# Clockwork at future lepton colliders, beam dumps, and SN1987

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Based on:

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# Outline

- Theory
    - Clockwork mechanism
    - Generalized Continuous Clockwork

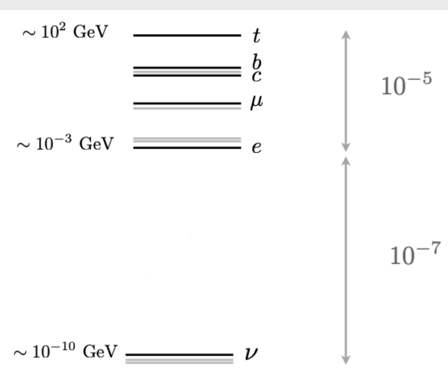
- Phenomenology
    - Future lepton colliders, beam dumps, cosmology
        - Randall-Sundrum
        - Linear Dilaton
        - Generalized Linear Dilaton

# Physics Beyond the Standard Model

SM contains several *hierarchies* between energy scales

$$\mathcal{L}_{SM} = \mathcal{L}_{kin} + gA_\mu \bar{F}\gamma_\mu F + Y_{ij}\bar{F}_i H F_j + \lambda (H^\dagger H)^2 + \ldots$$ **d=4**

- cosmological constant $+ c_0 \, \Lambda_{UV}^4 \sqrt{g}$ $\quad c_0 \sim -10^{-60} \left(\frac{\text{TeV}}{\Lambda_{UV}}\right)^4$ **d=0**
- hierarchy problem $+ c_2 \, \Lambda_{UV}^2 \, H^\dagger H$ $\quad c_2 \simeq 0.008 \left(\frac{\text{TeV}}{\Lambda_{UV}}\right)^2$ **d=2**
- strong CP problem $+ \theta \, \tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu}$ $\quad \theta \lesssim 10^{-10}$ **d=4**
- neutrino masses




Explaining *fine tuning*

- GUT, SUSY, XD, string theory landscape, anthropic principle,

**Clockwork:** generating hierarchy from O(1) numbers by asymmetric NN interactions

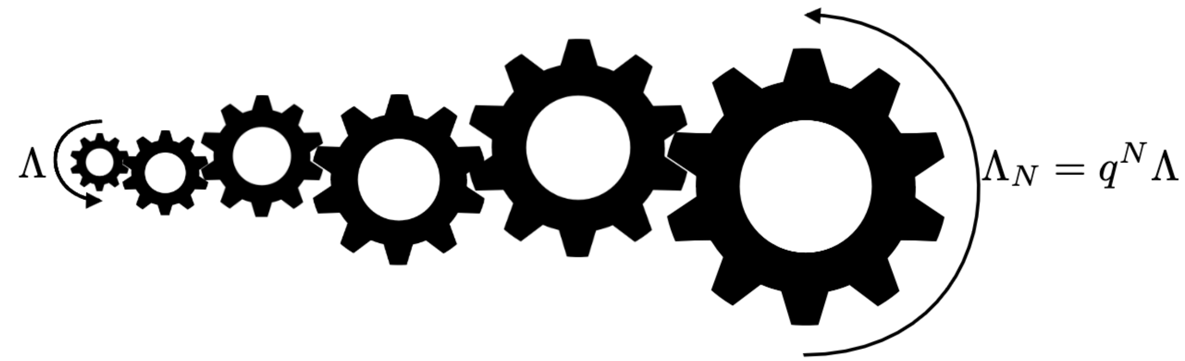
# Clockwork mechanism

- First studied in context of pseudoscalar (axion/relaxion) model building

  Kim, Nilles, Peloso 0409138, Dvali 0706.2050, K. Choi, H. Kim, S. Yun 1404.6209

  K. Choi, S. H. Im 1511.00132; Kaplan, Rattazzi 1511.01827

  - asymmetric NN interactions along the chain
  - exp. small couplings without large scales (e.g. large, super-Planckian, axion decay constant)




- General Clockwork

  Giudice, McCullough 1610.07962

  Only abelian symmetry can be Clockworked? Craig, Garcia, Sutherland, 1704.07831

- Generalized Continuous Clockwork

  K. Choi, S.H. Im, C. Shin 1711.06228, Giudice, Katz, McCullough, Torre, and Urbano 1711.08437

- UV completions

  Kehagias, Riotto 1710.04175, Antoniadis et al. 1710.05568, S. H. Im, H. Nilles, M. Olechowski 1811.11838

# Clockwork scalar (axion)

K. Choi and S. H. Im 1511.00132; Kaplan and Rattazzi 1511.01827

- $N+1$ complex scalars whose dynamics generate an exponentially suppressed interaction scale
  - $U(1)^{N+1}$ global symmetry broken at high scale $f$
- Add explicit breaking to $U(1)^0$ by asymmetric nearest neighbors couplings; SM coupled *only* to the last site

$$\mathscr{L} = -\frac{f^2}{2}\sum_{j=0}^{N} \partial_\mu U_j^\dagger \partial^\mu U_j + \frac{m^2 f^2}{2}\sum_{j=0}^{N-1}\left( U_j^\dagger U_{j+1}^q + \text{ h.c. } \right), \qquad q > 1$$

where $U_j(x) = e^{i\pi_j/j}$

$\pi_j \to \pi_j + a/q^j$ remains → N pseudo-Goldstone bosons, 1 massless Goldstone boson

Promote $a \to a(x)$. If $\pi_N$ coupled to SM by $1/f$, $a$ coupled by $1/(q^N f)$.

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where $U_j(x) = e^{i\pi_j/j}$

$\pi_j \to \pi_j + a/q^j$ remains

$$\mathscr{L}_{int} \simeq \frac{m^2}{2}\sum_{j=0}^{N}(\pi_j - q\pi_{j+1})^2 \simeq \frac{1}{2}\sum_{i,j=0}^{N}\pi_i\left(M_\pi^2\right)_{ij}\pi_j$$

$$M_\pi^2 = m^2\begin{pmatrix} 1 & -q & 0 & \cdots & & 0 \\ -q & 1+q^2 & -q & \cdots & & 0 \\ 0 & -q & 1+q^2 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ & & & & 1+q^2 & -q \\ 0 & 0 & 0 & \cdots & -q & q^2 \end{pmatrix}$$

$$m_0^2 = 0, \;\; m_k^2 = m^2\left[q^2 + 1 - 2q\cos\frac{k\pi}{N+1}\right]$$

massless Goldstone and gears

# Clockwork scalar - spectrum

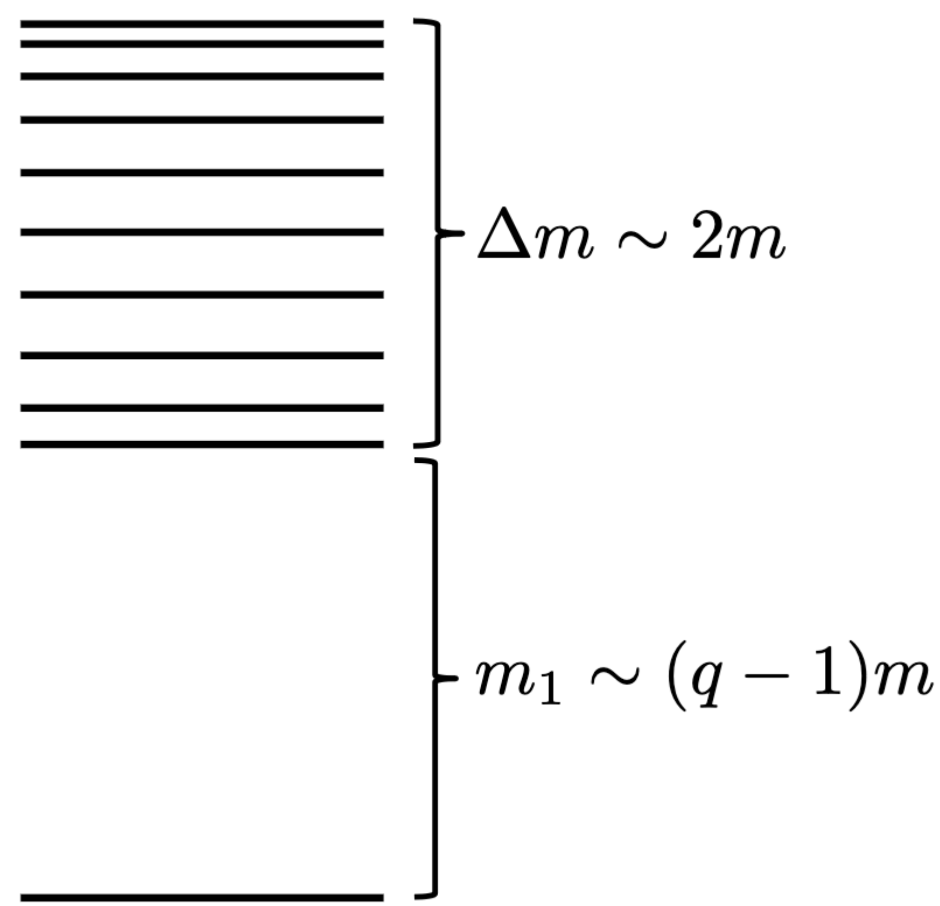
- At n-th site: $\pi_n = \sum_j O_{nj}\, a_j$

$$O_{j0} = \frac{\mathcal{N}_0}{q^j}, \quad O_{jk} = \mathcal{N}_k \left[ q \sin\frac{jk\pi}{N+1} - \sin\frac{(j+1)k\pi}{N+1} \right], \quad j = 0,..,N; \quad k = 1,..,N$$

localization of massless mode component along the gear → exponentially small coupling to SM (Nth site)

$$\mathcal{N}_0 \equiv \sqrt{\frac{q^2-1}{q^2-q^{-2N}}}, \quad \mathcal{N}_k \equiv \sqrt{\frac{2}{(N+1)\lambda_k}}$$

$$\lambda_k = 1 + q^2 - 2q\cos\theta_k, \qquad \theta_k = \frac{k\pi}{N+1}$$




- Mass gap Δm and the state density δm

$$\frac{\Delta m}{m_{a_1}} = 2(q-1),$$

$$\frac{\delta m_k}{m_{a_k}} \sim \frac{q\pi}{N\lambda} \sin\frac{k\pi}{N+1} \sim \mathcal{O}(1/N).$$

# Towards Continuous Clockwork

- How to take $n \to \infty$ limit (the chain as extra dim.)?

$$\mathscr{L}_{int} \simeq -\frac{m^2 f^2}{2} \sum_{j=0}^{N} \exp\left(\frac{i}{f}(\pi_j - q\pi_{j+1})^2\right) + \frac{\pi_j}{f} F\tilde{F}, \quad \pi_i \to \pi_i + \alpha/q^j$$

Craig, Garcia, Sutherland, 1704.07831

- $U(1)$ with charges: $1,\ldots,1/q^N$. Continuum limit with $q^N$ fixed $\to$ symmetry is non-compact (no charge quantization).
- Moreover, if the CW sym. group was not abelian, the generators would mix in the zero mode → all couplings have to be equal to 1.

- Notice: by field redefinition ($\pi_i \to \pi_i/q^i$)

$$\mathscr{L}_{int} \to -\frac{m^2 f^2}{2} \sum_{j=0}^{N} q^{-2j}(\partial_\mu \pi_j)^2 - \frac{m^2 f^2}{2} \exp\left(\frac{i}{q^j f}(\pi_j - q\pi_{j+1})^2\right) + \frac{\pi_j}{q^j f} F\tilde{F}$$

warping position-dependent coupling

Giudice, McCullough 1610.07962, K. Choi, S.H. Im, C. Shin 1711.06228, Giudice, Katz, McCullough, Torre, and Urbano 1711.08437

# Clockwork from 5th dim with LD

- Sites at $i = 0,\ldots,N \leftrightarrow$ points in 5-th dimension

$$y \leftrightarrow ja, \quad Na = \pi R, \quad \int \mathrm{dy} \leftrightarrow \sum, \quad \partial_y \phi \leftrightarrow \frac{1}{a}\left(\phi_{j+1} - \phi_j\right)$$

$$\mathscr{L}_{int} \to -\frac{m^2 f^2}{2} \sum_{j=0}^{N} q^{-2j}(\partial_\mu \pi_j)^2 - \frac{m^2 f^2}{2} \exp\left(\frac{i}{q^j f}(\pi_j - q\pi_{j+1})^2\right) + \frac{\pi_j}{q^j f} F\tilde{F}$$

$$\mathcal{L} = -\frac{1}{2} \int dy\, e^{-2ky} \left[\partial_\mu \pi\, \partial^\mu \pi + (\partial_y \pi)^2\right] + e^{-ky} \frac{\pi}{f} F\tilde{F}$$

- *Interpret* as Linear Dilaton background

$$\mathcal{S} = \int d^4x\, dy \sqrt{-g}\, \frac{M_5^3}{2} e^S \left(\mathcal{R} + g^{MN} \partial_M S\, \partial_N S + 4k^2\right)$$

From Einstein eqs: $\langle S \rangle = \pm\, 2ky$

- Works for any massless field (scalar, fermion, vector, graviton)
    - position-dependent hierarchy, zero-mode localisation, gear mass spectrum

# Clockwork for gravitons

$$\mathcal{L} = -\frac{m^2}{2}\sum_{j=0}^{N-1}\left(\left[h_j^{\mu\nu} - q h_{j+1}^{\mu\nu}\right]^2 - \left[\eta_{\mu\nu}(h_j^{\mu\nu} - q h_{j+1}^{\mu\nu})\right]^2\right)$$

- N+1 gravitons with diffeomorphism invariance

$$g^i_{\mu\nu} \to g^i_{\mu\nu} + \nabla_{(\mu} A^i_{\nu)}$$

NN interactions break it to (Linearized gravity $g^i_{\mu\nu} = \eta^i_{\mu\nu} + 2h^i_{\mu\nu}/M_{Pl}$)

$$g^i_{\mu\nu} \to g^i_{\mu\nu} + \frac{1}{q^i}\nabla_{(\mu}\tilde{A}_{\nu)}$$

SM fields only couple to the last graviton → hierarchy problem solved

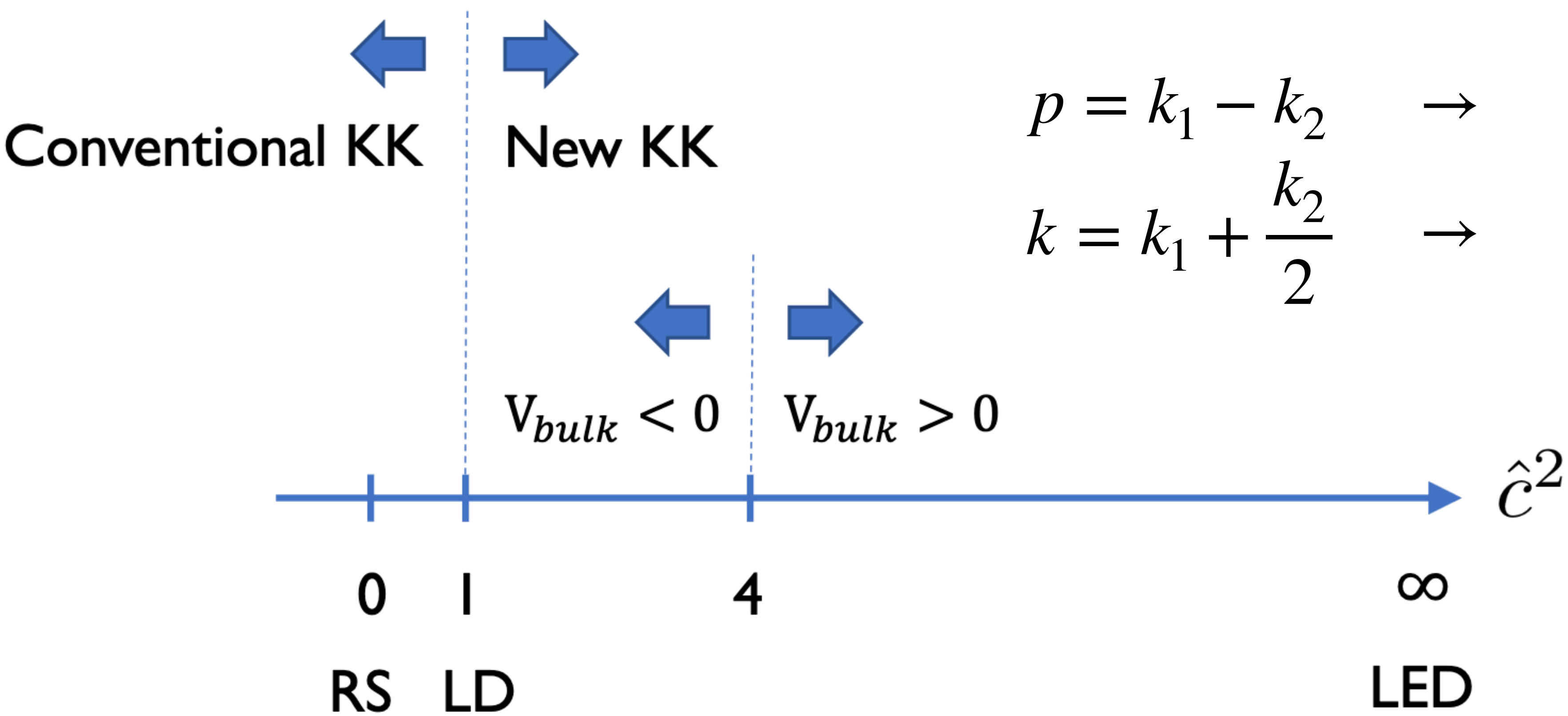
$$-\frac{1}{M_N} h_N^{\mu\nu} T_{\mu\nu} \to -\frac{1}{M_P}\tilde{h}_0^{\mu\nu} T_{\mu\nu} \qquad M_P = q^N M_N$$

Giudice, McCullough 1610.07962, K. Choi, S.H. Im, C. Shin 1711.06228, Giudice, Katz, McCullough, Torre, and Urbano 1711.08437

# Clockwork/5th dim. with GLD

$$ds^2 = e^{2k_1 y} dx^2 + e^{2k_2 y} dy^2, \qquad c^2 = \frac{k_2}{k_1}$$

gravity + dilaton + cosmological constants on 5D orbifold $M_4 \times S^1/Z_2$

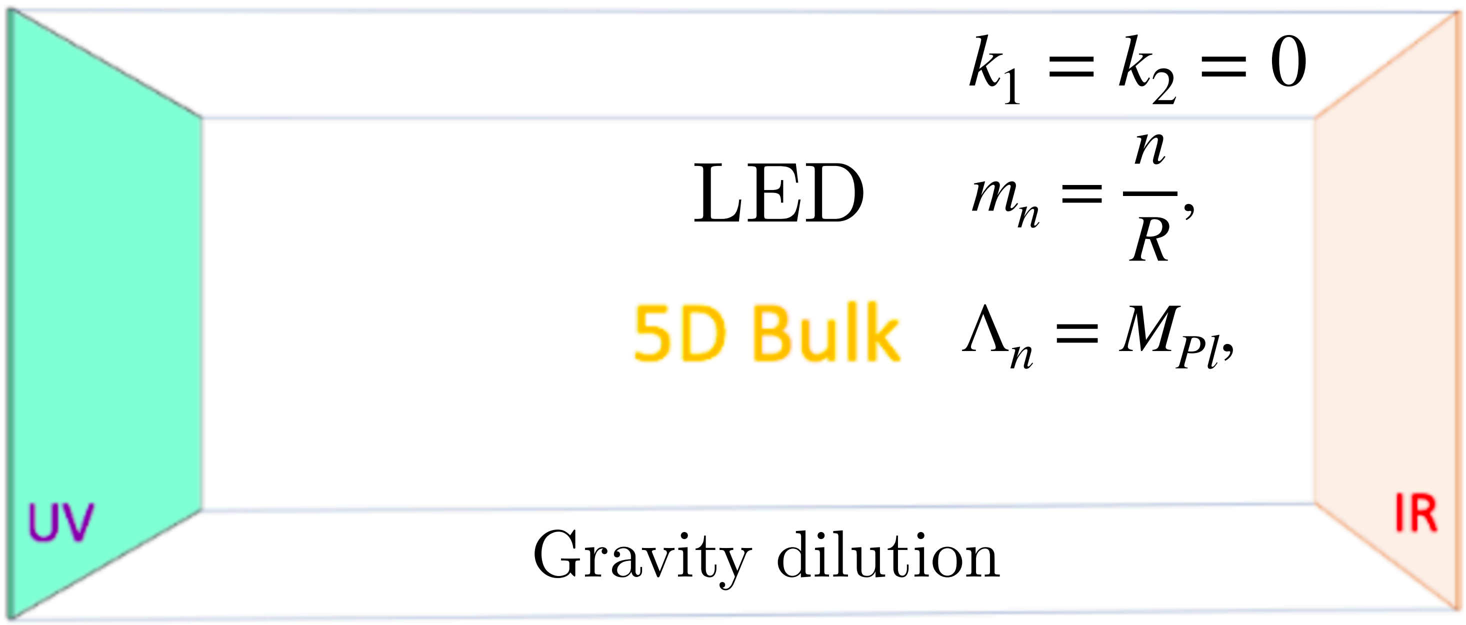


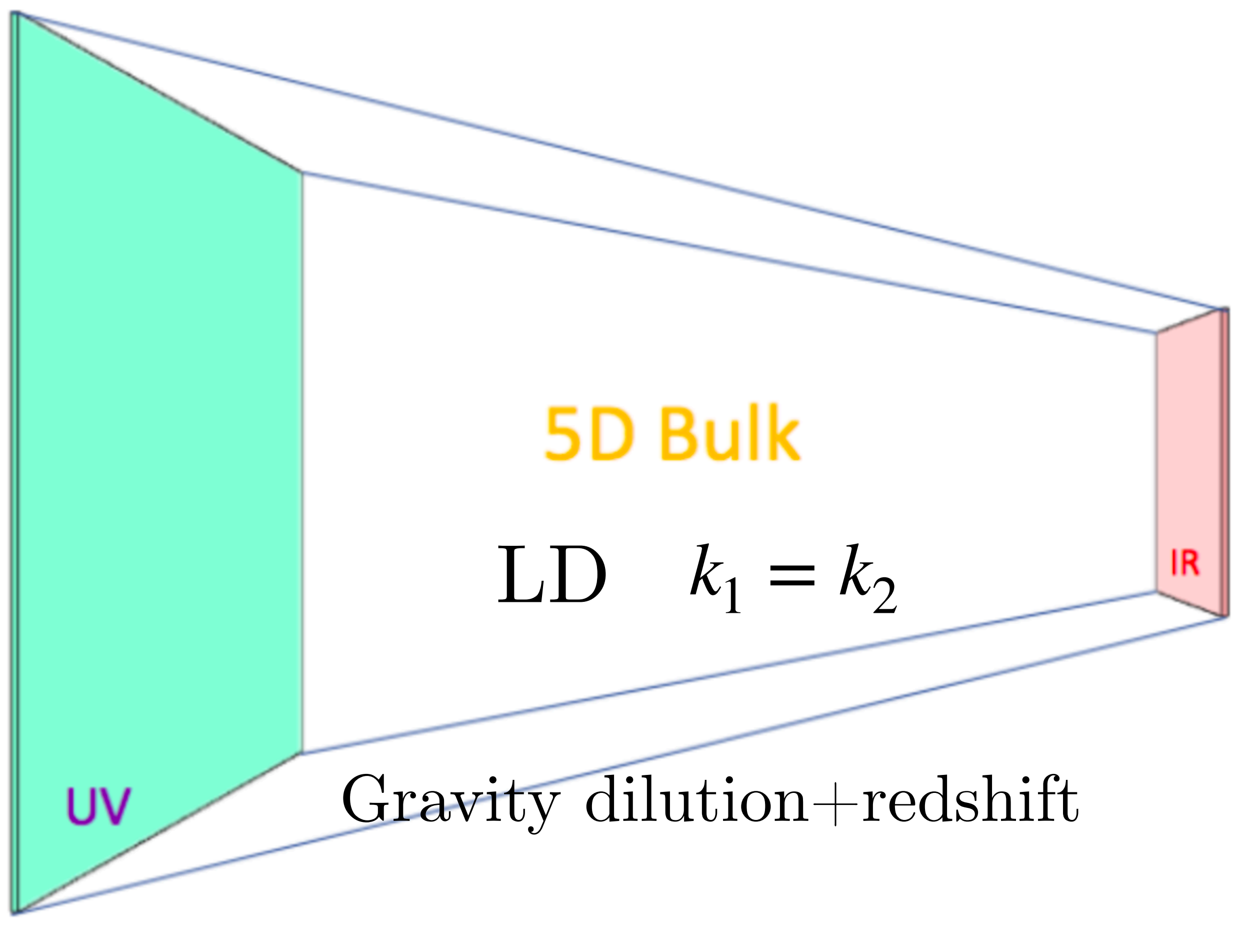

$p = k_1 - k_2 \quad \rightarrow$ KK masses

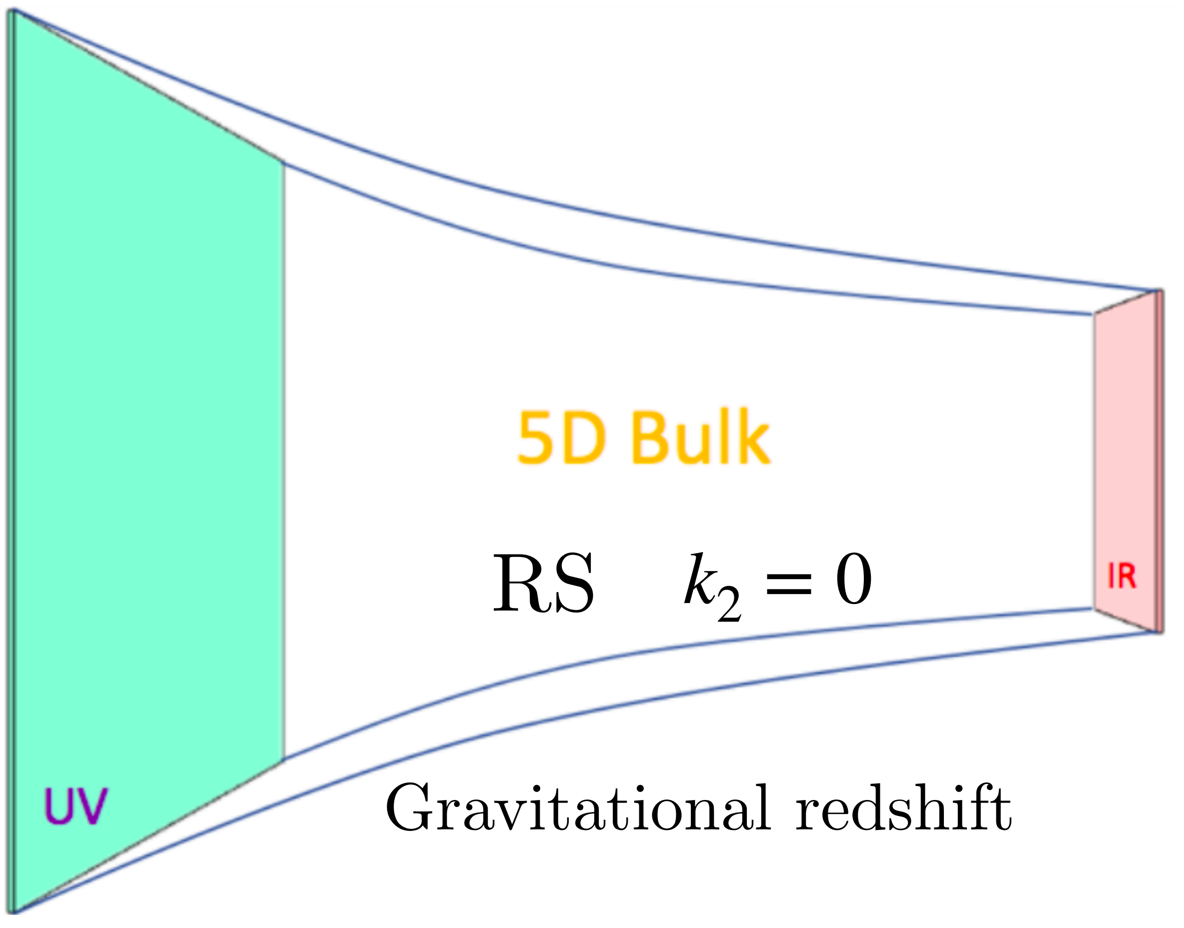
$k = k_1 + \frac{k_2}{2} \quad \rightarrow$ zero mode coupling hierarchy

S. H. Im, H. Nilles, M. Olechowski 1811.11838

# Geometry of the extra dimension



# Spectrum as a function of $k_{1,2}$

- There are three backgrounds as limiting cases

  Curved: Randall-Sundrum and Linear Dilaton

$$\text{RS}: \ m_n = \left(n + \frac{1}{4}\right)\pi k, \ \Lambda_n = \sqrt{\frac{M_5^3}{k}}, \qquad M_{\text{Pl.}} = \sqrt{\frac{M_5^3}{k}\left(e^{2k\pi R} - 1\right)}, \quad k_2 = 0,$$

$$\text{LD}: \ m_n = \sqrt{k^2 + \frac{n^2}{R^2}}, \quad \Lambda_n = \sqrt{M_5^3 \pi R \left(1 + \frac{k^2 R^2}{n^2}\right)}, \ M_{\text{Pl.}} = \sqrt{\frac{M_5^3}{k}\left(e^{2k\pi R} - 1\right)}, \ k_1 = k_2.$$

$\sim$ Flat: GLD

For hierarchy problem $kR \simeq 10$.

GLD: (for $1 < c < 2$)

$$m_n = \frac{\pi}{2}\left(k + k(-1+4n)\frac{|c^2-1|}{c^2+2}\right)\left(1 + \frac{kM_{Pl}^2}{M_5^3}\right)^{\frac{-|c^2-1|}{c^2+2}} \sim \frac{n}{R},$$

$$\Lambda_n = \frac{M_5}{C_n}, \qquad C_n = \frac{M_5^{3/2} 2^{2-\frac{c^2+2}{2|c^2-1|}} \pi^{\frac{c^2+2}{2|c^2-1|}} \left|c^2-1\right| \left(\frac{c^2+2}{4|c^2-1|} + n - \frac{1}{4}\right)^{\frac{c^2+2}{2|c^2-1|}}}{\Gamma\left(\frac{c^2+2}{2|c^2-1|}\right)\sqrt{(c^2+2)\left((4n-1)\left|c^2-1\right| + c^2+2\right)\left(kM_{Pl}^2 + M_5^3\right)}} \sim \frac{M_5}{M_{Pl}},$$

# Signatures

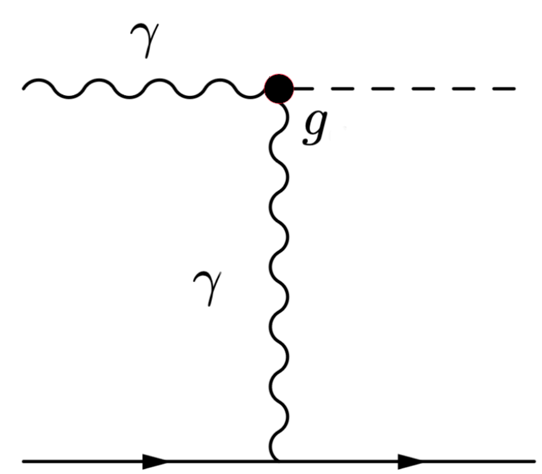
- We focus on two regimes for radion and KK gravitons: $\phi, G_k \to f\bar{f}, \gamma\gamma$
    - $c\tau\gamma \lesssim 1m$: FCC-ee, CLIC

$$\sigma\left(e^+e^- \to XG\right) = \int d\Omega \frac{d\sigma\left(e^+e^- \to GX\right)}{d\Omega}\left(1 - e^{-L_{\rm det}/L_G^{\perp}(\theta)}\right), \quad (2.2)$$

where $L_G^{\perp} = c\tau\gamma\beta\sin\theta$, $\gamma = (s - m_X + m_G^2)/(2m_G\sqrt{s})$ for $X = \gamma$, $Z$, and $\theta$ is the angle measured from the collider axis.

- $c\tau\gamma \gtrsim 1m$: SHiP, FASER, MATHUSLA and BBN/SN1987

$$N = \sum_{E,\theta} N_{\rm LLP}(E,\theta) \times \left(e^{-L_{\rm min}/d(E)} - e^{-L_{\rm max}/d(E)}\right)$$




Prod. from $\sigma_{\gamma N \to GN} \simeq \frac{\alpha_{em} g^2_{G\gamma\gamma} Z^2}{2}\left(\log\left(\frac{d}{1/a^2 + t_{max}}\right) - 2\right)$ and $Z \to Gf\bar{f}$.

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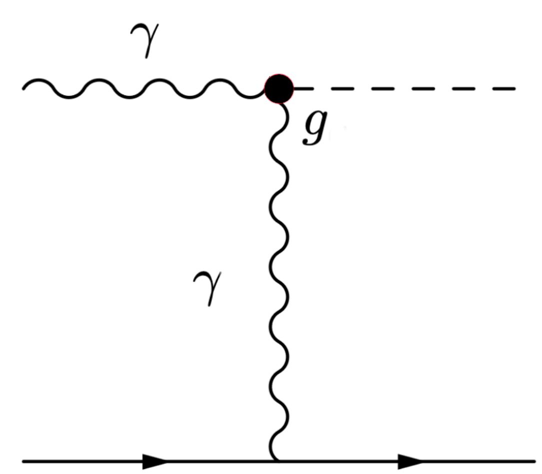


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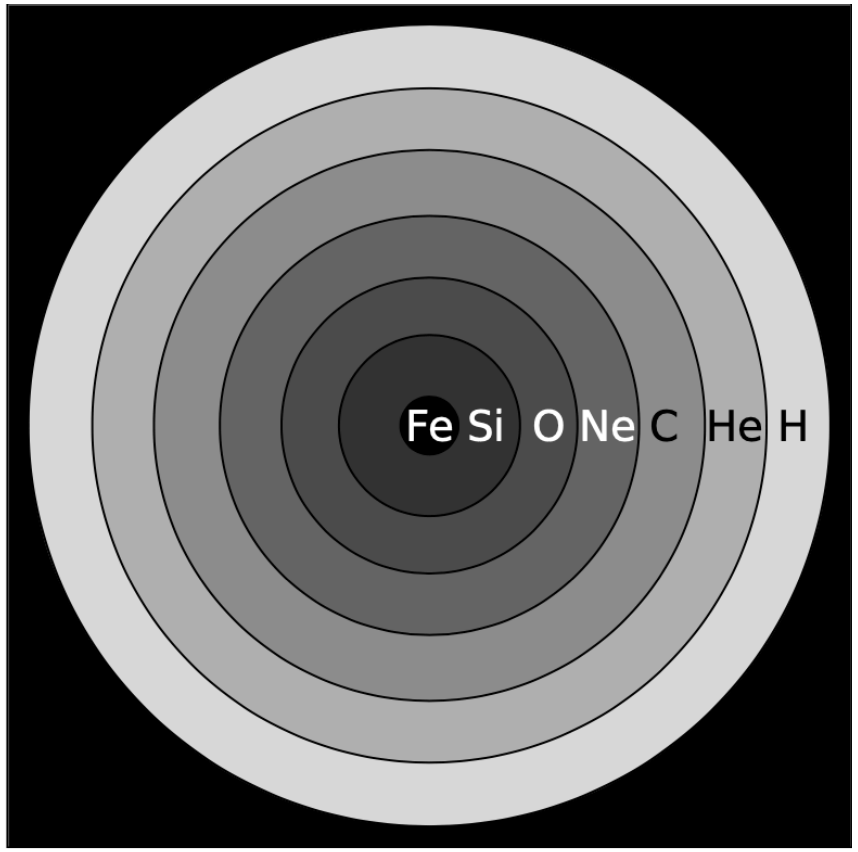
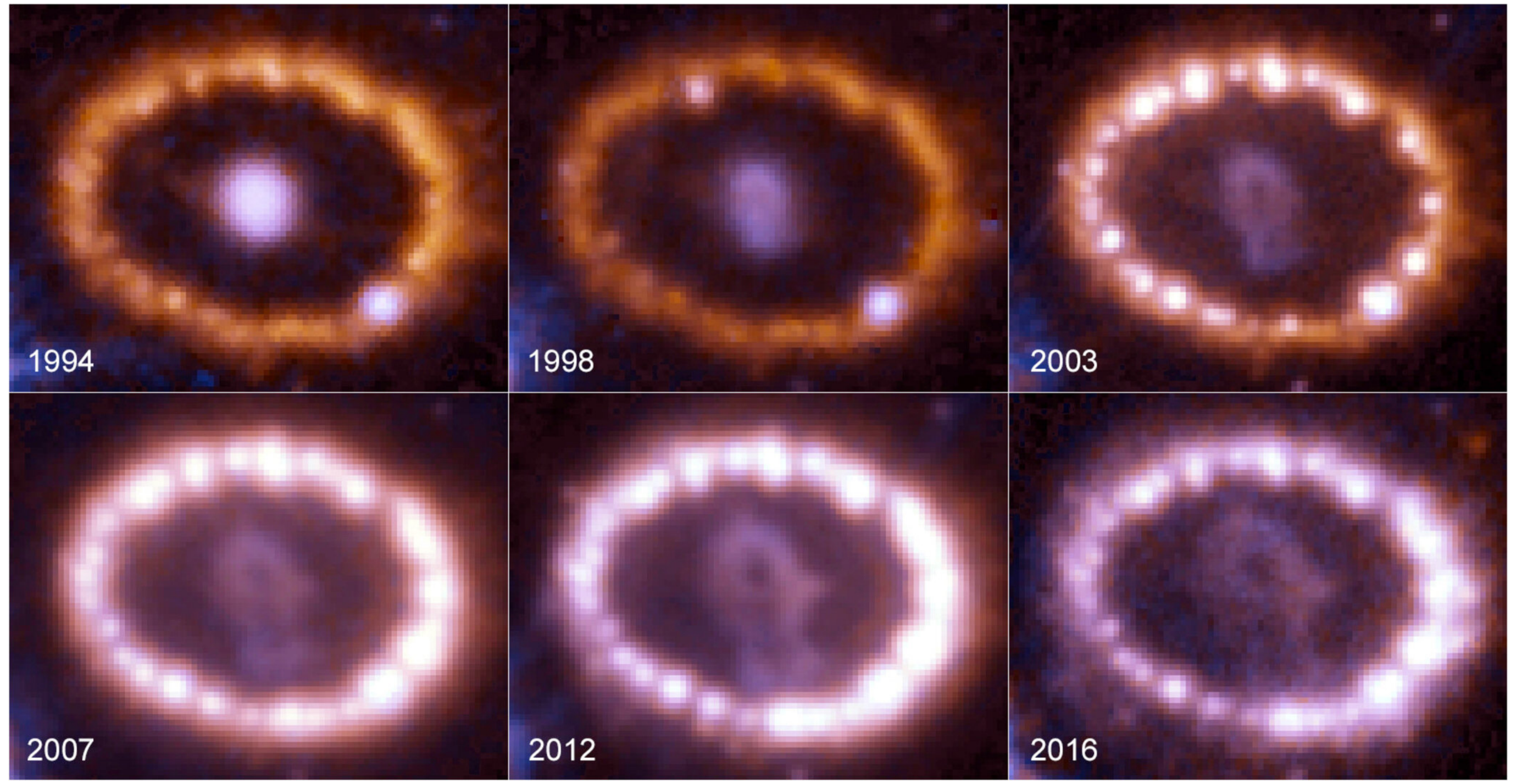
# SN1987




Chandrasekhar limit

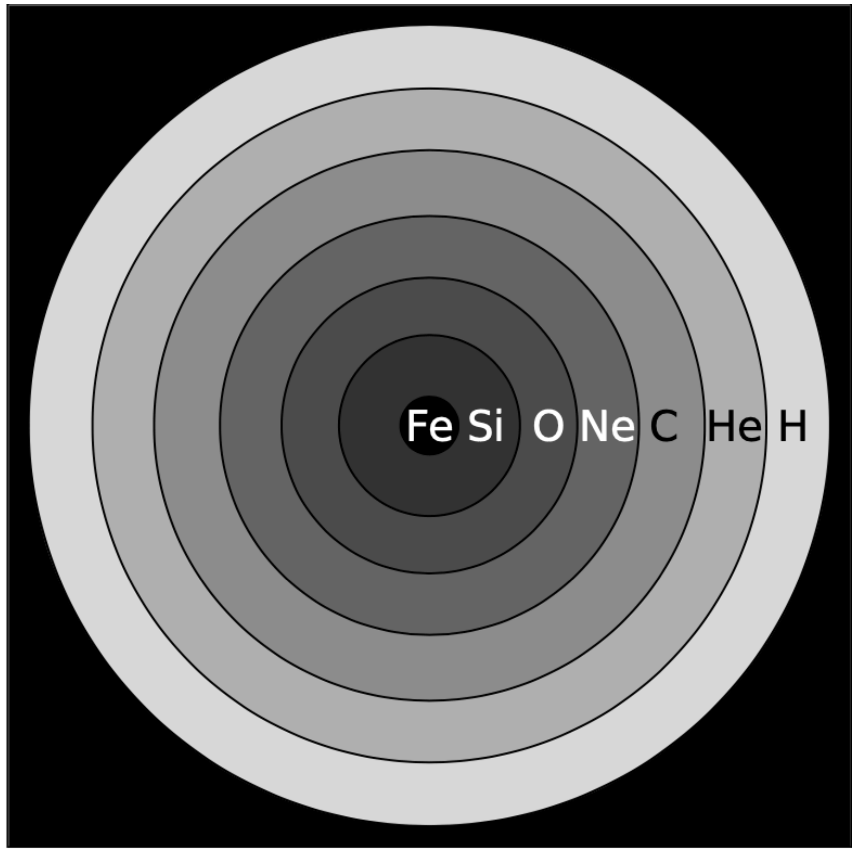
$$M_{\text{core}} > 1.44\, M_{\text{Sun}}$$

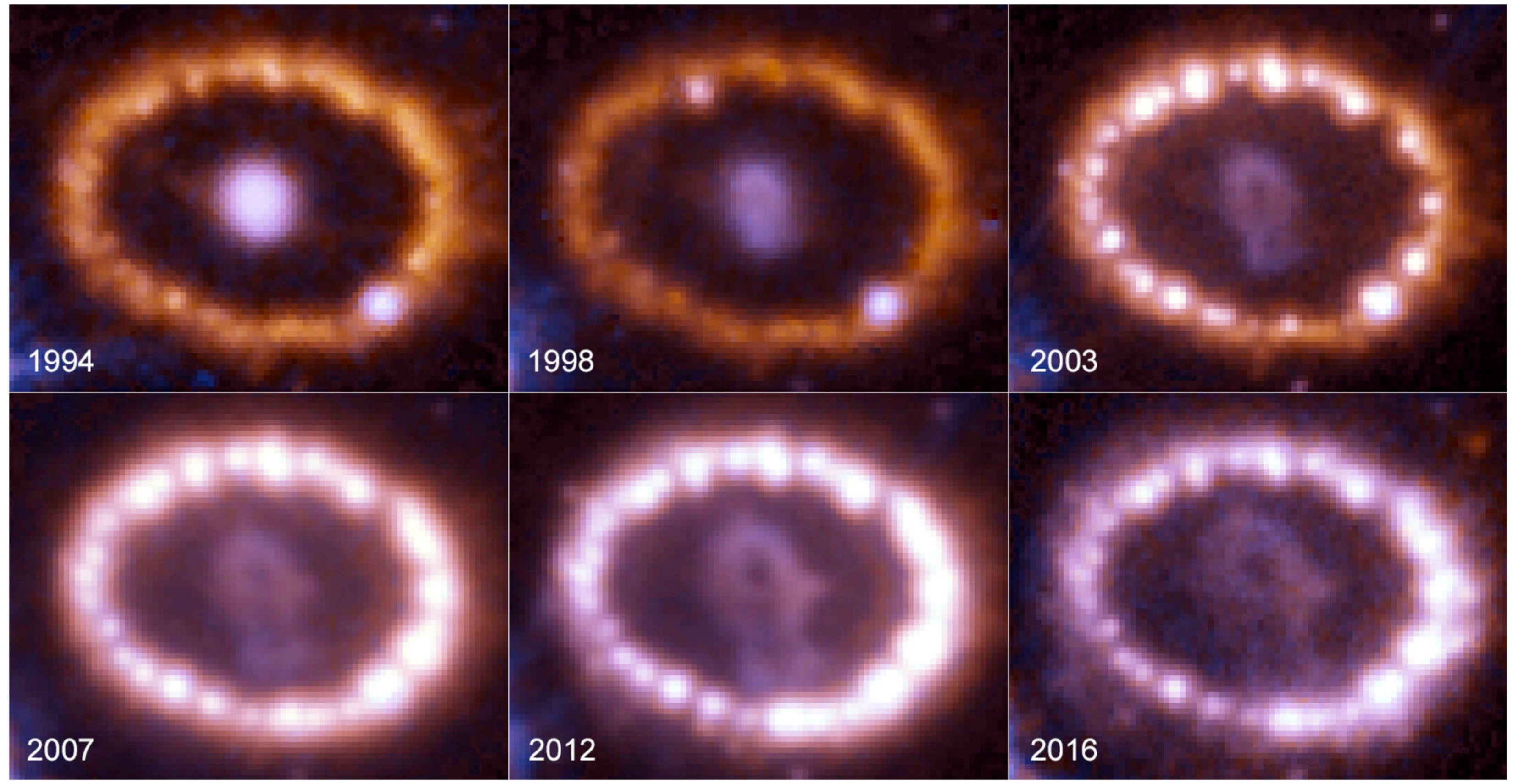



Core-collapse supernova

Binding energy $E \simeq \dfrac{G_N M^2}{R}$ gets released as neutrinos, which are emitted first, and photons.

# SN1987

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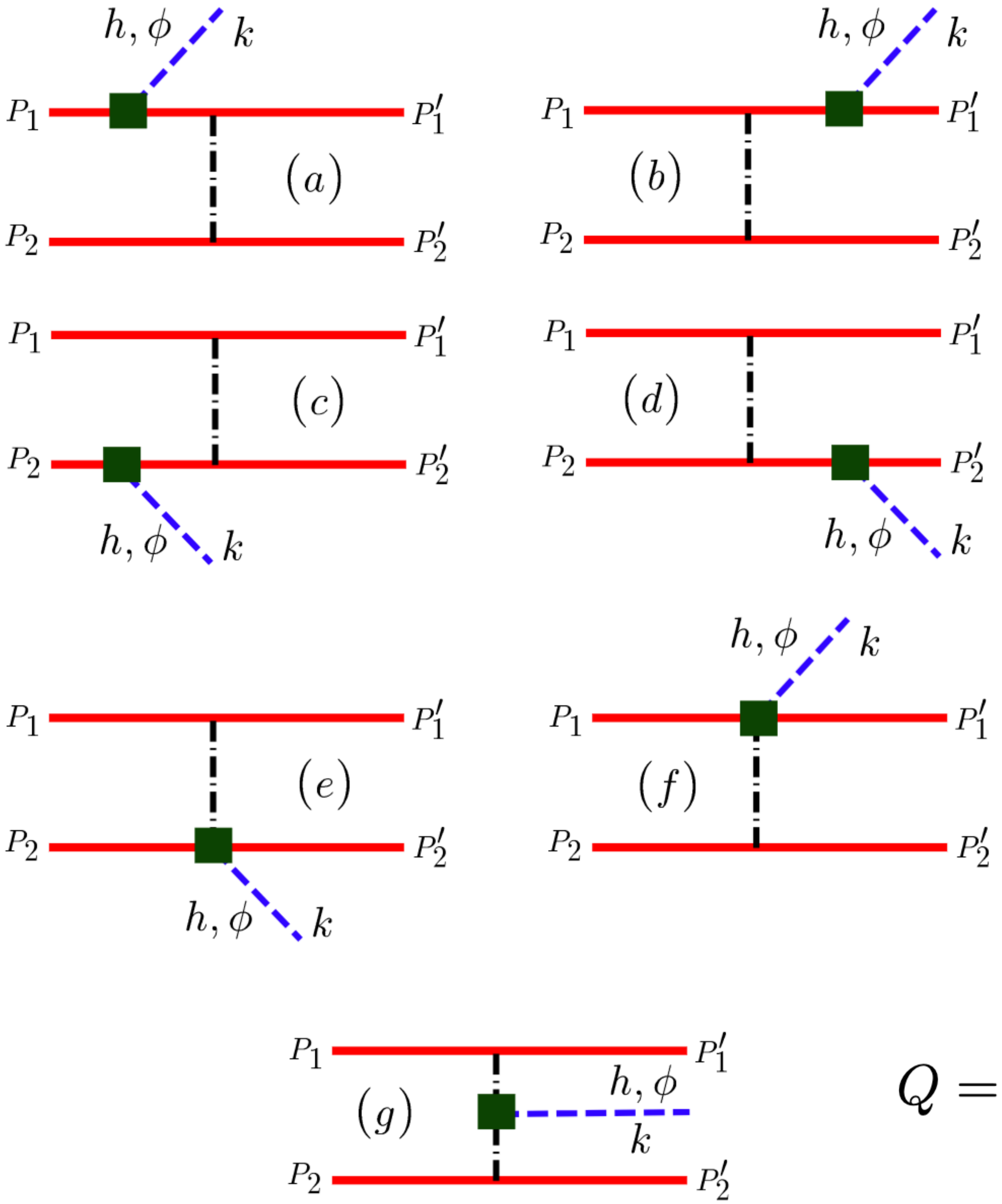
Neutrino luminosity $\mathscr{L}_\nu \simeq 3 \times 10^{53}\,\text{erg/s}$.

Raffelt criterion: $\mathscr{L}_{\text{BSM}} < \mathscr{L}_\nu$.

(assuming KK gravitons are not trapped inside the SN core)

$\lambda_{\text{MFP}} \simeq 1/(n\sigma) < R$.

# SN1987 for LED




$$N + N \to N + N + G_k$$

Energy loss rate for a single graviton

$$Q_m \propto \frac{1}{\Lambda(m)^2} \sigma_N n_B^2 T^{7/2} m_N^{-1/2}$$

$$T = 30\,\text{MeV}, \ \rho = 3\times10^{14}\ \text{g cm}^{-3}, \ \sigma_N = 25\ \text{mb}$$

Energy loss rate for all gravitons

$$S(-\omega) = \frac{1}{\omega^2} \frac{2}{1+e^{\omega/T}} \frac{1024\sqrt{\pi}}{5} \frac{\sigma_N n_B^2 T^{5/2}}{m_N^{1/2}} \frac{1}{\Lambda_n^2}$$

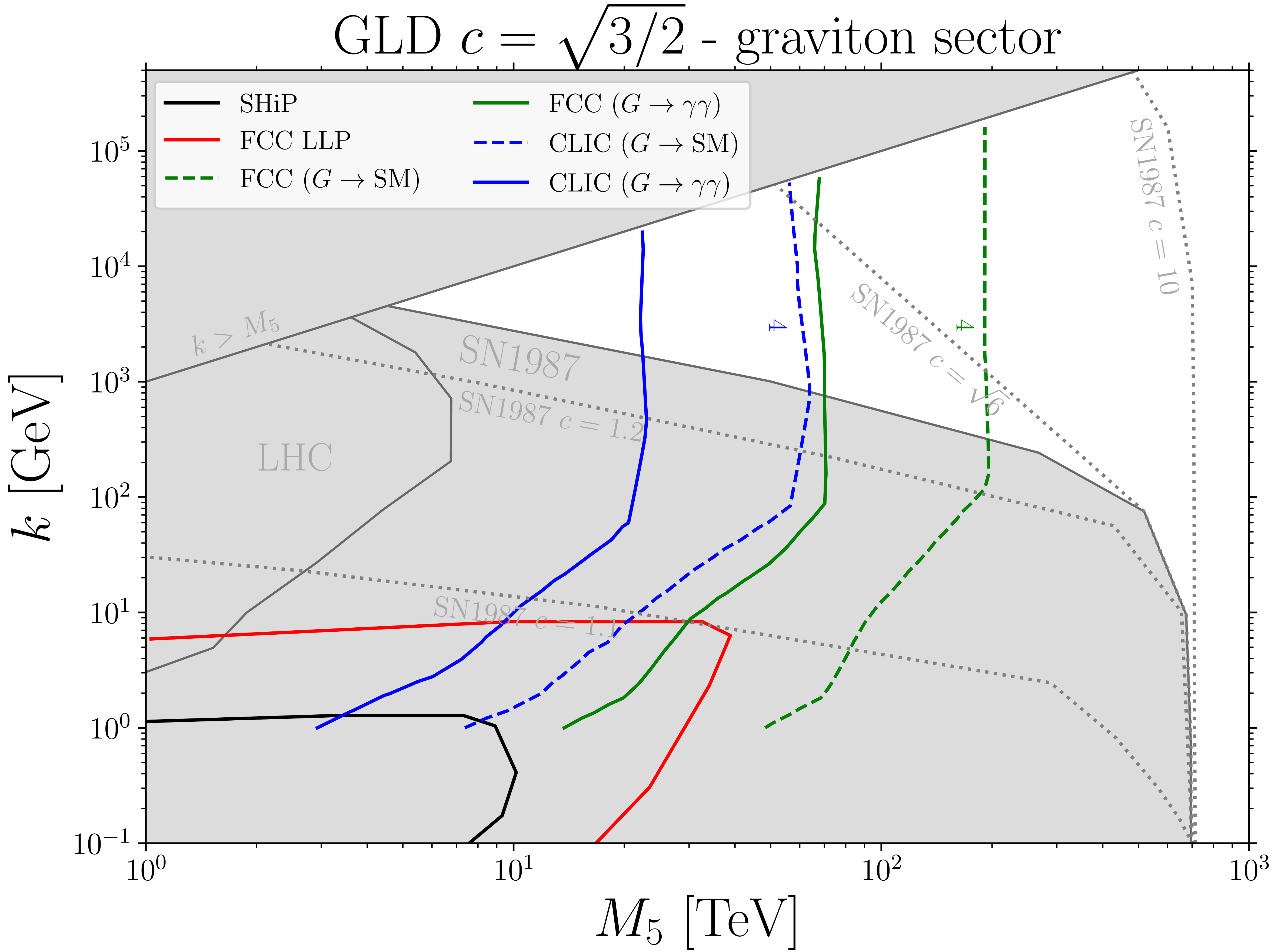
$$Q = \frac{2R}{(2\pi)^2} \int_0^\infty d\omega\, \omega\, S(-\omega) \int_0^\omega dm\, \rho(m) \left( \frac{19}{18} + \frac{11}{9}\frac{m^2}{\omega^2} + \frac{2}{9}\frac{m^4}{\omega^4} \right)$$

$\lambda_{\text{MFP}}$ due to $N + N + G_k \to N + N$ and decays

Hanhart, Phillips, Reddy, Savage 0007016

Hannestad, Raffelt 0103201, 0304029

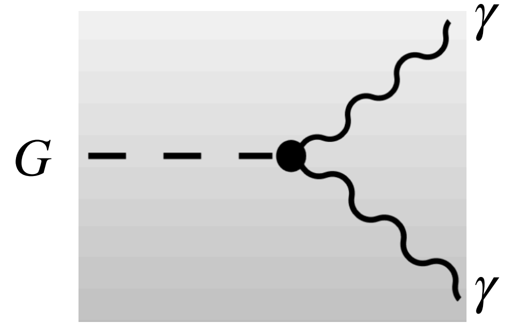
# SN1987 - Clockwork



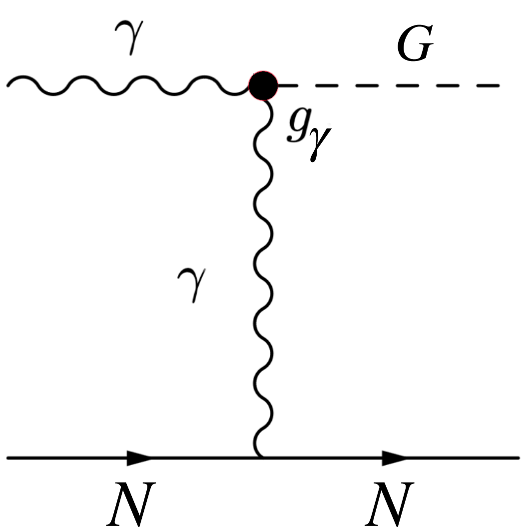
# Massive spin-2 mediator $c\tau\gamma \sim 100\,m$

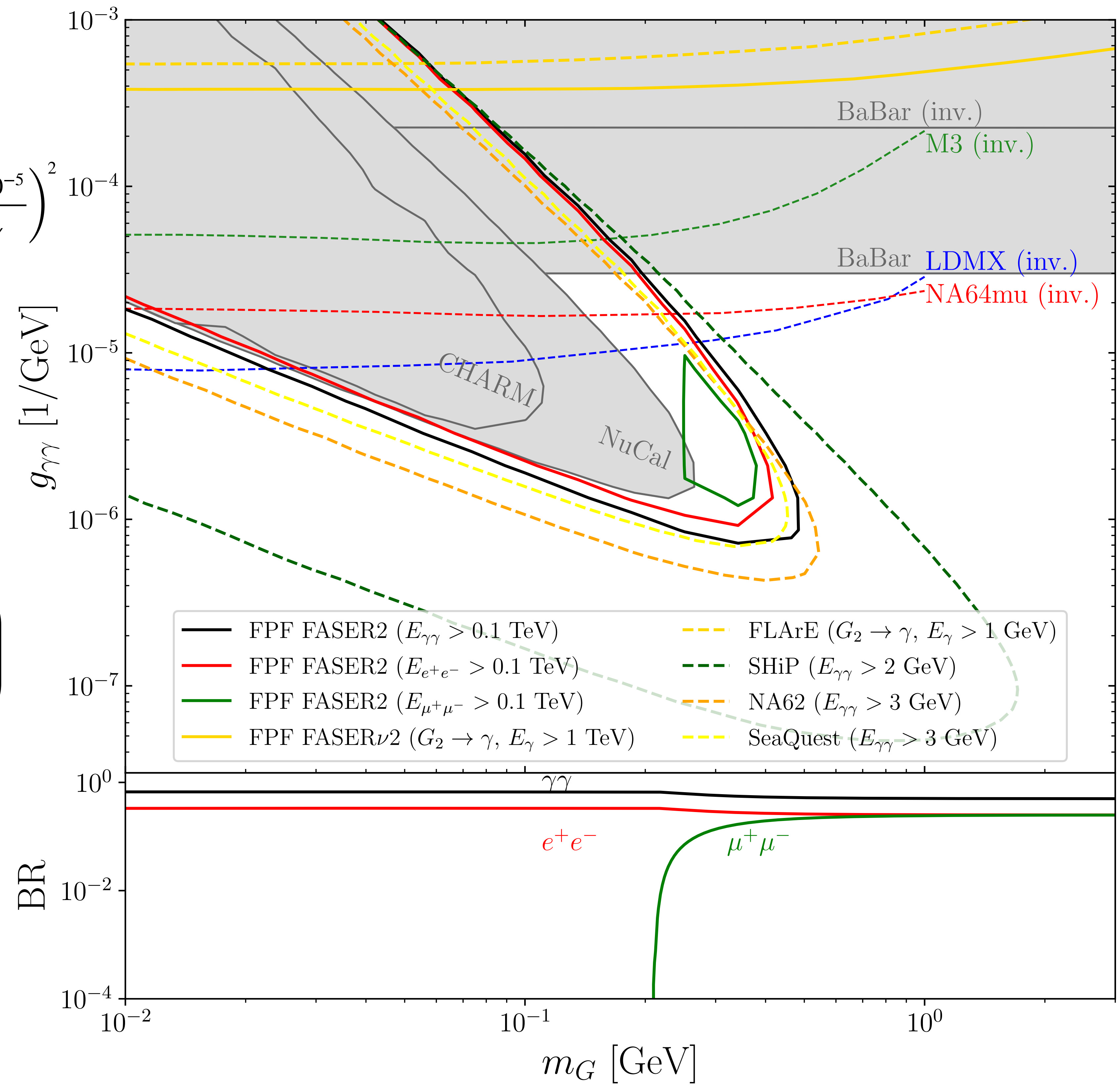
$$\mathscr{L} \supset g_\gamma G^{\mu\nu} T^{\rm EM}_{\mu\nu} + g_\ell G^{\mu\nu} T^{\rm matter}_{\mu\nu}$$

$$d_G = c\tau\beta\gamma \simeq 100\,m \times \left(\frac{E}{1000\,GeV}\right)\left(\frac{0.1\,GeV}{m_G}\right)^4\left(\frac{5\times10^{-5}}{g_{G\gamma\gamma}}\right)^2$$



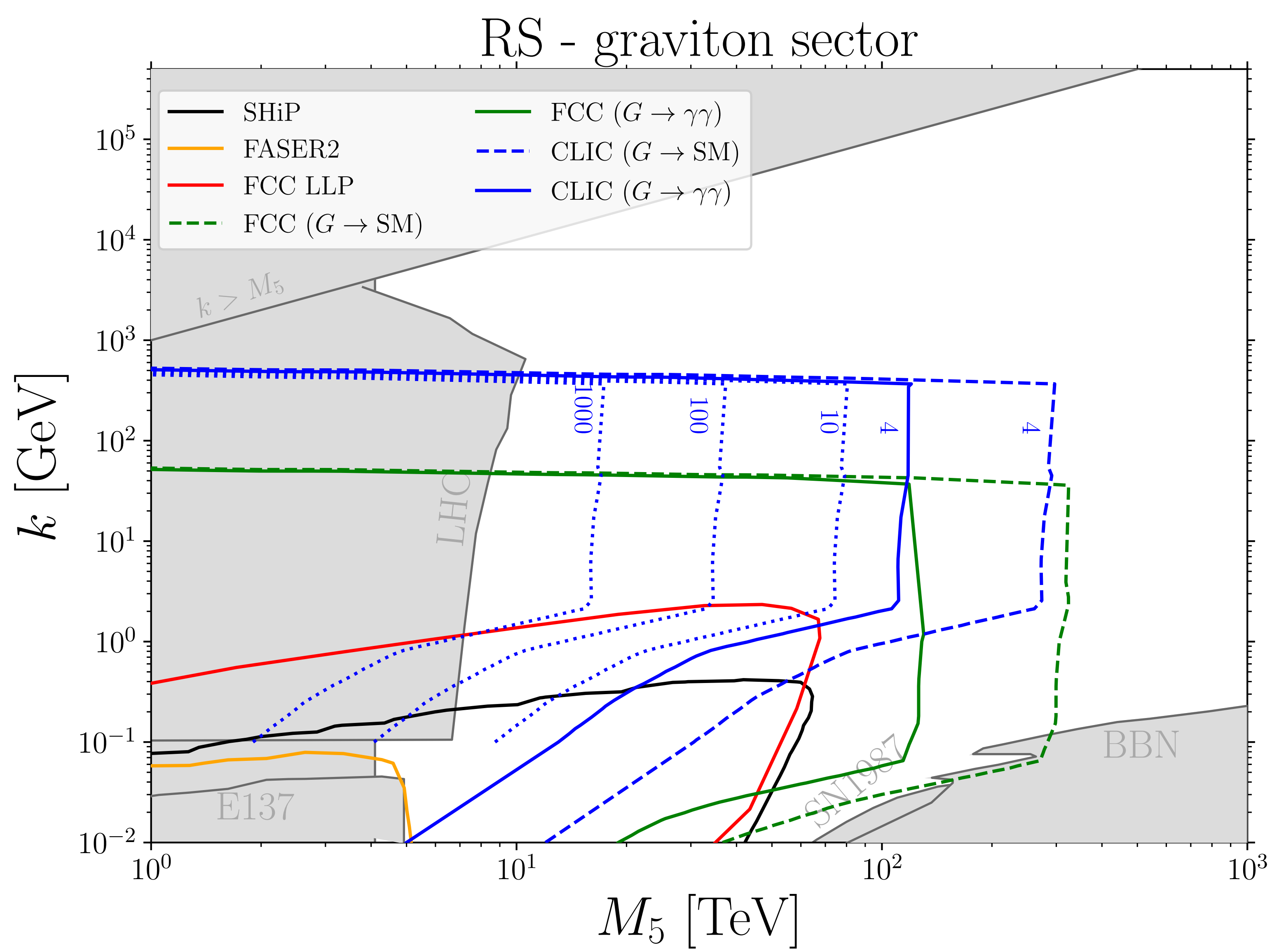

$$\sigma_{\gamma N\to GN} \simeq \frac{\alpha_{em} g^2_{G\gamma\gamma} Z^2}{2}\left(\log\left(\frac{d}{1/a^2 + t_{max}}\right) - 2\right)$$

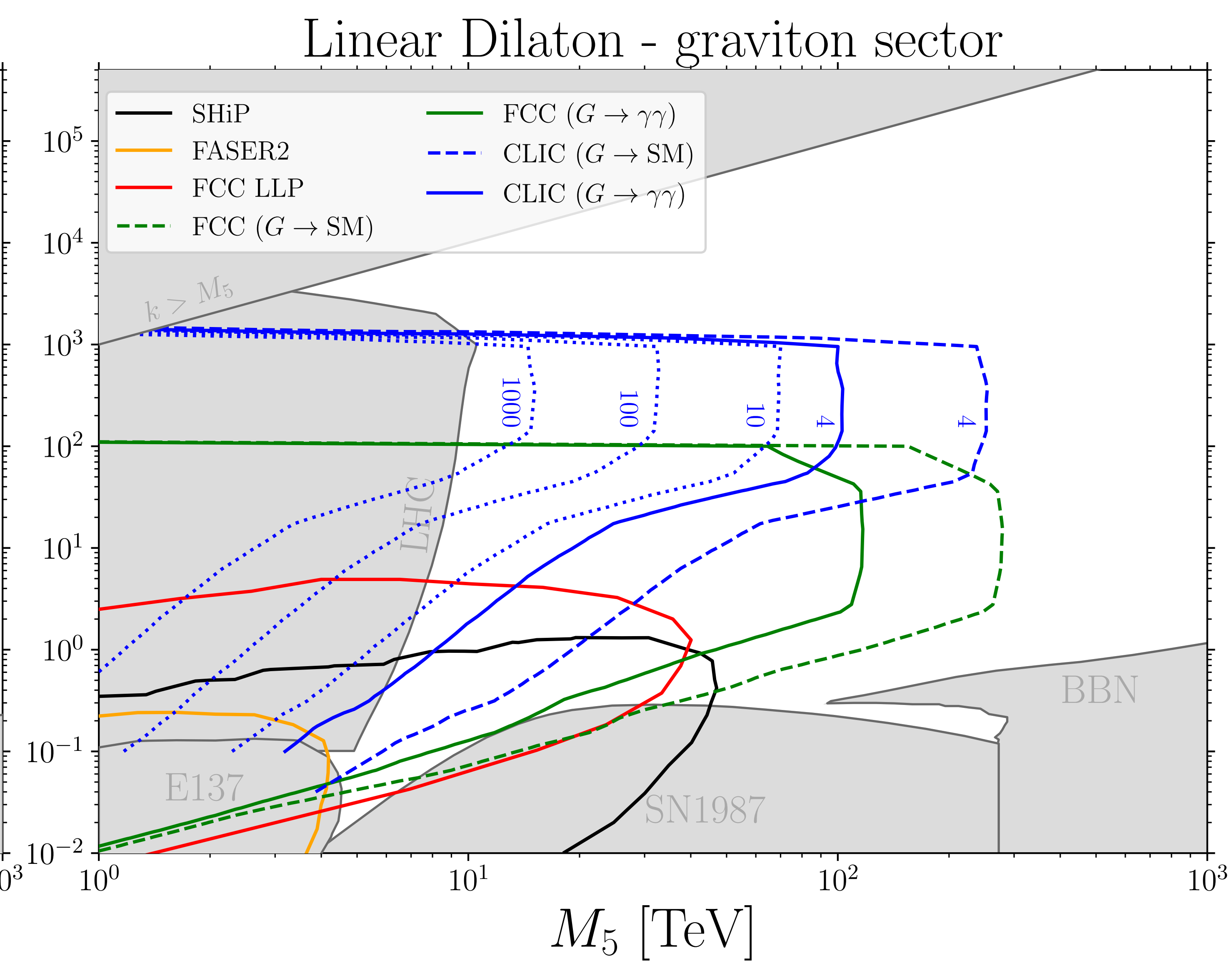



KJ, 2305.05710 (Phys. Rev. D 108, 115017)

# Results $c\tau\gamma \lesssim 1m$

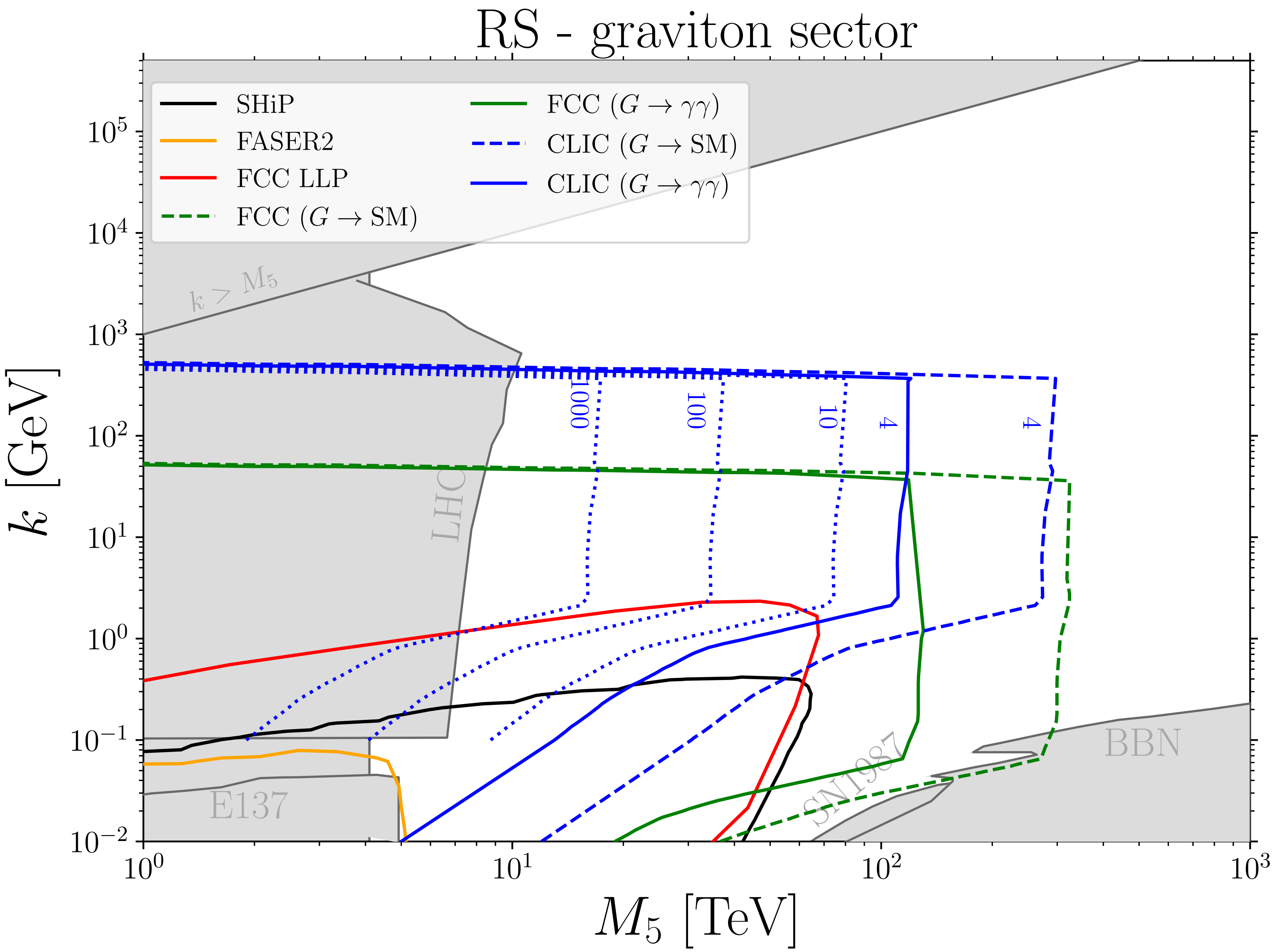


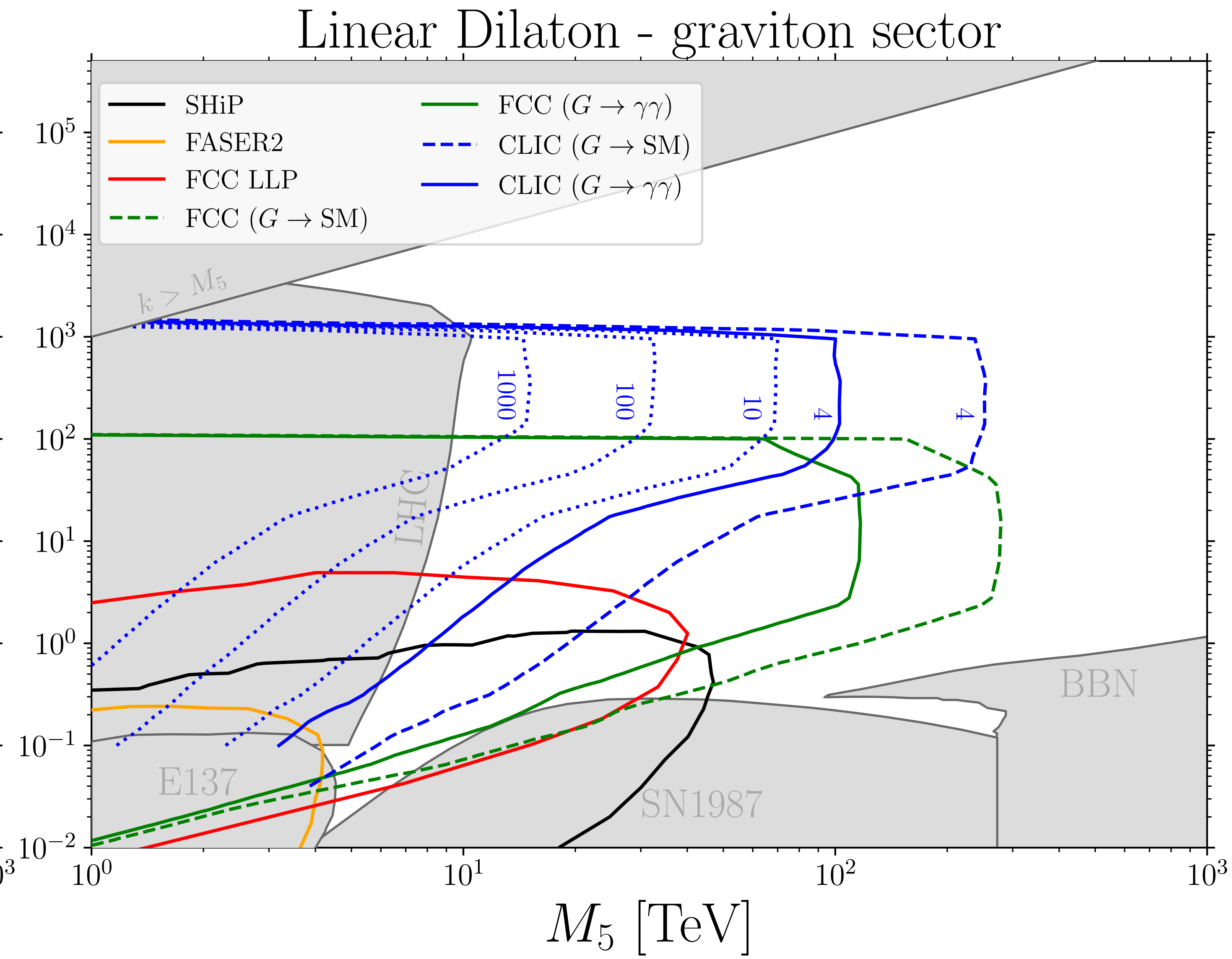




Short-lived regime → FCC-ee, CLIC $e^+e^- \to G\gamma, GZ, G$

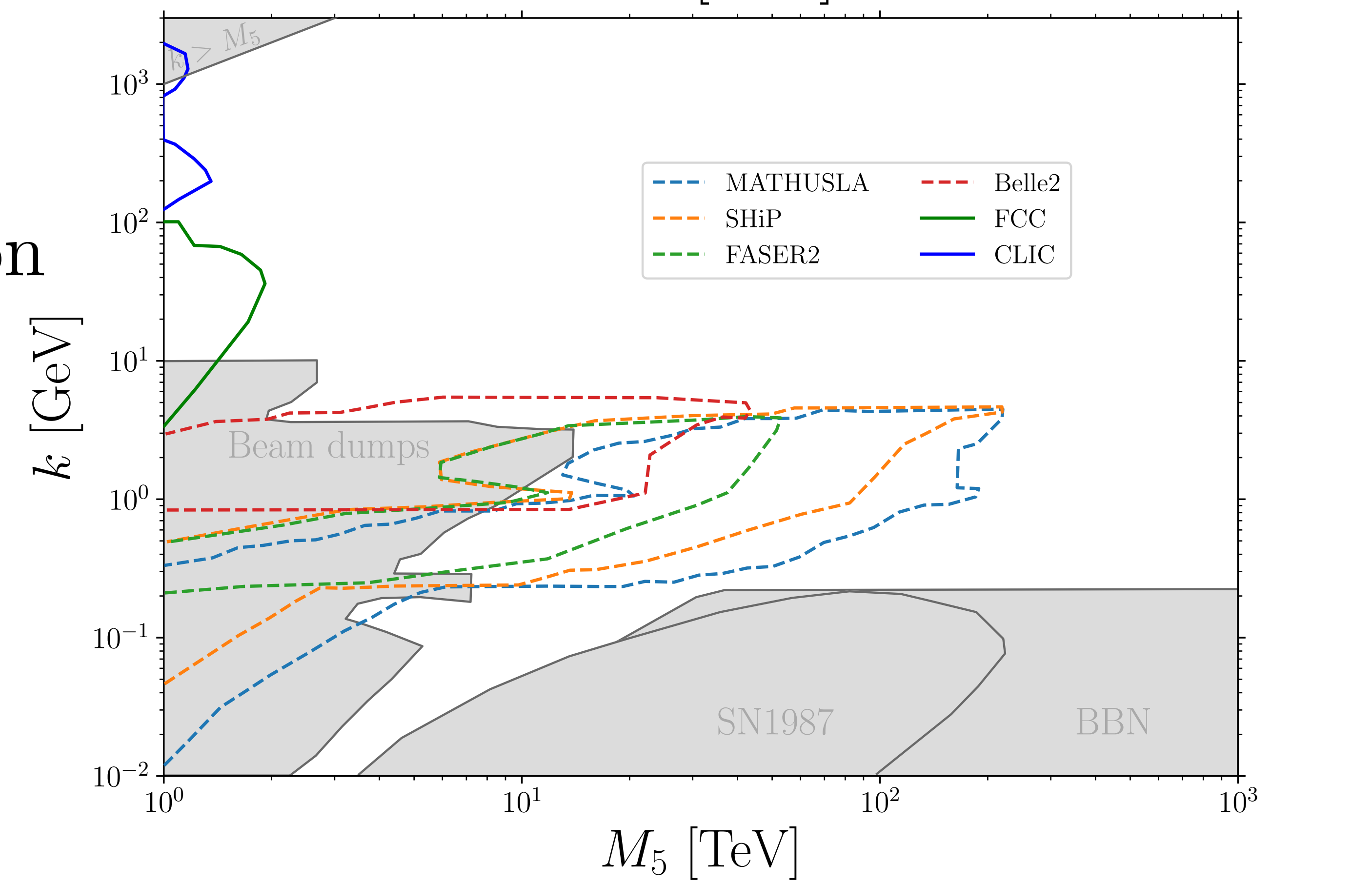
$e^+e^- \to Z,\ \ Z \to b\bar{b}G$

Long-lived regime → KK-gravitons from Primakoff scattering




Radion beam dump → efficient production in $B \to K\phi$ due to top coupling $\propto m_t$

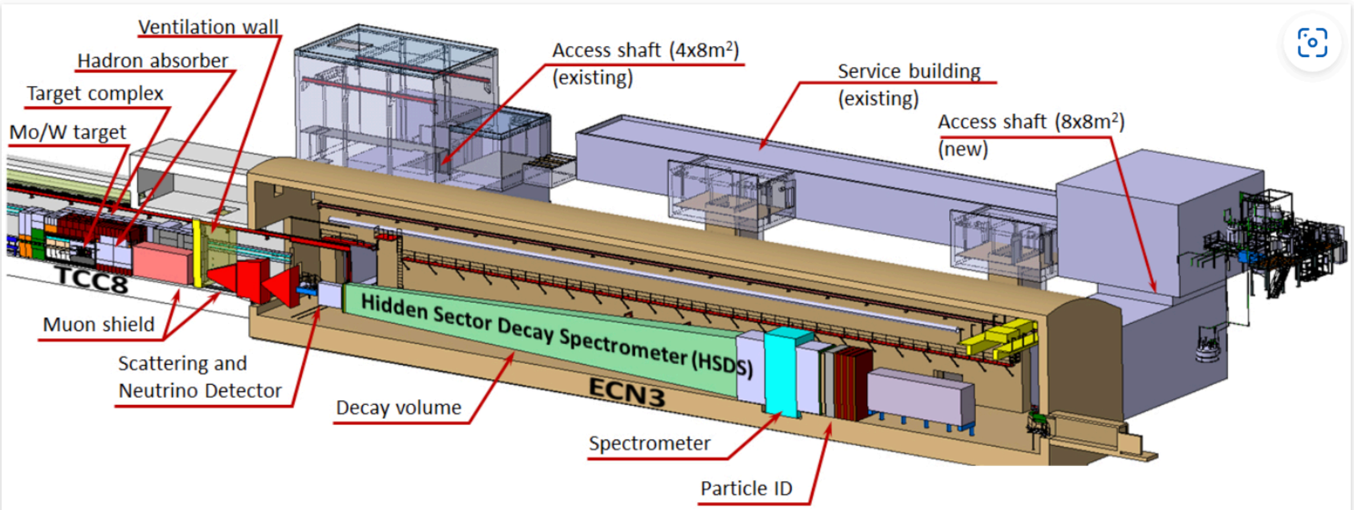


# Timeline

## SHiP sets sail to explore the hidden sector

**The experiment is designed to detect very feebly interacting particles, including candidate dark-matter particles**

19 APRIL, 2024 | By Corinne Pralavorio



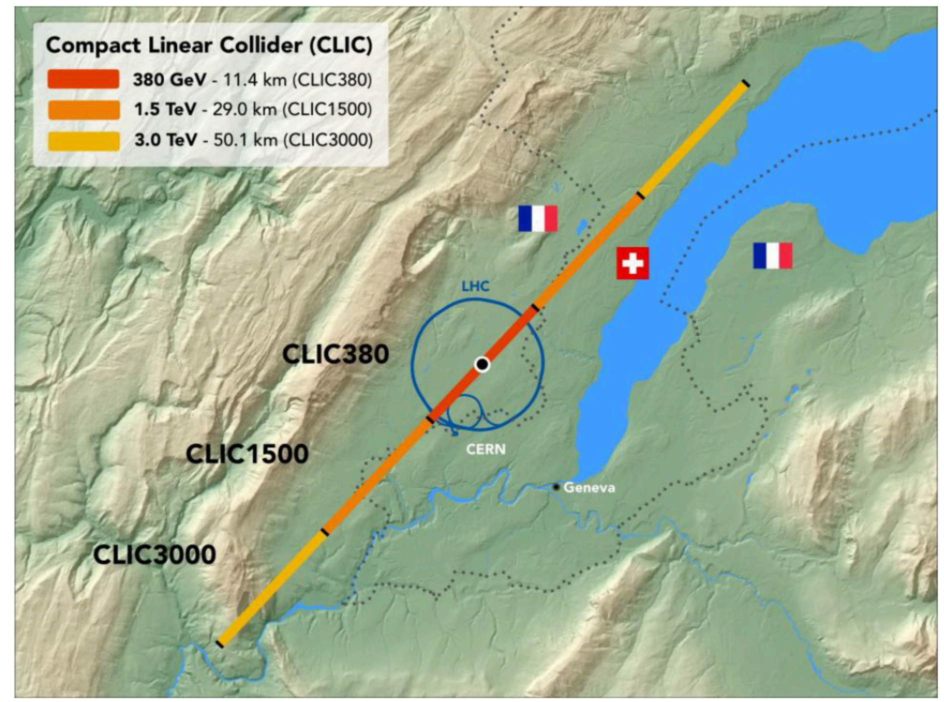

Layout of the SHiP experiment, with the target on the left. (Image : SHiP/CERN)

The SHiP (Search for Hidden Particles) collaboration was in high spirits at its annual meeting this week. Its project to develop a large detector and target to be installed in one of the underground caverns of the accelerator complex has been accepted by the CERN Research Board. Thus, SHiP plans to sail to explore the hidden sector in 2031. Scientists hope to capture particles that interact very feebly with ordinary matter – so feebly, in fact, that they have not yet been detected.

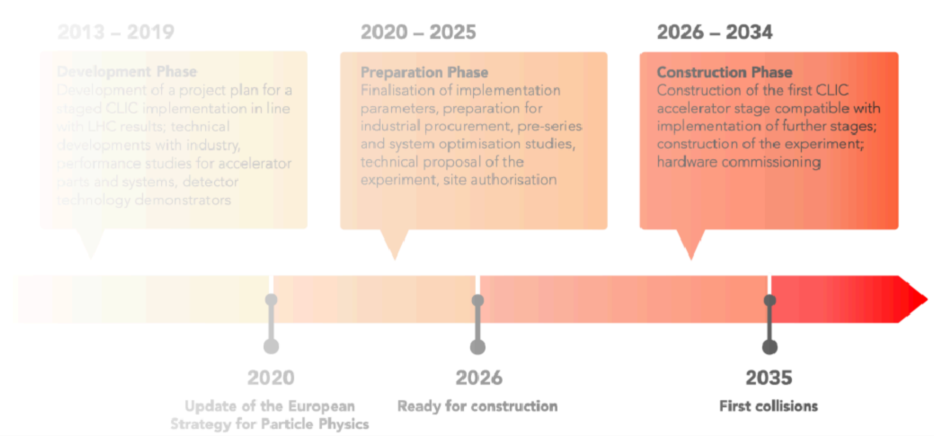
The tentative timeline is:

- **2025**: Completion of the FCC Feasibility Study
- **2027–2028**: Decision by the CERN Member States and international partners
- **2030s**: Start of construction
- **Mid-2040s**: **FCC-ee** begins operation and runs for approximately 15 years
- **2070s: FCC-hh** begins operation and runs for approximately 25 years

FCC-ee




CLIC



# Conclusions

- Clockwork is an interesting mechanism that can solve hierarchy problem. We studied its three benchmarks: RS, LD, and LED-like scenario of GLD.

- We found the sensitivities of the future lepton colliders: FCC and CLIC, which will cover the short-live regime up to $M_5 \sim 200$ TeV complementary to the long-lived regime, which will be also probed by FCC-LLP.

- Low curvature of LD is technically natural (approximate shift symmetry) and leads to light LLPs which will be probed by SHiP and FCC-LLP.

- For the RS with third brane, we updated the prospects of a sub-GeV radion to explain the NANOGrav gravitational wave signal by FOPT.