Multiple QCD axions

Maria Ramos @PASCOS 24

Based on JHEP04 (2024) 56

with Belén Gavela and Pablo Quílez

Why axions?

- Light window to deep UV physics:

$$m_a^2 \sim \frac{\Lambda_{\text{ins}}^4}{f_a^2}$$

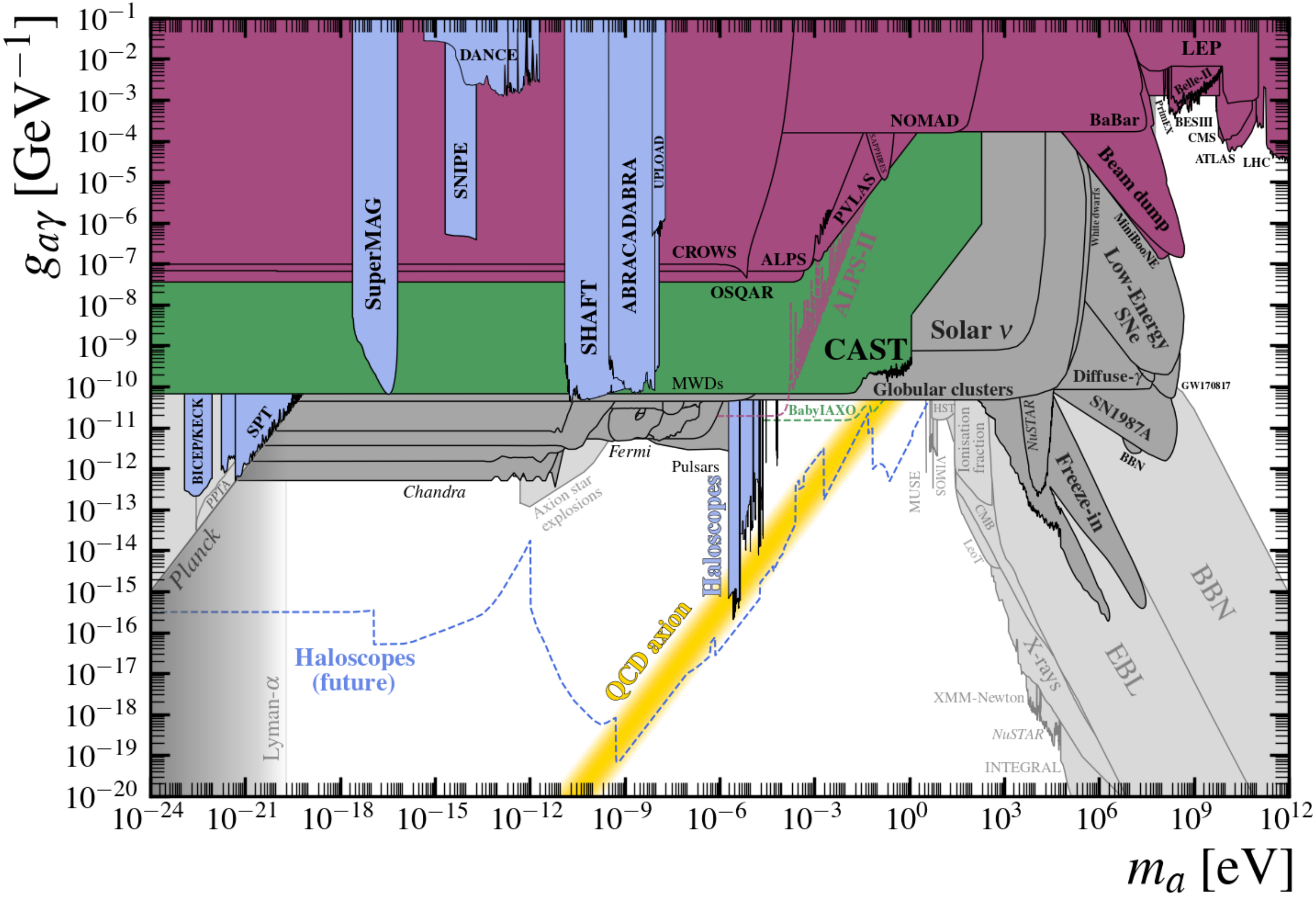
  - required by Quantum Gravity
    M. Reece 23
  - ubiquitous in String Theory
    P. Svrcek, E. Witten 06, Arvanitaki et al 09

- **Dark matter** candidate
  L.F. Abbott, P. Sikivie 82

$$\Omega_a h^2 \sim 0.1 \left(\frac{f_a}{10^{12}\,\text{GeV}}\right)^{7/6} \theta_i^2$$

Axion physics = special connection between particle phenomenology, fundamental theory and experiment, that we might uncover in the next years.

The axion-like particle landscape



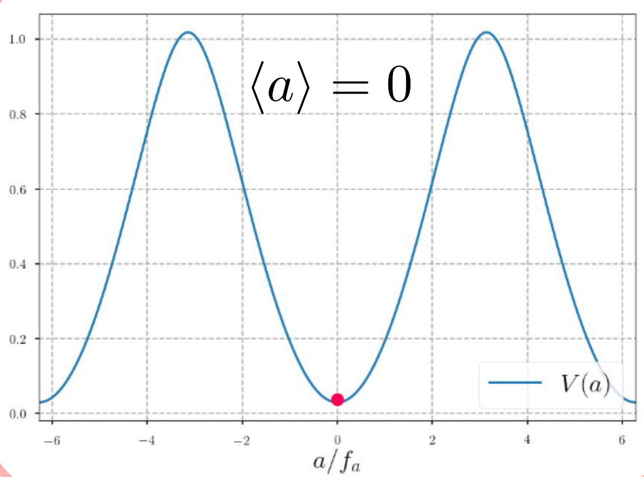
The canonical QCD axion

The strong CP problem: $\mathcal{L} \supset \frac{\alpha_s}{8\pi}\bar{\theta} G\widetilde{G}\,, \quad E(\theta) \geq E(\theta = 0)$

C. Vafa and E. Witten 84

A dynamical $U(1)_{PQ}$ solution:

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\underbrace{\left(\frac{a_{G\widetilde{G}}}{f_a} - \bar{\theta}\right)}_{a/f_a} G\widetilde{G}$$



$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{1}{2}\frac{a}{f_a}\right)}$$

Precise mass-scale relation:

$$m_a^2\, f_a^2 = \chi_{\rm QCD} \simeq m_\pi^2\, f_\pi^2\, \frac{m_u\, m_d}{(m_u + m_d)^2}$$

Challenging standard assumptions

❌ All PQ breaking comes from QCD

$$\partial_\mu j^\mu_{\mathrm{PQ}} = G\widetilde{G} + \textcolor{red}{\mathrm{other}}$$

[many others]
Gaillard, Gavela, Houtz, Rey, Quilez, 18
Csaki et al, 19
Gherghetta et al, 20
Hook, 18
Luzio, Gavela, Quilez, Ringwald 21
Valenti, Vecchi, Xu 22
Bedi, Gherghetta, Grojean, Guedes, Kley, Vuong 24

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✗ The axion is the only ALP in Nature

Additional ALPs can misalign the axion interaction and mass basis:

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_{G\widetilde{G}}}{F} - \bar{\theta}\right) G\widetilde{G} - V'(\hat{a}_{G\widetilde{G}}, \ldots, \hat{a}_N)$$

$$\Rightarrow m_i^2 f_i^2 = g_i \chi_{\rm QCD}$$

Peccei-Quinn condition

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\sum_{k=1}^{N}\frac{\hat{a}_k}{\hat{f}_k} - \bar{\theta}\right) G\widetilde{G} - V_B(\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_N) \to \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_{G\widetilde{G}}}{F} - \bar{\theta}\right) G\widetilde{G} - V_B^{\mathrm{R}}(\hat{a}_{G\widetilde{G}}, \ldots)$$

$$\mathbf{M}^2 = \mathbf{M}_A^2 + \mathbf{M}_B^2 = \begin{pmatrix} b_{11} & \mathbf{X}^\dagger \\ \mathbf{X} & \mathbf{M}_1^2 \end{pmatrix} = \frac{\chi_{\mathrm{QCD}}}{F^2}\begin{pmatrix} 1 & 0 \\ 0 & \mathbf{0} \end{pmatrix} + \begin{pmatrix} b_{11} - \frac{\chi_{\mathrm{QCD}}}{F^2} & \mathbf{X}^\dagger \\ \mathbf{X} & \mathbf{M}_1^2 \end{pmatrix}$$

$$\boxed{\exists\, U(1)_{PQ} \implies \lim_{\chi_{\mathrm{QCD}}\to 0} \det \mathbf{M}^2 = 0 \implies \det \mathbf{M}_B^2 = 0} \quad \langle \hat{a}_0 | a_{G\widetilde{G}} \rangle \neq 0$$

In terms of physical couplings,

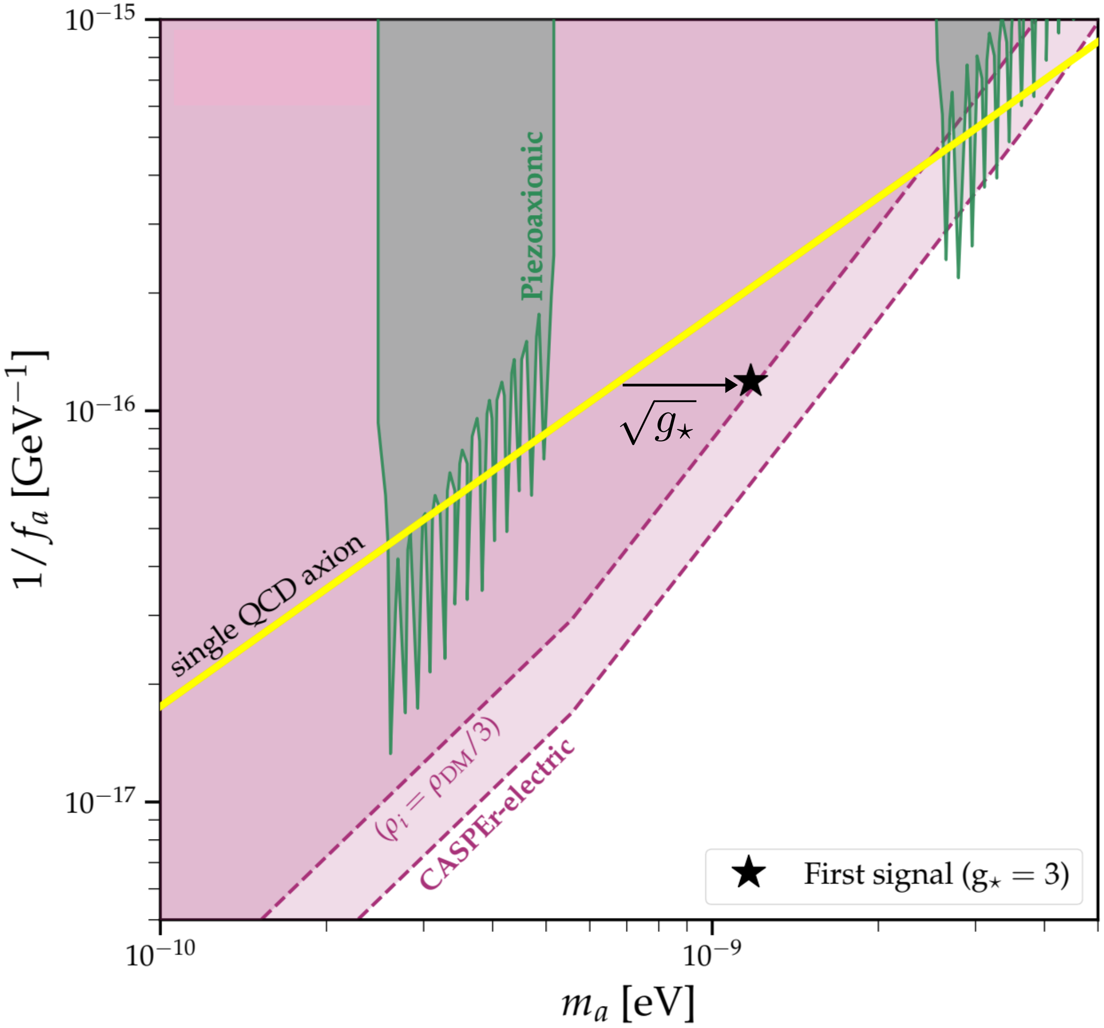
$$\mathcal{L}_{\mathrm{phy}} \supset \frac{\alpha_s}{8\pi}\frac{a_i}{f_i} G\widetilde{G}\,, \quad \text{with} \quad \frac{1}{f_i} = \frac{\langle \hat{a}_{G\widetilde{G}} | a_i \rangle}{F} \implies \sum_{i=1}^{N} \frac{1}{f_i^2} = \frac{1}{F^2}$$

Using eigenvector-eigenvalue theorem:

$$\frac{\det \mathbf{M}_1^2}{\det \mathbf{M}^2} = \sum_{i=1}^{N} \frac{|\langle \hat{a}_{G\widetilde{G}} | a_i \rangle|^2}{m_i^2} = \frac{F^2}{\chi_{\mathrm{QCD}}} \sum_{i=1}^{N} \frac{\chi_{\mathrm{QCD}}}{m_i^2 f_i^2} \Rightarrow \boxed{\sum_{i=1}^{N} \frac{1}{g_i} \equiv \sum_{i=1}^{N} \beta_i = 1}$$

QCD axion sum rule

New parameter space for QCD axions



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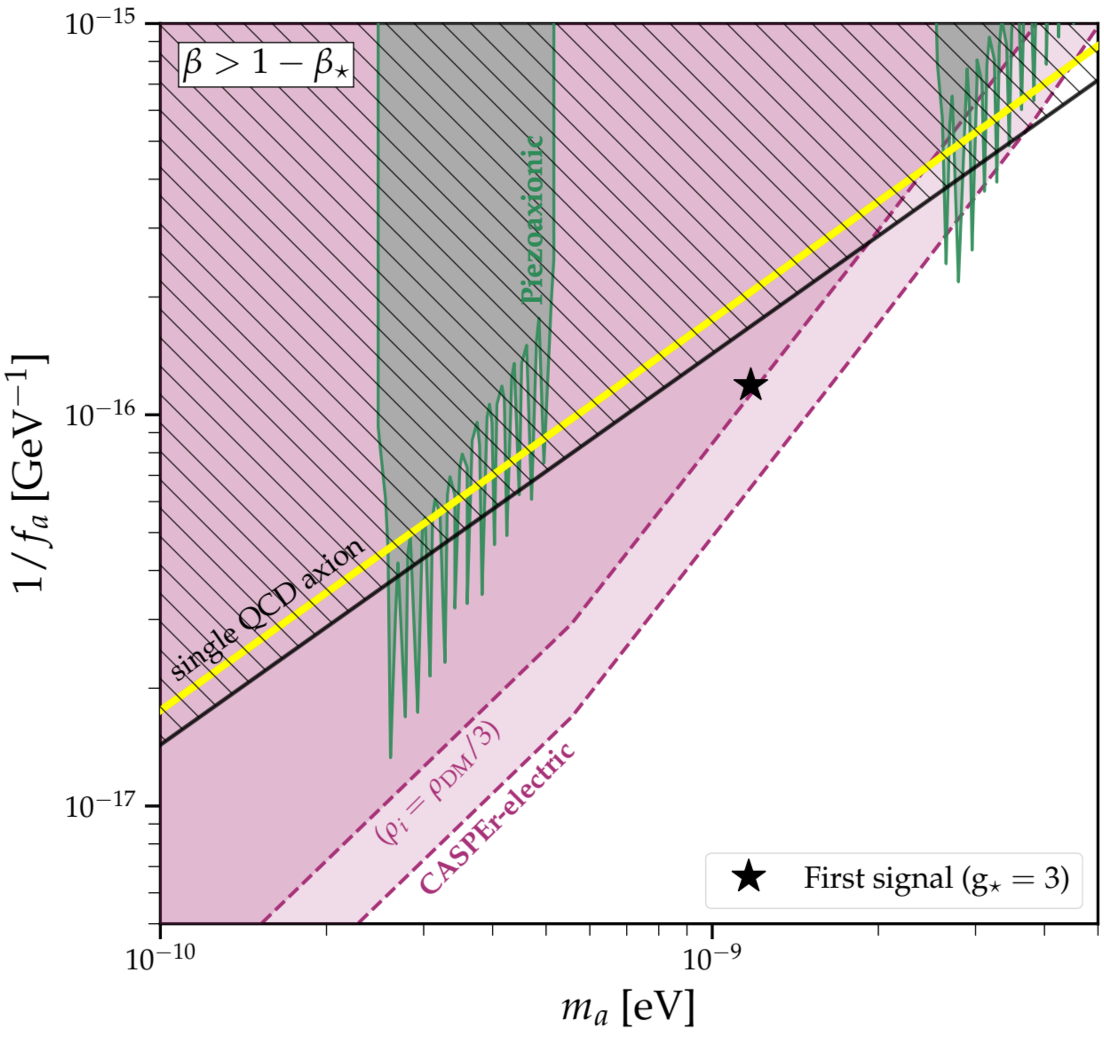
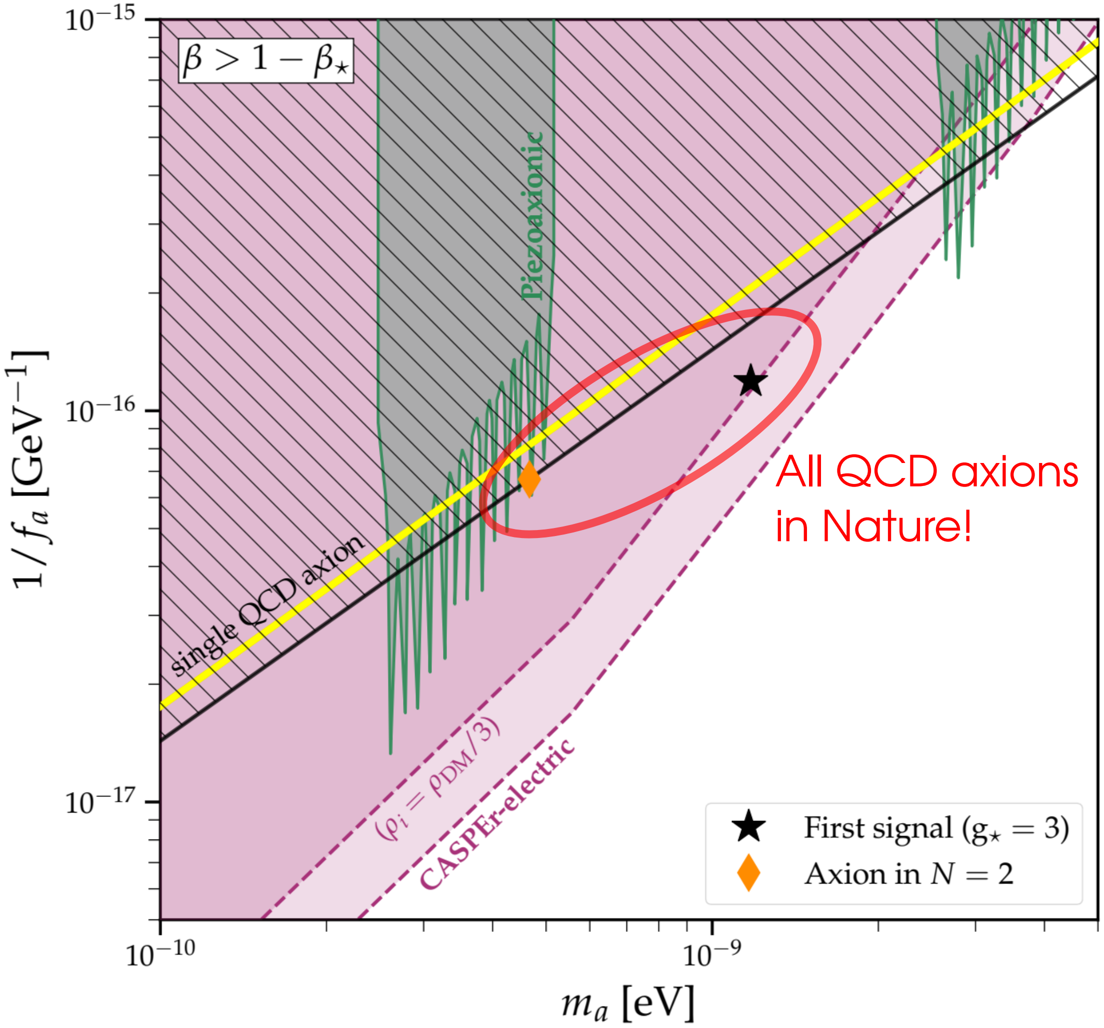


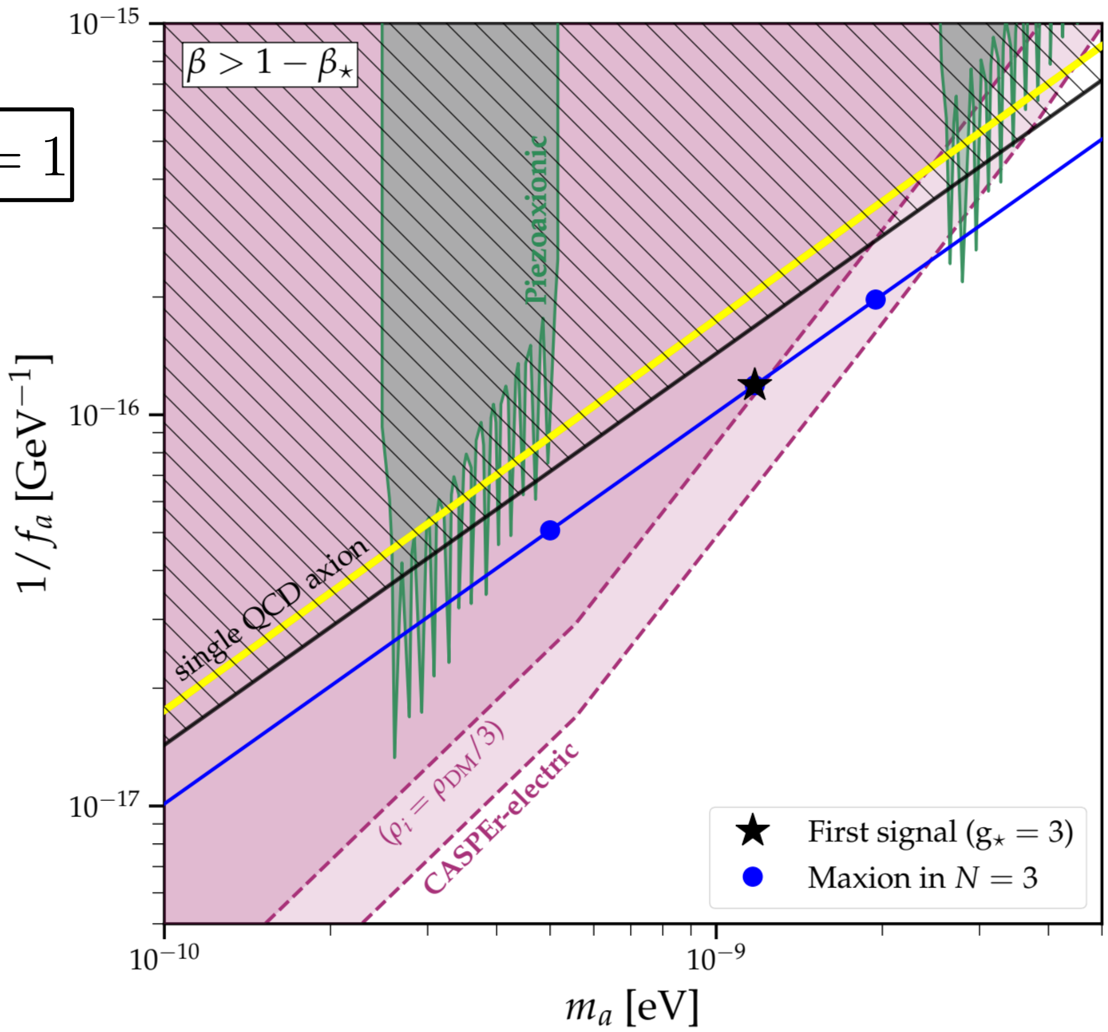
New parameter space for QCD axions

$$\beta_\star + \beta_2 = 1$$



New parameter space for QCD axions

$$\beta_\star + \beta_2 + \beta_3 = 1$$



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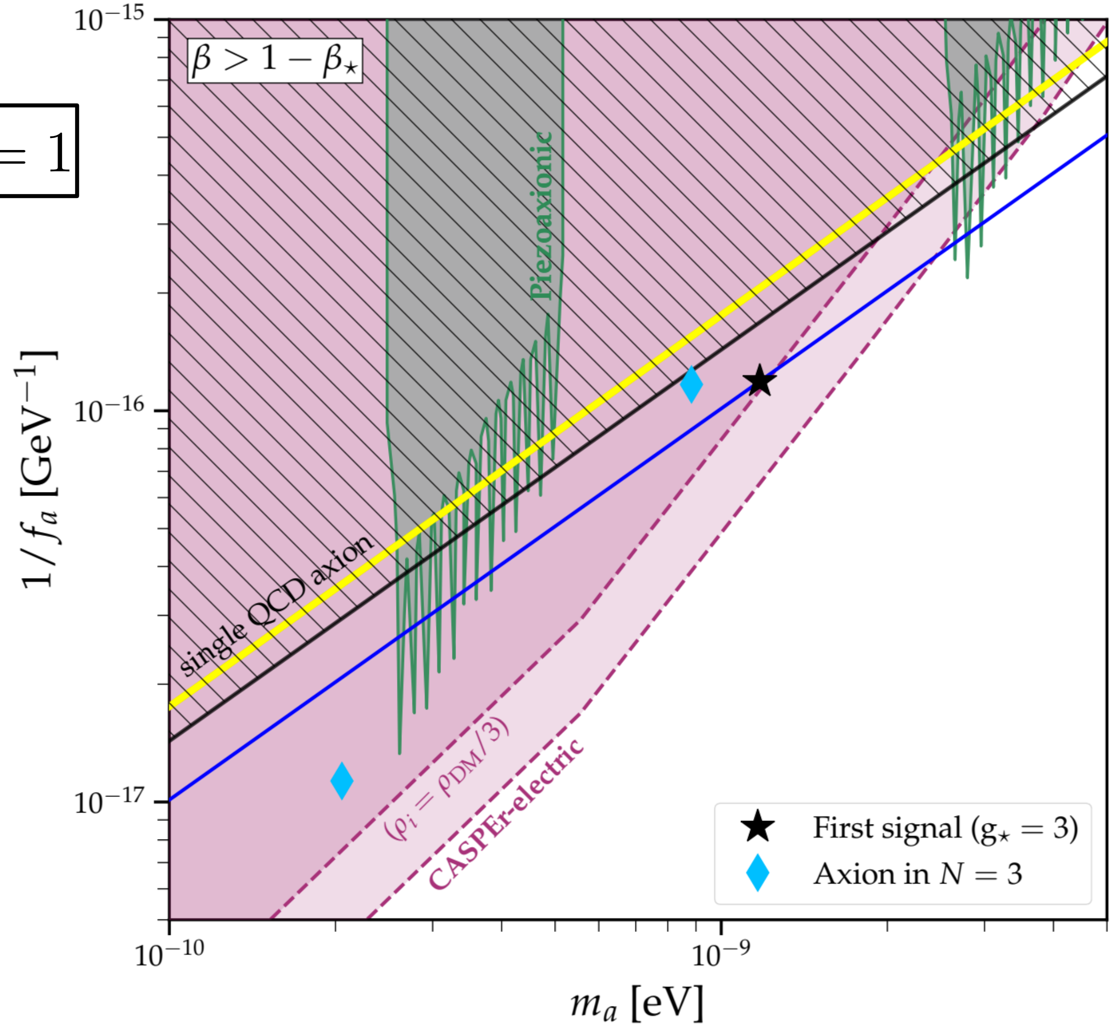
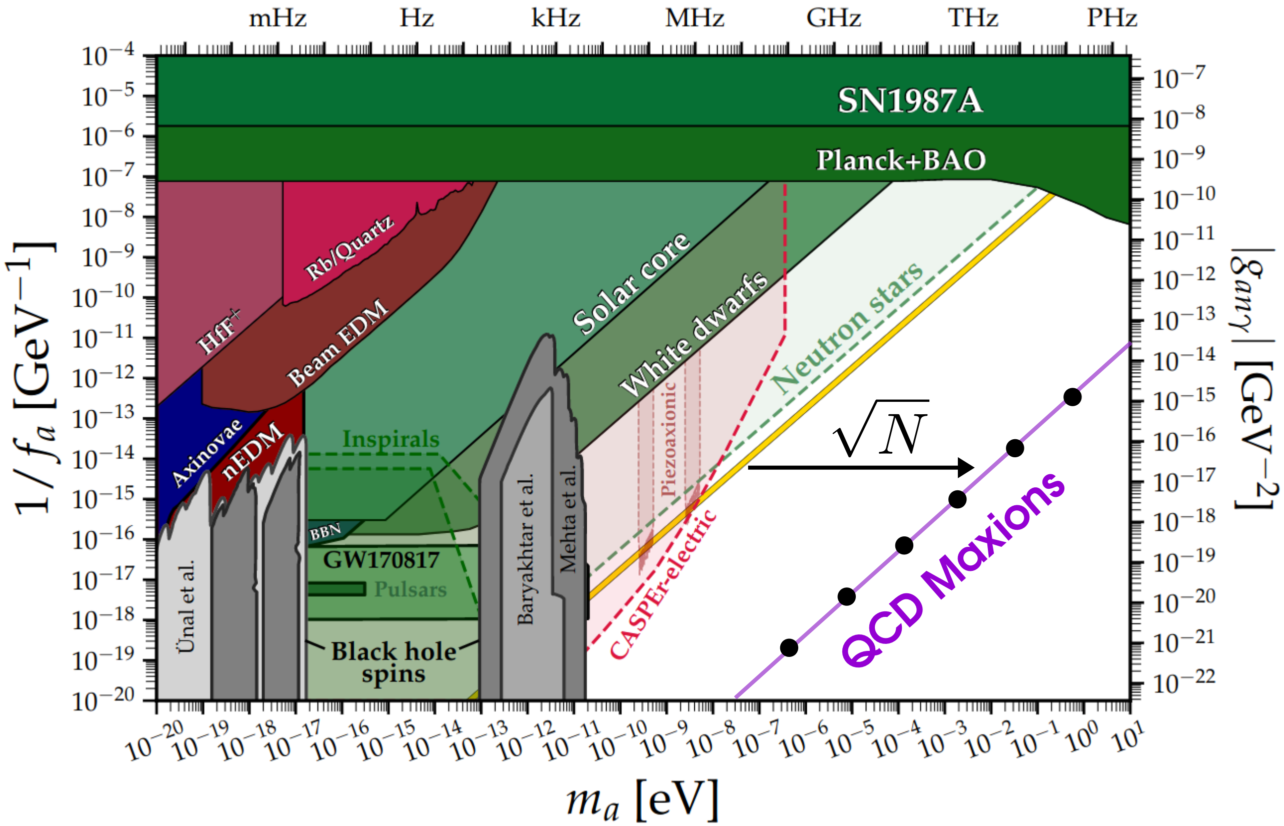


Maximally deviated signals

$$\max\left\{\min_i\{g_i\}\right\} = N \quad \Longrightarrow \quad g_i = N,\ \forall\, i$$



Implications for photon coupling

Assuming **universal** anomaly factors:

$$\mathcal{L} \supset \frac{1}{4}\sum_{k=1}^{N} g^{0}_{\hat{a}_k\gamma\gamma}\hat{a}_k F\widetilde{F} = \frac{\alpha_{em}}{8\pi}\sum_{k=1}^{N}\frac{E_k}{\mathcal{N}_k}\frac{\hat{a}_k}{\hat{f}_k}F\widetilde{F} \implies \frac{\alpha_{em}}{8\pi}\frac{E}{\mathcal{N}}\frac{a_{G\tilde{G}}}{F}F\widetilde{F}$$

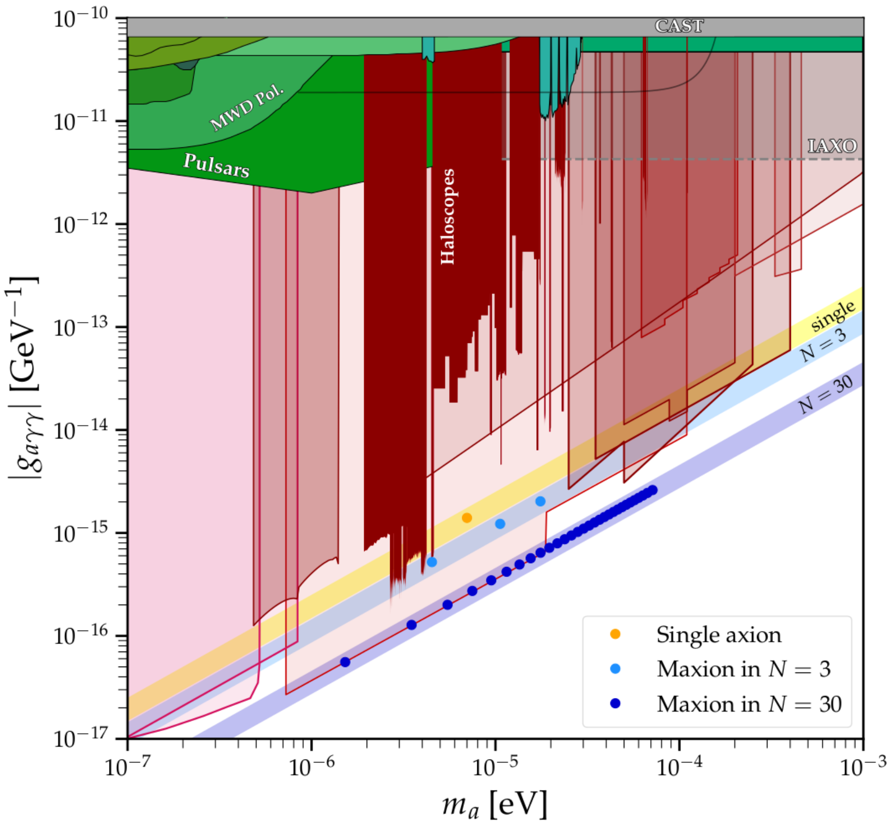
After $q = \begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\gamma_5 a_{G\tilde{G}}/(2F)Q_a}\begin{pmatrix} u \\ d \end{pmatrix}$:

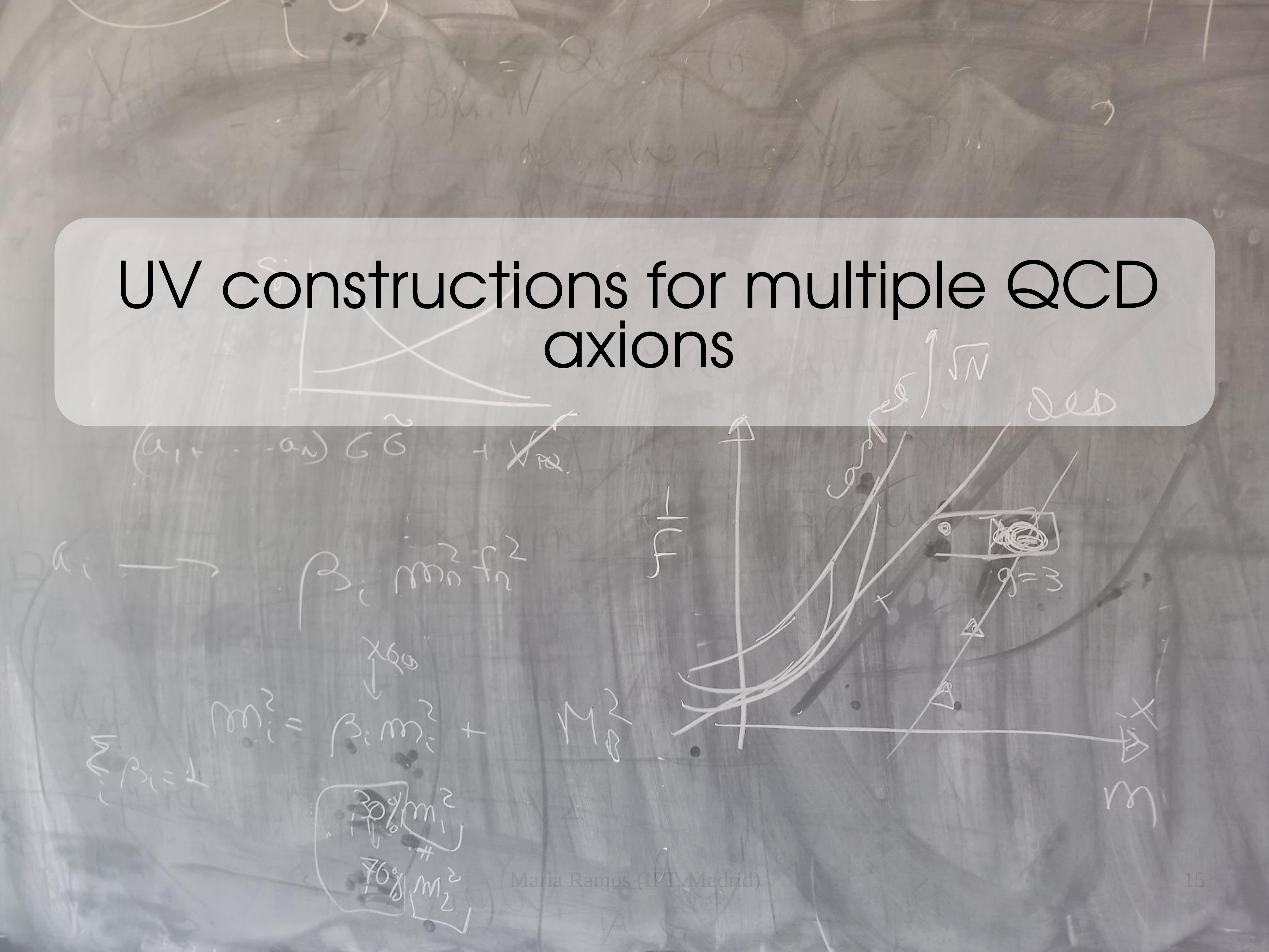
Di Cortona, Hardy, Vega, Villadoro 15

$$\mathcal{L} \supset \frac{\alpha_{em}}{8\pi}\left[\frac{E}{\mathcal{N}} - 1.92\right]\sum_i \frac{a_i}{f_i}F\widetilde{F}$$

$$\frac{m_i^2}{g^2_{a_i\gamma\gamma}} = \frac{m_a^2}{g^2_{a\gamma\gamma}}\Bigg|_{\text{single QCD axion}} \times g_i$$

$$\frac{(2\pi)^2}{\alpha_{em}^2}\left[\frac{E}{\mathcal{N}} - 1.92\right]^{-2}\sum_{i=1}^{N}\frac{g^2_{a_i\gamma\gamma}}{m_i^2} = 1$$





UV constructions for multiple QCD axions

Discrete models

$$\max \left\{ \min_i \{ g_i \} \right\} = N \qquad \Longrightarrow \quad g_i = N, \ \forall\, i$$

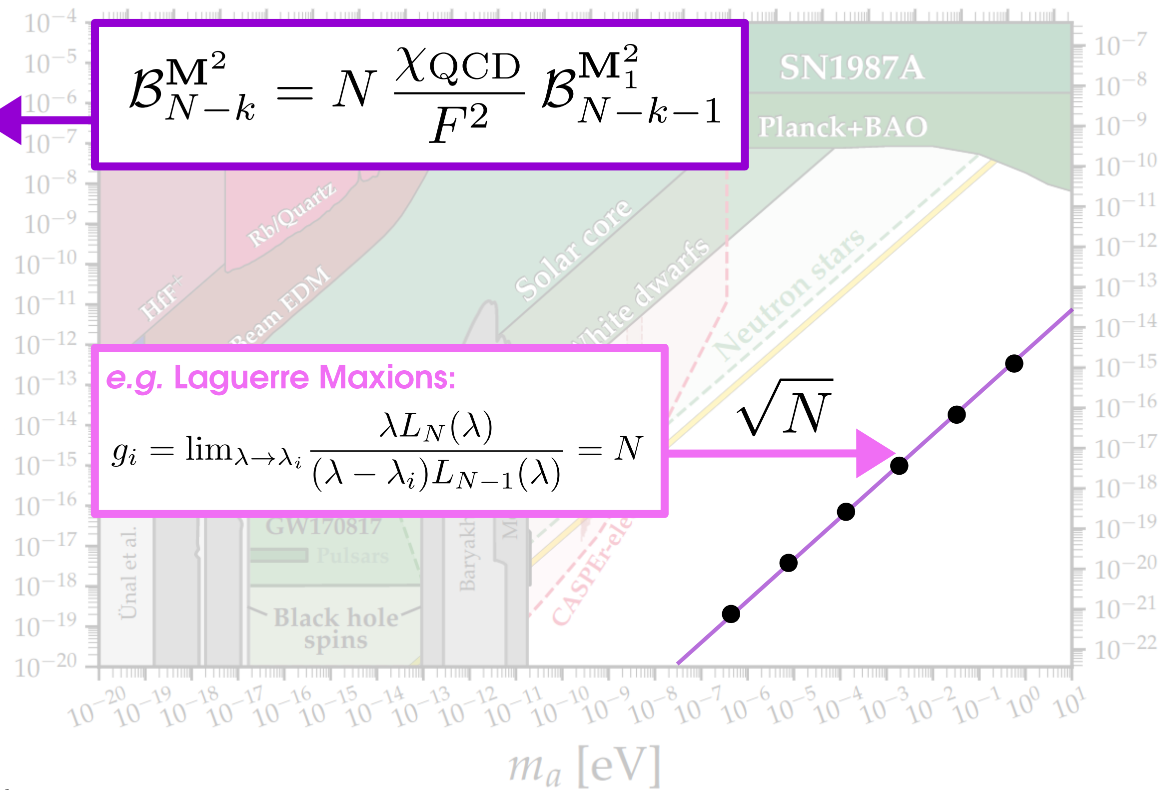
If the scales are the same, **N(N+1)/2** free parameters

$$\mathrm{tr}\, \mathbf{M}^2 = N \frac{\chi_{\rm QCD}}{F^2}$$

$$\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \frac{\chi_{\rm QCD}}{F^2}$$

$$\dots$$

$$\mathcal{B}^{\mathbf{M}^2}_{N-k} = N \frac{\chi_{\rm QCD}}{F^2} \mathcal{B}^{\mathbf{M}_1^2}_{N-k-1}$$



e.g. Laguerre Maxions:

$$g_i = \lim_{\lambda \to \lambda_i} \frac{\lambda L_N(\lambda)}{(\lambda - \lambda_i) L_{N-1}(\lambda)} = N$$

Bell polynomials:

$$p_{\mathbf{M}^2}(\lambda) = \sum_{k=0}^{N} \frac{(-1)^{N-k}}{(N-k)!} \mathcal{B}^{\mathbf{M}^2}_{N-k} \lambda^k$$

Extended KSVZ model

$$\mathcal{L}_{\rm UV} \supset -\left[y_1 \overline{\Psi}_{1,L} S_1 \Psi_{1,R} + y_2 \overline{\Psi}_{2,L} S_2 \Psi_{2,R} + \text{h.c.}\right] - V(|S_{1,2}|^2)$$

$$U(1)_{\rm PQ}: \quad \Psi_{j,L} \to e^{i\alpha_j/2}\Psi_{j,L}\,, \ \Psi_{j,R} \to e^{-i\alpha_j/2}\Psi_{j,R}\,, \ S_j \to e^{i\alpha_j} S$$

After SSB :
$$S_{1,2} = \frac{1}{\sqrt{2}}\left(\hat{f}_{1,2} + \rho_{1,2}\right) e^{i\hat{a}_{1,2}/\hat{f}_{1,2}}$$

Explicit breaking effects can reduce the symmetry:

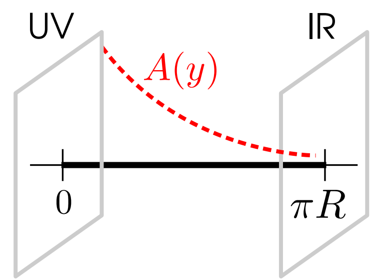
- $\mathcal{L}_{\rm UV} \supset \frac{\mu^2}{2} S_2 S_2 + \text{h.c.}$ — 2 maxions → $\mu^2 = \chi_{\rm QCD}/\hat{f}^2$

$$\mathcal{L}_{\rm eff} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_1}{\hat{f}} + \frac{\hat{a}_2}{\hat{f}} + \bar{\theta}\right) G\widetilde{G} - \mu^2 \hat{a}_2^2 \implies \mathbf{M}^2 = \begin{pmatrix} 2\frac{\chi_{\rm QCD}}{\hat{f}^2} + \mu^2 & \mu^2 \\ \mu^2 & \mu^2 \end{pmatrix}$$

- $\mathcal{L}_{\rm UV} \supset \mu \overline{\Psi}_{1,L} \Psi_{1,R}$ — 2 maxions → $\mu M_1^3 = 8\pi^3 \chi_{\rm QCD}$

Continuous models

$$ds^2 = \begin{cases} 1 \text{ (LED)} \\ A(y) = e^{-2k|y|} \text{ (RS)} \end{cases} \times \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$



Dienes, Dudas, Ghergghetta 99

$$S_5(a) \supset \int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{g} \left(\frac{1}{2} M_s g^{AB} \partial_A a \partial_B a - \frac{\alpha_s}{8\pi} \frac{a}{f_5} G_{\mu\nu} \widetilde{G}^{\mu\nu} \delta(y - \pi R) \right)$$

$$a(x,y) = \frac{1}{\sqrt{2\pi R M_s}} \sum a_i(x) \psi_i(y)$$

Integrating over the fifth dimension:

$$\mathcal{L} \supset \sum_n \left(\frac{1}{2} (\partial_\mu a_n)^2 - \frac{1}{2} \mu_n^2 a_n^2 \right) + \frac{1}{f_4} \frac{\alpha_s}{8\pi} \sum_n (a_n \psi_n^\pi) \tilde{G}_{\mu\nu} G^{\mu\nu}, \quad \mu_n^2 \sim n^2 \mu_1$$

Provides a predictive mixing structure for KK axions!

$$\mathbf{M}^2 = m_4^2 \begin{pmatrix} (\psi_0^\pi)^2 & \psi_0^\pi \psi_1^\pi & \psi_0^\pi \psi_2^\pi & \cdots \\ \psi_1^\pi \psi_0^\pi & (\psi_1^\pi)^2 + y^2 & \psi_1^\pi \psi_2^\pi & \cdots \\ \psi_2^\pi \psi_0^\pi & \psi_2^\pi \psi_1^\pi & (\psi_2^\pi)^2 + y^2 \left(\frac{\mu_2}{\mu_1} \right)^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

$$y \equiv \frac{\mu_1}{m_4} = \frac{\mu_1}{\sqrt{\chi}} f_4$$

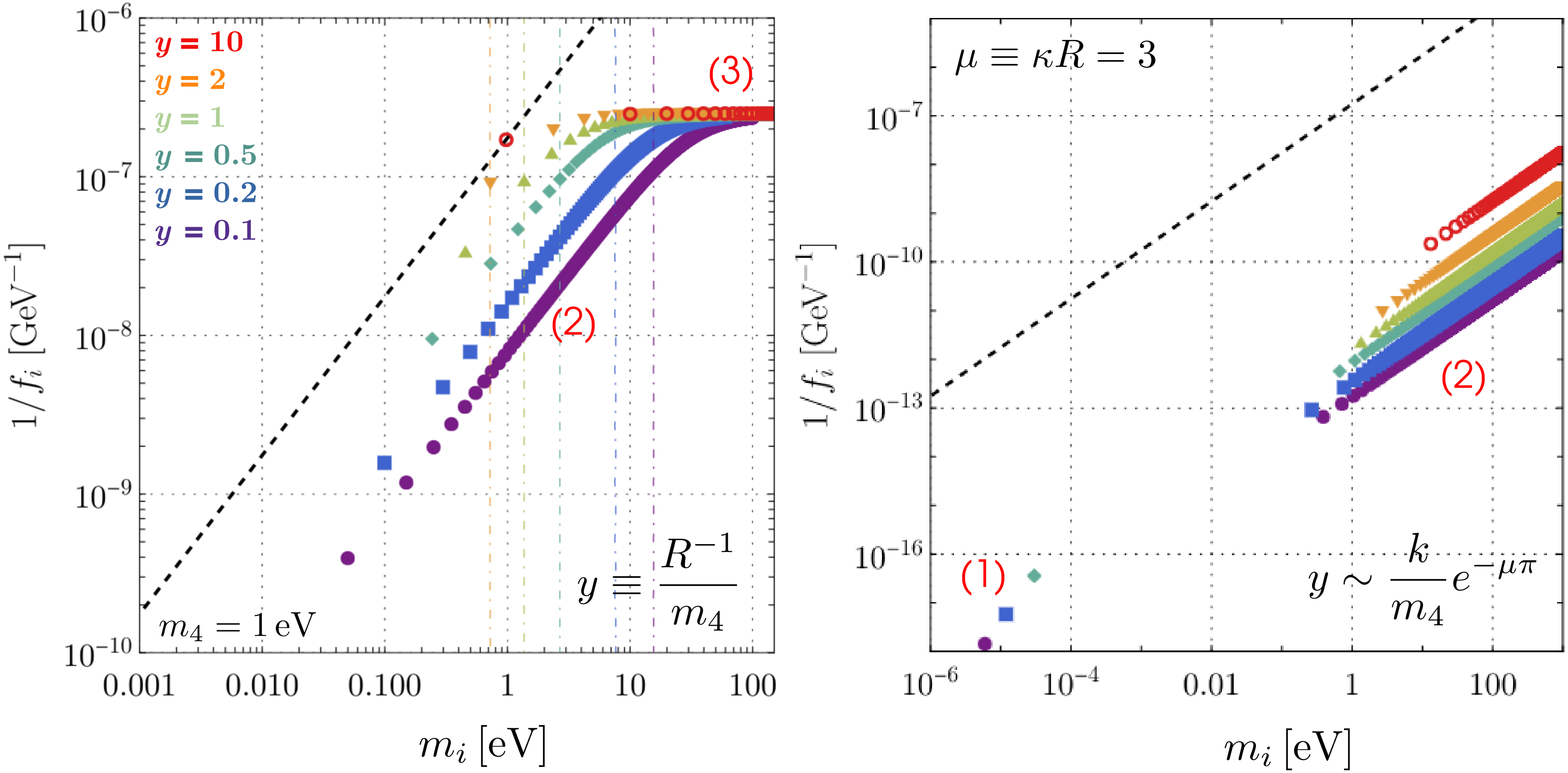
$$\psi_{n \geq 1}^{\text{LED}} = \mathcal{O}(1)$$

$$\psi_{n \geq 1}^{\text{RS}} = e^{\mu\pi}$$

preliminary

Maxions in extra dimensions

WORK IN PROGRESS with Arturo De Giorgi



Proof of existence: maxions are the non-canonical axions of ED models!

Signatures:

(1) k-suppressed eigenmode (2) Maxion band (3) Plateau of heavy modes

preliminary

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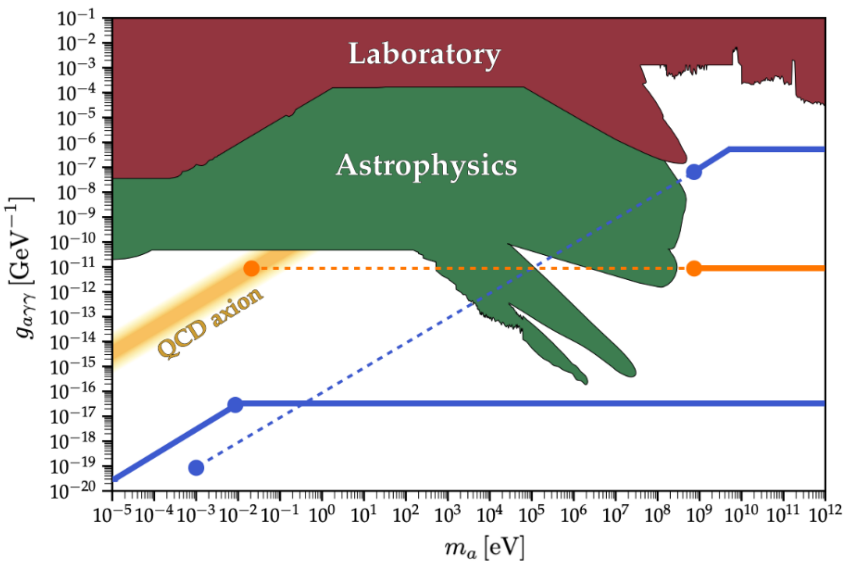
$$\psi_{n\geq 1}^{\text{RS}} = e^{\mu\pi}$$

To produce maxions:

$$\rho^2 \equiv \left(\frac{\psi_1}{y}\right)^2 = \left(\frac{\psi_1}{f_4}\right)^2 \frac{\chi_{\text{QCD}}}{\mu_1^2} \gtrsim 1$$



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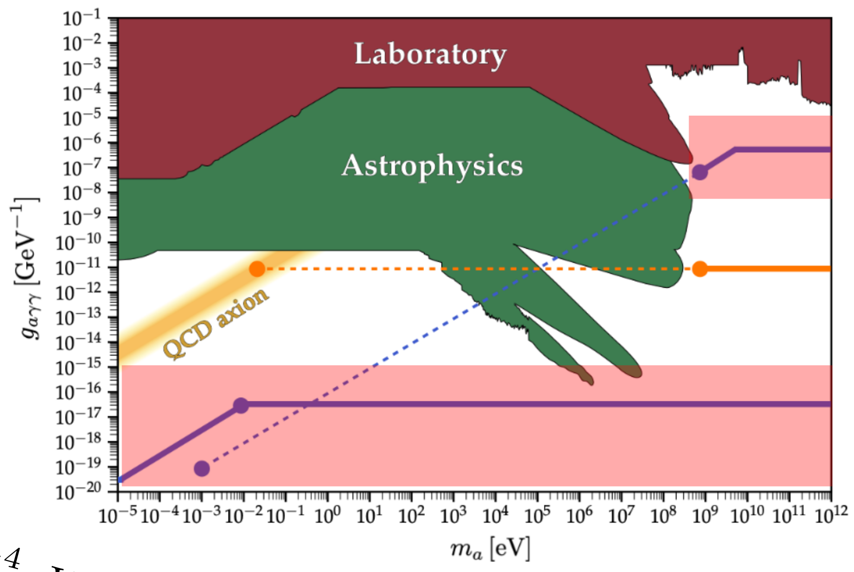
To treat KK tower perturbatively:

$$\rho^2 \sim g_0 \lesssim \left(\frac{1}{\Lambda = 10^3\,\mathrm{GeV}}\right)^3 \frac{\chi_{\mathrm{QCD}}}{\mu_1}$$

$$\ll 10^{-4}\,\mathrm{eV}$$



Lightest graviton mass largely ruled out...

preliminary

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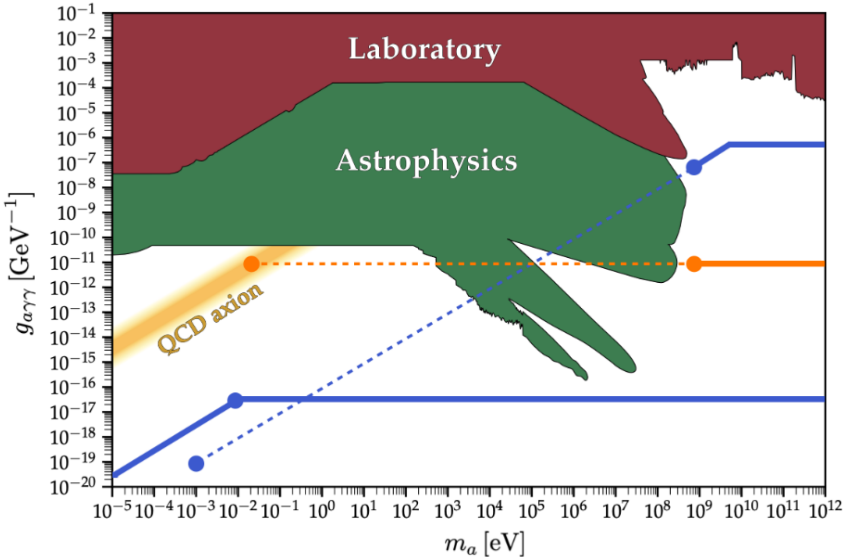


Non-universal mixing structures (e.g. in string theory) required to produce the **non-canonical patterns**!

Take home messages

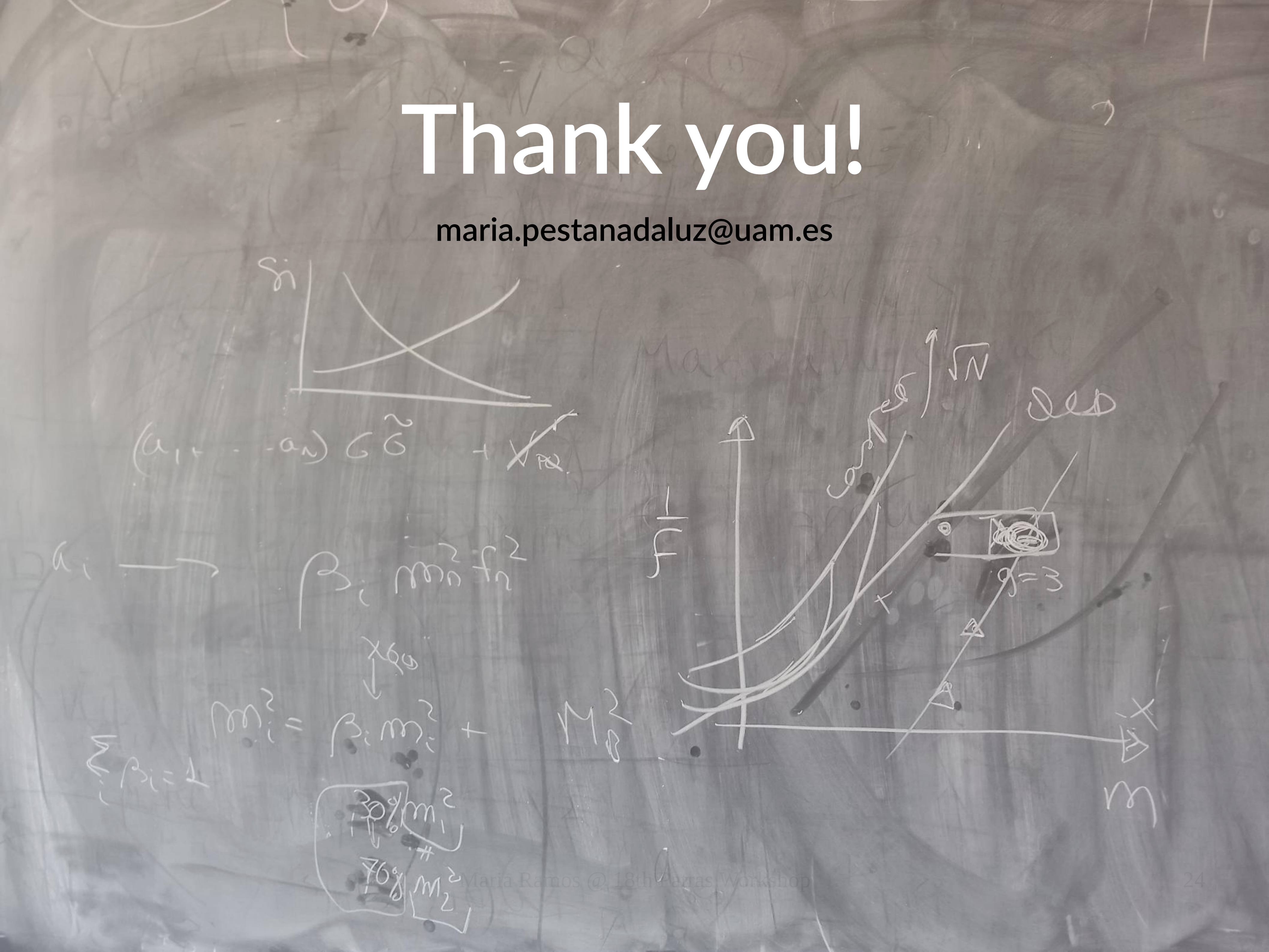
1. Any signal to the right of the canonical axion band can indicate a multiple QCD axion solution to the strong CP problem!

2. The QCD axion sum rule links the possible mass-scale values of the different axions, and allows us to count how many axions are involved in the solution to the strong CP problem.

3. The complete reconstruction of the multiple QCD axion may require a complementary search between different experiments.

4. The most exotic phenomenology consists on N aligned signals deviated from the QCD line by a factor of $\sqrt{N}$. **Extra dimensional models predict these solutions.**

5. Currently assessing UV setups and modified experimental bounds that take into account these multiple axions.



Thank you!

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