UV origin of modular flavor symmetries

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Based on collaborations with:
Y. Almumin, M.-C. Chen, V. Knapp-Pérez, X. Li, X.-G. Liu, O. Medina, H.P. Nilles, M. Ramos-Hamud, S. Ramos-Sánchez, S. Shukla, A. Trautner and P. Vaudrevange

Apologies & disclaimers

Apologies & disclaimers:

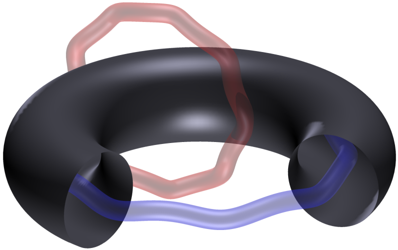
- This talk will not have extensive references to all activities that contribute to this exciting field, sorry!
- I will suppress many details and focus on the big picture instead.
- To fit the talk into the assigned slot I have to make tough choices and sweep certain crucial contributions and insights under the rug.

String Phenomenology

String compactifications

☞ Violins: need to be built in such a way that the oscillating strings produce the right sounds

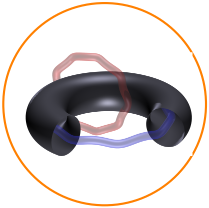
String compactifications



- Violins: need to be built in such a way that the oscillating strings produce the right sounds
- String compactifications: twist the strings in such a way that the excitations carry the quantum numbers of the standard model particles

String phenomenology

String phenomenology



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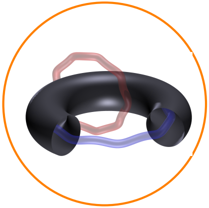

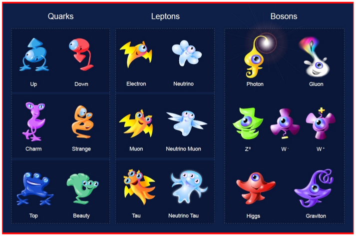

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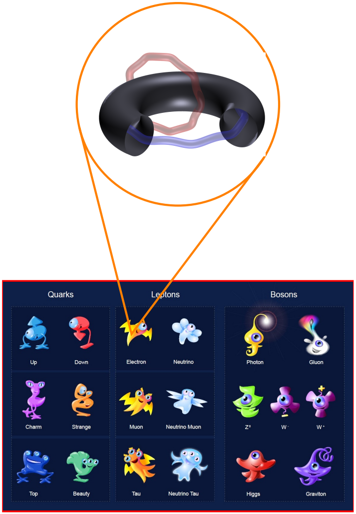

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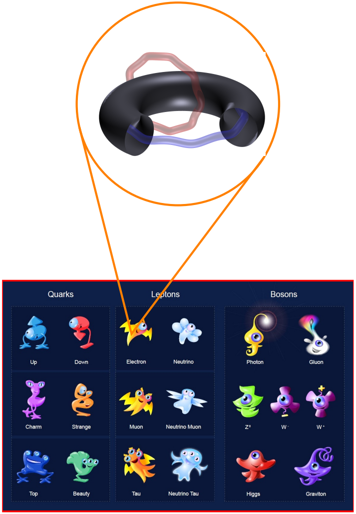

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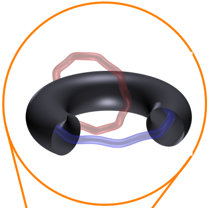
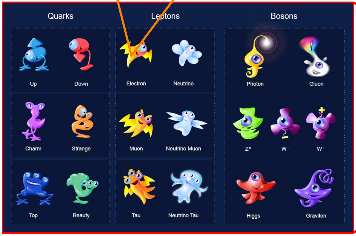
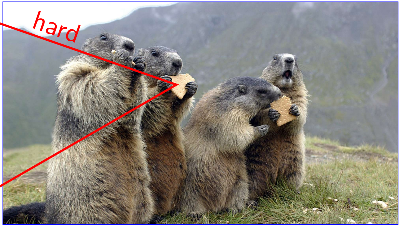
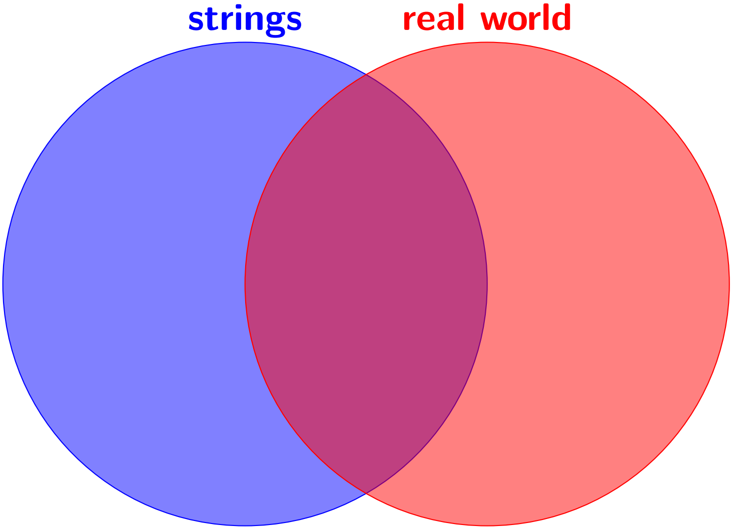


not easy either





Strings and the real world



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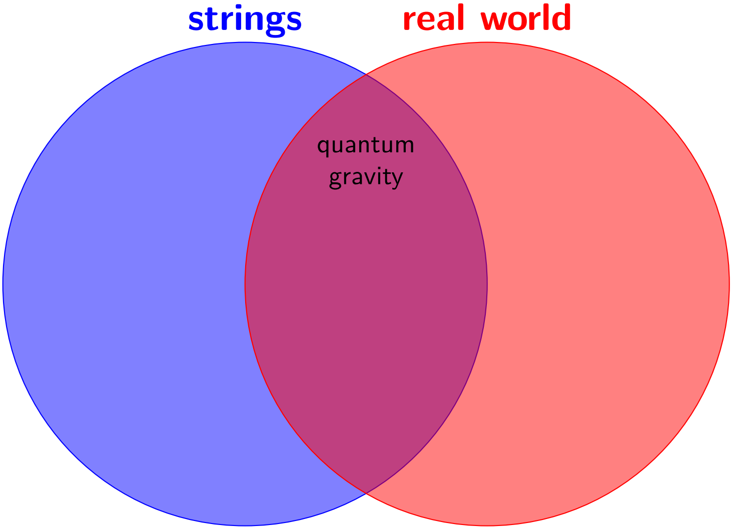

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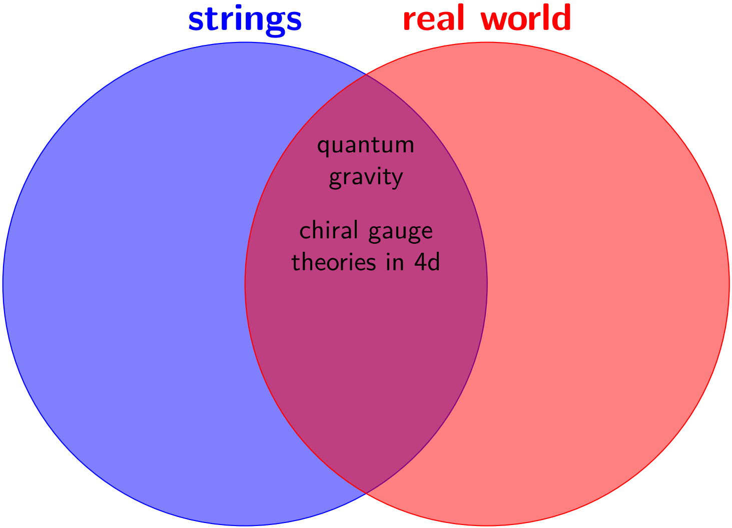

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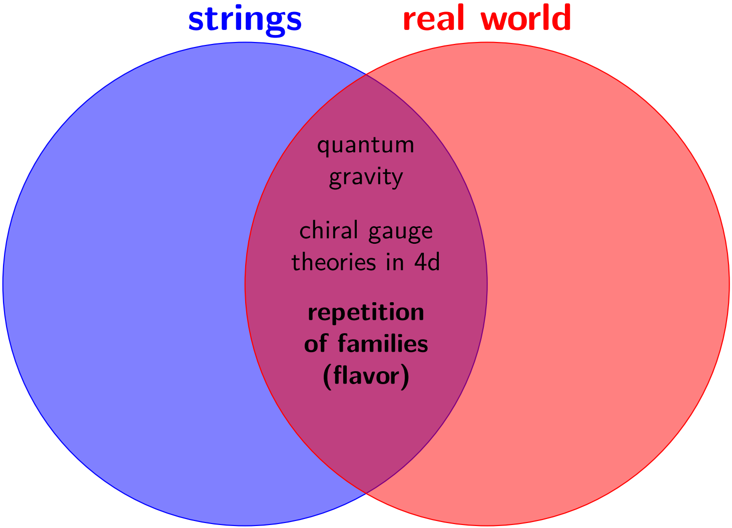
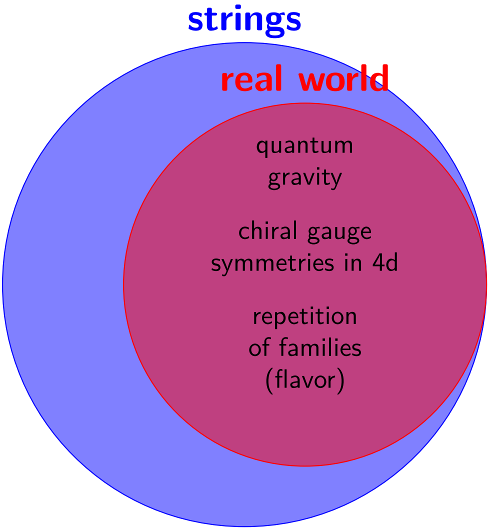


Strings and the real world from the viewpoint of string theorists



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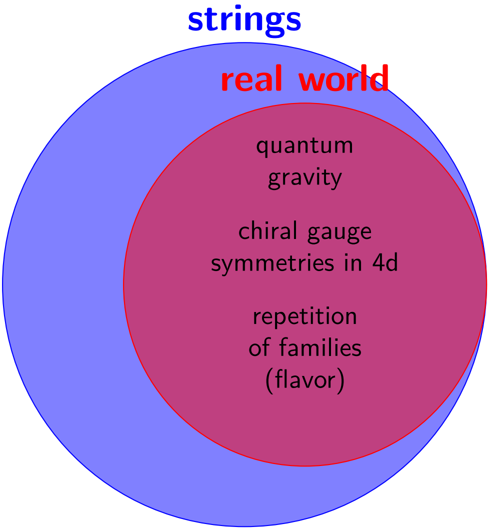

Aims of string phenomenology include understanding of

- fermion masses
- nature of neutrino masses
- mixing angles
- $\mathcal{CP}$ phases
- longevity of proton
- nature of dark matter
- origin of the baryon asymmetry of the universe
- inflation
- ...

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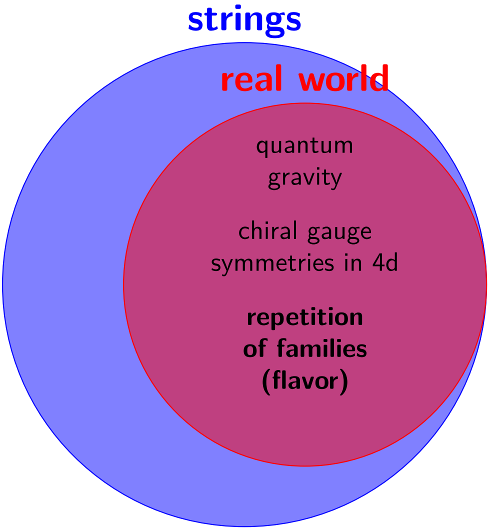

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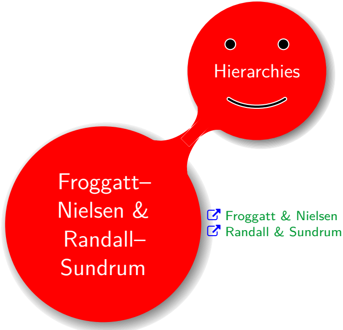
Theories of flavor

Froggatt–Nielsen & Randall–Sundrum

Froggatt & Nielsen [1979];...
Randall & Sundrum [1999];...

Theories of flavor



Froggatt & Nielsen [1979];...
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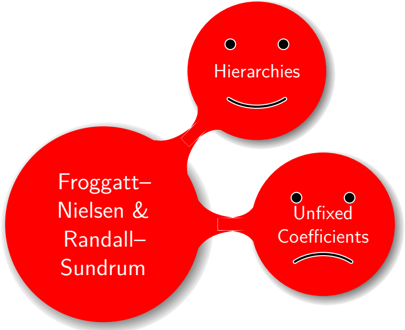

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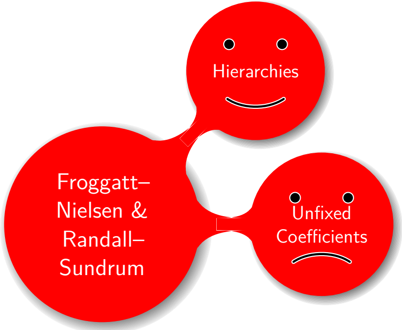
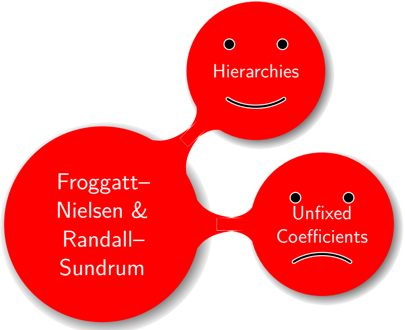


Kaplan & Schmaltz [1994]; . . .

Theories of flavor



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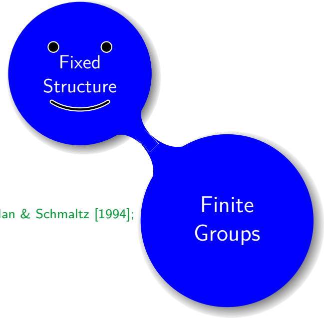

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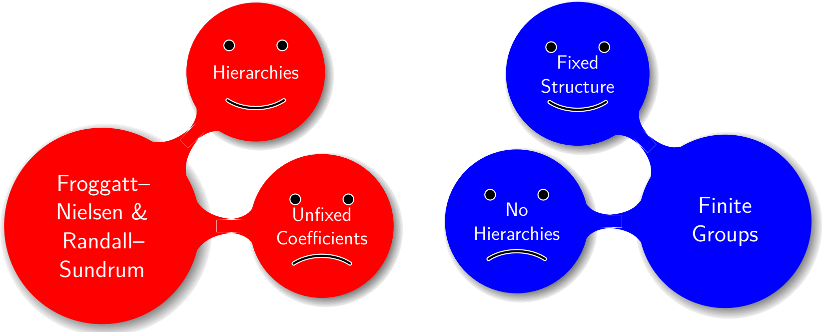
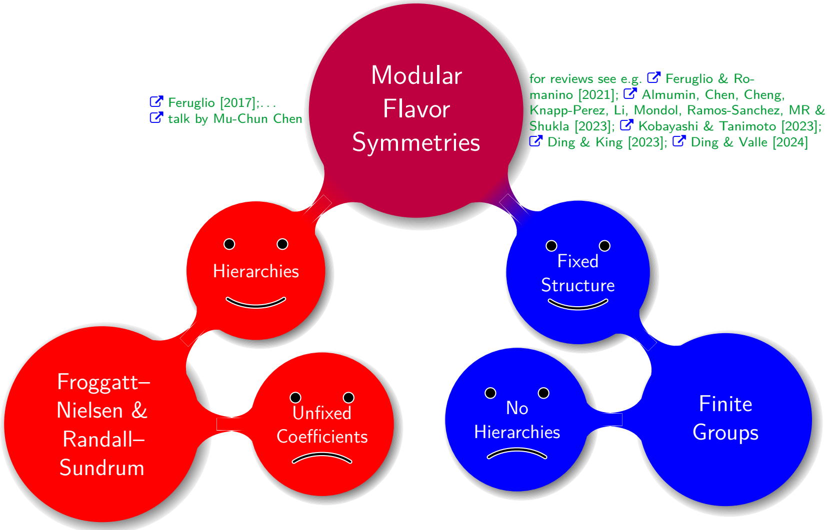
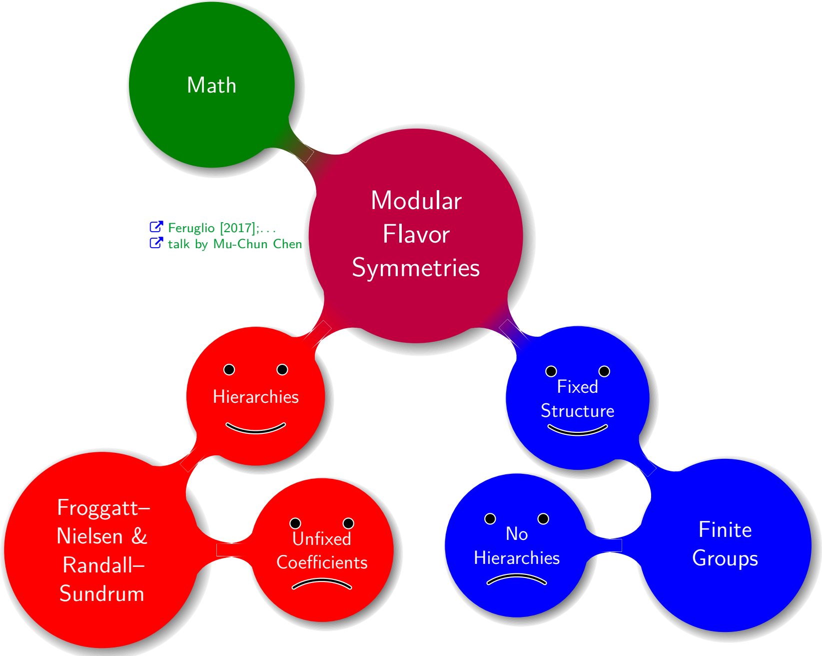


Theories of flavor

Feruglio [2017];. . .
talk by Mu-Chun Chen

for reviews see e.g. Feruglio & Romanino [2021]; Almumin, Chen, Cheng, Knapp-Perez, Li, Mondol, Ramos-Sanchez, MR & Shukla [2023]; Kobayashi & Tanimoto [2023]; Ding & King [2023]; Ding & Valle [2024]




Math
Modular Flavor Symmetries
Feruglio [2017];...
talk by Mu-Chun Chen
Hierarchies
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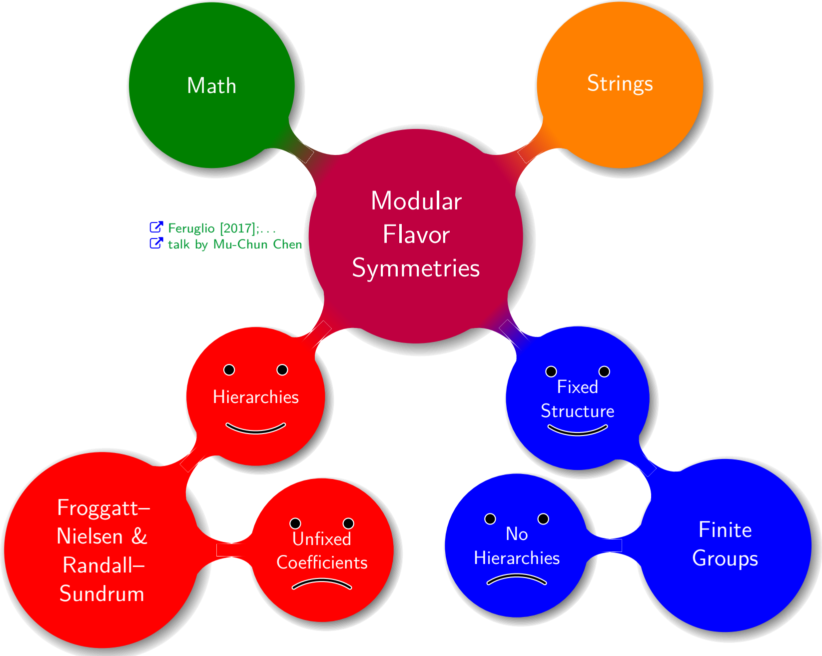
Froggatt–Nielsen & Randall–Sundrum
Unfixed Coefficients
No Hierarchies
Finite Groups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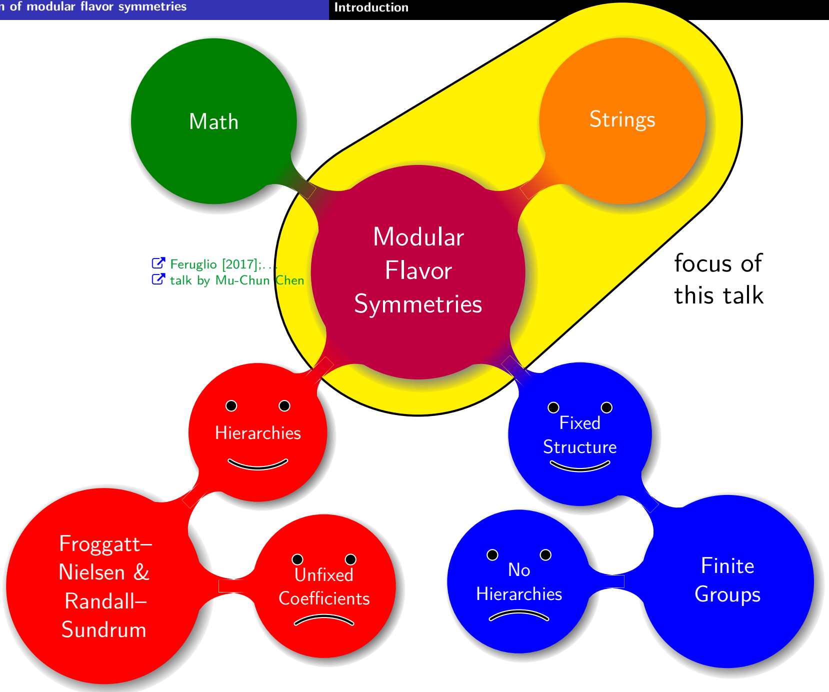
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Modular

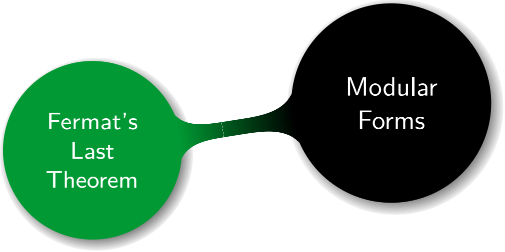
Forms

&

Flavor Symmetries

Modular forms

☞ Modular forms are well known in mathematics and some areas of physics



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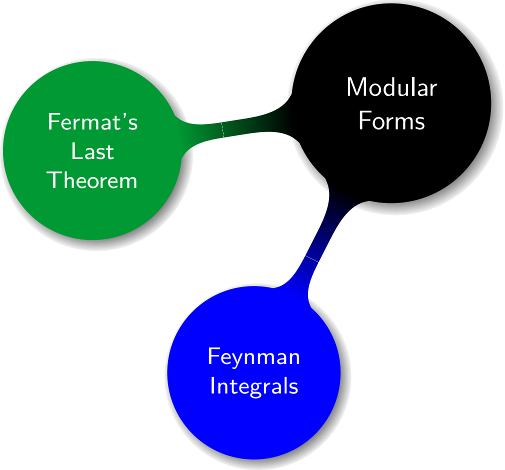

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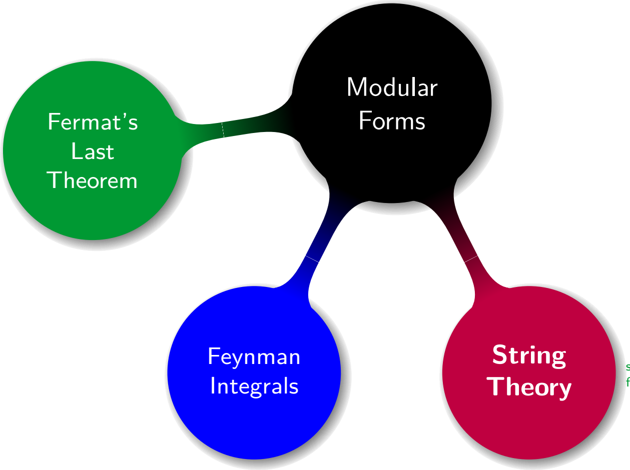

see e.g. D'Hoker & Kaidi [2022] for an overview

Modular forms & modular flavor symmetries

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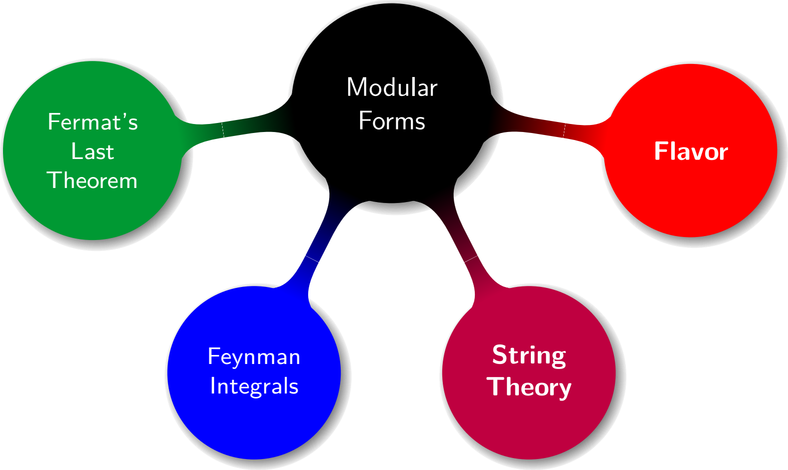

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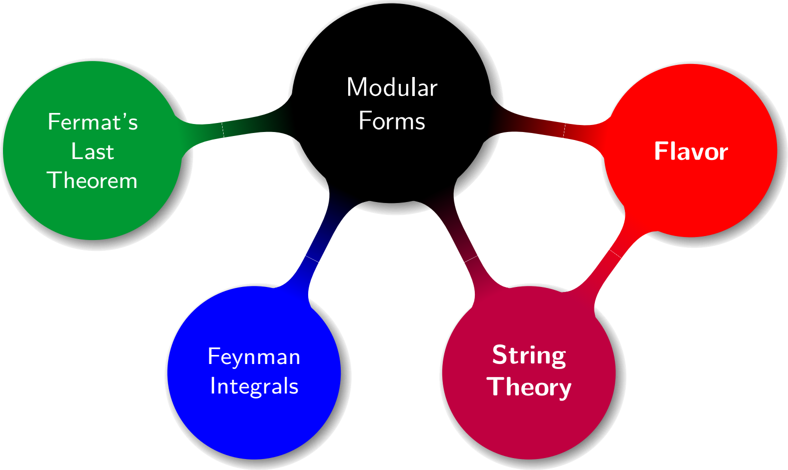
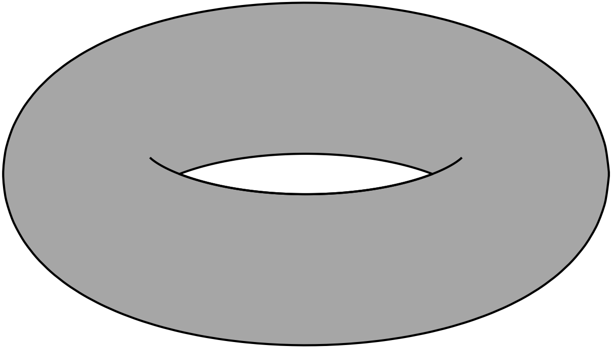


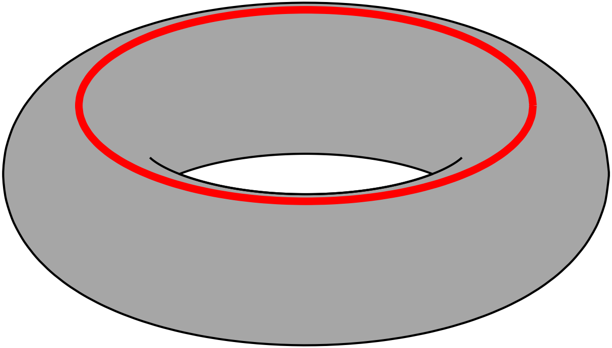
Tori

☞ Torus = donut



Tori

- ☞ Torus = donut
- ☞ Two cycles



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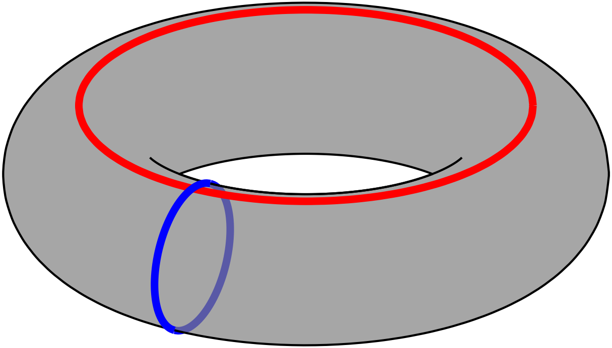
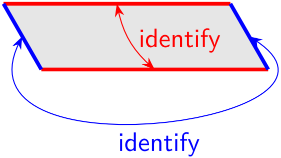


Tori

☞ Torus can be thought of as a parallelogram (which emerges by cutting the torus open along the red and blue cycles)

Tori

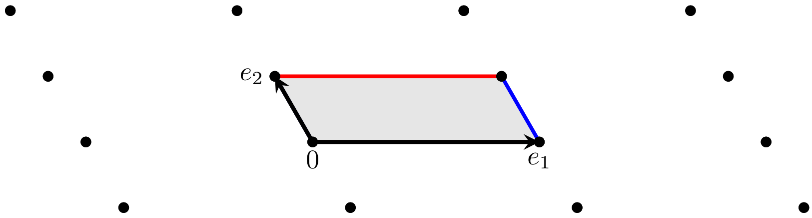


☞ Opposite edges get identified

Tori

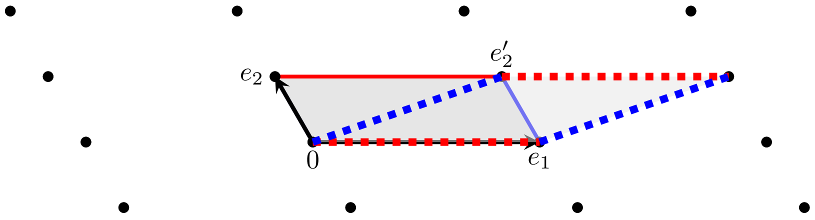
☞ Edges define basis vectors of a lattice

Tori



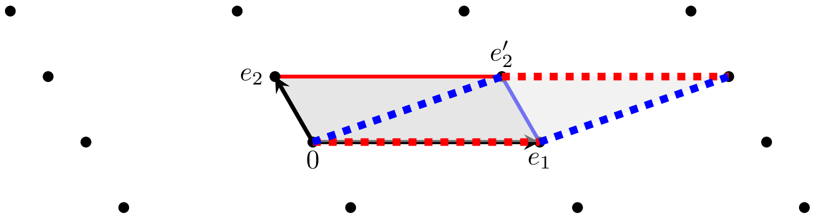
- Torus is $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$: two points in the plane get identified if they differ by a lattice translation

Tori



☞ Fundamental domain is not unique

Tori

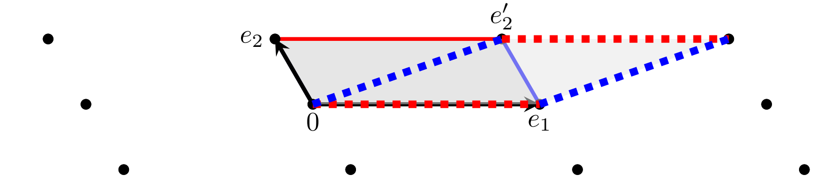


- Fundamental domain is not unique
- We can build linear combinations of the basis vectors

$$\begin{pmatrix} e_2 \\ e_1 \end{pmatrix} \xmapsto{\gamma} \begin{pmatrix} e_2' \\ e_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e_2 \\ e_1 \end{pmatrix} =: \gamma \begin{pmatrix} e_2 \\ e_1 \end{pmatrix}$$

$a, b, c, d \in \mathbb{Z}$

Tori



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- Volume of fundamental domain stays the same $\Leftrightarrow \det\gamma = 1 \curvearrowright$ $\gamma \in \mathrm{SL}(2,\mathbb{Z})$ (there is a superfluous sign, so $\gamma \in \Gamma = \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2$)

$\mathrm{SL}(2,\mathbb{Z})$

☞ Two basic transformations

$$T \;: e_2 \xmapsto{T} e_2' = e_2 + e_1 \qquad \curvearrowright \gamma = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} =: T$$

$$S \;: e_1 \xmapsto{S} e_1' = e_2 \quad \& \quad e_2 \mapsto e_2' = -e_1 \qquad \curvearrowright \gamma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} =: S$$

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$\tau \xmapsto{T} \tau + 1$ and $\tau \xmapsto{S} \dfrac{-1}{\tau}$

$\mathrm{SL}(2,\mathbb{Z})$ and modular flavor symmetries

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- S and T generate $\mathrm{SL}(2,\mathbb{Z})$ and $S^2 = -\mathbb{1}$ & $(S\,T)^3 = \mathbb{1}$

$\mathrm{SL}(2,\mathbb{Z})$ and modular flavor symmetries

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Modular flavor symmetries:

Identify finite groups with generators satisfying $\Gamma = \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2$ relations

$$S^2 = (S\,T)^3 = \mathbb{1} \quad \& \quad \text{additional relations}$$

Congruence subgroups of Γ & modular flavor symmetries

Feruglio [2017]

Congruence subgroups of Γ & modular flavor symmetries

Feruglio [2017]

☞ Congruence subgroups of $\Gamma := \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \; ; \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mod N \right\}$$

level

$\Gamma = \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2$

Congruence subgroups of Γ & modular flavor symmetries

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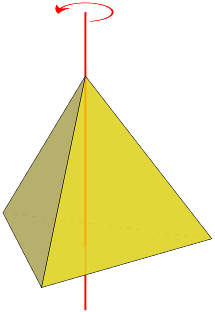

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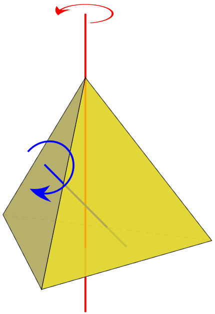

Vector-valued modular forms & modular flavor symmetries

- Traditional modular forms

$$f(\gamma\,\tau) = (c\tau + d)^k f(\tau)$$

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2$$

Vector-valued modular forms & modular flavor symmetries

☞ Traditional modular forms

$$f(\gamma\,\tau) = (c\tau + d)^k f(\tau)$$

$k \in \mathbb{N}$ modular weight

Vector-valued modular forms & modular flavor symmetries

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Liu & Ding [2022]

- Vector-valued modular forms

$$f_i(\gamma\,\tau) = (c\tau+d)^k\,[\rho_N(\gamma)]_{ij}\,f_j(\tau)$$

representation matrix of e.g. Γ_N
e.g. $\Gamma_3 = A_4$

- Conditions:
  - modular covariance

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Liu & Ding [2022]

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Liu & Ding [2022]

- Vector-valued modular forms

$$f_i(\gamma\,\tau) = (c\tau+d)^k\,[\rho_N(\gamma)]_{ij}\,f_j(\tau)$$

- Conditions:
  - modular covariance
  - meromorphy
  - no singularities

Vector-valued modular forms & modular flavor symmetries

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Crucial property of modular forms:

Given a finite modular group and a modular weight, the modular vector-valued modular form transforming in a given irrep $\boldsymbol{r}$ is *uniquely** determined.

Restrictions may apply.
Talk to your local mathematician to find out if modular forms are right for you.

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- E.g. modular A_4 model:

Feruglio [2017]; cf. talk by Mu-Chun Chen

$$\left.\begin{array}{r}\Lambda\\ \operatorname{Re}\tau\\ \operatorname{Im}\tau\end{array}\right\} \xrightarrow{\text{predict}} \left\{\begin{array}{l}\text{3 mass eigenvalues } m_i\\ \text{3 mixing angles } \theta_{ij}\\ \text{3 phases (1 Dirac \& 2 Majorana)}\end{array}\right.$$

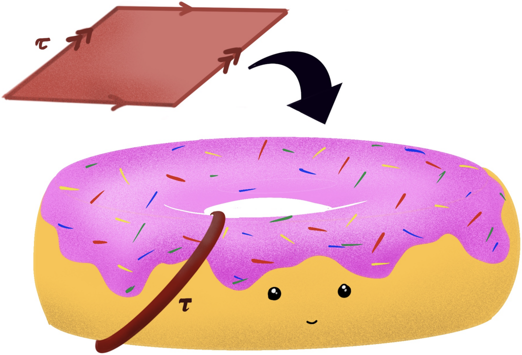
What do

Tori

have to do with

Flavor

???



Courtesy of Shreya Shukla

An instructive toy example

Bachas [1995a]

☞ We will discuss magnetized tori, which have a couple of interesting features:

- symmetries and chiral fermions which resemble the SM,
- explicit tori so rather obvious modular symmetries,
- field-theoretic description

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- symmetries and chiral fermions which resemble the SM,
- explicit tori so rather obvious modular symmetries,
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Bachas [1995b]

☞ However, this scheme also has some disturbing aspects:

- problems with the vacuum energy,
- so far no model that comes close to the SM

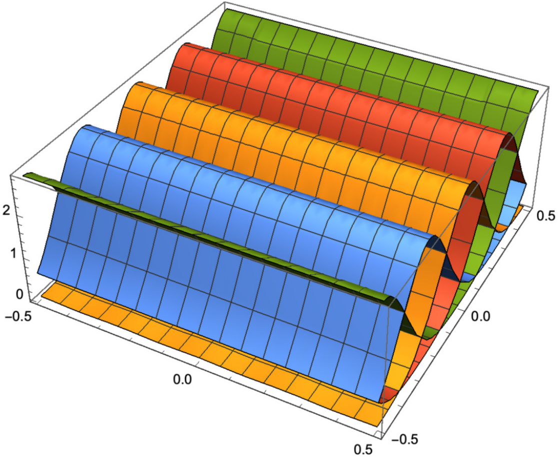
➥ This is just a **toy model**

Zero-modes on magnetized tori & modular weights

☞ Torus with magnetic flux ↷ chiral zero modes

Cremades, Ibáñez & Marchesano [2004]

$$\psi^{j,M}(z,\tau,\zeta) = \mathcal{N}\,\mathrm{e}^{\pi\,\mathrm{i}\,M\,(z+\zeta)\,\frac{\mathrm{Im}(z+\zeta)}{\mathrm{Im}\,\tau}}\,\vartheta\begin{bmatrix}\frac{j}{M}\\0\end{bmatrix}\bigl(M\,(z+\zeta), M\,\tau\bigr)$$



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Flux parameter $\curvearrowright$ # of zero modes

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"Wilson line"

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Jacobi ϑ-function

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$$\mathcal{N} = \left(\frac{2M\ \mathrm{Im}\,\tau}{\mathcal{A}^2}\right)^{1/4} \propto (\mathrm{Im}\,\tau)^{-1/4}$$

area of torus

$$\mathcal{A} = (2\pi R)^2\, \mathrm{Im}\,\tau$$

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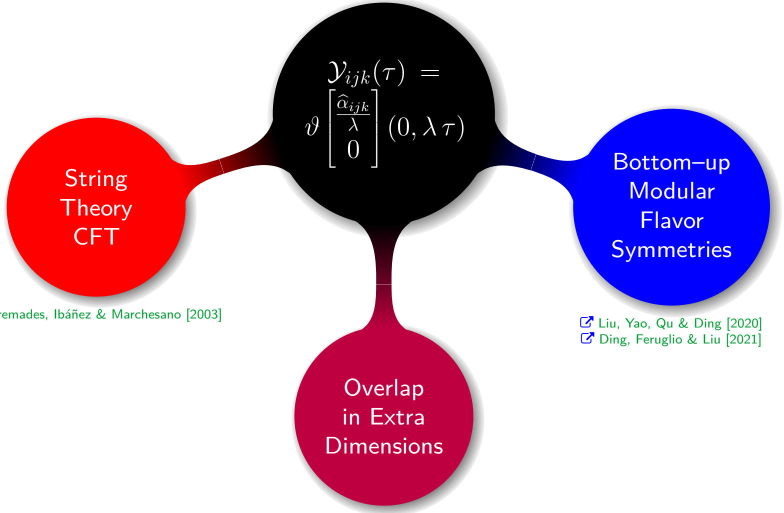
☞ Yukawa couplings can be computed via overlap integrals

➥ Yukawa couplings transform as vector-valued modular* forms

metaplectic

Three ways of computing Yukawa couplings

Almumin, Chen, Knapp-Pérez, Ramos-Sánchez, MR & Shukla [2021]

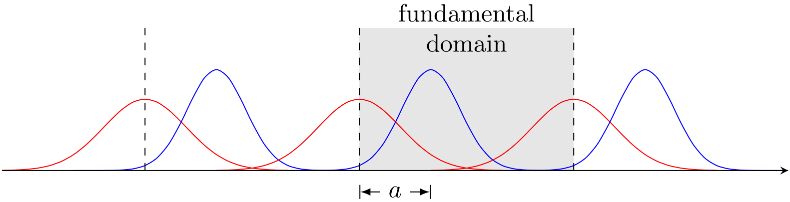


Cremades, Ibáñez & Marchesano [2003]

Liu, Yao, Qu & Ding [2020]
Ding, Feruglio & Liu [2021]

Cremades, Ibáñez & Marchesano [2004]

Benefits of the toy model with an explicit torus



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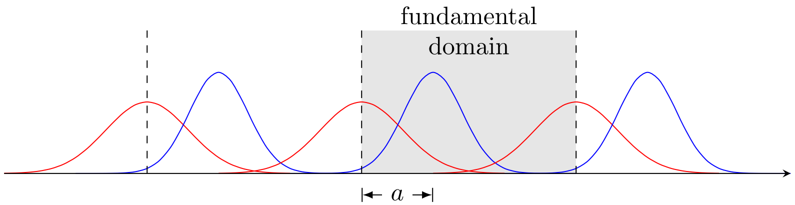

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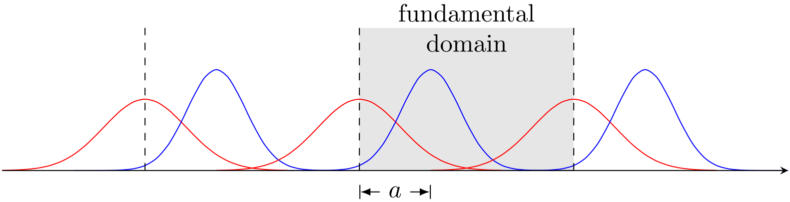


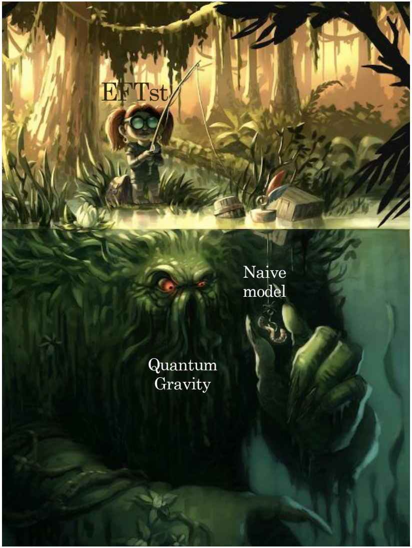
- Couplings of fields sitting at different positions are exponentially suppressed with the distance
- Unsuppressed couplings if all fields sit at the same place
- None of this is surprising but crucially the modular forms, which are uniquely determined by the metaplectic symmetries, have these features

Modular Flavor Symmetries

from

Strings

Strings and the real world



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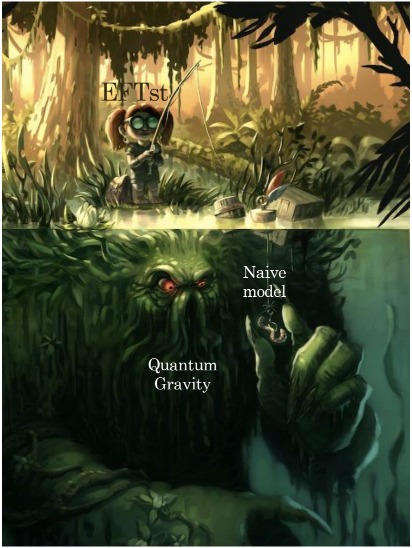

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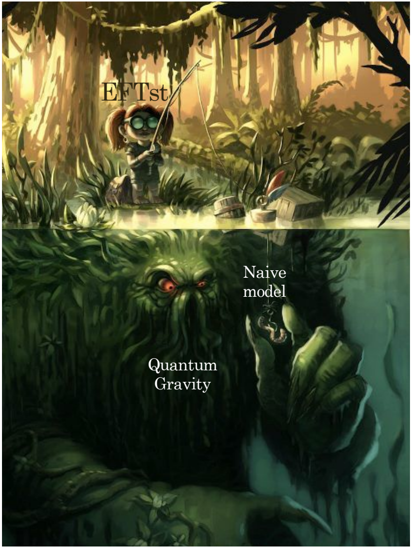

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☞ Naive bottom-up models failing to fulfill consistency conditions are ruled out. . .

. . . and so are top-down models which are inconsistent with observation

Particle physics from heterotic orbifolds

- ☞ Heterotic orbifolds have a long and rich history in providing us with potentially realistic models

  Ibáñez, Kim, Nilles & Quevedo [1987];...; Ramos-Sanchez & MR [2024]

as opposed to unrealistic

substantial literature

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- Explicit MSSM models with the following ingredients exist:
  - exactly 3 generations (vector-like exotics can be consistently decoupled)
  - no (fractionally charged or other) exotics
  - a "hidden" gauge group with a condensation scale suitable to explain TeV-scale soft masses
  - just one pair of of Higgs with the μ term of the order of the soft masses
  - proton decay sufficiently suppressed

R symmetries

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- proton decay sufficiently suppressed
- realistic top Yukawa coupling
- pseudo-anomalous $\mathrm{U}(1)$ symmetry

Modular flavor symmetries were hiding in plain sight

☞ (Yukawa) couplings *are* modular forms in heterotic orbifolds e.g. Quevedo [1996]

This is nothing but one of the $SL(2,\mathbf{Z})_{T,U}$ transformation for toroidal orbifold compactifications ($a = b = d = 1, c = 0$ in eq. (10)). Therefore the only conditions these symmetries impose on W is that it should transform as a modular form of a given weight ($W \to (cT+d)^{-3}W$ for the simplest toroidal orbifolds with T the overall size of the compactification space)[36]. In fact, explicit calculations for specific orbifold models show that

$$W_{tree}(T, Q^I) = \chi_{IJK}(T)\, Q^I Q^J Q^K + \cdots \tag{19}$$

with $\chi(T)$ a particular modular form of $SL(2,\mathbf{Z})$ or any other duality group and the ellipsis represent higher powers of Q, exponentially suppressed. The identification of $\chi(T)$ with modular forms was a highly nontrivial check of the explicit orbifold calculations which were preformed in refs. [37] without any relation (nor knowledge) of the underlying duality symmetry $SL(2,\mathbf{Z})$. This kind of symmetry puts also strong constraints to the higher order, nonrenormalizable, corrections to W, since each matter field Q transforms in a particular way under that symmetry ($Q \to (cT+d)^n Q$ with n the modular weight of Q). There are also other discrete symmetries, as those defined by the point group $\mathcal{P}$ and space group $\mathcal{S}$ of an orbifold which have to be respected by the superpotential W. These 'selection rules' are very important to find vanishing couplings and uncover flat directions which can be used to break the original gauge symmetries and construct quasi-realistic models.

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- These facts have been forgotten when non-Abelian flavor symmetries from strings were discussed more recently

  Kobayashi, Nilles, Plöger, Raby & MR [2007]

Eclectic flavor symmetries in heterotic orbifolds

Baur, Nilles, Trautner & Vaudrevange [2019]
Nilles, Ramos–Sánchez & Vaudrevange [2021]; Baur, Kade, Nilles, Ramos-Sanchez & Vaudrevange [2021]

☞ Discrete flavor symmetries are identified as the outer automorphisms of the Narain space group

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- Discrete flavor symmetries are identified as the outer automorphisms of the Narain space group
- These symmetries include:
  - traditional flavor symmetries
  - modular flavor symmetries
  - R symmetries (including non-Abelian discrete R symmetries)
  - $\mathcal{CP}$ symmetries and $\mathcal{CP}$-like transformations

Lessons from strings

Nilles & Ramos-Sanchez [2024]

	bottom-up	top-down
Modular flavor symmetries	✓	✓

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	bottom-up	top-down
Modular flavor symmetries	✓	✓
Modular weights	arbitrary	constrained

fractional modular weights & superpotential has nontrivial modular weight

Lessons from strings

Nilles & Ramos-Sanchez [2024]

	bottom-up	top-down
Modular flavor symmetries	✓	✓
Modular weights	arbitrary	constrained
Representations	arbitrary	constrained

representations are often linked to modular weight

Lessons from strings

Nilles & Ramos-Sanchez [2024]

	bottom-up	top-down
Modular flavor symmetries	✓	✓
Modular weights	arbitrary	constrained
Representations	arbitrary	constrained
Normalization of modular forms	?	✓

☞ Bottom-up proposal for normalization

Petcov [2024]

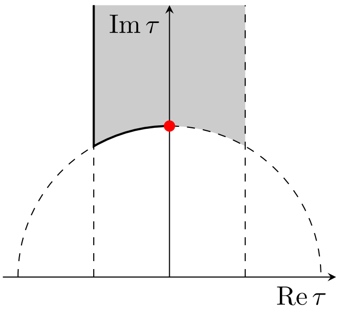
Modular flavor symmetries

☞ Modular flavor symmetries trade the VEV alignment conundrum for the question of stabilizing τ

Modular flavor symmetries

- Modular flavor symmetries trade the VEV alignment conundrum for the question of stabilizing τ
- In many phenomenologically viable scenarios the best-fit values of τ occur close to, but slightly away from, the critical points $\tau = \mathrm{i}$ or $\tau = \omega := \mathrm{e}^{2\pi\,\mathrm{i}\,/3}$

Feruglio [2023a]; Feruglio [2023b]; Ding, Feruglio & Liu [2024]; Petcov & Tanimoto [2023a]; Petcov & Tanimoto [2023b];...



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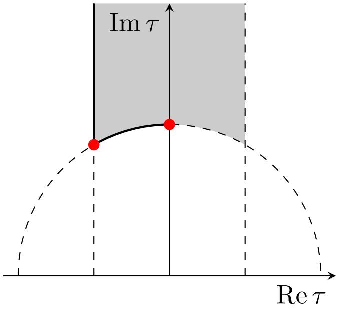

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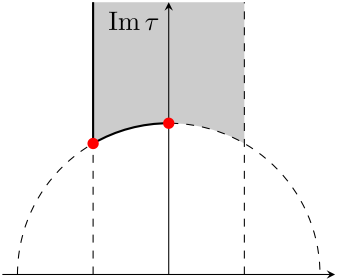

Feruglio [2023a]; Feruglio [2023b]; Ding, Feruglio & Liu [2024]; Petcov & Tanimoto [2023a]; Petcov & Tanimoto [2023b];...

- Challenge: find mechanisms that explain values of τ close to but not precisely at the critical points

Dent [2002]; Kobayashi, Shimizu, Takagi, Tanimoto & Tatsuishi [2019]; Ishiguro, Kobayashi & Otsuka [2021] Novichkov, Penedo & Petcov [2022]; Ishiguro, Okada & Otsuka [2022]; Leedom, Righi & Westphal [2023];...

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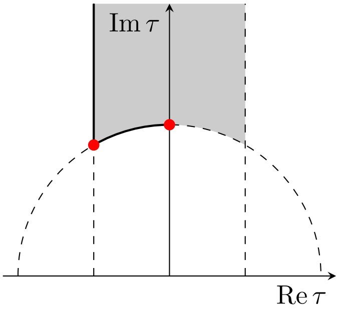

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- It is possible to relate the small departure of τ to supersymmetry breaking in de Sitter vacua with phenomenologically viable soft masses

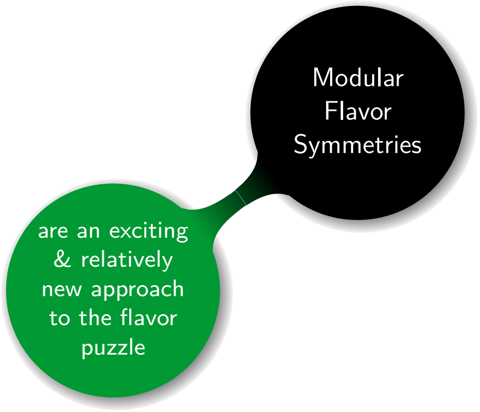
Knapp-Perez, Liu, Nilles, Ramos-Sanchez & MR [2023]

Summary

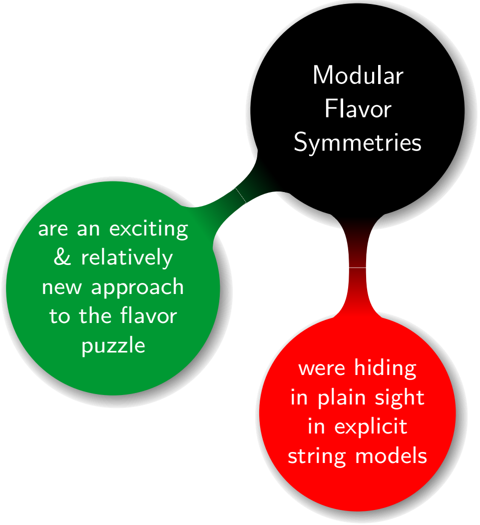
&

Outlook

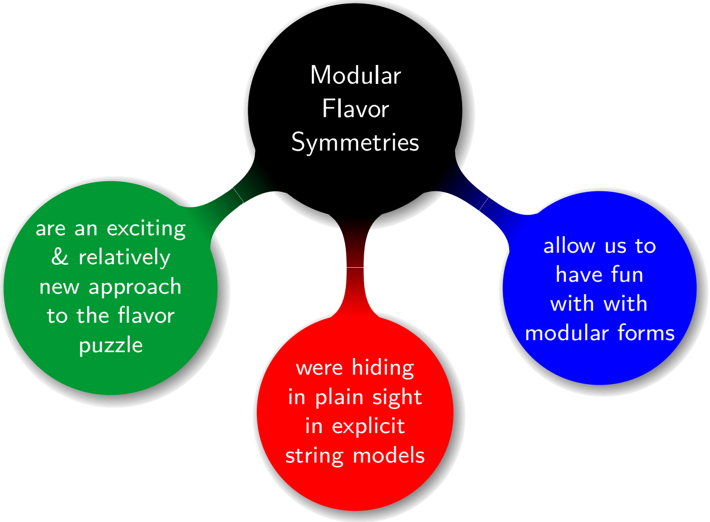
Summary


Modular Flavor Symmetries
are an exciting & relatively new approach to the flavor puzzle

Summary


Modular Flavor Symmetries
are an exciting & relatively new approach to the flavor puzzle
were hiding in plain sight in explicit string models

Summary


Modular Flavor Symmetries
are an exciting & relatively new approach to the flavor puzzle
were hiding in plain sight in explicit string models
allow us to have fun with with modular forms

Outlook

☞ Open questions:

1. precision

cf. talk by Mu-Chun Chen

$\mathrm{SL}(2,\mathbb{Z})$ has unfinitely many elements so the predictive power of modular flavor symmetries is generically enhanced w.r.t. traditional symmetries but still there aren't too many examples in which the precision of predictions is anywhere close to experimental uncertainties

Outlook

☞ Open questions:

1. precision
2. how indispensible is supersymmetry?
   - does modular invariance protect scalars?
     cf. Dienes [1994]; Cremades, Ibáñez & Marchesano [2004]; Buchmüller, Dierigl & Dudas [2018]; Abel & Dienes [2021]; Almumin, Chen, Knapp-Pérez, Ramos-Sánchez, MR & Shukla [2021]
   - more generally, does naive EFT work in more sophisticated settings?
     loosely related to talks by Krzysztof Jodlowski & Guilherme Guedes
   - can we find a good physics justification for using harmonic Maass forms?
     Qu & Ding [2024]
   - can we use, say, mock modularity for flavor?

Outlook

☞ Open questions:

1. precision
2. how indispensible is supersymmetry?
3. attempt to understand flavor in an otherwise consistent model
   explicit strings, no proton decay, no dangerous excotis, consistent cosmology, . . .

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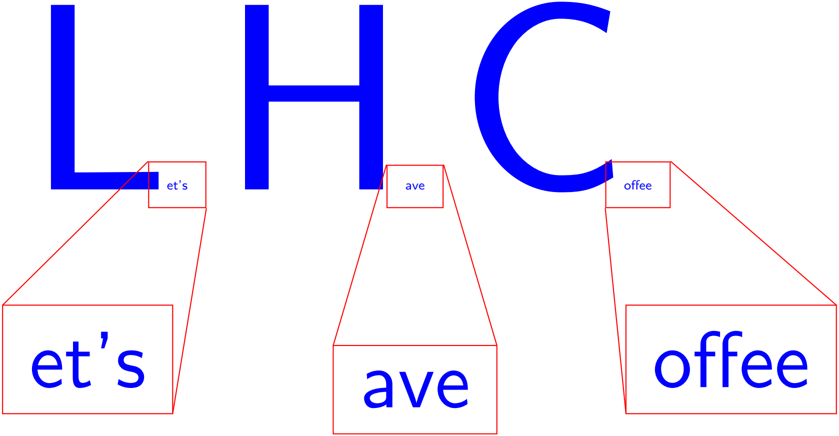

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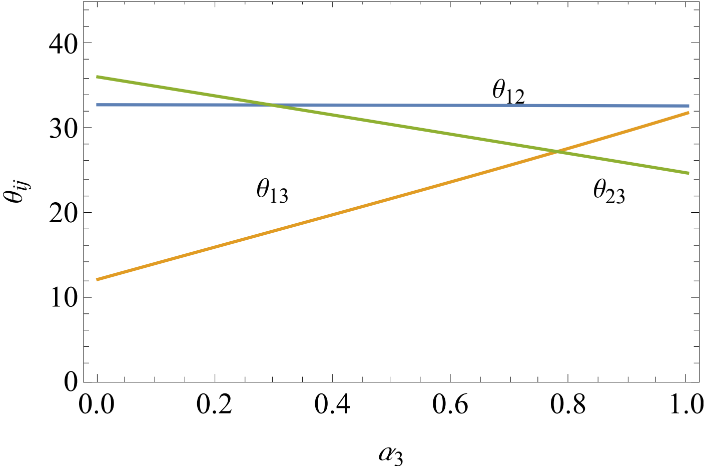
Thank you very much!

More open questions

☞ Extra parameters in the Kähler potential

Chen, Ramos-Sánchez & MR [2020]

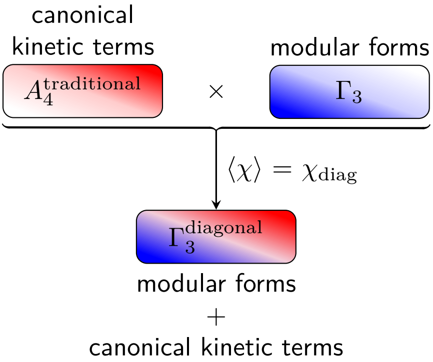


More open questions

- ☞ Extra parameters in the Kähler potential — Chen, Ramos-Sánchez & MR [2020]

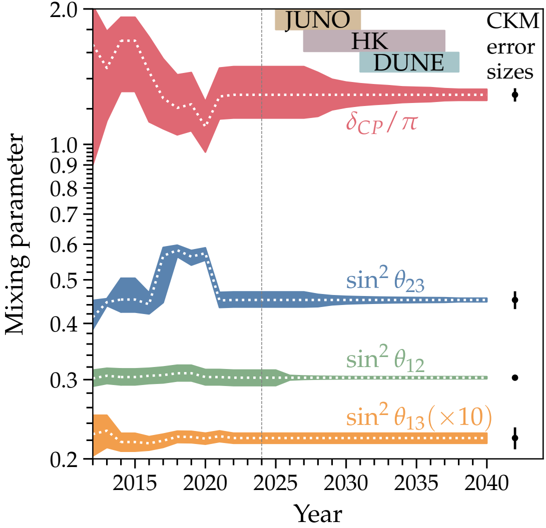
- ☞ Proof-of-principle solutions exist but are admittedly not too elegant

Chen, Knapp–Pérez, Ramos-Hamud, Ramos–Sánchez, MR & Shukla [2022]



Current and future precision of neutrino experiments

Song, Li, Argüelles, Bustamante & Vincent [2021]



courtesy of Shirley Li

Feruglio's model

Feruglio [2017]

☞ Neutrino mass in traditional A_4 models

$$m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2a & -c & -b \\ -c & 2b & -a \\ -b & -a & 2c \end{pmatrix} \qquad \text{(traditional)}$$

VEV of the u-type Higgs in the MSSM

Feruglio's model

Feruglio [2017]

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☞ Neutrino mass in a "modular" A_4 model

$$m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

known modular functions of τ

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} X_2^2 \\ \sqrt{2}X_1 X_2 \\ -X_1^2 \end{pmatrix} \text{ where } \begin{cases} X_1(\tau) := 3\sqrt{2}\dfrac{\eta^3(3\tau)}{\eta(\tau)} \\ X_2(\tau) := -3\dfrac{\eta^3(3\tau)}{\eta(\tau)} - \dfrac{\eta^3(\tau/3)}{\eta(\tau)} \end{cases}$$

Feruglio's model

Feruglio [2017]

- Neutrino mass in traditional A_4 models

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- Highly predictive:

$$\left.\begin{array}{r} \Lambda \\ \mathrm{Re}\,\tau \\ \mathrm{Im}\,\tau \end{array}\right\} \xrightarrow{\text{predict}} \left\{\begin{array}{l} \text{3 mass eigenvalues } m_i \\ \text{3 mixing angles } \theta_{ij} \\ \text{3 phases (1 Dirac \& 2 Majorana)} \end{array}\right.$$

Predictions of the Feruglio model

Esteban, Gonzalez-Garcia, Maltoni, Schwetz & Zhou [2020]
Song, Li, Argüelles, Bustamante & Vincent [2021]

Feruglio [2017]

NuFIT 5.2 (2022)

without SK atmospheric data

	Inverted Ordering ($\Delta\chi^2 = 2.3$)	
	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
$\sin^2\theta_{23}$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
$\theta_{23}/^\circ$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
$\sin^2\theta_{13}$	$0.02219^{+0.00060}_{-0.00057}$	$0.02047 \to 0.02396$
$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.11}$	$8.23 \to 8.90$
$\delta_{\mathrm{CP}}/^\circ$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$-2.498^{+0.032}_{-0.025}$	$-2.581 \to -2.408$

☞ Model predictions

$$\sin^2\theta_{12} = 0.295$$

$$\sin^2\theta_{23} = 0.651$$

$$\sin^2\theta_{13} = 0.0447$$

$$\delta_{\mathcal{CP}} = 279^\circ$$

$$\Bigg\} \frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}} = 0.0292$$

Predictive power of modular symmetries

☞ Ingredients

- modular invariance

Predictive power of modular symmetries

- Ingredients
- modular invariance
- holomorphy

Predictive power of modular symmetries

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} superpotential essentially $\mathscr{W}$ fixed

Predictive power of modular symmetries

☞ Ingredients

- modular invariance
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} superpotential essentially $\mathscr{W}$ fixed

☞ However, typical observables are not holomorphic

e.g. $$\left.\begin{aligned} \mathscr{W} &= \frac{\mathcal{M}(\tau)}{2}\,\Phi^2 \\ K &= \frac{1}{(-\mathrm{i}\,\tau+\mathrm{i}\,\overline{\tau})^{k_\Phi}}\overline{\Phi}\Phi \end{aligned}\right\} \curvearrowright\quad m_\text{physical} = m_\text{physical}(\overline{\tau},\tau) = |\mathcal{M}(\tau)|\,(-\mathrm{i}\,\tau+\mathrm{i}\,\overline{\tau})^{k_\Phi}$$

not entirely fixed by symmetries

Predictive power of modular symmetries

- Ingredients

modular invariance
holomorphy
finiteness

} superpotential essentially $\mathscr{W}$ fixed

- However, typical observables are not holomorphic

$$\left.\begin{aligned} \text{e.g. } \mathscr{W} &= \frac{\mathcal{M}(\tau)}{2}\,\Phi^2 \\ K &= \frac{1}{(-\mathrm{i}\,\tau + \mathrm{i}\,\overline{\tau})^{k_\Phi}}\overline{\Phi}\Phi \end{aligned}\right\} \curvearrowright \quad \begin{aligned} m_{\text{physical}} &= m_{\text{physical}}(\overline{\tau},\tau) \\ &= |\mathcal{M}(\tau)|\,(-\mathrm{i}\,\tau + \mathrm{i}\,\overline{\tau})^{k_\Phi} \end{aligned}$$

? Are there observables which fulfill , & ?

Modular invariant holomorphic observables

☞ Typical model

$$\mathscr{W}_{\text{lepton}} = Y_e^{ij}\, L_i \cdot H_d\, E_j + \frac{1}{2}\kappa'_{ij}(\tau)\, L_i \cdot H_u\, L_j \cdot H_u$$

Modular invariant holomorphic observables

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$$\mathscr{W}_{\text{lepton}} = Y_e^i \, L_i \cdot H_d \, E_i + \frac{1}{2}\kappa_{ij}(\tau) \, L_i \cdot H_u \, L_j \cdot H_u$$

diagonal

Modular invariant holomorphic observables

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- Modular invariant holomorphic observables

Chen, Li, Liu, Medina & MR [2024]

$$I_{ij}(\tau) := \frac{M_{ii}(\tau)\, M_{jj}(\tau)}{\bigl(M_{ij}(\tau)\bigr)^2} = \frac{\kappa_{ii}(\tau)\, \kappa_{jj}(\tau)}{\bigl(\kappa_{ij}(\tau)\bigr)^2} = \frac{m_{ii}(\tau,\overline{\tau})\, m_{jj}(\tau,\overline{\tau})}{\bigl(m_{ij}(\tau,\overline{\tau})\bigr)^2}$$

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- I_{ij} are entirely given by masses, mixing angles and phases, e.g.

$$I_{12} = \frac{a_0 \left[\widetilde{m}_1 \left(\mathrm{e}^{\mathrm{i}\,\delta} c_{23} s_{12} + c_{12} s_{13} s_{23}\right)^2 + \widetilde{m}_2 \left(\mathrm{e}^{\mathrm{i}\,\delta} c_{12} c_{23} - s_{12} s_{13} s_{23}\right)^2 + \mathrm{e}^{2\,\mathrm{i}\,\delta} m_3 c_{13}^2 s_{23}^2\right]}{c_{13}^2 \left[\widetilde{m}_1 c_{12} \left(\mathrm{e}^{\mathrm{i}\,\delta} c_{23} s_{12} + c_{12} s_{13} s_{23}\right) + \widetilde{m}_2 s_{12} \left(s_{12} s_{13} s_{23} - \mathrm{e}^{\mathrm{i}\,\delta} c_{12} c_{23}\right) - \mathrm{e}^{2\,\mathrm{i}\,\delta} m_3 s_{13} s_{23}\right]^2}$$

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$\widetilde{m}_1 := m_1\, \mathrm{e}^{\mathrm{i}\,\varphi_1}$

$\widetilde{m}_2 := m_2\, \mathrm{e}^{\mathrm{i}\,\varphi_2}$

Modular invariant holomorphic observables

☞ Typical model

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$c_{ij} := \cos\theta_{ij}$

Modular invariant holomorphic observables

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$$s_{ij} := \sin\theta_{ij}$$

Modular invariant holomorphic observables

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$$a_0 := \left(\widetilde{m}_1 c_{12}^2 + \widetilde{m}_2 s_{12}^2\right) c_{13}^2 + \mathrm{e}^{2\,\mathrm{i}\,\delta} m_3 s_{13}^2$$

Modular invariant holomorphic observables

☞ Typical model

$$\mathscr{W}_{\text{lepton}} = Y_e^i\, L_i \cdot H_d\, E_i + \frac{1}{2}\kappa_{ij}(\tau)\, L_i \cdot H_u\, L_j \cdot H_u$$

☞ Modular invariant holomorphic observables

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☞ I_{ij} are invariant under the renormalization group

Chang & Kuo [2002]

Invariants in the Feruglio model

- Invariants

$$I_{12}(\tau) = -2$$

Invariants in the Feruglio model

☞ Invariants

$$\left.\begin{aligned} I_{12}(\tau) &= -2 \\ I_{13}(\tau) &= -2\left(1+\frac{1}{3}j_3(\tau)\right)^3 \\ I_{23}(\tau) &= -\frac{32}{I_{13}} = \frac{16}{\left(1+\frac{1}{3}j_3(\tau)\right)^3} \end{aligned}\right\} \curvearrowright\ I_{13}\,I_{23} = -32$$

Hauptmodul of $\Gamma(3)$
$j_3(\tau) := \eta(\tau/3)^3/\eta(3\tau)^3$

Invariants in the Feruglio model

☞ Invariants

$$\left.\begin{aligned} I_{12}(\tau) &= -2 \\ I_{13}(\tau) &= -2\left(1+\frac{1}{3}j_3(\tau)\right)^3 \\ I_{23}(\tau) &= -\frac{32}{I_{13}} = \frac{16}{\left(1+\frac{1}{3}j_3(\tau)\right)^3} \end{aligned}\right\} \curvearrowright \; I_{13}\, I_{23} = -32$$

☞ A_4 sum rule

Chuliá Centelles, Cepedello & Medina [2022]; Centelles Chuliá, Kumar, Popov & Srivastava [2024]

$$m_3 = \begin{cases} m_2 + m_1 & \text{for normal ordering (NO)} \\ m_2 - m_1 & \text{for inverted ordering (IO)} \end{cases}$$

Invariants in the Feruglio model

- Invariants

$$\begin{aligned} I_{12}(\tau) &= -2 \\ I_{13}(\tau) &= -2\left(1+\frac{1}{3}j_3(\tau)\right)^3 \\ I_{23}(\tau) &= -\frac{32}{I_{13}} = \frac{16}{\left(1+\frac{1}{3}j_3(\tau)\right)^3} \end{aligned} \Bigg\} \curvearrowright \; I_{13}\, I_{23} = -32$$

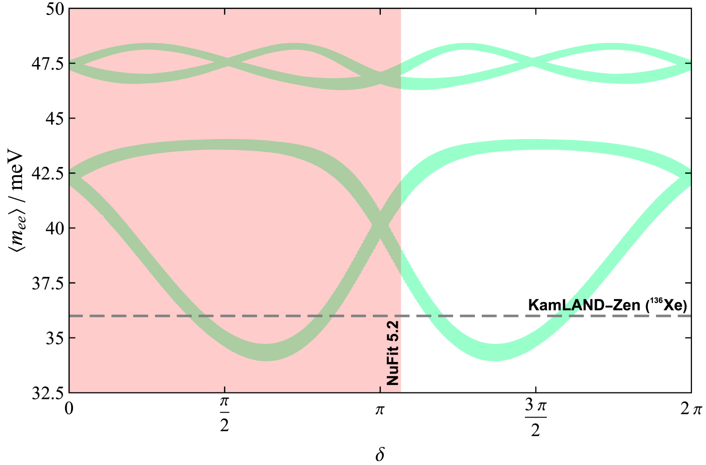
- A_4 sum rule

Chuliá Centelles, Cepedello & Medina [2022]; Centelles Chuliá, Kumar, Popov & Srivastava [2024]

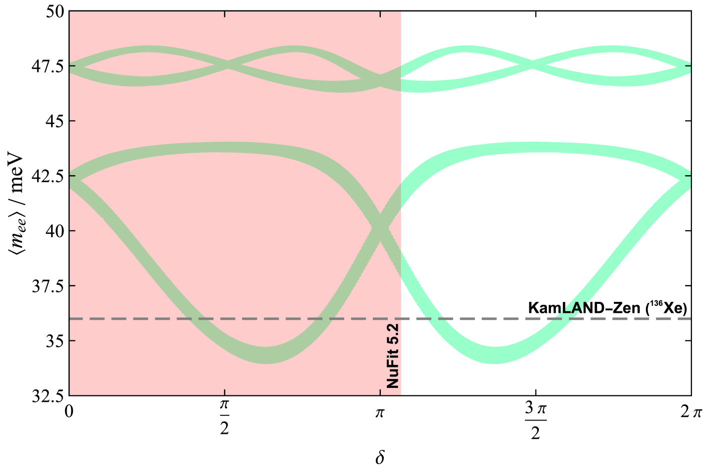
$$m_3 = \begin{cases} m_2 + m_1 & \text{for normal ordering (NO)} \\ m_2 - m_1 & \text{for inverted ordering (IO)} \end{cases}$$

➥ Imposing $I_{12} = -2$ and experimental constraints leaves one free parameter, e.g. δ

An application: m_{ee} as a function of δ



An application: m_{ee} as a function of δ



☞ Imposing $I_{13}\, I_{23} = -32$ rules out the model for all τ

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