Testing the neutrino mass sum rules with present data and future prospects

Quang Anh Nguyen[1] Dr. Son Cao[2]

[1]University of Science and Technology of Hanoi

[2]Institute for Interdisciplinary Research in Science and Education

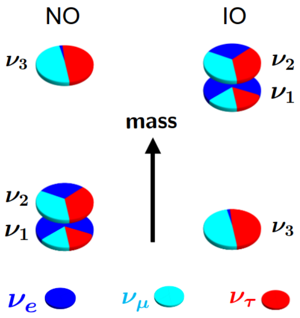
ICISE, July 2024

Neutrino Mass Ordering

- Neutrinos have non-zero mass.
- According to neutrino oscillation data, the mass squared difference:

$$\left|\Delta m^2_{31}\right| = \left|m^2_3 - m^2_1\right| \approx 2.5 \times 10^{-3}\text{eV}^2$$
$$\Delta m^2_{21} = m^2_2 - m^2_1 \approx 7.5 \times 10^{-5}\text{eV}^2$$

- Only the mass scale is known, the exact mass value yet be able to be measured.



PMNS matrix parameterization

- The matrix can be parameterized with three mixing angles $\theta_{13}, \theta_{12}, \theta_{23}$ and one irreducible Dirac CP-violation phase δ_{CP}
- $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$
- If the neutrino is a Majorana particle, there are two extra CP-violation phases, ρ_1 and ρ_2, which do not affect neutrino oscillations.

$$U = U(\theta_{23})U(\theta_{13}, \delta_{\mathrm{CP}})U(\theta_{12})U_M(\alpha_1, \alpha_2)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\mathrm{CP}}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Constraints on neutrino mass

- From cosmology: `arxiv.org/abs/1807.06209`
  (Results from Planck and Baryon Acoustics Oscillation)

$$\sum_{N_\nu} m_{\nu_i} = m_{\nu_1} + m_{\nu_2} + m_{\nu_3} < 0.12\text{eV at } 90\% \text{ C.L}$$

where $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$ are three mass eigenstates of neutrino

- Beta decay (KATRIN experiment): `arxiv.org/abs/2404.08733v1`

$$(A, Z) \to (A, Z+1) + e^- + \bar{\nu}_i$$

$$\langle m_{\nu_\beta} \rangle = \sqrt{|U_{e_1}|^2 m_{\nu_1}^2 + |U_{e_2}|^2 m_{\nu_2}^2 + |U_{e_3}|^2 m_{\nu_3}^2}$$

$$\langle m_{\nu_\beta} \rangle < 0.8\text{eV at } 90\% \text{ C.L}$$

where $U_{e_1}, U_{e_2}, U_{e_3}$ are elements of the PMNS matrix

Constraints on neutrino mass

- Neutrinoless double beta decay: `arxiv.org/abs/2406.11438v1` (KAMLand-Zen experiment, only if neutrinos are Majorana particles)

$$(A, Z) \rightarrow (A, Z+2) + 2e$$

$$\left\langle m_{\beta\beta}^{0\nu} \right\rangle = \sum_i U_{ei}^2 m_{\nu_i} < 28 - 122\text{meV at 90\% C.L}$$

$$= \left| m_{\nu_1} cos^2\theta_{13} cos^2\theta_{12} + m_{\nu_2} cos^2\theta_{13} sin^2\theta_{12} e^{i\rho_1} + m_{\nu_3} cos^2\theta_{13} e^{i\rho_2} \right|$$

where: θ_{13}, θ_{12} are the mixing angles

ρ_1, ρ_2 are two Majorana phases, from 0 to π

Emergence of ν mass sum rule from flavor symmetries

- Neutrino mass matrix M_ν, in general, depends on six complex parameters
- Unitary transformation diagonalises M_ν and leads to three complex mass eigenvalues, which in principle, is uncorrelated.
- When a flavor symmetry is assumed to govern the gauge transformation of the leptons, electroweak symmetry breaking will result in an effective neutrino mass matrix $M_\nu^{eff.}$, which is parameterized with fewer parameters
- Diagonalising the $M_\nu^{eff.}$ will lead to three complex mass eigenstates $(\tilde{m}_1, \tilde{m}_2, \tilde{m}_3)$ and additional relationship(s) among these complex mass eigenstates $\rightarrow$ neutrino mass sum rules

Sum rules

Sum Rule	Group	Seesaw Type
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	A_4[158, 247–251]; S_4[252]; A_5[75]	Weinberg
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	$\Delta(54)$[253]; S_4[254]	Type II
$\tilde{m}_1 + 2\tilde{m}_2 = \tilde{m}_3$	S_4[255]	Type II
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4 [117, 118, 158, 237, 248–251, 256–262]	Weinberg
	S_4[133, 263]; T'[156, 246, 264–267]; T_7[268]	
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4[269]	Type II
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	S_4[270]	Dirac
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	$L_e - L_\mu - L_\tau$[271]	Type II
$\tilde{m}_1 + \frac{\sqrt{3}+1}{2}\tilde{m}_3 = \frac{\sqrt{3}-1}{2}\tilde{m}_2$	A_5'[272]	Weinberg
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	A_4[158]; S_4[247, 254]; A_5[78, 166]	Type I
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	S_4[254]	Type III
$2\tilde{m}_2^{-1} + \tilde{m}_3^{-1} = \tilde{m}_1^{-1}$	A_4[118, 158, 235, 237, 238, 245, 273–281]; T'[246]	Type I
$\tilde{m}_1^{-1} + \tilde{m}_3^{-1} = 2\tilde{m}_2^{-1}$	A_4[282–284]; T'[285]	Type I
$\tilde{m}_3^{-1} \pm 2i\tilde{m}_2^{-1} = \tilde{m}_1^{-1}$	$\Delta(96)$[286]	Type I
$\tilde{m}_1^{1/2} - \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4[236]	Type I
$\tilde{m}_1^{1/2} + \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4[287]	Scotogenic
$\tilde{m}_1^{-1/2} + \tilde{m}_2^{-1/2} = 2\tilde{m}_3^{-1/2}$	S_4[240]	Inverse

Table 1. Sum rules for complex light neutrino mass eigenvalues ($\tilde{m}_i$) derived from combinations of neutrino mass generation mechanisms and discrete flavor symmetries (`arxiv.org/pdf/2310.20681`)

Complex mass in mass rules

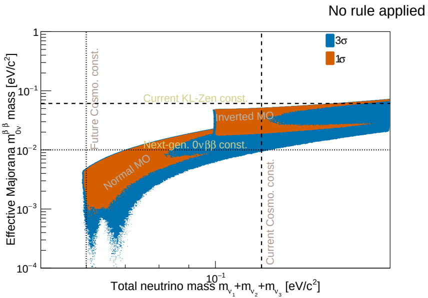
- Complex mass in the polar form $\tilde{m}_i = m_i e^{i\phi_i}$, where m_i is the physical mass eigenstate; in general form, $\phi_i \in [0; 2\pi]$ but α_{21} and α_{31} is from $[0; \pi]$ following the reference `arxiv.org/abs/0804.3627`
- It connects to standard parameterization α_{21} and α_{31} by the PMNS matrix
- Assume a sum rule $\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$
  $\rightarrow m_1 e^{i\phi_1} + m_2 e^{i\phi_2} = m_3 e^{i\phi_3}$
  $\rightarrow m_1 + m_2 e^{i(\phi_2 - \phi_1)} = m_3 e^{i(\phi_3 - \phi_1)}$
  $\rightarrow m_1 + m_2(cos\alpha_{21} + isin\alpha_{21}) = m_3(cos\alpha_{31} + isin\alpha 31)$
- Two relations can be concluded:
  Real: $m_1 + m_2 cos\alpha_{21} = m_3 cos\alpha_{31}$
  Imaginary: $m_2 sin\alpha_{21} = m_3 sin\alpha_{31}$
- $m_2, m_3 = \mathrm{f}(m_1, \alpha_{21}, \alpha_{31})$

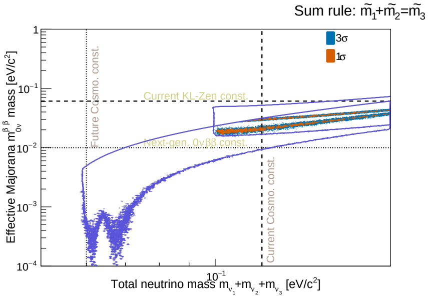
Input constraints

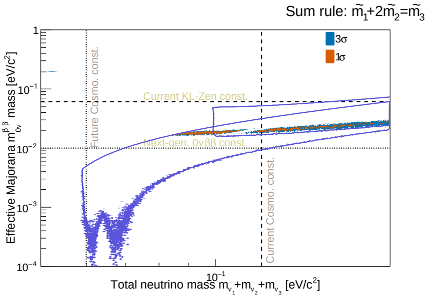
	Current (eV)	Future (eV)
Beta decay (KATRIN)	0.8	0.2
Neutrinoless double beta decay (KAMLand-Zen2016)	0.061 - 0.175	0.02
Cosmology	0.12	0.06

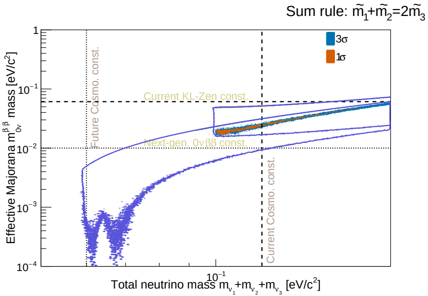
NuFIT 5.0 (2020)

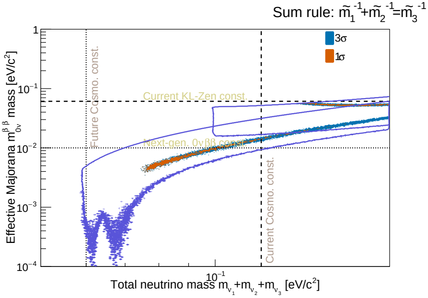
without SK atmospheric data	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ eV^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ eV^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

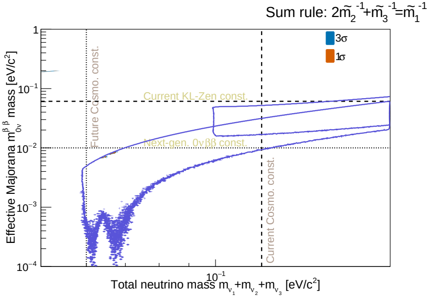

No rule applied
3σ
1σ
Current KL-Zen const.
Next-gen. 0νββ const.
Future Cosmo. const.
Current Cosmo. const.
Inverted MO
Normal MO
Effective Majorana $m^{\beta\beta}_{0\nu}$ mass [eV/c^2]
Total neutrino mass $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}$ [eV/c^2]

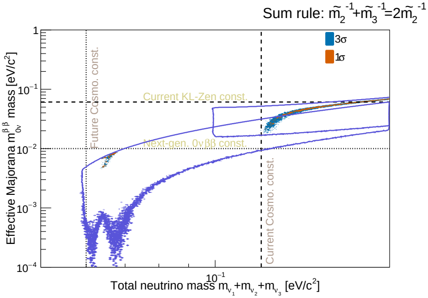

Sum rule: $\tilde{m}_1+\tilde{m}_2=\tilde{m}_3$
3σ
1σ
Current KL-Zen const.
Next-gen. 0νββ const.
Future Cosmo. const.
Current Cosmo. const.
Effective Majorana $m^{\beta\beta}_{0\nu}$ mass [eV/c^2]
Total neutrino mass $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}$ [eV/c^2]

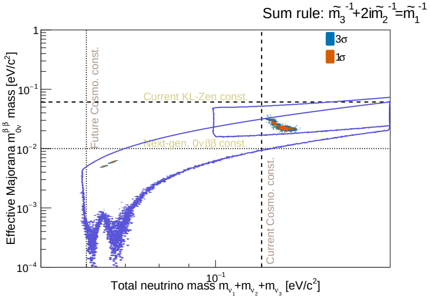

Sum rule: $\tilde{m}_1+2\tilde{m}_2=\tilde{m}_3$
3σ
1σ
Current KL-Zen const.
Next-gen. 0νββ const.
Future Cosmo. const.
Current Cosmo. const.
Effective Majorana $m^{\beta\beta}_{0\nu}$ mass [eV/c^2]
Total neutrino mass $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}$ [eV/c^2]

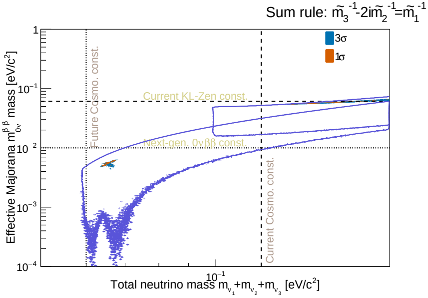

Sum rule: $\tilde{m}_1+\tilde{m}_2=2\tilde{m}_3$
3σ
1σ
Current KL-Zen const.
Next-gen. 0νββ const.
Future Cosmo. const.
Current Cosmo. const.
Effective Majorana $m^{\beta\beta}_{0\nu}$ mass [eV/c^2]
Total neutrino mass $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}$ [eV/c^2]

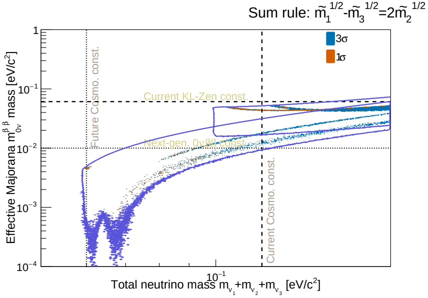

Sum rule: $\tilde{m}_1^{-1}+\tilde{m}_2^{-1}=\tilde{m}_3^{-1}$
3σ
1σ
Future Cosmo. const.
Current Cosmo. const.
Current KL-Zen const.
Next-gen. 0νββ const.
Effective Majorana $m_{0\nu}^{\beta\beta}$ mass [eV/c^2]
Total neutrino mass $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}$ [eV/c^2]

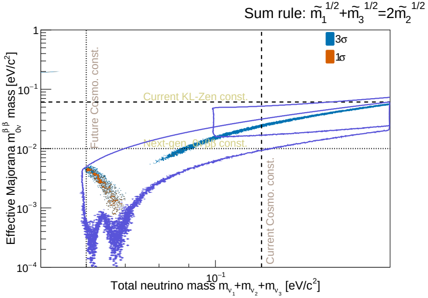

Sum rule: $2\tilde{m}_2^{-1}+\tilde{m}_3^{-1}=\tilde{m}_1^{-1}$
3σ
1σ
Future Cosmo. const.
Current Cosmo. const.
Current KL-Zen const.
Next-gen. 0νββ const.
Effective Majorana $m_{0\nu}^{\beta\beta}$ mass [eV/c^2]
Total neutrino mass $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}$ [eV/c^2]

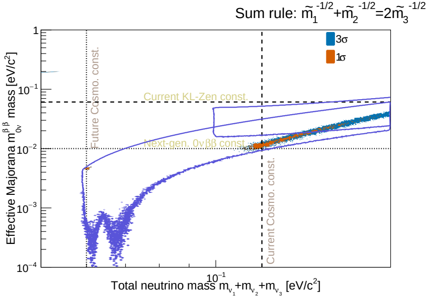

Sum rule: $\tilde{m}_2^{-1}+\tilde{m}_3^{-1}=2\tilde{m}_2^{-1}$
3σ
1σ
Future Cosmo. const.
Current Cosmo. const.
Current KL-Zen const.
Next-gen. 0νββ const.
Effective Majorana $m_{0\nu}^{\beta\beta}$ mass [eV/c^2]
Total neutrino mass $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}$ [eV/c^2]


Sum rule: $\tilde{m}_3^{-1}+2i\tilde{m}_2^{-1}=\tilde{m}_1^{-1}$
3σ
1σ
Future Cosmo. const.
Current Cosmo. const.
Current KL-Zen const.
Next-gen. 0νββ const.
Effective Majorana $m_{0\nu}^{\beta\beta}$ mass [eV/c^2]
Total neutrino mass $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}$ [eV/c^2]


Sum rule: $\tilde{m}_3^{-1}-2i\tilde{m}_2^{-1}=\tilde{m}_1^{-1}$
3σ
1σ
Current KL-Zen const.
Next-gen. 0νββ const.
Future Cosmo. const.
Current Cosmo. const.
Effective Majorana $m_{0\nu}^{\beta\beta}$ mass [eV/c^2]
Total neutrino mass $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}$ [eV/c^2]


Sum rule: $\tilde{m}_1^{1/2}-\tilde{m}_3^{1/2}=2\tilde{m}_2^{1/2}$
3σ
1σ
Current KL-Zen const.
Next-gen. 0νββ const.
Future Cosmo. const.
Current Cosmo. const.
Effective Majorana $m_{0\nu}^{\beta\beta}$ mass [eV/c^2]
Total neutrino mass $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}$ [eV/c^2]


Sum rule: $\tilde{m}_1^{1/2}+\tilde{m}_3^{1/2}=2\tilde{m}_2^{1/2}$
3σ
1σ
Current KL-Zen const.
Next-gen. 0νββ const.
Future Cosmo. const.
Current Cosmo. const.
Effective Majorana $m_{0\nu}^{\beta\beta}$ mass [eV/c^2]
Total neutrino mass $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}$ [eV/c^2]


Sum rule: $\tilde{m}_1^{-1/2}+\tilde{m}_2^{-1/2}=2\tilde{m}_3^{-1/2}$
3σ
1σ
Current KL-Zen const.
Next-gen. 0νββ const.
Future Cosmo. const.
Current Cosmo. const.
Effective Majorana $m_{0\nu}^{\beta\beta}$ mass [eV/c^2]
Total neutrino mass $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}$ [eV/c^2]

Conclusion

Sum Rule	Group	Seesaw Type	
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	A_4[158, 247–251]; S_4[252]; A_5[75]	Weinberg	Largely allowed
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	$\Delta(54)$[253]; S_4[254]	Type II	
$\tilde{m}_1 + 2\tilde{m}_2 = \tilde{m}_3$	S_4[255]	Type II	Largely allowed
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4 [117, 118, 158, 237, 248–251, 256–262]	Weinberg	
	S_4[133, 263]; T'[156, 246, 264–267]; T_7[268]		Disfavored
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4[269]	Type II	
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	S_4[270]	Dirac	Largely allowed
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	$L_e - L_\mu - L_\tau$[271]	Type II	
$\tilde{m}_1 + \frac{\sqrt{3}+1}{2}\tilde{m}_3 = \frac{\sqrt{3}-1}{2}\tilde{m}_2$	A_5'[272]	Weinberg	Disfavored
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	A_4[158]; S_4[247, 254]; A_5[78, 166]	Type I	Largely allowed
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	S_4[254]	Type III	Largely allowed
$2\tilde{m}_2^{-1} + \tilde{m}_3^{-1} = \tilde{m}_1^{-1}$	A_4[118, 158, 235, 237, 238, 245, 273–281]; T'[246]	Type I	Marginally allowed
$\tilde{m}_1^{-1} + \tilde{m}_3^{-1} = 2\tilde{m}_2^{-1}$	A_4[282–284]; T'[285]	Type I	
$\tilde{m}_3^{-1} \pm 2i\tilde{m}_2^{-1} = \tilde{m}_1^{-1}$	$\Delta(96)$[286]	Type I	Marginally allowed
$\tilde{m}_1^{1/2} - \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4[236]	Type I	Largely allowed
$\tilde{m}_1^{1/2} + \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4[287]	Scotogenic	Largely allowed
$\tilde{m}_1^{-1/2} + \tilde{m}_2^{-1/2} = 2\tilde{m}_3^{-1/2}$	S_4[240]	Inverse	Marginally allowed

Here we just look at the sum rules in their most basic form, without any correction.

Thank you for your listening!