# Small-instanton induced flavor invariants and the axion potential

**Based on 2402.09361**

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# The QCD axion

- A dynamical solution: introduce a spontaneously broken $U(1)_A$

$$\mathcal{L} \supset \left(\bar{\theta} + \frac{a}{f_a}\right) \frac{g_s}{16\pi^2} G\tilde{G}$$

- Axion is the Goldstone boson, whose shift symmetry allows to absorb θ effects:

$$\theta_{\text{eff}} \equiv \frac{\langle a \rangle}{f_a}$$

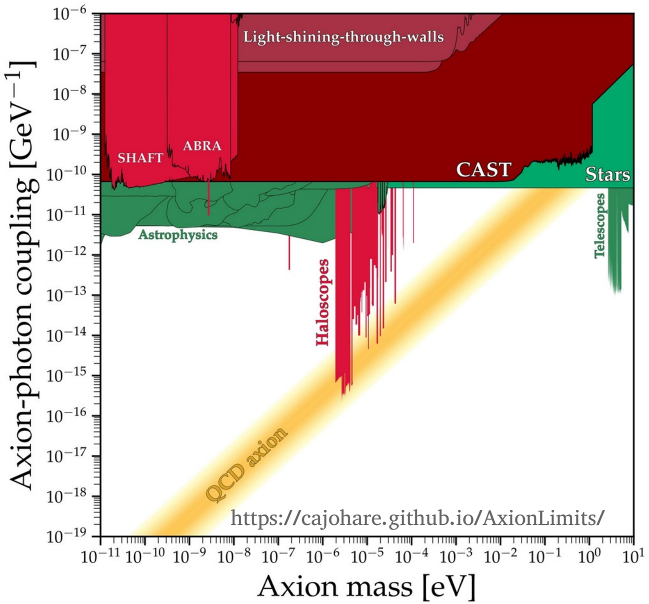
Strong CP-problem is now a question about the vev of the axion

# The axion potential

- QCD effects break shift-symmetry and generate an axion potential, with $\langle a \rangle = 0$ Vafa, Witten 84
- Through $\chi_{\rm PT}$ axion mass can be related to pion mass

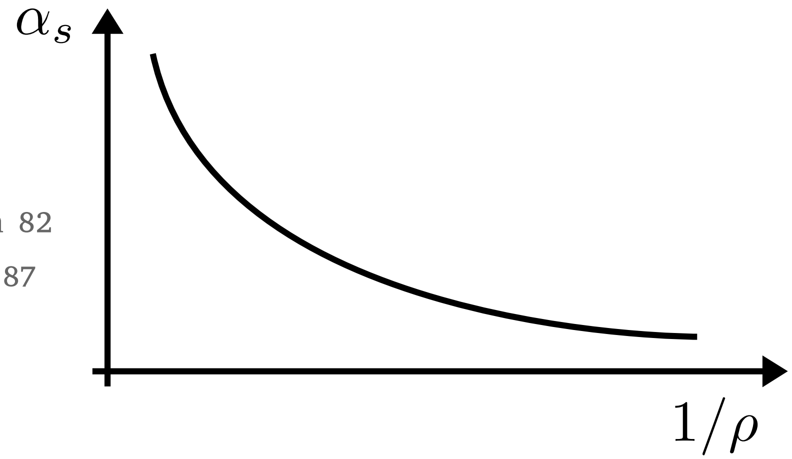
$$m_a^2 f_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} m_\pi^2 f_\pi^2$$

Cortona, Hardy, Vega, Villadoro 1511.02867




<u>How stable is this prediction?</u>

# Enhancing the axion mass




Holdom and Peskin 82

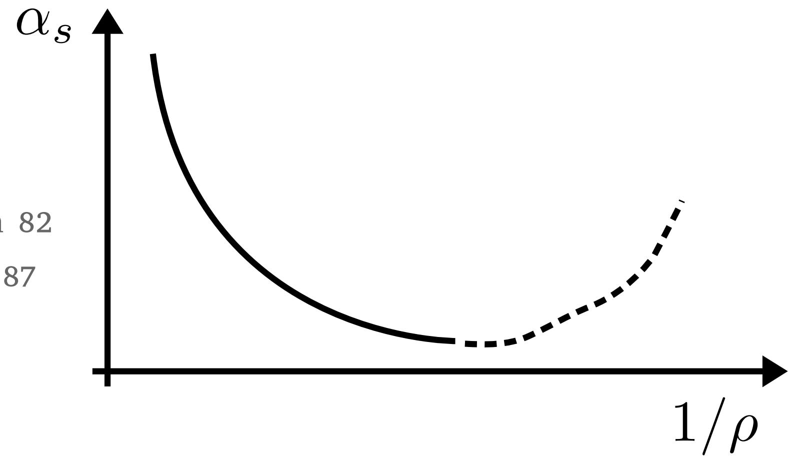
Flynn and Randall 87

Instanton contributions scale as:

$$V(a) \sim e^{-\frac{8\pi}{\alpha_s(1/\rho)}}$$

$$m_a = m_{\text{QCD}}$$

# Enhancing the axion mass




Holdom and Peskin 82

Flynn and Randall 87

Instanton contributions scale as:

$$V(a) \sim e^{-\frac{8\pi}{\alpha_s(1/\rho)}}$$

$$m_a = m_{\mathrm{QCD}} + m_{\mathrm{UV}}$$

May be relevant. Calculable in one-instanton limit

# Product gauge groups

Agrawal and Howe 1710.04213

$$SU(3)_1 \times SU(3)_2 \to SU(3)_c$$

$$\frac{1}{\alpha_1(M)} + \frac{1}{\alpha_2(M)} = \frac{1}{\alpha_s(M)}$$

- Each sector is more strongly coupled than QCD and therefore has more relevant instanton effects
- Stronger effects when one considers k-product of $SU(3)$

Csáki, Ruhdorfer, Shirman 1912.02197

# CP-violation (SM)

- Breaking of shift-symmetry and CP-violation result in linear term for the potential

$$V(a) = \chi_{\mathcal{O}}(0)\frac{a}{f_a} + \frac{1}{2}\chi(0)\left(\frac{a}{f_a}\right)^2$$

- This linear term shifts the minimum

$$\theta_{\rm ind} \equiv -\frac{\chi_{\mathcal{O}}(0)}{\chi(0)}$$

<u>CP-violating effects induce contributions to θ</u>

# CP-violation (SM)

- CP violation in the SM is parameterized by

$$J_4 = \mathrm{Im}\left(\mathrm{Tr}\left[Y_u Y_u^\dagger, Y_d Y_d^\dagger\right]\right)$$

Jarlskog 85
Bernabeu, Branco, Gronau 85

- The misalignment of the axion potential from SM CPV is too small to be observed:

$$\theta_{\mathrm{eff}}^{\mathrm{SM}} \sim \frac{G_F^2}{m_c^2} J_{\mathrm{CKM}} f_\pi^4 \Lambda_\chi^2 \sim 10^{-19}$$

Georgi, Randall 86
Luzio, Gisbert, Levati, Paradisi, Sørensen 2312.17310

- Had $J_4$ been larger, PQ solution would not have worked
- Observation of axion and $\theta > 10^{-19}$ implies new sources of CPV.

# New sources of CPV – SMEFT

- Using the SMEFT expansion to parameterize new physics:

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{\mathcal{L}_6}{\Lambda^2} + \mathcal{O}(1/\Lambda^4)$$

$$\mathcal{L}_d = c_i \mathcal{O}_i \qquad [\mathcal{O}_i] = d$$

- Many potential sources of CPV – can they affect the QCD axion solution?
- Basis of Jarlskog-like CPV invariants for the SMEFT:

$$L_{abcd}(C) = \text{Im}\left[\text{Tr}\left(X_{\text{u}}^a X_{\text{d}}^b X_{\text{u}}^c X_{\text{d}}^d C\right)\right]$$

$$X_{u,d} \equiv Y_{u,d} Y_{u,d}^\dagger$$

Bonnefoy, Gendy, Grojean, Ruderman
2112.03889, 2302.07288

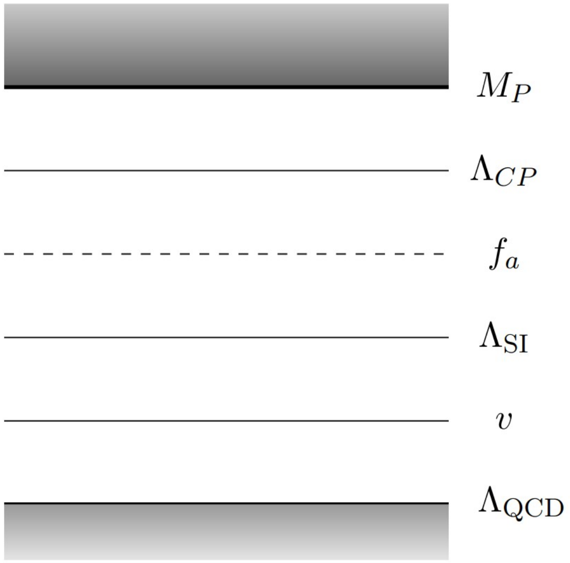
# Why does this matter?

- CP-violation effects with only QCD : $\sim \dfrac{\Lambda_{\text{QCD}}}{\Lambda_{\text{CPV}}}$
- In the presence of high-energy instantons, contributions to enhanced EDMs

$$\sim \frac{\Lambda_{\text{SI}}}{\Lambda_{\text{CPV}}} \lesssim 10^{-10}$$

<u>Do these models still work?</u>

<u>Can CPV invariants be useful?</u>




Bedi, Gherghetta, Pospelov
2205.07948

# Instanton computations

- Vacuum to vacuum transition in instanton background

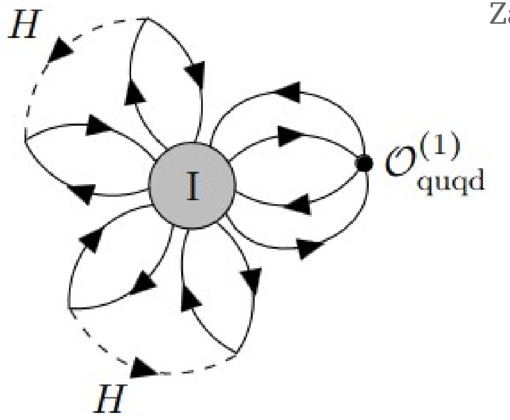
$$\langle 0|0\rangle\big|_{\text{1-inst.}} = e^{-i\theta_{\rm QCD}} \int d^4x_0 \int \frac{d\rho}{\rho^5}\, d_N(\rho) \int \prod_{f=1}^{N_f} \left(\rho\, d\xi_f^{(0)}\, d\bar{\xi}_f^{(0)}\right) e^{-\bar{\psi}J\psi + \text{h.c.}}$$

't Hooft 76
Shifman Vainshtein, Zakharov 79

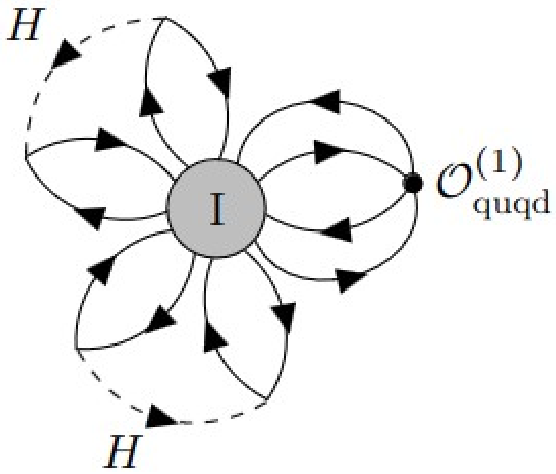
Instanton configuration

$$G^a_{\mu\nu}\big|_{\text{1-inst.}} = -4\eta_{a\mu\nu}\frac{\rho^2}{[(x-x_0)^2+\rho^2]^2}\,.$$

collective coordinates

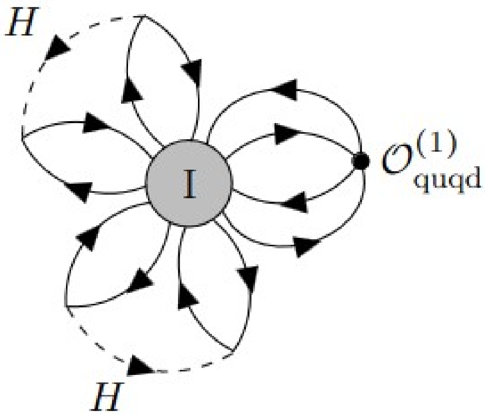


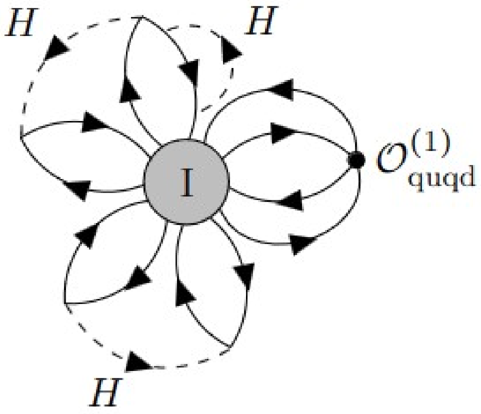
# Determinant-like invariants




$$\mathcal{I}(C^{(1,8)}_{\rm quqd}) = {\rm Im}\left[e^{-i\theta_{\rm QCD}}\epsilon^{ABC}\epsilon^{abc}\epsilon^{DEF}\epsilon^{def}Y_{{\rm u},Aa}Y_{{\rm u},Bb}C^{(1,8)}_{{\rm quqd},CcDd}Y_{{\rm d},Ee}Y_{{\rm d},Ff}\right]$$

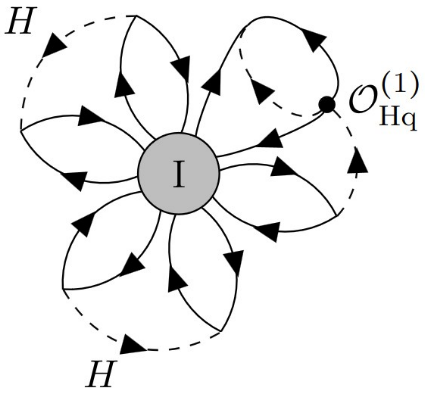
# Determinant-like invariants

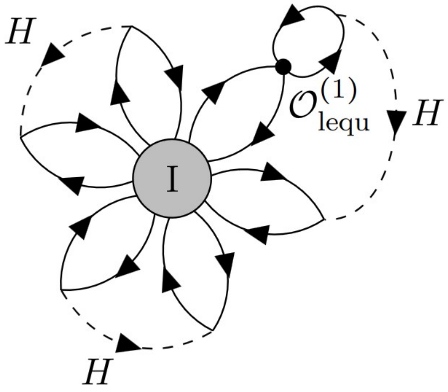



$$\mathcal{A}^{a_1,b_1,c_1,d_1}_{a_2,b_2,c_2,d_2}(C^{(1,8)}_{\rm ququd}) = \mathrm{Im}\Big[e^{-i\theta_{\rm QCD}}\epsilon^{ABC}\epsilon^{abc}\epsilon^{DEF}\epsilon^{def}Y_{{\rm u},Aa}Y_{{\rm u},Bb}\left(X^{a_1}_{\rm u}X^{b_1}_{\rm d}X^{c_1}_{\rm u}X^{d_1}_{\rm d}\right)_C{}^{C'} \times C^{(1,8)}_{{\rm ququd},C'cD'd}\left(X^{a_2}_{\rm u}X^{b_2}_{\rm d}X^{c_2}_{\rm u}X^{d_2}_{\rm d}\right)_D{}^{D'}Y_{{\rm d},Ee}Y_{{\rm d},Ff}\Big],$$

# Determinant-like invariants




$$\mathcal{I}_{abcd}(C_{\mathrm{Hq}}^{(1,3)}) \equiv \mathrm{Im}\left[e^{-i\theta_{\mathrm{QCD}}}\epsilon^{IJK}\epsilon^{ijk}Y_{\mathrm{u},Ii}Y_{\mathrm{u},Jj}\left(X_{\mathrm{u}}^{a}X_{\mathrm{d}}^{b}X_{\mathrm{u}}^{c}X_{\mathrm{d}}^{d}\,C_{\mathrm{Hq}}^{(1,3)}\,Y_{\mathrm{u}}\right)_{Kk}\det Y_{\mathrm{d}}\right]$$

$$\mathcal{I}_{abcd}^{f}(C_{\mathrm{lequ}}^{(1,3)}) \equiv \mathrm{Im}\left[e^{-i\theta_{\mathrm{QCD}}}\epsilon^{IJK}\epsilon^{ijk}Y_{\mathrm{u},Ii}Y_{\mathrm{u},Jj}\left(X_{\mathrm{u}}^{a}X_{\mathrm{d}}^{b}X_{\mathrm{u}}^{c}X_{\mathrm{d}}^{d}\right)_{K}{}^{L}\left(Y_{\mathrm{e}}^{\dagger}X_{\mathrm{e}}^{f}\right)^{mN}C_{\mathrm{lequ},NmLk}^{(1,3)}\det Y_{\mathrm{d}}\right]$$

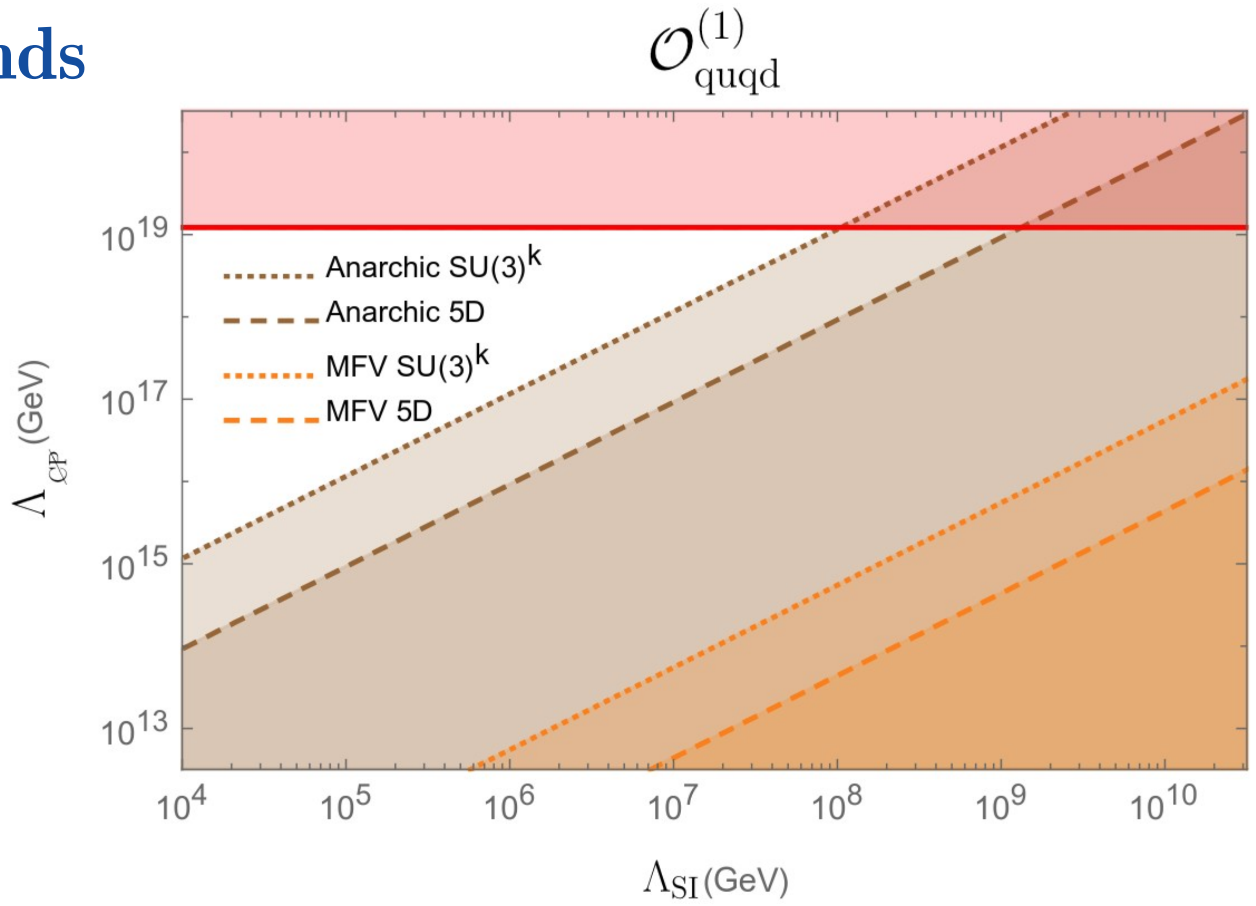
# NDA

- Result proportional to invariants
  - Easily test flavor assumptions of the WC
- Refine naive dimensional analysis estimates

Csáki, D'Agnolo, Kuflik, Ruhdorfer, 2311.09285

- Extend to other operators
  - While $\mathcal{O}_{qudq}$ seems to give the larger contribution, might be a suppressed in the UV

# Bounds



$\mathcal{O}_{\rm ququd}^{(1)}$
Anarchic SU(3)$^k$
Anarchic 5D
MFV SU(3)$^k$
MFV 5D
$10^{19}$
$10^{17}$
$10^{15}$
$10^{13}$
$\Lambda_{\not{C}\not{P}}$ (GeV)
$10^4$
$10^5$
$10^6$
$10^7$
$10^8$
$10^9$
$10^{10}$
$\Lambda_{\rm SI}$ (GeV)

# Axion EFT – Shift-breaking effects

- Shift-symmetric ALP, derivative basis:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) + \frac{\partial_\mu a}{f} \sum_{\psi \in \text{SM}} \bar{\psi} c_\psi \gamma^\mu \psi$$

- Shift-breaking ALP effects, Yukawa-like basis:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{a}{f}\left(\bar{Q}\tilde{Y}_u \tilde{H} u + \bar{Q}\tilde{Y}_d H d + \bar{L}\tilde{Y}_e H e + \text{h.c.}\right)$$

# Axion EFT – Shift-breaking effects

- Yukawa-like basis is the more general, and can capture shift-symmetric effects
  - Performing a field-redefinition on the fermions $\psi' \equiv e^{-i\frac{a}{f}c_\psi}\psi$

$$\tilde{Y}_{u,d} = i(Y_{u,d}c_{u,d} - c_Q Y_{u,d}) \ , \quad \tilde{Y}_e = i(Y_e c_e - c_L Y_e)$$

Chala, G. G., Ramos, Santiago 2012.09017

- To verify whether a specific value of $\tilde{Y}$ respects shift-symmetry one needs invariants!

Bonnefoy, Grojean, Kley 2206.04182

# Axion EFT – Shift-breaking invariants

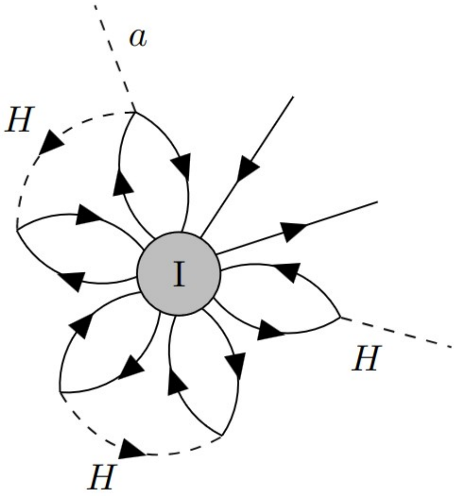
- There are 13 invariants in the fermionic sector
  - Only 1 is CP-even:

Bonnefoy, Grojean, Kley 2206.04182

$$I_{ud}^{(4)} = \mathrm{Im}\,\mathrm{Tr}\left(\left[X_u, X_d\right]^2 \left(\left[X_d, \tilde{Y}_u Y_u^\dagger\right] - \left[X_u, \tilde{Y}_d Y_d^\dagger\right]\right)\right)$$

If small instantons enhance axion mass, will they enhance other shift-breaking invariants?

# Axion EFT – generating shift-breaking invariants




Work in progress

If small instantons enhance axion mass, will they enhance other shift-breaking invariants?

# Conclusions

- Small-instantons can be responsible for relevant UV contributions to the axion mass
- In the presence of CP violating physics, this enhancement will also induce contributions to nEDM
  - The estimation of these effects can be made easier with the help of determinant-like invariants
- Shift-breaking effects can also be generated in ALP coupling to fermions

# Thanks

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