Constraints on large scalar multiplets added to the Standard Model

Darius Jurčiukonis and Luís Lavoura

Vilnius University, Institute of Theoretical Physics and Astronomy
darius.jurciukonis@tfai.vu.lt
Universidade de Lisboa, Instituto Superior Técnico, CFTP,
balio@cftp.tecnico.ulisboa.pt

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- We consider the SM + a single $SU(2)$ multiplet χ of complex scalar fields.
- We consider the weak isospin J of that multiplet, up to $J = 7/2$.
- We allow the multiplet to have arbitrary weak hypercharge Y.

We have the 4 multiplets, where χ has $n = 2J + 1$ components χ_I (I is the third component of isospin)

$$H = \begin{pmatrix} a \\ b \end{pmatrix}, \quad \tilde{H} = \begin{pmatrix} b^* \\ -a^* \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi_J \\ \vdots \\ \chi_{-J} \end{pmatrix}, \quad \tilde{\chi} = \begin{pmatrix} \chi^*_{-J} \\ \vdots \\ (-1)^{2J}\, \chi^*_J \end{pmatrix}.$$

Then, the potential $V = V_2 + V_4$ with $V_2 = -\mu_1^2 F_1 + \mu_2^2 F_2$ and

$$V_4 = \frac{\lambda_1}{2} F_1^2 + \lambda_3 F_1 F_2 + \lambda_4 F_4 + \text{terms four-linear in the } \chi_I,$$

where invariants are defined following

$$F_1 \equiv \left(H \otimes \tilde{H}\right)_{\mathbf{1}} = |a|^2 + |b|^2, \quad F_2 \equiv (\chi \otimes \tilde{\chi})_{\mathbf{1}} = \sum_{I=-J}^{J} |\chi_I|^2$$

$$F_4 \equiv \frac{|a|^2 - |b|^2}{2} \sum_{I=-J}^{J} I\, |\chi_I|^2 + \frac{z + z^*}{2}, \quad z \equiv a b^* \sum_{I=1-J}^{J} \chi_I^*\, \chi_{I-1} \sqrt{J^2 - I^2 + J + I}.$$

Masses

From the previous equations the mass-squared of the scalar χ_I is

$$m_I^2 = \mu_2^2 + \left(\lambda_3 - \frac{\lambda_4}{2}\, I\right) |v|^2 .$$

This implies that the difference between the masses-squared of χ_I and χ_{I+1} is

$$\Delta m^2 = \frac{\left|\lambda_4 v^2\right|}{2},$$

which is I-independent. An upper bound on $|\lambda_4|$ is therefore equivalent to an upper bound on Δm^2.

H has VEV $v \approx 174\,\text{GeV}$ and χ has no VEV. The VEV of V is

$$\langle 0 \,|V|\, 0 \rangle = -\mu_1^2 v^2 + \frac{\lambda_1}{2}\, v^4 .$$

The mass-squared of the Higgs particle is $m_H^2 = 2\lambda_1 v^2$. Since experimentally $m_H \approx 125\,\text{GeV}$, one has $\lambda_1 \approx 0.258$.

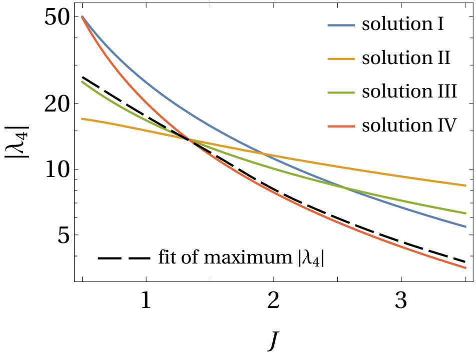
To compute UNI conditions, we consider the scattering of a pair of scalars of χ to another pair of scalars, both pairs having the same I and Y.

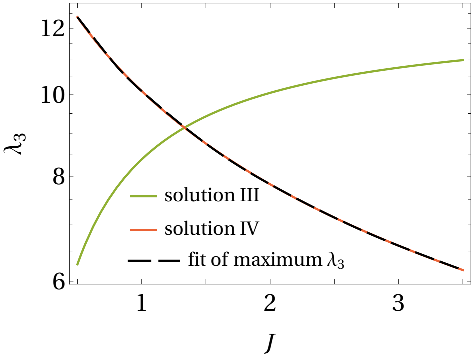
UNI

$$\begin{aligned} |\lambda_3| + \frac{J+1}{2}|\lambda_4| &< 8\pi, \\ 3|\lambda_1| + \sqrt{9\lambda_1^2 + 8(2J+1)\lambda_3^2} &< 16\pi, \\ |\lambda_1| + \sqrt{\lambda_1^2 + \frac{2}{3}J(J+1)(2J+1)\lambda_4^2} &< 16\pi. \end{aligned}$$

BFB

$$\begin{aligned} \lambda_1 &\geq 0, \\ \lambda_3 &\geq 0, \\ |\lambda_4| &\leq \frac{2\lambda_3}{J}. \end{aligned}$$





Full potential

The product $\chi \otimes \chi$ of two identical multiplets of $SU(2)$ only has a symmetric component—the anti-symmetric component vanishes because the two multiplets are equal.

$$(\chi \otimes \chi)_{\text{symmetric}} = \begin{pmatrix} c_{2J} \\ c_{2J-1} \\ \vdots \\ c_{-2J} \end{pmatrix} \oplus \begin{pmatrix} d_{2J-2} \\ d_{2J-3} \\ \vdots \\ d_{2-2J} \end{pmatrix} \oplus \begin{pmatrix} e_{2J-4} \\ e_{2J-5} \\ \vdots \\ e_{4-2J} \end{pmatrix} \oplus \cdots$$

The two-field states in each multiplet in the right-hand side of equation above are evaluated by using Clebsch–Gordan coefficients in the standard fashion. Then the invariants

$$F_5 \equiv \sum_{I=2-2J}^{2J-2} |d_I|^2 , \quad F_6 \equiv \sum_{I=4-2J}^{2J-4} |e_I|^2 , \quad \cdots , \quad F_{t+3} \equiv \begin{cases} |q_1|^2 + |q_0|^2 + |q_{-1}|^2 & \Leftarrow \quad J \in \mathbb{Q}, \\ |q_0|^2 . & \Leftarrow \quad J \in \mathbb{Z}. \end{cases}$$

The quartic part of the scalar potential thus is quartic χ terms in red

$$V_4 = \frac{\lambda_1}{2} F_1^2 + \frac{\lambda_2}{2} F_2^2 + \lambda_3 F_1 F_2 + \lambda_4 F_4 + \sum_{i=5}^{t+3} \lambda_i F_i, \quad \text{where} \quad t = \operatorname{ceil}(n/2) .$$

UNI

$$\begin{aligned} |\lambda_1| &< 8\pi, \\ |\lambda_2| &< 8\pi, \\ |\lambda_2 + 2\lambda_i| &< 8\pi \quad (i = 5, \ldots, t+3). \end{aligned}$$

$$|\text{eignval}(\mathcal{S})| < 8\pi, \quad \mathcal{S} = \begin{pmatrix} 2\lambda_1 & \lambda_1 & \Sigma_1 \\ \lambda_1 & 2\lambda_1 & \Sigma_2 \\ \Sigma_1^T & \Sigma_2^T & \Lambda \end{pmatrix},$$

where the $1 \times n$ and $n \times n$ submatrices are given by

$$\begin{aligned} (\Sigma_1)_{1k} &= \lambda_3 + \frac{\lambda_4}{2}\,(J + 1 - k)\,, \\ (\Sigma_2)_{1k} &= \lambda_3 - \frac{\lambda_4}{2}\,(J + 1 - k)\,, \\ \Lambda_{kl} &= \lambda_2\,(1 + \delta_{kl}) + 4\sum_{i=5}^{t+3}\lambda_i \sum_m \text{CG}^{i,J}_{k,l,m}. \end{aligned}$$

BFB

necessary BFB

$$\begin{aligned} \lambda_1 &\geq 0, \quad \lambda_2 \geq 0, \\ \widehat{\lambda}_i &\geq 0, \quad \widehat{\lambda}_i \equiv \lambda_2 + q_i, \quad q_i \equiv \frac{2J^2}{\kappa_i}\,\lambda_i \\ \lambda_3 &\geq -\sqrt{\lambda_1\lambda_2}, \quad \kappa_i = (i-4)\,(4J + 9 - 2i)\,, \\ |\lambda_4| &\leq \frac{2}{J}\left(\lambda_3 + \sqrt{\lambda_1\lambda_2}\right). \end{aligned}$$

sufficient BFB

$$\text{either} \quad \lambda_i > 0, \quad \text{or} \quad \lambda_i\Lambda_i < 0,$$

$$\text{or} \quad \sqrt{\frac{\widehat{\lambda}_i}{q_i\,\Lambda_i}} > \frac{2}{J\,|\lambda_4|}, \quad \text{or} \quad \lambda_3 \geq -\sqrt{\frac{\widehat{\lambda}_i\,\Lambda_i}{q_i}},$$

respectively, where $\Lambda_i \equiv \frac{J^2}{4}\,\lambda_4^2 + q_i\lambda_1$.

Results

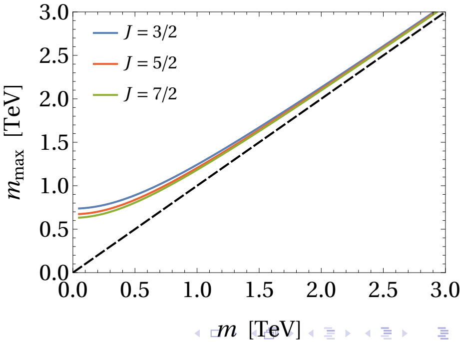
J	**1/2**	**1**	**3/2**	**2**	**5/2**	**3**	**7/2**
maximum $\lvert\lambda_4\rvert$	26.46	17.49	11.96	8.10	5.97	4.65	3.76
maximum λ_3	12.37	10.10	8.75	7.82	7.14	6.61	6.19
minimum λ_3	−1.46	−1.26	−1.13	−1.03	−0.95	−0.89	−0.84

Table: The maximum allowed value of $|\lambda_4|$, and the maximum and minimum allowed values of λ_3, for various values of J.

The maximum possible mass of a multiplet of scalars as a function of its minimum mass m.

$$m_{\max} = \sqrt{m^2 + Jv^2 \left|\lambda_4\right|_{\mathrm{maximal}}}.$$

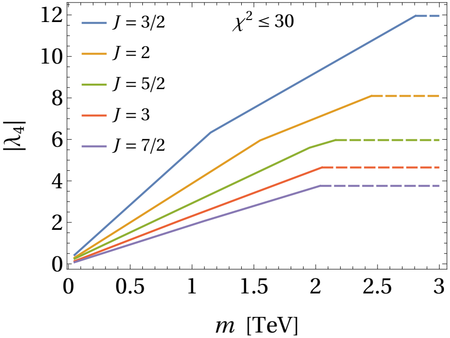
One sees that heavy scalar multiplets tend to be almost degenerate; for $m \gtrsim 2$ TeV, $m_{\max} - m \sim 100$ GeV.

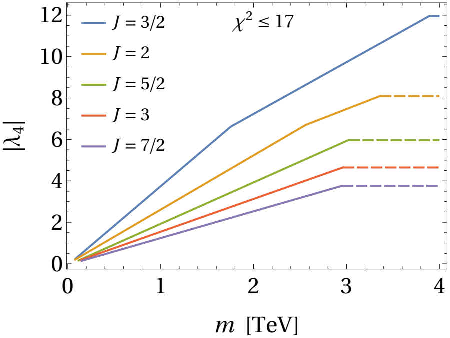


Numerical results

- If the new scalars are very heavy, and they cannot be produced at the LHC, then they will make themselves felt only indirectly through their oblique corrections.
- We parameterize those corrections through six oblique parameters (OPs) S, T, U, V, W, and X. An electroweak observable O obeys

$$\frac{O_{\mathrm{NP}}}{O_{\mathrm{SM}}} = 1 + c_S^O S + c_T^O T + c_U^O U + c_V^O V + c_W^O W + c_X^O X$$





Numerical results

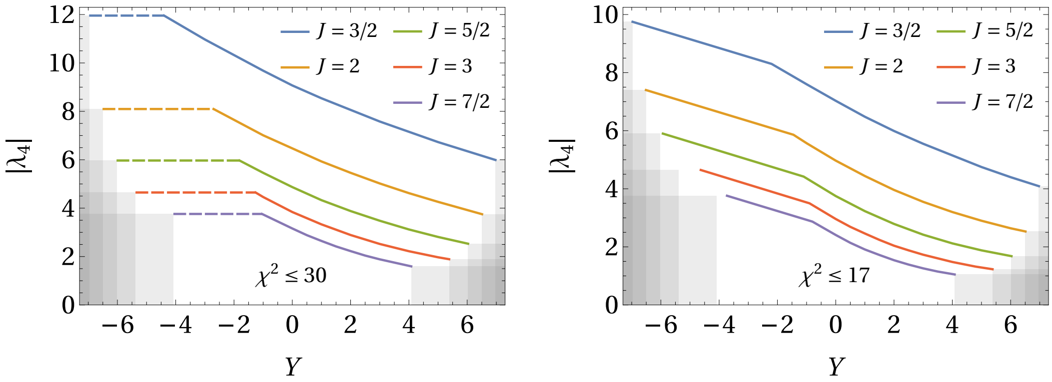


Figure: The upper bound on $|\lambda_4|$ *versus* the hypercharge Y, for various values of J, for $m = 3$ TeV, and for fits with $\chi^2 \leq 30$ (left panel) or $\chi^2 \leq 17$ (right panel). The horizontal dashed lines indicate the upper bounds from the UNI and BFB conditions, and the curved lines indicate the upper bounds from the OPs. The gray bands indicate the J-dependent restrictions on Y derived in arXiv:2403.12914.

RGEs

- In order to derive the one-loop RGEs we have used package SARAH.
- The dimensionless couplings that we take into account are g_1, g_2, g_3, y_t, and λ_i.
- The SARAH model files and output files, and the expressions of the RGEs are available at https://github.com/jurciukonis/RGEs-for-multiplets.

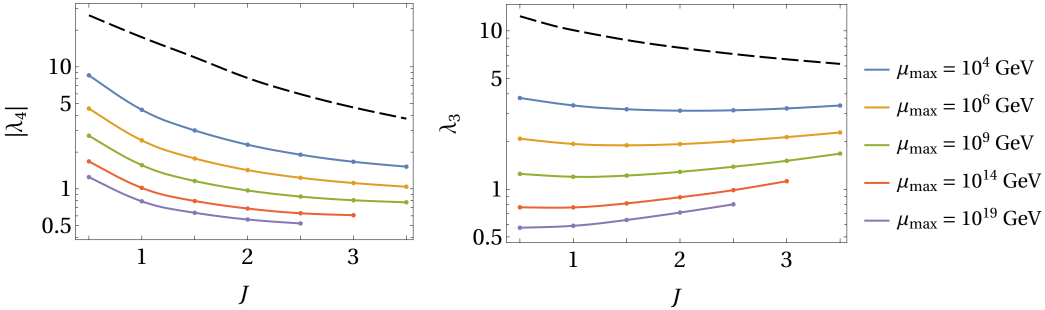


Figure: The maximum allowed values of $|\lambda_4|$ (left panel) and λ_3 (right panel) *versus* J for different cut-off scales $\mu_{\max}$.

SM + quadruplet

We study the scalar potential of the extension of the SM through a scalar quadruplet with hypercharge either $Y = 1/2$ or $Y = 3/2$ [arXiv:2406.01628].
In those cases there are extra couplings in the scalar potential. For instance, if $Y = 1/2$,

$$V_4 = \frac{\lambda_1}{2} F_1^2 + \frac{\lambda_2}{2} F_2^2 + \lambda_3 F_1 F_2 + \lambda_4 F_4 + \lambda_5 F_5 + \left(\sum_{p=6}^{8} \frac{\lambda_p}{2} \mathcal{F}_p + \mathrm{H.c.} \right).$$

The gauge-invariant quantities are generated by multiplying each triplet by the complex conjugate of another triplet

$$\begin{aligned}
\mathcal{F}_6 &= (\chi \otimes H)^\dagger_{\mathbf{3}} (H \otimes H)_{\mathbf{3}} , \\
\mathcal{F}_7 &= (\chi \otimes H)^\dagger_{\mathbf{3}} (\chi \otimes \chi)_{\mathbf{3}} , \\
\mathcal{F}_8 &= (H \otimes H)^\dagger_{\mathbf{3}} (\chi \otimes \chi)_{\mathbf{3}} .
\end{aligned}$$

UNI

There are only two scattering channels to be considered:

- the channel of two-field states with $I = Y = 1$,
- the channel of two-field states with $I = Y = 0$.

All the other channels yield UNI conditions that just reproduce some of those that we derive through these two channels.
The moduli of the eigenvalues of the scattering matrices must be smaller than $M = 8\pi$.

BFB

- We assume that the coefficients $\lambda_{6\text{–}8}$ are *real*,
- we define three vectors $\vec{v}_{1,2,3}$ in $\mathbb{R}^6$ expressed by the scalar fields of the Higgs doublet and quadruplet.
- then $V_4 = \cdots + \lambda_6\, \vec{v}_1 \cdot \vec{v}_2 + \lambda_7\, \vec{v}_2 \cdot \vec{v}_3 + \lambda_8\, \vec{v}_1 \cdot \vec{v}_3$.

Recipe for BFB

- We firstly minimize V_4 relative to the angles among the six-vectors $\vec{v}_{1,2,3}$,
- we minimize V_4 relative to $F_{1,2,4,5}$ following previous scheme and using conditions for positivity of quartic polynomials,
- then we perform numerical scan over domain of two parameters given by the straight line $y = 0$ and by the parabola $y = (9/5)\, x\, (1 - x)$ for $x \in [0, 1]$.

Results

The other authors use different definitions, for instance

$$\lambda_1 = 2\lambda_H, \quad \lambda_6 = \frac{2}{\sqrt{3}}\,\hat{\lambda}_{H\phi}.$$

Motivation for further analysis

- Heavy particles can indirectly influence the couplings of the Higgs boson, especially its self-couplings.
- The UNI and BFB conditions can strongly constrain the Higgs couplings [arXiv:2311.17995].
- We are working on computing the Higgs cubic and quartic couplings within the framework of this model.
- Work in progress...

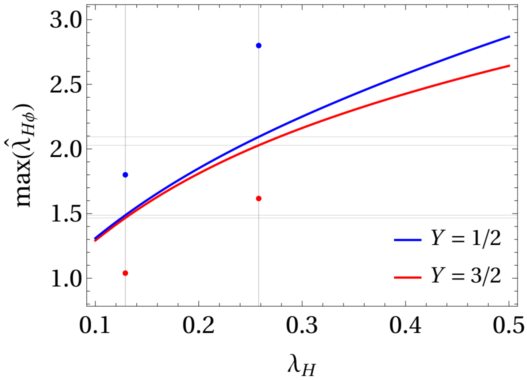


Figure: Upper bounds on $\hat{\lambda}_{H\phi}$ as functions of λ_H. The blue and red dots represent the upper bounds given by other authors for the cases $Y = 1/2$ and $Y = 3/2$, respectively. The lines depict the upper bounds that we have computed from both our UNI and BFB conditions.

Conclusions

- We have found that, upper bound on Δm^2 depends crucially, not just on the UNI conditions, but also on the BFB ones.
- We have been able to derive *necessary and sufficient* BFB conditions for this model, allowing for the most general terms in the scalar potential.
- Our study can be understood as a step towards the understanding of more specific models that will have specific values of J and Y.
- Analysis on SM + quadruplet for $Y = 1/2$ shows that inclusion of all terms in the scalar potential leads to stronger restrictions on the model parameters.