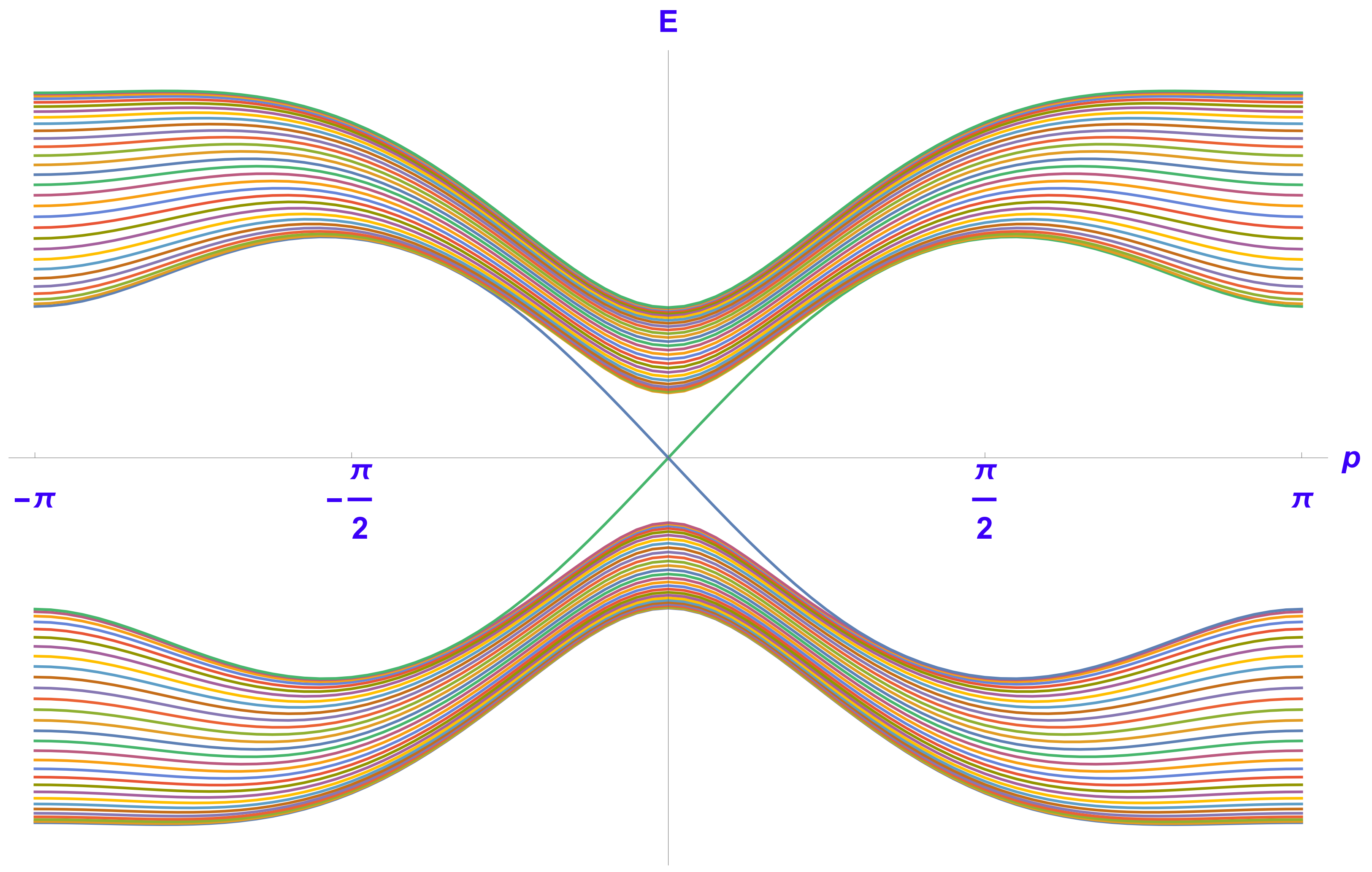
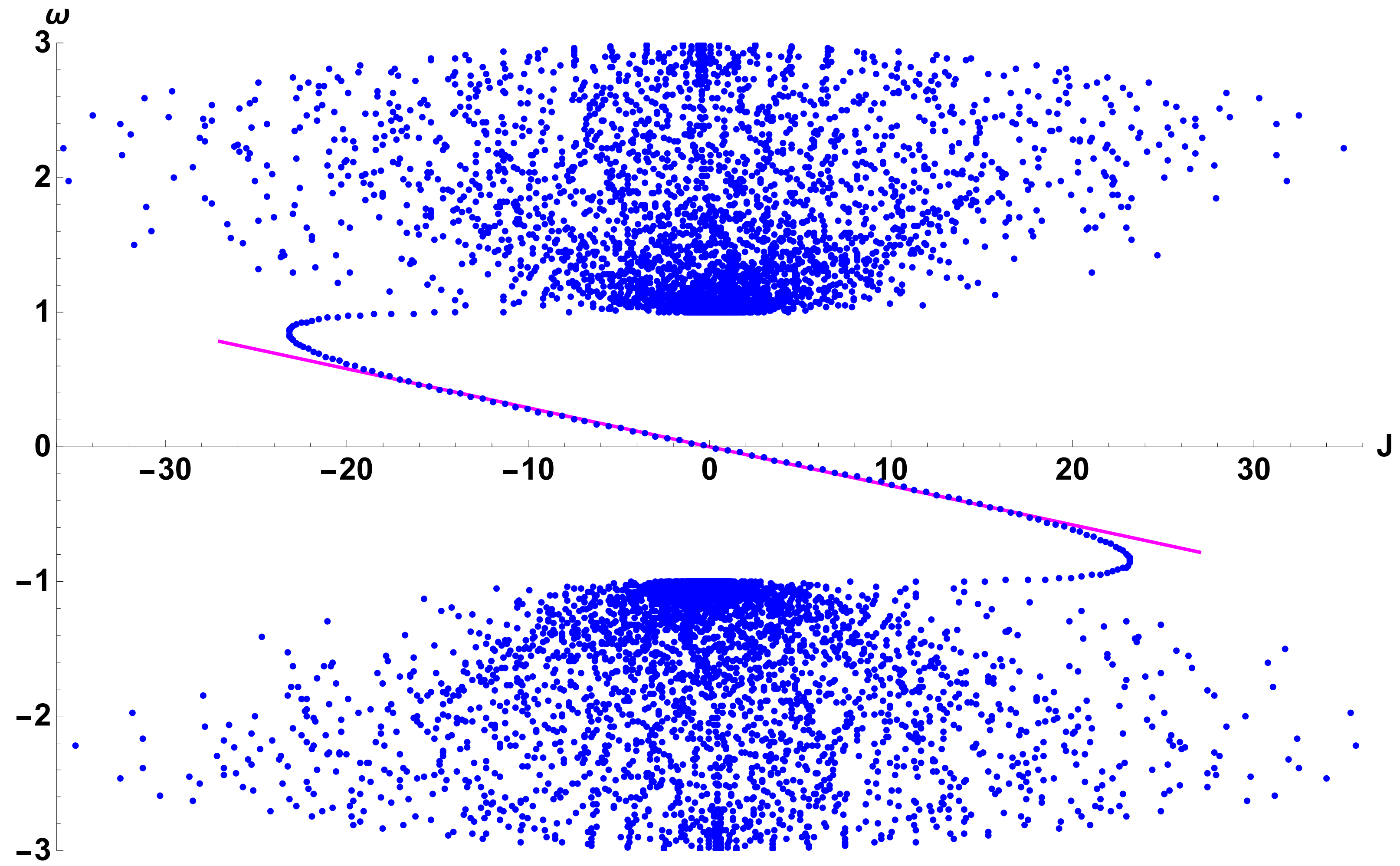
The Standard Model on the Lattice





What this talk is not:

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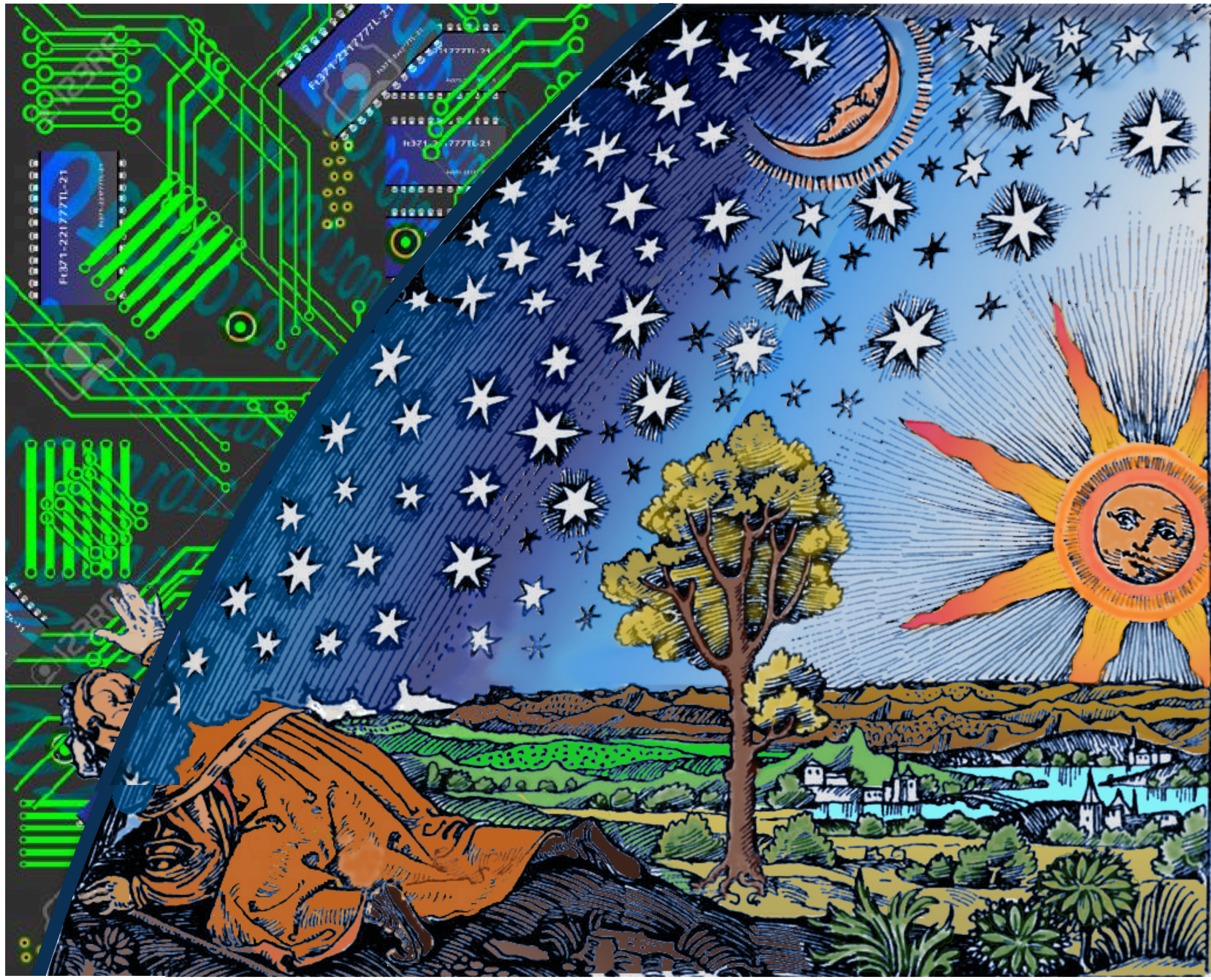
What this talk is:

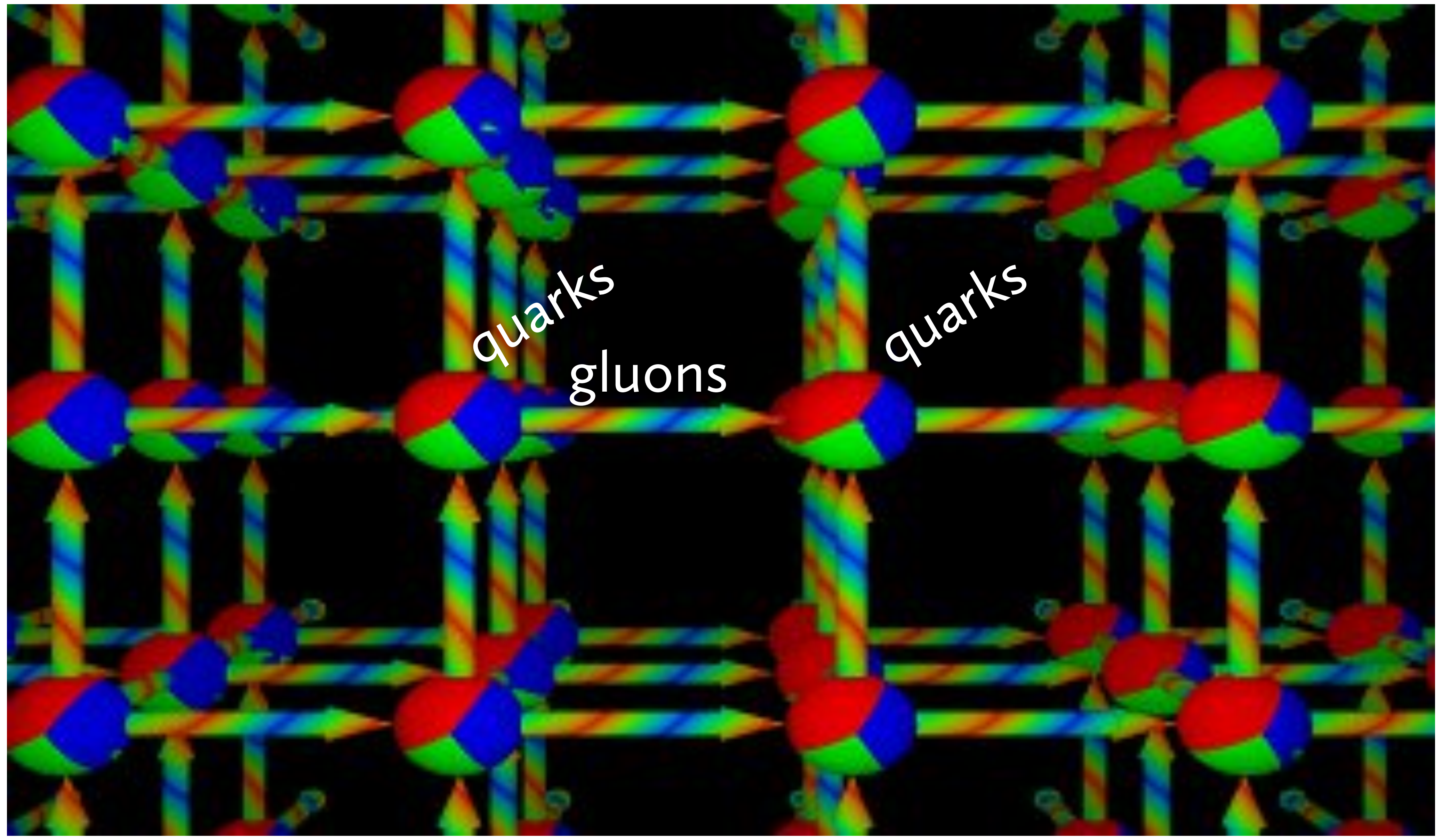
- Why it is important to develop a nonperturbative (lattice) regulator for the Standard Model
- Why that has been an impossibility for the 50 years since Wilson invented lattice field theory
- The secret seems to lie in the sorts of topological materials condensed matter theorists have been discussing since discovery of the Integer Quantum Hall Effect
- Possible implications for BSM physics?

Infinities are endemic to quantum field theories because we like to particle couplings are point-like.

The usual renormalization procedure was developed in perturbation theory to hide these infinities

Ken Wilson reinvented quantum field theory because he wanted to formulate QFT on a computer, with no room for infinities





Lattice QCD is now a standard computational tool… but does not extend to the whole Standard Model

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- Pauli-Villars doesn't work — breaks gauge symmetry
- Dimensional regularization not known to work past 2 loops: can't analytically extend γ_5 to non-integer dimensions
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The Standard model is not currently a calculational scheme that can be extended to arbitrary precision!

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- Perhaps there are known nonperturbative effects we would like to compute numerically, such as electroweak baryon violation in early universe?
- There is no foundation beneath our theory of the micro world.

Nielsen-Ninomiya theorem:

consider Euclidian fermion action on a lattice:

$$S = \int \frac{d^d p}{(2\pi)^d} \bar{\Psi}(-p)\widetilde{D}(p)\Psi(p)$$

wanted: massless Dirac fermion with chiral symmetry

1. $\tilde{D}(\mathbf{p})$ is a periodic, analytic function of p_μ; ☜ locality
2. $D(\mathbf{p}) \propto \gamma_\mu p_\mu$ for $a|p_\mu| \ll 1$; ☜ correct continuum limit
3. $\tilde{D}(\mathbf{p})$ invertible everywhere except $p_\mu = 0$; ☜ no doublers
4. $\{\gamma_5, \tilde{D}(\mathbf{p})\} = 0$. ☜ exact chiral symmetry

Nielsen-Ninomiya theorem: one can have *at most* 3 of these 4 desired attributes

Need #4 to project out a Weyl fermion from a massless Dirac fermion to simulate SM

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1. Decouple them by breaking gauge symmetry and giving them a mass; restore gauge symmetry in continuum limit
Golterman, Shamir

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3. Eliminate mirror fermions by sacrificing locality (this talk)

Edge states and topological phases

Chirality can occur in nature in surprising places

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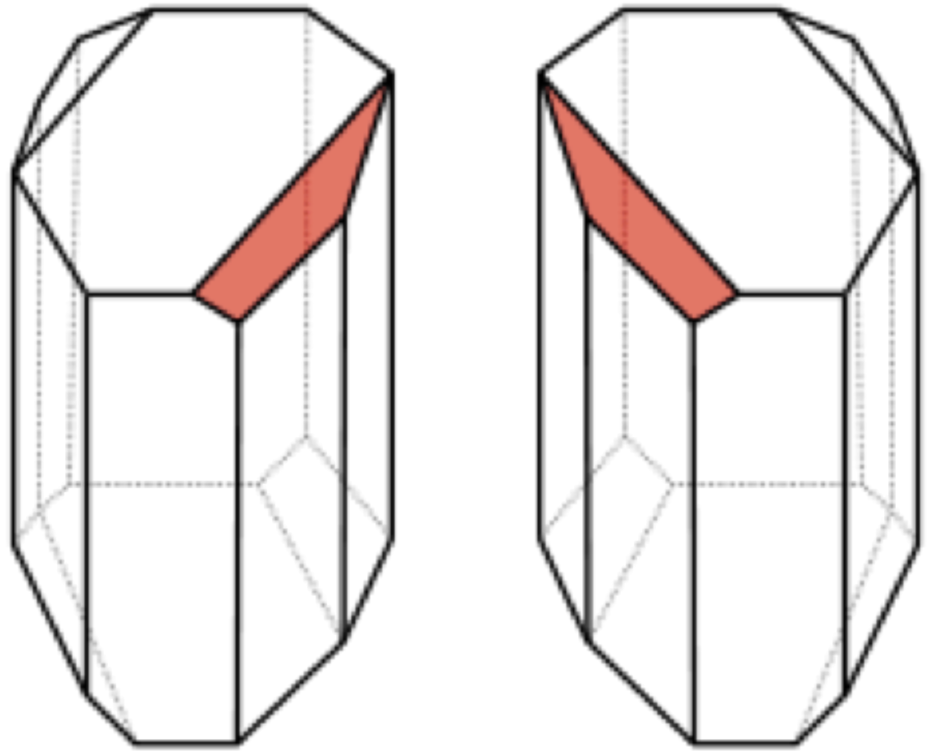
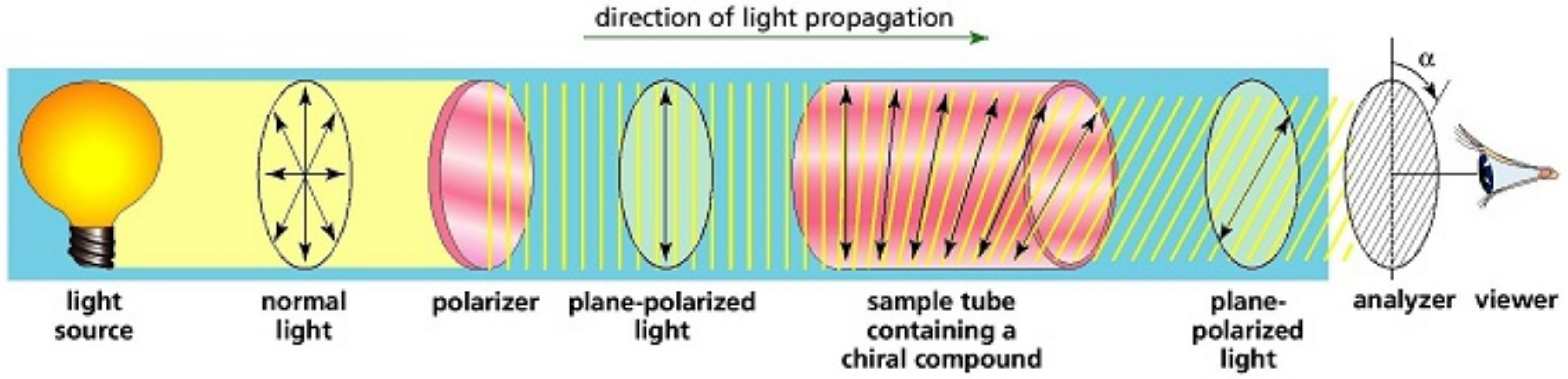
Chirality can occur in nature in surprising places

Louis Pasteur

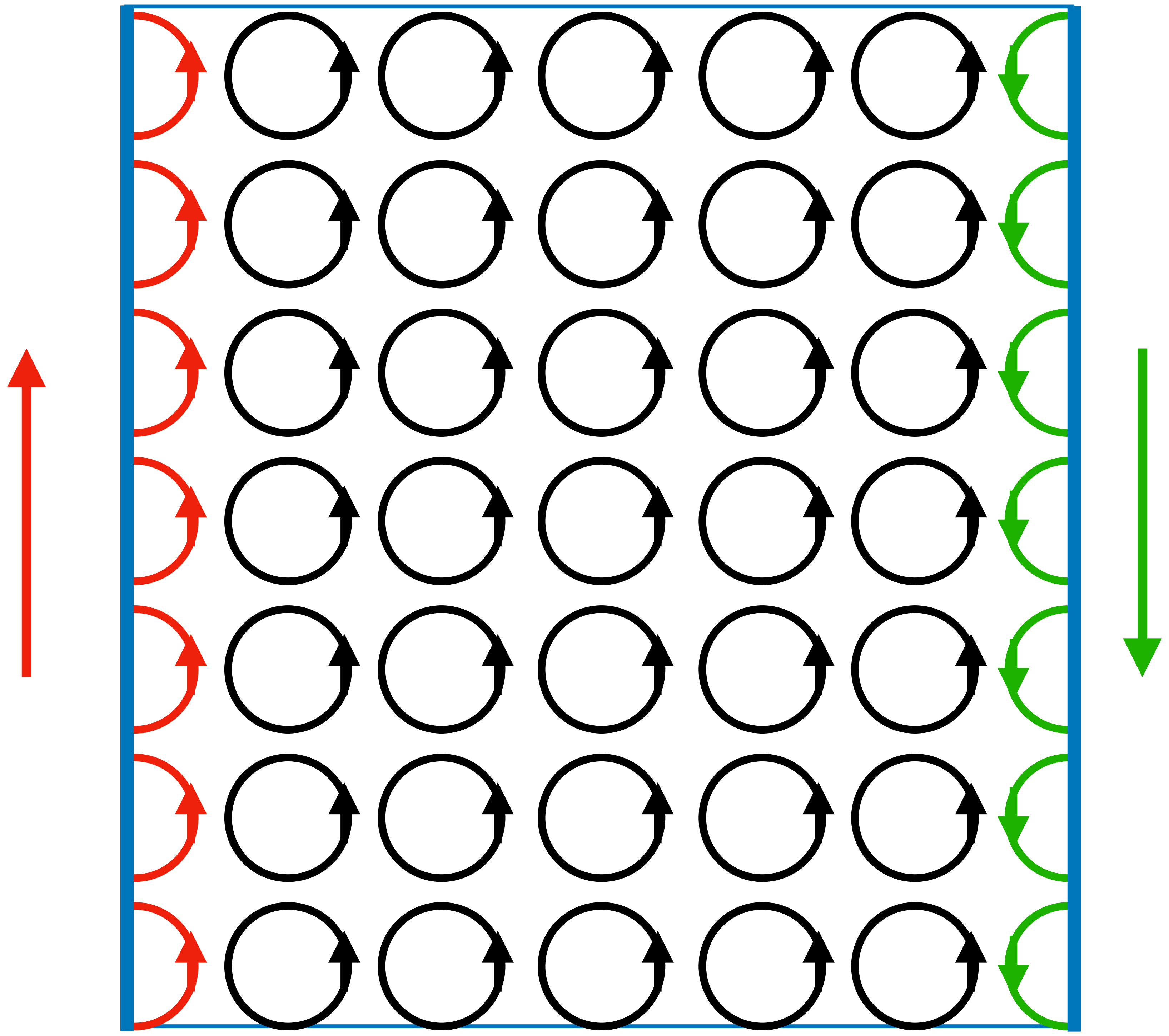


Left- and right-handed tartaric acid crystals



In physics:

Chiral edge states appear naturally in the Integer Quantum Hall Effect:



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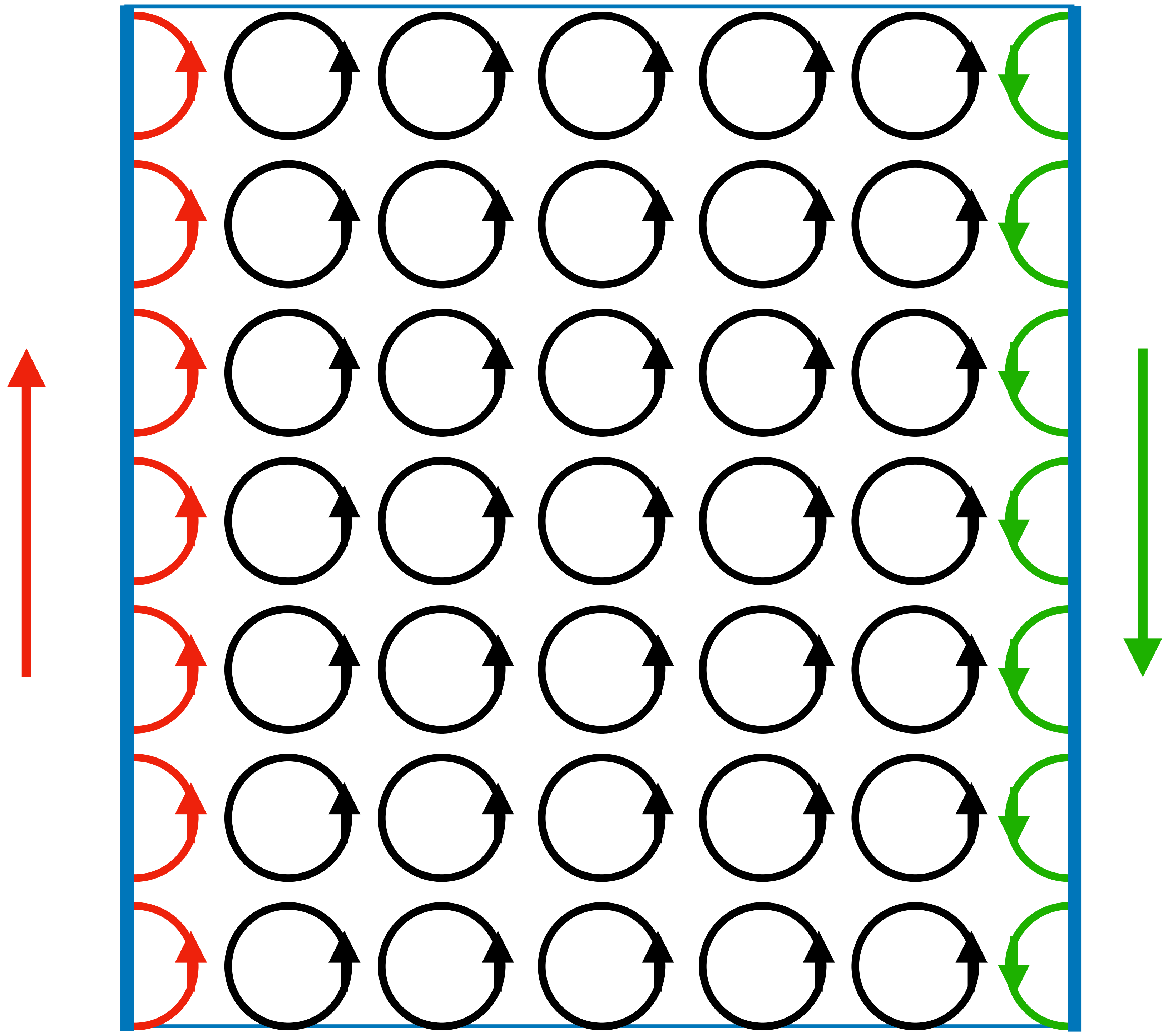


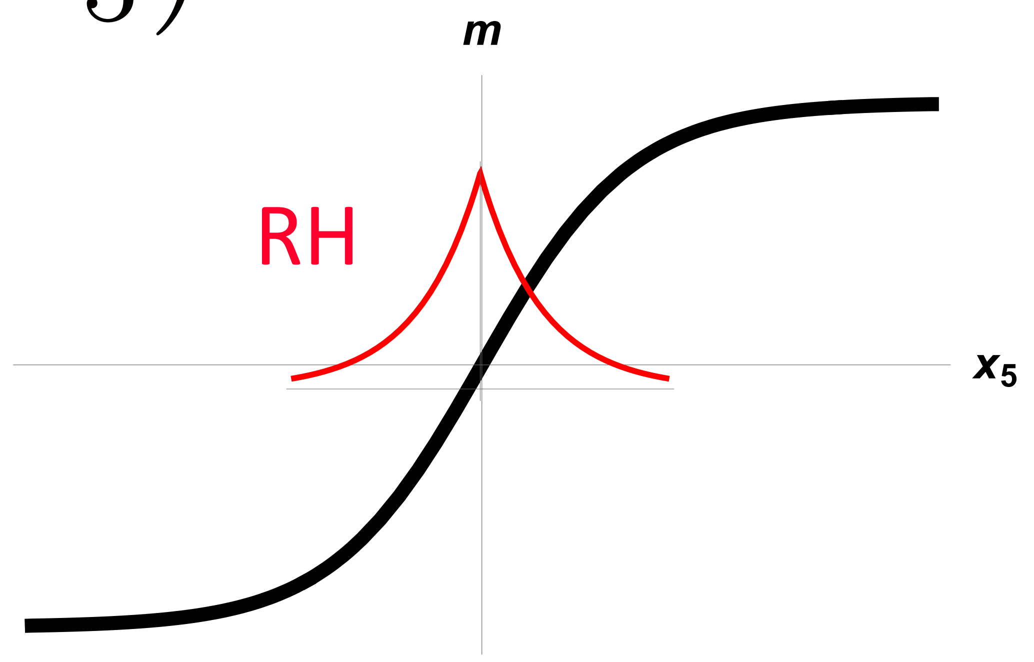
Dirac fermions with domain wall mass [Jackiw & Rebbi]:

$$\left[\partial\!\!\!/ + \gamma_5 \partial_5 + m(x_5)\right] \Psi = 0$$

Has solutions: $\Psi = \phi_\pm(x_5)\chi_\pm$

$$\gamma_5 \chi_\pm = \pm \chi_\pm$$

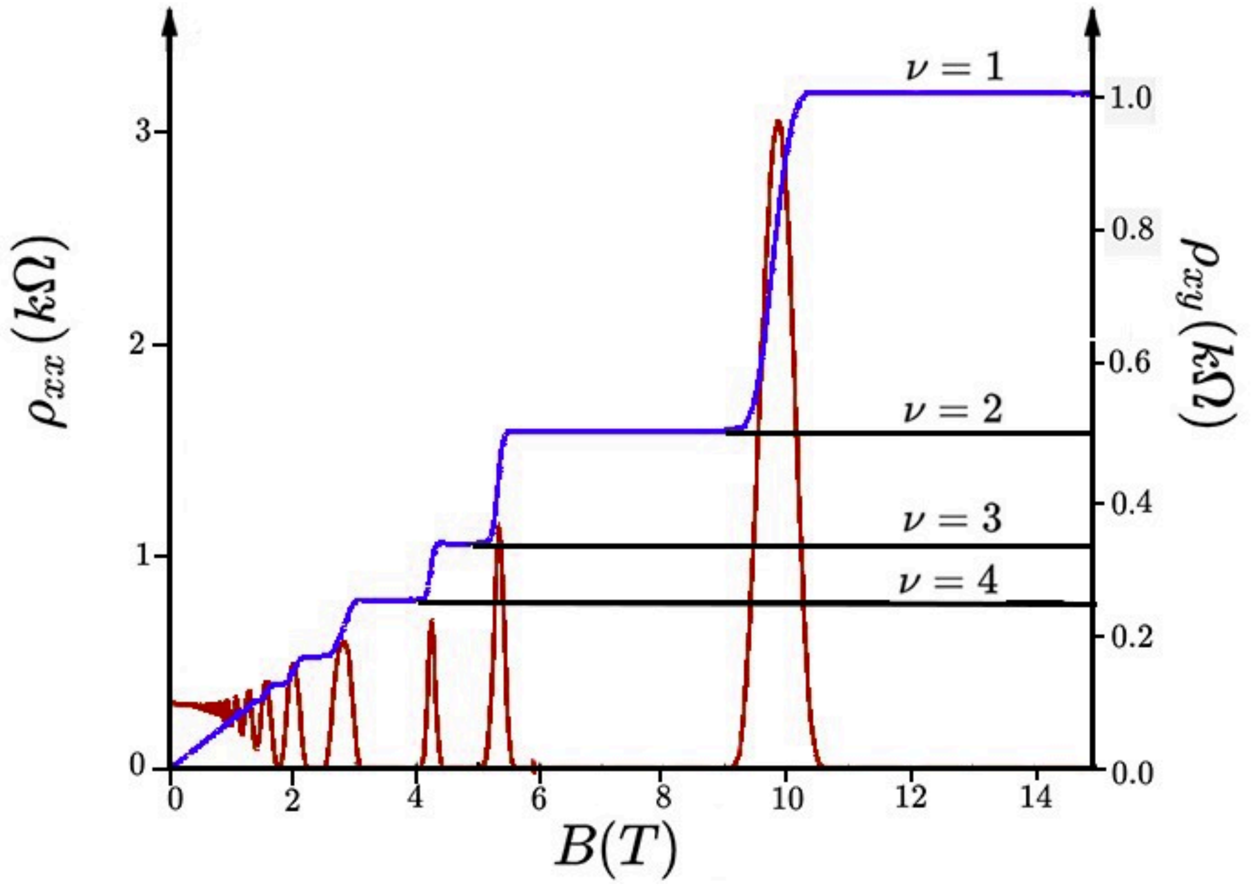
$$\phi_\pm(x_5) = e^{\mp \int^{x_5} m(s)\, ds}$$



Only one solution is normalizable

With this domain wall mass profile, ϕ_+ is normalizable ➡ massless chiral edge state

Why does the Dirac equation have a massless chiral edge state? Same reason as the appearance of edge states in Integer Quantum Hall Effect: **TOPOLOGY**.



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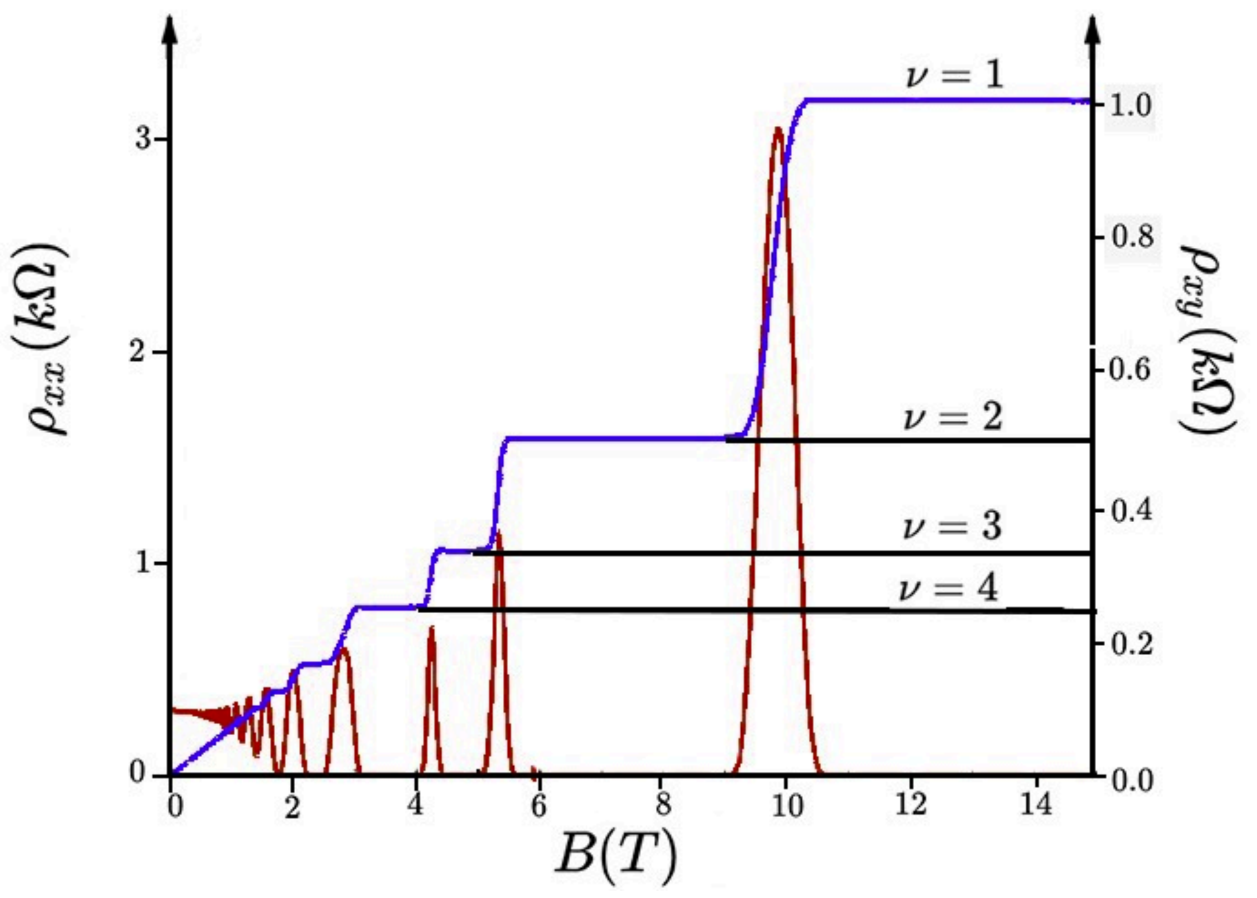

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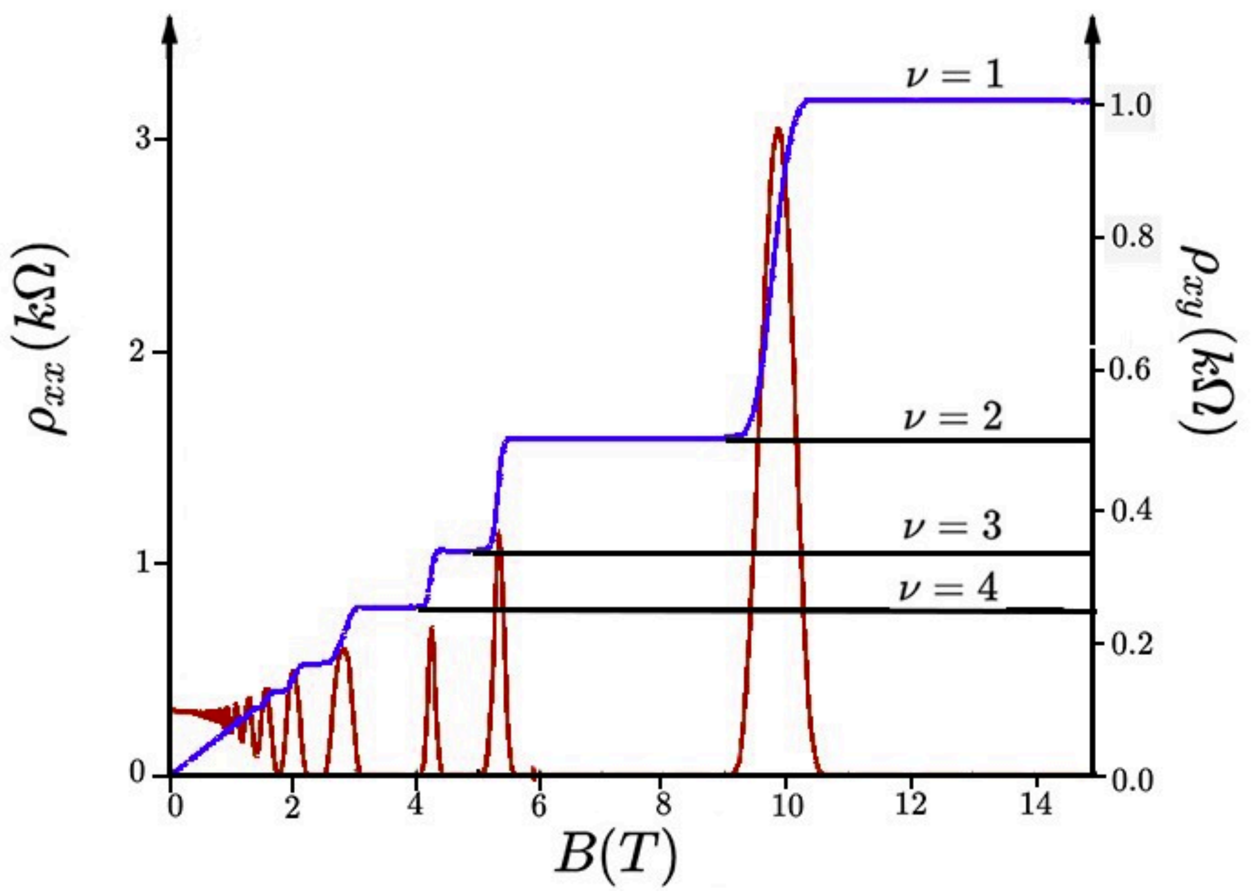

Thouless et al. explained the quantized resistivity of the Integer Quantum Hall Effect in terms of topology.

For the Dirac analog, the topology is in the behavior of fermion spin as one moves through a finite (regulated) momentum space.

Chiral edge states naturally arise at the boundary between regions in different topological phases.

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Yes: if you want chiral fermions on a 4d edge, look at massive Dirac fermions in 5d.

Integrate them out of the theory in the presence of gauge fields: obtain a Chern-Simons operator, $\varepsilon_{abcde} A_a \partial_b A_c \partial_d A_e$.

Its coefficient is quantized in integer units of e^2/h (von Klitzing conductivity!) and independent under continuous deformations of parameters of the theory.

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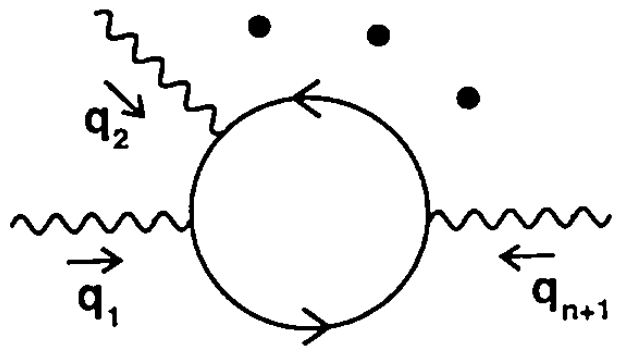
How does topology result from a 1-loop Feynman diagram??

Using Ward identity, Chern-Simons coefficient in d= 2n+1 is proportional to



$$\epsilon_{\mu_1\ldots\mu_d}\int\frac{d^dp}{(2\pi)^d}\,\mathrm{Tr}\;S(p)\frac{\partial S^{-1}(p)}{\partial p_{\mu_1}}\cdots S(p)\frac{\partial S^{-1}(p)}{\partial p_{\mu_d}}$$

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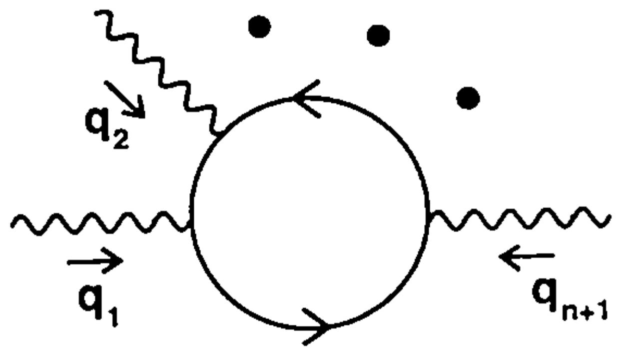

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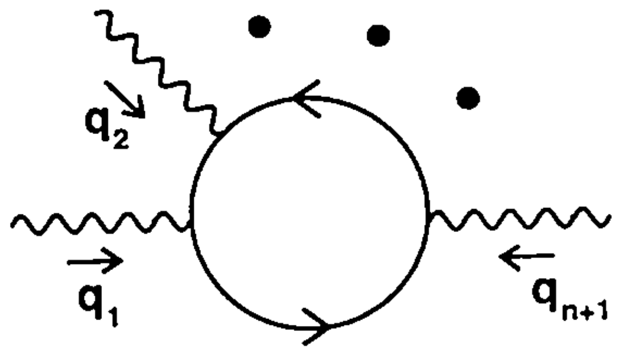

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Example: free Dirac fermion $S(p) = \frac{1}{i\not{p} + m}$ $\frac{\partial S^{-1}(p)}{\partial p_\alpha} = i\gamma^\alpha$

Feynman diagram computes the winding number of S(p) as a map from momentum space to Dirac spinor space… much more general than just for free Dirac fermion — also true for fermions on a lattice.

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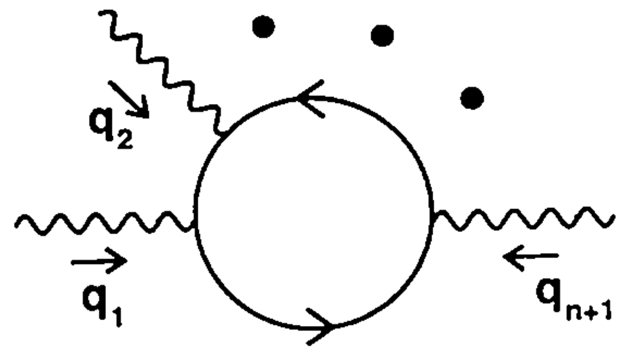

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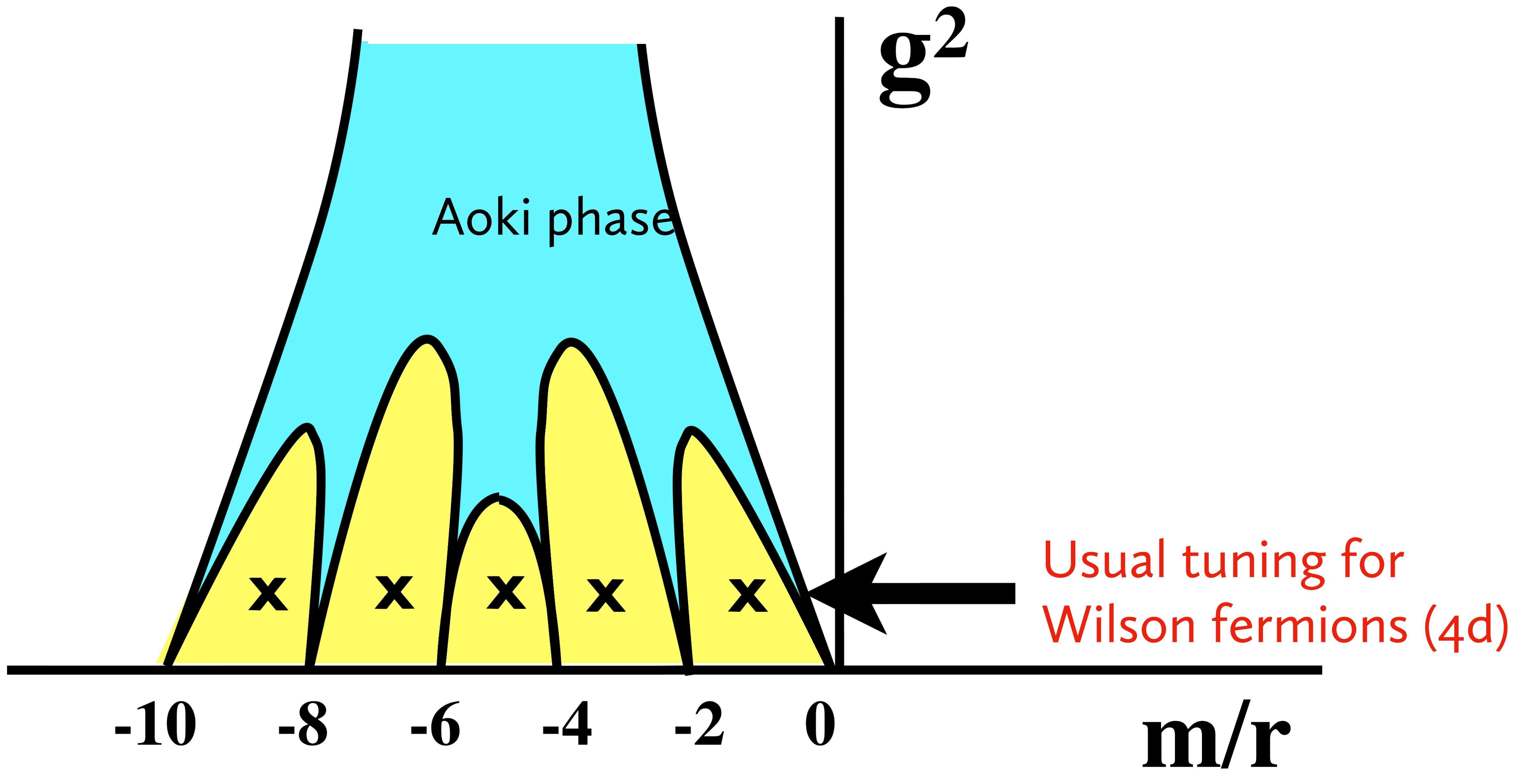
Massless chiral fermions will appear at interface between regions with different Chern-Simons coefficients.

Phase diagram for lattice QCD with Wilson fermions in 5d Euclidian spacetime



S Aoki, Prog Th Phys 122 (1996) 179

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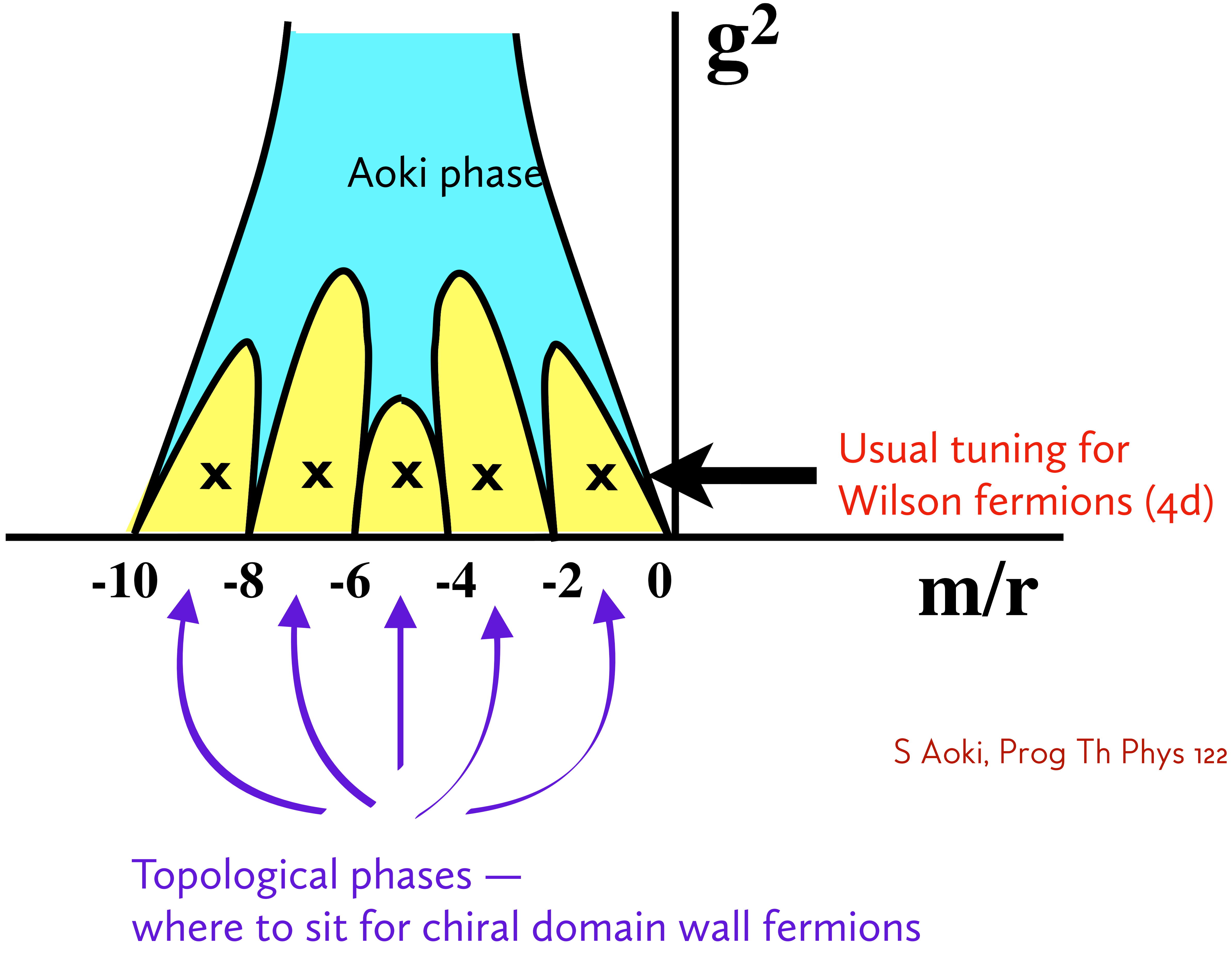


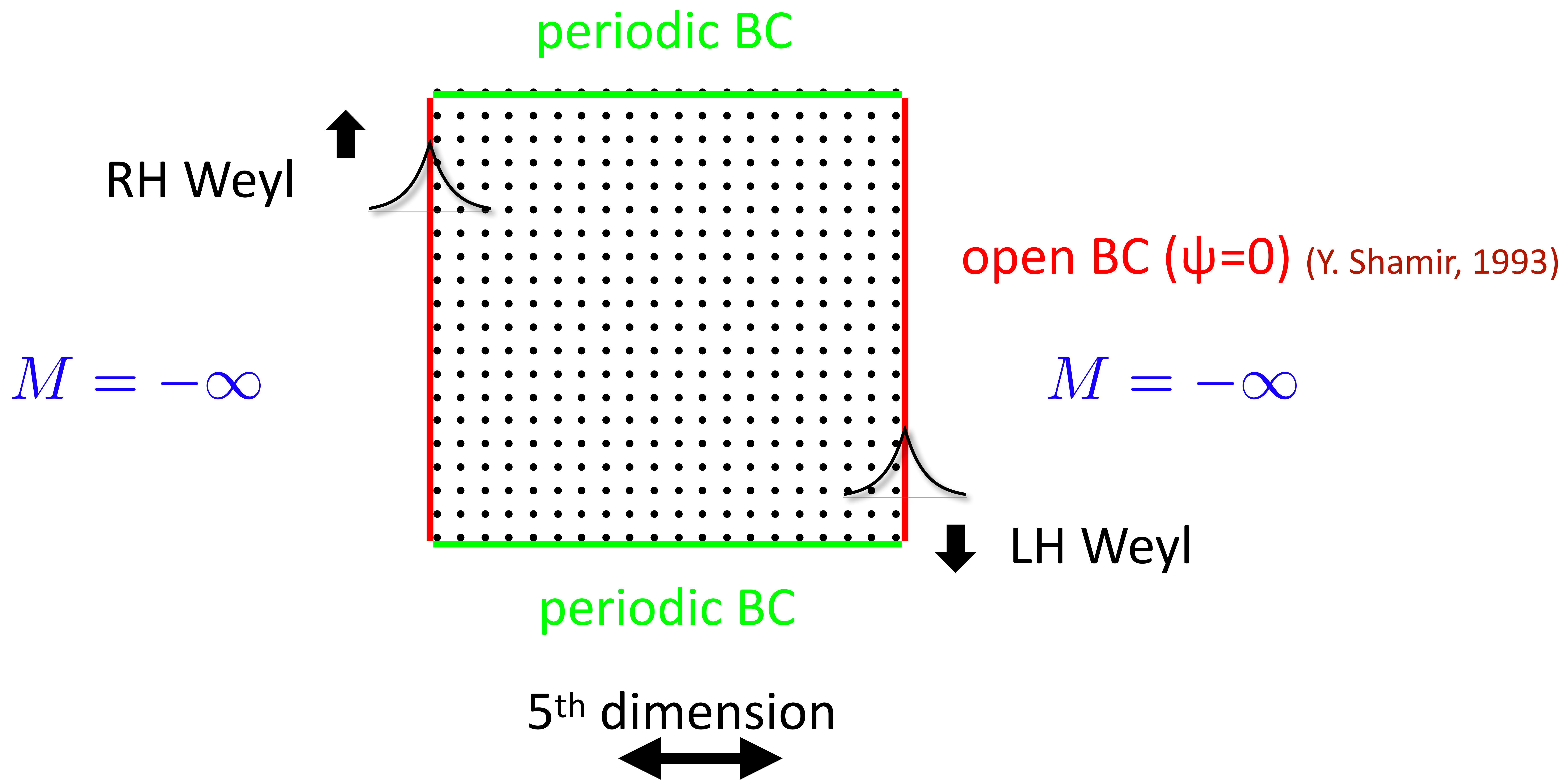
S Aoki, Prog Th Phys 122 (1996) 179

The phenomenon of massless edge states at topological phase boundaries exists for lattice fermions.

DBK, Phys. Lett. B 288 (1992) 342

M. Golterman, K. Jansen, DBK, Phys. Lett. B 301 (1993) 219

A 5d strip of lattice with 4d boundaries is now often used to simulate lattice QCD with very good chiral symmetry, useful for many applications.

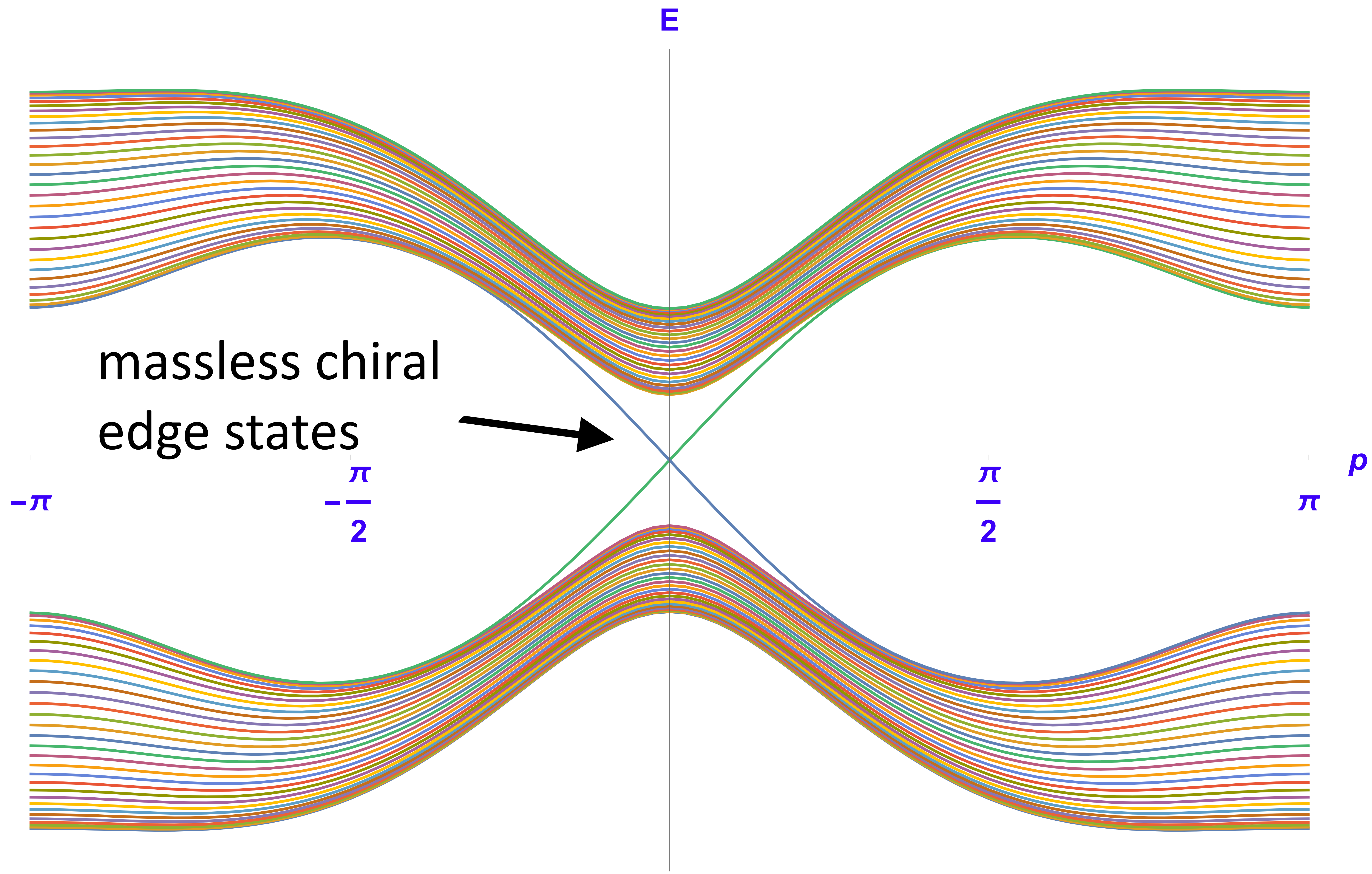


Obtain *almost* massless RH & LH Weyl fermions… mass $\propto e^{-2ML}$

QCD gauge fields are taken to be independent of the 5th dimension

The spectrum for Wilson fermions on the 5d strip

massive bulk states



Wilson fermions on the 5d strip are:

- Useful for performing lattice QCD computations
- *Useless* for simulating a chiral gauge theory
  - vector-like theory (LH and RH fermions have same gauge charges)
  - chiral symmetry is broken by a tiny amount (exponentially small in size of 5th dimension) — not exact as needed for chiral gauge theory

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But what happens on lattice with a single boundary between topological phases?

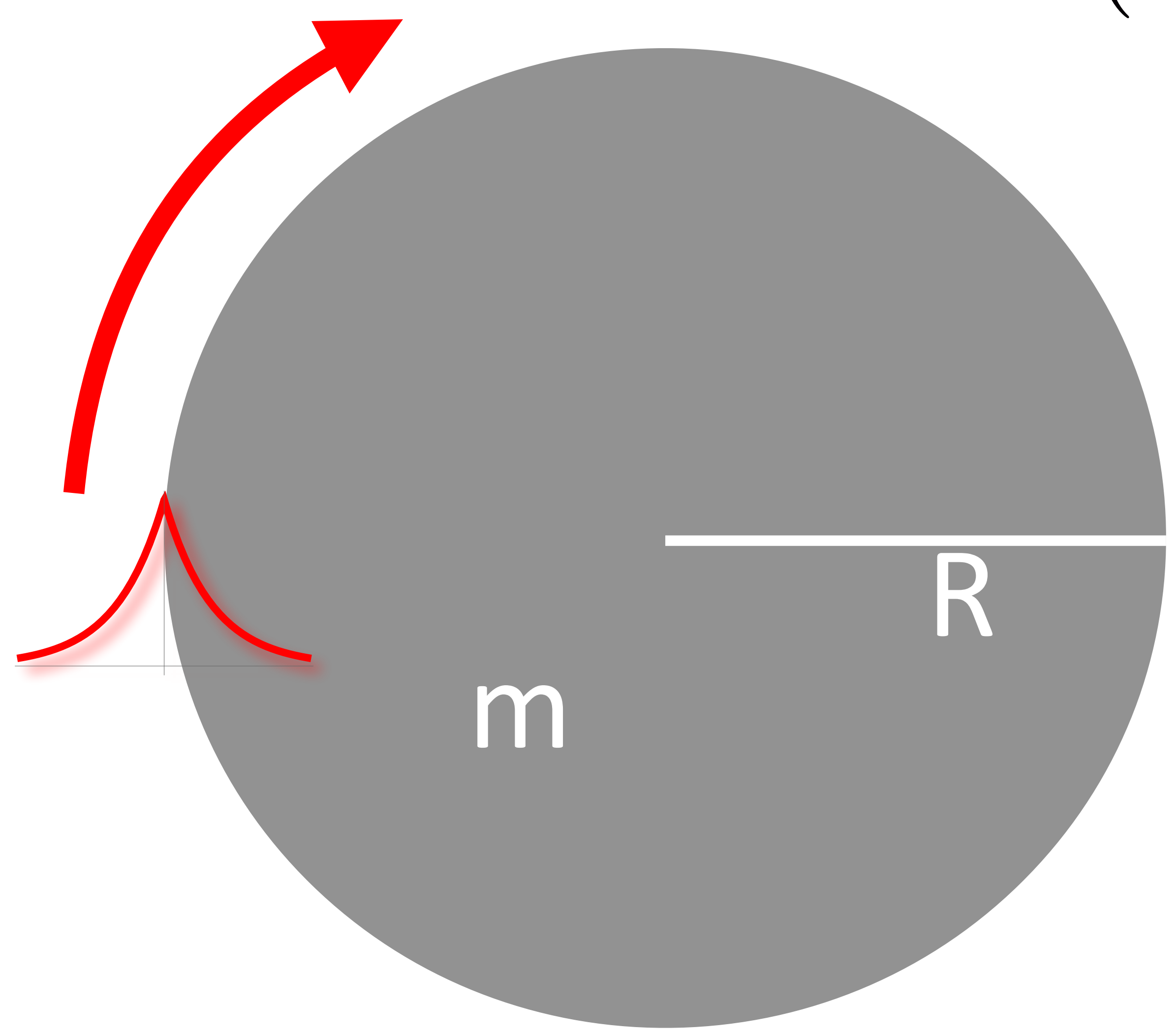
DB Kaplan: Phys. Rev. Lett. 132 (2024) 141603, arXiv:2312.01494
DB Kaplan, S. Sen: Phys. Rev. Lett. 132 (2024) 141604, arXiv:2312.04012

Edge states on manifold with a **single** boundary:

Consider Dirac fermion an a disk:

$$m(r) = \begin{cases} m & r < R \\ -M & r > R \end{cases}$$

$$-M \to -\infty$$



Shouldn't this have a single Weyl fermion edge state?

Which must be exactly massless?

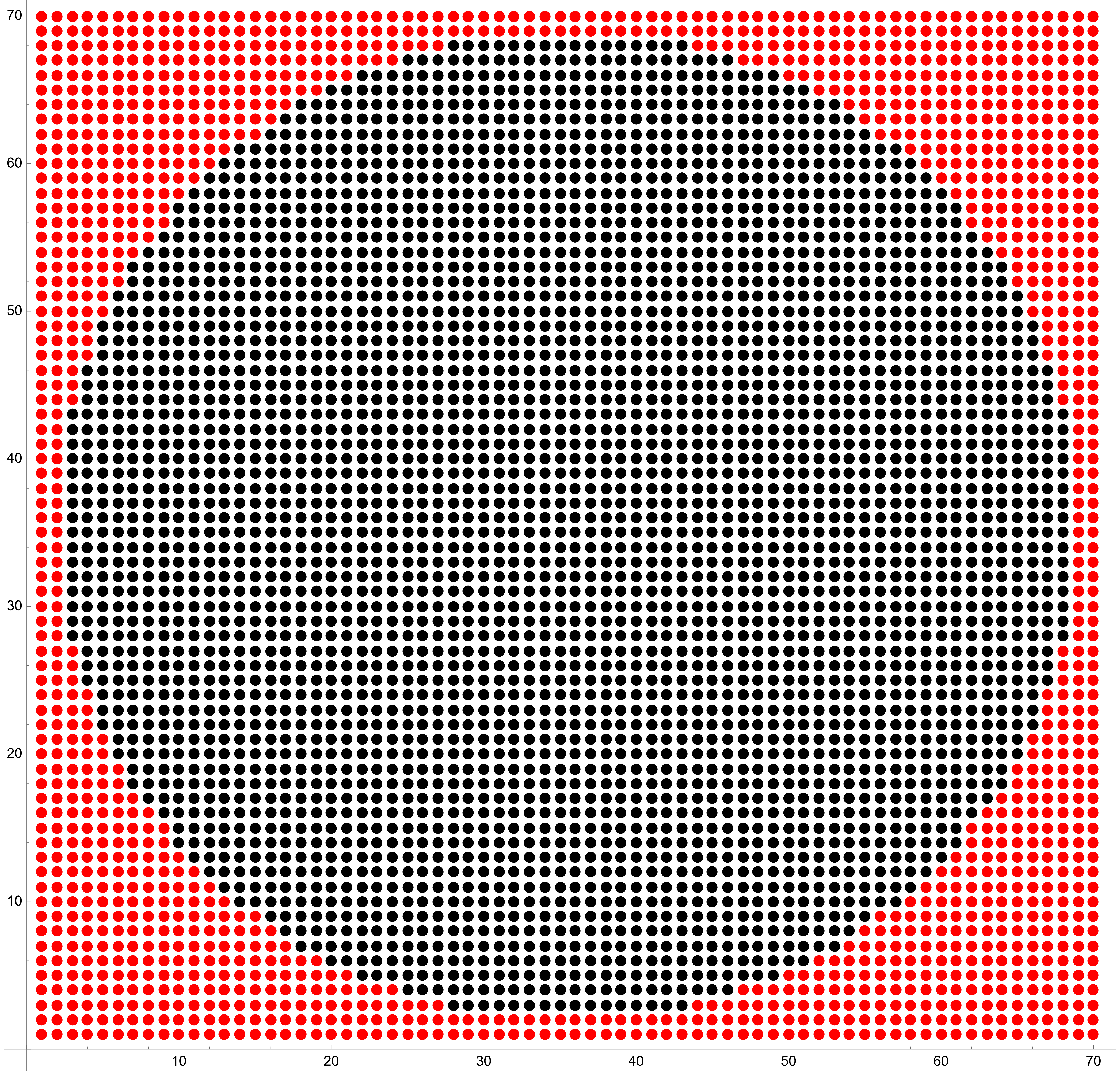
Which can be realized with Wilson fermions on a lattice?

Weyl edge state?
Look at 1+1 dispersion relation

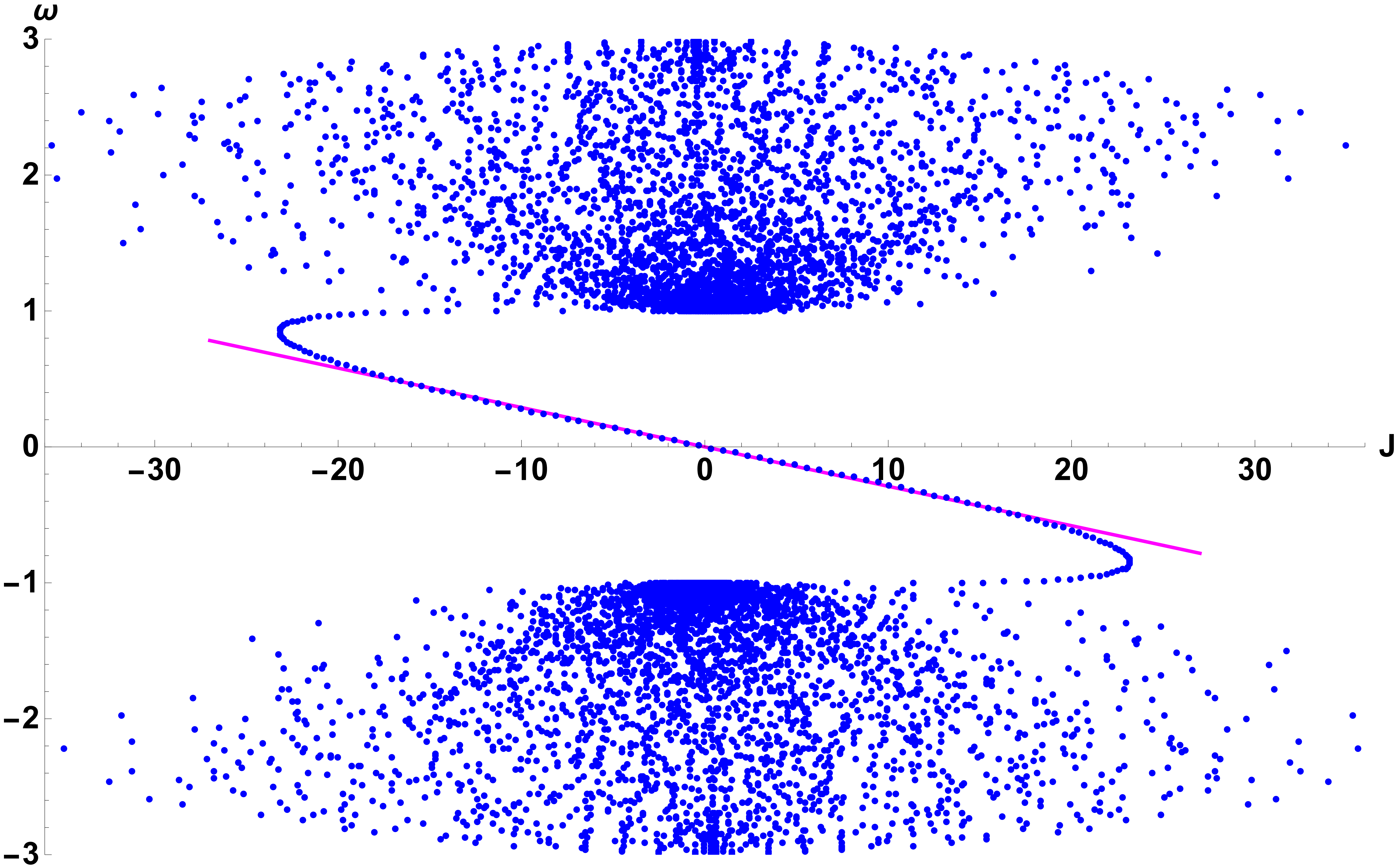
Work on a lattice disc with open BC

R = 34 lattice sites

*If you want E vs p for the edge state, **plot E vs J/R***



Energy eigenvalue ω_n



Angular momentum $\langle\psi_n|\hat{J}|\psi_n\rangle$

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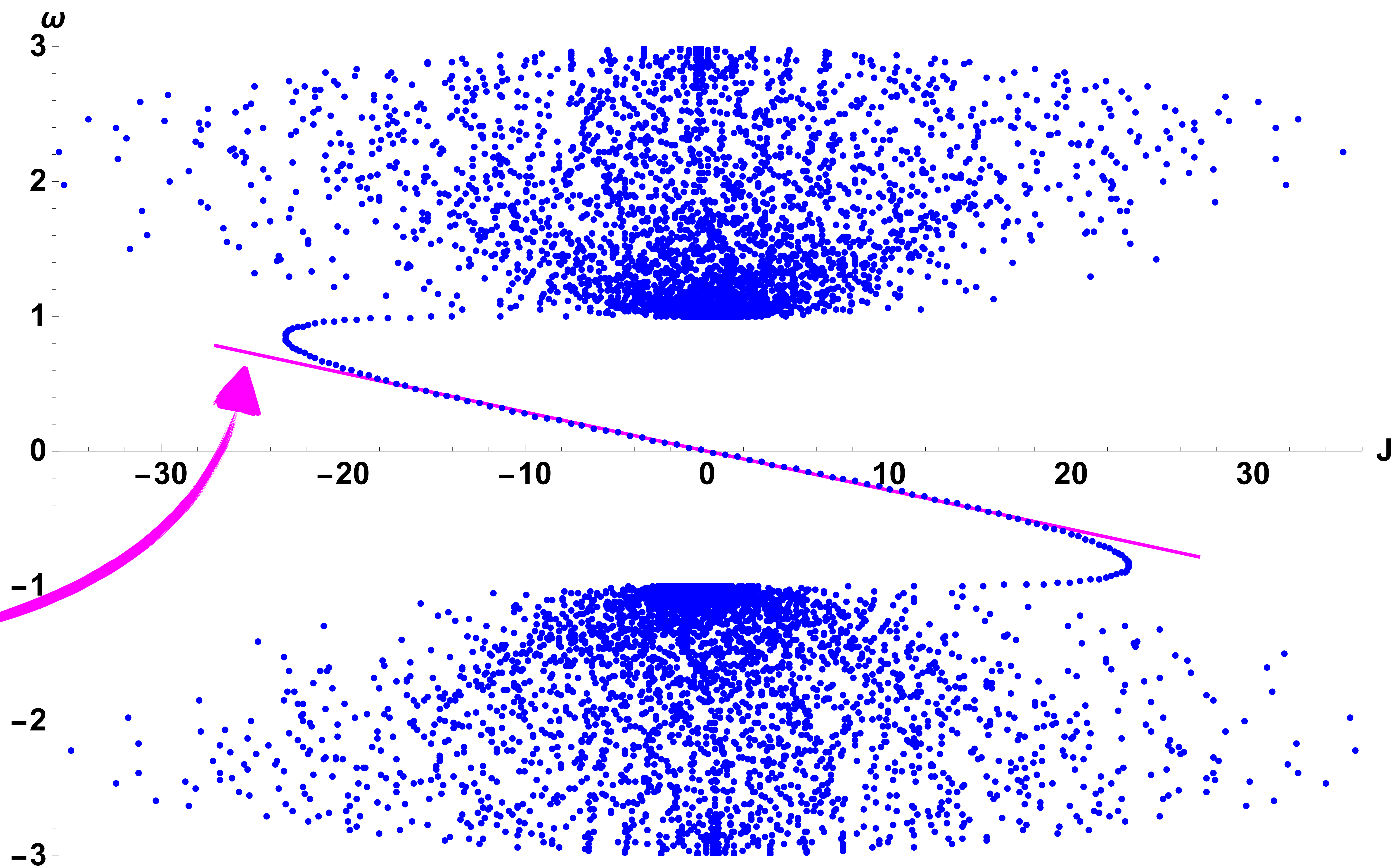
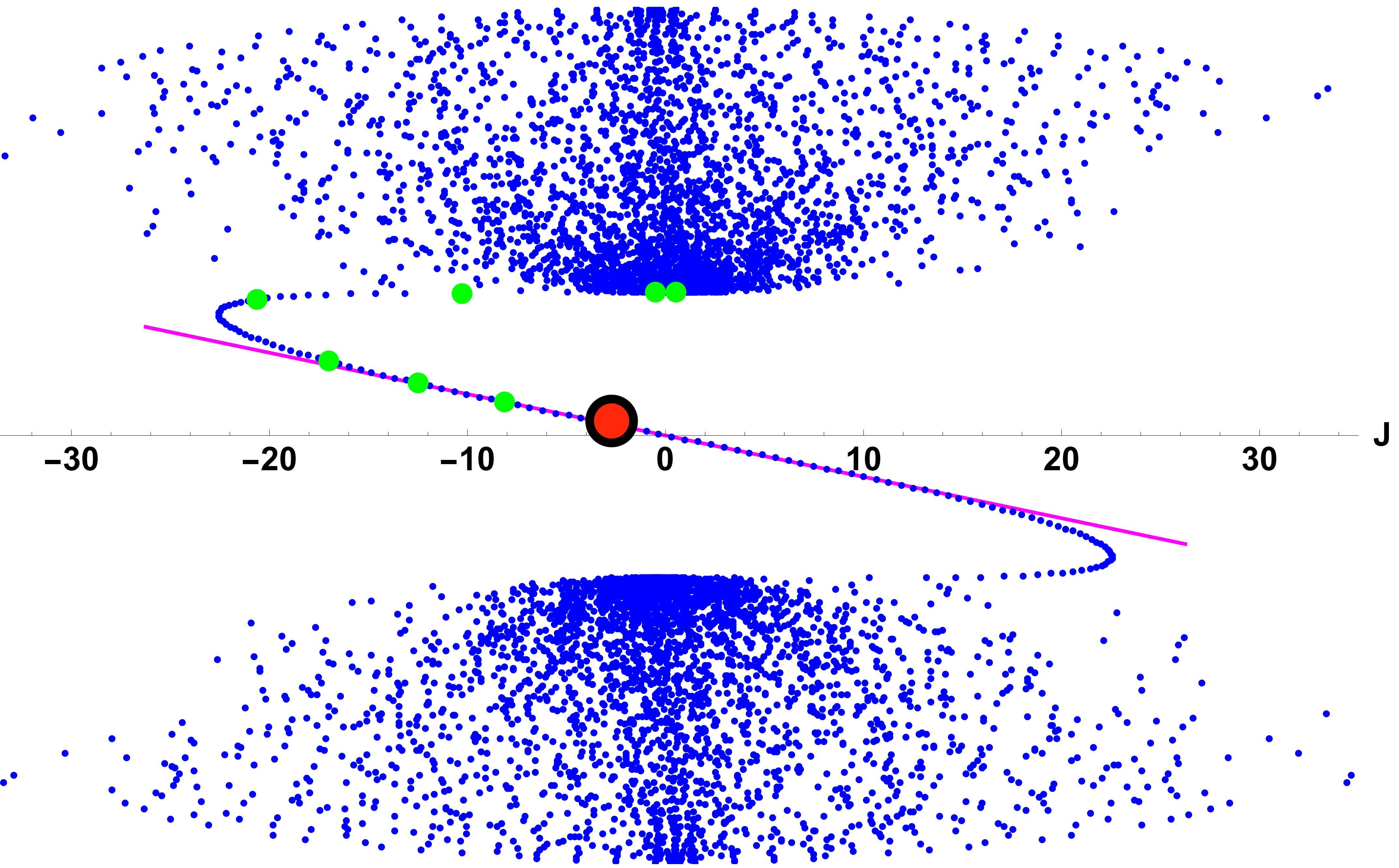


$$\omega = -\frac{J}{R}$$

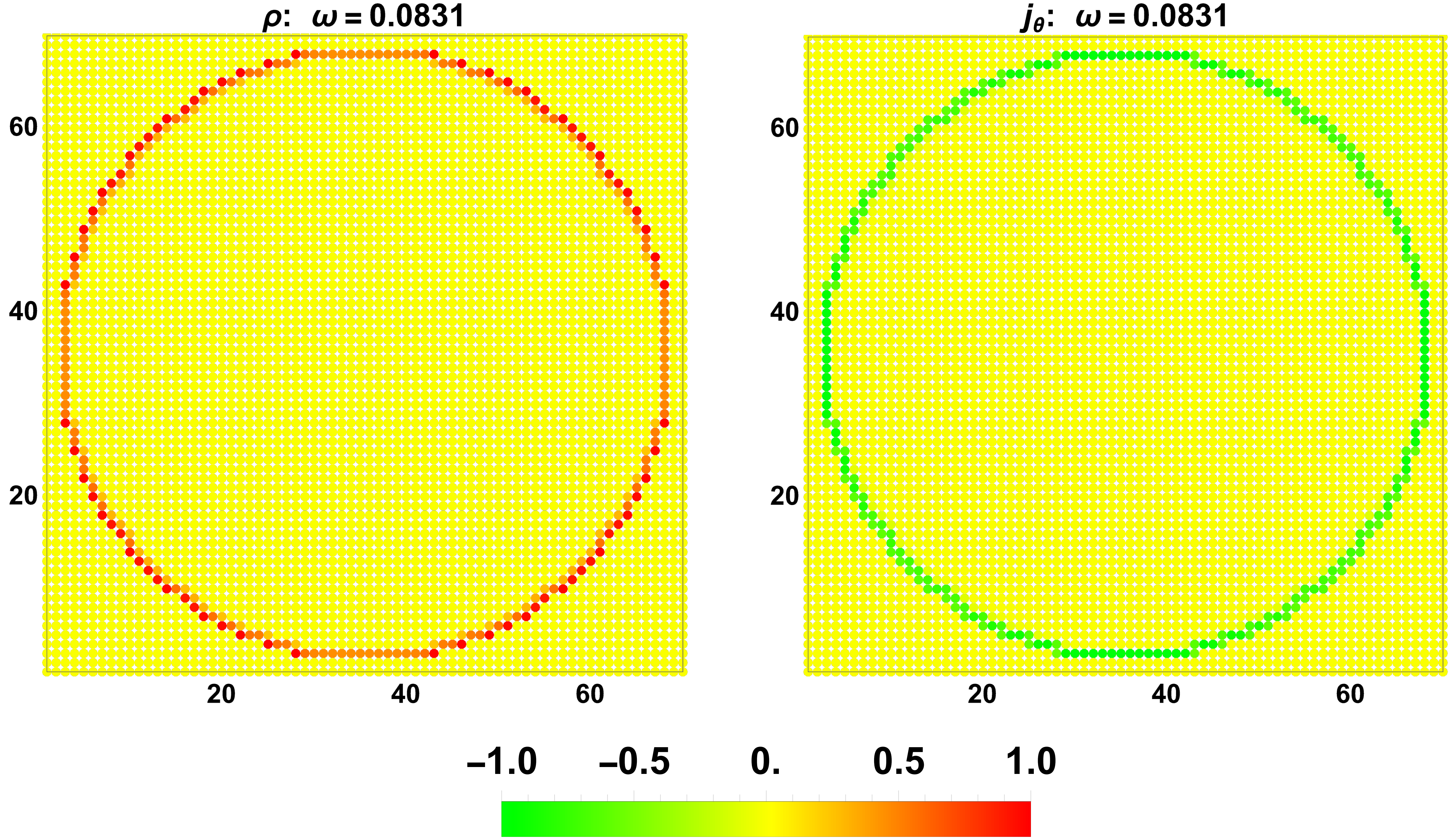
Angular momentum $\langle\psi_n|\hat{J}|\psi_n\rangle$

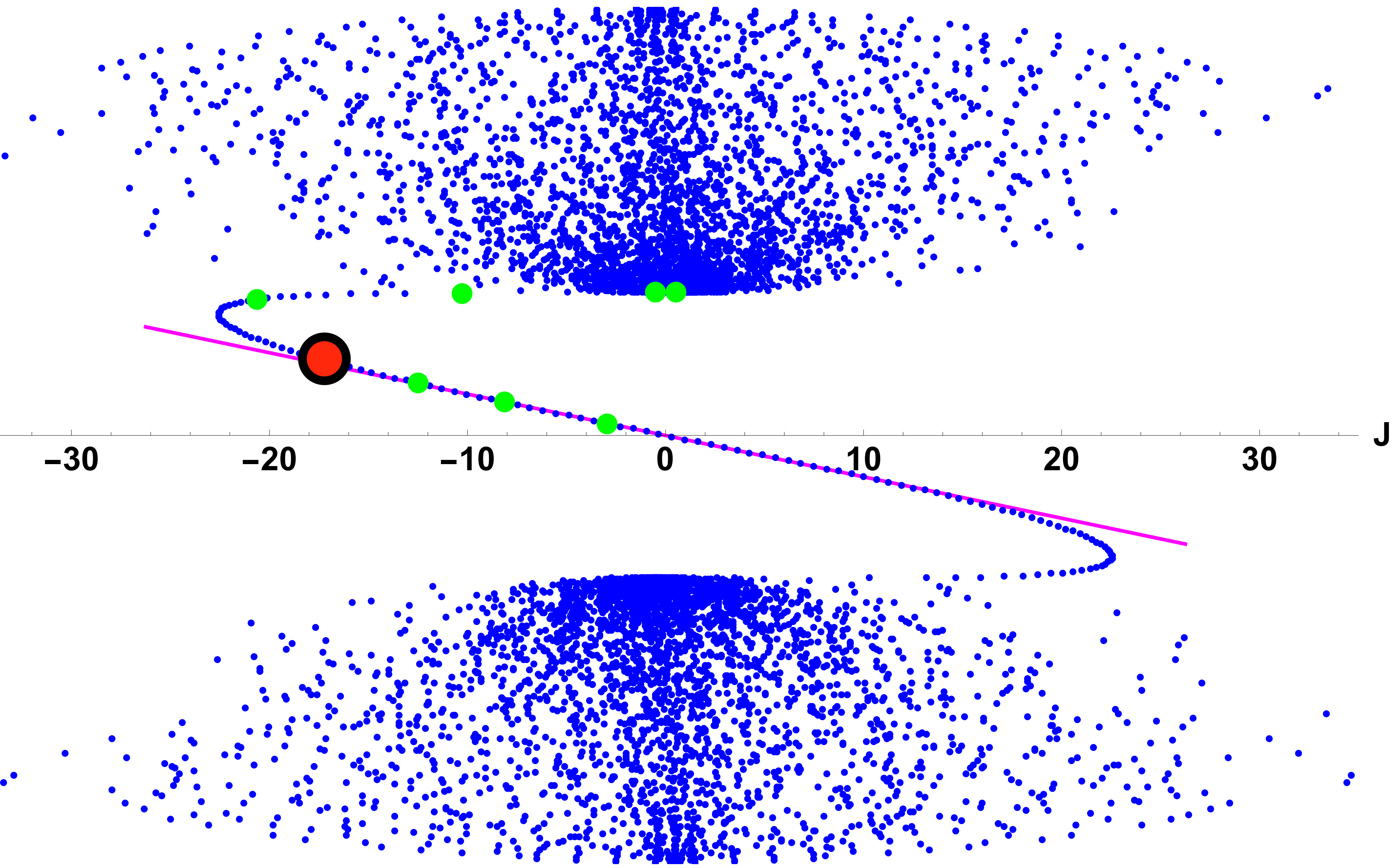
Nielsen-Ninomiya would have you believe this is not possible for sensible system



charge density ρ

current density j_θ





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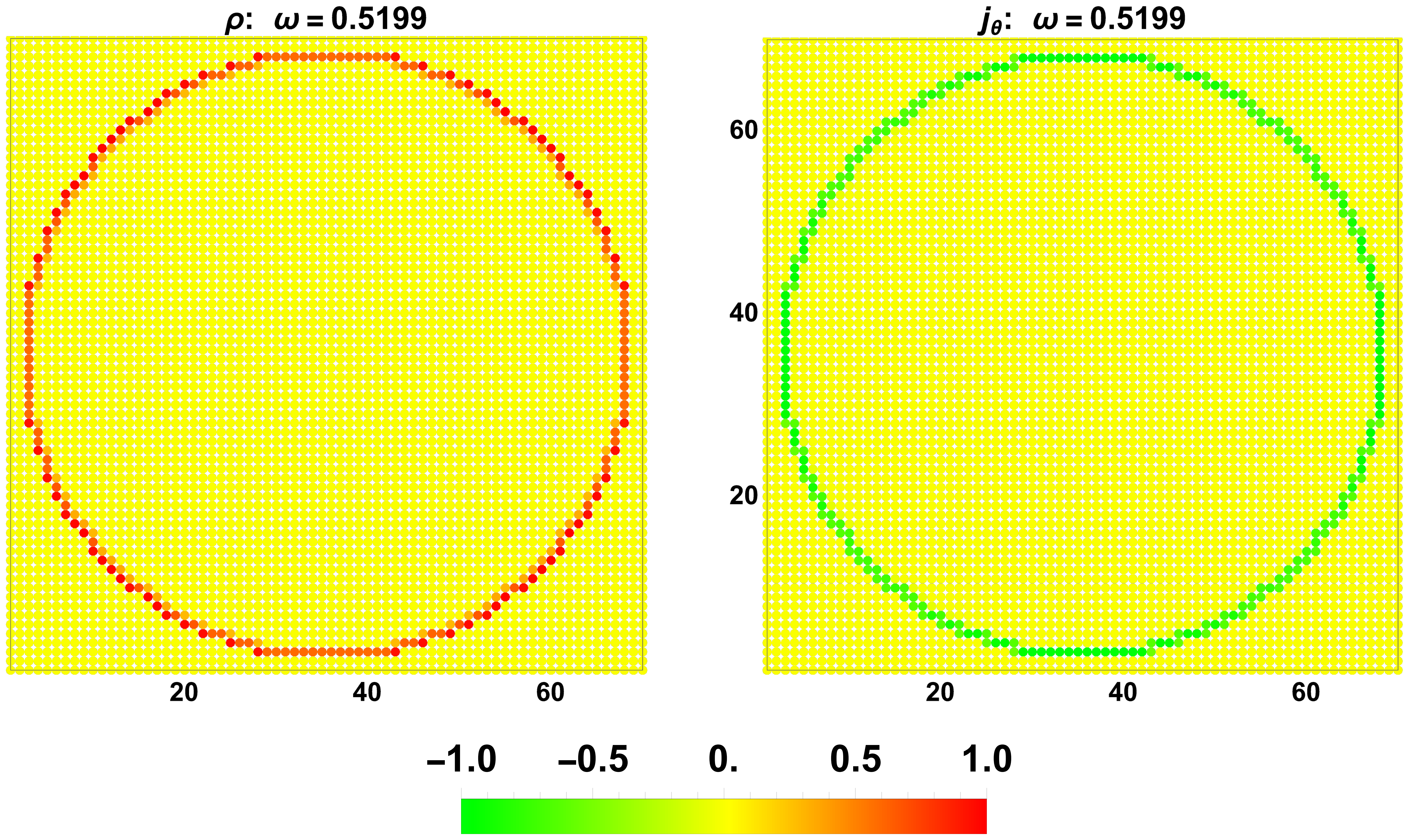

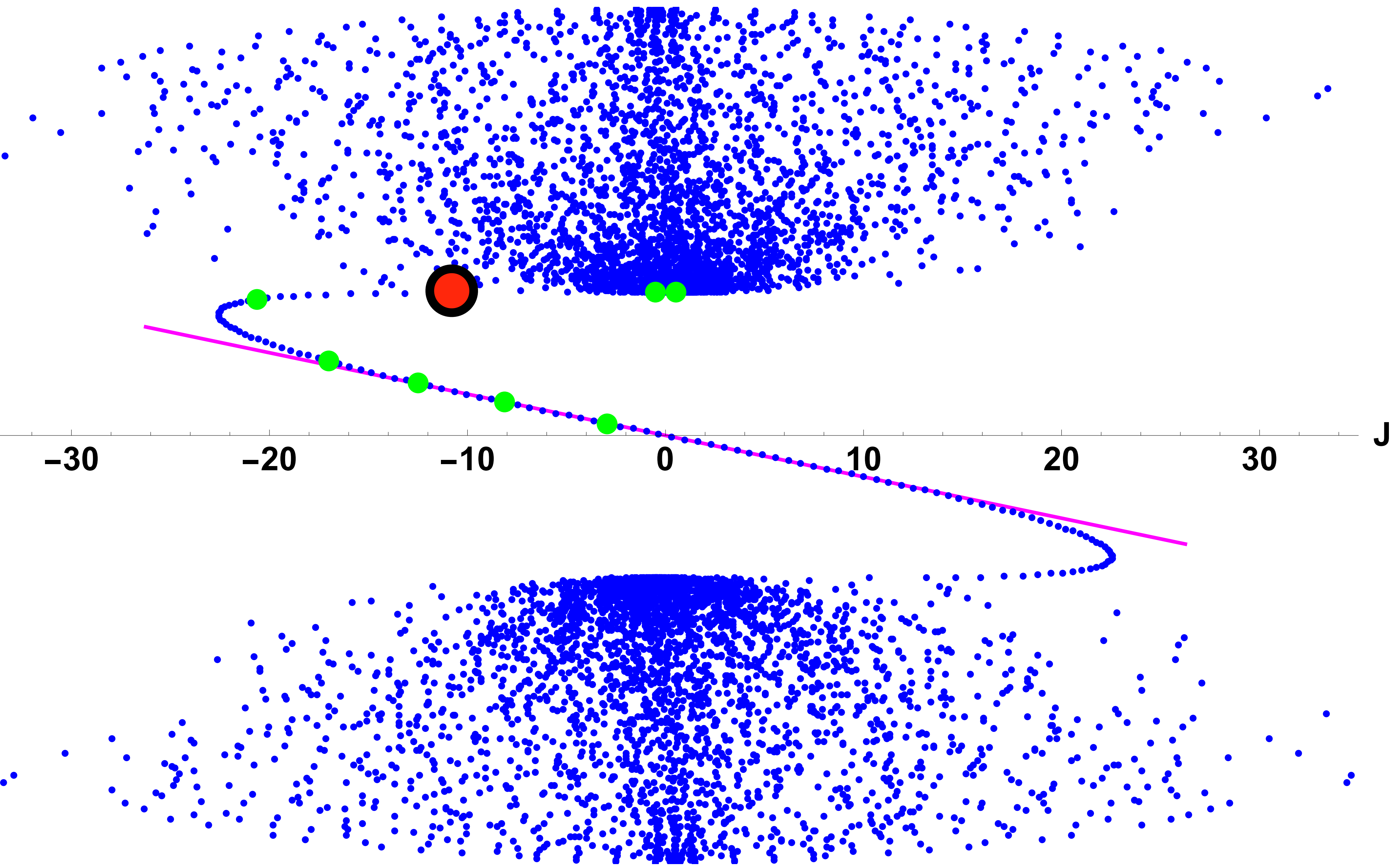

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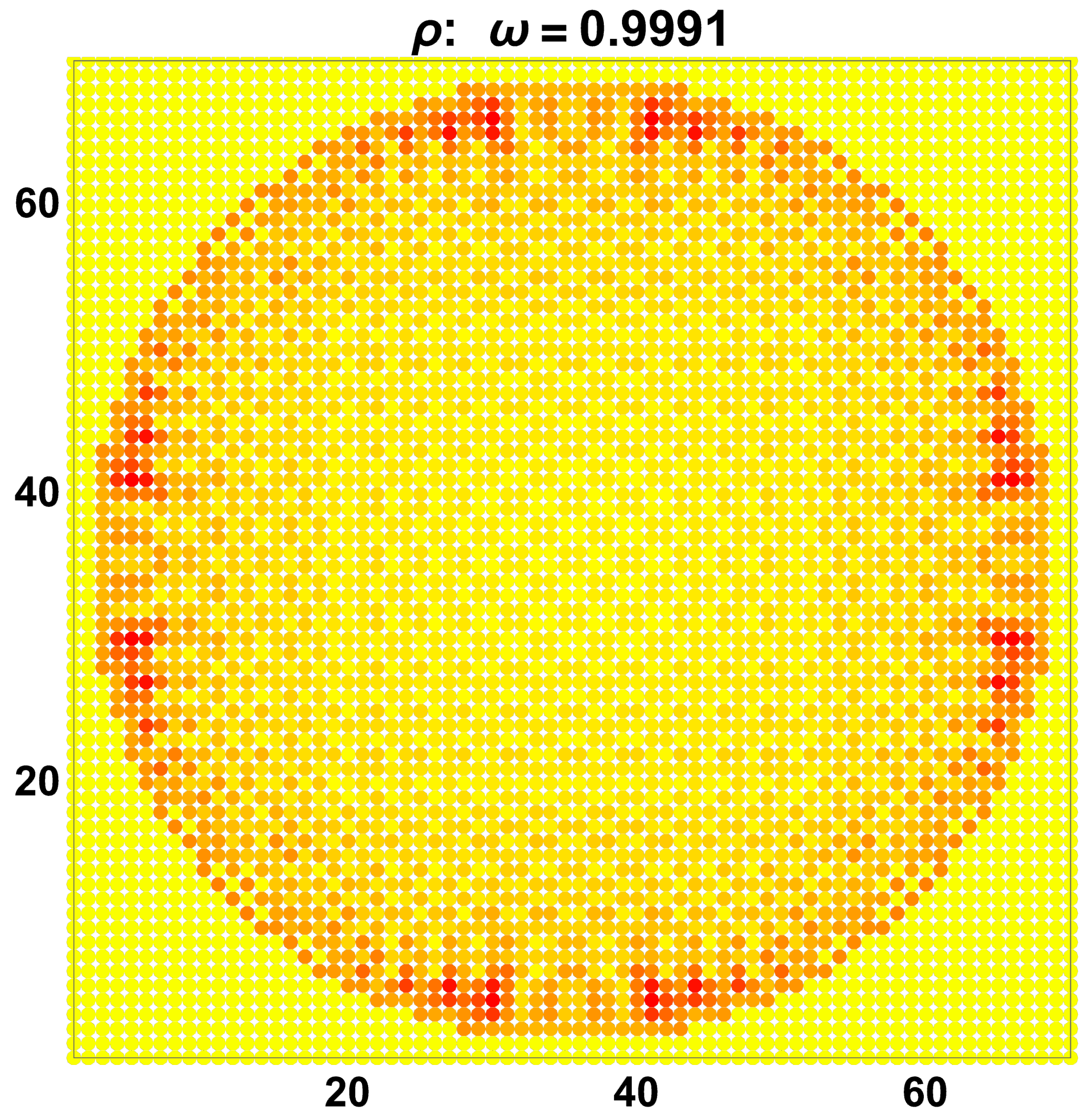

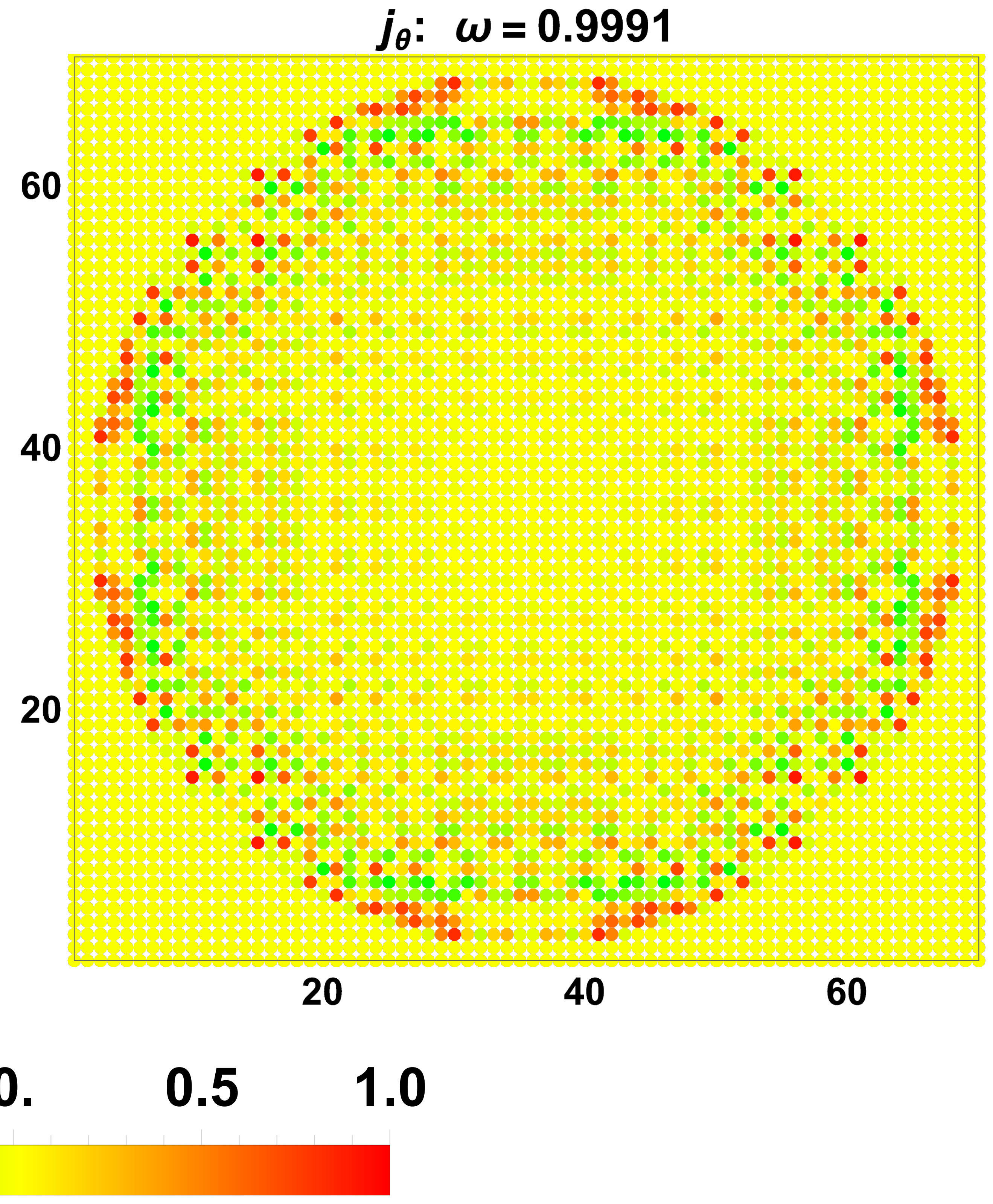

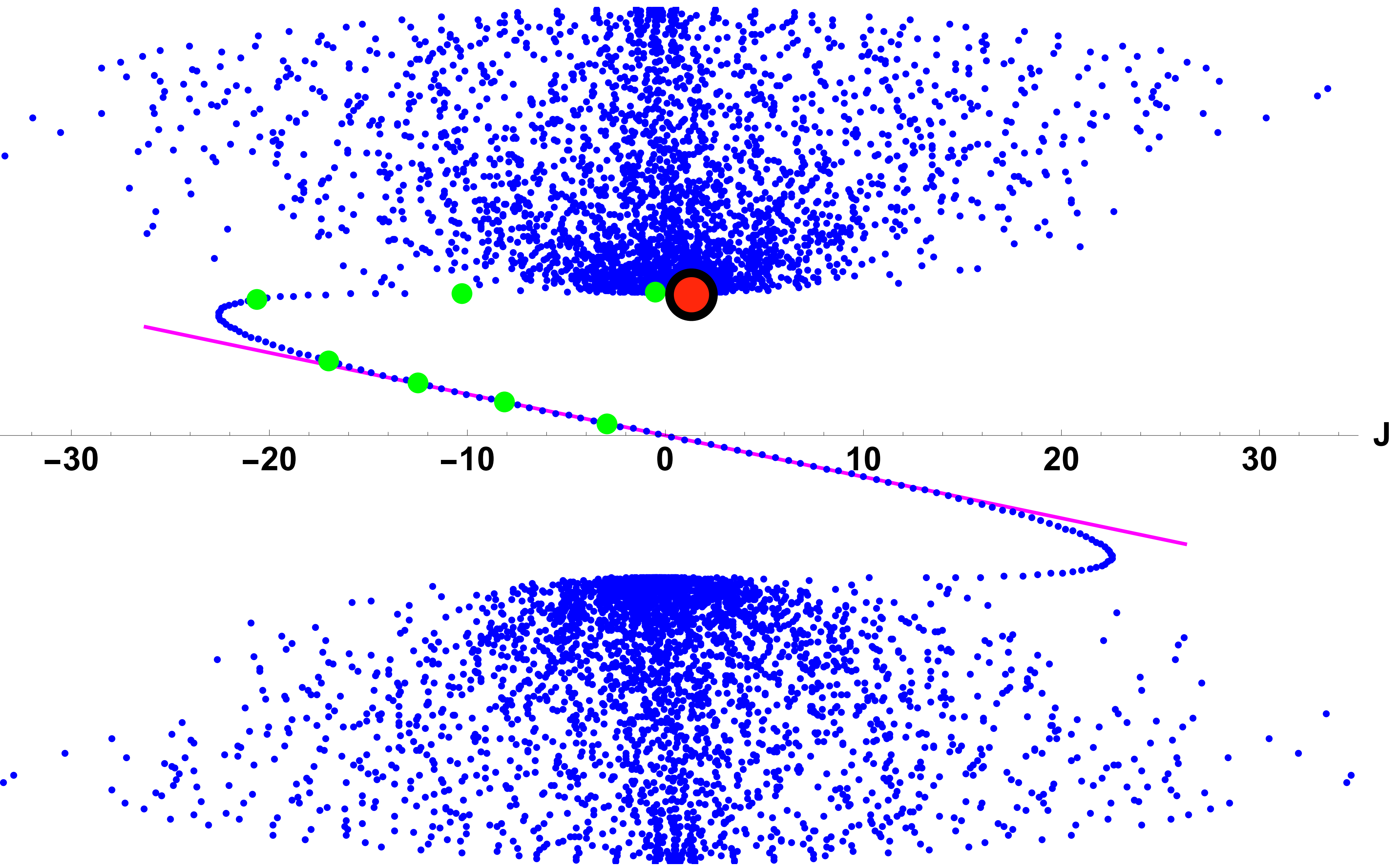

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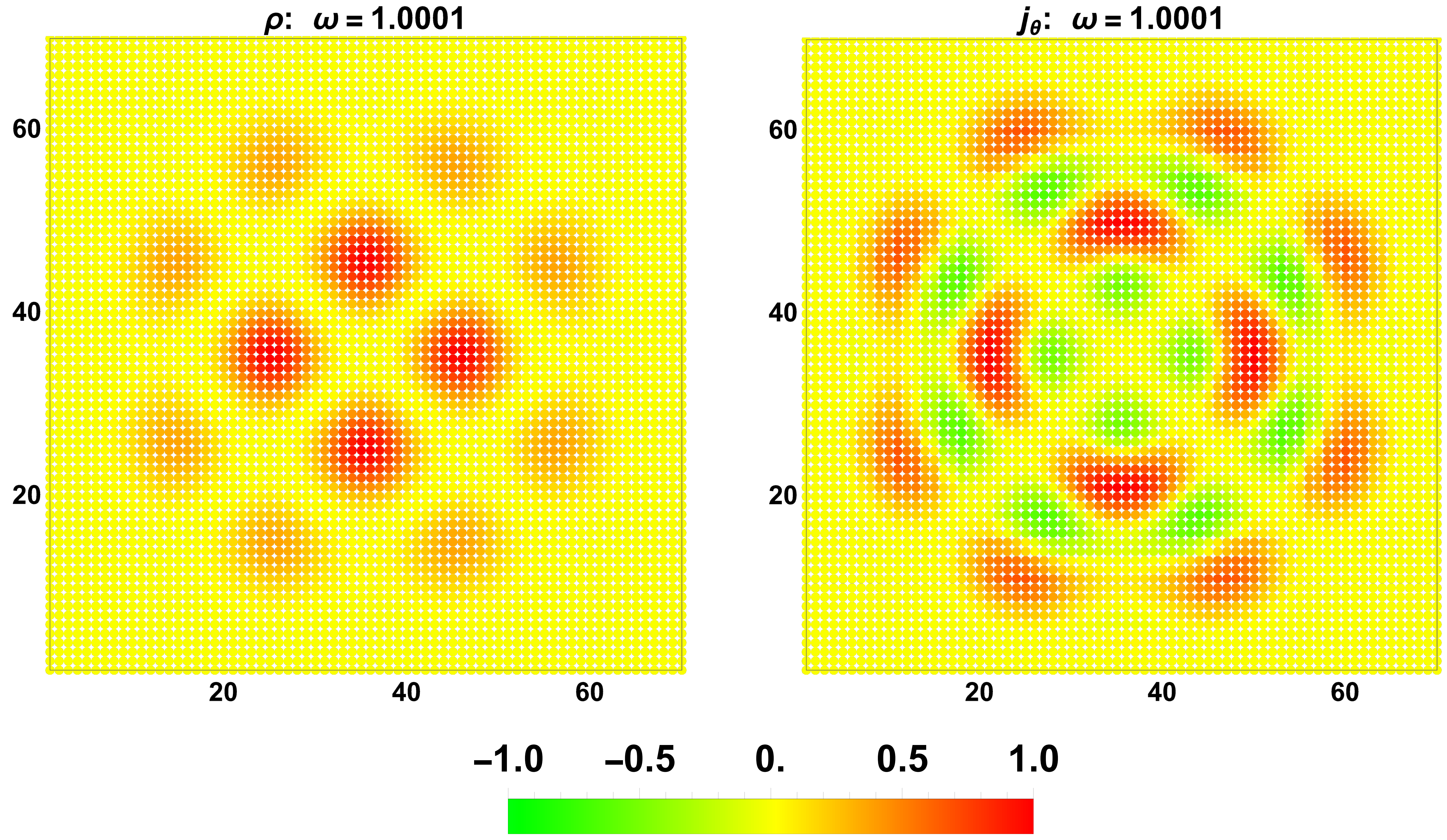

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Recipe: Define bulk gauge fields B_μ to be functionals of the boundary values A_μ ; integrate only over the A_μ in the path integral

$$B_\mu(\mathbf{x}_\perp, r, \theta)\Big|_{r=R} = A_\mu(\mathbf{x}_\perp, \theta)$$

For example, B_μ can be solution to Euclidian YM eq. subject to this BC.

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Bulk fermion modes generate a Chern Simons operator in the bulk which is a function of B_μ and therefore a nonlocal functional of the edge gauge fields A_μ

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This theory will be a local d-dimensional theory in the infrared *iff* the chiral gauge theory is anomaly-free (like the SM!)

The whole story? No.

Golterman & Shamir arXiv:1404.16372 (2024): $U(1)_A$ behaves wrong: ’t Hooft operators from instantons involve spurious fermion zeromodes in 5d bulk.

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Are there other unexpected predictions, nonperturbative effects?

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Golterman & Shamir arXiv:1404.16372 (2024): $U(1)_A$ behaves wrong: ’t Hooft operators from instantons involve spurious fermion zeromodes in 5d bulk.

Possible solutions might exist… but only for the case $\theta_{QCD}=0$? Could this be a prerequisite for defining the SM nonperturbatively? Too early to say, work in progress.

Are there other unexpected predictions, nonperturbative effects?

The first task is to reproduce QCD effects (or 1+1 dimension analogs) with a setup like this.

Summary

Like condensed matter system, topological "matter" is ubiquitous in relativistic quantum field theories with a gap, and such materials support chiral edge states

These topological phases can be exploited on the lattice for simulating Weyl fermions, defying the Nielsen-Ninomiya theorem by violating some of its assumptions.

It look like it may be possible to gauge such theories as local 4d theories if the gauge anomalies cancel (as they do in the SM).

Hopefully before long a simulation of nonperturbative effects in the SM might be possible.

Question for PASCOS 2024:

A fifth dimension was introduced as a “trick” for nonperturbatively defining the Standard Model on a lattice…

…but if it turns out to be the *only* feasible way to define the SM, should we take the hint that this might be how the real world works?

Can a cosmology for such a world make sense (remember — the gauge fields are weird)?

…or is there perhaps a more natural formulation to confine the propagating gauge fields to the boundary?

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➤ Do ``non-universal’’ features of a regulator for the Standard Model tell us about constraints on the world?

- Constraints on θ_{QCD}?
- The world as a 4D boundary of a 5D universe?