

Halo-independent bounds of WIMP-nucleon couplings from direct detection and neutrino observations

Sunghyun Kang

Based on

JCAP03(2023)011(arXiv: 2212.05774) & JCAP11(2023)077(arXiv: 2308.13203)

in collaboration with Stefano Scopel and Arpan Kar

2024 PASCOS



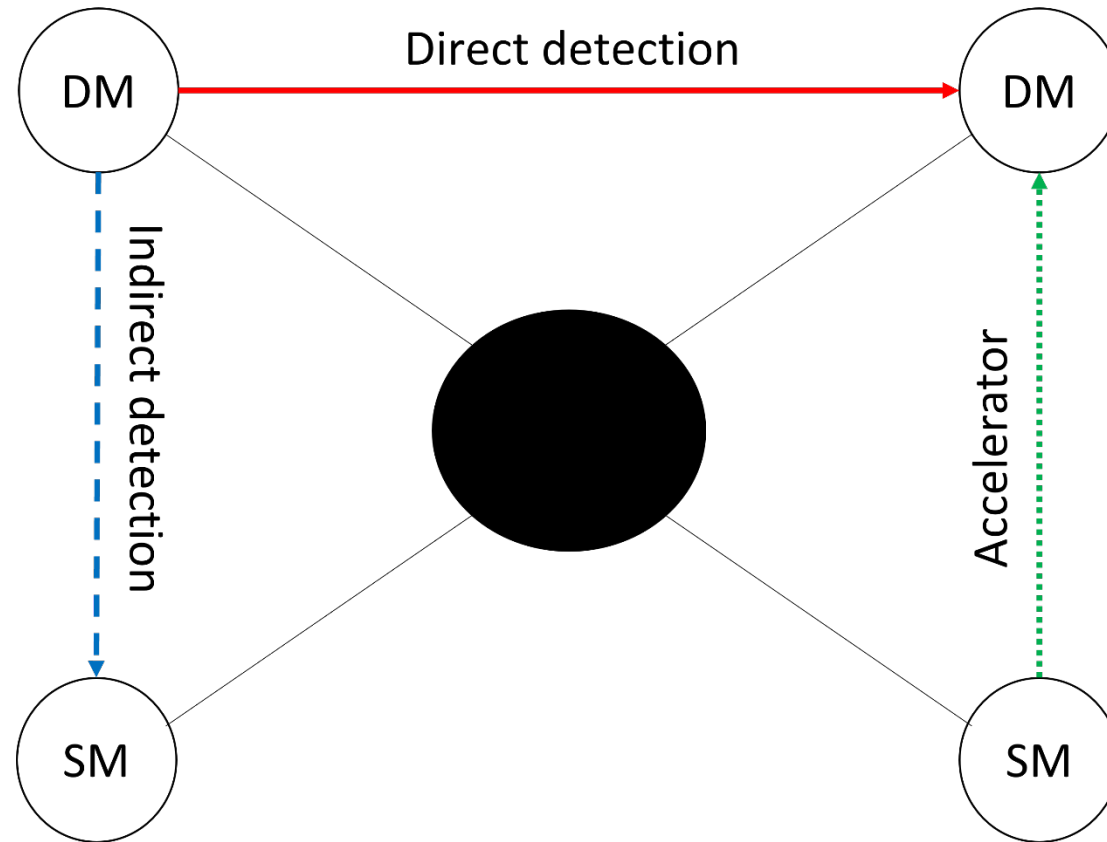
SOGANG UNIVERSITY



Dark Matter and WIMPs

- Many evidences of Dark Matter
 - Galaxy rotational curve
 - CMB
 - Lensing effect
- Many candidates
 - Neutrino
 - Cold Dark Matter (CDM)
 - Weakly Interacting Massive Particle (WIMP)
 - Weak-type interaction
 - no electric charge, no color
 - Mass range in GeV-TeV range
 - WIMP miracle
 - correct relic abundance is obtained at $\langle \sigma v \rangle = \textit{weak scale}$
 - most extensions of SM are proposed independently at that scale.

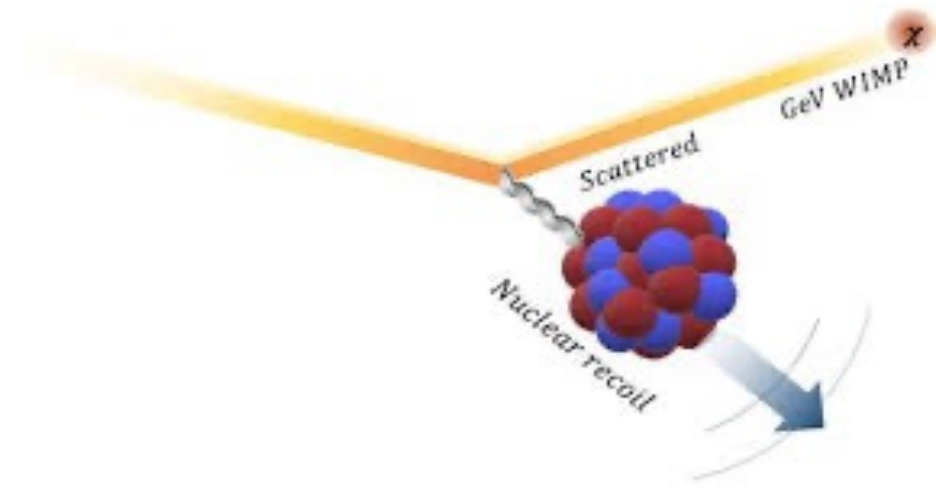
Detection strategies



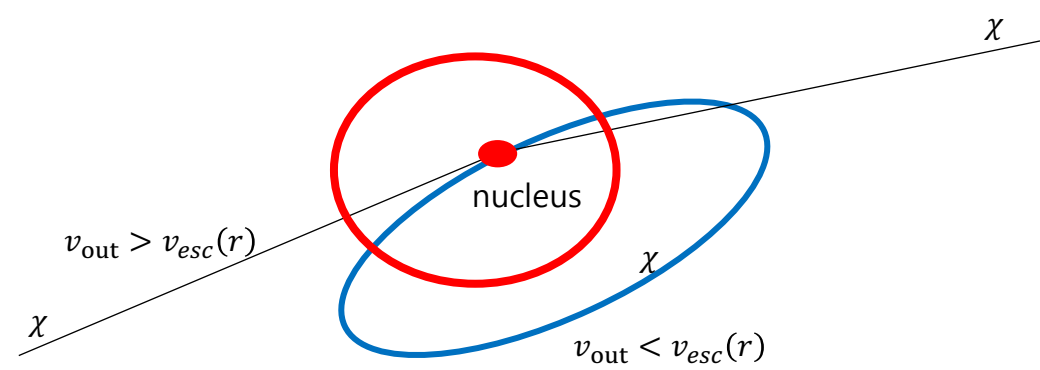
- Direct detection: DM interacts with SM particles (left to right)
- Indirect detection: DM annihilation (top to bottom)
- Accelerator: DM creation (bottom to top)

Direct Detection (DD)

- The signals are WIMP-nucleus recoil events
- Low probability requires high exposure
- Underground to avoid background
- Depend on features of targets and experimental set-ups
- Different nuclear targets and background subtraction:
 - COSINE, ANAIS, DAMA, LZ, PandaX-4T, XENON-nT, PICO-60 and ect.



Indirect Detection (NT)



- Capture rate in the celestial body
- WIMP scatters off nucleus at distance r inside celestial body
 - same interaction probed by DD
- If its outgoing speed v_{out} is below the escape velocity $v_{esc}(r)$, it gets locked into gravitationally bound orbit and keeps scattering again and again
- Capture process is favored for low (even vanishing) WIMP speeds

Non-Relativistic Effective Theory (NREFT)

- WIMP is slow, so that the recoil events are non-relativistic
- NREFT provides a general and efficient way to characterize results with mass of WIMP and coupling constants

Operators spin up to 1/2

- Hamiltonian: $\sum_{i=1}^N (c_i^n \mathcal{O}_i^n + c_i^p \mathcal{O}_i^p)$

- Non-relativistic process

- all operators must be invariant by Galilean transformations

($v \sim 10^{-3}c$ in galactic halo)

- Building operators using:

$$i \frac{\vec{q}}{m_N}, \vec{v}^\perp, \vec{S}_\chi, \vec{S}_N$$

$$\mathcal{O}_1 = 1_\chi 1_N; \quad \mathcal{O}_2 = (v^\perp)^2; \quad \mathcal{O}_3 = i \vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp \right)$$

$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N; \quad \mathcal{O}_5 = i \vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp \right); \quad \mathcal{O}_6 = \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \right) \left(\vec{S}_N \cdot \frac{\vec{q}}{m_N} \right)$$

$$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp; \quad \mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp; \quad \mathcal{O}_9 = i \vec{S}_\chi \cdot \left(\vec{S}_N \times \frac{\vec{q}}{m_N} \right)$$

$$\mathcal{O}_{10} = i \vec{S}_N \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{11} = i \vec{S}_\chi \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{12} = \vec{S}_\chi \cdot \left(\vec{S}_N \times \vec{v}^\perp \right)$$

$$\mathcal{O}_{13} = i \left(\vec{S}_\chi \cdot \vec{v}^\perp \right) \left(\vec{S}_N \cdot \frac{\vec{q}}{m_N} \right); \quad \mathcal{O}_{14} = i \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \right) \left(\vec{S}_N \cdot \vec{v}^\perp \right)$$

$$\mathcal{O}_{15} = - \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \right) \left(\left(\vec{S}_N \times \vec{v}^\perp \right) \cdot \frac{\vec{q}}{m_N} \right),$$

Non-Relativistic Effective Theory (NREFT)

- Scattering amplitude:

$$\frac{1}{2j_\chi + 1} \frac{1}{2j_N + 1} \Sigma_{spins} |M|^2 \equiv \Sigma_k \Sigma_{\tau=0,1} \Sigma_{\tau'=0,1} R_k^{\tau\tau'} \left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}, \{c_i^\tau, c_j^{\tau'}\} \right) W_k^{\tau\tau'}(\mathbf{y})$$

- Differential cross section : $\frac{d\sigma}{dE_R} = \frac{1}{10^6} \frac{2m_N c^2}{4\pi v^2} \left[\frac{1}{2j_\chi+1} \frac{1}{2j_N+1} \Sigma_{spin} |M|^2 \right]$

- Differential rate : $\frac{dR}{dE_R} = N_T \int_{v_{min}}^{v_{esv}} \frac{\rho_\chi}{m_\chi} v \frac{d\sigma}{dE_R} f(v) dv$

- With $E_R = \frac{\mu_{\chi N}^2 v^2}{m_N}$, $v_{min} = \frac{1}{\sqrt{2m_N E_R}} \left| \frac{m_N E_R}{\mu_{\chi N}} + \delta \right|$

Non-Relativistic Effective Theory (NREFT)

- Scattering amplitude:

$$\frac{1}{2j_\chi + 1} \frac{1}{2j_N + 1} \sum_{spins} |M|^2 \equiv \sum_k \sum_{\tau=0,1} \sum_{\tau'=0,1} R_k^{\tau\tau'} \left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}, \{c_i^\tau, c_j^{\tau'}\} \right) W_k^{\tau\tau'}(y)$$

- $R_k^{\tau\tau'}$: WIMP response function

- Velocity dependence: $\mathcal{R}_k^{\tau\tau'} = \mathcal{R}_{k,0}^{\tau\tau'} + \mathcal{R}_{k,1}^{\tau\tau'}(v^2 - v_{min}^2)$

- $W_k^{\tau\tau'}$: nuclear response function

- $y = (qb/2)^2$

- b: harmonic oscillator size parameter

- $k = M, \Delta, \Sigma', \Sigma'', \tilde{\Phi}'$ and Φ''

- allowed responses assuming nuclear ground state is a good approximation of P and T

DD event rate (elastic scattering)

- DD event rate

$$R_{DD} = M\tau_{exp} \frac{\rho_\chi}{m_\chi} \int du f(u) u \Sigma_T N_T \int_{E_{R,th}}^{2\mu_{\chi T}^2 u^2 / m_T} dE_R \zeta_{exp} \frac{d\sigma_T}{dE_R}$$

- $M\tau_{exp}$: exposure
- $E_{R,th}$: experimental energy threshold
- ζ_{exp} : experimental features such as quenching, resolution, efficiency, etc.

- $R_{DD} = \int_0^{u_{esc}} f(u) H_{DD}(u)$

$$H_{DD}(u) = u M\tau_{exp} \frac{\rho_\chi}{m_\chi} \Sigma_T N_T \int_{E_{R,th}}^{2\mu_{\chi T}^2 u^2 / m_T} dE_R \zeta_{exp} \frac{d\sigma_T}{dE_R}$$

Capture rate (elastic scattering)

- Capture rate

$$C_{\odot} = \frac{\rho_{\chi}}{m_{\chi}} \int du f(u) \frac{1}{u} \int_0^{R_{\odot}} dr 4\pi r^2 w^2 \Sigma_T \rho_T(r) \Theta(u_T^{C-max} - u) \int_{m_{\chi} u^2 / 2}^{2\mu_{\chi T}^2 w^2 / m_T} dE_R \frac{d\sigma_T}{dE_R}$$

- ρ_T : the number of density of target
 - r : distance from the center of the Sun for Standard Solar Model AGSS09ph
 - u : DM velocity asymptotically far away from the Sun
 - $v_{esc}(r)$: escape velocity at distance r
 - $w^2(r) = u^2 + v_{esc}^2(r)$
- Neutrino Telescope (NT):
 - the neutrino flux from the annihilation of WIMPs captured in the Sun
 - DM annihilations into $b\bar{b}$

Capture rate (elastic scattering)

- with assumption of equilibrium between capture and annihilation:

$$\Gamma_{\odot} = C_{\odot}/2$$

- $C_{\odot} = \int_0^{u^{c-max}} du f(u) H_C(u)$

$$H_C = \frac{\rho_{\chi}}{m_{\chi}} \frac{1}{u} \int_0^{R_{\odot}} dr 4\pi r^2 w^2 \Sigma_T \rho_T(r) \Theta(u_T^{c-max} - u) \int_{m_{\chi} u^2 / 2}^{2\mu_{\chi T}^2 w^2 / m_T} dE_R \frac{d\sigma_T}{dE_R}$$

- $u_T^{c-max} = v_{esc}(r) \sqrt{\frac{4m_{\chi}m_T}{(m_{\chi}-m_T)^2}}$: maximum WIMP speed for capture possible

Halo independent approach

- Scattering count rate:

$$R \sim \int dv H(v) f(v)$$

interaction

velocity distribution

- Two parts of interaction and velocity distribution
 - needs to avoid uncertainty
 - interaction: include all possible interaction types
 - velocity distribution: halo independent approaches
- Model independent method: the most general scenarios

Halo independent approach

- Halo independent approach with arbitrary speed distribution, $f(u)$
 - The only constraint: $\int_{u=0}^{u_{max}} f(u) du = 1$
- Direct detection experiments have a threshold $u > u_{th}^{DD}$
 - Due to the energy threshold of experimental detectors
- Capture in the Sun is favored for low WIMP speeds
 - $u < u_T^{C-max}$
- In order to cover full speed range: combine DD and capture

Halo independent approach

- Considering one effective coupling (c_i) at a time:
 - $R_{exp}(c_i^2) = \int du f(u) H_{exp}(c_i^2, u) \leq R_{max}$
 - R_{max} : corresponding maximum experimental bound
- Using relation : $H(c_i^2, u) = c_i^2 H(c_i = 1, u)$
 - $H(c_{i,max}^2(u), u) = R_{max}$
 - $c_{i,max}^2(u) = \frac{R_{max}}{H(c_i=1, u)}$
 - $c_{i,max}(u)$: upper limit on c_i at single speed stream u

Halo independent approach

- $R_{exp}(c_i^2) = \int du f(u) H_{exp}(c_i^2, u) \leq R_{max}$

$$\begin{aligned} R(c_i^2) &= \int_0^{u_{max}} du f(u) H(c_i^2, u) \\ &= \int_0^{u_{max}} du f(u) \frac{c_i^2}{c_{i,max}^2(u)} H(c_{i,max}^2(u), u) \\ &= \int_0^{u_{max}} du f(u) \frac{c_i^2}{c_{i,max}^2(u)} R_{max} \leq R_{max} \end{aligned}$$

- upper limit on i-th coupling c_i :

$$c_i^2 \leq \left[\int_0^{u_{max}} du \frac{f(u)}{c_{i,max}^2(u)} \right]^{-1}$$

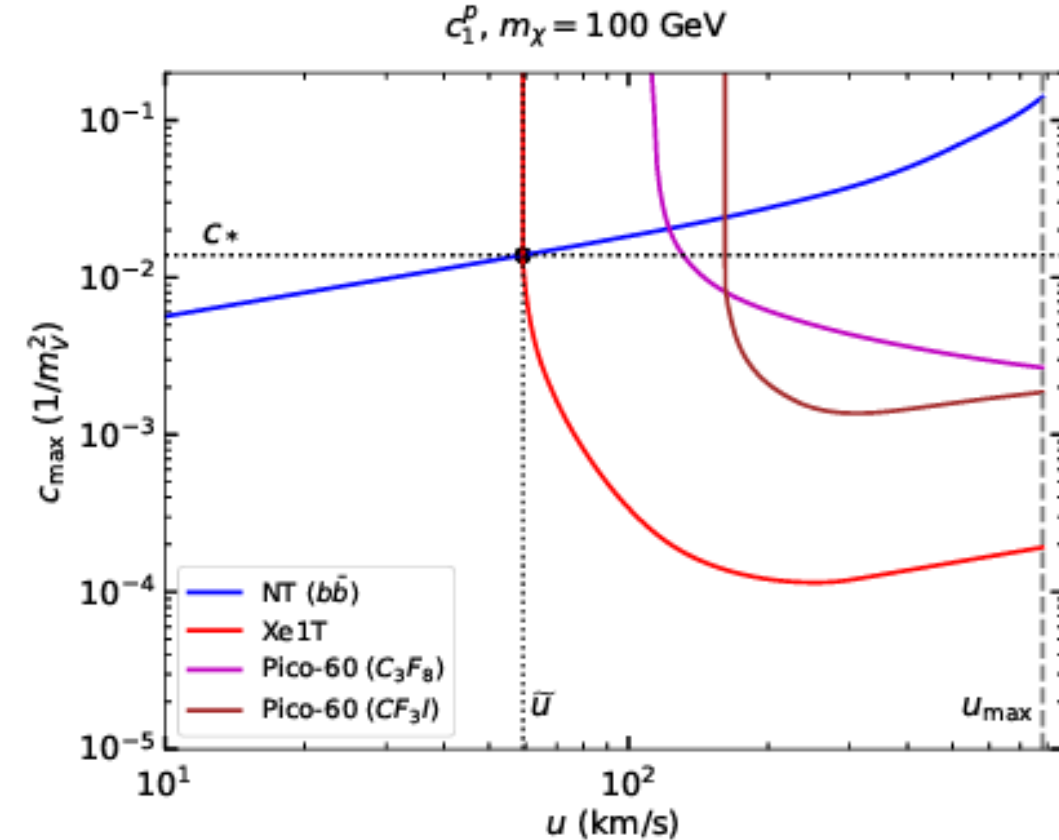
Halo independent approach

- $c_* = c_{max}^{NT}(\tilde{u}) = c_{max}^{DD}(\tilde{u})$: halo independent limit
- \tilde{u} : intersection speed of NT and DD

- To cover whole speed range, one may combine DD and NT

- $u_T^{C-max} = v_{esc}(r) \sqrt{\frac{4m_\chi m_T}{(m_\chi - m_T)^2}}$

- $(u_{th}^{DD})^2 = \frac{m_T}{2\mu_{\chi T}^2} E_{R,th}$



S. Kang, A. Kar, S. Scopel, JCAP03(2023)011

Halo independent approach

- Intersection:

$$(c^{NT})_{max}^2(u) \leq c_*^2 \quad \text{for } 0 \leq u \leq \tilde{u}$$

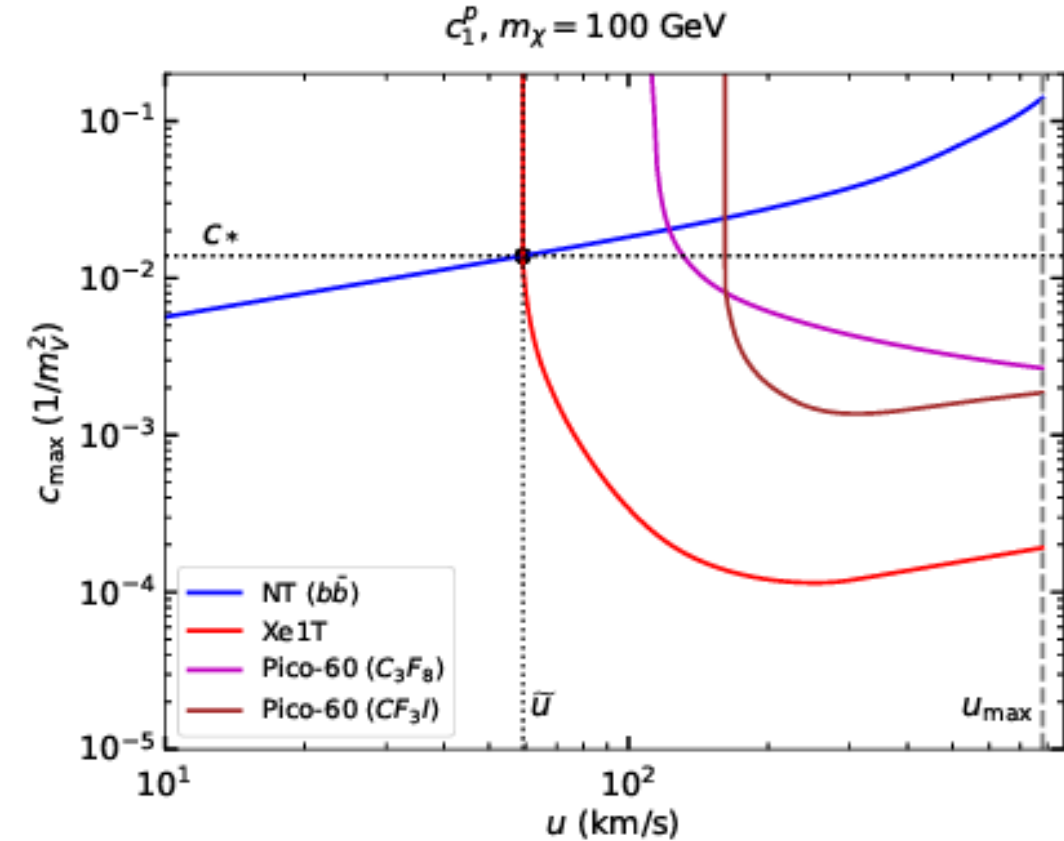
$$(c^{DD})_{max}^2(u) \leq c_*^2 \quad \text{for } \tilde{u} \leq u \leq u_{max}$$

$$c^2 \leq c_*^2 \left[\int_0^{\tilde{u}} du f(u) \right]^{-1} = \frac{c_*^2}{\delta}$$

$$c^2 \leq c_*^2 \left[\int_{\tilde{u}}^{u_{max}} du f(u) \right]^{-1} = \frac{c_*^2}{1-\delta}$$

$$\delta = \int_0^{\tilde{u}} du f(u)$$

$$c^2 \leq 2c_*^2$$



S. Kang, A. Kar, S. Scopel, JCAP03(2023)011

Halo independent approach

- If $(c^{DD})_{max}^2(u) > c_*^2$ at $u = u_{max}$:

$$c^2 \leq c_*^2 \left[\int_0^{\tilde{u}} du f(u) \right]^{-1} = \frac{c_*^2}{\delta}$$

$$c^2 \leq (c^{DD})_{max}^2(u_{max}) \left[\int_{\tilde{u}}^{u_{max}} du f(u) \right]^{-1} = \frac{(c^{DD})_{max}^2(u_{max})}{1-\delta}$$

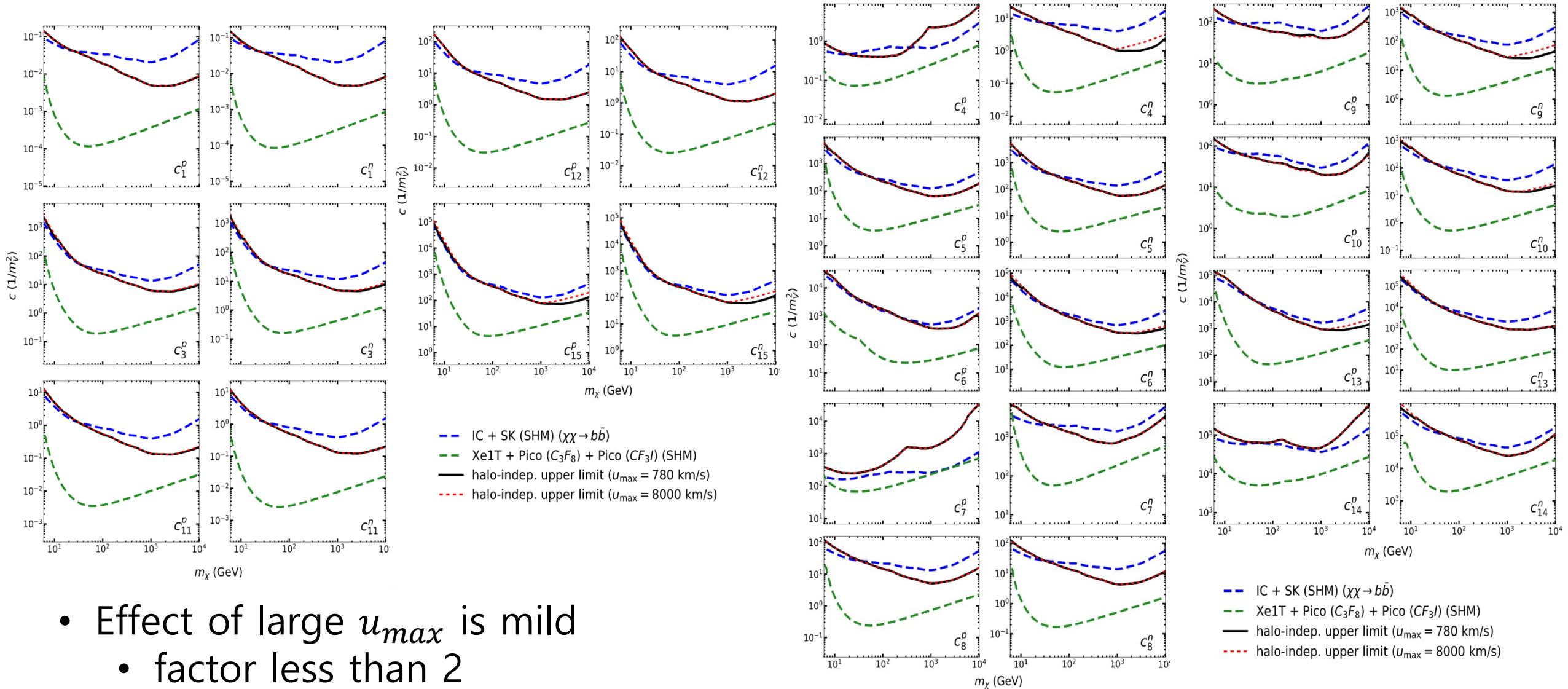
$$c^2 \leq (c^{DD})_{max}^2(u_{max}) + c_*^2$$

- If $u_{th}^{DD} > u_{max}$:

$$c^2 \leq (c^{NT})^2(u_{max})$$

- Halo independent limit may depend on u_{max}

Halo independent approach



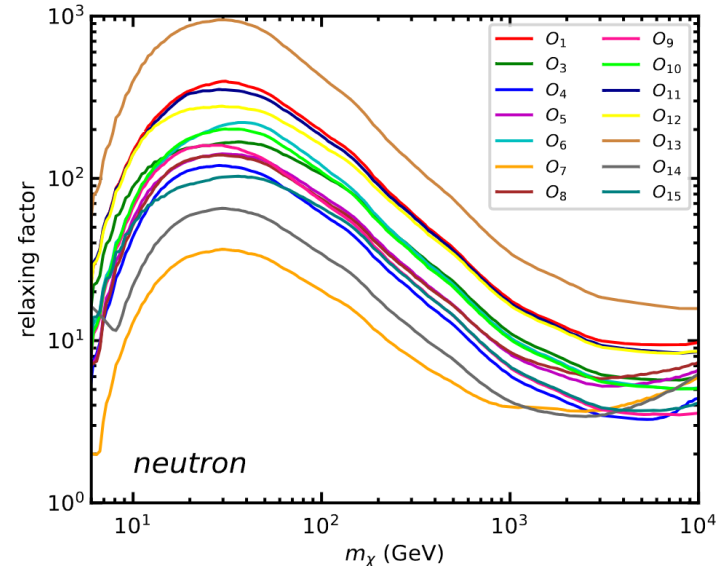
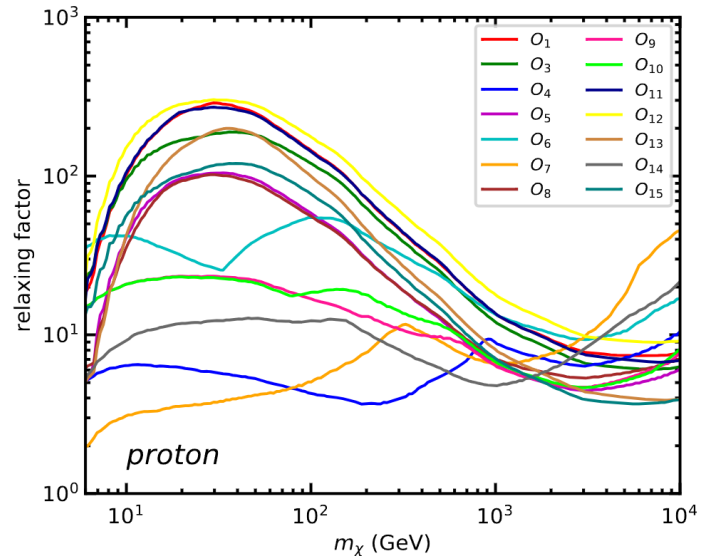
- Effect of large u_{max} is mild
- factor less than 2

Halo independent approach

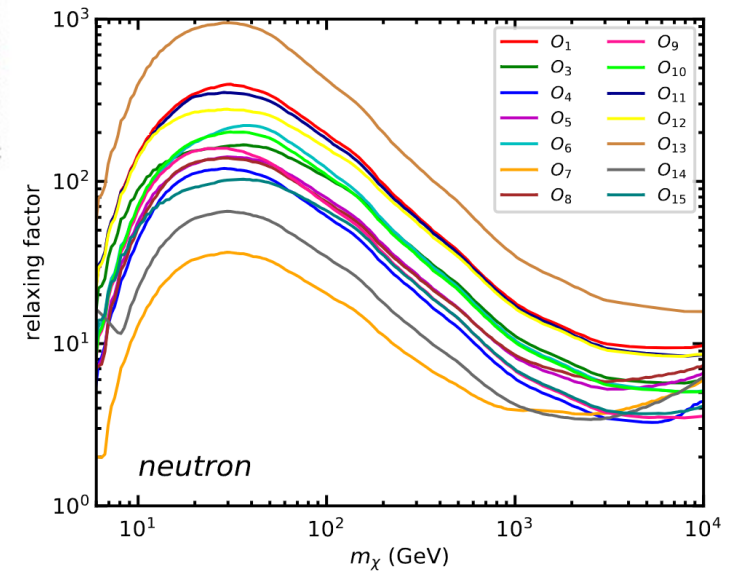
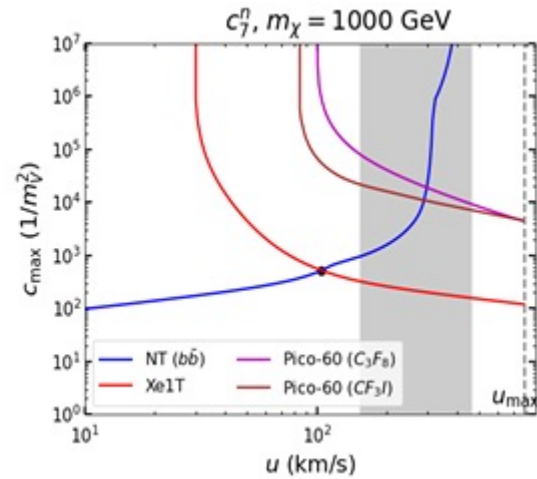
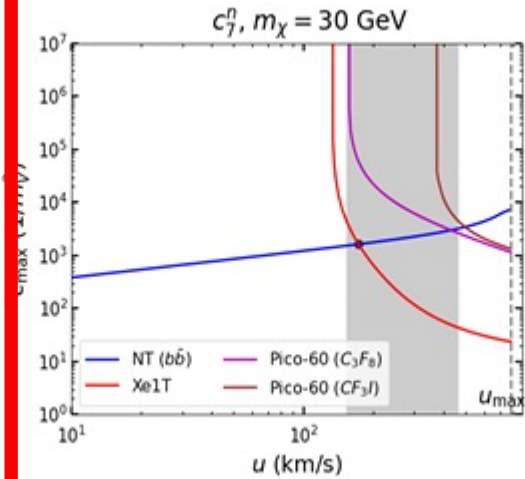
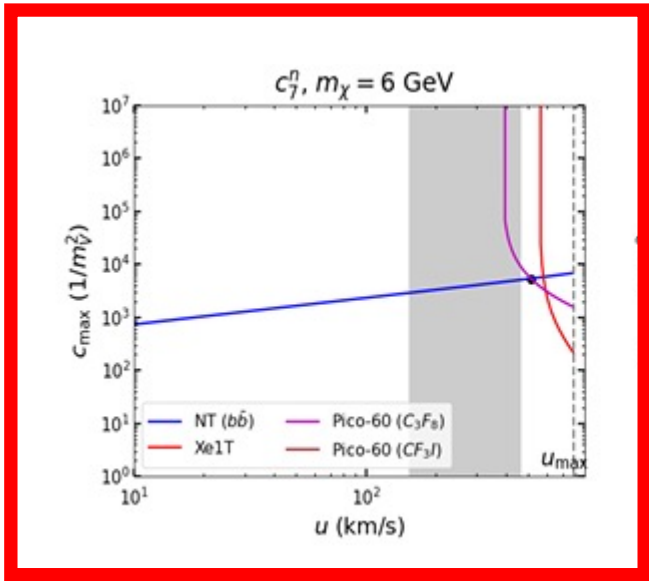
- Relaxing factor

$$r_f^2 = \frac{2c_*^2}{(c_{SHM}^{exp})^2} = 2c_*^2 \int_0^{u_{max}} du \frac{f_M(u)}{(c^{exp})_{max}^2(u)} = 2c_*^2 \left\langle \frac{1}{(c^{exp})_{max}^2} \right\rangle \cong 2c_*^2 \left\langle \frac{1}{(c^{exp})_{max}^2} \right\rangle_{bulk}$$

$$\int_{bulk} du f_M(u) \approx 0.8$$



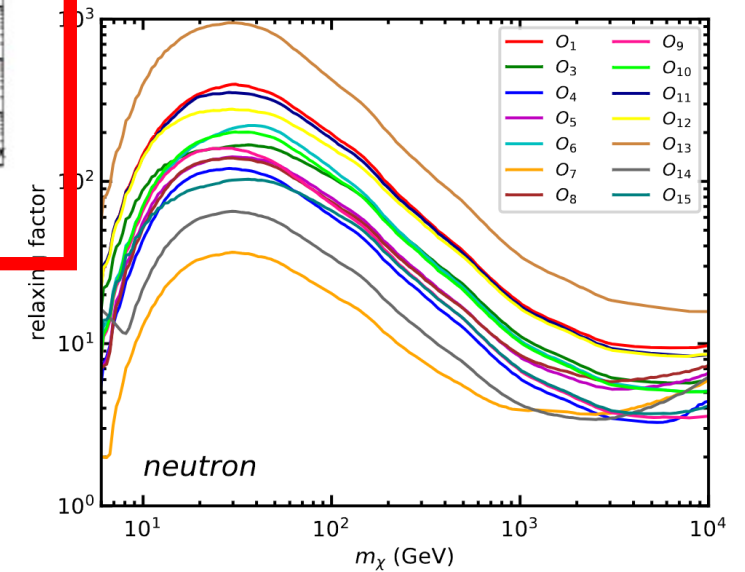
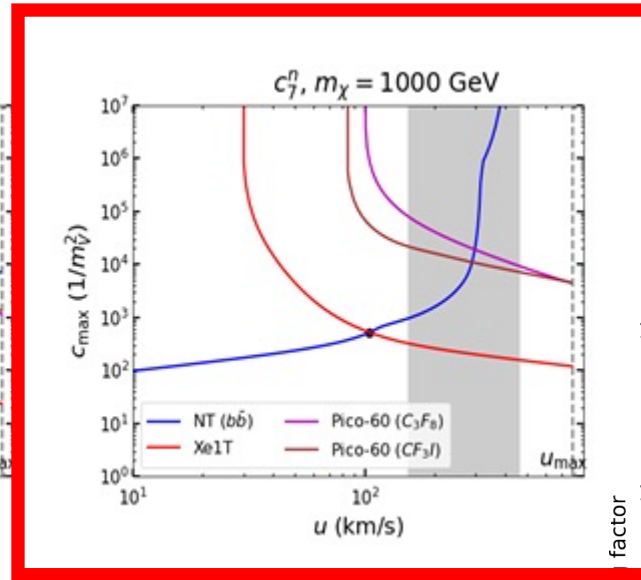
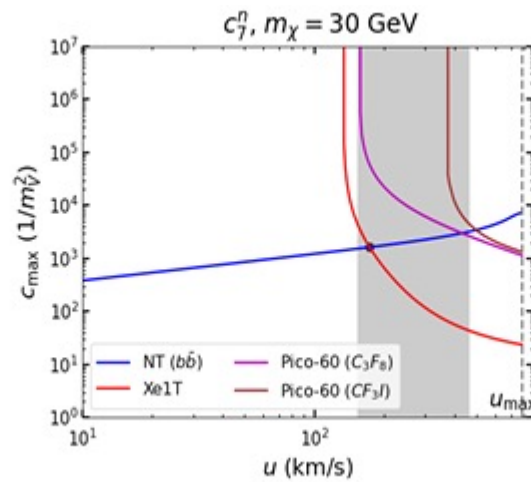
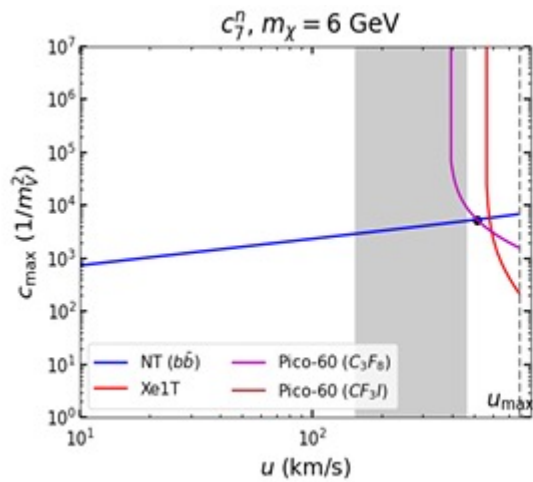
Halo independent approach



- small or large mass range
 - outside the bulk of Maxwellian
 - smooth dependence on u
- intermediate range (10 ~ 200 GeV)
 - inside the bulk of Maxwellian
 - steep dependence on u

$$r_f^2 \simeq 2c_*^2 \left\langle \frac{1}{(c^{\text{exp}})_{\text{max}}^2} \right\rangle_{\text{bulk}},$$

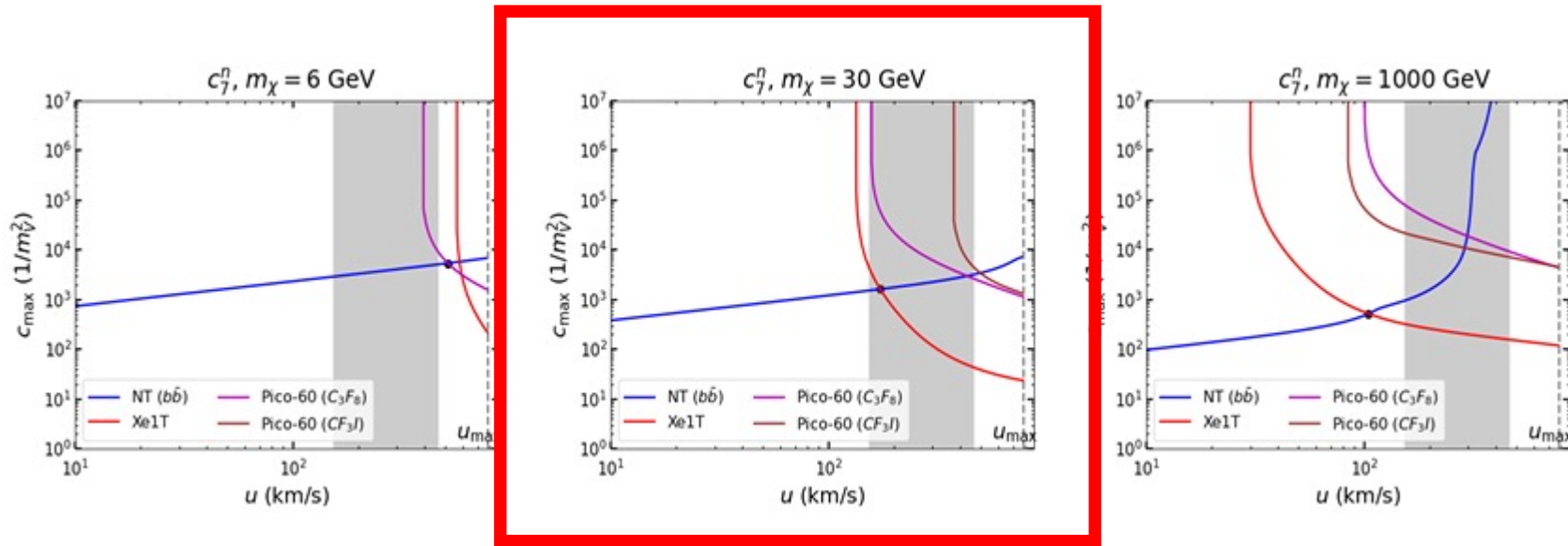
Halo independent approach



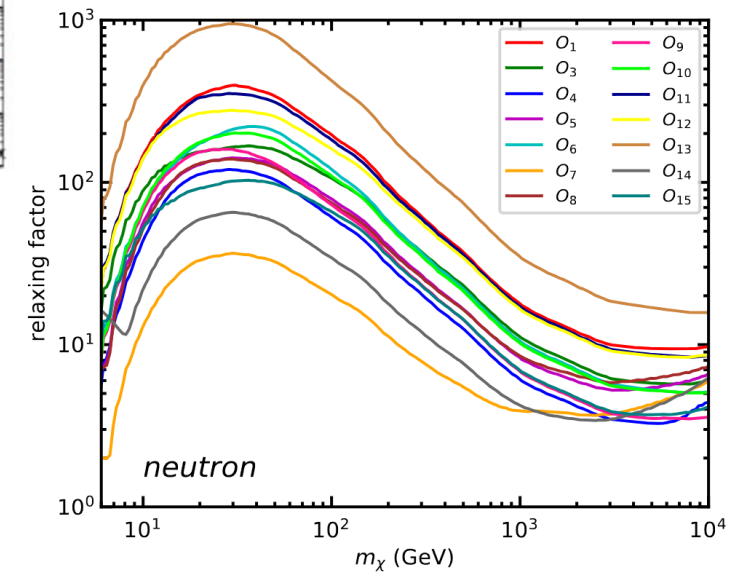
- small or large mass range
 - outside the bulk of Maxwellian
 - smooth dependence on u
- intermediate range (10 ~ 200 GeV)
 - inside the bulk of Maxwellian
 - steep dependence on u

$$r_f^2 \simeq 2c_*^2 \left\langle \frac{1}{(c^{\text{exp}})_{\text{max}}^2} \right\rangle_{\text{bulk}},$$

Halo independent approach



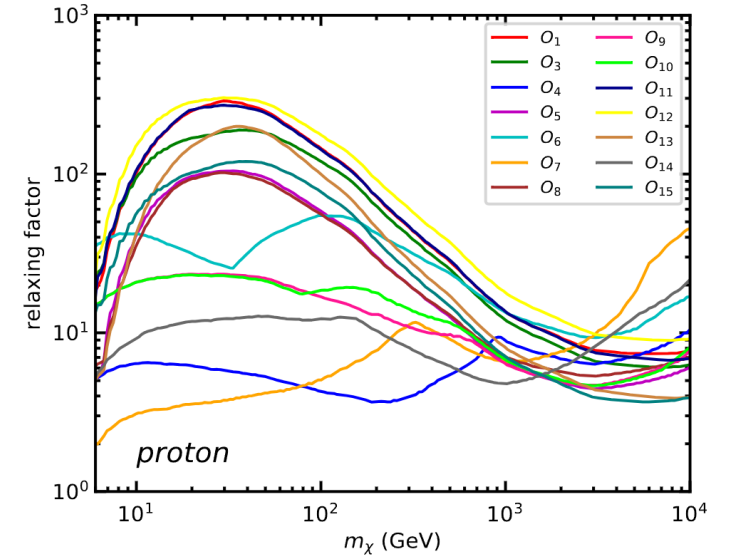
- small or large mass range
 - outside the bulk of Maxwellian
 - smooth dependence on u
- intermediate range (10 ~ 200 GeV)
 - inside the bulk of Maxwellian
 - steep dependence on u



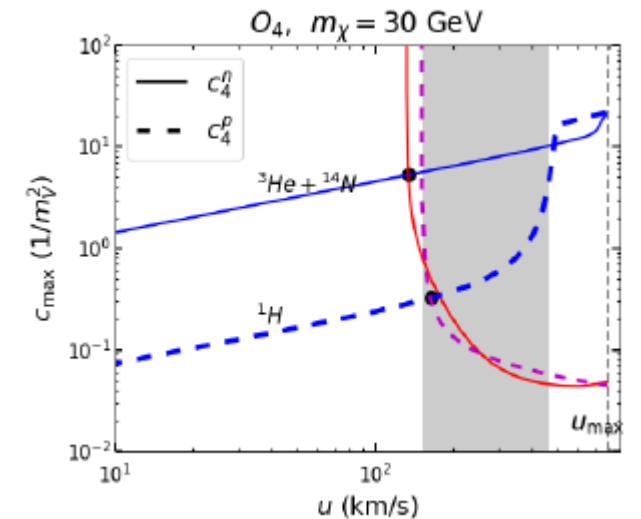
$$r_f^2 \simeq 2c_*^2 \left\langle \frac{1}{(c^{\text{exp}})_{\text{max}}^2} \right\rangle_{\text{bulk}},$$

Halo independent approach

- small relaxing factors
 - $O_{4,7}$: SD with no q suppression
 - $O_{9,10,14}$: SD with q^2 suppression
 - O_6 : SD with q^4 suppression



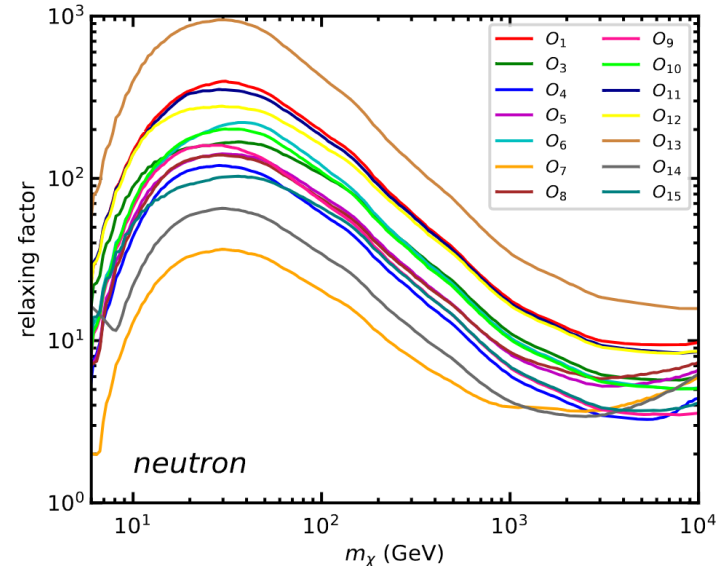
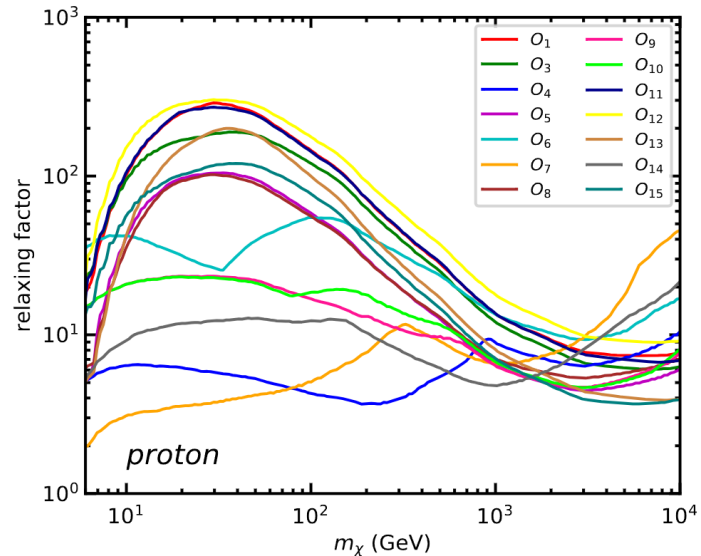
operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0), \Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2), \tilde{\Phi}'(q^2)$	$\Sigma''(q^0), \Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$



Halo independent approach

- High relaxing factor:
the halo-independent method can weaken the bound

$$r_f^2 = \frac{2c_*^2}{(c_{SHM}^{exp})^2} = 2c_*^2 \int_0^{u_{max}} du \frac{f_M(u)}{(c^{exp})_{max}^2(u)} = 2c_*^2 \left\langle \frac{1}{(c^{exp})_{max}^2} \right\rangle \cong 2c_*^2 \left\langle \frac{1}{(c^{exp})_{max}^2} \right\rangle_{bulk}$$



DD event rate (inelastic scattering)

- DD event rate

$$R_{DD} = M \tau_{exp} \frac{\rho_\chi}{m_\chi} \int du f(u) u \Sigma_T N_T \int_{E_{R,th}}^{2\mu_{\chi T}^2 u^2 / m_T} dE_R \zeta_{exp} \frac{d\sigma_T}{dE_R}$$



$$R_{DD} = M \tau_{exp} \frac{\rho_\chi}{m_\chi} \int du f(u) u \Sigma_T N_T \Theta(u^2 - v_{T*}^2) \int_{E_{min}}^{E_{max}} dE_R \zeta_{exp} \frac{d\sigma_T}{dE_R}$$

$$E_{max,min}(u) = \frac{\mu_{\chi T}^2 u^2}{2 m_T} \left(1 \pm \sqrt{1 - \frac{2\delta}{\mu_{\chi T} u^2}} \right)^2$$

$$v_{T*}^2 = \sqrt{\frac{2\delta}{\mu_{\chi T}}}$$

Capture rate (inelastic scattering)

- Capture rate

$$C_{\odot} = \frac{\rho_{\chi}}{m_{\chi}} \int du \frac{f(u)}{u} \int_0^{R_{\odot}} dr 4\pi r^2 w^2 \Sigma_T \rho_T(r) \Theta(u_T^{C-max} - u) \int_{m_{\chi} u^2 / 2}^{2\mu_{\chi T}^2 w^2 / m_T} dE_R \frac{d\sigma_T}{dE_R}$$



$$C_{\odot} = \frac{\rho_{\chi}}{m_{\chi}} \int du \frac{f(u)}{u} \int_0^{R_{\odot}} dr 4\pi r^2 w^2 \Sigma_T \rho_T(r) \Theta(\omega^2 - v_{T*}^2) \Theta(E_{max}^{\chi} - E_{cap}^{\chi}) \int_{\max(E_{min}^{\chi}, E_{cap}^{\chi})}^{E_{max}^{\chi}} dE_R \frac{d\sigma_T}{dE_R}$$

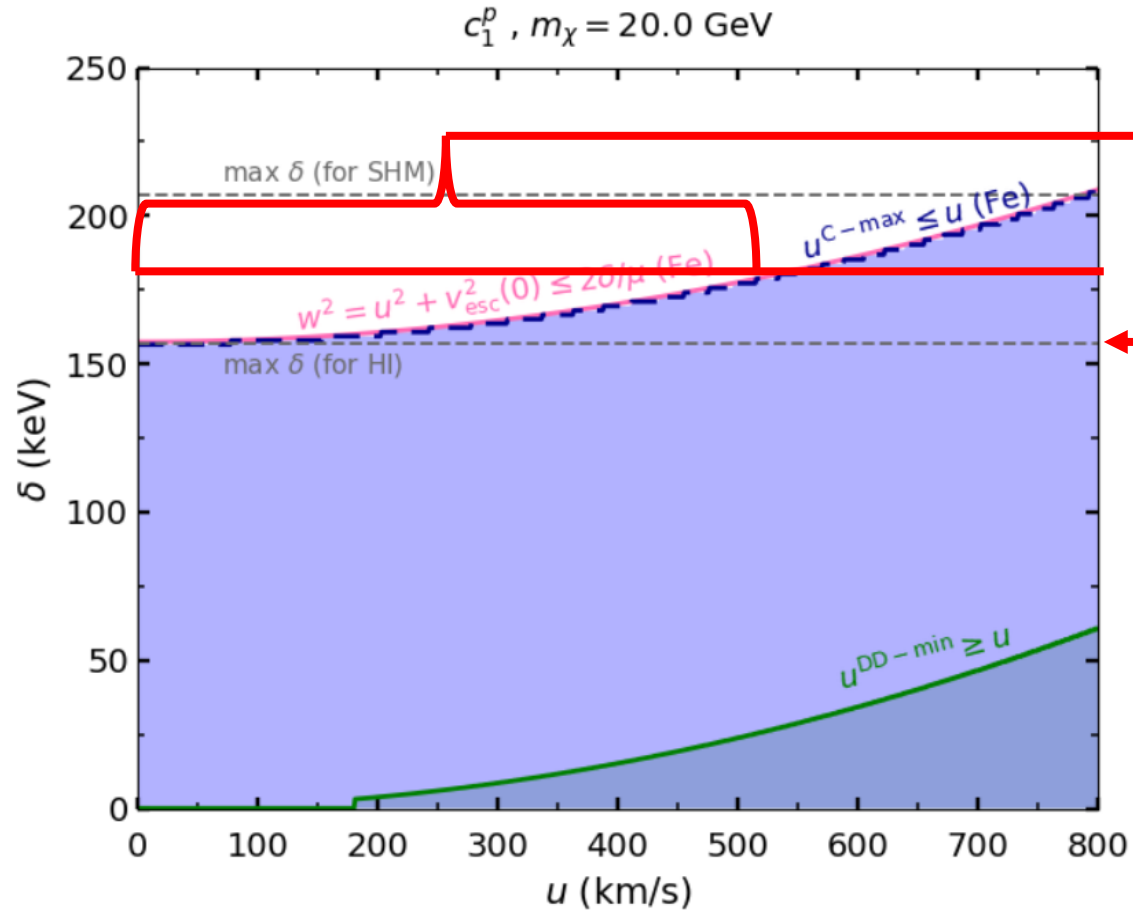
$$E_{max,min}(u) = \frac{1}{2} m_{\chi} \omega^2 \left(1 - \frac{\mu_{\chi T}^2}{m_T^2} \left(1 \pm \frac{m_T}{m_{\chi}} \sqrt{1 - \frac{v_{T*}^2}{\omega^2}} \right)^2 \right)^2$$

$$E_{cap}^{\chi}(u) = \frac{1}{2} m_{\chi} u^2 - \delta$$

Kinematic conditions

- $u^2 + v_{esc}^2(r = 0) > v_{T*}^2$
 - for inelastic scattering process to be kinematically possible
- $u > u^{DD-min}$
 - for the recoil energy to be above the DD experimental threshold
- $u < u^{C-max}$
 - for outgoing speed to be below the escape velocity in the Sun
- $u^{DD-min} < u^{C-max}$
 - for DD and capture intersect

Determining δ_{max}

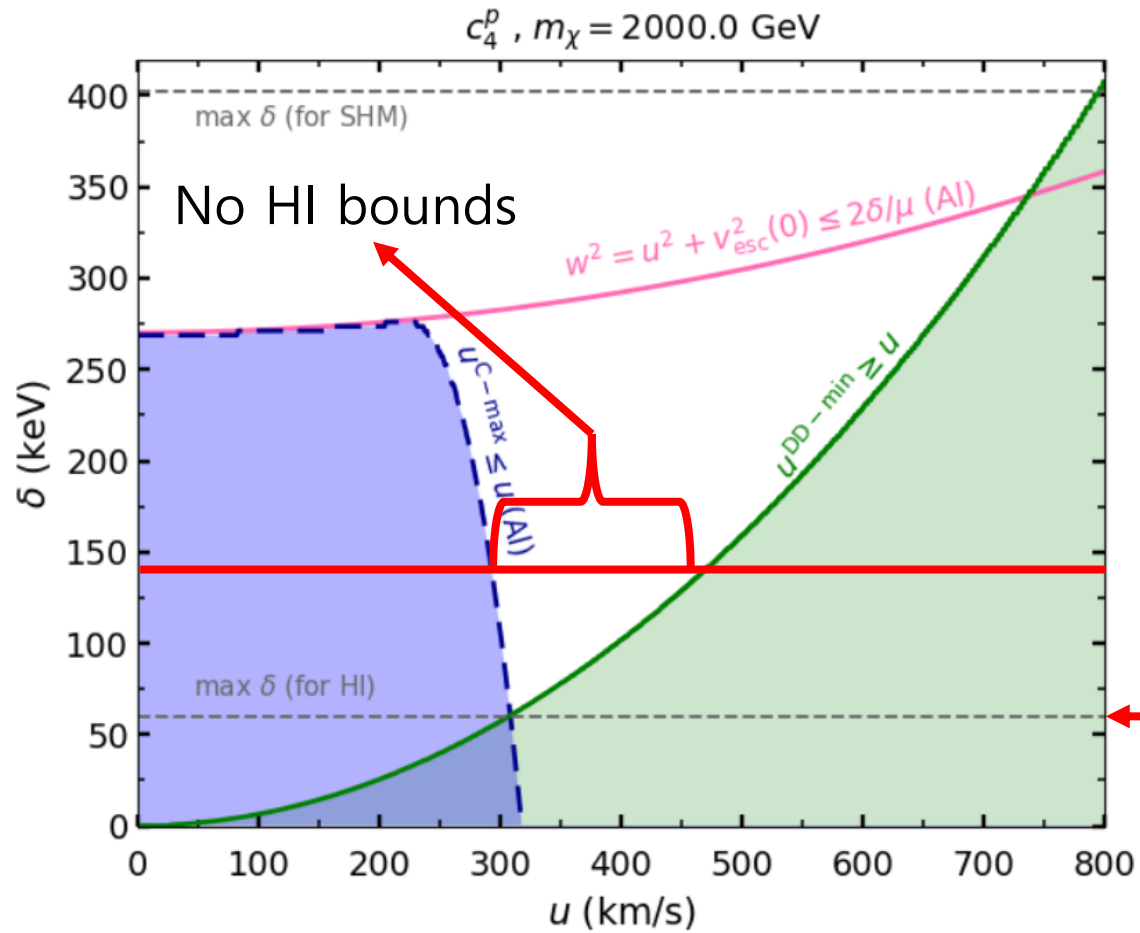


No HI bounds

δ^{max} for HI

- Low m_χ region, capture can cover full speed range alone up to δ^{max}
- Capture alone can determine δ^{max}
- Above δ^{max} , no HI bounds

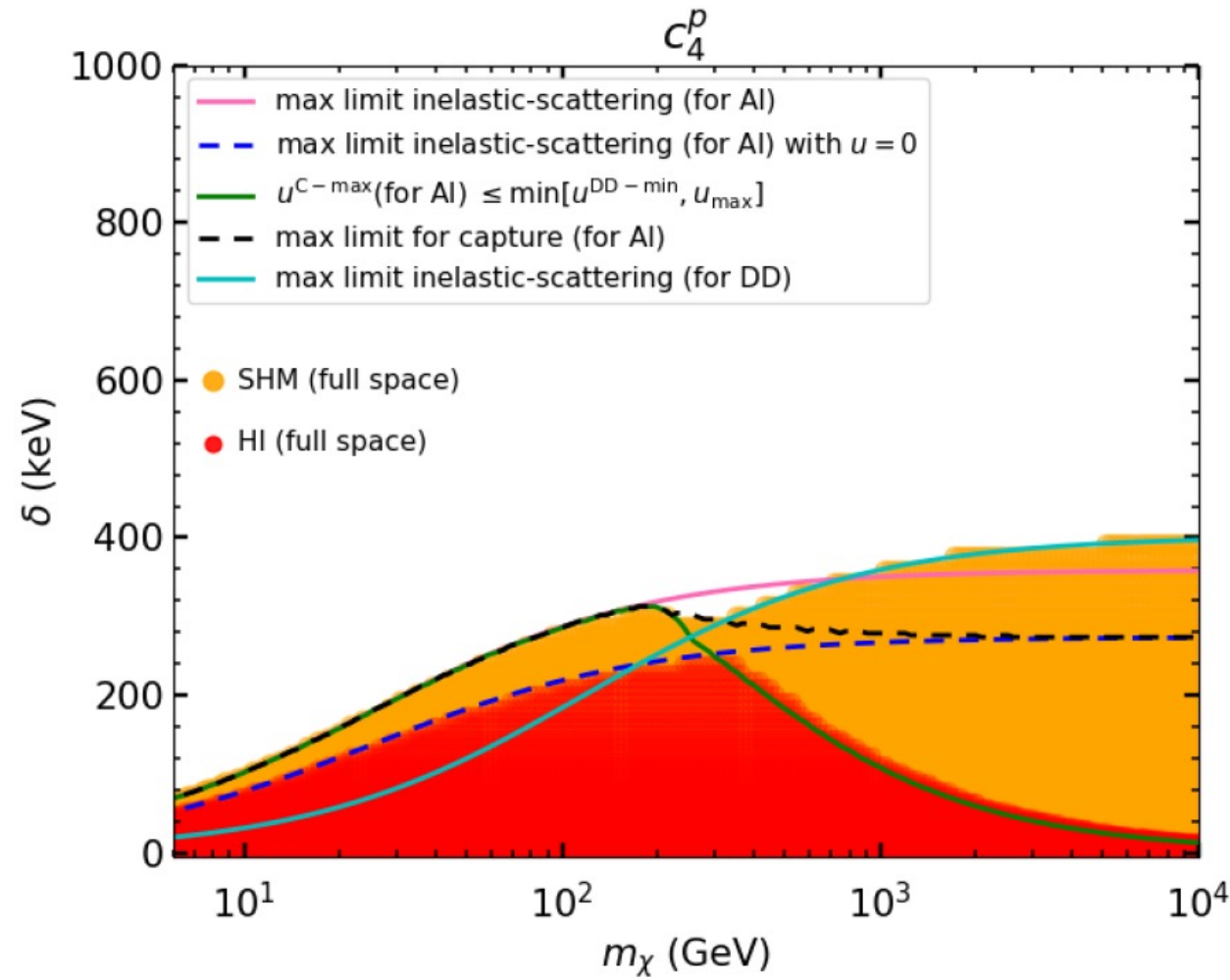
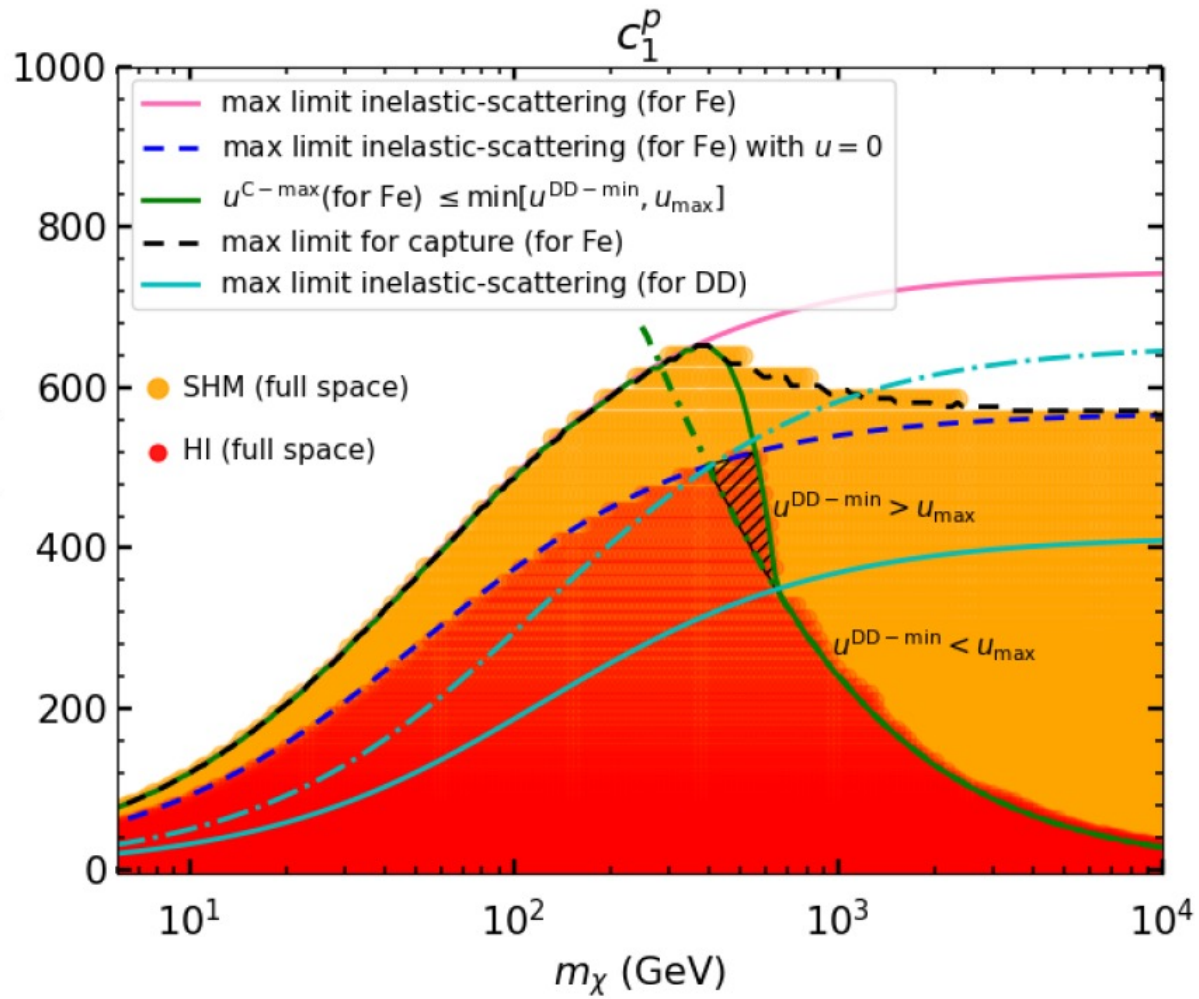
Determining δ_{max}



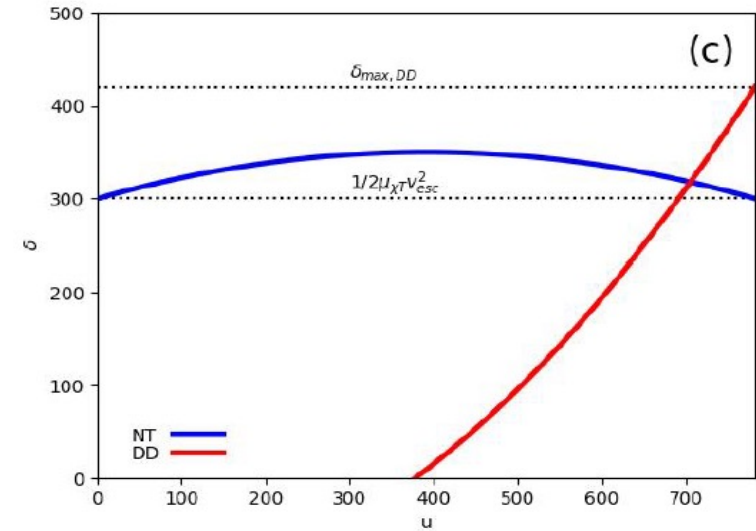
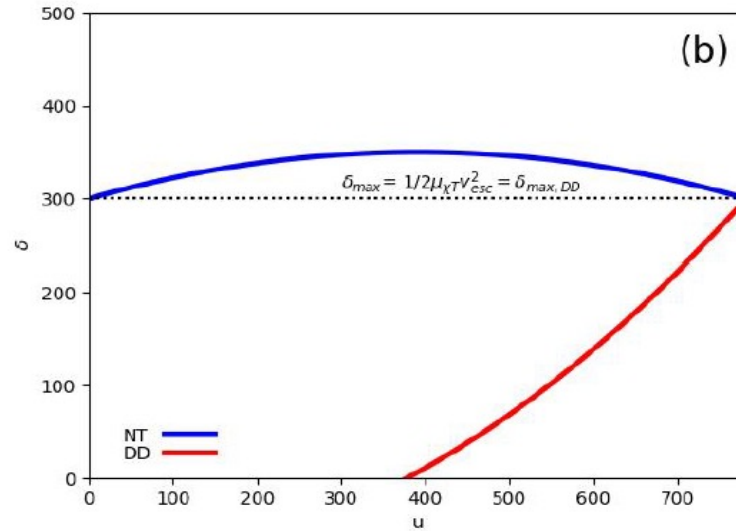
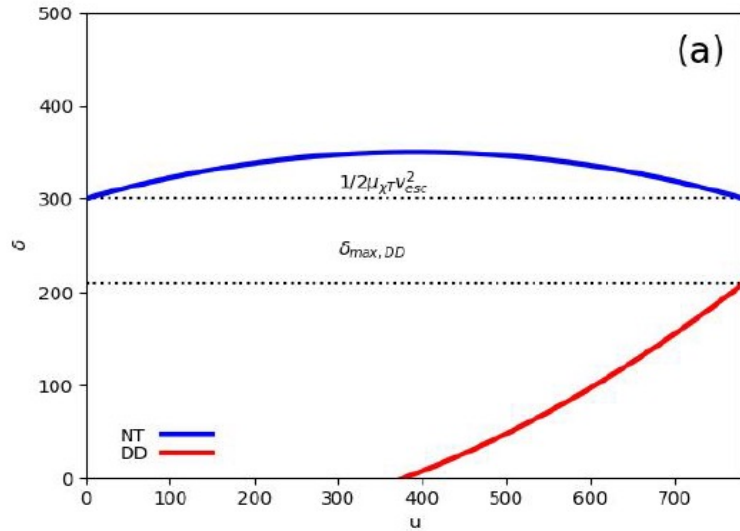
- High m_χ region, DD and capture decide δ^{max} together
- δ^{max} is determined by a combination of DD and capture
- Above δ^{max} , no HI bounds

δ^{max} for HI

$m_\chi - \delta$ planes

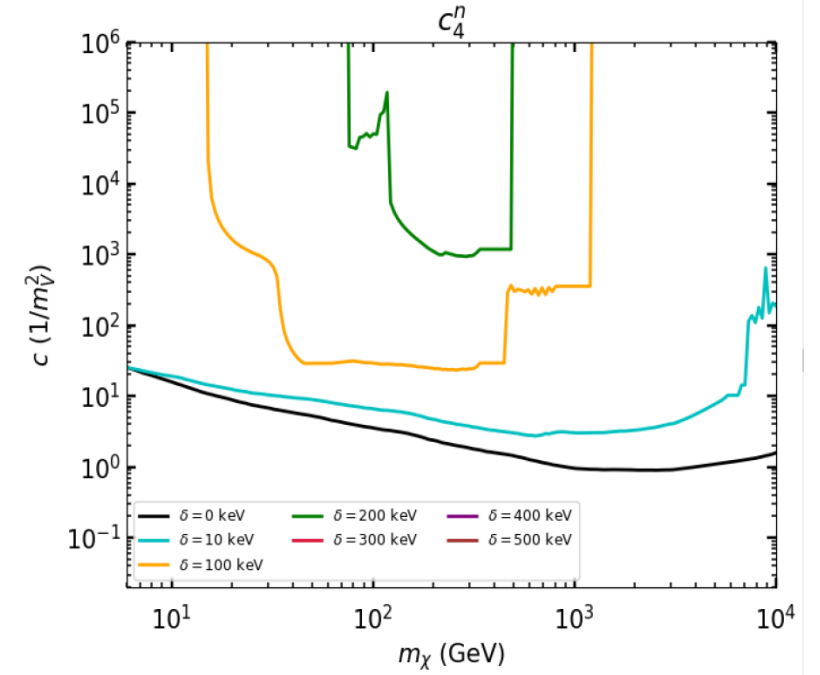
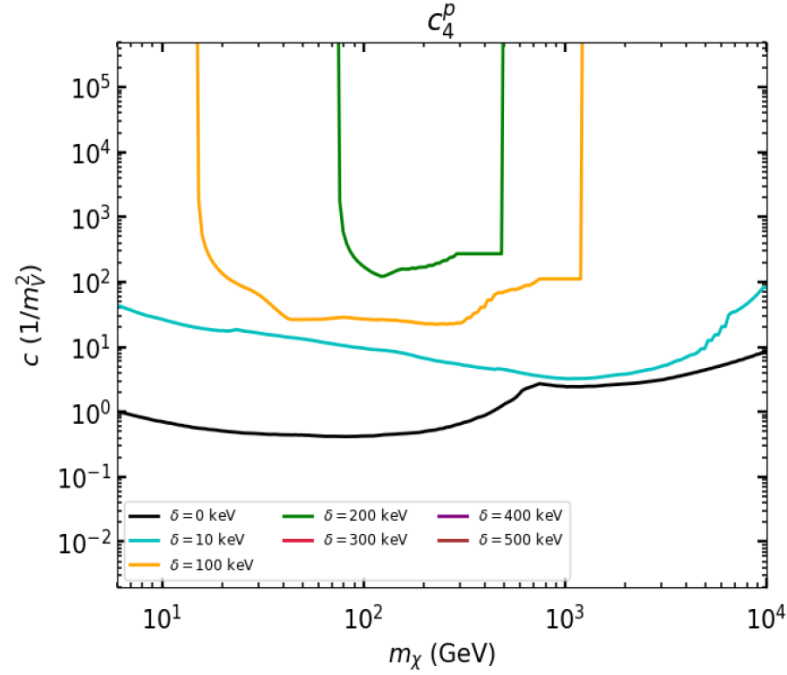
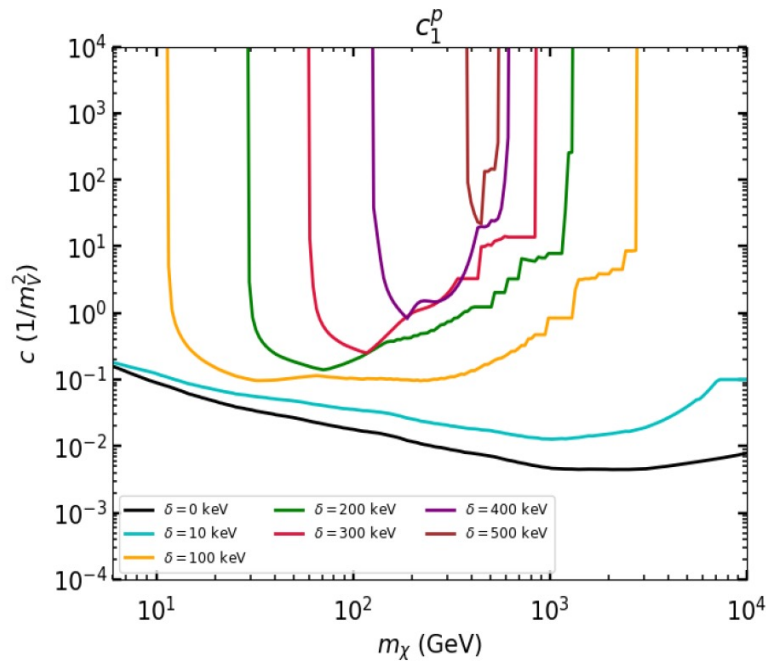


Role of DD



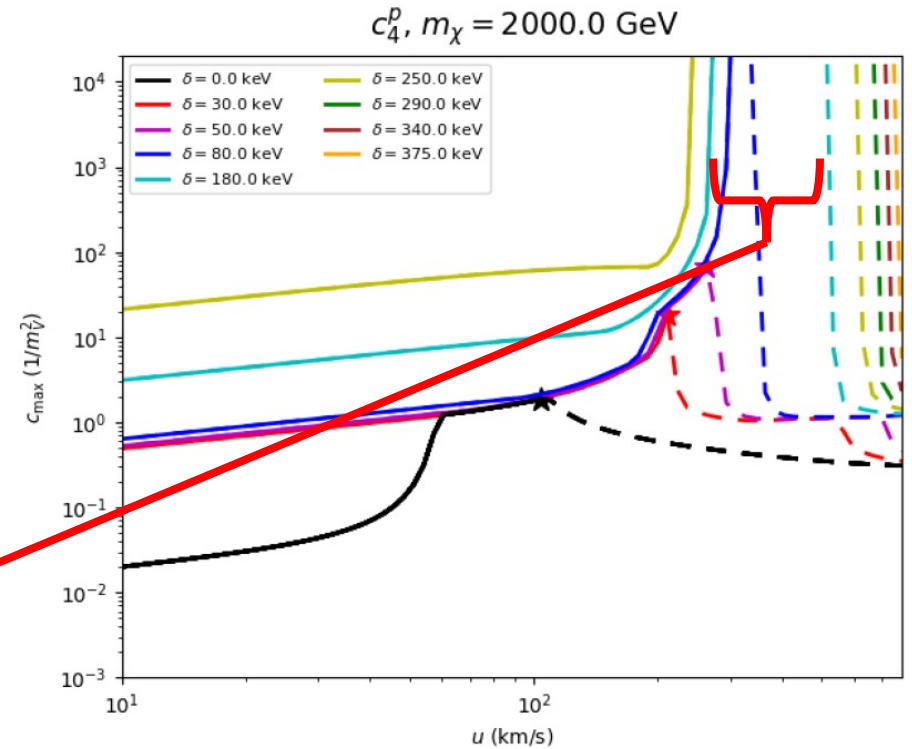
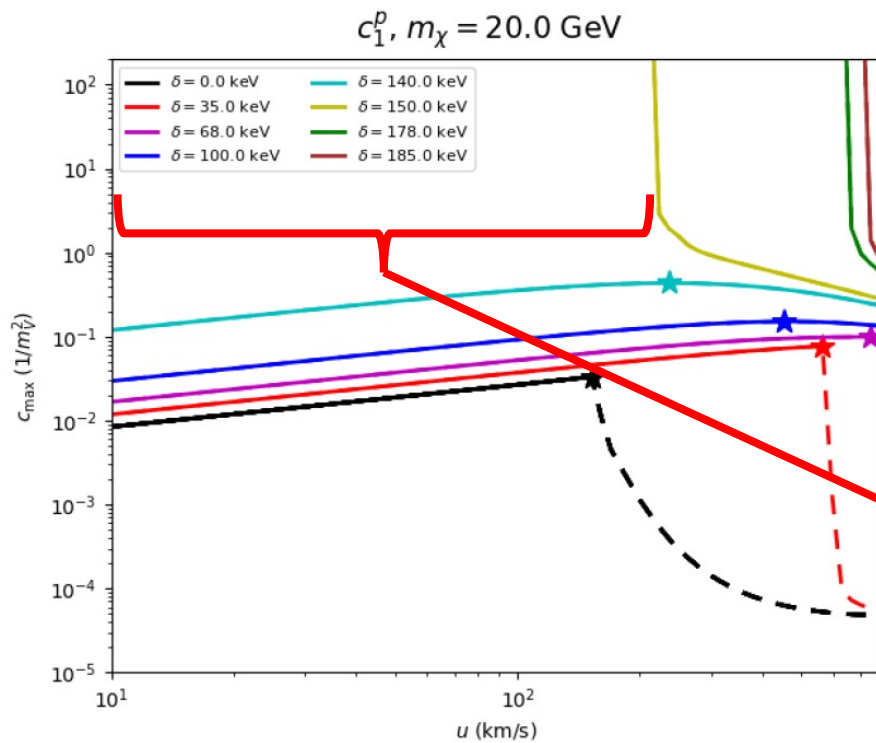
- $r = \frac{m_N}{m_T} \geq r_{min} \simeq 3.9$
 - below this value, only Capture determines δ_{max}
- SI ($T = {}^{56}Fe, N = Xe$)
 - $r \simeq 2.3$
- SD ($T = {}^{27}Al, N = Xe (I)$)
 - $r \simeq 4.86$ (4.7)

Halo-independent exclusion plots



- HI exclusion plots of SI and SD couplings
- For increasing δ the constraints get weaker and WIMP mass range for HI bounds is shrinking

Halo-independent exclusion plots

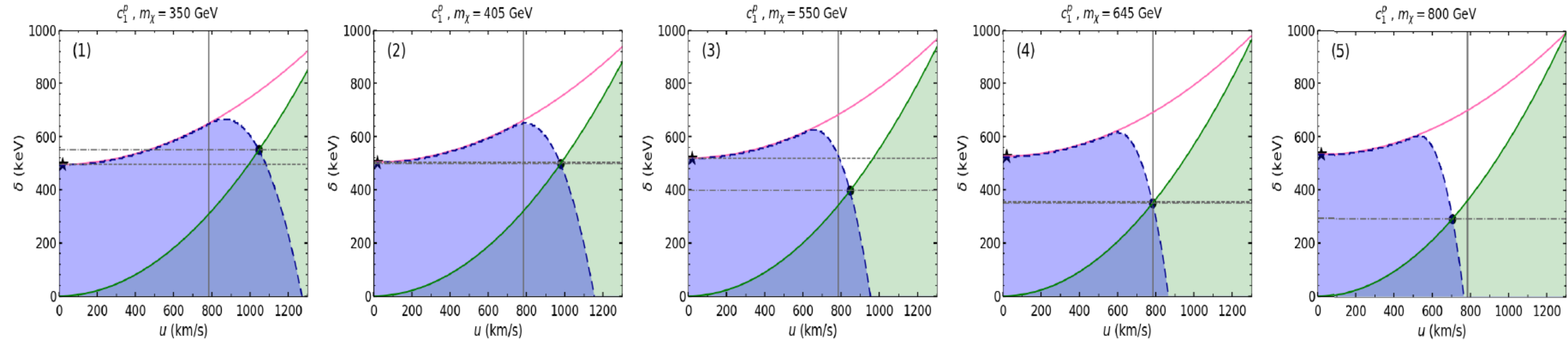
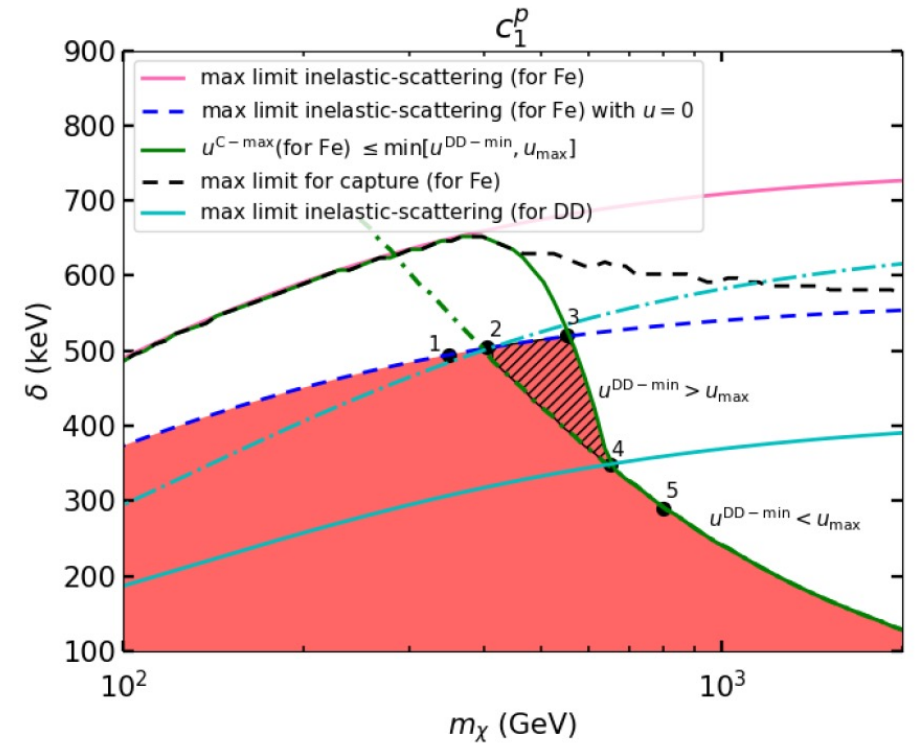


No HI bounds

- low m_χ : capture is kinematically impossible
- high m_χ : DD and capture doesn't intersect

Dependence on u_{max}

- HI might be sensitive to u_{max}
- except point (3), δ_{max} does not change even if we extend the value of u_{max}
- Only happens is SI case and δ_{max} does not decrease dramatically



WimPyDD

- User-friendly Python code
- Calculates expected rates in any scenarios:
 - arbitrary spins
 - inelastic scattering
 - generic WIMP velocity distribution
- Published and can be downloaded:
 - <https://wimpydd.hepforge.org/>

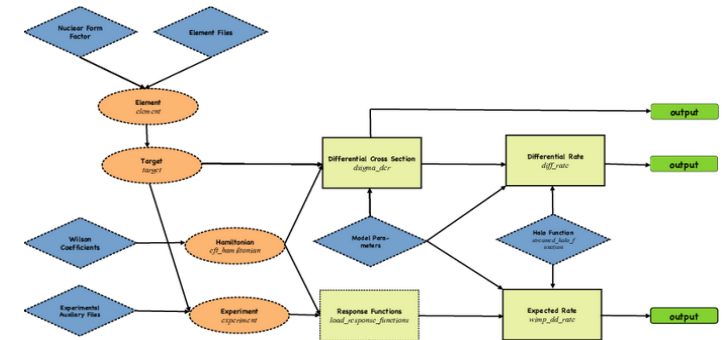
- Home
- Download
- The code
 - Getting Started
 - Examples
 - Nuclear Targets
 - Effective Hamiltonian
 - Experiment
 - Nuclear Response Functions
 - Halo Function
 - Formulas and definitions
- Contact



WimPyDD

WimPyDD is an object-oriented and customizable Python code that calculates accurate predictions for the expected rates in WIMP direct-detection experiments within the framework of Galilean-invariant non-relativistic effective theory. WimPyDD handles different scenarios including **inelastic scattering**, **WIMP of arbitrary spin** and a **generic velocity distribution** of WIMP in the Galactic halo.

WimPyDD is written by **Stefano Scopel, Gaurav Tomar, Sunghyun Kang, and Injun Jeong.**



Summary

- Halo independent method can be applied to any speed distribution
- Combining results of direct detection experiments and capture in the Sun may provide halo-independent bounds according to the value of δ
- In elastic scattering,
 - In most cases the relaxation of halo independent bounds is moderate in low and high m_χ
 - More moderate values of the relaxation is obtained with c_{SD}^p
 - High relaxing factor: halo independent method weaken the bounds
→ sensitive on speed distribution
- In inelastic scattering,
 - There is a specific region in $m_\chi - \delta$ plane where Halo independent bounds is possible
 - Unless $\frac{m_N}{m_T}$ is larger than about 4, direct detection does not play any role