Magnetogenesis from an Anisotropic Universe

Based on arXiv:2311.07685 (Phys.Rev.D 109 (2024) 8, 083507)

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10th July, 2024

PASCOS 2024, 7-13 July, ICISE, Quy Nhon, Vietnam

Overview

1. Introduction
2. Inflationary magnetogensis
3. Post inflationary evolution
4. Conclusion

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- It suffers from strong coupling and backreaction problems.
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- Anisotropic inflation has been proposed in the literature to explain the cold spots in CMB.

Background Spacetime

- We work with a Bianchi-I-type metric

$$ds^2 = a^2(\eta)\left[-d\eta^2 + b^2(\eta)dx^2 + dy^2 + dz^2\right] \tag{1}$$

$$\eta \to \text{Conformal time} \; ; a(\eta) \to \text{Scale factor}$$

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- No explicit coupling of the inflaton field to the gauge field, free action:

$$S = -\frac{1}{4}\int d^4x\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $A_\mu \rightarrow$ Magnetic vector potential.

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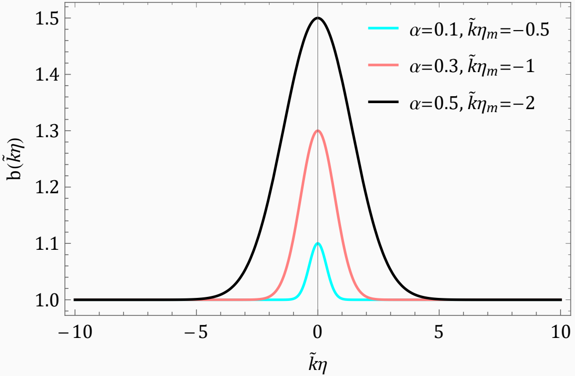

Figure 1: Variation of anisotropic factor *b* with time.

Evolution equations

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$$A''_n + \frac{b'}{b}A'_n + \tilde{g}_{jn}\tilde{g}'^{jk}A'_k - \tilde{g}^{im}\partial_i F_{mn} \quad = \quad 0 \tag{4}$$

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- Evolution in the momentum space:

$$u''_n + \frac{b'}{b}u'_n + \tilde{g}'^{jl}\tilde{g}_{jn}u'_l + \tilde{g}^{im}\left(k_m k_i u_n - k_n k_i u_m\right) = 0. \tag{6}$$

- Evolution of mode functions:

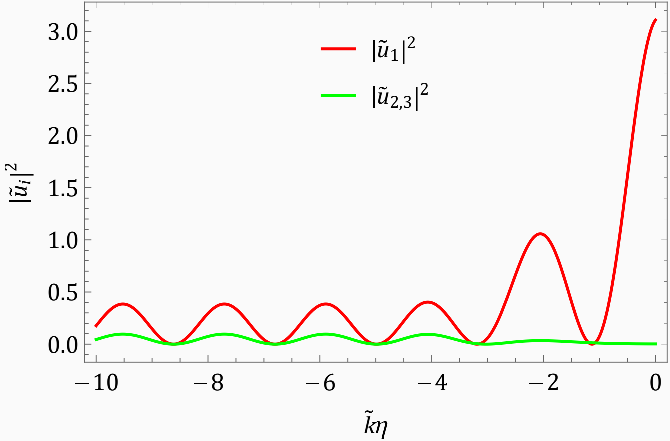

Figure 2: Evolution of mode functions with time for anisotropic free parameters $\alpha = 3, \tilde{k}\eta_m = -2$ ($\tilde{u}_i = \sqrt{\tilde{k}}u_i$).

EM power spectra

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- In terms of mode functions:

$$\mathcal{P}_E(\eta,\tilde{k}) = \frac{\tilde{k}^3}{2\pi^2 a^4}\left(\frac{|u_1'(\eta)|^2}{b^2} + |u_2'(\eta)|^2 + |u_3'(\eta)|^2\right) \tag{7}$$

$$\mathcal{P}_B(\eta,\tilde{k}) = \frac{\tilde{k}^5}{2\pi^2 a^4}\left[\frac{1}{b^2}\left(2|u_1|^2 + |u_2|^2 + |u_3|^2 - 2\Re(u_1 u_2^*) - 2\Re(u_1 u_3^*)\right) + \left(|u_2|^2 + |u_3|^2 - 2\Re(u_2 u_3^*)\right)\right] \tag{8}$$

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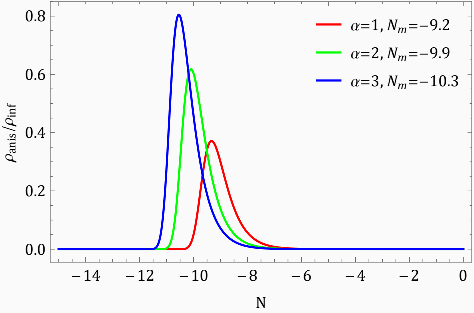

Figure 3: Evolution of the ratio of anisotropic energy density to inflationary energy density with e-folding number.

$$N = \ln\left(a/a_{\rm end}\right)\ ;a = -1/H\eta$$

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$$\frac{\rho_E + \rho_B}{\rho_{\text{inf}}} \sim 10^{-9};$$

- No backreaction from produced EM field during inflation.
- Anisotropy does not affect inflation.

- The universe becomes radiation-dominated instantly after inflation.

Post inflationary evolution in instant reheating

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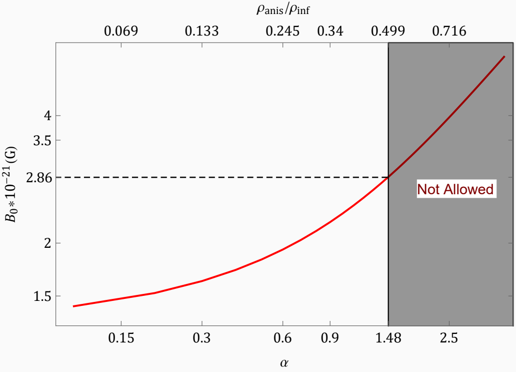

Figure 4: Variation of present strength of magnetic field with anisotropic parameter α for $\tilde{k}\eta_m = -2$.

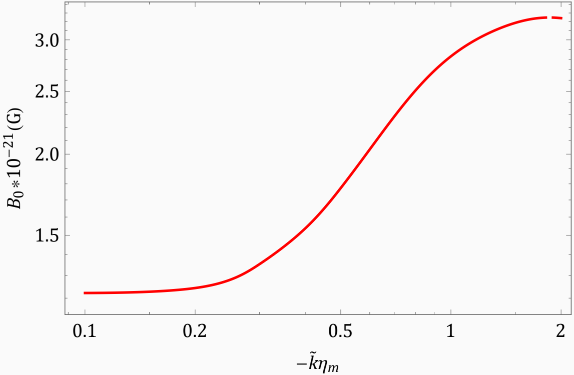

Figure 5: Variation of present strength of magnetic field with $\tilde{k}\eta_m$ for $\alpha = 1.45$.

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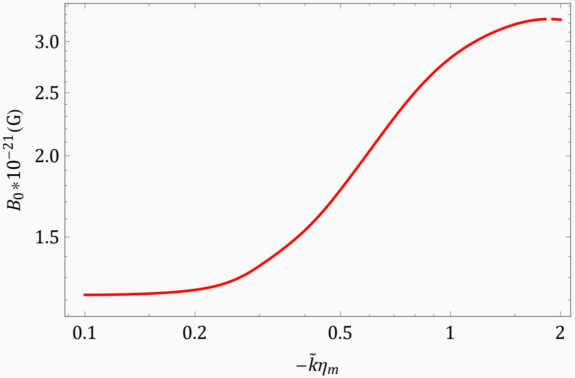

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- Maximum magnetic field value obtained through instant reheating: $B_0 \sim 3 \times 10^{-21}$ G.

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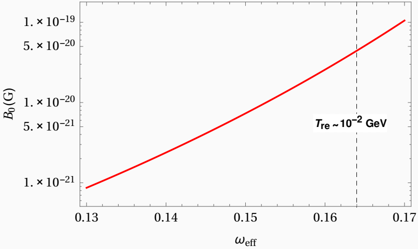

Figure 6: Variation of present-day magnetic field with effective equation of state ω_{eff}.

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- We get a rather tight constraint on $0.132 < \omega_{eff} < 0.164$.

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Thank You!

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