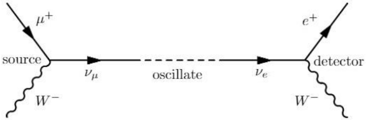
Ultimate Precision of the Leptonic Mixing Angle θ_{23} and its Implications for the Leptonic Flavor Models

Phan To Quyen(1,2)
S. Cao(2), N. T. Hong Van(3), A. Nath(4) and T.V.Ngoc(5)

(1) Graduate University of Science and Technology (GUST, VAST, VietNam).
(2) Institute for Interdisciplinary Research in Science and Education (IFIRSE, ICISE, VN).
(3) Institute of Physics, Vietnam Academy of Science and Technology.
(4) Department of Physics, Namrup College, Assam, India.
(5) Department of Physics, Kyoto University, Kyoto, Japan

Neutrino oscillation framework

- Neutrino oscillation: one type of neutrino flavor "oscillate" to another type of flavor during propagation



- PMNS framework: standard 3-flavor neutrino oscillation

flavor eigenstates **mass eigenstates**

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta_{CP}} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Atmospheric sector **Reactor sector** **Solar sector**

- Oscillation probability depends on oscillation parameters

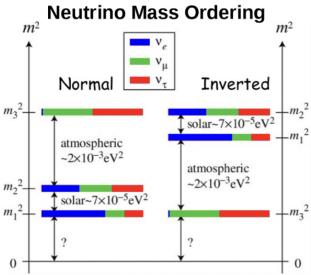
$$P(\nu_\alpha \to \nu_\beta) = P(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP}, \Delta m^2_{21}, \Delta m^2_{31}, L, E, \rho)$$

More detail in talks: José W.F. Valle, Son Cao, Jae Yu... (PASCOS July 8, 2024).

Present landscape of the leptonic mixing

NUFIT 5.2

	Parameter	Best fit	3σ C.L. range
	$\sin^2\theta_{12}$	0.303	0.270-0.341
	$\sin^2\theta_{13}(\times 10^{-2})$	2.203	2.0-2.4
③	$\sin^2\theta_{23}$	0.572	0.406-0.620
①	$\delta_{CP}(^\circ)$	197	108-404
②	$\Delta m^2_{21}(10^{-5}\mathrm{eV}^2/c^4)$	7.41	6.82-8.03
	$\Delta m^2_{31}(10^{-3}\mathrm{eV}^2/c^4)$	2.511	2.428-2.597



Three unsolved questions in neutrino oscillation:

- CP-violation phase in the leptonic mixing matrix?
- Neutrino mass ordering?
- Whether the leptonic mixing angle θ_{23} is maximal or not?

Our objective is the θ_{23} precise measurement

Three unsolved questions in neutrino oscillation:

- CP-violation phase in the leptonic mixing matrix?
- Neutrino mass ordering?
- Whether the leptonic mixing angle θ_{23} is maximal or not?

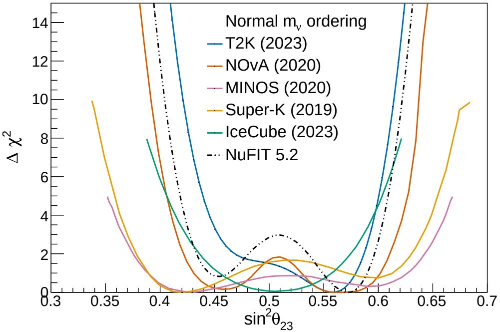
If $\theta_{23} = \pi/4$

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & -\frac{\sqrt{2}}{2}\left(s_{12} + c_{12}s_{13}e^{-i\delta}\right) & \frac{\sqrt{2}}{2}\left(s_{12} - c_{12}s_{13}e^{-i\delta}\right) \\ s_{12}c_{13} & \frac{\sqrt{2}}{2}\left(c_{12} - s_{12}s_{13}e^{-i\delta}\right) & -\frac{\sqrt{2}}{2}\left(c_{12} + s_{12}s_{13}e^{-i\delta}\right) \\ s_{13}e^{i\delta} & \frac{\sqrt{2}}{2}c_{13} & \frac{\sqrt{2}}{2}c_{13} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

→ υ_3 shares the same fractions of flavor-muon and flavor-tau.

→ The value of θ_{23} allows us to test a class of lepton flavor models where the leptonic mixing pattern can be emerged

Current understanding of the θ_{23} mixing angle



	T2K	NOνA	MINOS	Super-K	IceCube	NuFIT 5.2
Constraint $\sin^2\theta_{13}/10^{-2}$	2.18 ± 0.07	2.10 ± 0.11	2.10 ± 0.11	2.10 ± 0.11	2.224 ± 0.11	2.203 ± 0.0575
Best fit $\sin^2\theta_{23}$	$0.561^{+0.019}_{-0.038}$	$0.57^{+0.03}_{-0.04}$	$0.43^{+0.20}_{-0.04}$	$0.425^{+0.051}_{-0.034}$	0.51 ± 0.05	$0.572^{+0.018}_{-0.023}$
Maximal rej.$[\sigma]$	1.22	1.29	0.90	1.25	0.28	1.69
Wrong-octant rej.$[\sigma]$	1.22	0.37	0.53	0.85	0	0.89

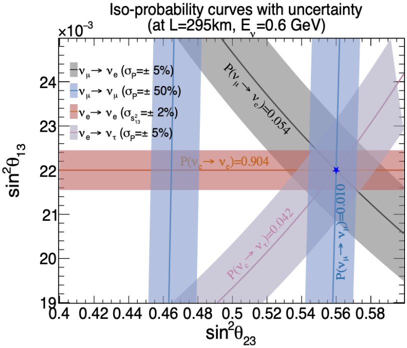
- The current data is consistent to the maximal mixing. However, we cannot tell whether θ_{23} is in the lower octant (LO) or higher octant (HO).

Overall samples to measure θ_{23} mixing angle

○Most effective strategy:
combine disappearance ($\upsilon_\mu \rightarrow \upsilon_\mu$) — appearance ($\upsilon_\mu \rightarrow \upsilon_e$) samples in the accelerator/atmospheric -based exp.

- ○ *Use both neutrinos and anti-neutrinos.*
- ○ *Combine ($\bar{\upsilon}_e \rightarrow \bar{\upsilon}_e$) sample to constrain on θ_{13} from reactor-based exp and improve the θ_{23}-θ_{13} degeneracy.*

○Using ($\upsilon_e \rightarrow \upsilon_\tau$) sample (in neutrino factory) can help.
○Using other baseline/energy also can help.



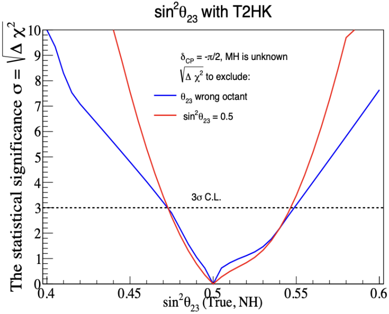
- This job investigates θ_{23} measurement with the accelerator exp. (T2HK) Ref[1] (Jae Yu's talk). The contribution of the samples mentioned above is studied as well.

Two hypothesis tests for θ_{23} precise measurement

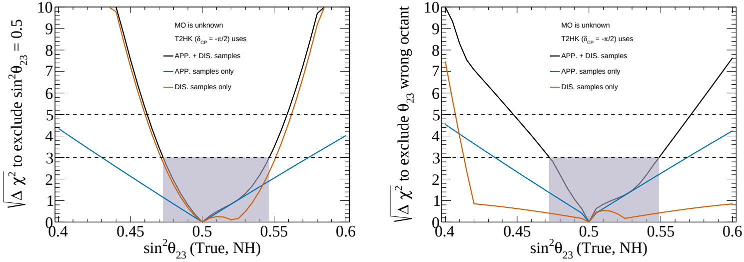
Two approaches for the statistical test in θ_{23} determination:

- First, perform $\sin^2\theta_{23} = 0.5$ hypothesis test by the statistical significance to exclude $\sin^2\theta_{23} = 0.5$.
- We then find the "right" octant by excluding the "wrong" octant hypothesis.

The statistical significance for the former test is typically higher than the later test at > 3σ.



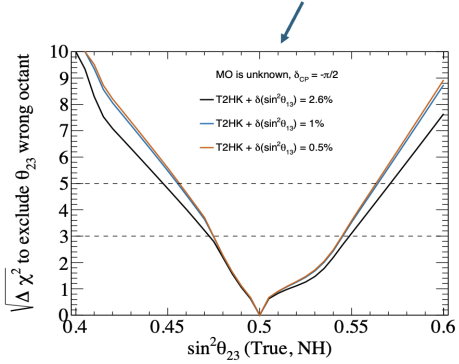
θ_{23} determination with T2HK

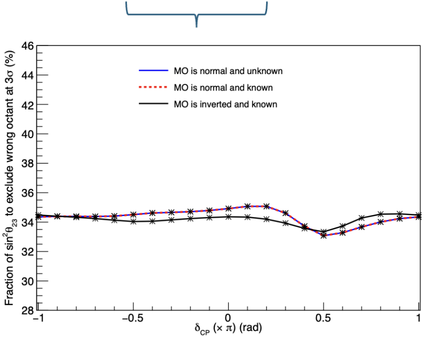


- For excluding $\sin^2\theta_{23} = 0.5$, main contribution is from disappearance measurement unless $0.5 < \sin^2\theta_{23} \leq 0.53$.
- For octant resolving, mostly driven by the appearance. And octant resolvability is better if $\sin^2\theta_{23}$ lies in the lower octant.
- At 3σ, the maximal - enclosed region of $\sin^2\theta_{23}$ is $\sin^2\theta_{23} = [0.473, 0.547]$ while the blind - octant region of $\sin^2\theta_{23}$ is $\sin^2\theta_{23} = [0.473, 0.549]$.

Effects from θ_{13}, δ_{CP} and MO to the octant resolving

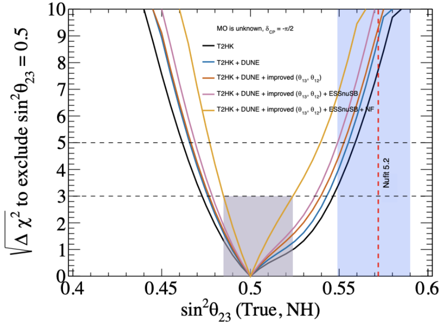
$$P(\nu_\mu \to \nu_e) \approx \boxed{\sin^2 2\theta_{13}}\, \sin^2\theta_{23} \sin^2\frac{\Delta m_{31}{}^2 L}{4E} + \text{Terms}\,\underbrace{(\boxed{\text{CPV}} + \text{CPC} + \boxed{\text{Matter}})}_{}$$





- Improvement of $\sin^2\theta_{13}$ (2.6% to 1.0%), T2HK can narrow the octant-blinded region down to 7.9%(12.1%) at $(3\sigma)(5\sigma)$ respectively.
- $OR_{\theta_{23}}$ has marginal dependence on δ_{CP} and MO determination.
- Systematic and statistics improvement (if possible) impact on the octant

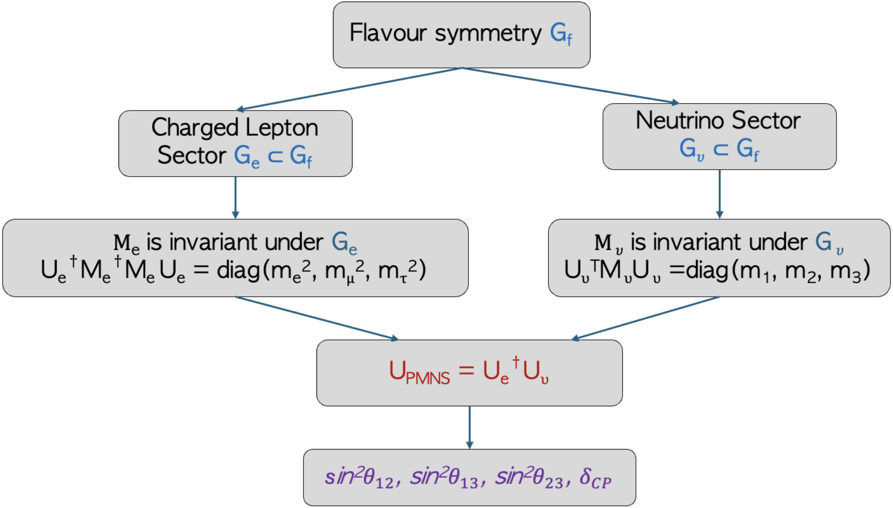
Ultimate reach for θ_{23} precision measurement



Joint analysis	The maximal - enclosed region		
	$\sin^2\theta_{23}(3\sigma)$	$\sin^2\theta_{23}(5\sigma)$	Improved percent at $3\sigma(5\sigma)$ compared to T2HK only
T2HK	0.473 - 0.547	0.462 - 0.559	–
+ DUNE	0.476 - 0.543	0.466 - 0.555	+9.5%(8.2%)
+ improved $(\theta_{13}, \theta_{12})$	0.477 - 0.540	0.467 - 0.553	+14.9%(11.3%)
+ ESSnuSB	0.479 - 0.536	0.469 - 0.549	+23%(17.5%)
+ Neutrino Factory	0.485 - 0.524	0.477 - 0.539	+47.3%(36%)

- The current bound (Nufit 5.2) can be excluded from $\sin^2\theta_{23} = 0.5$ with $\sim 4\sigma$ from (T2HK + DUNE + Reactor) and $\sim 7\sigma$ by joint analysis data.
- The range of $\sin^2\theta_{23} = [0.477, 0.540](3\sigma)$ is a challenge to future experiments (T2HK, DUNE, Reactor) in addressing the θ_{23} maximal mixing.
- Using additional data from ESSnuSB and NF, the maximal-enclosed region will be narrowed down to $\sin^2\theta_{23} = [0.485, 0.524](3\sigma)$ (improve to 47.3% over T2HK only).

Lepton flavour models



Flavour symmetry G_f

Charged Lepton Sector $G_e \subset G_f$

Neutrino Sector $G_\upsilon \subset G_f$

M_e is invariant under G_e

$U_e^\dagger M_e^\dagger M_e U_e = \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)$

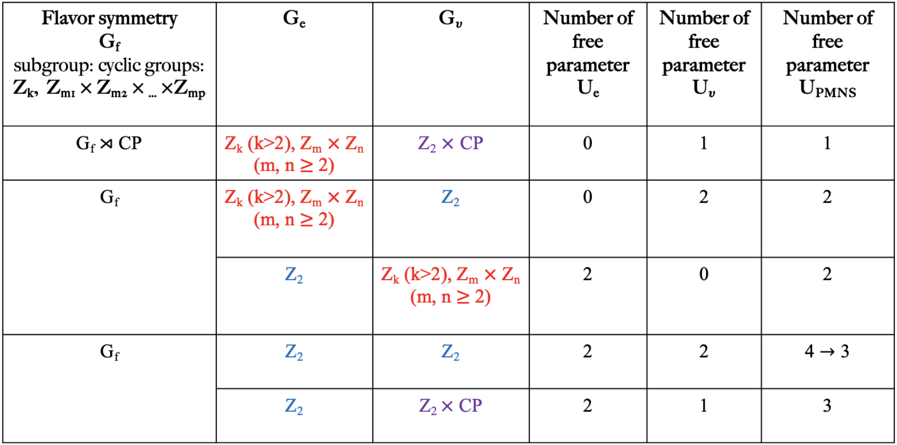
M_υ is invariant under G_υ

$U_\upsilon^T M_\upsilon U_\upsilon = \mathrm{diag}(m_1, m_2, m_3)$

$U_{PMNS} = U_e^\dagger U_\upsilon$

$sin^2\theta_{12}, sin^2\theta_{13}, sin^2\theta_{23}, \delta_{CP}$

Lepton flavour models - $G_f = A_5, S_4$



Flavor symmetry G_f subgroup: cyclic groups: Z_k, $Z_{m1} \times Z_{m2} \times ... \times Z_{mp}$	G_e	G_υ	Number of free parameter U_e	Number of free parameter U_υ	Number of free parameter U_{PMNS}
$G_f \rtimes CP$	Z_k (k>2), $Z_m \times Z_n$ (m, n $\geq$ 2)	$Z_2 \times CP$	0	1	1
G_f	Z_k (k>2), $Z_m \times Z_n$ (m, n $\geq$ 2)	Z_2	0	2	2
	Z_2	Z_k (k>2), $Z_m \times Z_n$ (m, n $\geq$ 2)	2	0	2
G_f	Z_2	Z_2	2	2	4 → 3
	Z_2	$Z_2 \times CP$	2	1	3

Testing one - free parameter models(S_4, A_5) Ref[5]

11 cases are taken in Ref[5]:

○ 6 cases with $G_f = S_4$

+ Case I, Case II , Case IV

+ Case V: $\sin^2\theta_{23} >$ 0.7 (excluded from NuFIT 5.2)

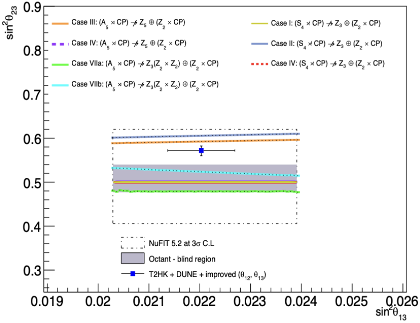
+ Case VI-a, VI-b: $\sin^2\theta_{12} < 0.26$ (excluded fr. NuFIT 5.2)

○ 5 cases with $G_f = A_5$

+ Case II: $\sin^2\theta_{12} < 0.26$ (excluded fr. NuFIT 5.2)

+ Case III, Case IV, Case VIIa, Case VIIb

→ 7 cases are kept in this test.



- The value of θ_{23} and its precision are a useful key for excluding the flavor models.
- Cases: IV(A5), (IV and I)(S4) predict the maximal mixing $\sin^2\theta_{23} = 0.5$.
  Cases: II(S_4), (III & VIIb)(A5) prefer $\sin^2\theta_{23}(HO)$ & Case: VIIa(A_5): $\sin^2\theta_{23}(LO)$.
- Cases: (VIIa and VIIb) (A_5) lie in octant - blind region which the combination T2HK + DUNE + futute improved $(\theta_{12}, \theta_{23})$ precision can't solve yet.
- At best-fit: $\sin^2\theta_{23} = 0.572$, θ_{23} precision from (T2HK + DUNE + Reactor) allows us to exclude all models.

Testing two - free parameter models (S_4, A_5) Ref[5]

8 cases are taken in Ref[5]:

- ○ 2 cases with $G_f = S_4, G_e = Z_3, G_\nu = Z_2$
- ○ 6 cases with $G_f = A_5$

+ 2 cases: ($G_e = Z_2, G_\nu = Z_3$) predict:

($\sin^2\theta_{23}, \cos\delta_{CP}$) ~($0.554, 0.695$), ($0.446, -0.698$)

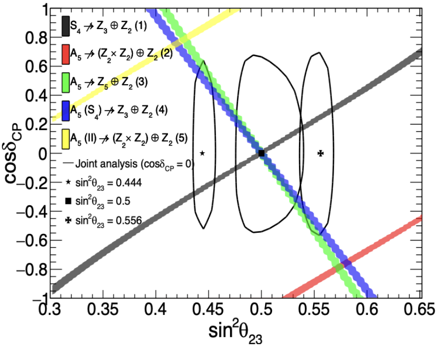
($\cos\delta_{CP} \notin \sin^2\theta_{23}$).

+ 1 case: $G_e = Z_3, G_\nu = Z_2$

+ 1 case: $G_e = Z_5, G_\nu = Z_2$

+ 2 cases: $G_e = Z_2 \times Z_2, G_\nu = Z_2$

→ 6 cases: $\cos\delta_{CP} = f(\sin^2\theta_{23})$



- δ_{CP} and θ_{23} are important information for addressing the flavor models.
- (2): $\cos\delta_{CP} = [-1, -0.6], \sin^2\theta_{23}(HO)$ & (5): $\cos\delta_{CP} = [0.65, 1], \sin^2\theta_{23}(LO)$. (1): $\cos\delta_{CP} = [-0.45, 0.55]$ & (3) and (4): $\cos\delta_{CP} = [-1, 1]$.
- At $\cos\delta_{CP} = 0$ $\sin^2\theta_{23} = [0.444, 0.556]$, Cases (1), (3), (4) have not distinguished with (T2HK + DUNE + improved (θ_{12}, θ_{13})) data yet.

Three free parameters models - Sum rules Ref[5]

- The PMNS matrix get the forms as: $U_{PMNS} = U_e^\dagger U_\nu = (\tilde{U}_e)^\dagger \psi \tilde{U}_\nu Q_0$ and

$$\tilde{U}_e = R_{23}^{-1}(\theta_{23}^e) R_{12}^{-1}(\theta_{12}^e), \ \ \tilde{U}_\nu = R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) \ \ (\theta_{23}^\nu = -\pi/4)$$

(θ_{12}^ν gives $\tilde{U}_\nu$ forms: TBM, BM, GRA, GRB, HG) leads to solar sum rule:

$$\cos\delta_{CP} = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta_{12}^\nu + (\sin^2\theta_{12} - \cos^2\theta_{12}^\nu)(1 - \cot^2\theta_{23}\sin^2\theta_{13})\right]$$

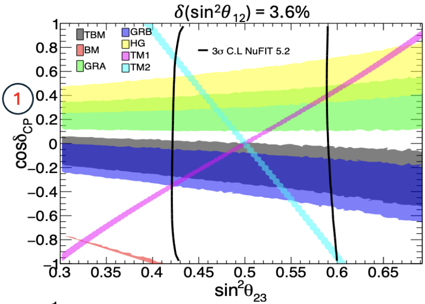
- If the lepton mixing to be of the $TM1$ or $TM2$ form then result to atmospheric sum rules.

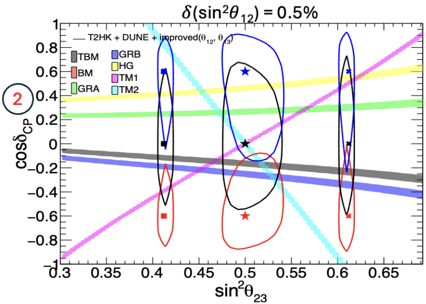
$$U_{\mathrm{TM1}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix} \quad U_{\mathrm{TM2}} \approx \begin{pmatrix} - & \sqrt{\frac{1}{3}} & - \\ - & \sqrt{\frac{1}{3}} & - \\ - & -\sqrt{\frac{1}{3}} & - \end{pmatrix}$$

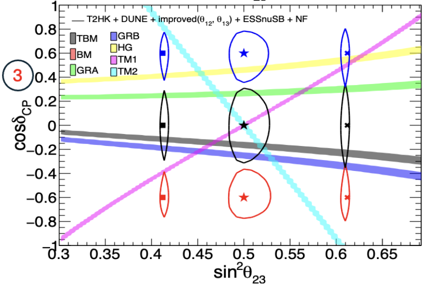
$$\cos\delta_{CP} = -\frac{(1 - 5\sin^2\theta_{13})\cot 2\theta_{23}}{2\sqrt{2}\sin\theta_{13}\sqrt{1 - 3\sin^2\theta_{13}}} \qquad \cos\delta_{CP} = \frac{\cos 2\theta_{13}\cot 2\theta_{23}}{\sin\theta_{13}\sqrt{2 - 3\sin^2\theta_{13}}}$$

(TM1) (TM2)

Testing sum rules







- ○ δ_{CP} and θ_{23} are important information for addressing the models.
- ○ Constrain on θ_{12} from JUNO effects to the distinguish possibility of the models significantly.
- ○ At $\cos\delta_{CP} = 0$, $\sin^2\theta_{23} \sim [0.44, 0.56]$ joint analysis data from (T2HK + DUNE + improved $(\theta_{12}, \theta_{13})$ cannot distinguish 6 models.
- ○ High precision of δ_{CP} allows ESSnuSB to test the models effectively (NF has marginal effect to the sensitivity improvement).

Conclusion

1. Ultimate Precision of the Leptonic Mixing Angle θ_{23}
    - The θ_{23} precise determination does not depend on δ_{CP} and the unknown neutrino mass ordering.
    - The ultimate sub-percent constrain on the $\sin^2\theta_{13}$ and systematic - statistic improvement are helpful to leverage the octant resolving capability.
    - The range of $\sin^2\theta_{23} = [0.479, 0.538](3\sigma)$ is a challenge to future experiments (T2HK, DUNE) in addressing $\sin^2\theta_{23}$ is maximal or/and the θ_{23} octant.
    - With joint possible future neutrino facilities (ESSnuSB and NF),the blind regions will be narrowed down to $\sin^2\theta_{23} = [0.488, 0.516](3\sigma)$.
2. Testing the leptonic flavor models
    - δ_{CP} and $\sin^2\theta_{23}$ are the important key to address the flavor models.
    - High precision on δ_{CP} allows ESSnuSB to test the models effectively.

Acknowledgement

• Phan To Quyen was funded by the Master, PhD Scholarship Programme of Vingroup Innovation Foundation (VINIF), code VINIF.2023.TS.095.

• This study is funded by the National Foundation for Science and Technology Development (NAFOSTED) of Vietnam under Grant No. 103.99-2023.144

References

[1] K. Abe and and others, (2018),
Hyper-Kamiokande Design Report
arXiv:hep-ph 1805.04163.

[2] Strait, James and others, (2016),
Long-Baseline Neutrino Facility (LBNF) and Deep Underground Neutrino Experiment (DUNE)
arXiv:hep-ph 1601.05823.

[3] Rosauro Alcaraz and others, (2022),
Physics potential of the ESSnuSB
doi 10.22323/1.402.0063.

[4] Choubey, S. and others, (2011),
International Design Study for the Neutrino Factory, Interim Design Report
arXiv:hep-ph 1112.2853.

[5] Mattias Blennow, Monojit Ghosh, Tommy Ohlsson, Arsenii Titov, (2020),
Testing Lepton Flavor Models at ESSnuSB
arXiv:hep-ph 2004.00017.

Thank you very much for your attention!