Viable anisotropic k-inflation models in the presence of one-form or two-form fields

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Based on

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(in collab. with Tuyen M. Pham, Duy H. Nguyen, and W. F. Kao)

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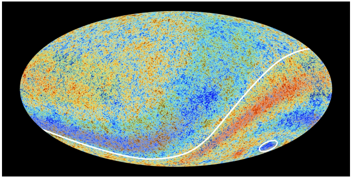
I. Motivations

- Cosmological Principle: our universe is just simply homogeneous and isotropic on large scales as described by the spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime:

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right),$$

- The cosmological principle has played as a basic assumption of all standard inflationary models.
- Two anomalies, the hemispherical asymmetry and the cold spot of the cosmic microwave background (CMB) temperature, which was firstly observed by WMAP and then confirmed by Planck, cannot be explained by all standard inflationary models based on Cosmological Principle.



Two CMB anamolies. (Credit: Planck collaboration)

I. Motivations

- If the cosmological principle was broken down in the early universe, would it still be invalid in the late time universe ?
- Recently, there have been some recent observational studies claiming that the current universe might be anisotropic, in contrast to the prediction of the cosmic no-hair conjecture !? c.f. Colin et al., A&A631(2019)L13

A&A 631, L13 (2019)

Letter to the Editor

Evidence for anisotropy of cosmic acceleration*

Jacques Colin[1], Roya Mohayaee[1], Mohamed Rameez[2] and Subir Sarkar[3]

- There has existed the cosmic no-hair conjecture proposed by Hawking and his colleagues claiming that a final state of our universe should be homogeneous and isotropic, regardless of any inhomogeneous and/or anisotropic initial states. A complete proof for this conjecture, however, has been a great challenge for several decades.

  c.f. Gibbons and Hawking, PRD15(1977)2738; Hawking and Moss, PLB110(1982)35; Wald, PRD28(1983)2118; Kleban and Senatore, JCAP10(2016)022; Carroll and Chatwin-Davies, PRD97(2018)046012.

- The above anomalies suggest that the cosmic no-hair conjecture may be violated !
  $\rightarrow$ How to violate this conjecture ?

I. Motivations

- Recently, a counterexample to the cosmic no-hair conjecture has been found by Kanno, Soda, and Watanabe (KSW) in a model, in which an unusual, supergravity-motivated coupling between scalar and vector fields, $f^2(\phi)F_{\mu\nu}F^{\mu\nu}$, is introduced,

$$S_{\rm KSW} = \int d^4x\sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right].$$

c.f. Watanabe, Kanno, and Soda, PRL102(2009)191302; Kanno, Soda, and Watanabe, JCAP12(2010)024.

- Other counterexamples have also been confirmed to exist in some non-canonical extensions of the KSW model, e.g., the string-inspired Dirac-Born-Infeld model.

c.f. Do and Kao, PRD84(2011)123009; Ohashi, Soda, and Tsujikawa, PRD88(2013)103517.

- Stable anisotropic inflation has been shown to appear in a modified scenario of KSW model, in which the one-form field (a.k.a. vector field) is replaced by a two-form field.

c.f. Ohashi, Soda, and Tsujikawa, PRD87(2013)083520; JCAP12(2013)009; Ito and Soda, PRD92(2015)123533.

- What if a combination of non-canonical scalar and two-form fields admits stable anisotropic inflation ?
- Which field is the better player in the light of Planck data, one-form field or two-form field ?

II. Non-canonical anisotropic inflation models

- A general action of non-canonical anisotropic inflation based on one-form field (a.k.a. vector field) A_μ is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + P(\phi, X) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right].$$

- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the one-form field A_μ.
- A general action of non-canonical anisotropic inflation based on two-form field (a.k.a. Kalb-Ramond field) $B_{\mu\nu}$ is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + P(\phi, X) - \frac{1}{12}f^2(\phi)H_{\mu\nu\rho}H^{\mu\nu\rho}\right].$$

- $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ is the field strength of $B_{\mu\nu}$.
- $P(\phi, X)$ is an arbitrary function of scalar field ϕ and its kinetic term defined as $X \equiv -(1/2)\partial_\mu\phi\partial^\mu\phi$.

  c.f. Armendariz-Picon, Damour, and Mukhanov, PLB458(1999)209.

- We consider the form of $P(\phi, X)$ coming from the well-known k-inflation PLB458(1999)209:

$$P(\phi, X) = K(\phi)X + L(\phi)X^2,$$

here $K(\phi)$ and $L(\phi)$ are arbitrary functions of ϕ.

II. Non-canonical anisotropic inflation models

- Bianchi type I spacetime, which is homogeneous but anisotropic:

$$ds^2 = -dt^2 + \exp\left[2\alpha(t) - 4\sigma(t)\right] dx^2 + \exp\left[2\alpha(t) + 2\sigma(t)\right] \left(dy^2 + dz^2\right).$$

- $\sigma(t)$ acts as a deviation from the isotropy determined by $\alpha(t)$, i.e., $\sigma(t) \ll \alpha(t)$.
- One-form field: $A_\mu(t) = [0, A_x(t), 0, 0]$.
- Two-form field: $\frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^\nu = v_B(t) dy \wedge dz$.
- Choosing the following ansatz:

$$\alpha = \zeta \log(t), \;\; \sigma = \eta \log(t), \;\; \phi = \xi \log(t) + \phi_0$$

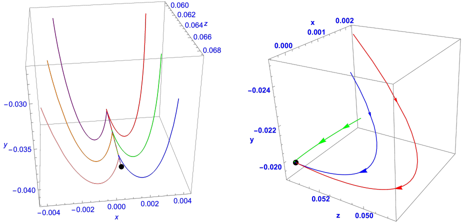
along with exponential functions:

$$K(\phi) = k_0 e^{\kappa\phi}, \;\; L(\phi) = l_0 e^{\lambda\phi}, \;\; f(\phi) = f_0 e^{\rho\phi}$$

$\rightarrow$ power-law inflation: $\exp\left[2\alpha(t) - 4\sigma(t)\right] = t^{2\zeta - 4\eta}$, $\exp\left[2\alpha(t) + 2\sigma(t)\right] = t^{2\zeta + 2\eta}$.

II. Non-canonical anisotropic inflation models

- Non-trivial couplings between scalar and one-(two-)form fields all admit stable anisotropic inflationary solutions:



(From left to right) Attractive behavior of anisotropic inflationary solution of k-inflation one-form and two-form models, respectively.

- Small anisotropy:

$$\frac{\Sigma}{H} \equiv \frac{\dot{\sigma}}{\dot{\alpha}} = \frac{\eta}{\zeta} \simeq -\frac{\lambda}{4\rho} \quad (k\text{-inflation one-form field model})$$

$$\frac{\Sigma}{H} \equiv \frac{\dot{\sigma}}{\dot{\alpha}} = \frac{\eta}{\zeta} \simeq \frac{3\lambda}{19\rho} \quad (k\text{-inflation two-form field model})$$

$\rightarrow |\Sigma/H| \ll 1$ if $|\lambda| \ll \rho$ during the inflationary phase.

III. Tensor-to-scalar ratio

- Tensor-to-scalar ratios: The most important test of any inflationary models.
- Since the statistical isotropy of CMB is broken, the scalar power spectrum is modified as Ackerman, Carroll, & Wise, PRD75(2007)083502

$$\boxed{\mathcal{P}_k^{\zeta(0)} \to \mathcal{P}_{k,\mathrm{ani}}^{\zeta} = \mathcal{P}_k^{\zeta(0)} \left(1 + g_* \cos^2\theta\right)}.$$

  - g_* characterizes the deviation from the spatial isotropy, i.e., $|g_*| < 1$.
  - θ is the angle between the comoving wave number **k** with the *privileged* direction **V** close to the ecliptic poles.
  - $\mathcal{P}_k^{\zeta(0)}$, the isotropic scalar power spectrum ($g_* = 0$), for non-canonical scalar field is defined as

$$\boxed{\mathcal{P}_k^{\zeta(0)} = \mathcal{P}_{k,\mathrm{nc}}^{\zeta(0)} = \frac{1}{8\pi^2 M_p^2} \frac{H^2}{c_s \epsilon}\bigg|_{c_s^* k_* = a_* H_*}}$$

  where $c_s^2 \equiv \partial_X p / \partial_X \rho \leq 1$ is the speed of sound of scalar perturbation and $\epsilon \equiv -\dot{H}/H^2 \ll 1$ is the slow-roll parameter.
  c.f. Armendariz-Picon, Damour, & Mukhanov, PLB458(1999)219

III. Tensor-to-scalar ratio

- Observational constraints of g_*:
  - $g_* = 0.29 \pm 0.031$ at 9σ using the 5-year WMAP data.
    c.f. Groeneboom, Ackerman, Wehus, & Eriksen, AJ722(2010)452.
  - $g_* = 0.002 \pm 0.016$ at 68% CL using the Planck 2013 data.
    c.f. Kim & Komatsu, PRD88(2013)011301(R).
  - $|g_*| < 0.072$ at 95% CL using the 9-year WMAP data.
    c.f. Ramazanov & G. Rubtsov, PRD89(2014)043517.
  - $-0.041 < g_* < 0.036$ at 95% CL using the Planck 2015 data.
    c.f. Ramazanov, Rubtsov, Thorsrud, & Urban, JCAP03(2017)039.
  - $-0.09 < g_* < 0.08$ at 95% CL using the LSS surveys data.
    c.f. Sugiyama, Shiraishi, & Okumura, MNRAS473(2018)2737.

- Our goal: Calculate the corresponding g_* for non-canonical anisotropic inflation using the standard Bunch-Davies (BD) vacuum state for the non-canonical scalar field:

$$\boxed{\zeta_{\rm nc}^{(0)}(k,\eta) = \frac{H}{2\sqrt{c_s\epsilon}M_p k^{3/2}}\left(1 + ic_s k\eta\right)e^{-ic_s k\eta}},$$

where superscript (0) denotes the (approximated) de Sitter background.

c.f. Chen, Huang, Kachru, & Shiu, JCAP01(2007)002

III. Tensor-to-scalar ratio (one-form field case)

- The full power spectrum in the Heisenberg interaction picture for the scalar perturbation, up to the second order, is given by

$$\langle 0|\hat{\zeta}_{\rm nc}(k,\eta)\hat{\zeta}_{\rm nc}(k',\eta)|0\rangle \simeq \frac{2\pi^2}{k^3}\delta^3(\mathbf{k}+\mathbf{k}')\mathcal{P}^{\zeta(0)}_{k,{\rm nc}} + \frac{2\pi^2}{k^3}\delta^3(\mathbf{k}+\mathbf{k}')\frac{c_s^4 E_x^2 N_{c_s k}^2}{\pi^2\epsilon^2 M_p^4}\sin^2\theta,$$

with $N_{c_s k} \simeq 60$ the e-fold number and $E_x \equiv (f/a^2)A_x^{(0)\prime}$. This implies

- $\mathcal{P}^{\zeta}_{k,{\rm nc}} = \mathcal{P}^{\zeta(0)}_{k,{\rm nc}}\left(1+\frac{8c_s^5 E_x^2 N_{c_s k}^2}{\epsilon M_p^2 H^2}\sin^2\theta\right) \simeq \boxed{\mathcal{P}^{\zeta(0)}_{k,{\rm nc}}\left(1-\frac{8c_s^5 E_x^2 N_{c_s k}^2}{\epsilon M_p^2 H^2}\cos^2\theta\right)}.$
- $\boxed{g_* = -c_s^5\frac{8E_x^2 N_{c_s k}^2}{\epsilon M_p^2 H^2} = c_s^5 g_*^0 < 0}$,

  where $\boxed{g_*^0 = -\frac{8E_x^2 N_{c_s k}^2}{\epsilon M_p^2 H^2} < 0}$ for canonical anisotropic inflation.

  $\rightarrow |g_*| \ll |g_*^0|$ if $c_s^2 \ll 1$.
- Scalar spectral index:

$$n_s - 1 \equiv \left.\frac{d\ln\mathcal{P}^{\zeta}_{k,{\rm nc}}}{d\ln k}\right|_{c_s^* k_* = a_* H_*} \simeq \boxed{-2\epsilon-\tilde{\eta}-s+\left(\frac{2}{N_{c_s k}}-5s\right)\frac{2g_*}{3-2g_*}}.$$

III. Tensor-to-scalar ratio (one-form field case)

- The full tensor power spectrum for the non-canonical scalar field is given by

$$\langle 0|\hat{h}_{ij}(\mathbf{k})\hat{h}_{ij}(\mathbf{k}')|0\rangle = \frac{2\pi^2}{k^3}\delta^3(\mathbf{k}+\mathbf{k}')\left(\mathcal{P}^{h(0)}_{k,\mathrm{nc}} + \frac{4E_x^2 N_k^2}{\pi^2 M_p^4}\sin^2\theta\right),$$

which implies $\boxed{\mathcal{P}^{h}_{k,\mathrm{nc}} \simeq \mathcal{P}^{h(0)}_{k,\mathrm{nc}}\left(1 - \frac{\epsilon g_*^0}{4}\sin^2\theta\right)}$, similar to that of canonical scalar field.

- Here, $\boxed{\mathcal{P}^{h(0)}_{k,\mathrm{nc}}(\mathbf{k}) = \frac{2}{\pi^2}\frac{H^2}{M_p^2}\Big|_{k_*=a_*H_*} = 16c_s\epsilon\mathcal{P}^{\zeta(0)}_{k,\mathrm{nc}}(\mathbf{k})}$ is the isotropic tensor power spectrum for non-canonical scalar field.
c.f. Armendariz-Picon, Damour, & Mukhanov, PLB458(1999)219.

- Tensor spectral index: $n_t \equiv \frac{d\ln\mathcal{P}^h_{k,\mathrm{nc}}}{d\ln k}\Big|_{k_*=a_*H_*} \simeq -2\epsilon.$

III. Tensor-to-scalar ratio (one-form field case)

- For non-canonical isotropic inflation: $r_{\rm nc}^{iso} = 16c_s\epsilon$.

  c.f. Armendariz-Picon, Damour, & Mukhanov, PLB458(1999)219.

- The full tensor-to-scalar ratio for non-canonical anisotropic inflation:

$$r_{\rm nc}^{\rm one-form} \equiv \frac{\mathcal{P}^{h}_{k,{\rm nc}}}{\mathcal{P}^{\zeta}_{k,{\rm nc}}} = 16c_s\epsilon\frac{1-\frac{1}{4}\epsilon g_*^0\sin^2\theta}{1-c_s^5 g_*^0\sin^2\theta} \simeq \boxed{16c_s\epsilon\frac{6-\epsilon g_*^0}{6-4c_s^5 g_*^0}},$$

  with the average value of $\sin^2\theta$ as $\langle\sin^2\theta\rangle = 2/3$ Ohashi, Soda, &Tsujikawa, JCAP12(2013)009.

- In the canonical limit $c_s \to 1$, the above formula will recover that derived for canonical scalar field.

  c.f. Ohashi, Soda, &Tsujikawa, JCAP12(2013)009.

III. Tensor-to-scalar ratio (two-form field case)

- The corresponding full tensor-to-scalar ratio for non-canonical anisotropic inflation when the one-form field is replaced by the two-form field:

$$r_{\rm nc}^{\rm two-form} = \frac{16c_s\epsilon}{1+c_s^5 g_*^0 \cos^2\theta} = \boxed{16 c_s \epsilon \frac{3}{3+c_s^5 g_*^0}},$$

with the average value $\langle \cos^2\theta \rangle = 1/3$ Ohashi, Soda, &Tsujikawa, JCAP12(2013)009.

- Scalar spectra index: $n_s - 1 \simeq \boxed{-2\epsilon - \tilde{\eta} - s - \left(\frac{2}{N_{c_s k}} - 5s\right)\frac{g_*}{3+g_*}}$.

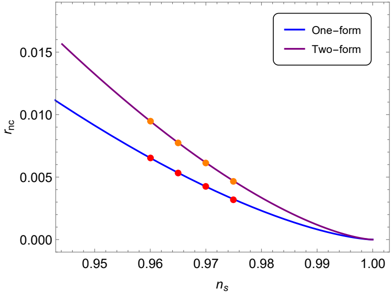
- NOTE: $\boxed{g_* = c_s^5 g_*^0 > 0}$, with $\boxed{g_*^0 = \frac{2E_{yz}^2 N_{c_s k}^2}{\epsilon H^2 M_p^2} > 0}$, in contrast ot the one-form field case.

- In the canonical limit $c_s \to 1$, the above formula will recover that derived for canonical scalar field.

  c.f. Ohashi, Soda, &Tsujikawa, JCAP12(2013)009.

III. Tensor-to-scalar ratio: one-form field vs. two-form field

- For the power law inflation:
  - One-form field case: $c_s^2 \simeq -\frac{\lambda}{48\rho} \ll 1$, $\epsilon \simeq -\frac{\lambda}{\rho} \ll 1$, $\tilde{\eta} = s = 0$.
  - Two-form field case: $c_s^2 \simeq -\frac{5\lambda}{228\rho} \ll 1$, $\epsilon \simeq -\frac{\lambda}{2\rho} \ll 1$, $\tilde{\eta} = s = 0$.



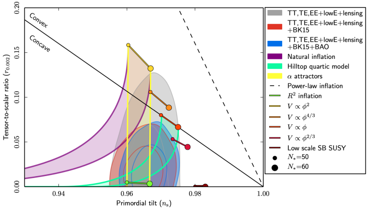


Fig. 8. Marginalized joint 68 % and 95 % CL regions for n_s and r at $k = 0.002\,\mathrm{Mpc}^{-1}$ from *Planck* alone and in combination with BK15 or BK15+BAO data, compared to the theoretical predictions of selected inflationary models. Note that the marginalized joint 68 % and 95 % CL regions assume $dn_s/d\ln k = 0$.

Tensor-to-scalar ratios of anisotropic inflation of k-inflation one-form and two-form models are highly consistent with the latest date of the Planck 2018 [A.&A.641(2020)A10]. The parameters have been chosen as $g_*^0 = -0.03$ (one-form field case), $g_*^0 = +0.03$ (two-form field case), $10^{-2} \leq c_s \leq 10^{-1}$.

- Planck 2018: $r < 0.063$ at 95% CL [A.&A.641(2020)A10].
- CMB-S4: target $r > 0.003$ at $> 5\sigma$ CL [Ap.J.926(2022)54].

IV. Conclusions

- Both one-form and two-form fields can cause stable spatial anisotropies during the inflationary phase due to their non-minimal coupling with the scalar field of k-inflation model.

- Anisotropic k-inflation models in the presence of one-form or two-form fields turn out to be cosmologically viable.

- One-form field seems to be a better player than two-form field in the light of the data of Planck 2018.

Thank you all for your attention !