On the Backreaction of Dirac Matter in JT Gravity and SYK Model

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Phys.Lett.B 853 (2024) 138702

July 10, 2024

JT Gravity and SYK Model

JT Gravity [Teitelboim 1983] [Jackiw 1985]

$$
\begin{aligned}
&-\frac{1}{16\pi G_2}\int d^2x\sqrt{|\det g_{\mu\nu}|}\;\phi(R-2\Lambda) \\
&-\frac{1}{8\pi G_2}\int du\sqrt{|\det h_{uu}|}\;\phi K
\end{aligned}
\tag{1}
$$

SYK Model

$$
\sum_{1\le i_1<i_2<i_3<i_4\le N} j_{i_1 i_2 i_3 i_4}\psi_{i_1}\psi_{i_2}\psi_{i_3}\psi_{i_4} \tag{2}
$$

JT Gravity and SYK Model

JT Gravity [Teitelboim 1983] [Jackiw 1985]

$$-\frac{1}{16\pi G_2}\int d^2x\sqrt{|\det g_{\mu\nu}|}\ \phi(R-2\Lambda) \qquad (1)$$
$$-\frac{1}{8\pi G_2}\int du\sqrt{|\det h_{uu}|}\ \phi K$$

SYK Model

$$\sum_{1\le i_1<i_2<i_3<i_4\le N} j_{i_1 i_2 i_3 i_4}\psi_{i_1}\psi_{i_2}\psi_{i_3}\psi_{i_4} \qquad (2)$$

- solvable in strong coupling limit ($N \gg \beta J \gg 1$)
- maximally chaotic (Quantum Chaos)
- emergent conformal symmetry (Emergence)

Information Loss

von Neumann Algebra [Leutheusser, Liu 2021] [Witten 2021]

- I: algebra of all bounded operators (finite N)
- II: loss of pure states (large N expansion to all orders);
  II_1: allowance of maximally mixed state
  II_∞: no maximally mixed state
- III: loss of pure state, density matrix, and trace operations
  ($N \to \infty$ or large N expansion up to some orders)

SYK Model and Backreaction

- Extension of SYK Model

$$\frac{i}{\sqrt{M}}\sum_{j=1}^{M}\sum_{1\leq i_1<i_2\leq N}\left(g_{i_1 i_2,j}\psi_{i_1}\psi_{i_2}\Psi_j^{\dagger}+g^{*}_{i_1 i_2,j}\psi_{i_1}\psi_{i_2}\Psi_j\right), \tag{3}$$

where the distribution is

$$\exp\left(-\sum_{j=1}^{M}\sum_{1\leq i_1<i_2\leq N} g^{*}_{i_1 i_2,j}\, g_{i_1 i_2,j}\frac{N}{2g^2}\right) \tag{4}$$

- Dirac fermion Matter (from the extension)

$$S_{\mathrm{DF}}=\frac{\lambda}{32\pi G_2\tilde{M}}\sum_{k=1}^{\tilde{M}}\int d^2x\sqrt{|\det g_{\rho\sigma}|}\;\bar{\Psi}_k\bar{\gamma}^{\mu}\overleftrightarrow{D}_{\mu}\Psi_k, \tag{5}$$

where $\tilde{M}=M/2$

Maximally Mixed State

- simulations suggest EE with a fixed initial state, $|0 \cdots 0\rangle$ by tracing over the matter sector:

  (i) first increases and then saturates

  (ii) saturates to the maximally allowed value of $M \ln 2$ in the large-N limit

  (iii) saturation time is finite in the large-N limit

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  (i) first increases and then saturates

  (ii) saturates to the maximally allowed value of $M\ln 2$ in the large-N limit

  (iii) saturation time is finite in the large-N limit
- state is well approximated by one that is maximally entangled

$$|\psi\rangle = \frac{1}{\sqrt{2^M}}\sum_{j=1}^{2^M} |j_M\rangle \otimes |j_D\rangle \,. \tag{6}$$

when reaching the saturation regime in the large-N limit

Symmetry

- Hamiltonian do not commute with the particle-hole operator

$$\mathcal{H} \equiv K \prod_{j=1}^{T_d} (c_j + c_j^\dagger), \tag{7}$$

where $T_d \equiv N/2 + M$, and K is an anti-linear operator

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- charge-conjugation operators:

$$\mathcal{P} \equiv K \prod_{j=1}^{T_d} \gamma_{2j-1}; \; \mathcal{R} \equiv K \prod_{j=1}^{T_d} i\gamma_{2j}, \tag{8}$$

where $T_d \equiv N/2 + M$, commute with the Hamiltonian

Random Matrix

- N/M is large enough:
  - $T_d \mod 4 = 0, (\mathcal{P}^2, \mathcal{R}^2) = (1, 1)$, no degeneracy, GOE;
  - $T_d \mod 4 = 1, (\mathcal{P}^2, \mathcal{R}^2) = (1, -1)$, no degeneracy, GUE;
  - $T_d \mod 4 = 2, (\mathcal{P}^2, \mathcal{R}^2) = (-1, -1)$, twofold degeneracy, GSE;
  - $T_d \mod 4 = 3, (\mathcal{P}^2, \mathcal{R}^2) = (-1, 1)$, no degeneracy, GUE,

Thank you!