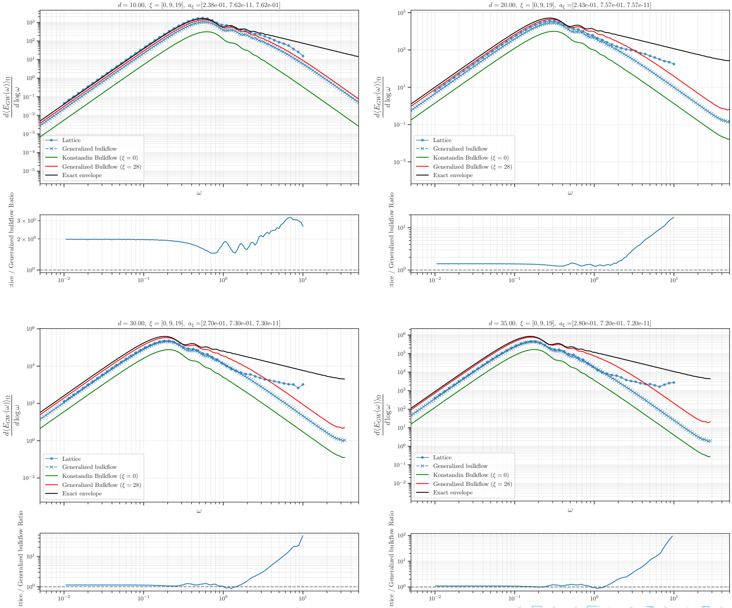


Figure 10: The surface tension for various bubble separations d, together with the fitting functions with the chosen powers $\xi \in [0, 9, 19]$.

Exact vs Generalized Bulk-flow



Sensitivity of fitting coefficients w.r.t d

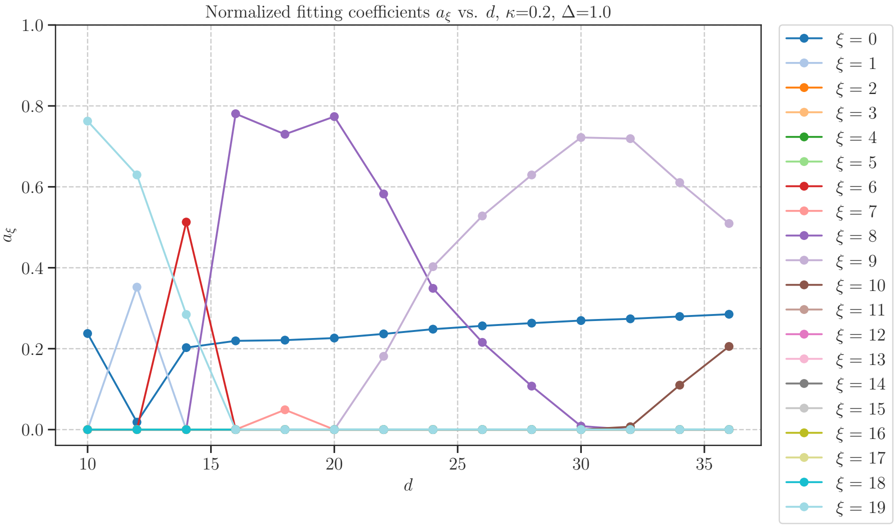


Figure 12: The sensitivity of fitting coefficients a_ξ for $\xi = 0 \ldots 19$.

Sensitivity of fitting coefficients w.r.t d

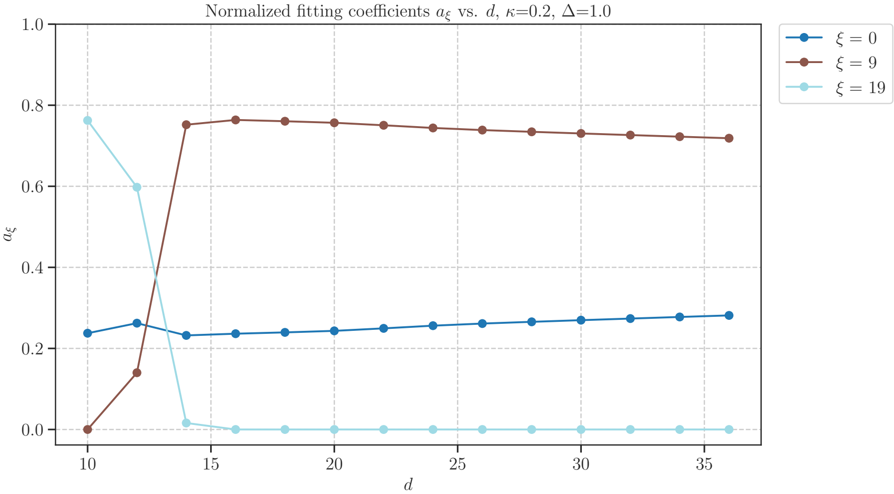


Figure 13: The sensitivity of fitting coefficients a_ξ with the fitting function corresponds to $\xi = 0, 9, 19$.

Summary

What we covered:

- Two bubbles system greatly simplifies the study of GW spectrum on the lattice.
- Generalized bulk-flow approximation is a powerful to quickly and correctly extract the GW spectrum from FOPTs.

What we're still working on:

- Extend the power spectrum computation towards many bubbles system.
- When does bulk-flow approximation break down? (e.g existing of long-live trapped vacuum.)
- Is there an even better way to formulate the bulk-flow contribution? (e.g dependence on angle between colliding walls.)

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Thank you
for
your attention!