The puzzles surrounding the muon anomalous magnetic moment

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No evidence for Beyond-Standard Model particles from collider experiments

Overwhelming evidence for dark sector from astrophysical observations

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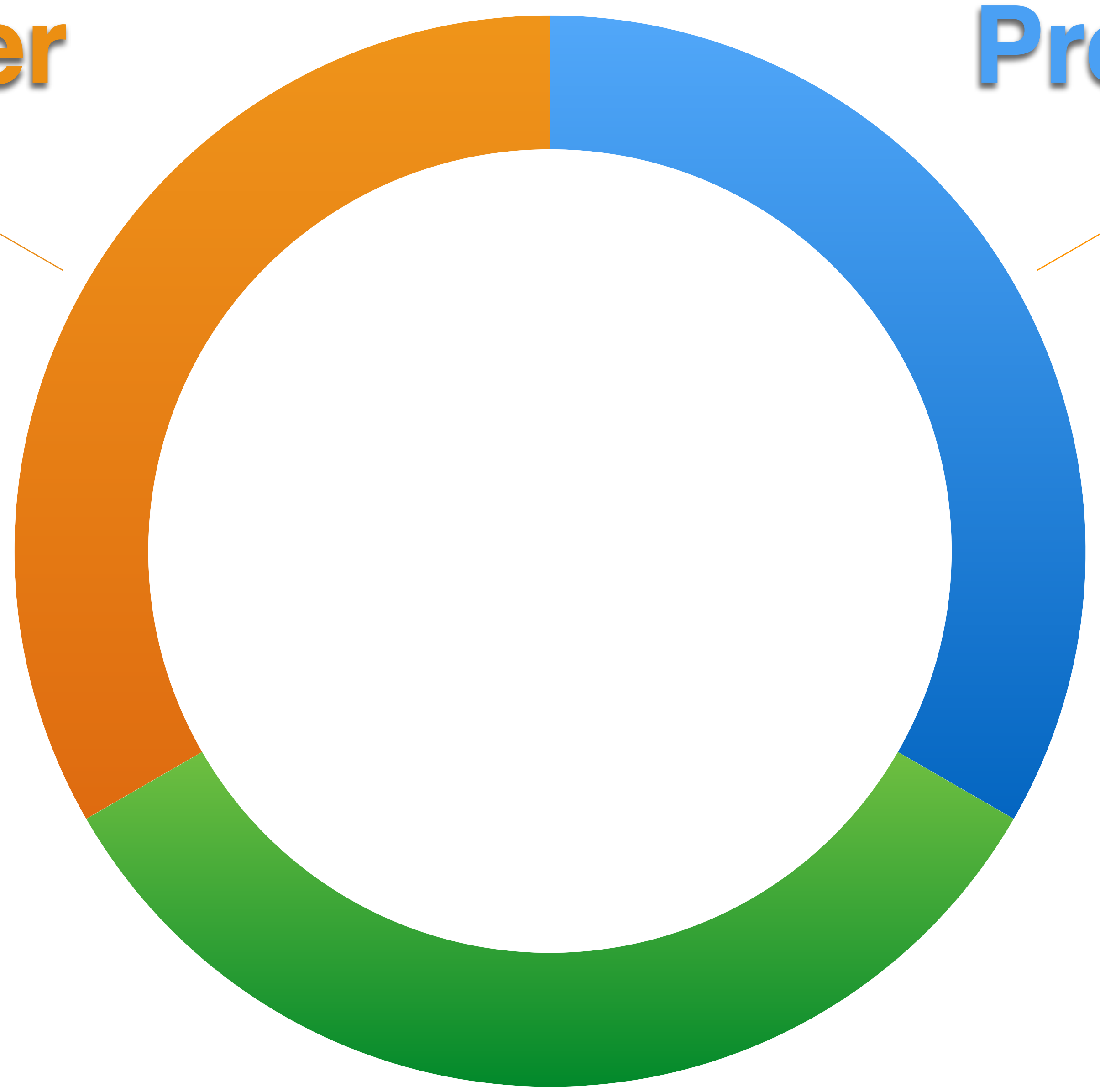

Lepton anomalous magnetic moments as probes for New Physics

Magnetic moment of particle with spin $\vec{S}$ and charge e : $\vec{M} = g \frac{e\hbar}{2m} \vec{S}$

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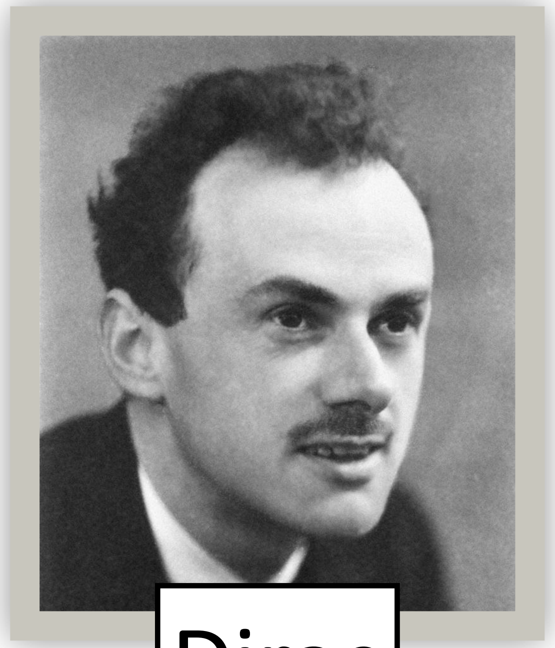
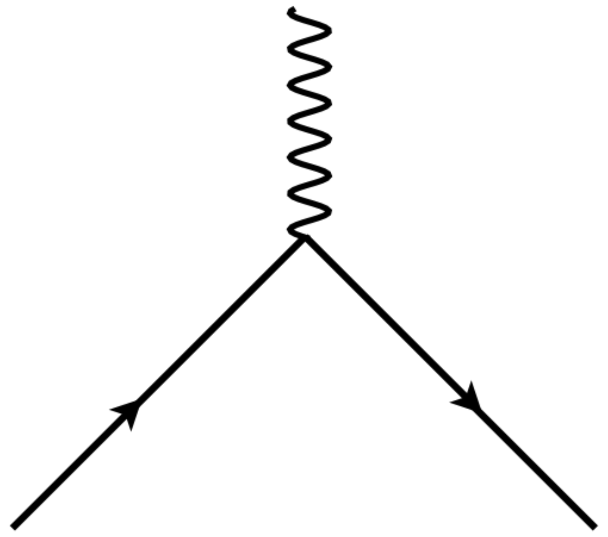


Dirac

$g = 2$



tree level

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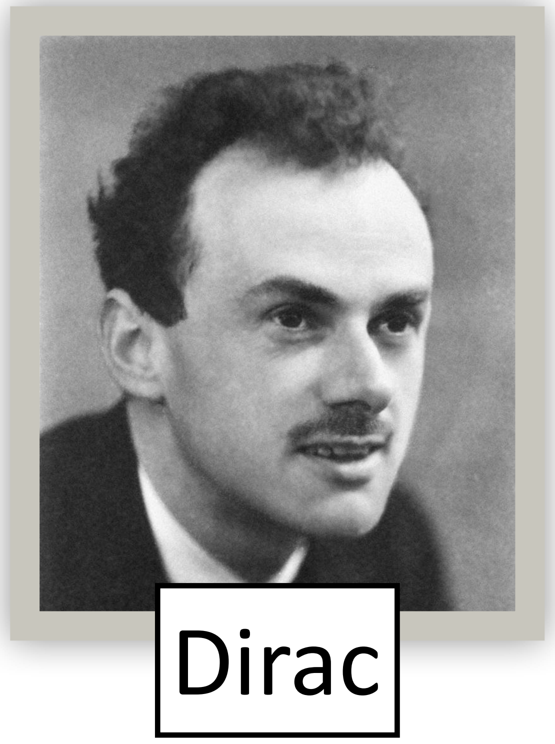

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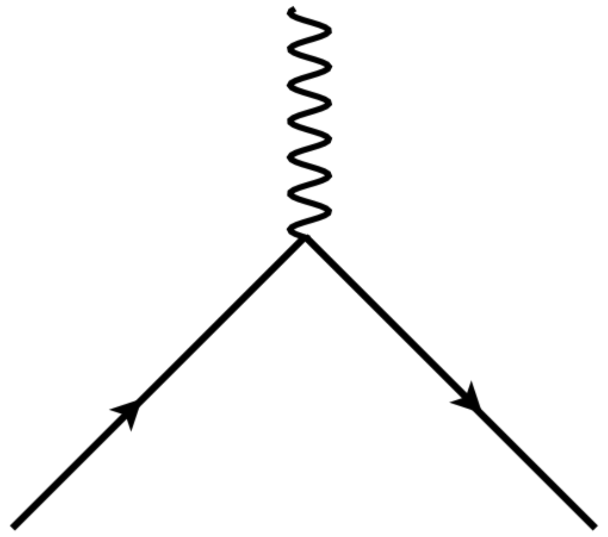

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Quantum corrections modify Dirac's prediction $g = 2$

$$g = 2(1 + a), \quad a :$$ anomalous magnetic moment

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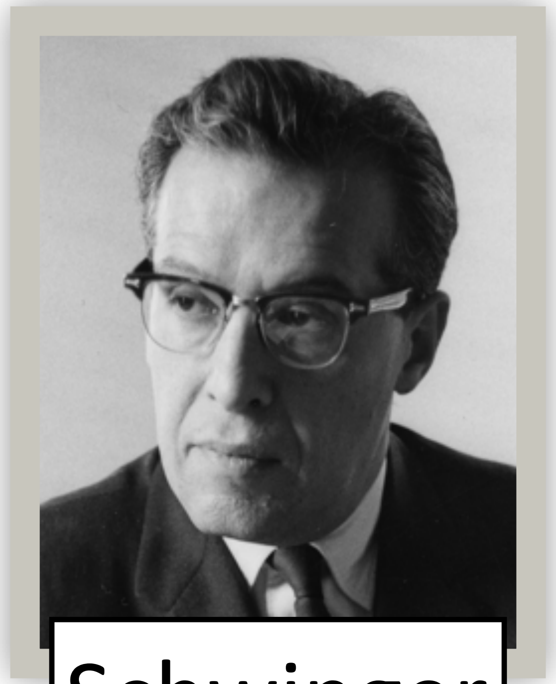
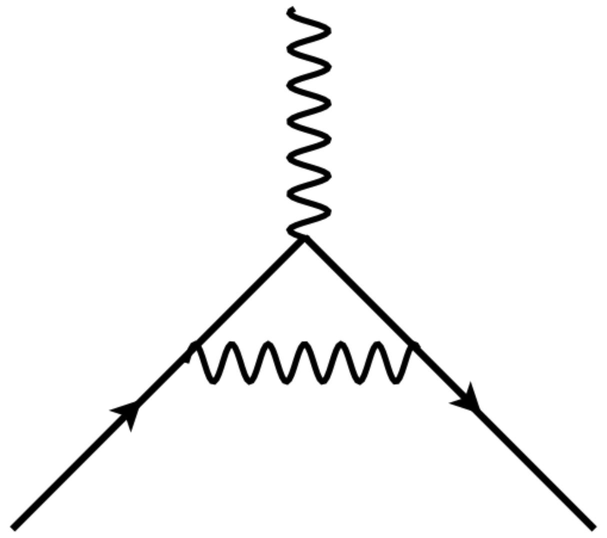


Schwinger

$$g = 2\left(1 + \frac{\alpha}{2\pi}\right)$$



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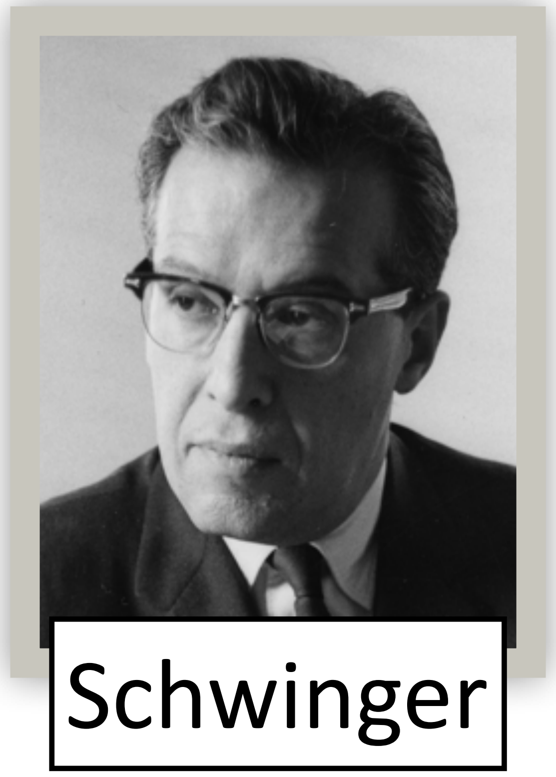

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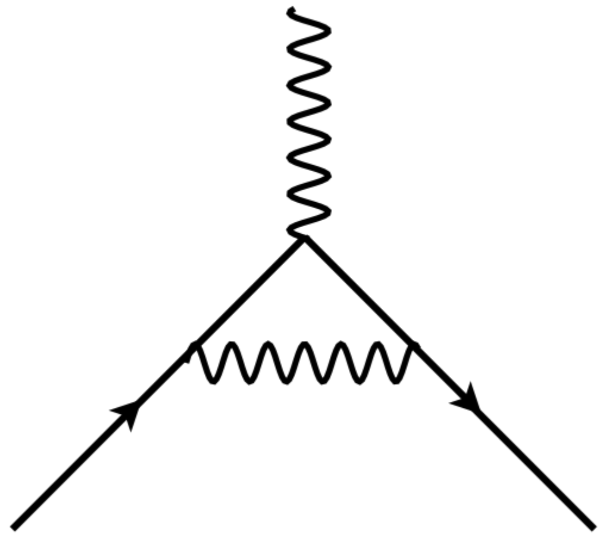

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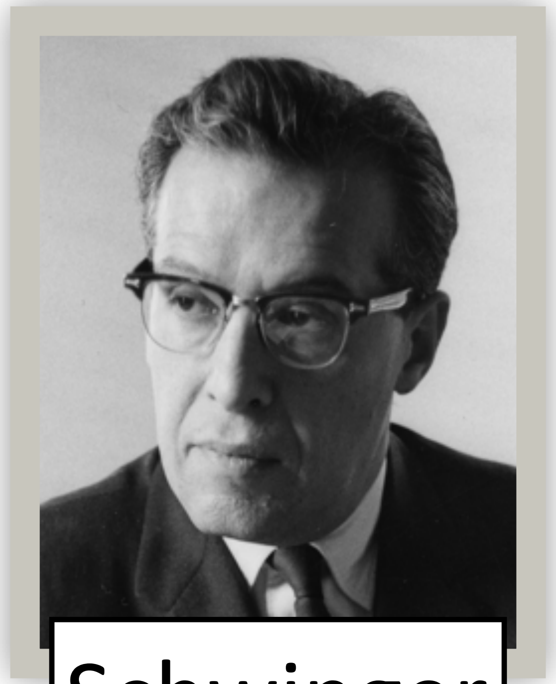
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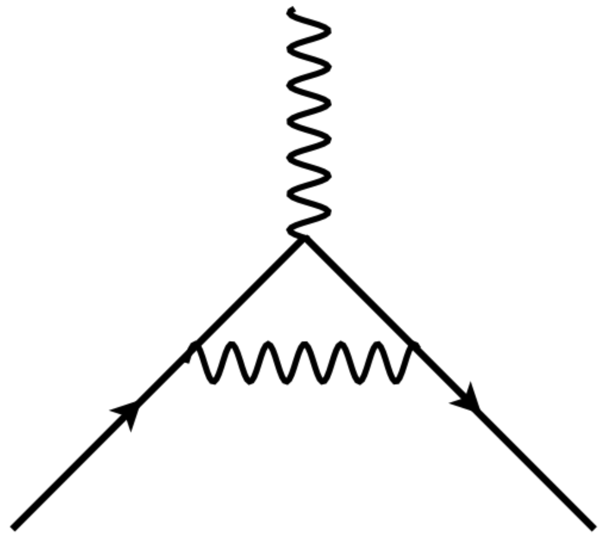

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Beyond leading order: distinct values of a_e, a_μ and a_τ

Muon $g-2$ Theory Initiative

- Founded in 2017
- Agree on common SM prediction
- Focus on hadronic contributions
- White Paper published in 2020
- Update foreseen in early 2025

Mainz 2018

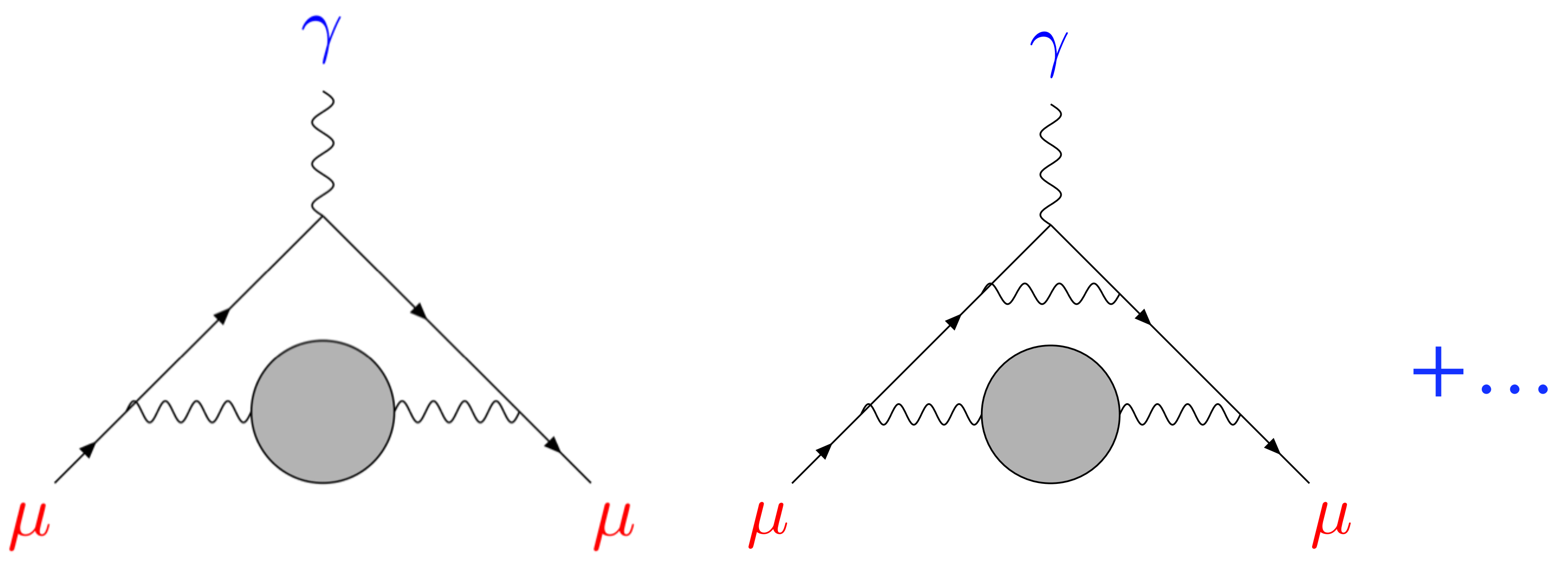

Zoom 2020


Bern 2023

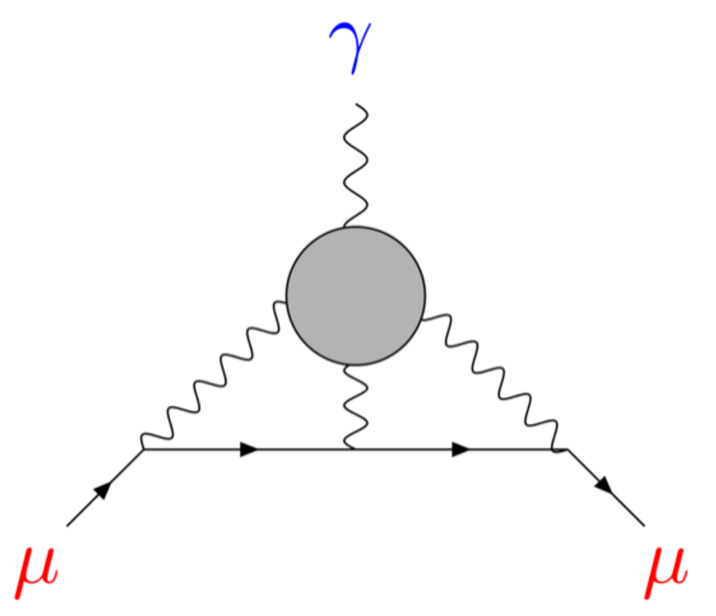
Standard Model prediction for muon $g-2$ [2020 White Paper]

QED:	$116\,584\,718.9\,(1) \times 10^{-11}$	0.001 ppm	
Weak:	$153.6(1.0) \times 10^{-11}$	0.01 ppm	
Hadronic vacuum polarisation:	$6845(40) \times 10^{-11}$	0.34 ppm	$[0.6\%]$
Hadronic light-by-light scattering:	$92(18) \times 10^{-11}$	0.15 ppm	$[20\%]$

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm hvp} + a_\mu^{\rm hlbl} = 116\,591\,810(43) \times 10^{-11} \qquad 0.37\ \text{ppm}$$



Hadronic vacuum polarisation (HVP)



Hadronic light-by-light scattering (HLbL)

[Aoyama et al., Phys. Rep. 887 (2020) 1, arXiv:2006.04822]

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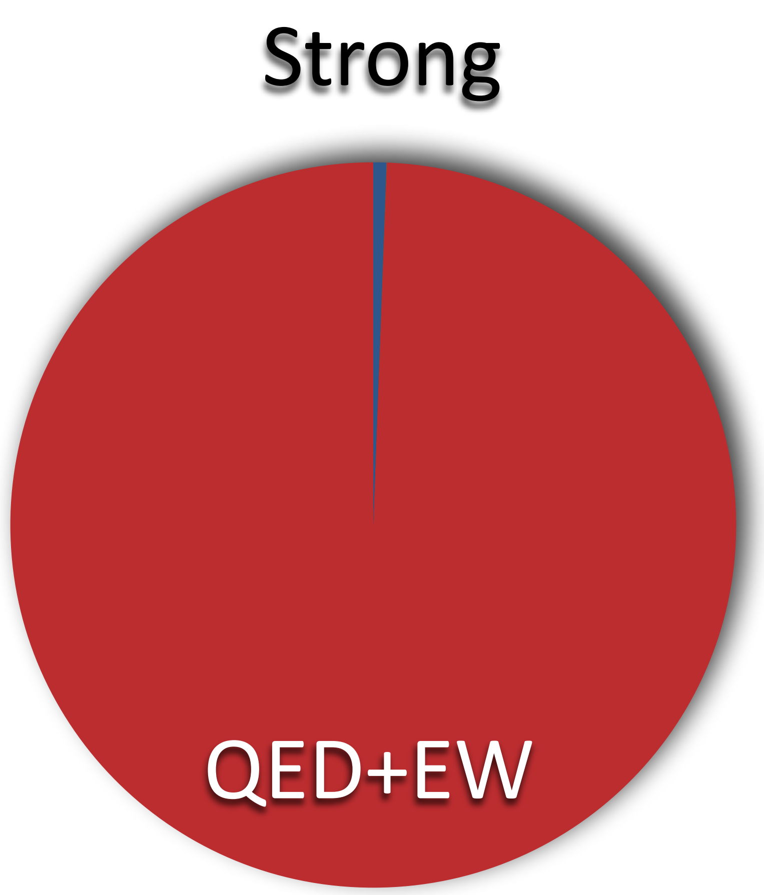
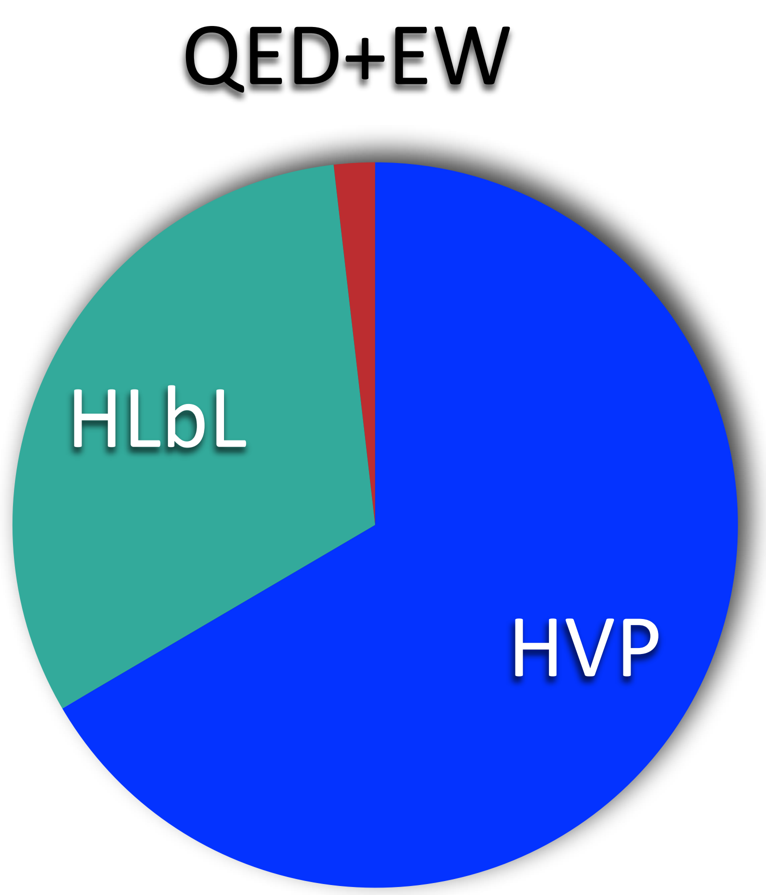
- QED and electroweak contributions account for 99.994% of the SM prediction for a_μ
- Error is dominated by strong interaction effects





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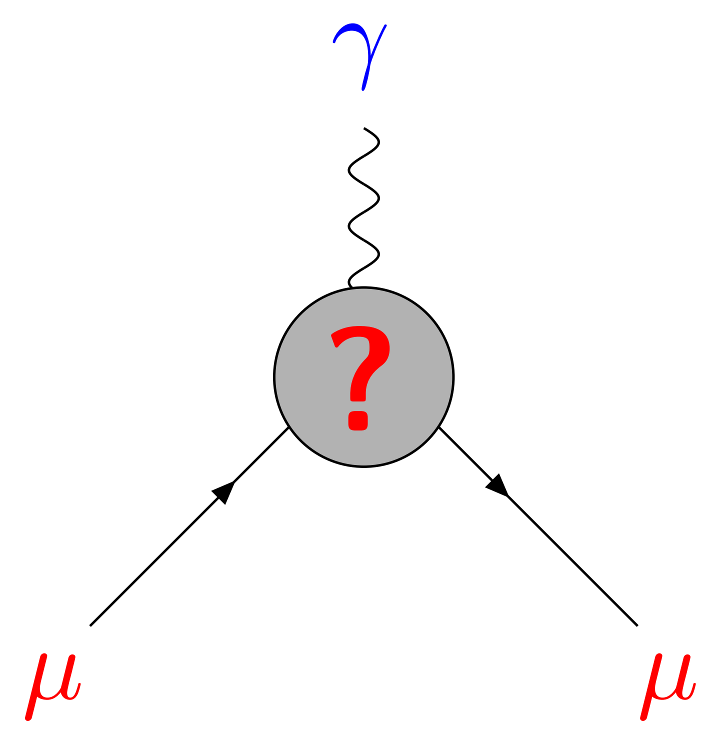

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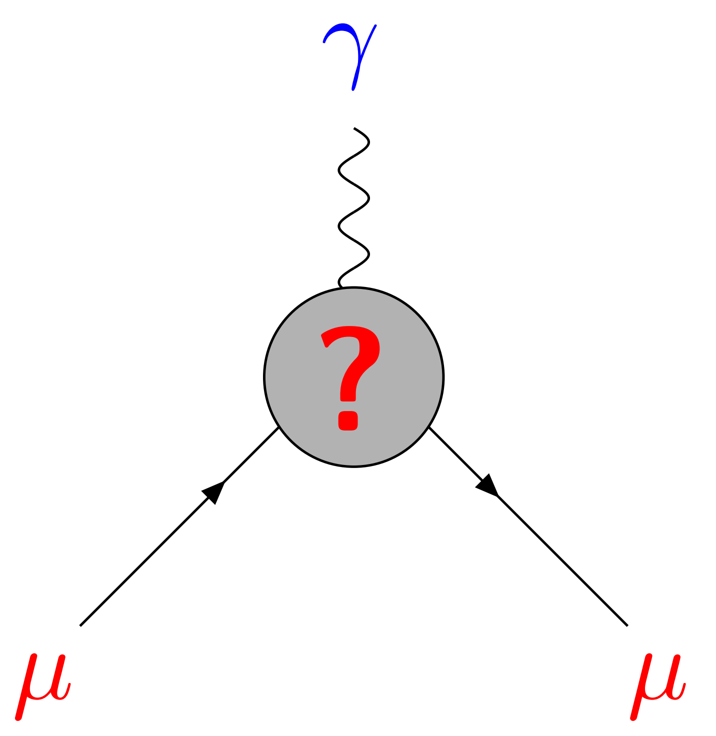


Why the muon?

$$a_\ell^{\rm BSM} \propto m_\ell^2 / M_{\rm BSM}^2 \qquad \ell = e,\, \mu,\, \tau$$

$\rightarrow$ sensitivity of a_μ enhanced by $(m_\mu/m_e)^2 \approx 4.3 \times 10^4$ relative to a_e

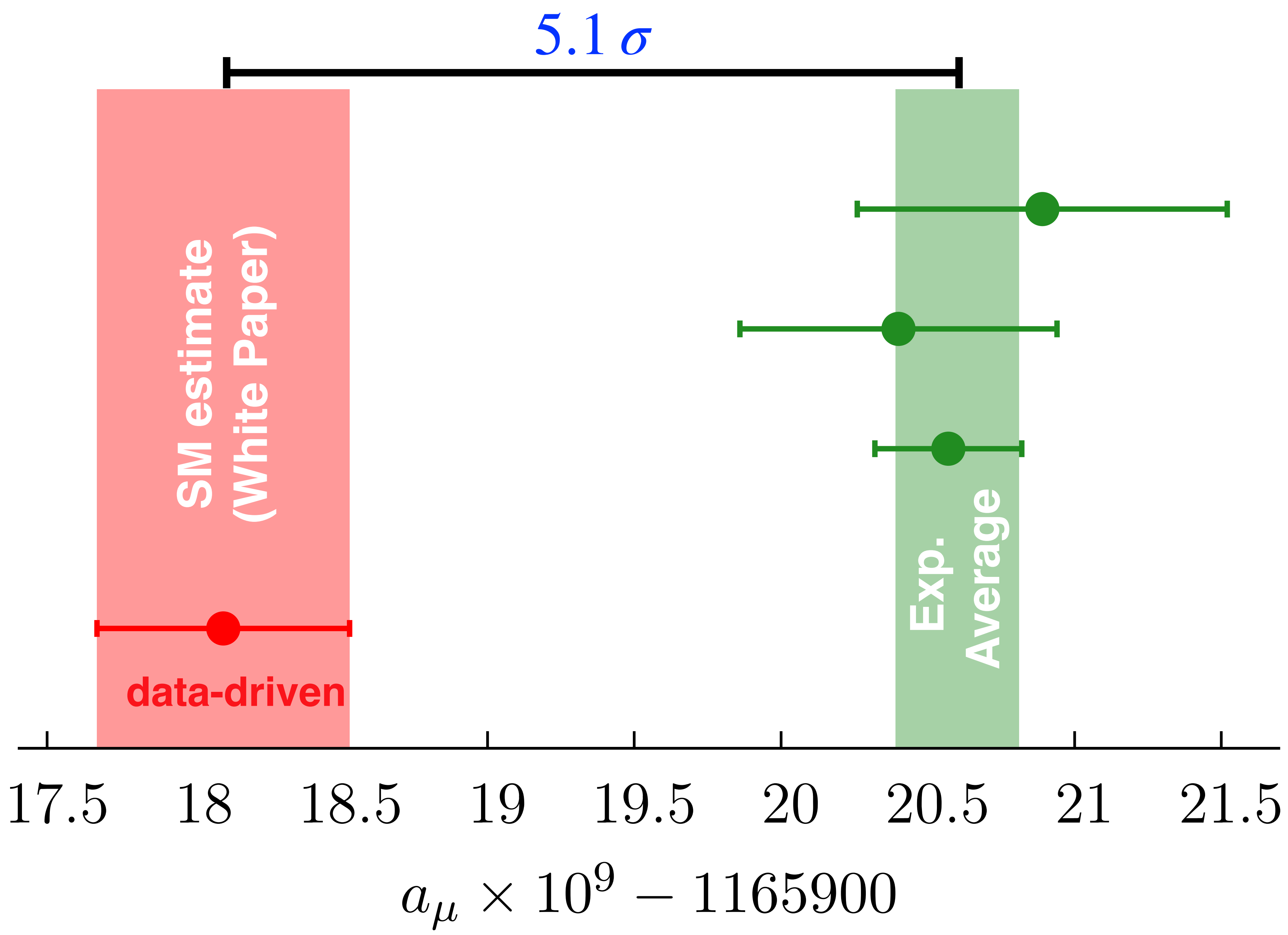
New Physics on the horizon?

Confronting the SM prediction with the E989 measurement

$$a_\mu^{\rm exp} = 116\,592\,049(22) \times 10^{-11} \quad [0.19\,\text{ppm}]$$

$$a_\mu^{\rm SM} = 116\,591\,810(43) \times 10^{-11} \quad [0.37\,\text{ppm}]$$

$$\Rightarrow \quad a_\mu^{\rm exp} - a_\mu^{\rm SM} = (249 \pm 48) \cdot 10^{-11} \quad [5.1\sigma]$$



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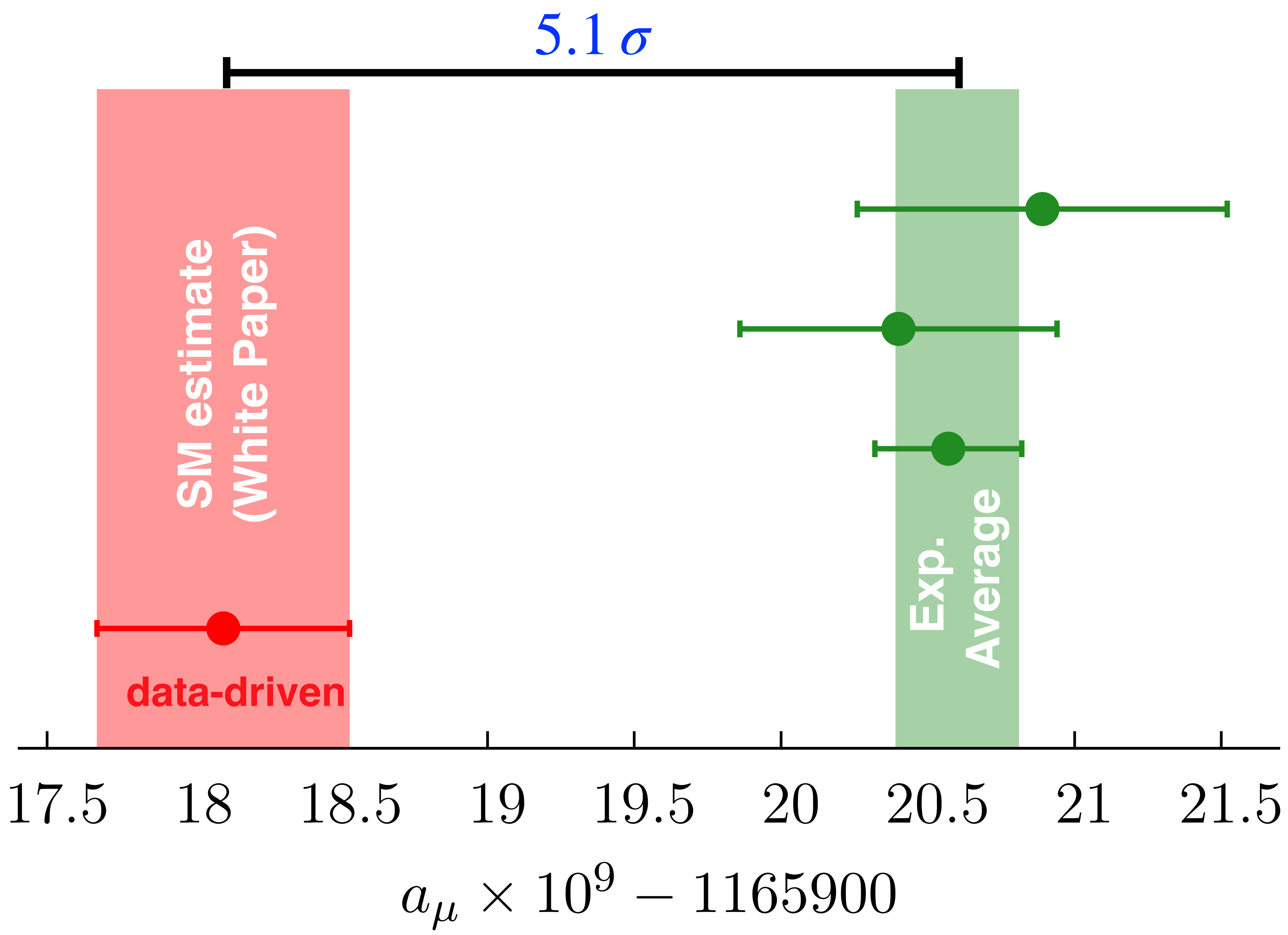

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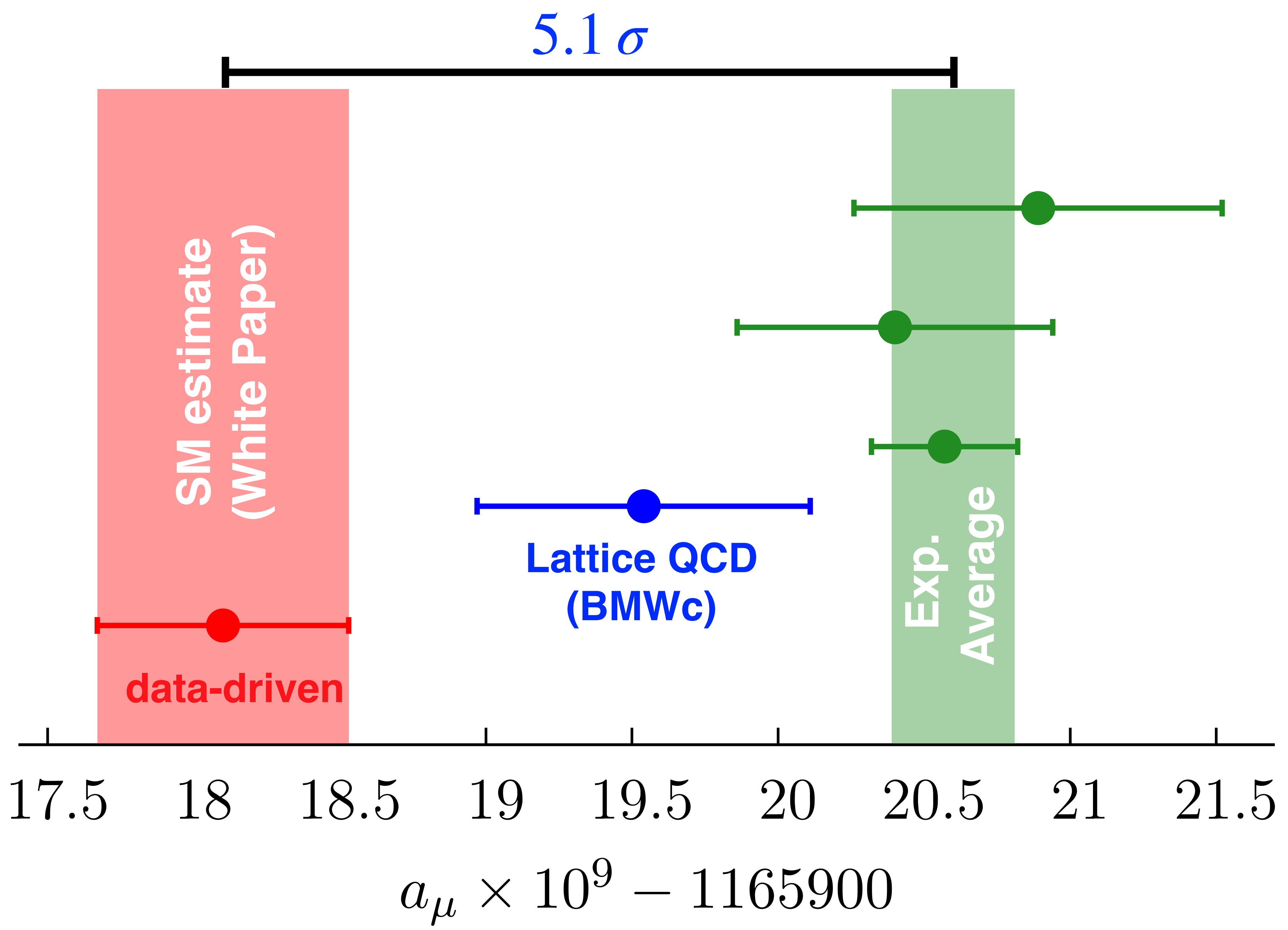

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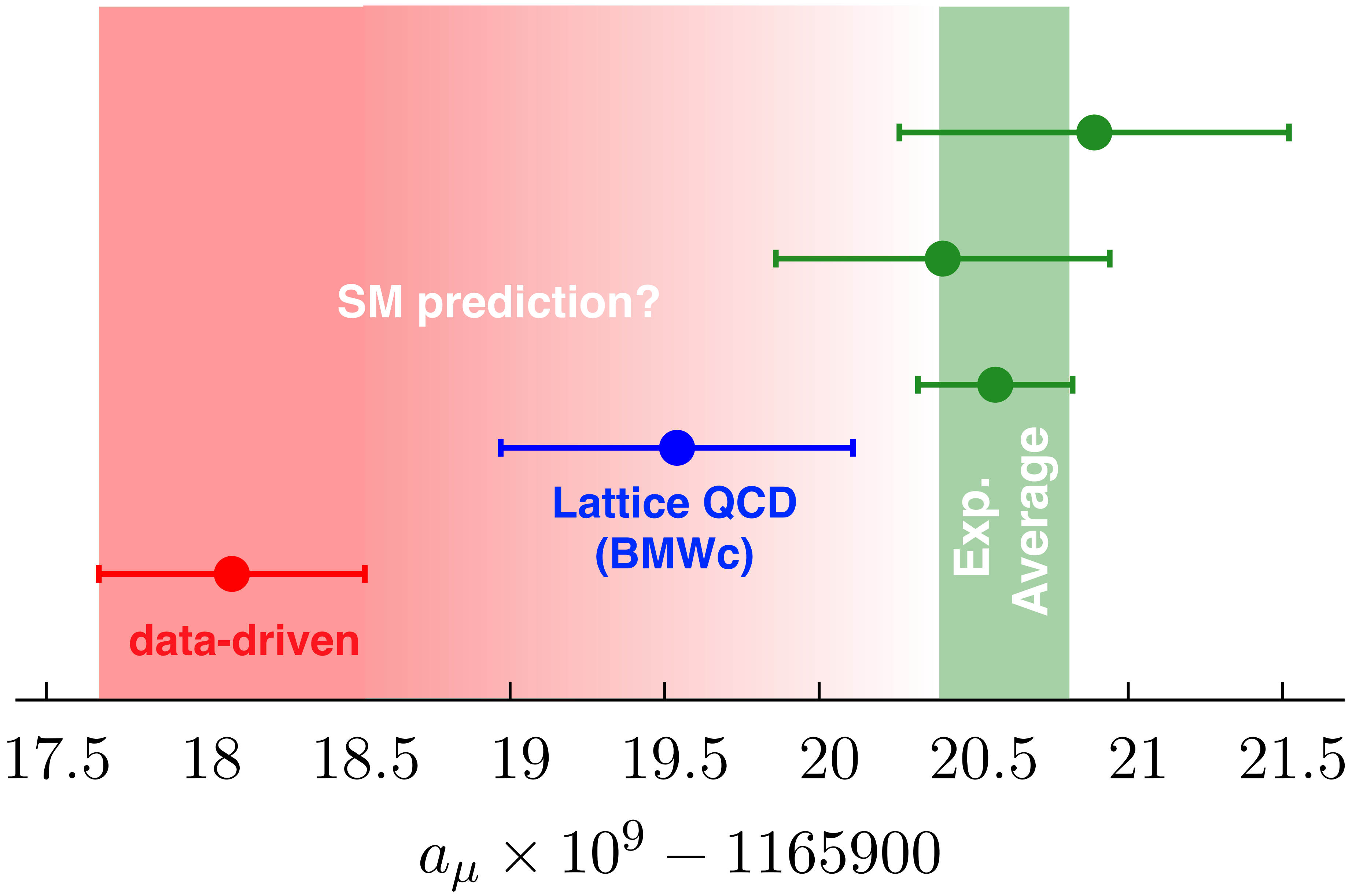

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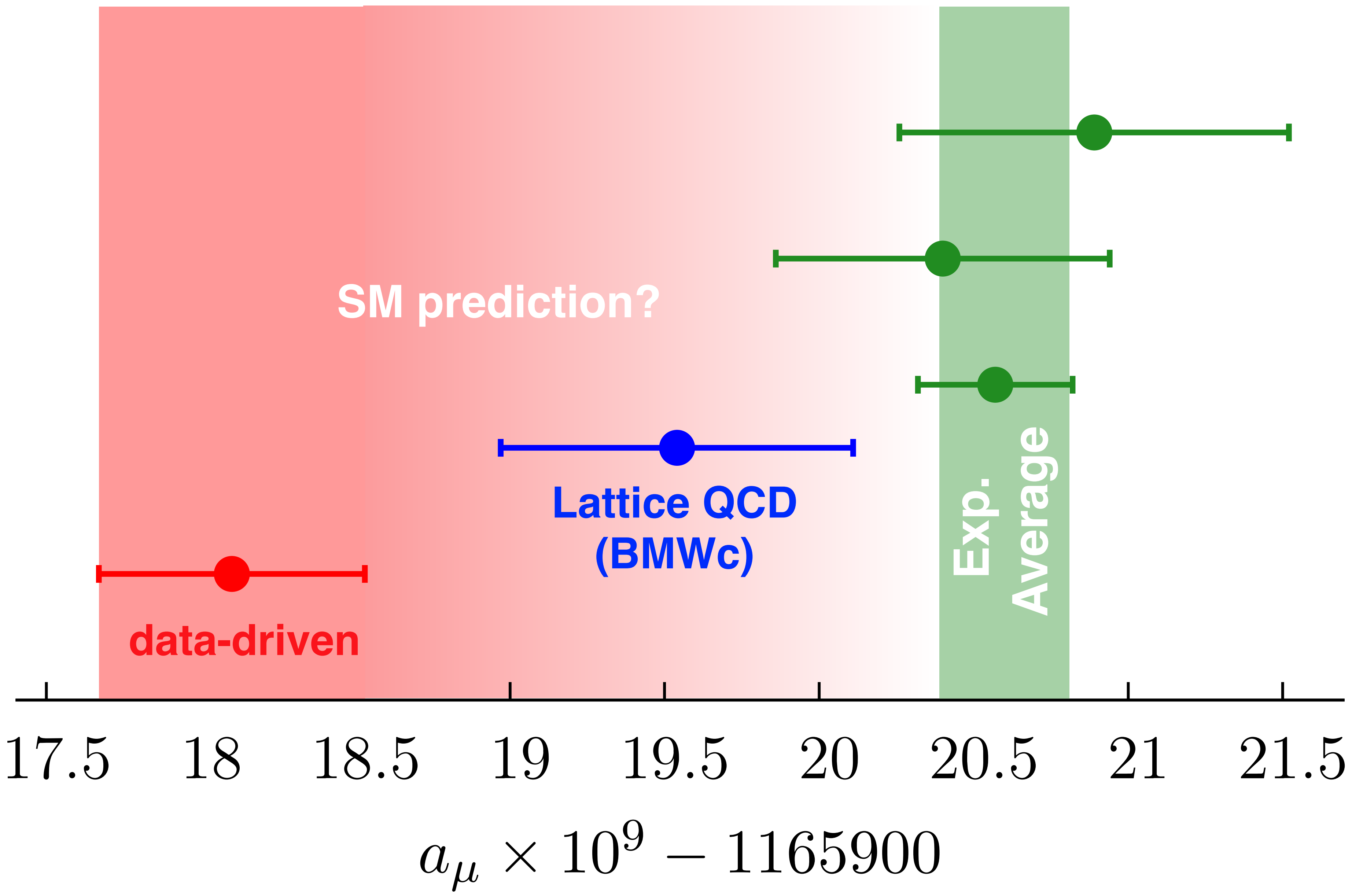

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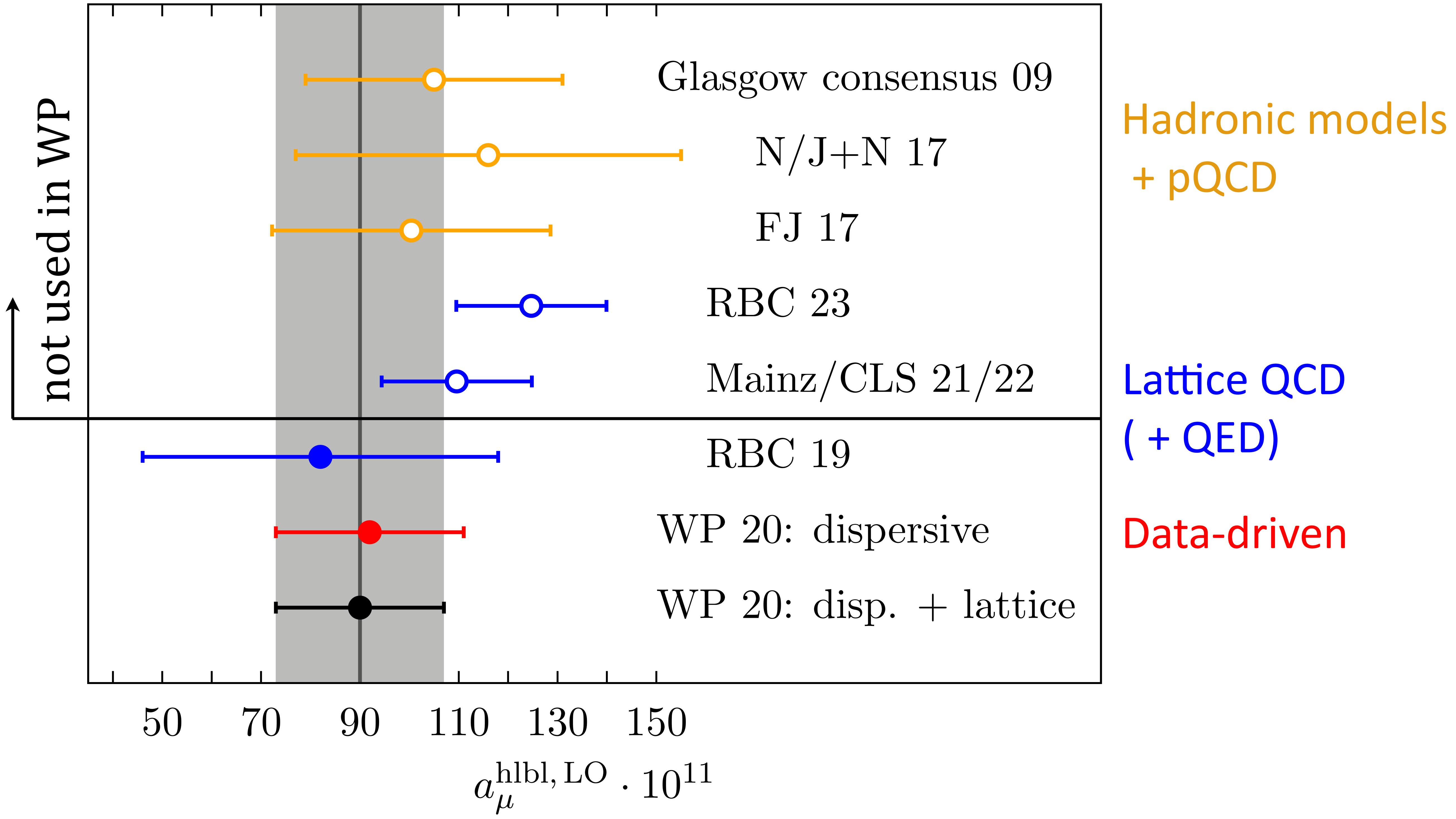
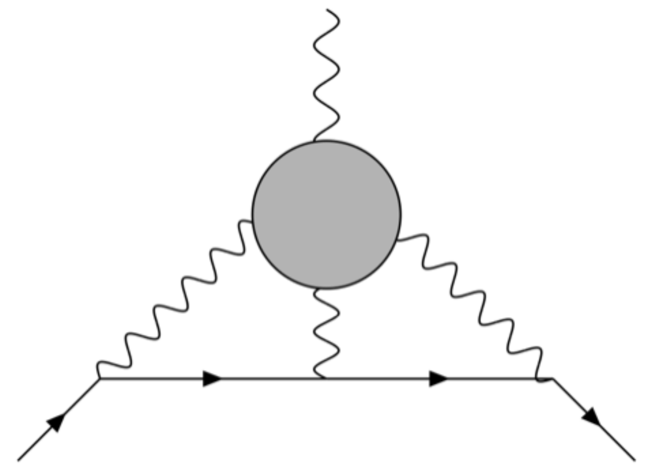
Requires independent confirmation

Hadronic light-by-light scattering

[Aoyama et al., Phys. Rep. 887 (2020) 1; Colangelo et al., arXiv:2203.15810]





Hadronic models, data-driven method and Lattice QCD produce compatible results

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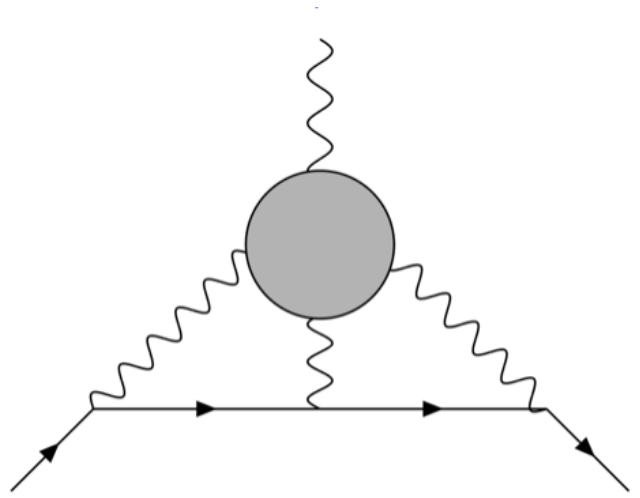

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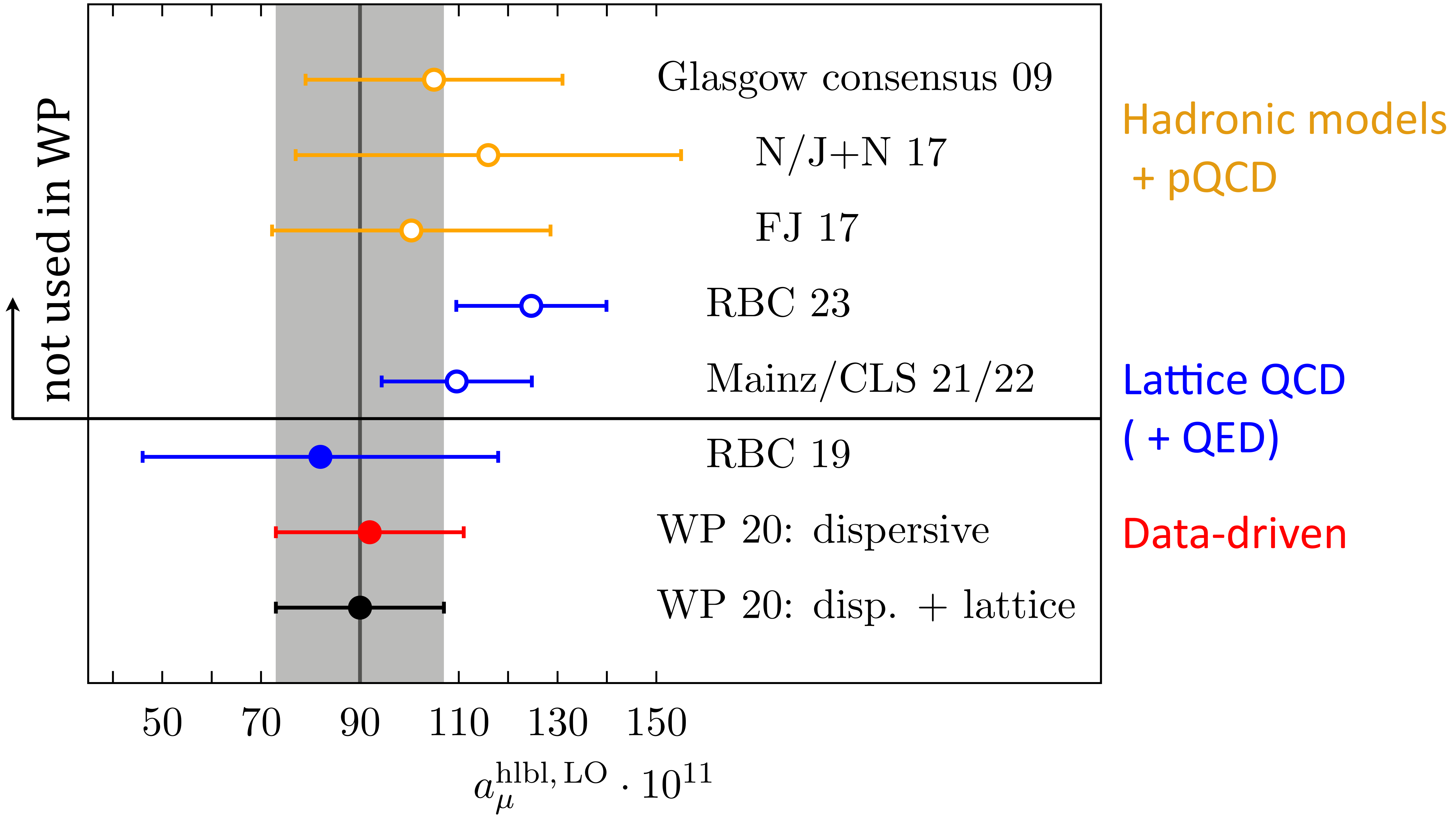

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Recent lattice calculations:

$$a_\mu^{\mathrm{hlbl,\,LO}} = \begin{cases} (109.6 \pm 14.7) \cdot 10^{-11} & \text{Mainz/CLS} \\ (124.7 \pm 15.2) \cdot 10^{-11} & \text{RBC} \end{cases}$$

[Chao et al., EPJC 81 (2021) 651; EPJC 82 (2022) 664; Blum et al., arXiv:2304.04423]

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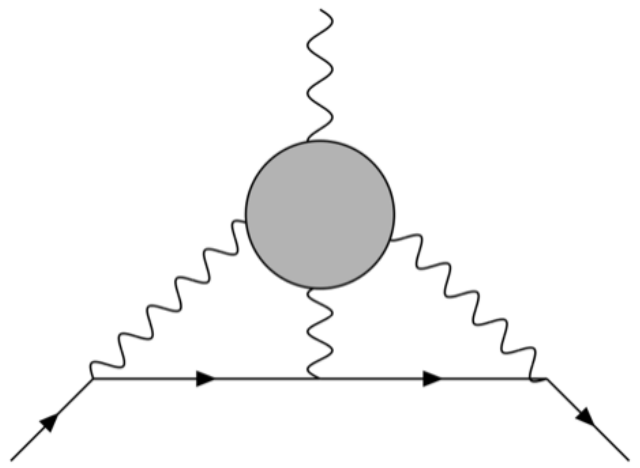

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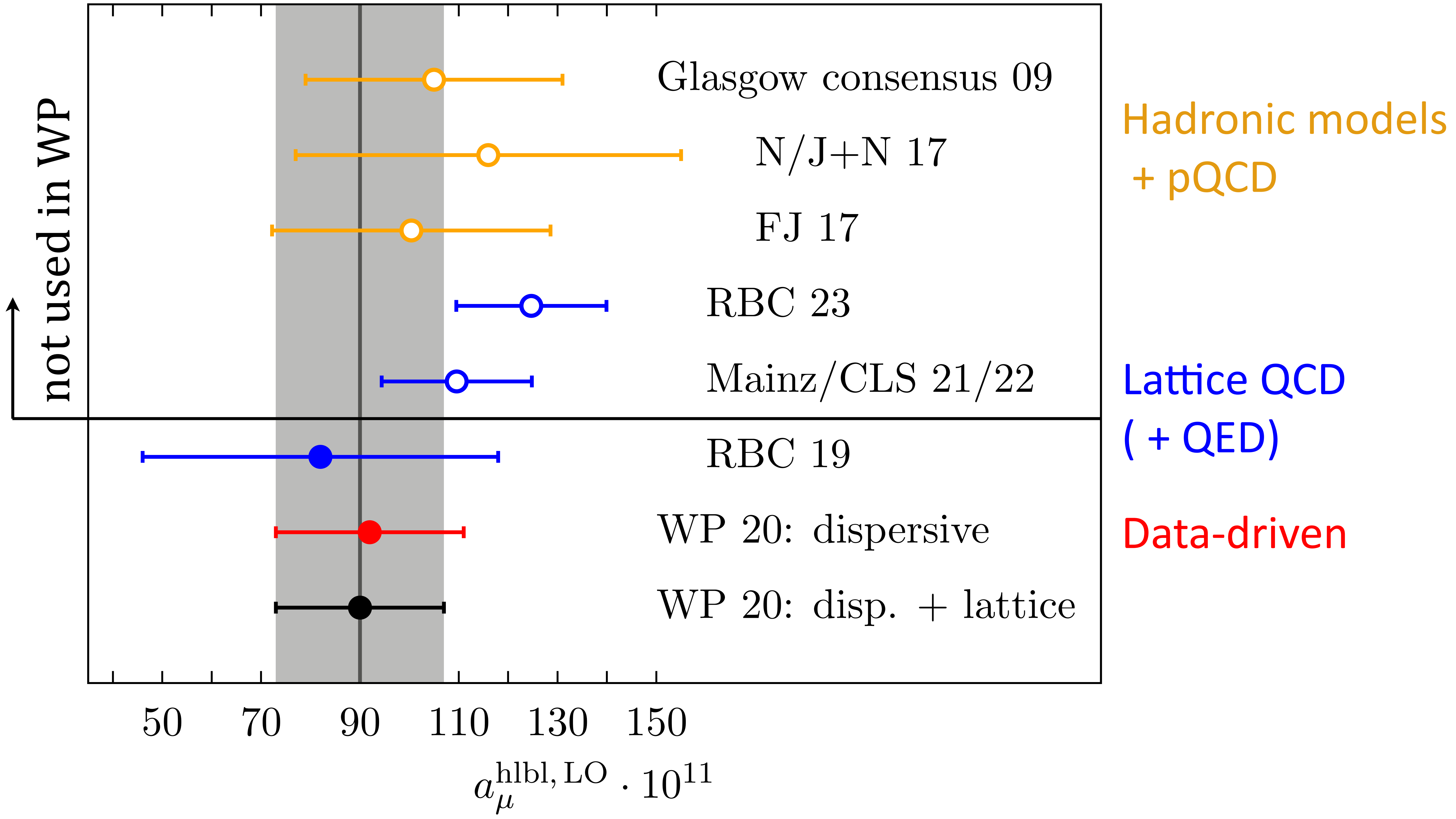

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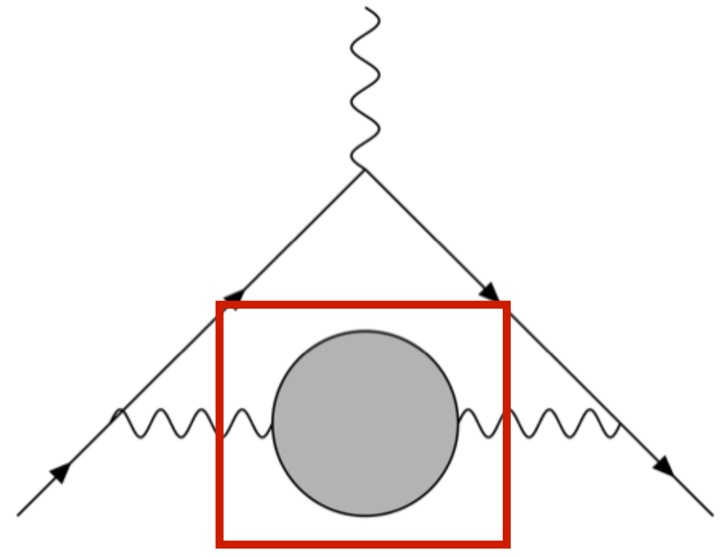
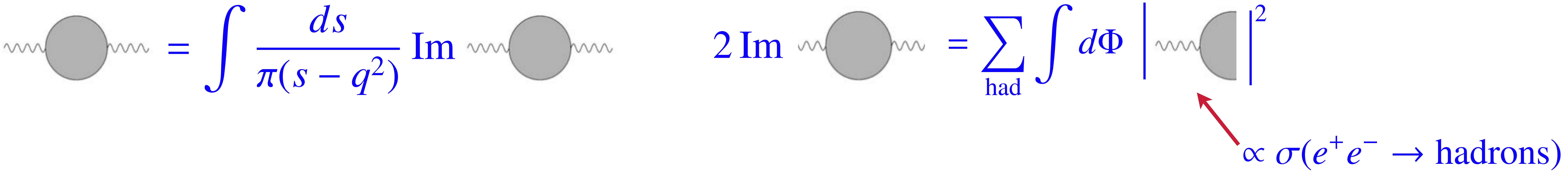
a_μ^{hlbl} : **Uncontroversial** — contributes $0.15\,\text{ppm}$ to the total SM uncertainty of $0.37\,\text{ppm}$

$\rightarrow$ Focus on refinements and further reduction of uncertainty

Hadronic vacuum polarisation from dispersion theory



Analyticity, unitarity & optical theorem imply:



$$a_\mu^{\text{hvp, LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds \, \frac{R_{\text{had}}(s)\hat{K}(s)}{s^2}, \qquad R_{\text{had}}(s) = \frac{3s}{4\pi\,(\alpha(s))^2}\,\sigma(e^+e^- \to \text{hadrons})$$

"R-ratio"

Hadronic effects cannot be treated in perturbation theory

- Use experimental data for $R_{\text{had}}(s)$ in the low-energy regime ("data-driven approach")
- Standard Model prediction is subject to experimental uncertainties

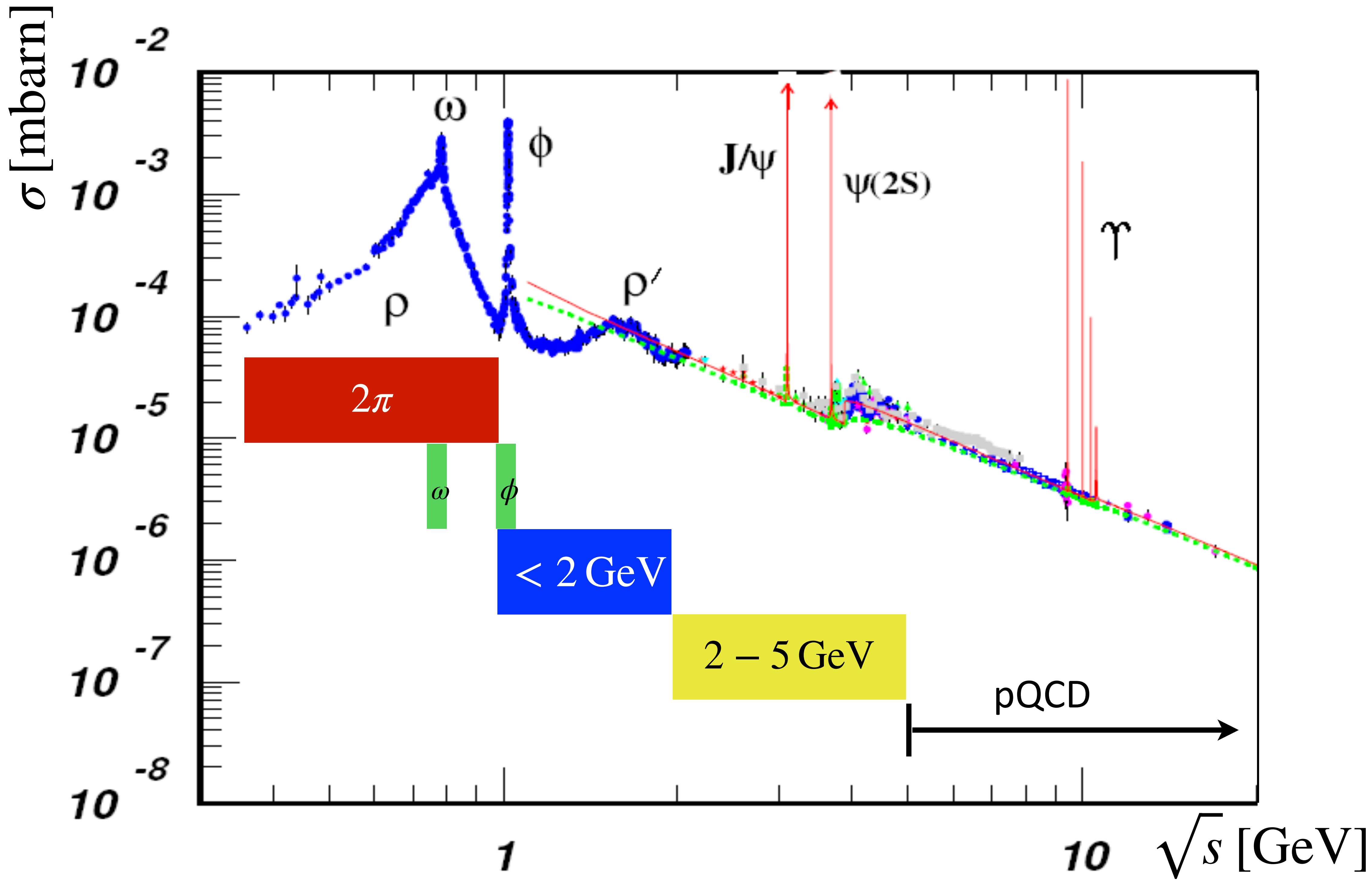
Hadronic vacuum polarisation: Data-driven approach

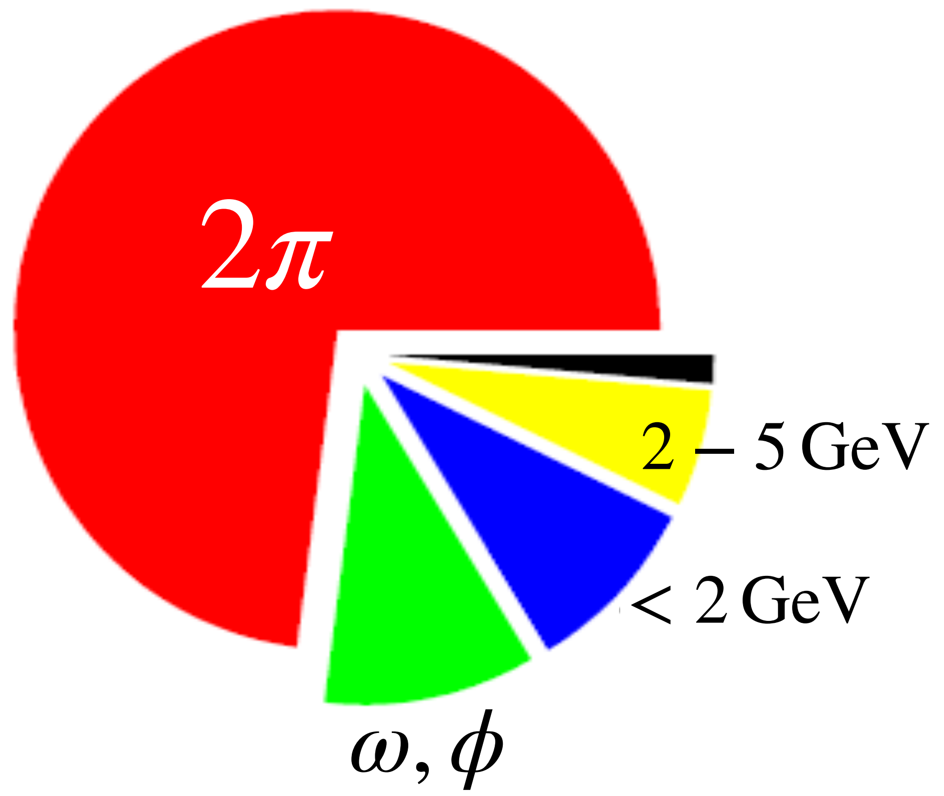
Decade-long effort to measure e^+e^- cross sections

$$a_\mu^{\rm hvp,\,LO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R_{\rm had}(s)\hat{K}(s)}{s^2}$$

$\sqrt{s} \lesssim 2\,\mathrm{GeV}$: sum of exclusive channels

$\sqrt{s} > 2\,\mathrm{GeV}$: inclusive channels, narrow resonances, perturbative QCD





Two-pion channel contributes $\approx 70\,\%$

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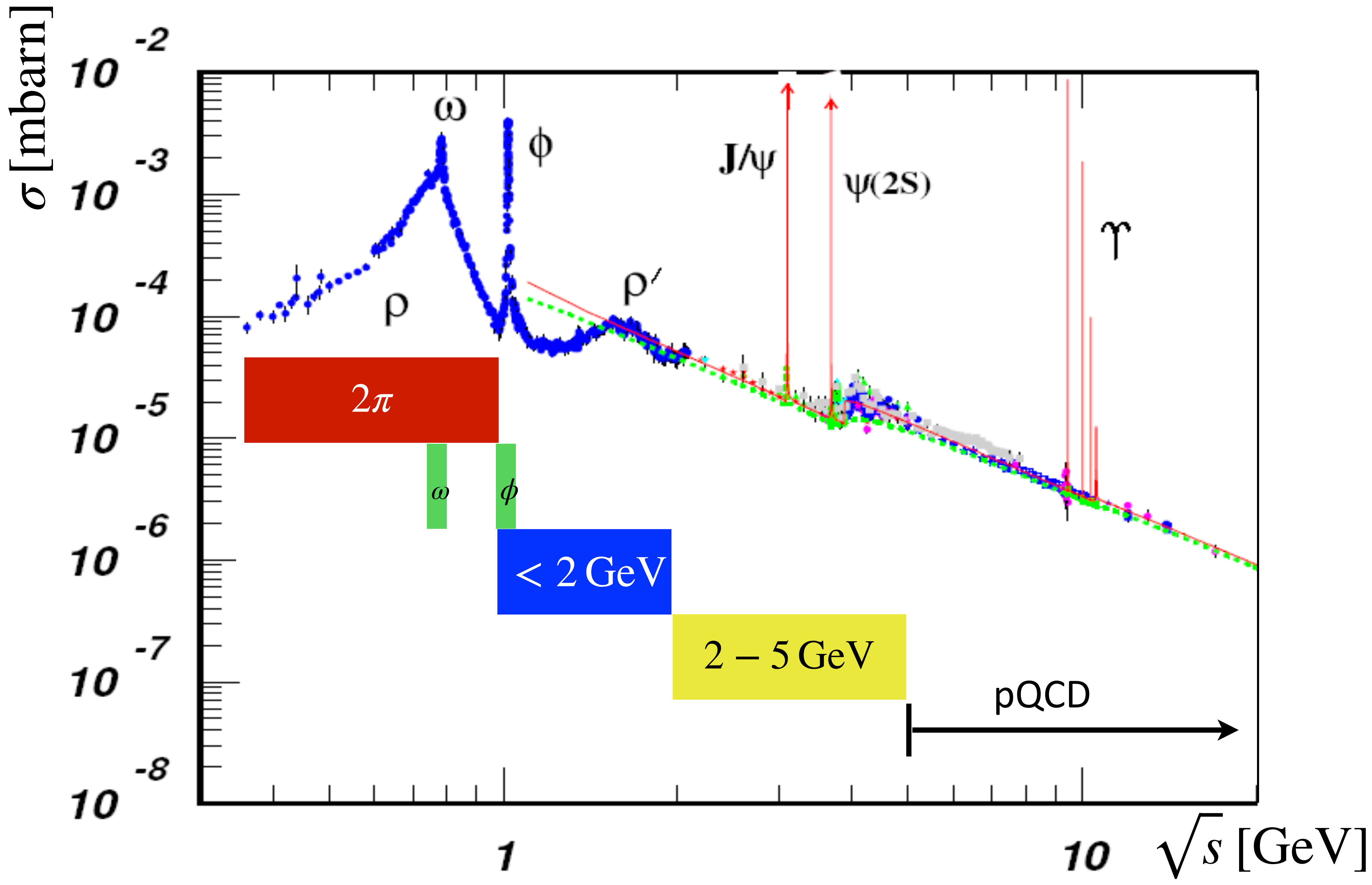

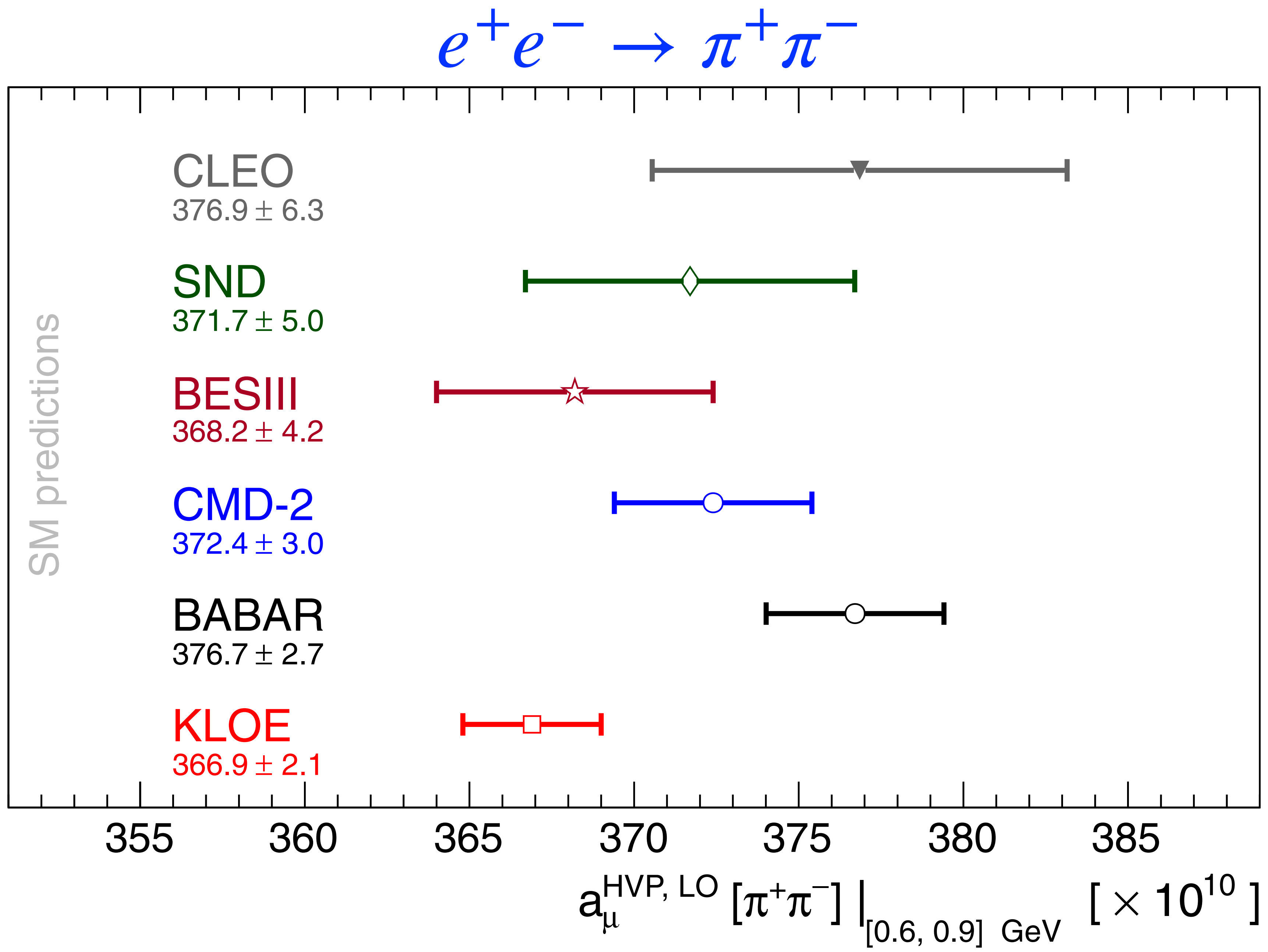

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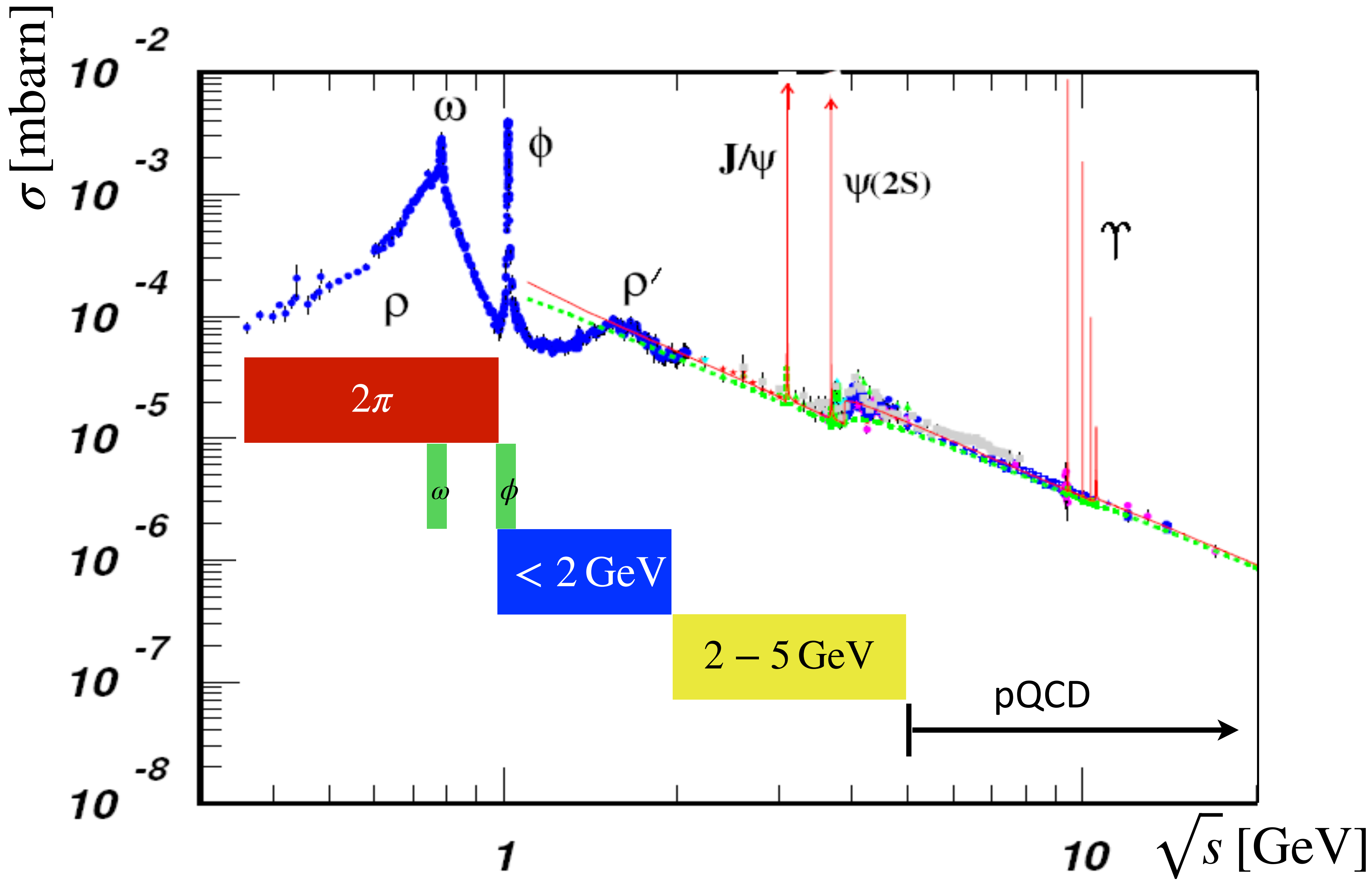
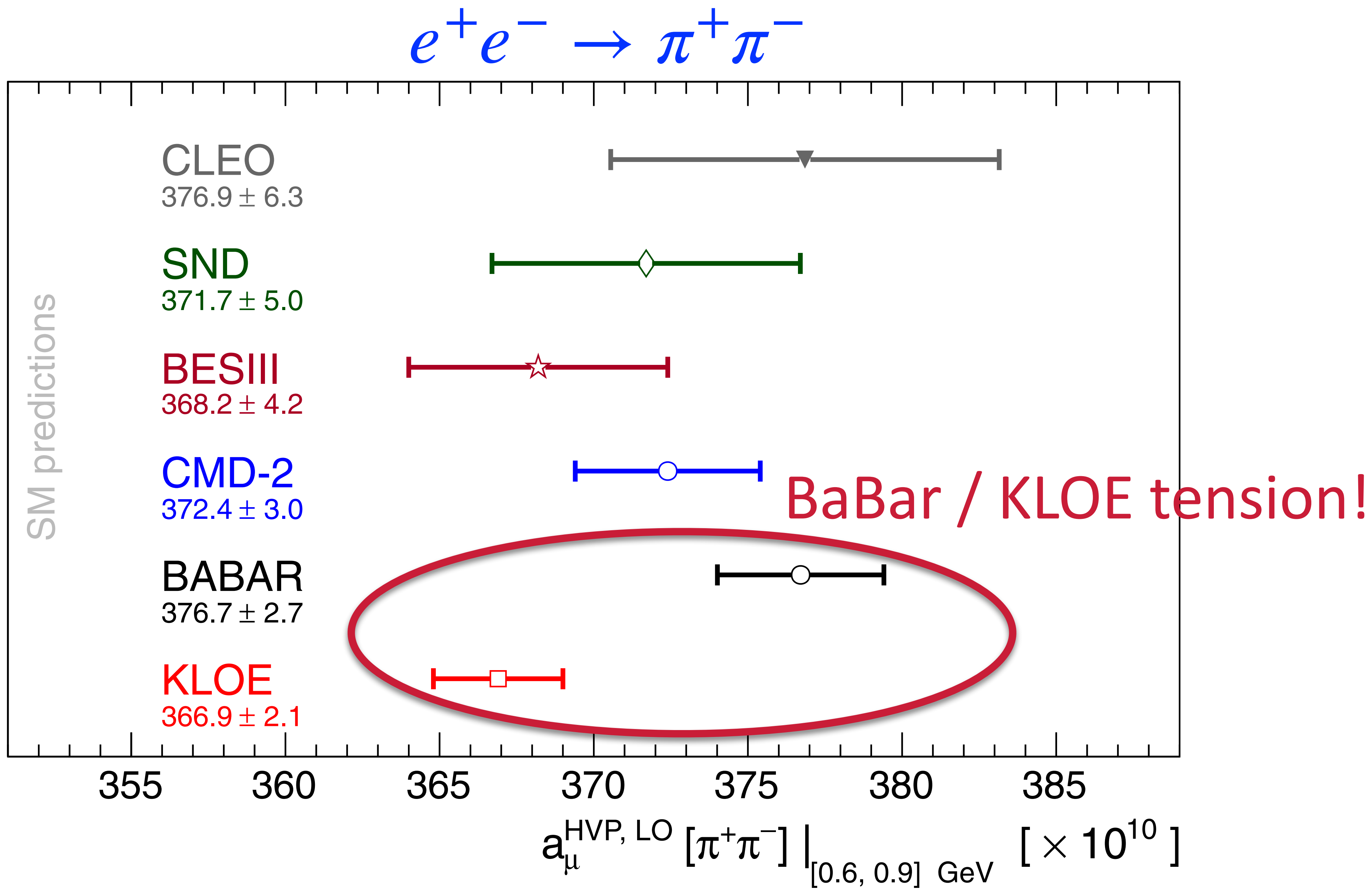
$$a_\mu^{\mathrm{hvp,\,LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R_{\mathrm{had}}(s)\hat{K}(s)}{s^2}$$

- $\sqrt{s} \lesssim 2\,\mathrm{GeV}$: sum of exclusive channels
- $\sqrt{s} > 2\,\mathrm{GeV}$: inclusive channels, narrow resonances, perturbative QCD





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	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\rm HVP,\ LO}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\rm DV+QCD}$	692.8(2.4)	1.2

[Aoyama et al., Phys. Rep. 887 (2020) 1, arXiv:2006.04822]

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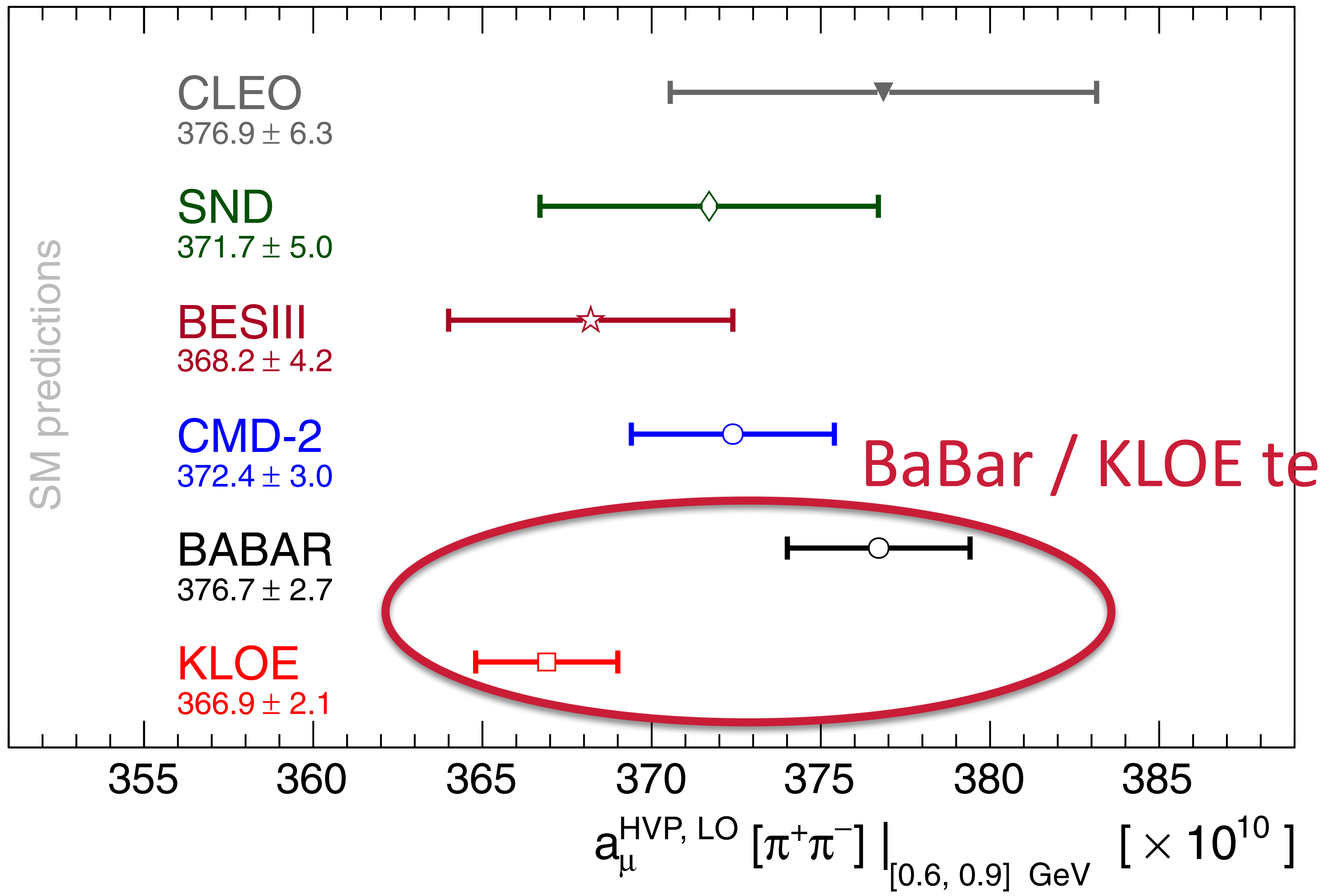
- White Paper recommended value (2020):

$$a_\mu^{\rm hvp,\,LO} = 693.1(2.8)_{\rm exp}(2.8)_{\rm syst}(0.7)_{\rm DV+QCD} \times 10^{-10}$$

$$= 693.1(4.0) \times 10^{-10} \qquad [0.6\%]$$

(accounts for tensions in the data and differences between analyses)

$e^+e^- \to \pi^+\pi^-$



BaBar / KLOE tension!

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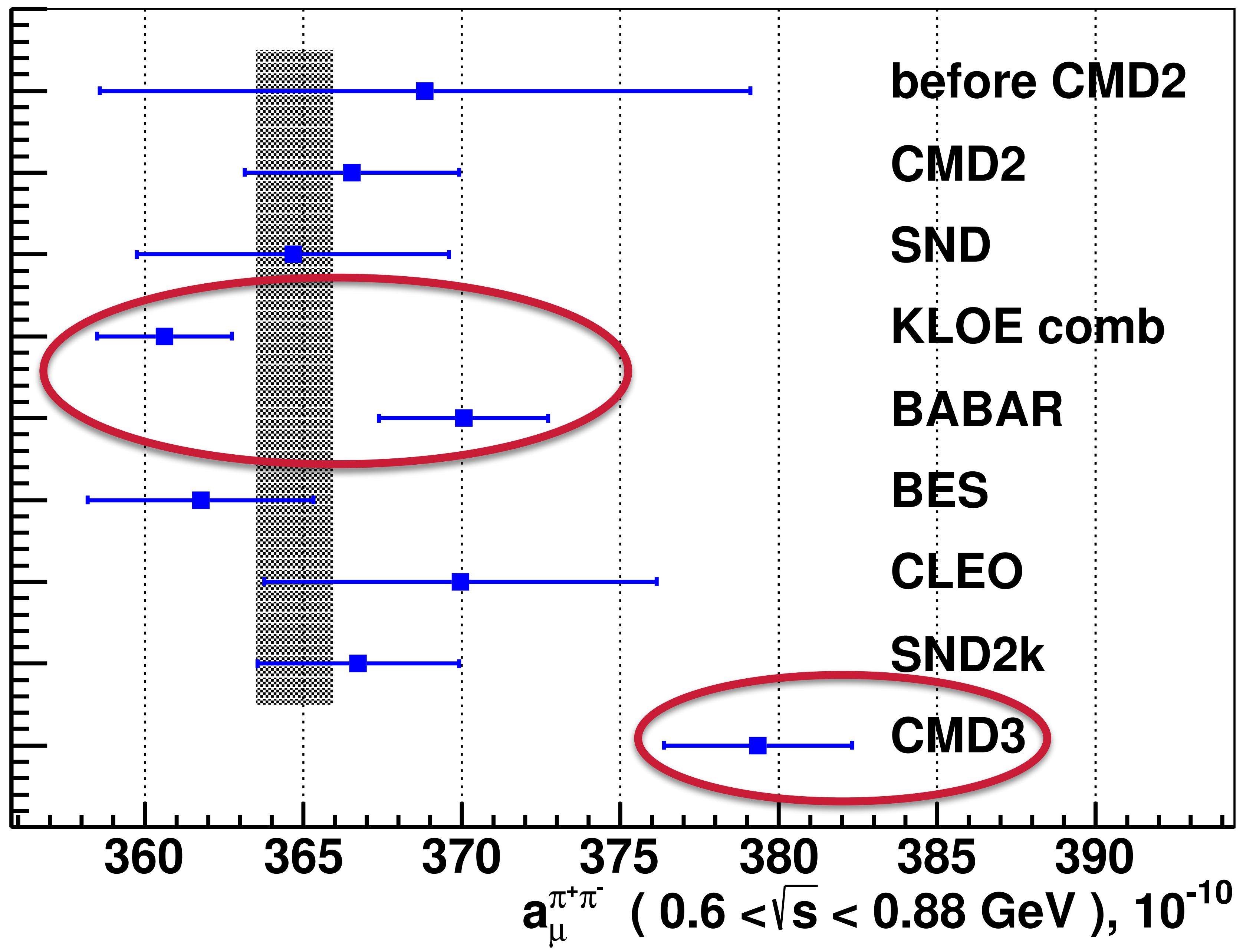
- Recent results in the $\pi^+\pi^-$ channel by CMD-3:

$\rightarrow$ further tension among e^+e^- data

$$a_\mu^{\rm hvp,\,LO} = 707.6(3.4)_{\rm exp}(0.7)_{\rm DV+QCD} \times 10^{-10}$$

(my own estimate)



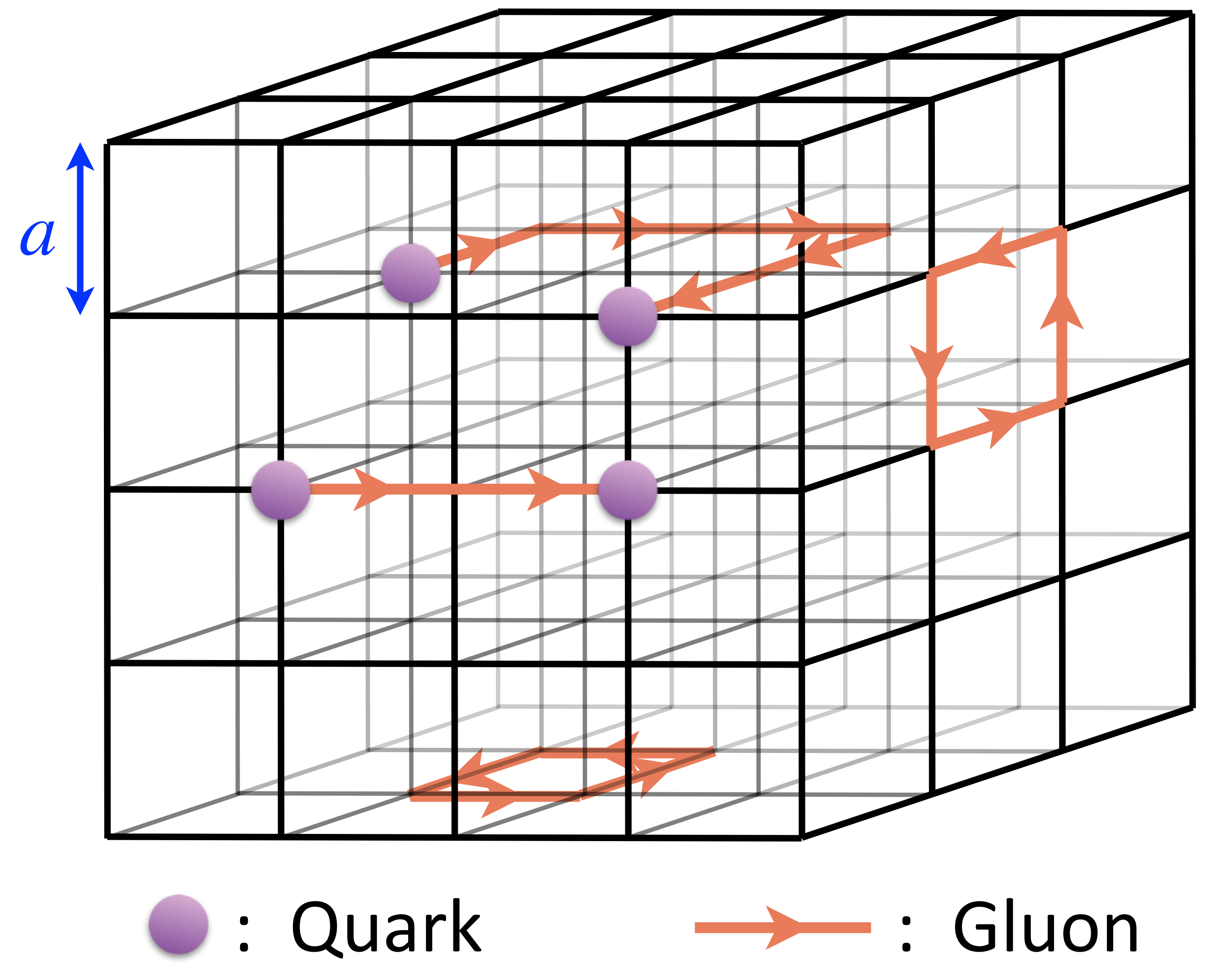
[Ignatov et al. (CMD-3 Collab.), Phys. Rev. D109 (2024) 112002]

Lattice QCD

Non-perturbative treatment of strong interaction via regularised Euclidean path integrals

Lattice spacing: $a, \quad x_\mu = n_\mu a, \quad a^{-1} = \Lambda_{\mathrm{UV}}$

Expectation value: $\langle \Omega \rangle = \frac{1}{Z} \int \prod_{x,\mu} dU_\mu(x)\, \Omega\, \mathrm{e}^{-S_{\mathrm{G}}^{\mathrm{eff}}[U]}$



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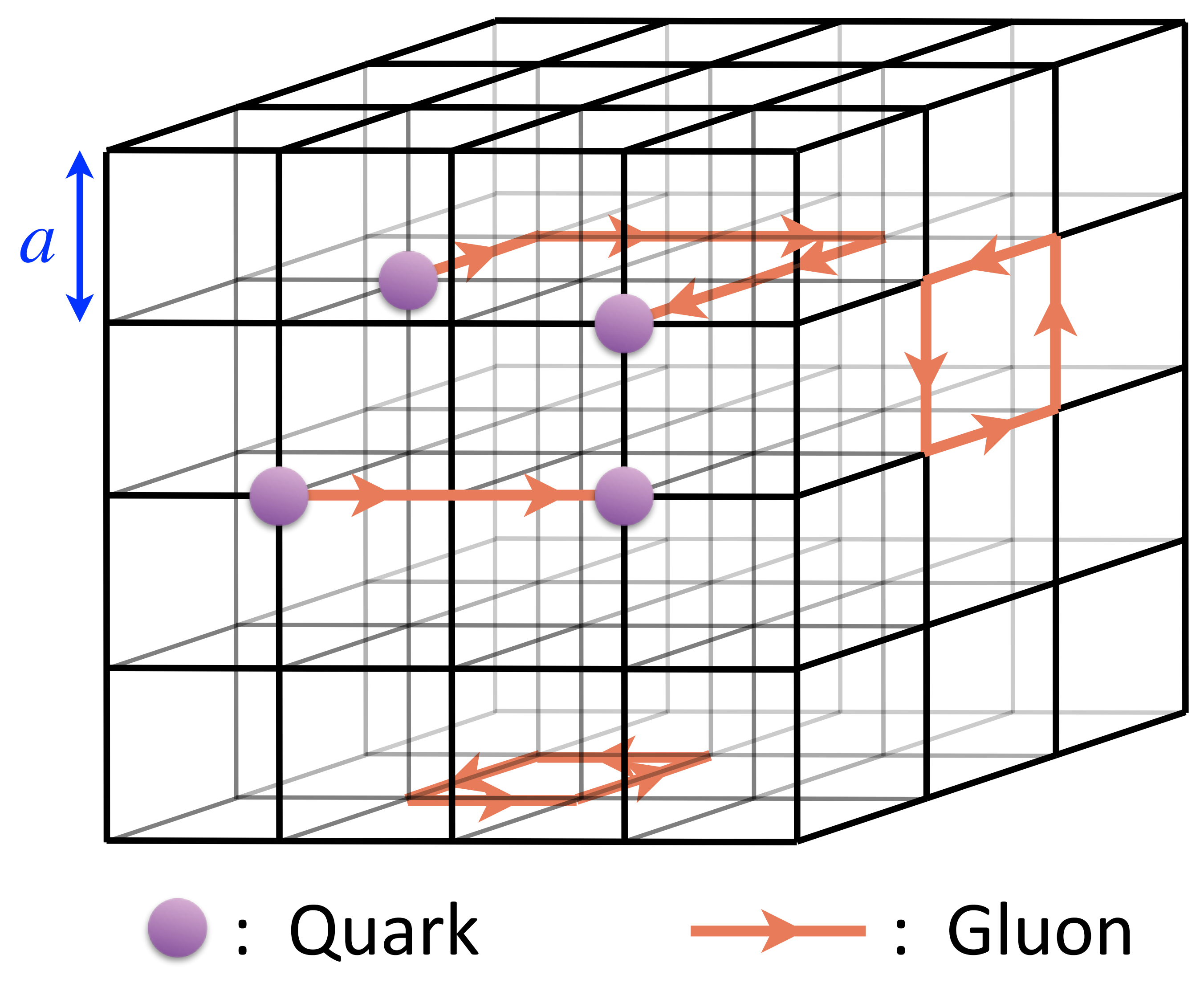


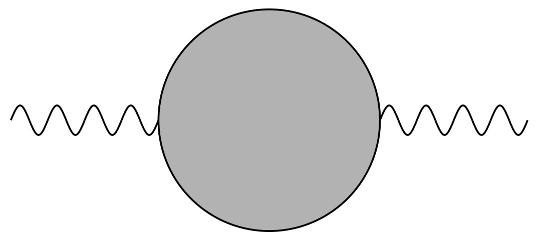
Procedure:

- Choose discretisation of QCD action
- Evaluate $\langle\Omega\rangle$ via Monte Carlo Integration:
  generate ensembles of gauge configurations via a Markov chain
- Ensemble average: $\langle\Omega\rangle \simeq \overline{\Omega}$ Statistical error: $\sqrt{\overline{\Omega^2} - \overline{\Omega}^2} \propto 1/N_{\rm cfg}^{1/2}$
- Extrapolate observables to the continuum limit: $a \to 0$ and tune quark masses to physical values

Hadronic vacuum polarisation from Lattice QCD

Lattice QCD does NOT determine the R-ratio from first principles

Time-momentum representation (TMR): *[Bernecker & Meyer EPJA 47 (2011) 148]*



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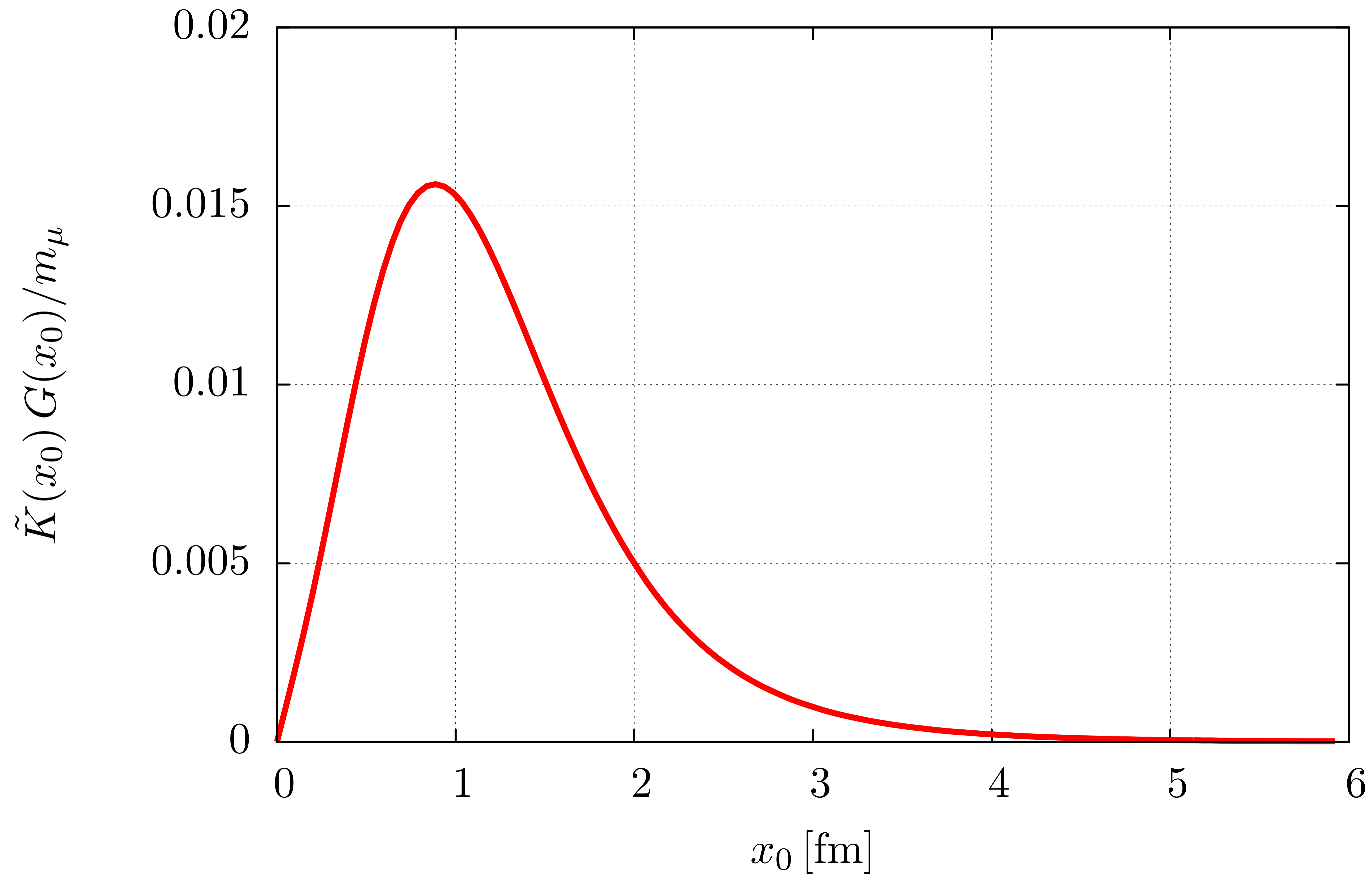
($\tilde{K}(t)$: known analytically)

- No reliance on experimental data, except for simple input quantities → scale setting, calibration
- Not sensitive to exclusive hadronic channels



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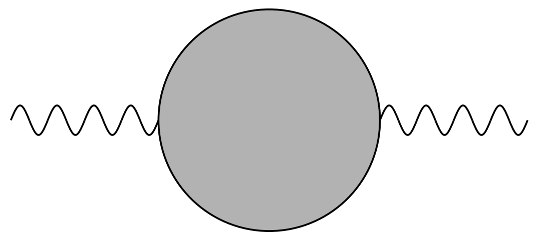

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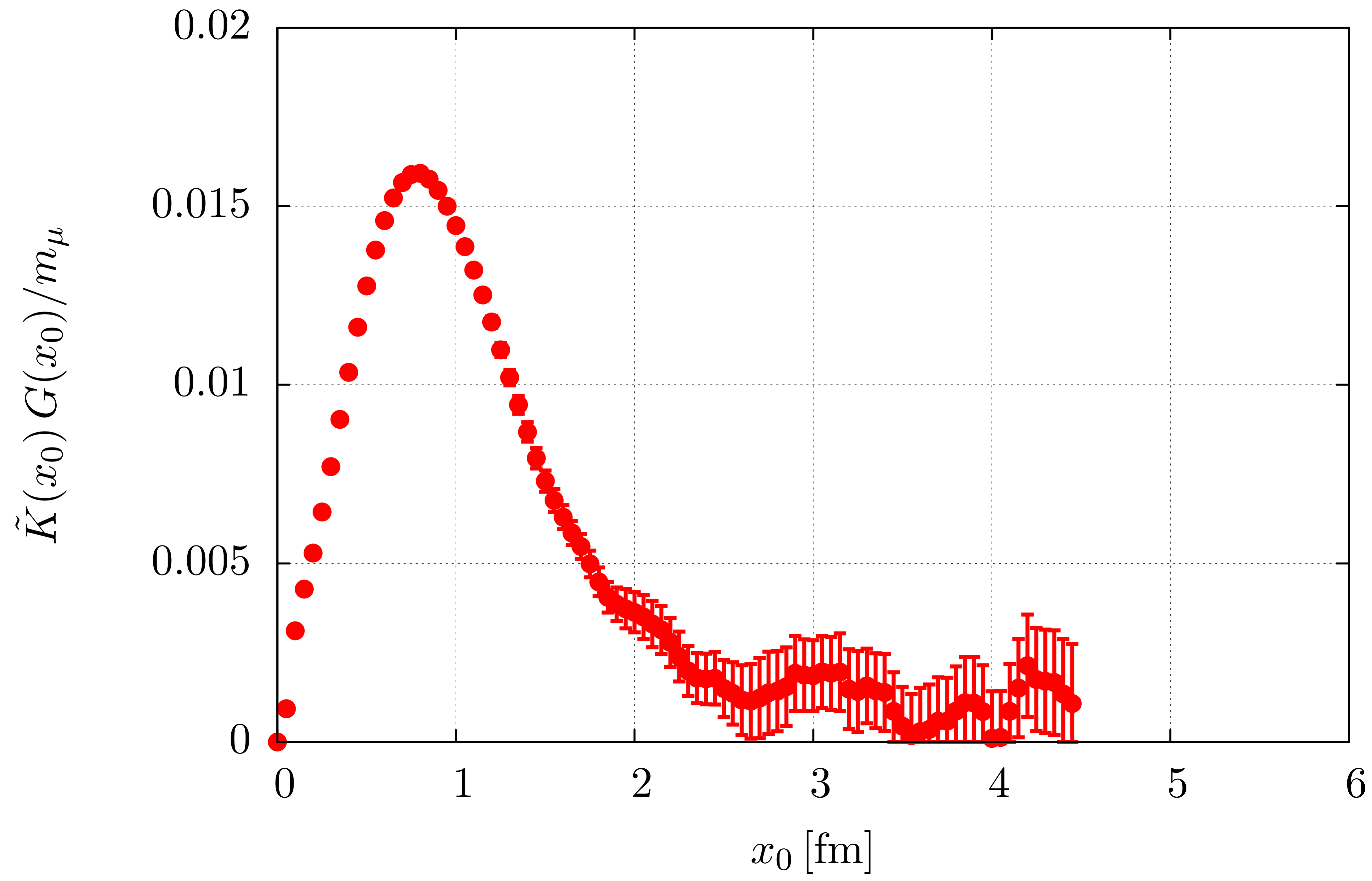

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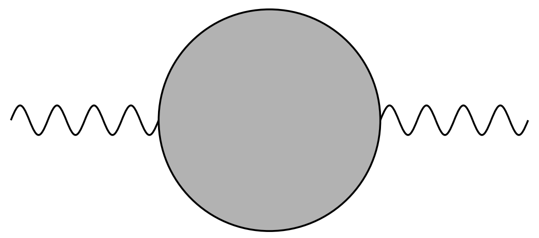

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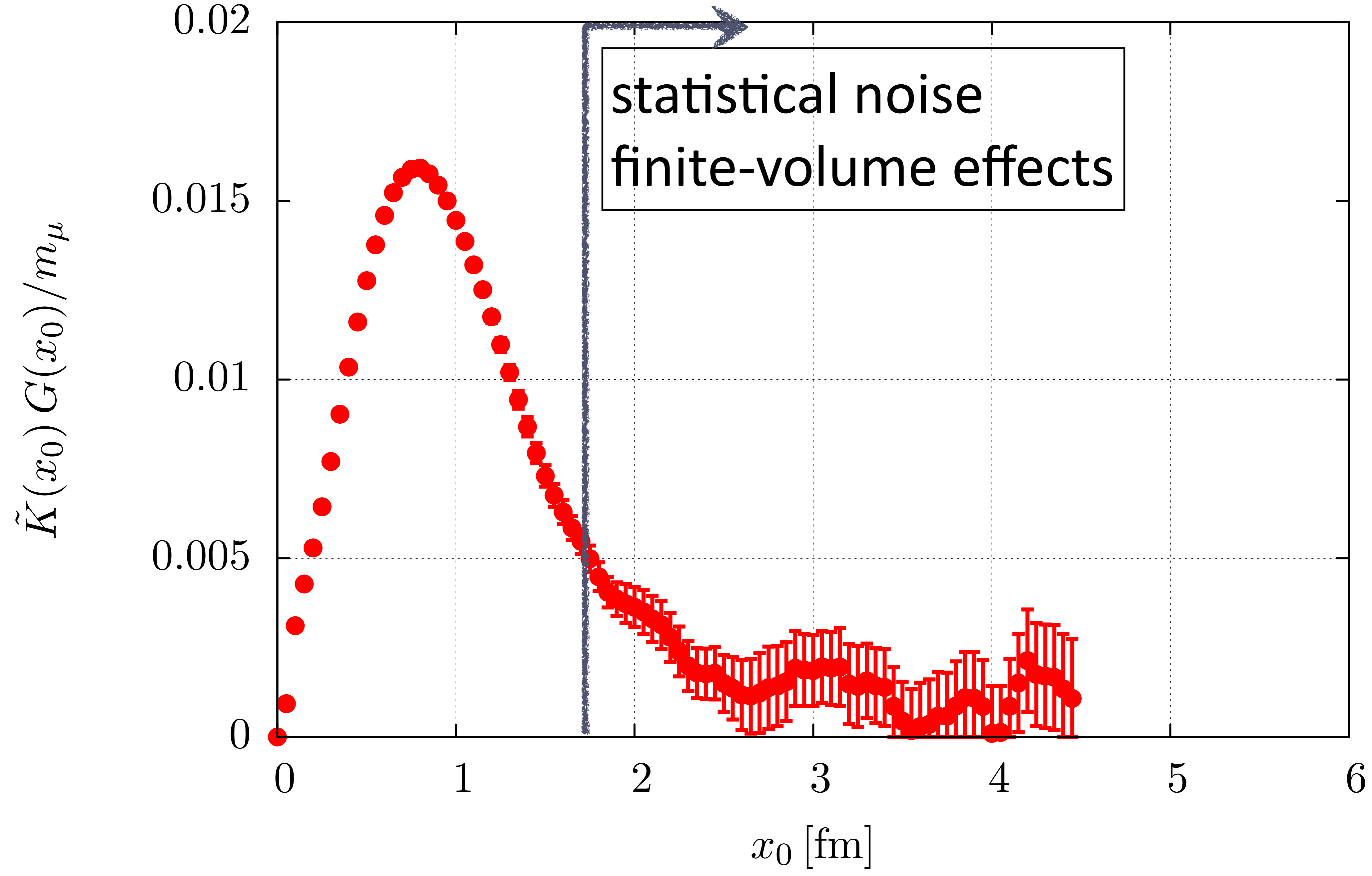

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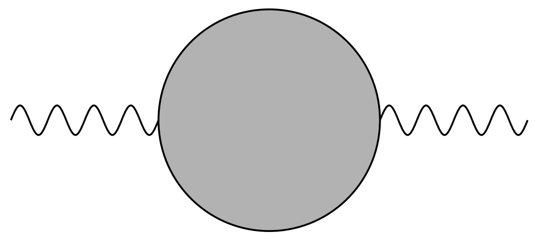

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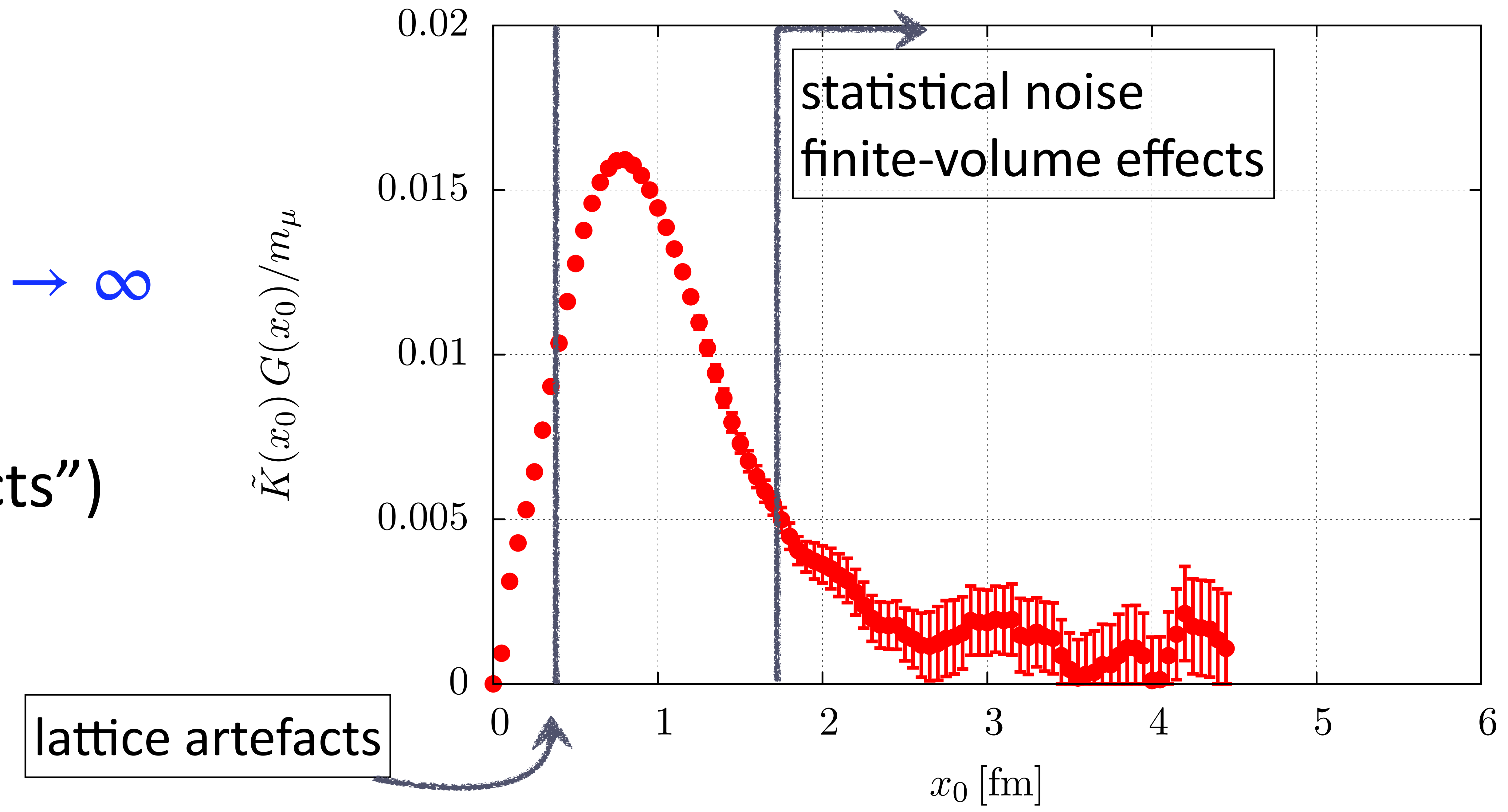

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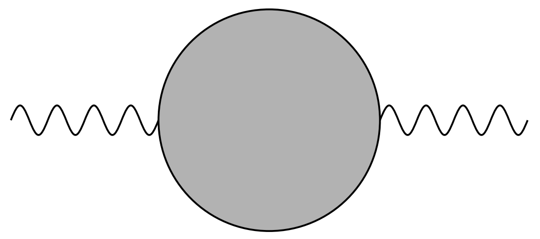

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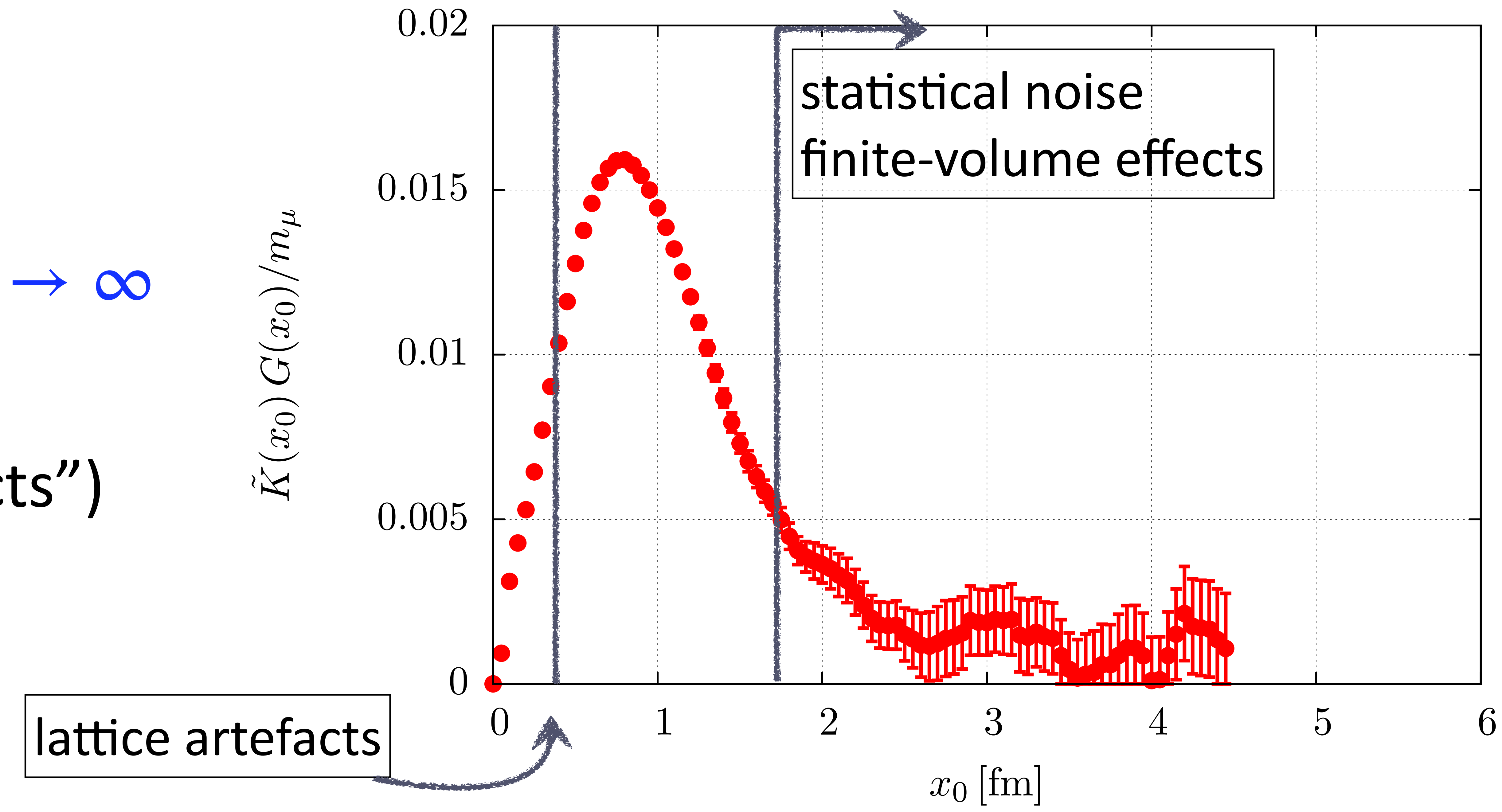
- Control discretisation effects ("lattice artefacts")
- Include isospin-breaking corrections



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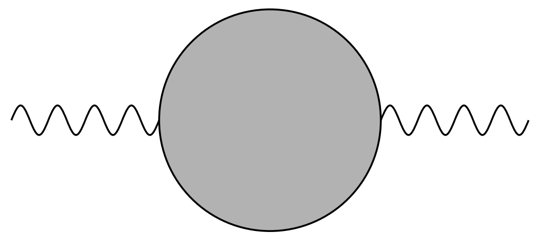

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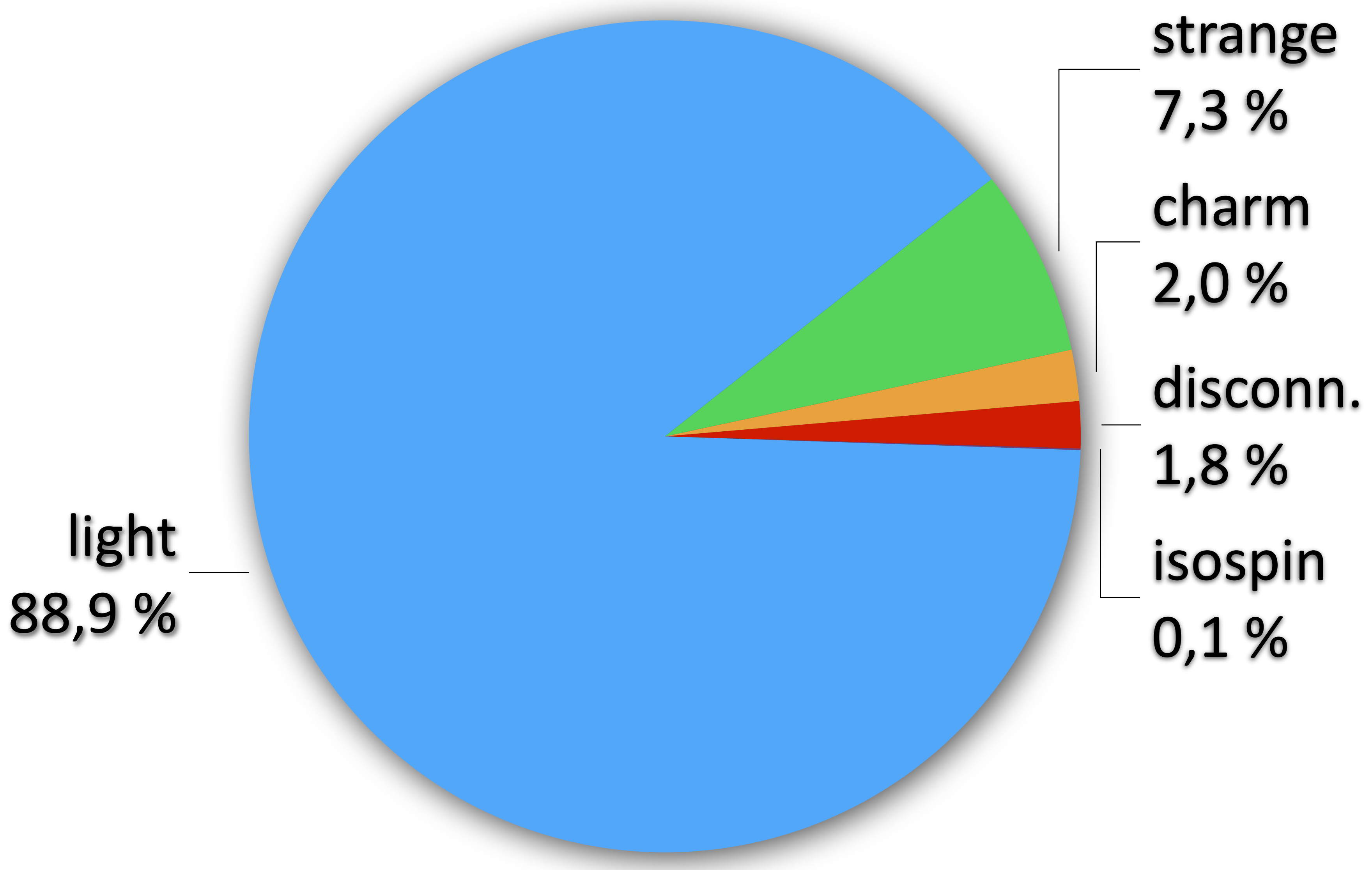
Light-quark connected contribution dominates



Common discretisations of the quark action

Computational cost depends significantly
on the chosen discretisation

"Fermion doubling problem"

computational cost

Common discretisations of the quark action

Computational cost depends significantly on the chosen discretisation

"Fermion doubling problem"

Rooted staggered quarks:

- remnant fermion doublers — "tastes"
- correct analytically for taste-induced lattice artefacts
- used by:
  BMW, Fermilab-HPQCD-MILC, ABGP,…

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Wilson quarks:

- no doublers; chiral symmetry broken explicitly
- "exceptional configurations": negative eigenvalues of Wilson-Dirac operator
- used by: Mainz/CLS, ETM, PACS

computational cost

Common discretisations of the quark action

Computational cost depends significantly on the chosen discretisation

"Fermion doubling problem"

Domain wall /overlap quarks:

- no doublers; chiral symmetry breaking exponentially small
- live in five dimensions (dwf)
- evaluate sign function of "conventional" action (ovlp)
- used by: RBC/UKQCD, χQCD,...

Rooted staggered quarks:

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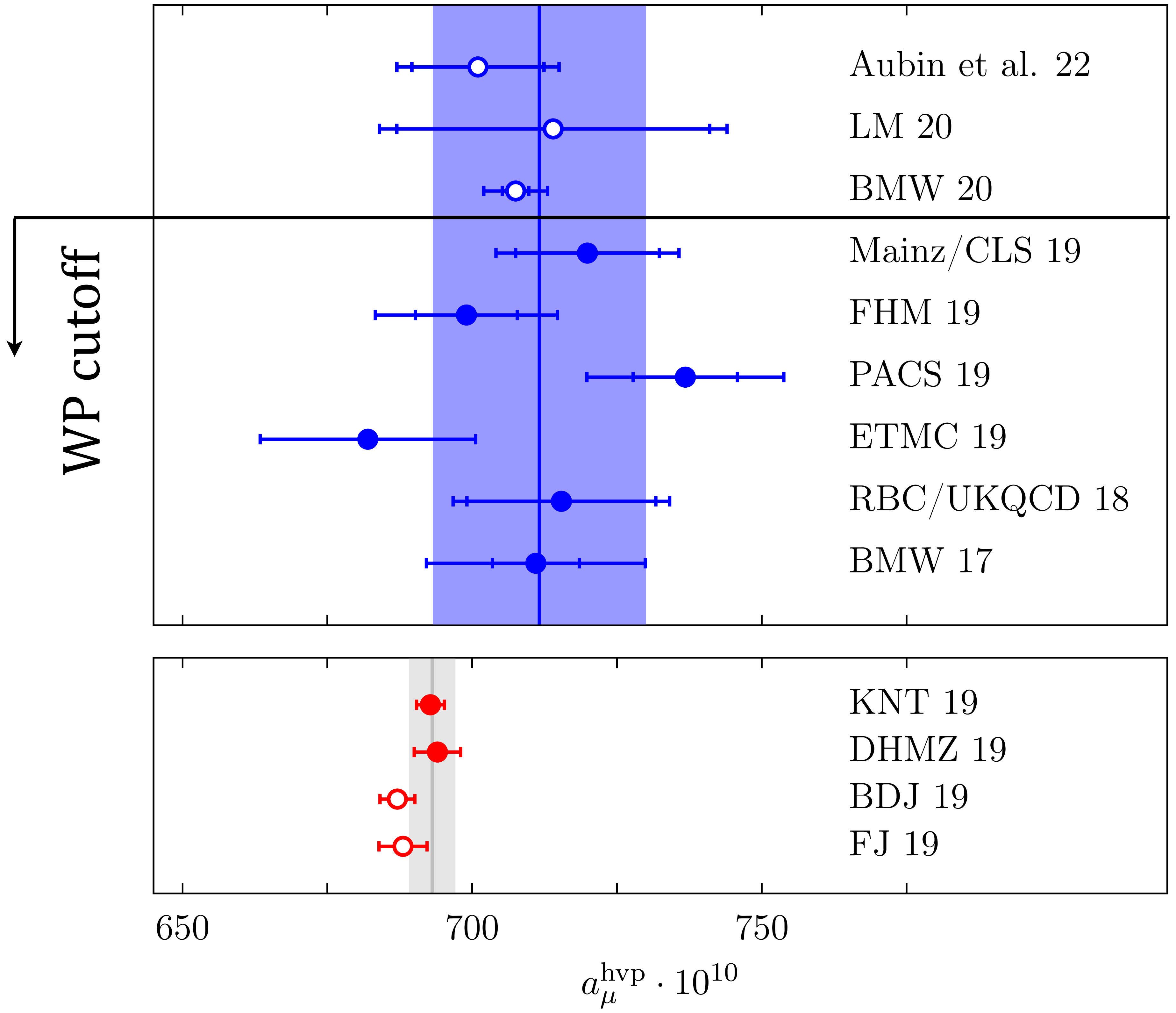
HVP in Lattice QCD

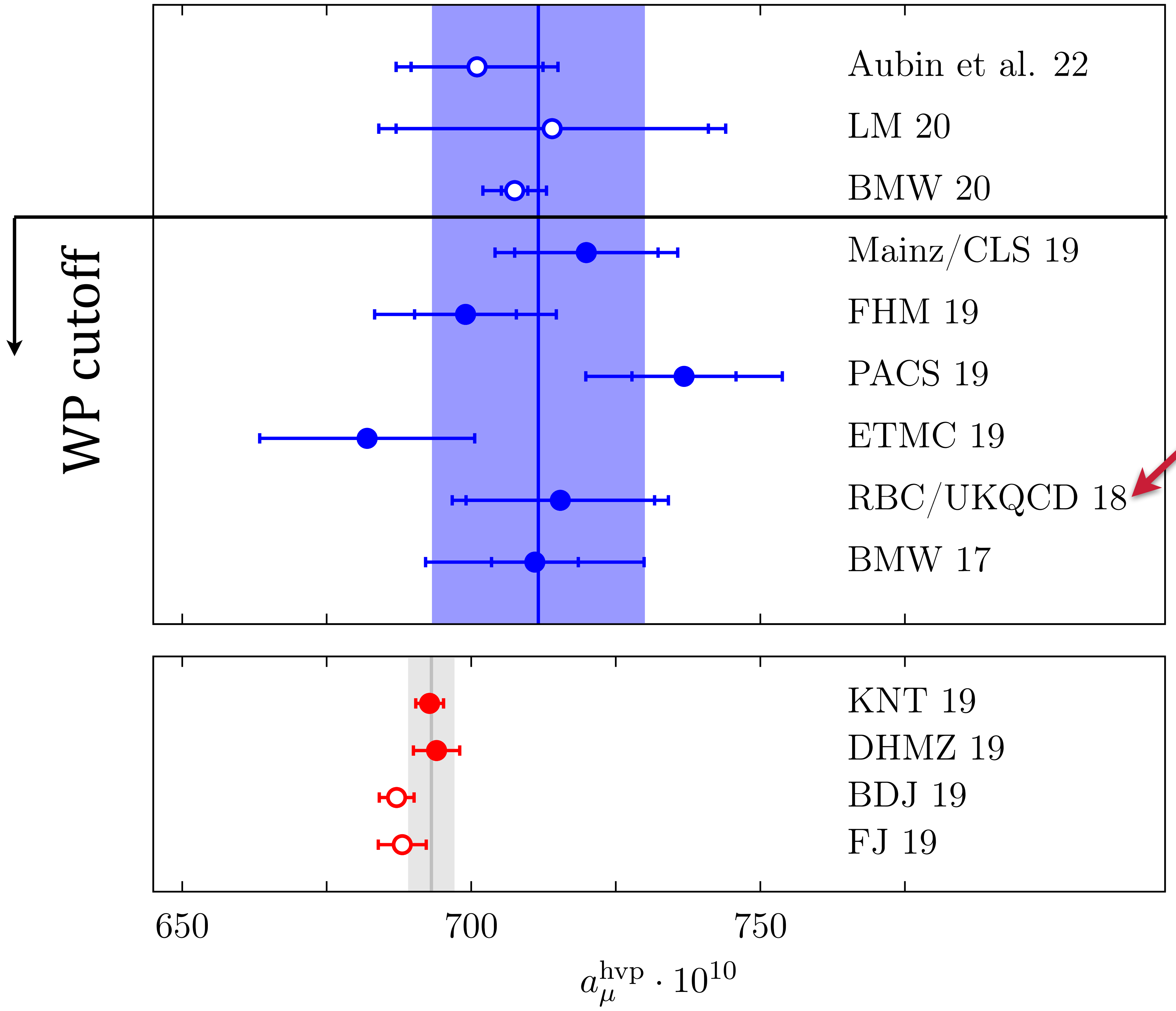


White Paper:

R-ratio: $a_\mu^{\rm hvp,\,LO} = (693.1 \pm 4.0) \cdot 10^{-10}$ [0.6%]

LQCD: $a_\mu^{\rm hvp,\,LO} = (711.6 \pm 18.4) \cdot 10^{-10}$ [2.6%]

HVP in Lattice QCD



RBC/UKQCD *[Blum et al., Phys. Rev. Lett. 121 (2018) 022003]*

- Domain wall fermions
- Two ensembles: $a = 0.114,\ 0.084\,\mathrm{fm}$ at $m_\pi^{\rm phys}$
- Leading isospin-breaking corrections included
- Naive continuum extrapol'n in a^2 including estimated a^4-term

$$a_\mu^{\rm hvp,\,LO} = (715.4 \pm 16.3 \pm 9.2) \cdot 10^{-10} \quad [2.6\%]$$

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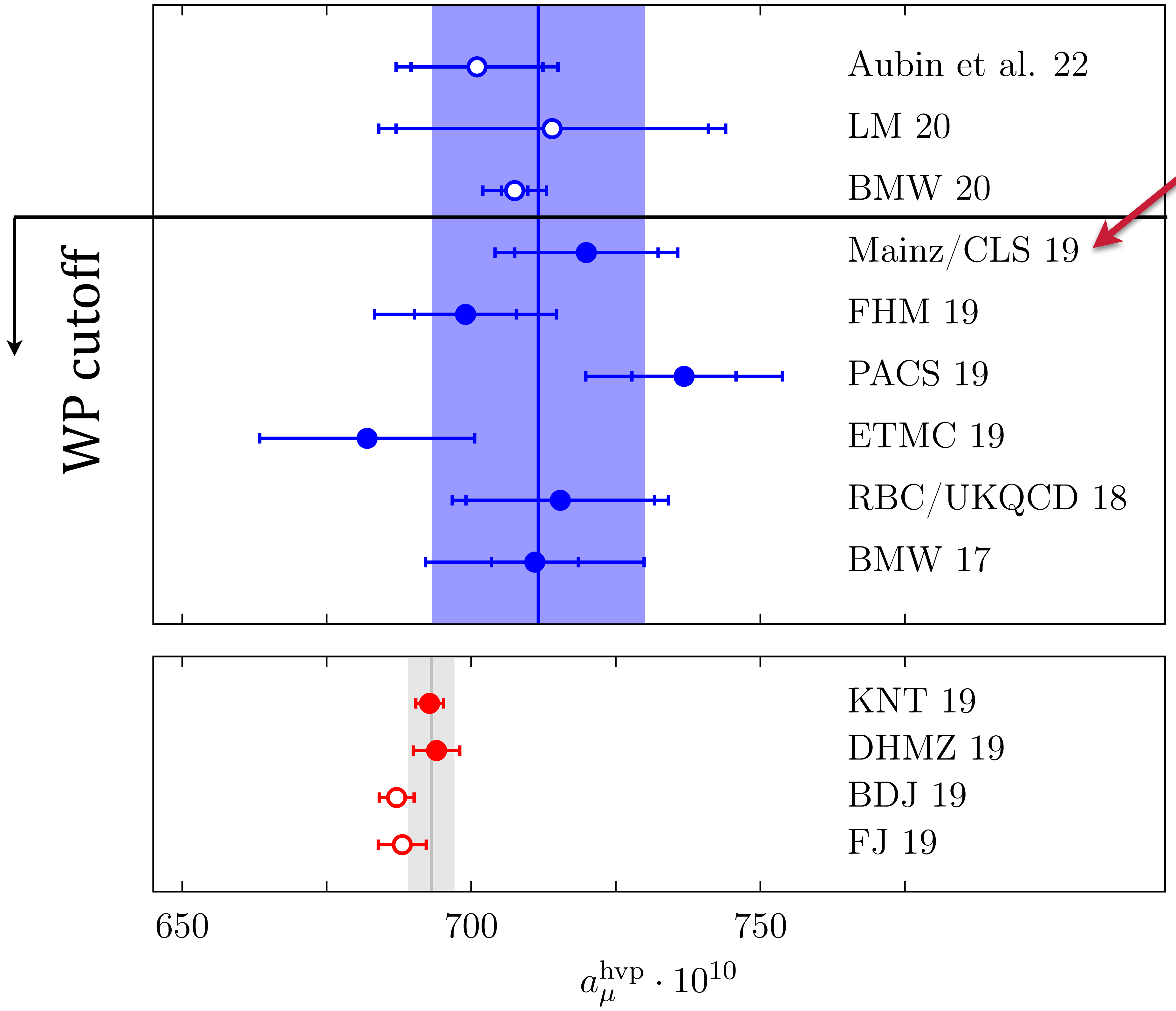

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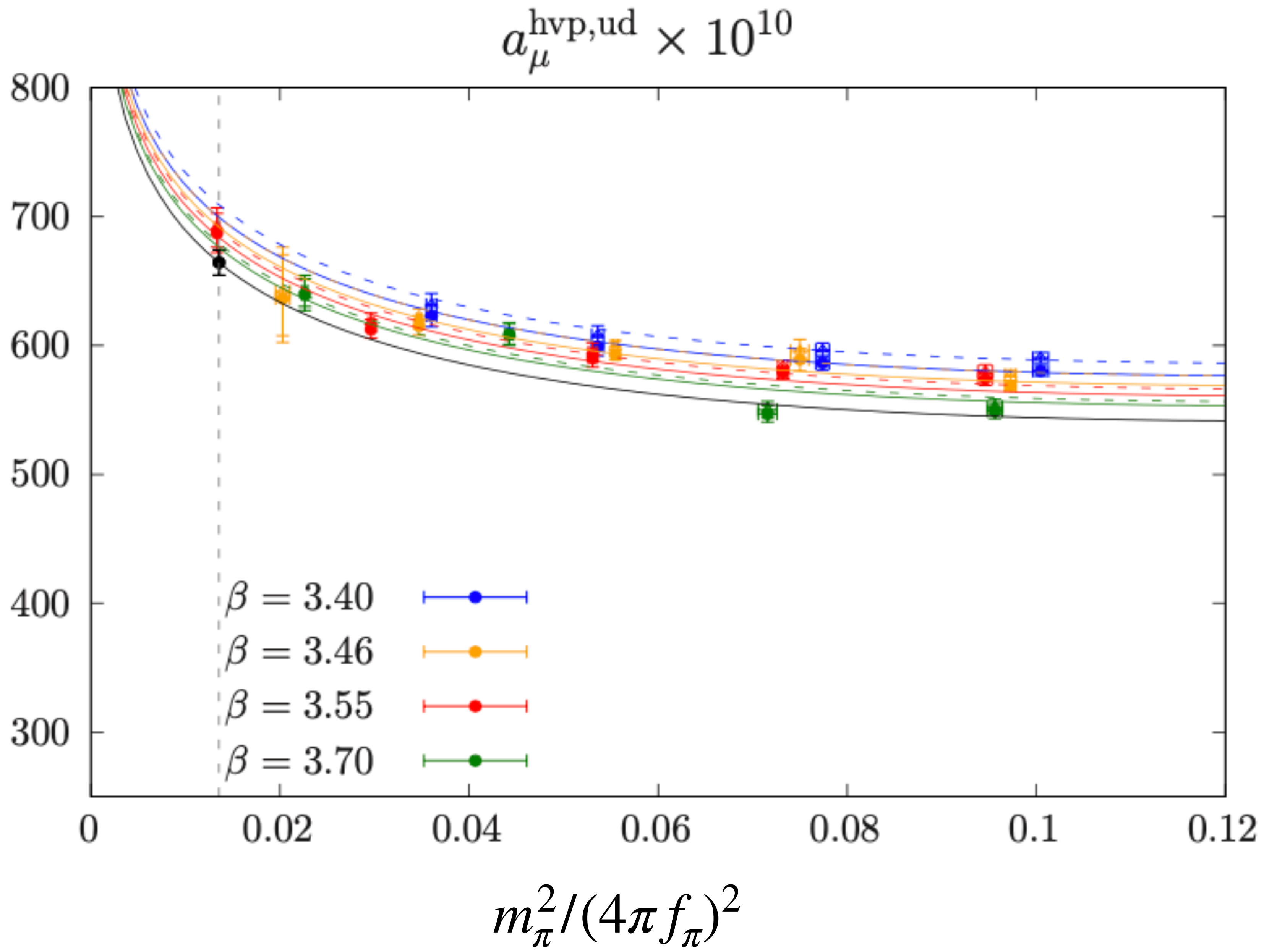
Mainz/CLS

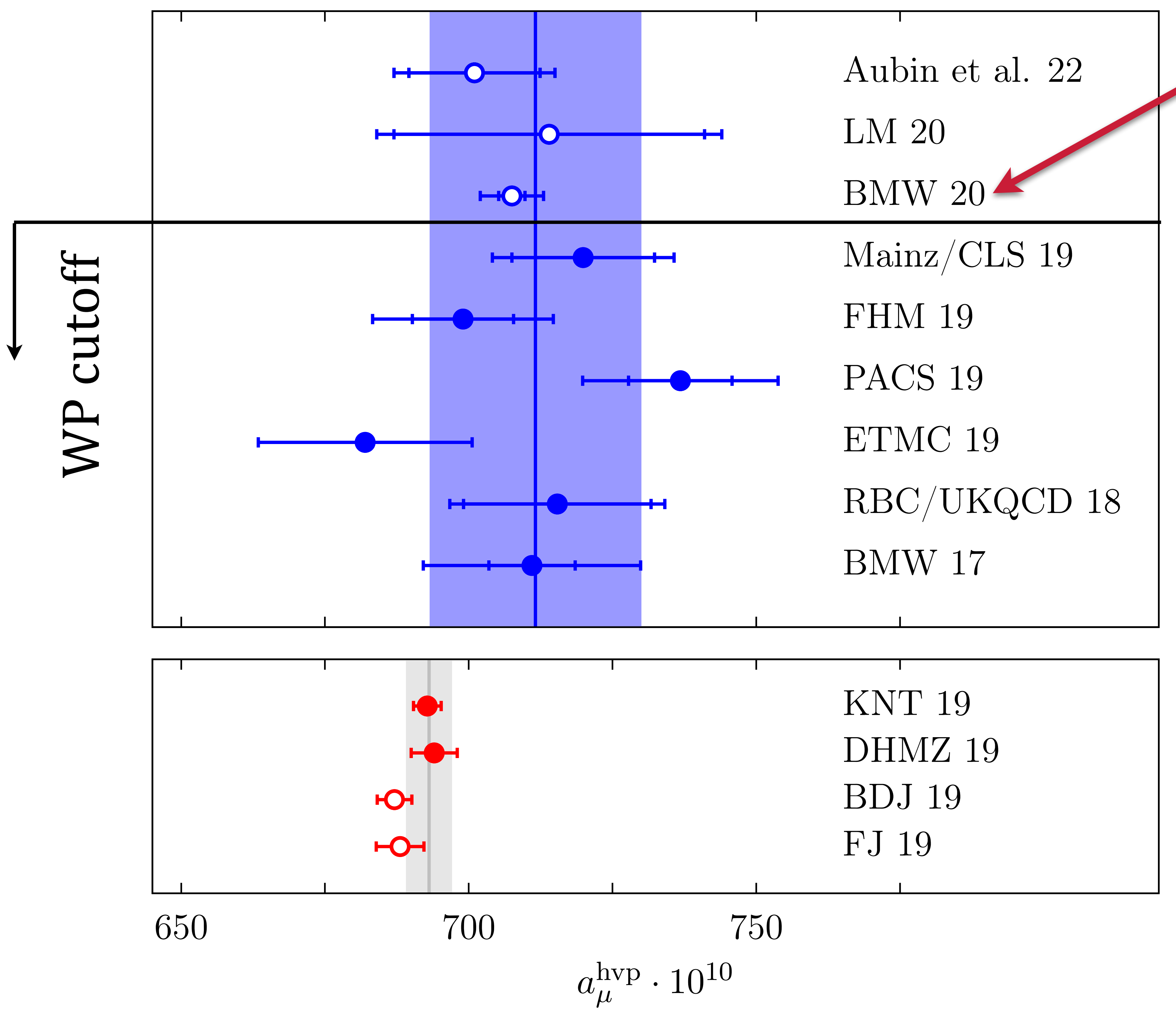
[Gérardin et al., Phys. Rev. D 100 (2019) 014510]

- $\mathrm{O}(a)$ improved Wilson fermions
- Four lattice spacings: $a = 0.085 - 0.050\,\mathrm{fm}$
- Pion masses $m_\pi = 130 - 420\,\mathrm{MeV}$
- Isospin-breaking correction by ETMC added to error
- Simultaneous chiral and continuum extrapolation



$a_\mu^{\rm hvp,\,LO} = (720.0 \pm 12.6 \pm 9.9) \cdot 10^{-10}$ [2.2%]

HVP in Lattice QCD



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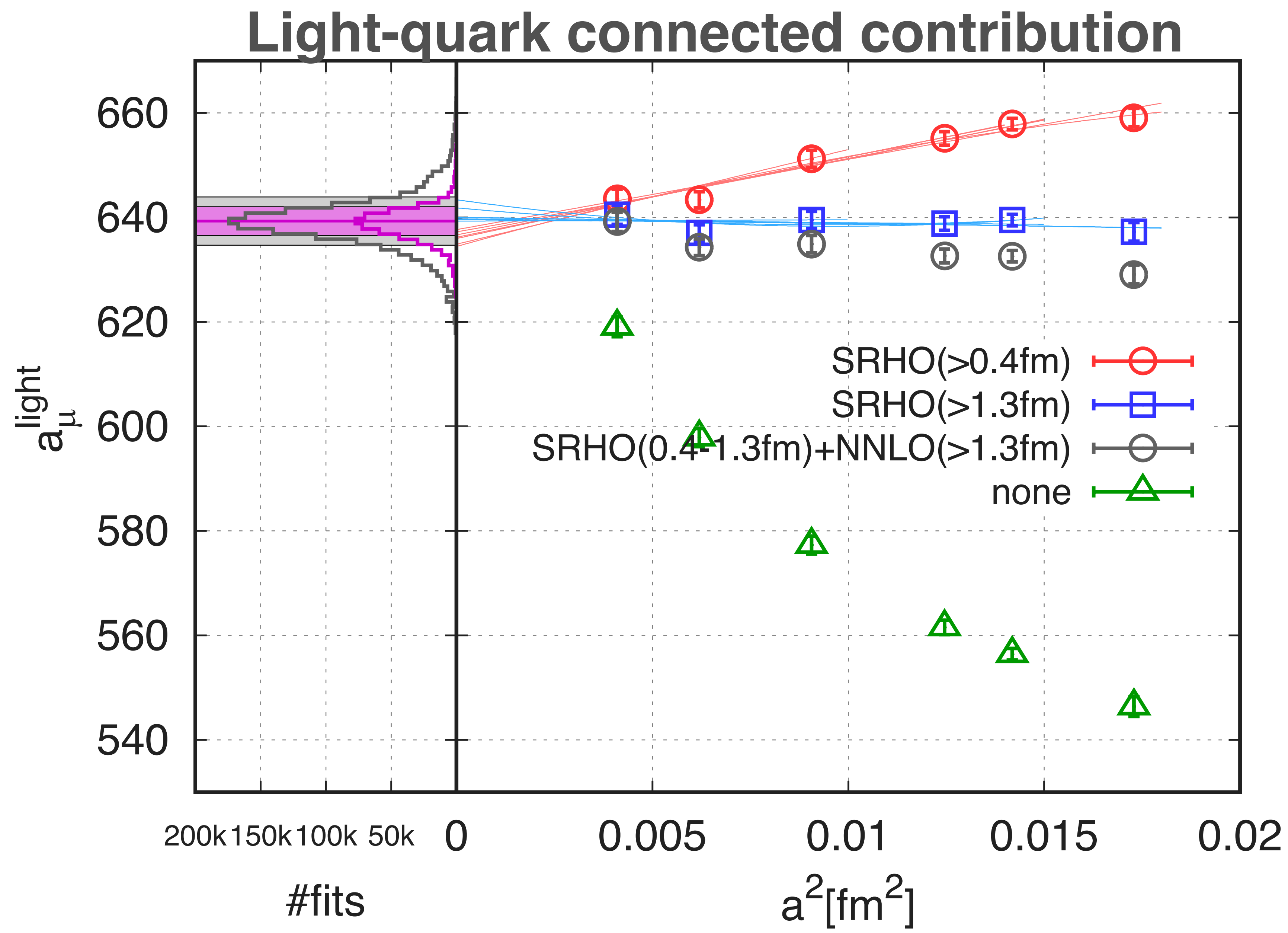
BMWc [Borsányi et al., Nature 593 (2021) 7857]

- Rooted staggered fermions
- Six lattice spacings: $a = 0.132 - 0.064\,\mathrm{fm}$
- Physical pion mass throughout
- Correct for taste-breaking before continuum extrapol'n
- Final result selected from distribution of different fits



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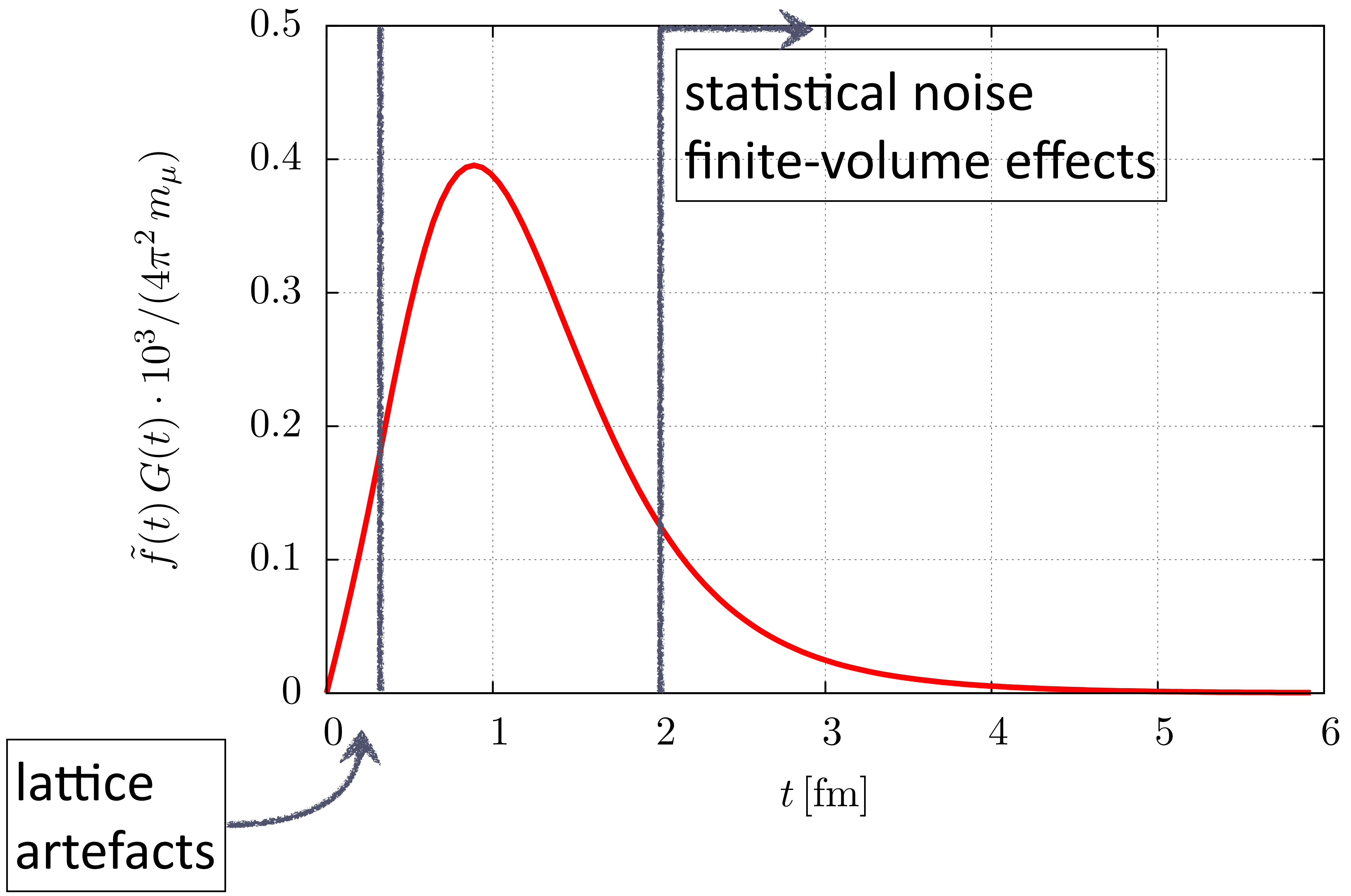
Window observables

[Blum et al., Phys. Rev. Lett. 121 (2018) 022003]

Idea: restrict integration to "unproblematic" regions

→ reduce statistical fluctuations and systematic effects

$$a_\mu^{\text{hvp, win}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{K}(t)\, G(t)\, W(t;\, t_0,\, t_1)$$



Intermediate-distance window:

$$W^{\text{ID}}(t;\, t_0,\, t_1) = \Theta(t,\, t_0, \Delta) - \Theta(t,\, t_1, \Delta)$$

$$\Theta(t,\, t', \Delta) = \tfrac{1}{2}\left[1 + \tanh(t - t')/\Delta\right]$$

$$t_0 = 0.4\,\text{fm},\ t_1 = 1.0\,\text{fm},\ \Delta = 0.15\,\text{fm}$$

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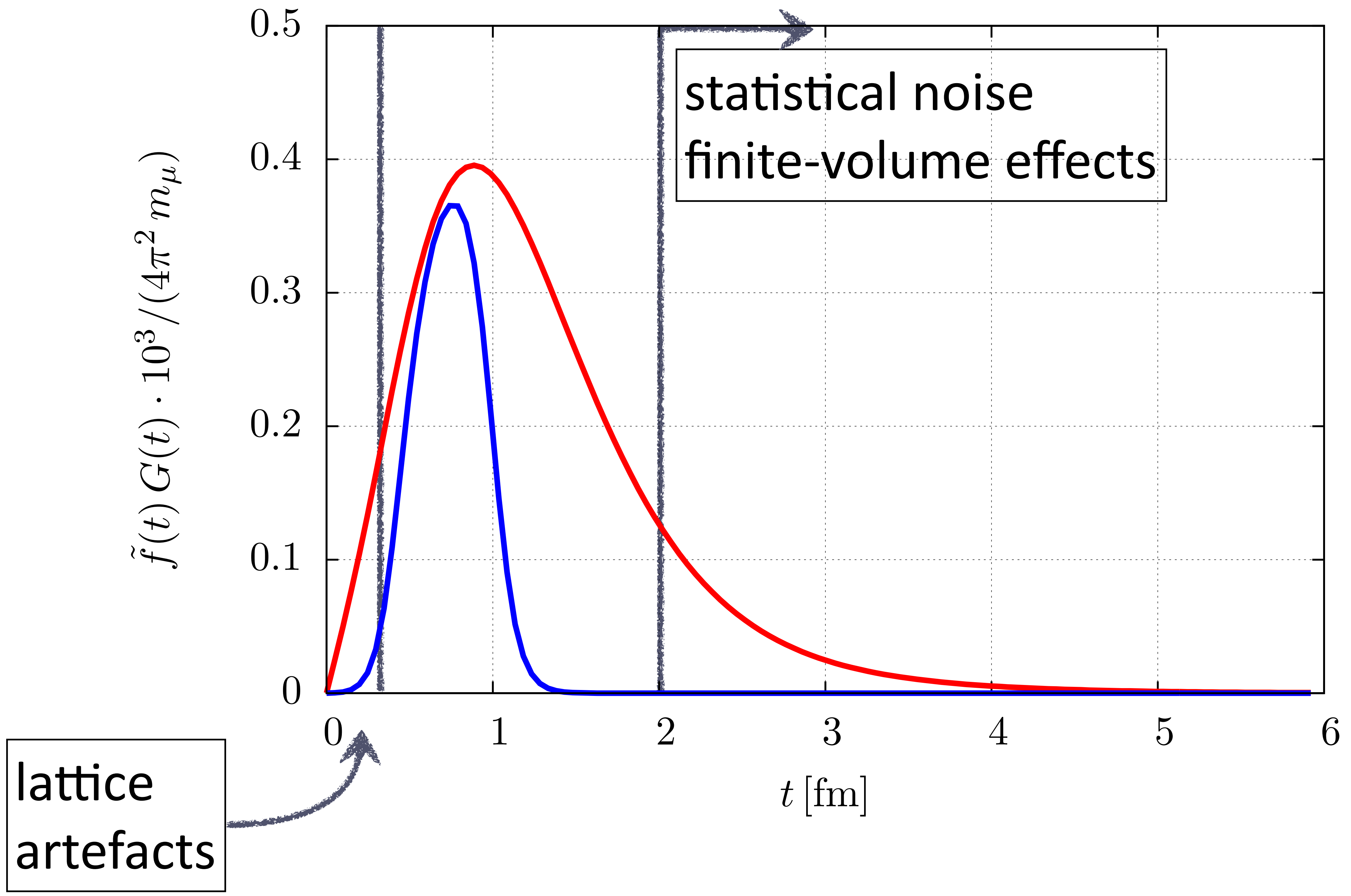

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- Uncertainty dominated by statistics

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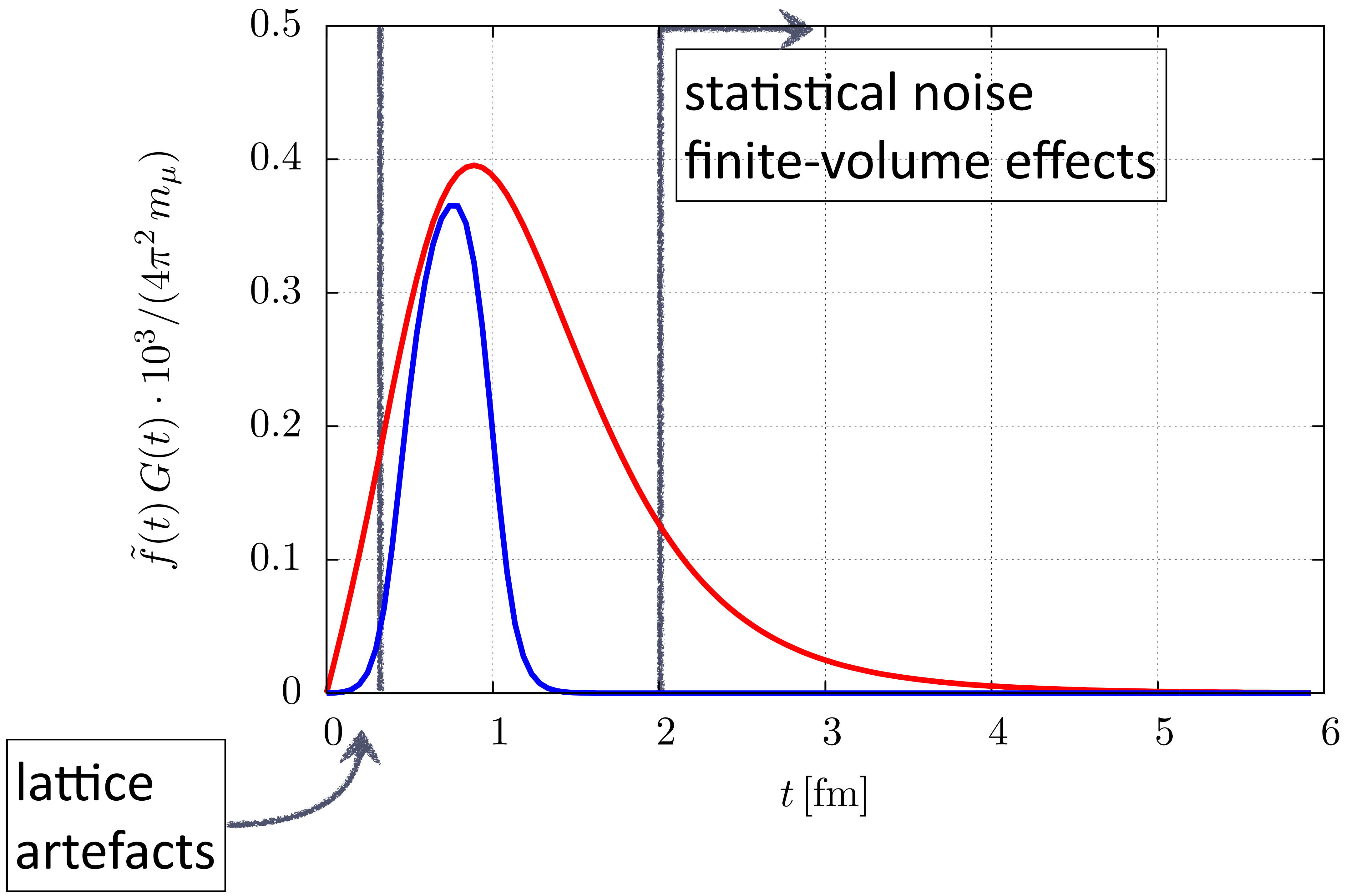

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→ Benchmark quantity for sub-contribution of HVP

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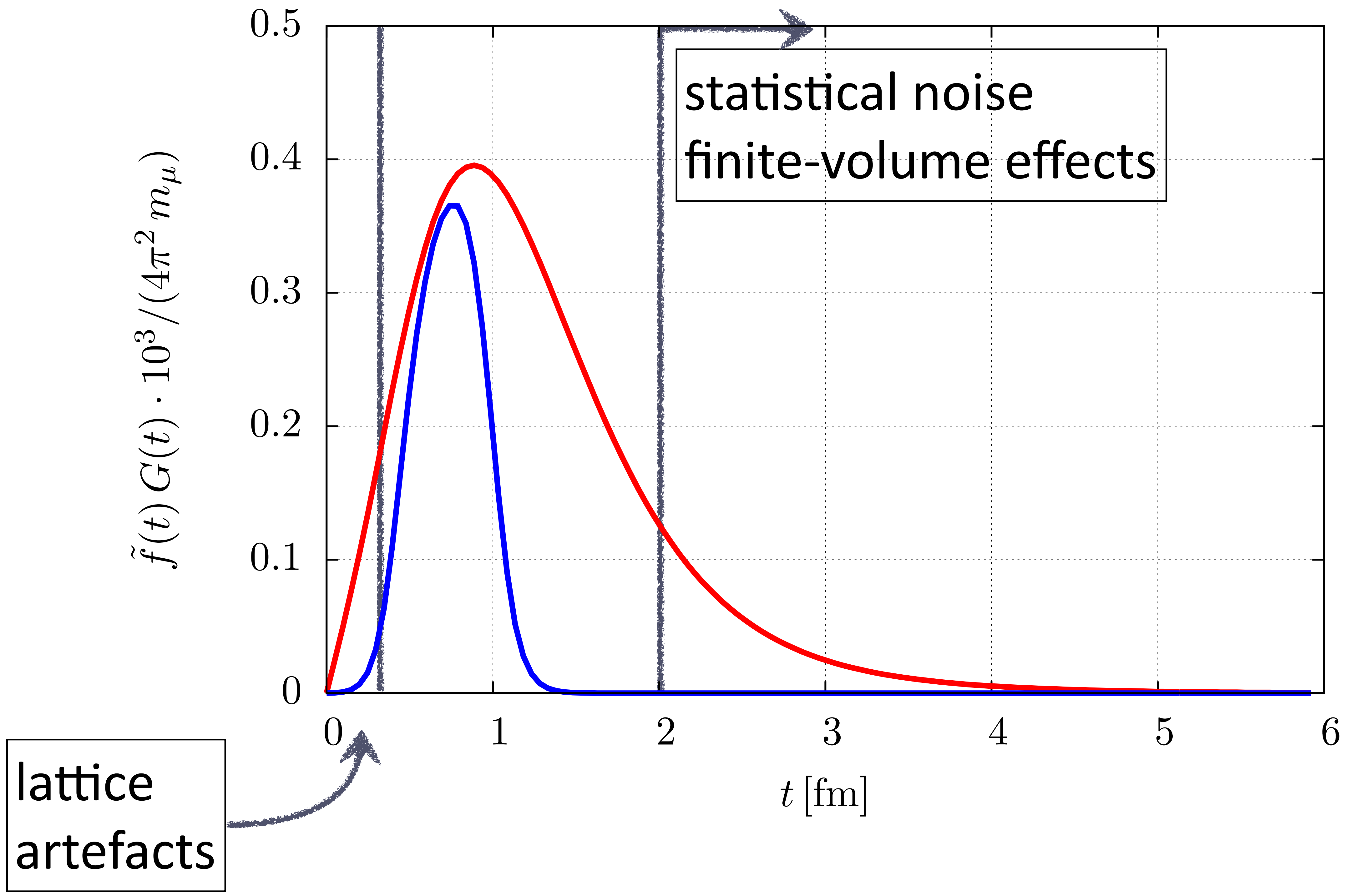

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$$\Theta(t,\, t', \Delta) = \tfrac{1}{2}\left[1 + \tanh(t - t')/\Delta\right]$$

$$t_0 = 0.4\,\text{fm},\;\; t_1 = 1.0\,\text{fm},\;\; \Delta = 0.15\,\text{fm}$$

- Finite-volume correction reduced to 0.25%
- Uncertainty dominated by statistics

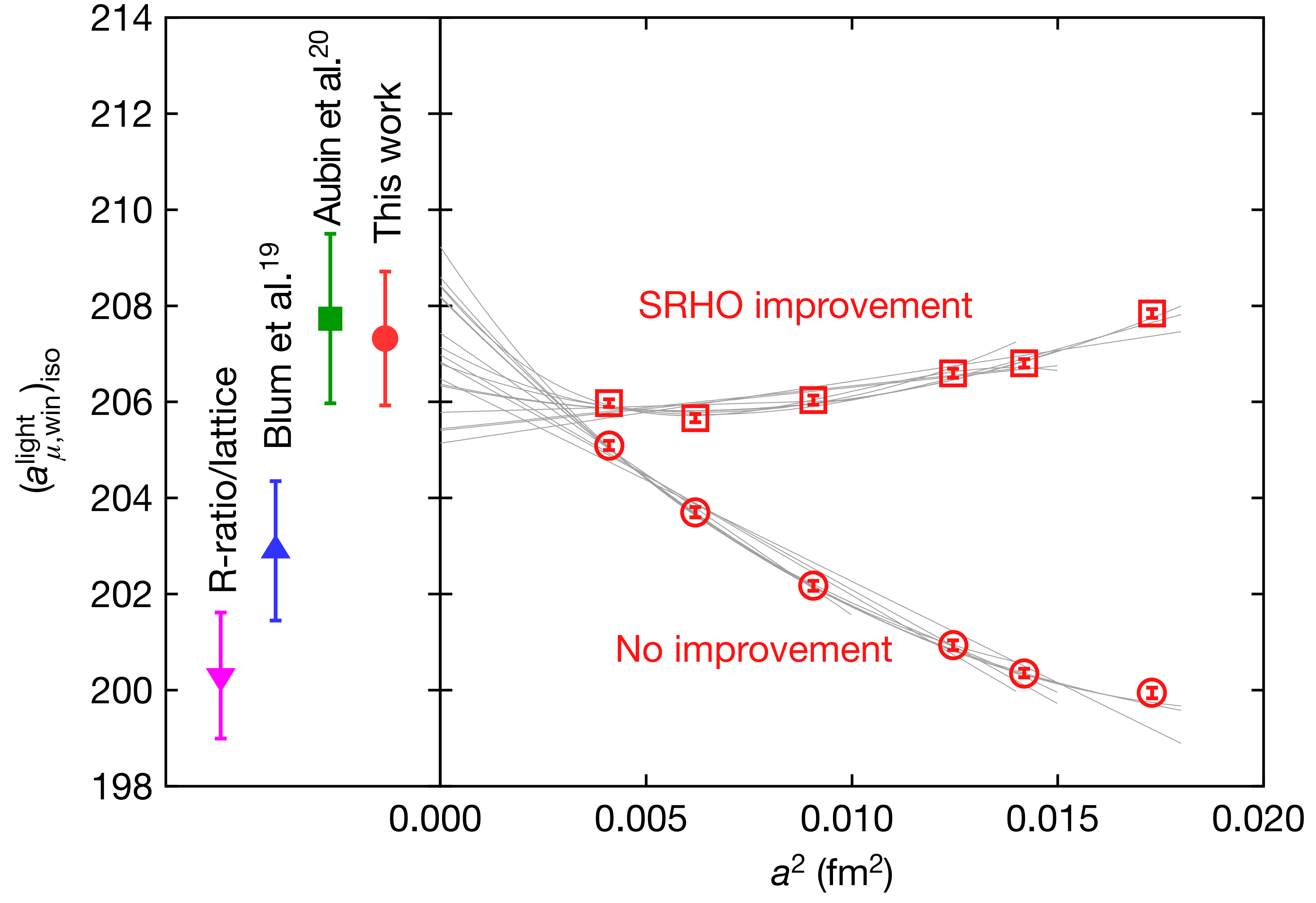
→ Benchmark quantity for sub-contribution of HVP

Data-driven approach: $a_\mu^{\text{win}} = (229.4 \pm 1.4) \cdot 10^{-10}$ [Colangelo et al., Phys Lett B833 (2022) 137313]

(Excluding the 2023 CMD-3 result for $e^+e^- \to \pi^+\pi^-$)

Intermediate window observable in Lattice QCD

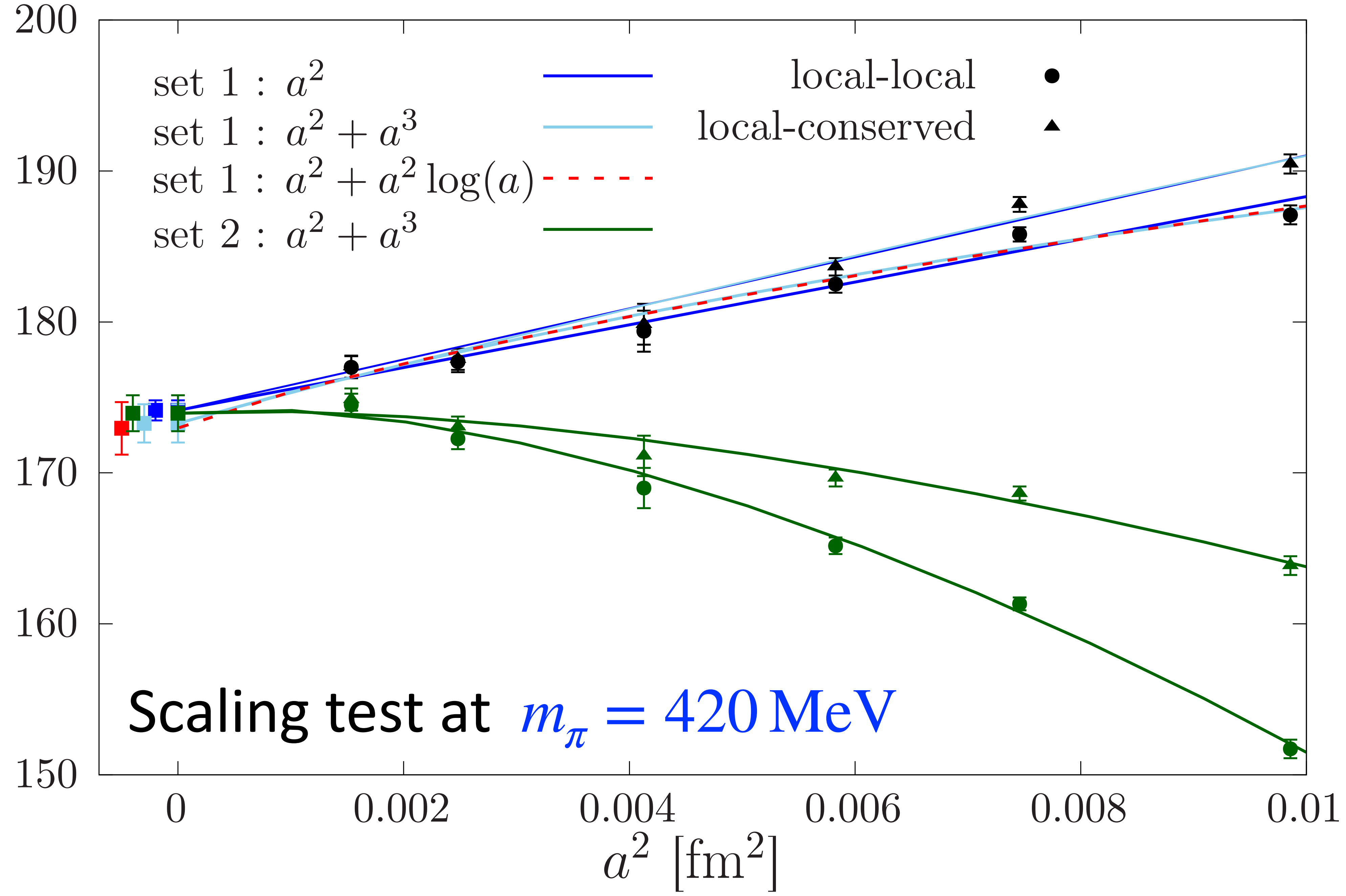
BMWc: Rooted staggered quarks



$$a_\mu^{\mathrm{win,ud}} = (207.3 \pm 0.4 \pm 1.3) \cdot 10^{-10}$$

[Borsányi et al., Nature 593 (2021) 7857]

Mainz/CLS: $\mathrm{O}(a)$ improved Wilson quarks



[Cè et al., Phys Rev D106 (2022) 114502]

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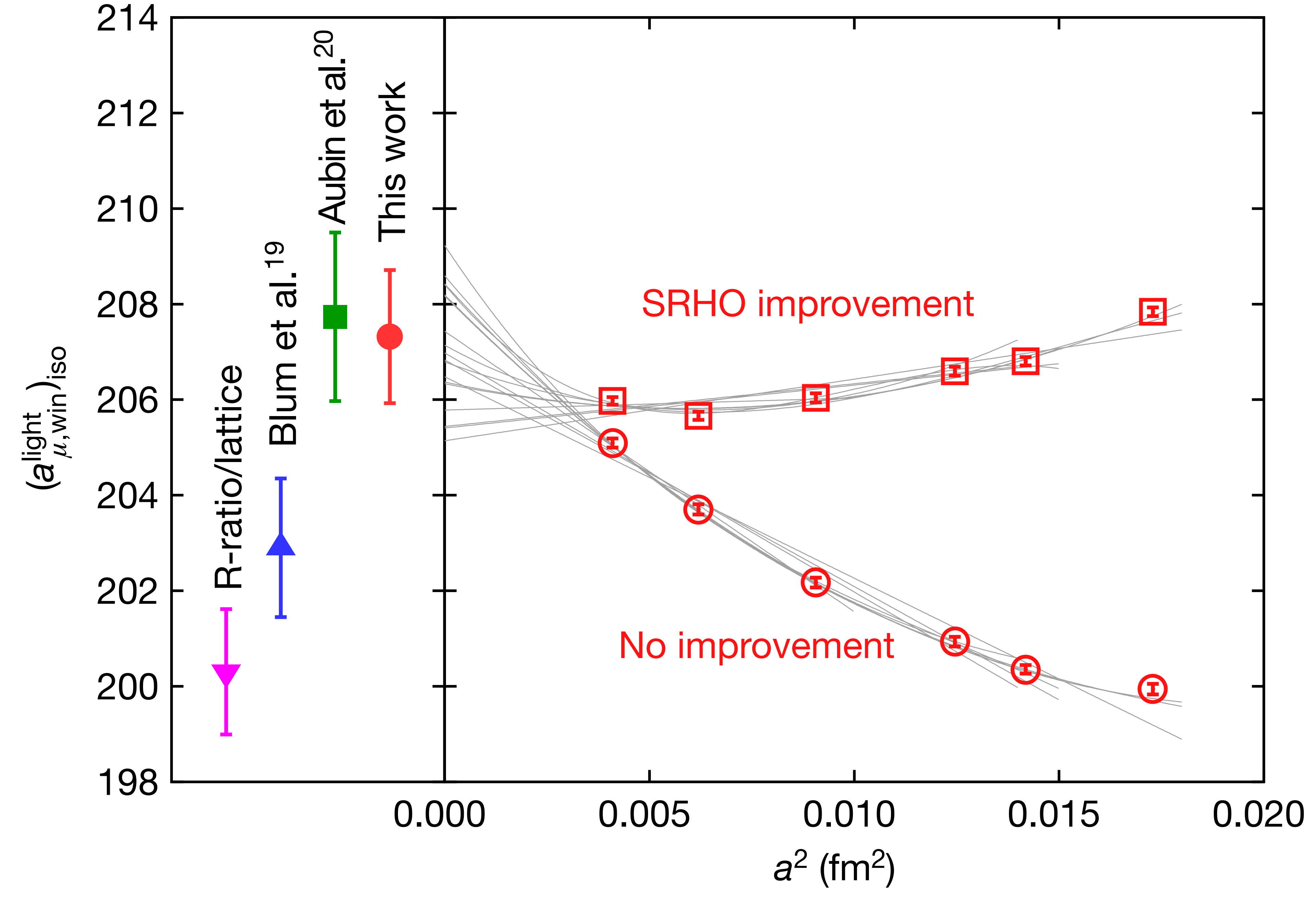

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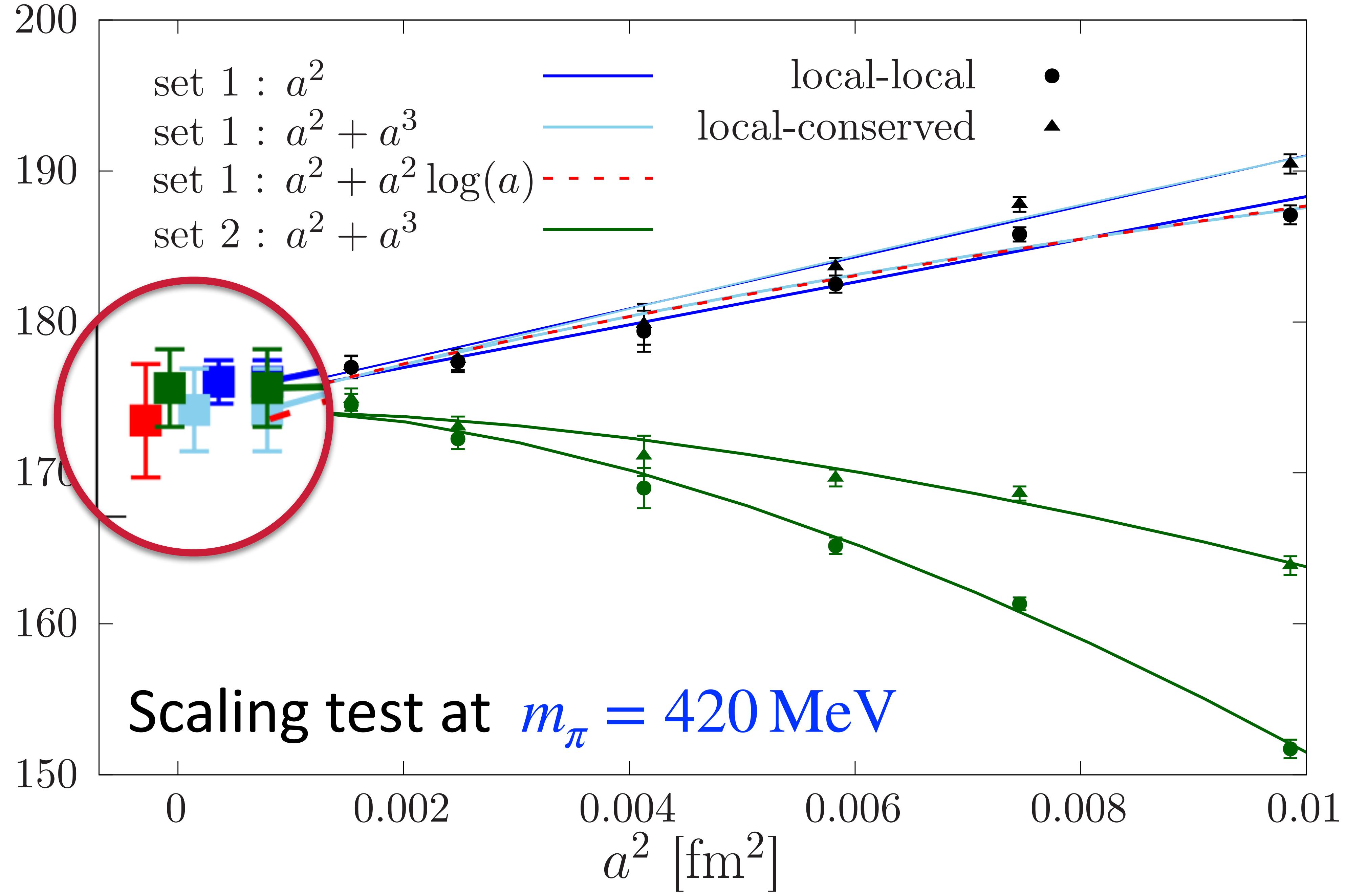

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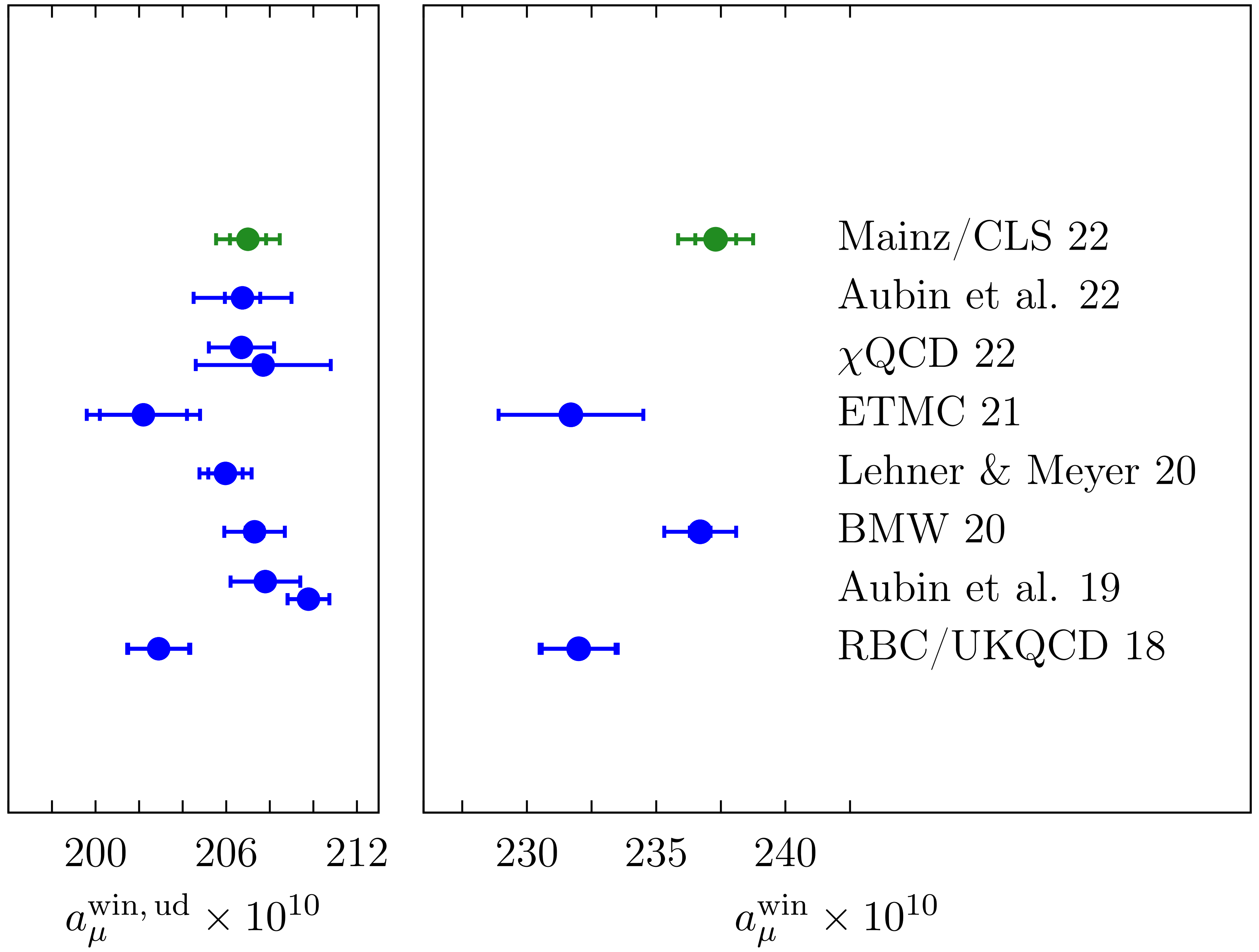
[Cè et al., Phys Rev D106 (2022) 114502]

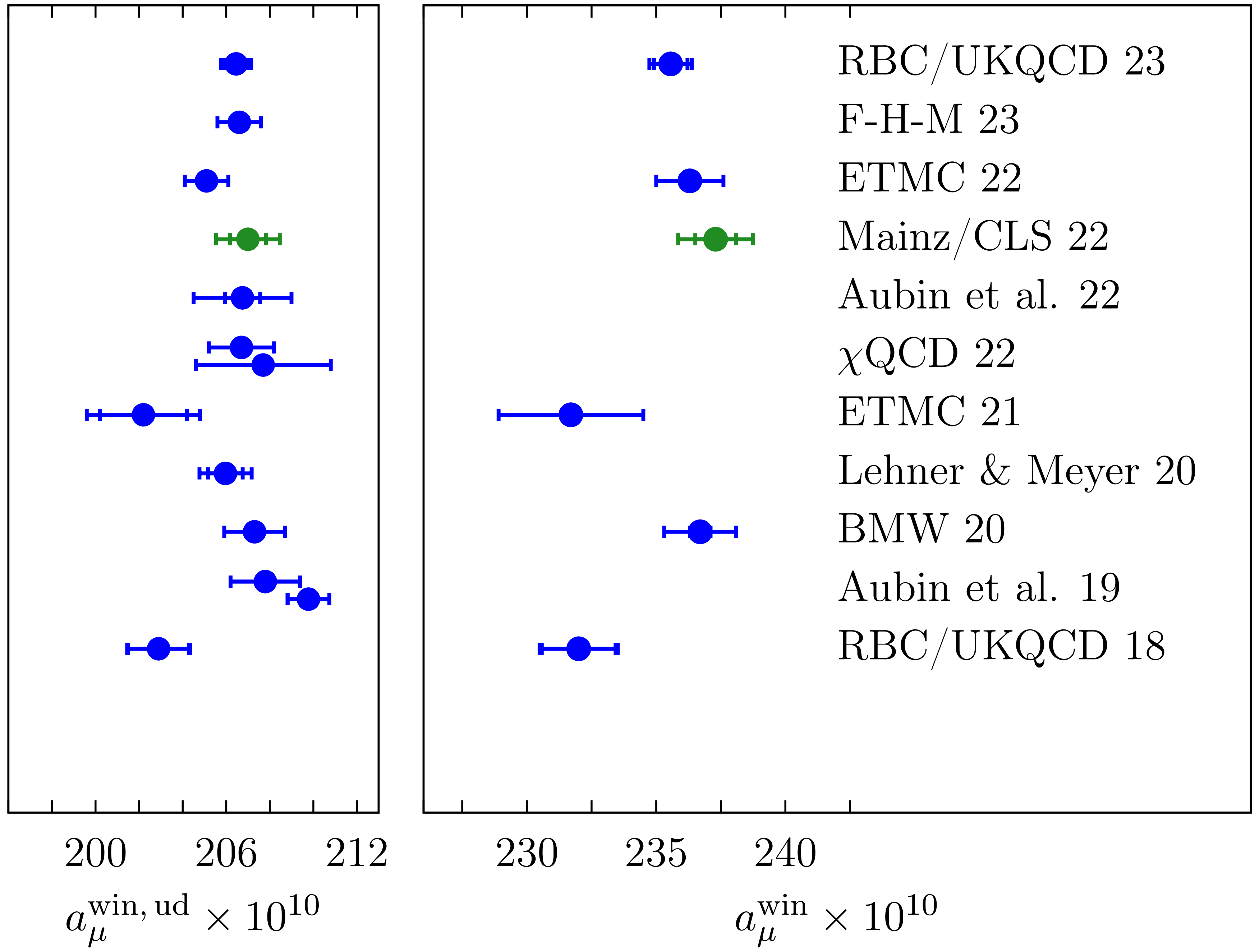
Window observable: Lattice QCD vs. R-ratio



Left: dominant light-quark contribution to $a_\mu^{\rm win}$

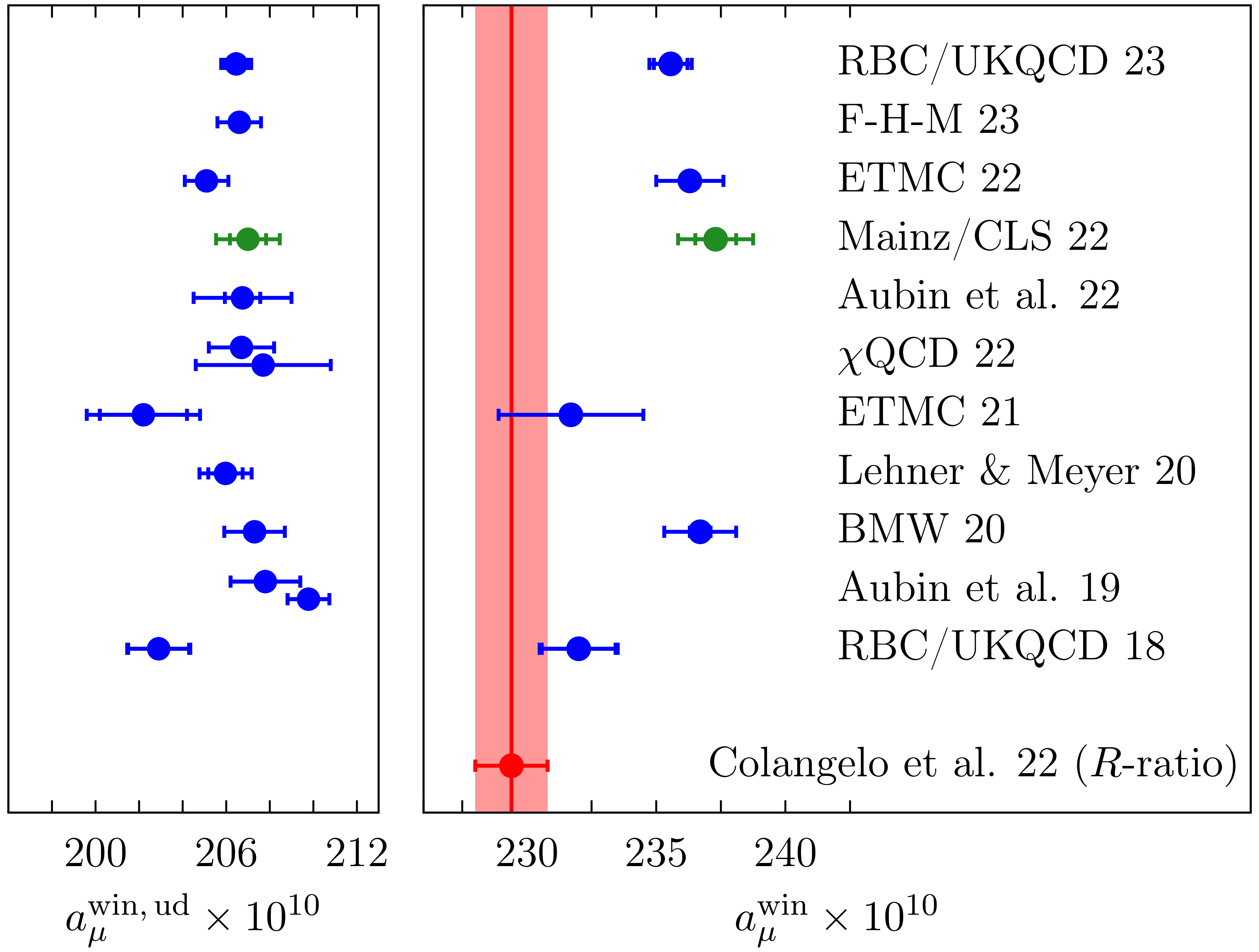
Right: including sub-leading contributions

Window observable: Lattice QCD vs. *R*-ratio



- Dominant light-quark contribution confirmed for wide range of discretisation with sub-percent precision

Window observable: Lattice QCD vs. R-ratio



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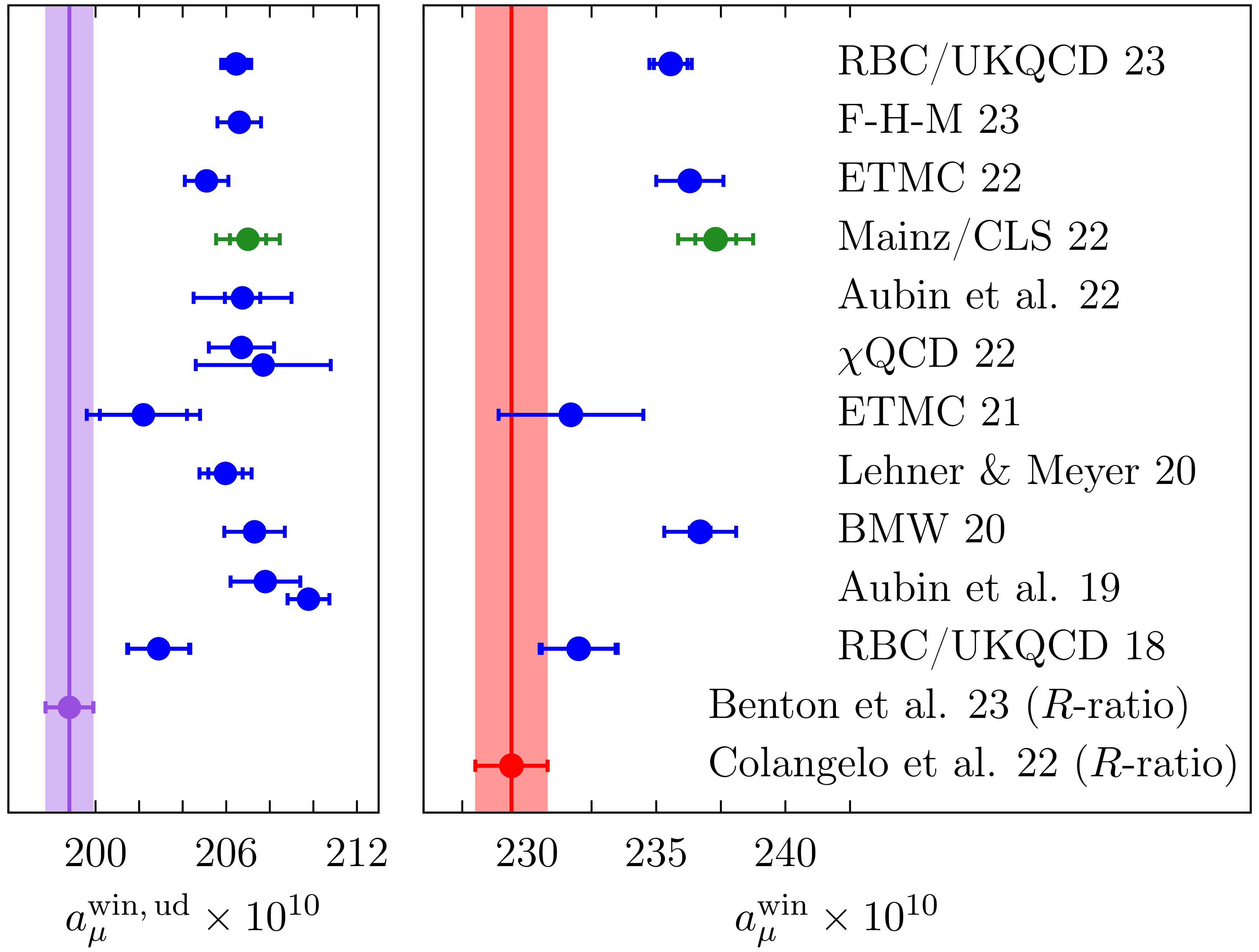

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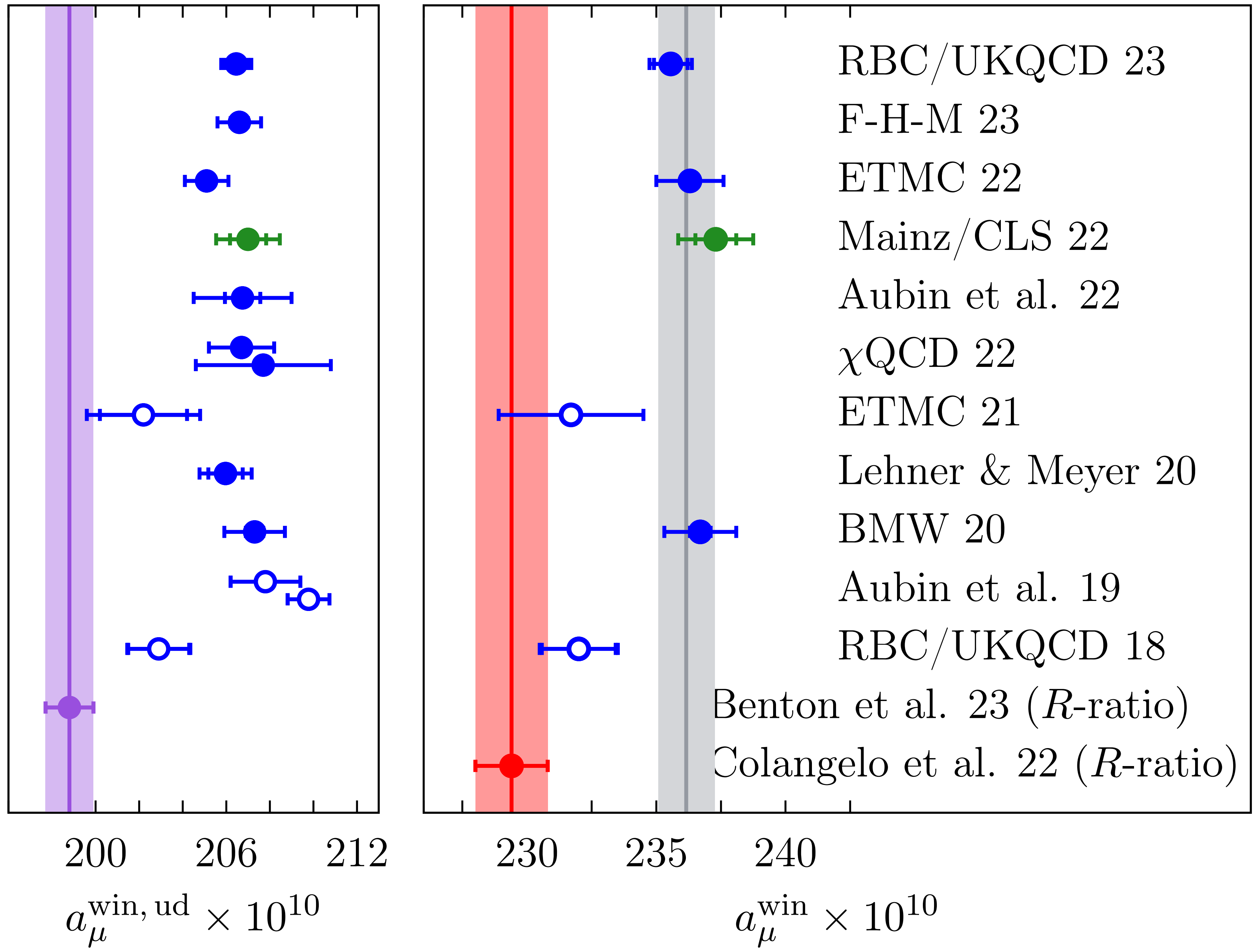

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Lattice average: $a_\mu^{\mathrm{win}} = (236.16 \pm 1.09) \cdot 10^{-10}$

(RBC/UKQCD 23, ETMC 22, Mainz/CLS 22, BMW 20)

[HW, arXiv:2306.04165]

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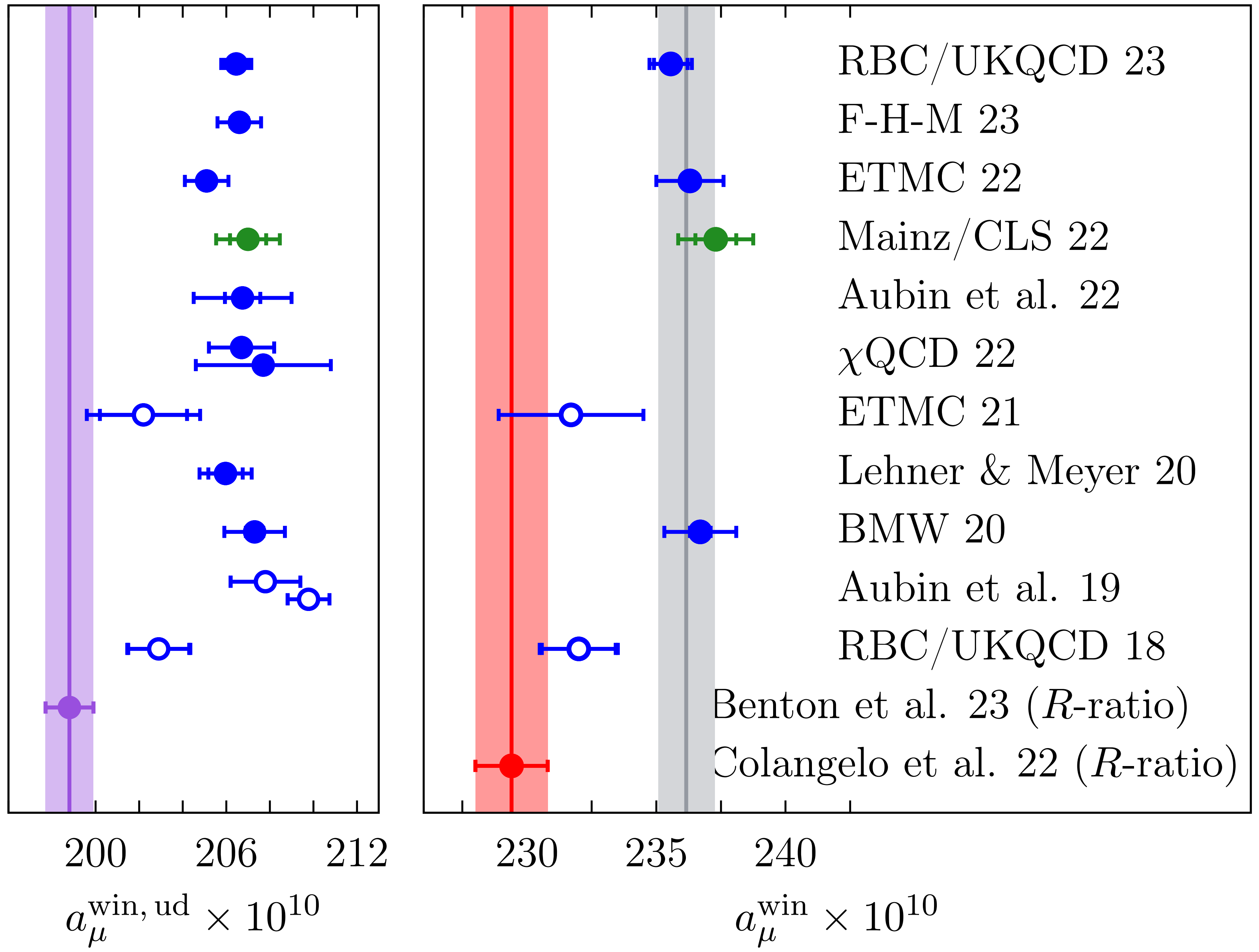

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$$a_\mu^{\rm win}\Big|_{\langle \rm lat\rangle} - a_\mu^{\rm win}\Big|_{e^+e^-} = (6.8 \pm 1.8) \cdot 10^{-10} \quad [3.8\,\sigma]$$

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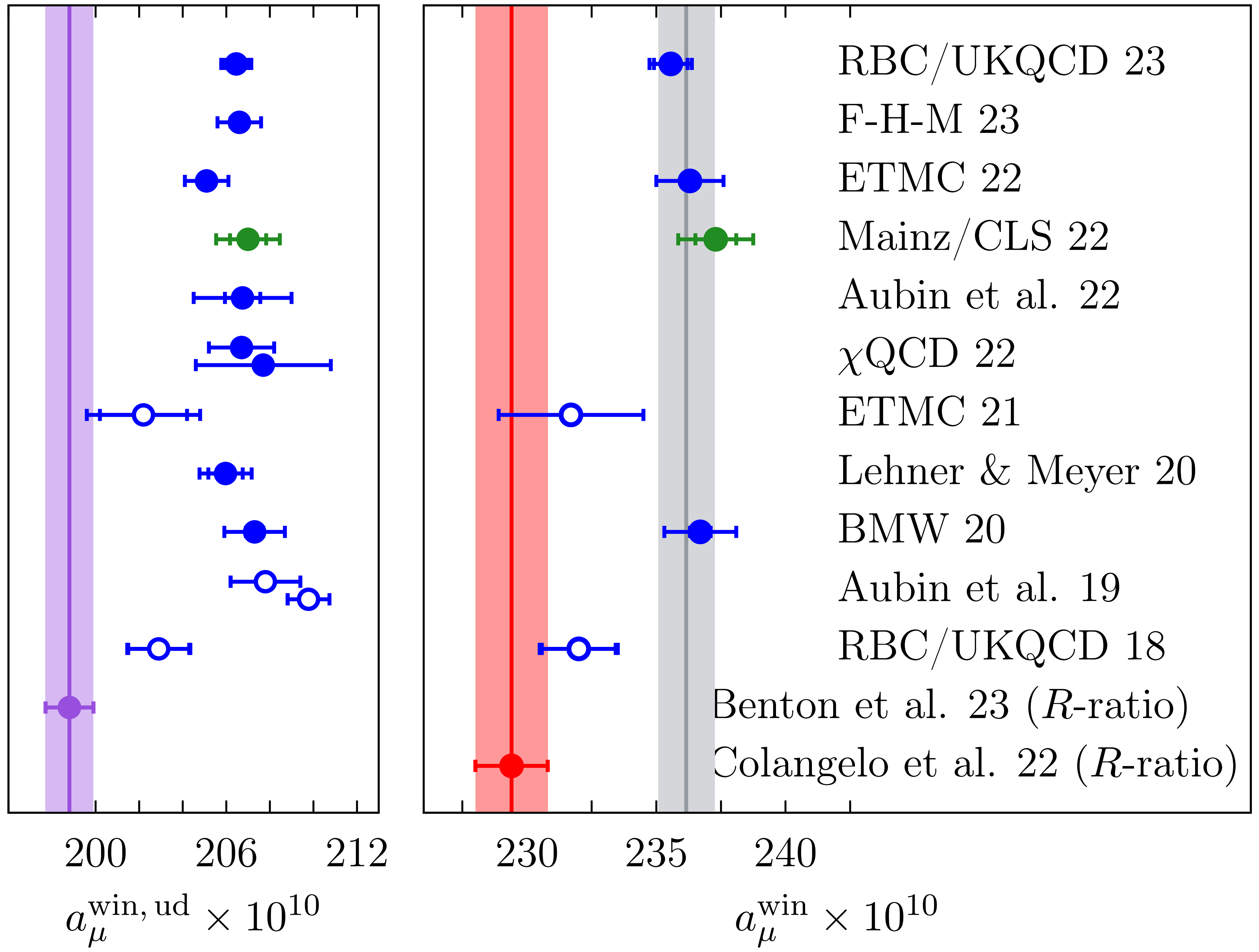

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- Subtract R-ratio result $a_\mu^{\rm win}\big|_{e^+e^-}$ from WP estimate and replace by lattice average $a_\mu^{\rm win}\big|_{\langle{\rm lat}\rangle}$:

$$a_\mu^{\rm exp} - a_\mu^{\rm SM}\Big|^{\rm win}_{e^+e^-\to\langle{\rm lat}\rangle} = (18.1 \pm 4.8)\cdot 10^{-10} \quad [3.8\,\sigma]$$

---

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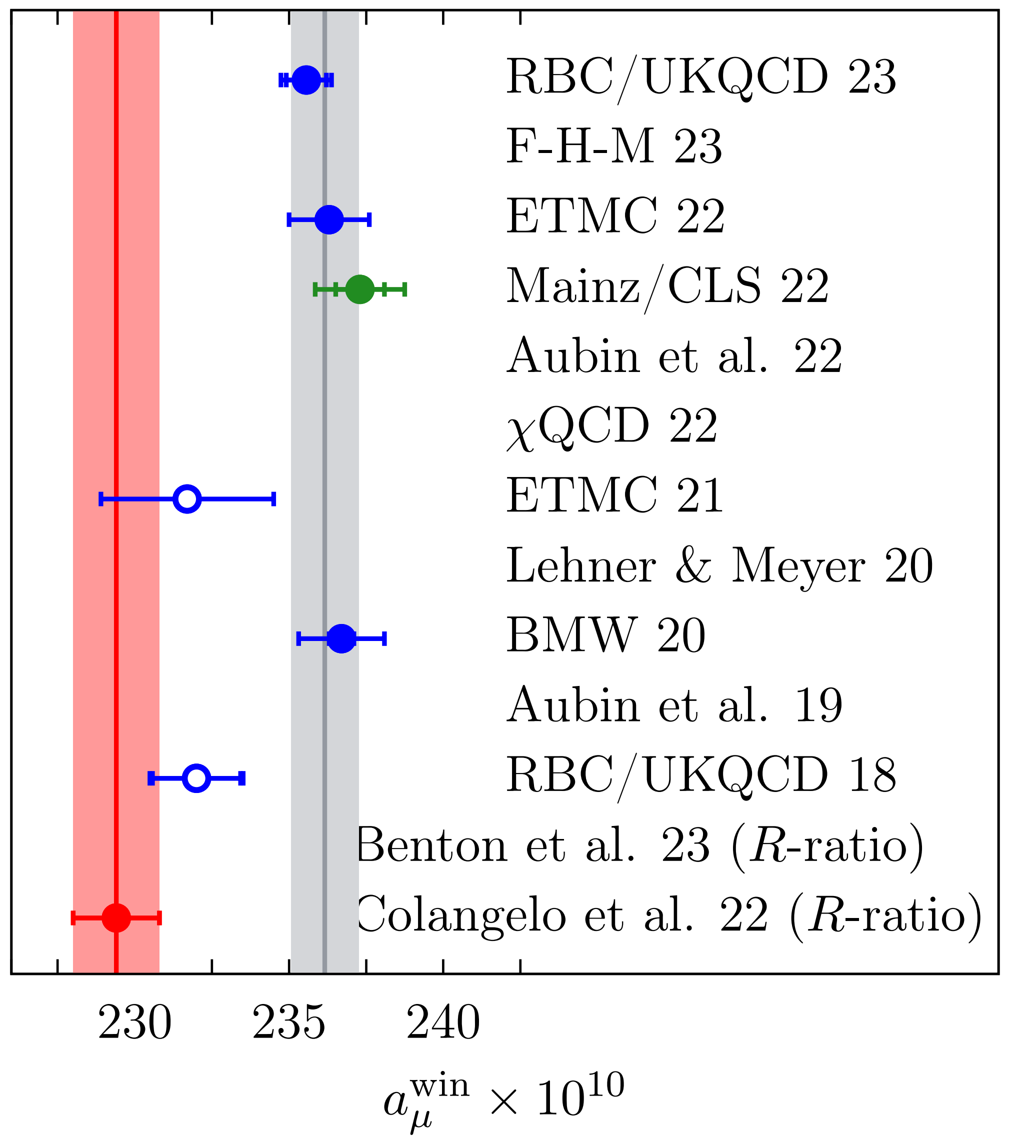
What can we learn from $a_\mu^{\rm win}$?



Primary observable in lattice calculations: vector correlator $G(t)$

$$G(t) \equiv -\frac{a^3}{3}\sum_k\sum_{\vec{x}}\left\langle j_k^{\rm em}(\vec{x},t)\,j_k^{\rm em}(0)\right\rangle = \frac{1}{12\pi^2}\int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s})\,R(s)^{\rm lat}\,s\,{\rm e}^{-\sqrt{s}t}$$

$a_\mu^{\rm win}\big|_{\rm lat} > a_\mu^{\rm win}\big|_{e^+e^-}$ implies that $R(s)^{\rm lat} > R(s)^{e^+e^-}$ in some interval of $\sqrt{s}$

Energy interval $600 \leq \sqrt{s} \leq 900\,{\rm MeV}$ contributes the same fraction to $a_\mu^{\rm hvp}$ and $a_\mu^{\rm win}$

$\sqrt{s}$ interval	$a_\mu^{\rm hvp}$	$(a_\mu^{\rm hvp})^{\rm SD}$	$(a_\mu^{\rm hvp})^{\rm ID}$	$(a_\mu^{\rm hvp})^{\rm LD}$	$\bar{\Pi}(1\ {\rm GeV}^2)$
Below 0.6 GeV	15.5	1.5	5.5	23.5	8.2
0.6 to 0.9 GeV	58.3	23.1	54.9	65.4	52.6
Above 0.9 GeV	26.2	75.4	39.6	11.1	39.2
Total	100.0	100.0	100.0	100.0	100.0

[Cè et al., Phys Rev D106 (2022) 114502]

What can we learn from a_μ^{win} ?

- Phenomenological model for R-ratio predicts *[Mainz/CLS, Cè et al., Phys Rev D 106 (2022) 114502]*

$$\sqrt{s} = 600 - 900\,\text{MeV:} \qquad \frac{R(s)^{\text{lat}}}{R(s)^{e^+e^-}} = 1 + \epsilon \quad \Rightarrow \quad \frac{(a_\mu^{\text{hvp}})^{\text{lat}}}{(a_\mu^{\text{hvp}})^{e^+e^-}} \approx \frac{(a_\mu^{\text{win}})^{\text{lat}}}{(a_\mu^{\text{win}})^{e^+e^-}} = 1 + 0.6\epsilon$$

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  $\Rightarrow$ $R(s)^{\rm lat}$ is enhanced by 5% relative to $R(s)^{e^+e^-}$ for $\sqrt{s} = 600 - 900\,\text{MeV}$

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Similar conclusions

- Dispersive treatment of pion form factor [Colangelo, Hoferichter, Stoffer, PLB 814 (2021) 136073]
- "Energy-smeared" R-ratio from lattice data [ETMC, Alexandrou et al., PRL 130 (2023) 241901]

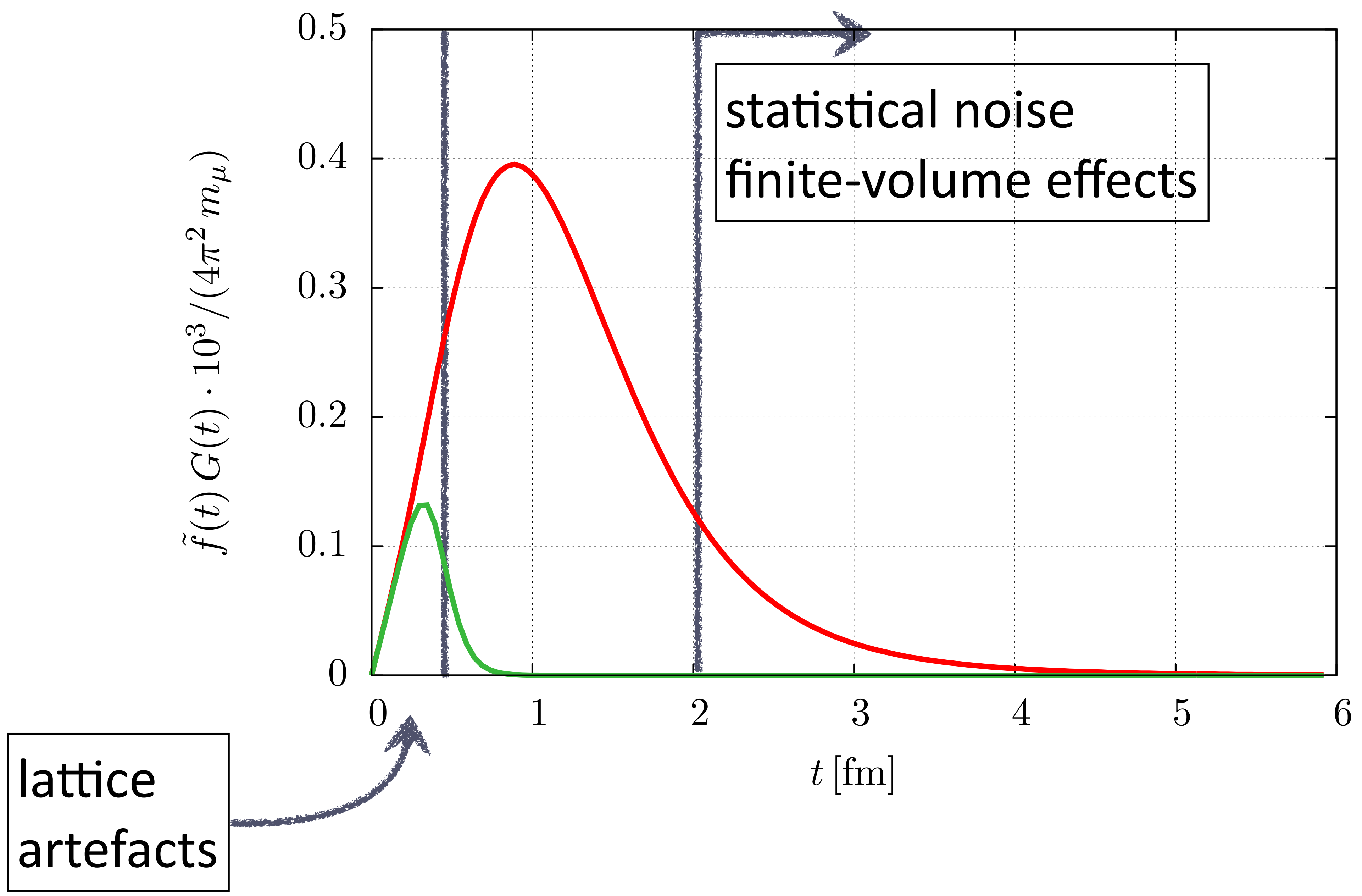
More windows….

Short-distance window:

- Finite-volume correction negligible
- Uncertainty dominated by control over lattice artefacts

$$(a_\mu^{\rm win})^{\rm SD} = (68.85 \pm 0.15 \pm 0.42) \times 10^{-10}$$

[Kuberski et al., JHEP 03 (2024) 172, arXiv:2401.11895]



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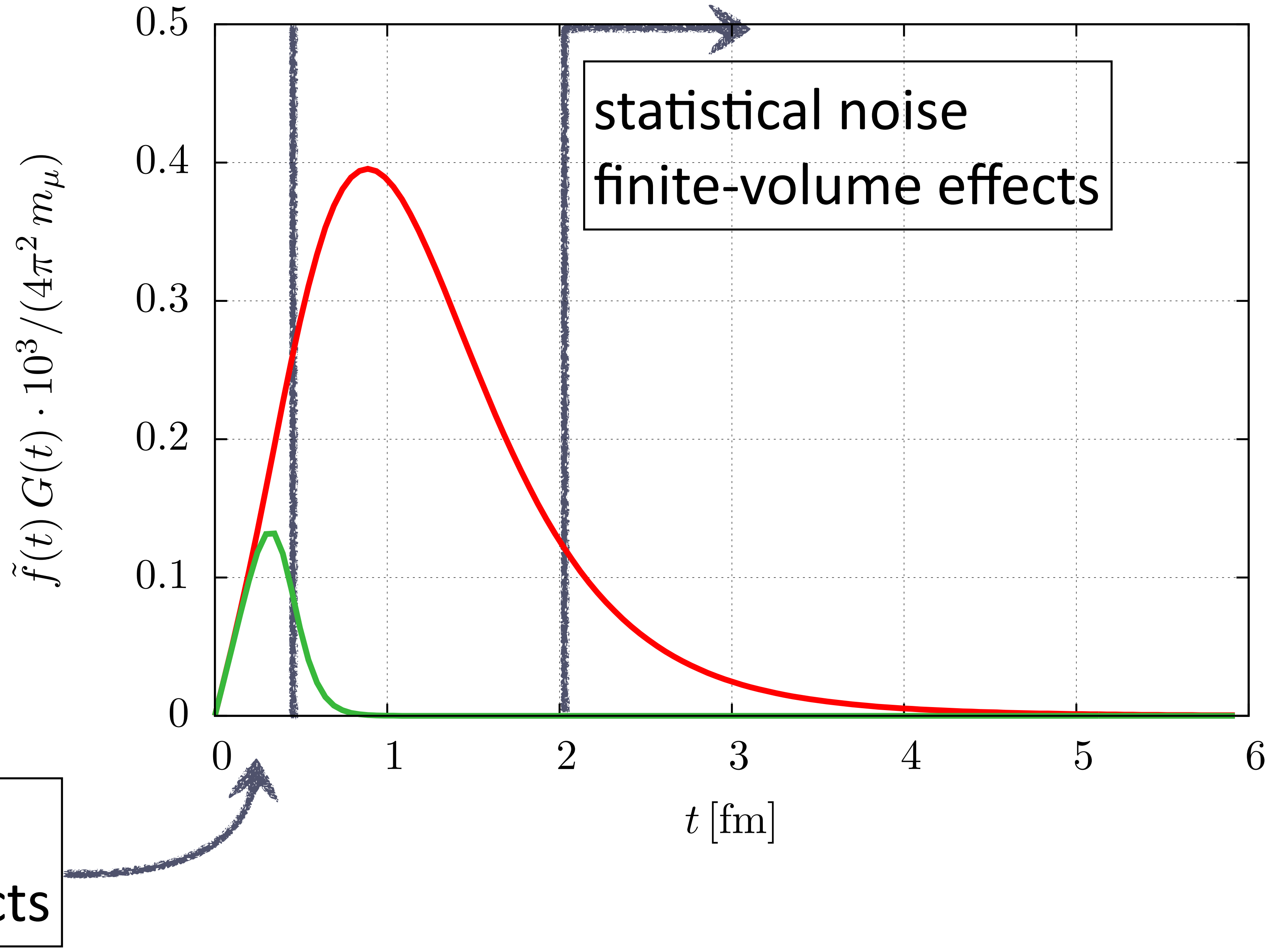
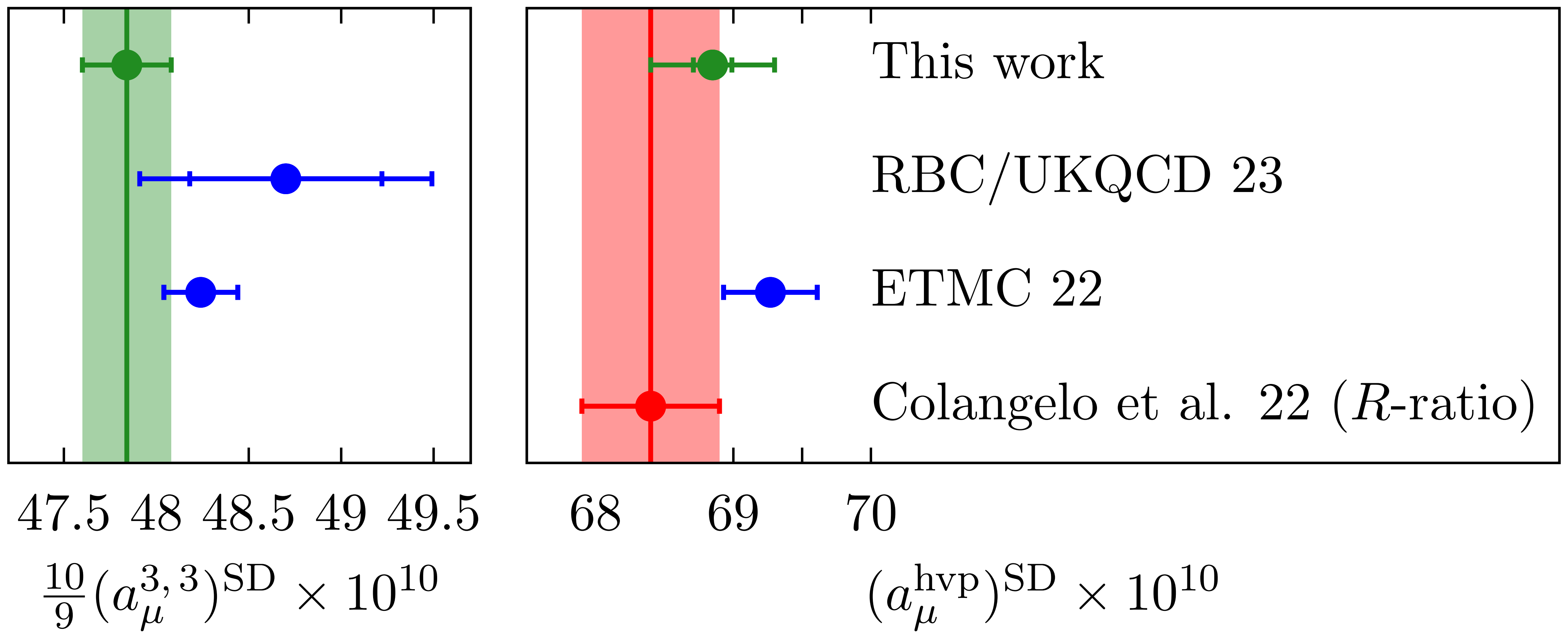


Hadronic model:

- 5% enhancement of $R(s)^{\rm lat}$ for $0.6\,{\rm GeV} \leq \sqrt{s} \leq 0.9\,{\rm GeV}$ increases $(a_\mu^{\rm win})^{\rm SD}$ by $+1 \times 10^{-10}$
- Expectation confirmed by lattice calculations



Hadronic running of electromagnetic coupling

Electromagnetic coupling is energy-dependent:

$$\alpha^{-1} = 137.035\,999\ldots \qquad \alpha(q^2) = \frac{\alpha}{1-\Delta\alpha(q^2)} \qquad \alpha^{-1}(M_Z^2) = 127.951 \pm 0.009$$

Correlation between $a_\mu^{\rm hvp}$ and the hadronic running of $\Delta\alpha_{\rm had}$:

$$\Delta\alpha_{\rm had}(q^2) = -\frac{\alpha\, q^2}{3\pi}\,\mathcal{P}\!\!\int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R(s)}{s(s-q^2)}\,, \qquad a_\mu^{\rm hvp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R(s)\,\hat{K}(s)}{s^2}$$

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Euclidean momenta

$\Delta\alpha_{\mathrm{had}}(-Q^2)$ accessible in lattice QCD via the same correlator $G(t)$ with a different kernel function:

$$\Delta\alpha_{\mathrm{had}}(-Q^2) = \frac{\alpha}{\pi}\frac{1}{Q^2}\int_0^{\infty} dt\, G(t)\left[Q^2t^2 - 4\sin^2\left(\tfrac{1}{2}Q^2t^2\right)\right]$$

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Hadronic running at Z-pole: $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ $\rightarrow$ key quantity in global electroweak fit

Evaluation of $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ and comparison with EW precision data

Adler function approach, aka. “Euclidean split technique”

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \Delta\alpha_{\rm had}^{(5)}(-Q_0^2) \quad \leftarrow \text{ lattice QCD}$$

$$+[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)] \quad \leftarrow \text{ perturbative Adler function}$$

$$+[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)] \quad \leftarrow \text{ pQCD}$$

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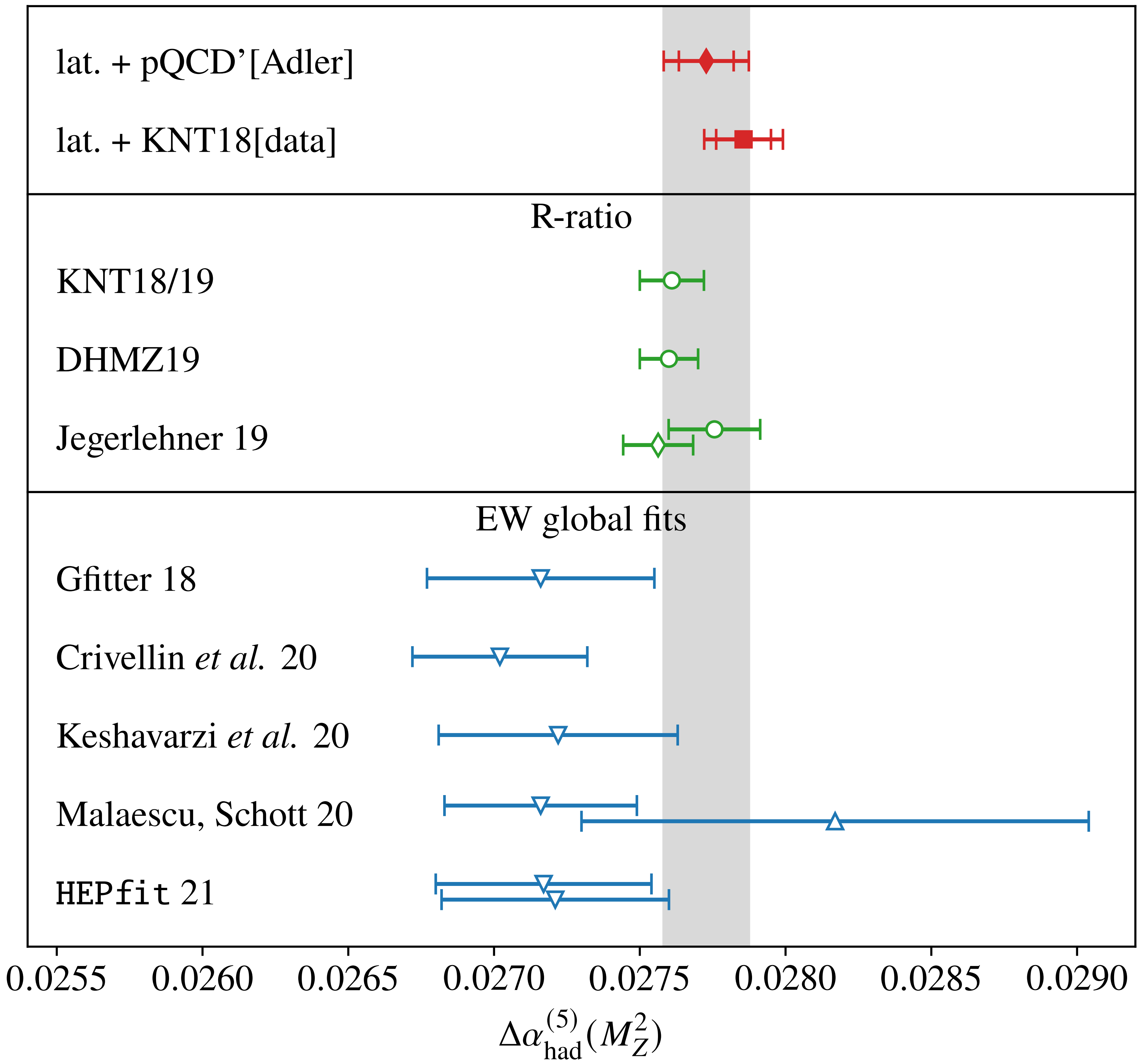
[Mainz/CLS, Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]



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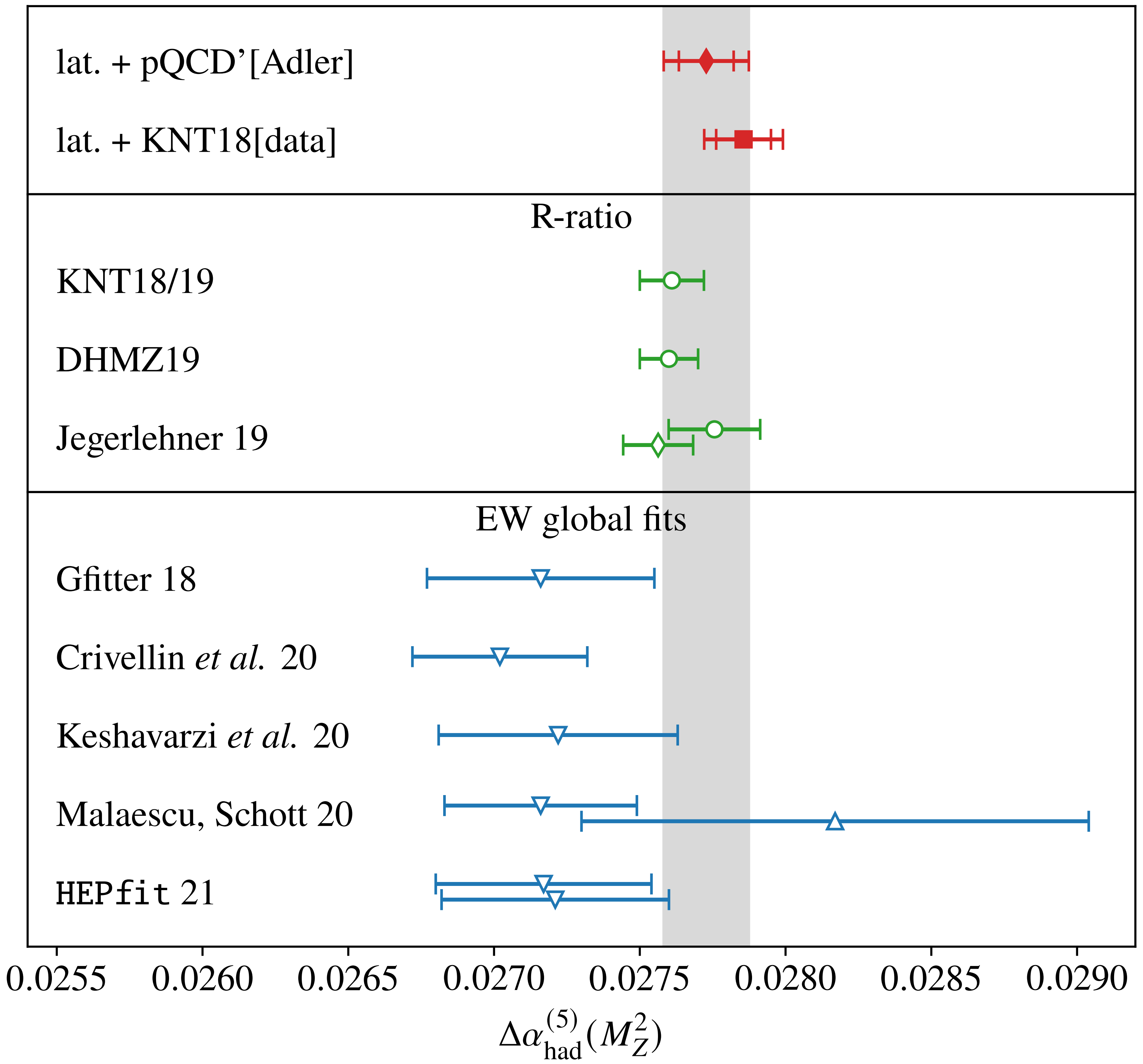

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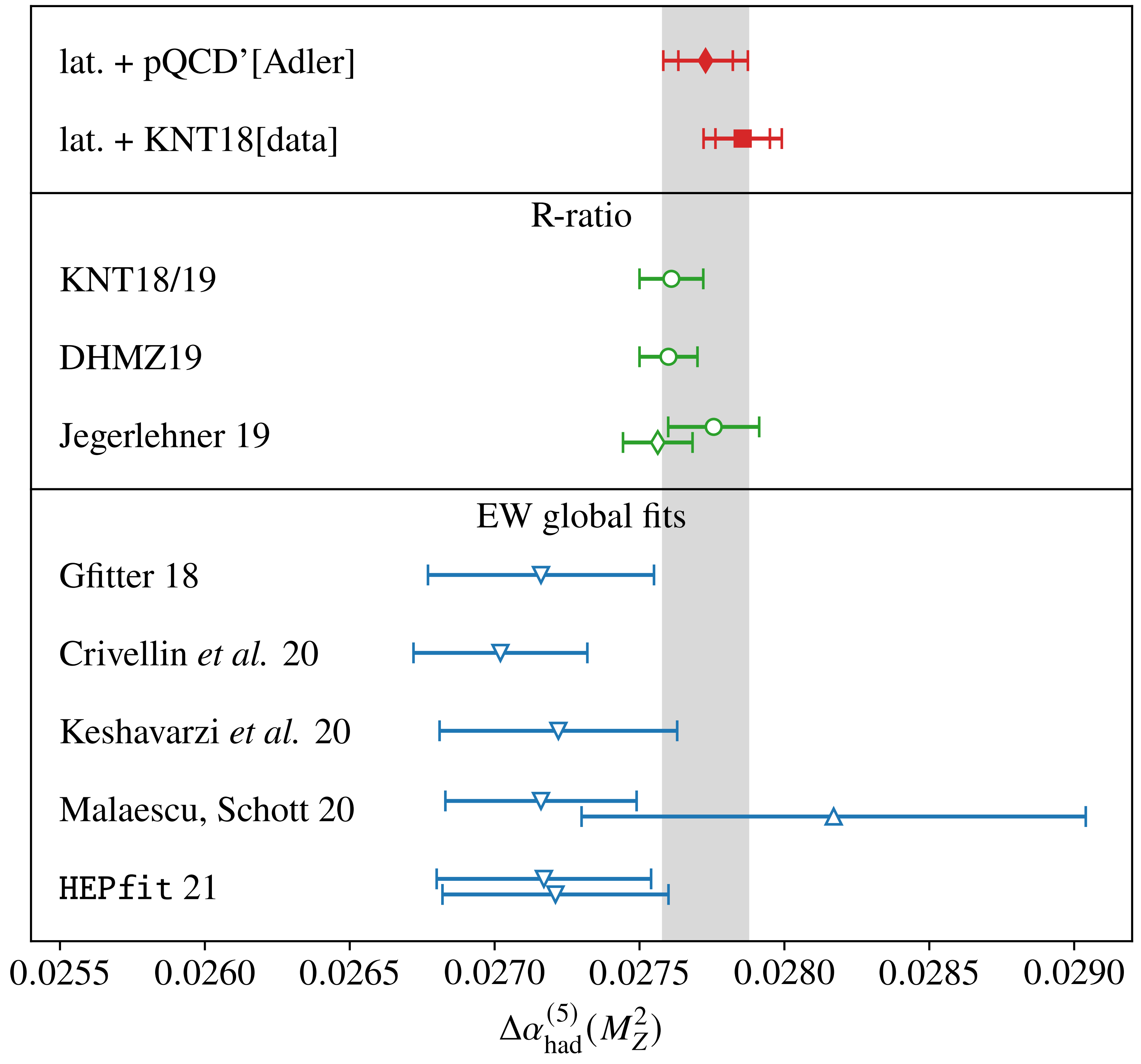

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Standard Model can accommodate a larger value for a_μ without contradicting electroweak precision data

Summary and outlook

No straightforward theoretical interpretation of the Fermilab E989 experiment

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- Tensions between lattice QCD and e^+e^- hadronic cross sections*
- Tension in $\pi^+\pi^-$channel between BaBar vs. KLOE and CMD-3 vs. all other results

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Fermilab E989 prepares to release result including data from Runs 4–6

Update of White Paper expected by $\approx$ Dec 2024, including new lattice results(?)

*pre-2023


7th Plenary Workshop of the Muon g−2 Theory Initiative
September 9-13, 2024 @ KEK, Tsukuba, Japan
https://conference-indico.kek.jp/event/257
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