Dark Matter theory
(From heavy to light dark matter)

Celine Boehm

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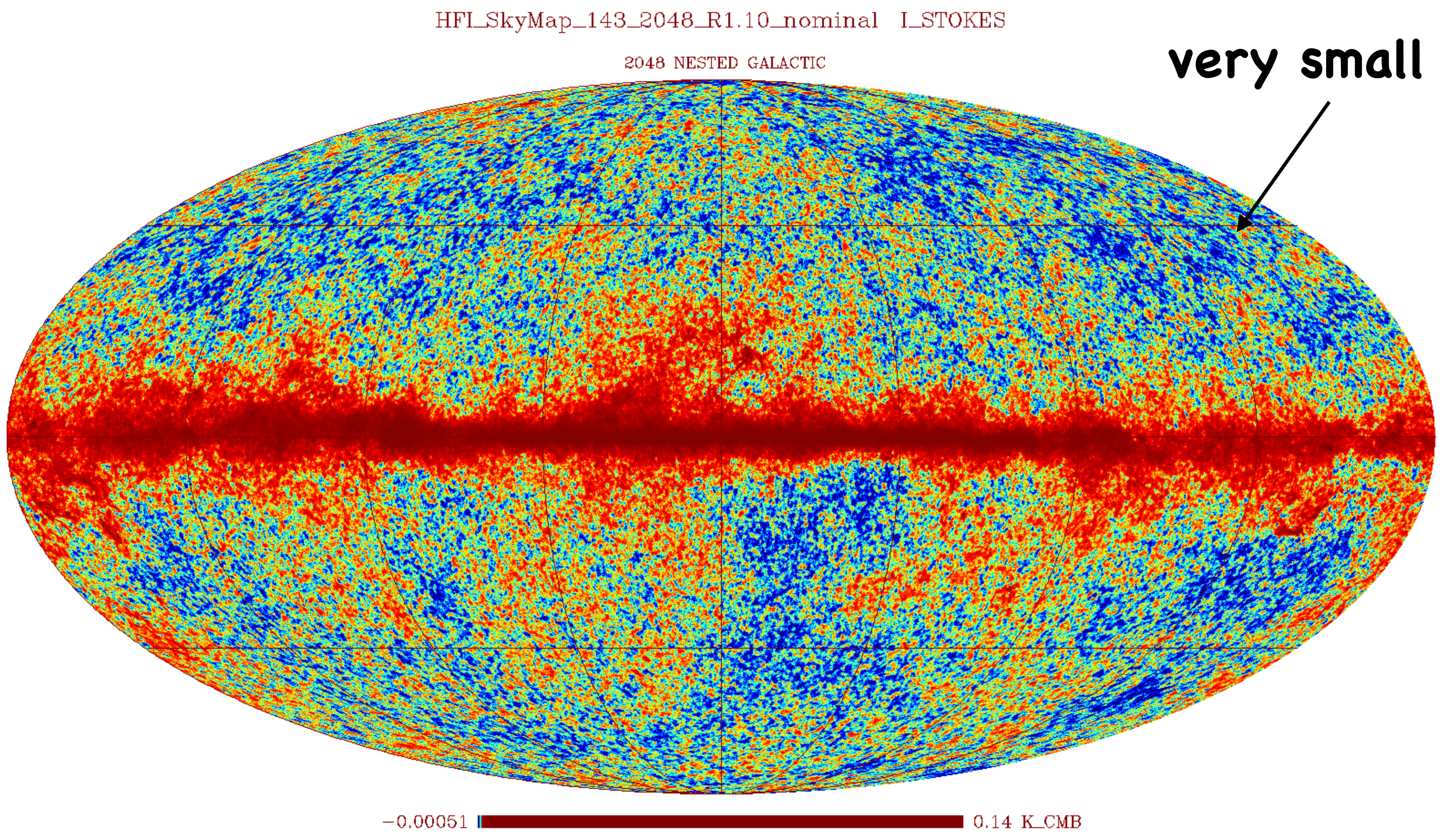
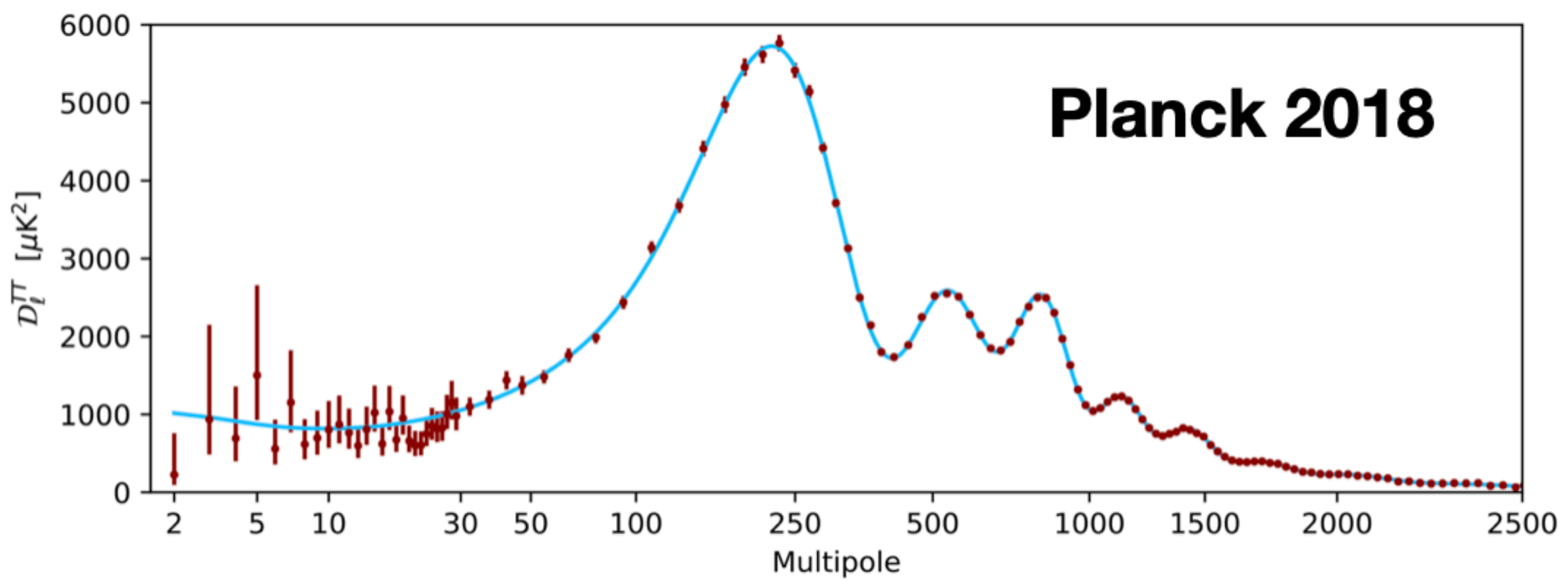
Recontres du Vietnam, 8 August 2023

Courtesy Millie McDonald, Whisky Bay in Wilsons Promontory, 45 frames, each a stack of 4x 6s exposures at ISO 800

DM needed for CMB & Structure formation



very small scales

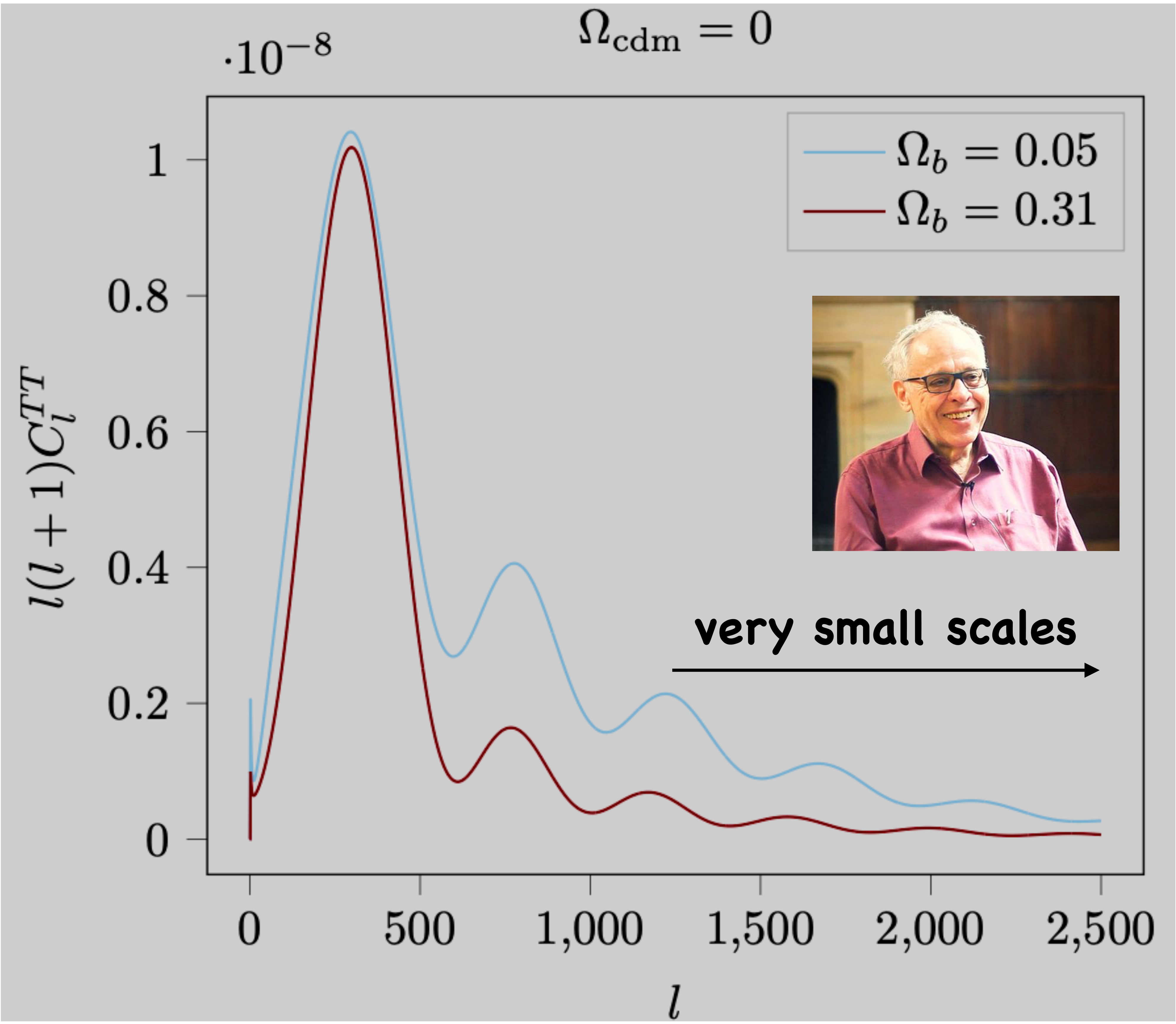


$$\dot{\theta}_b = k^2\psi - \mathcal{H}\theta_b + c_s^2 k^2 \delta_b - R^{-1}\dot{\kappa}(\theta_b - \theta_\gamma)$$

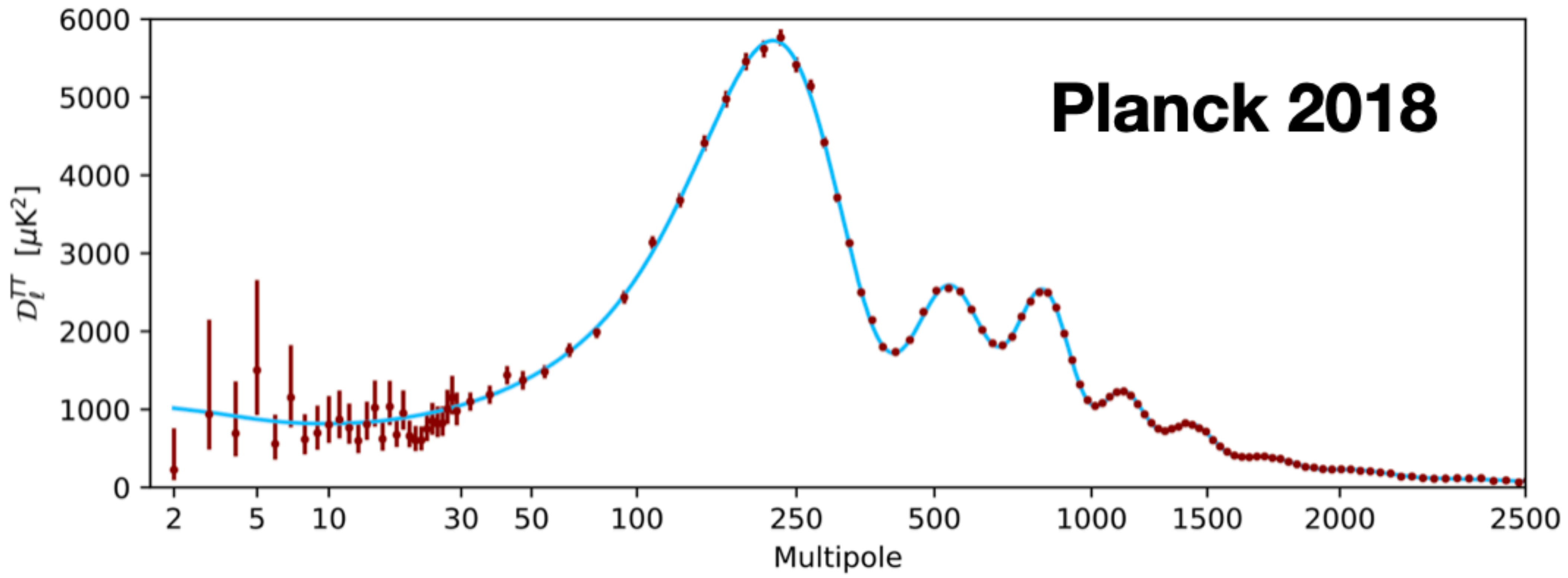
Ordinary matter

$$\dot{\theta}_\gamma = k^2\psi + k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) - \dot{\kappa}(\theta_\gamma - \theta_b)\,,$$

Photons



DM needed for CMB & Structure formation



$$\dot{\theta}_b = k^2\psi - \mathcal{H}\theta_b + c_s^2 k^2 \delta_b - R^{-1}\dot{\kappa}(\theta_b - \theta_\gamma)$$

$$\dot{\theta}_\gamma = k^2\psi + k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) - \dot{\kappa}(\theta_\gamma - \theta_b)\ ,$$

$$\boxed{\dot{\theta}_{DM} = k^2\psi - \mathcal{H}\theta_{DM}\ ,}$$

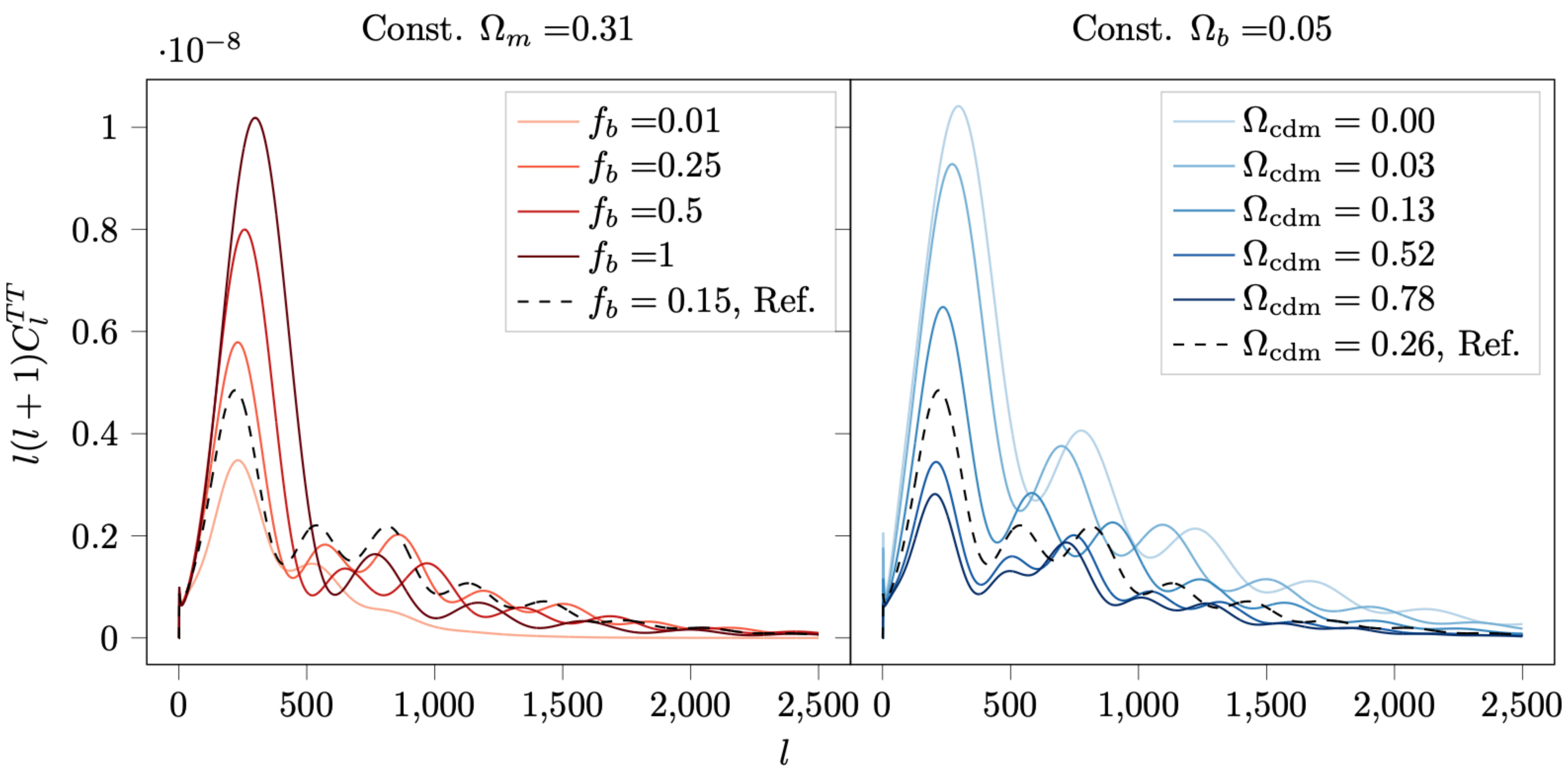
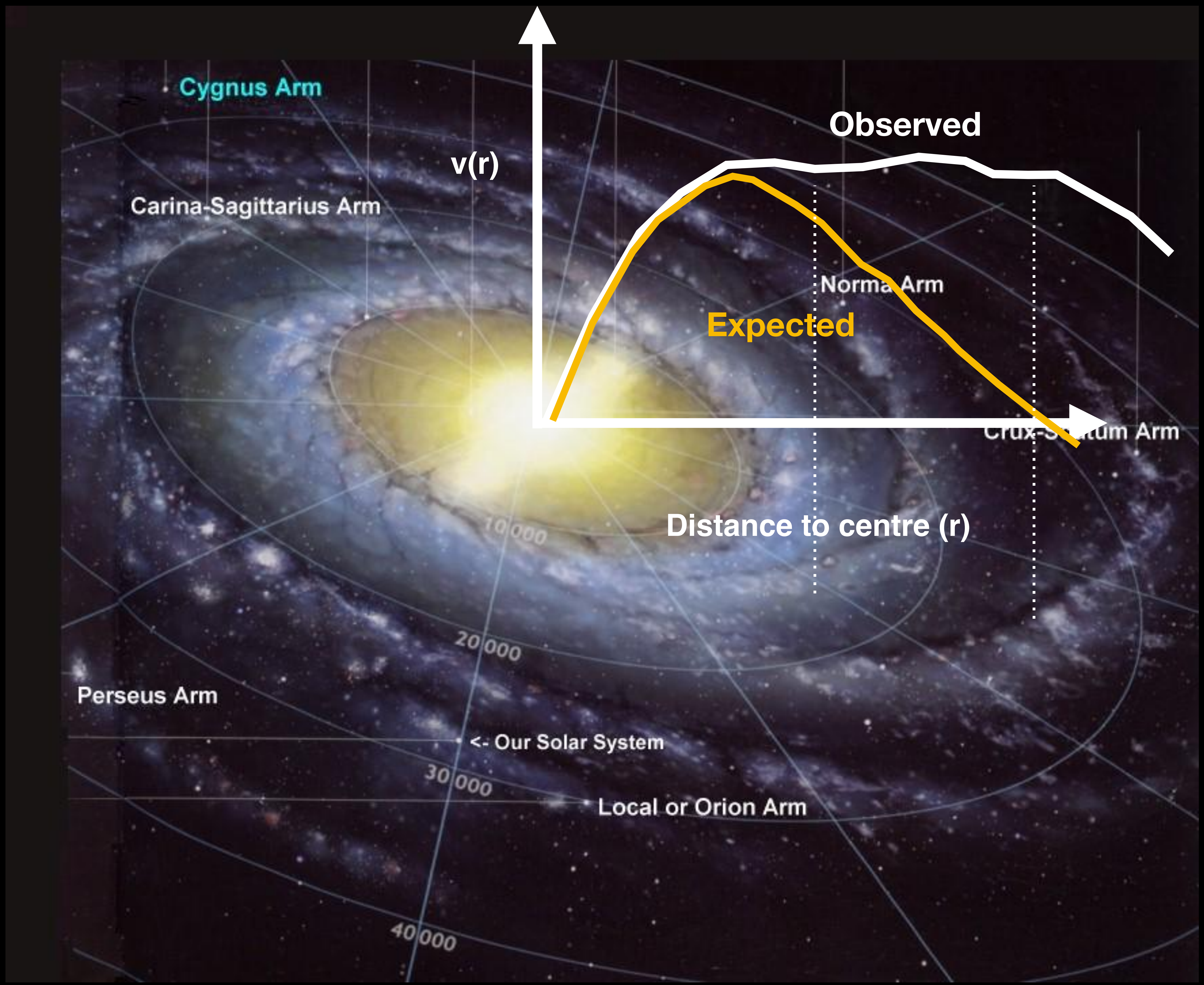


Figure 1.6: *The impact on the CMB TT spectrum of changing the baryon fraction while keeping the total matter density constant (left) as well as changing the dark matter density while keeping the baryon density constant (right). In the latter, the Hubble rate today is kept constant, while Ω_Λ varies to maintain flatness. Ref. notes the values inferred from observations [5].*

DM needed for CMB & Structure formation



- What drives structures to form?
- What prevents structures to collapse?

- Missing mass
- Lack of dissipation

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Solutions?

	Deeper gravitational potential	Fighting Dissipation
Modifying gravity	✓ (Acceleration)	Hard :(Require a relativistic theory
Adding mass/particles	✓ if massive	✓ if weak interactions

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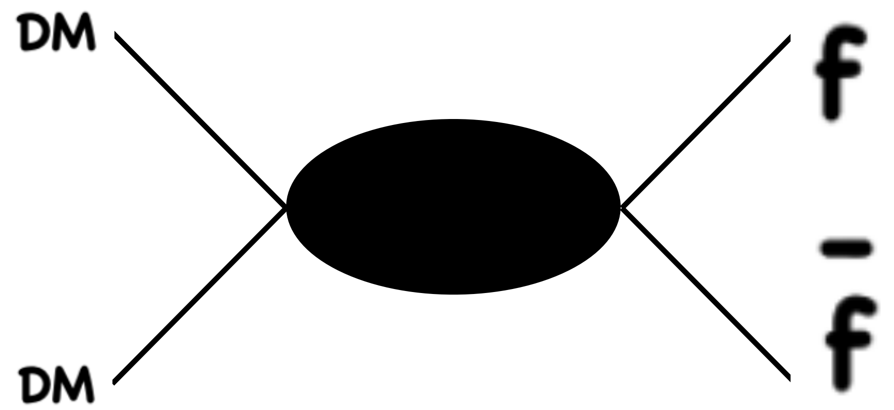
The first guiding principle

- mass and no dissipation = weakly interacting massive particles

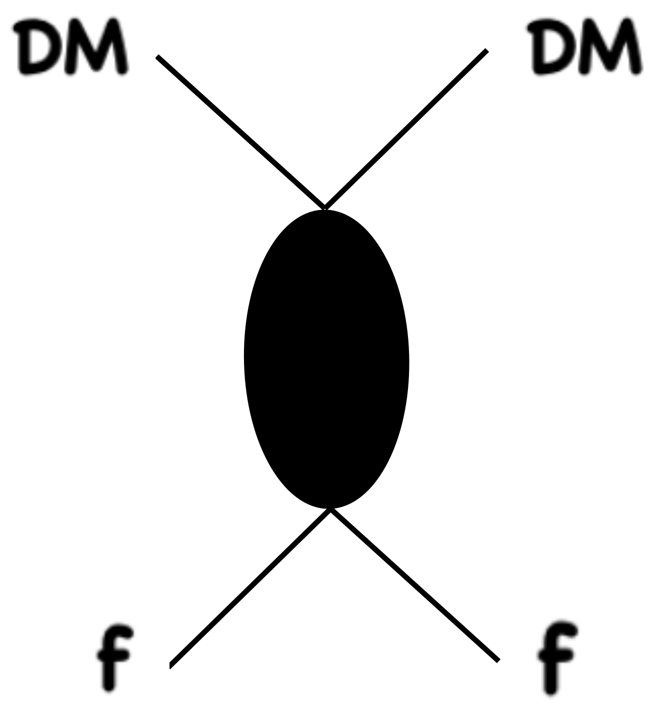
Weakly Interacting Massive Particles

Assuming similar behaviour as SM particles



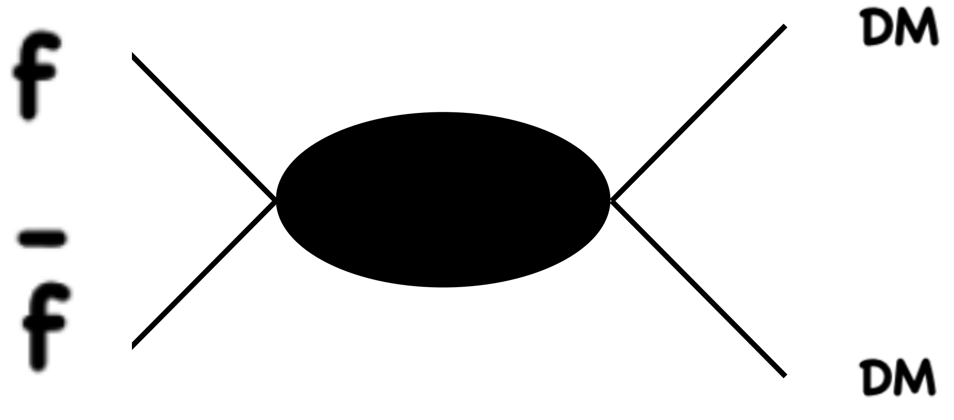
Relic density + indirect detection

cosmology



Structure formation + direct detection

nuclear/particle



Collider /
Particle Physics experiments

particle

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The second guiding principle

- Relic density
  (the corresponding energy density changes the geometry of the Universe)

(Thermal) Heavy DM

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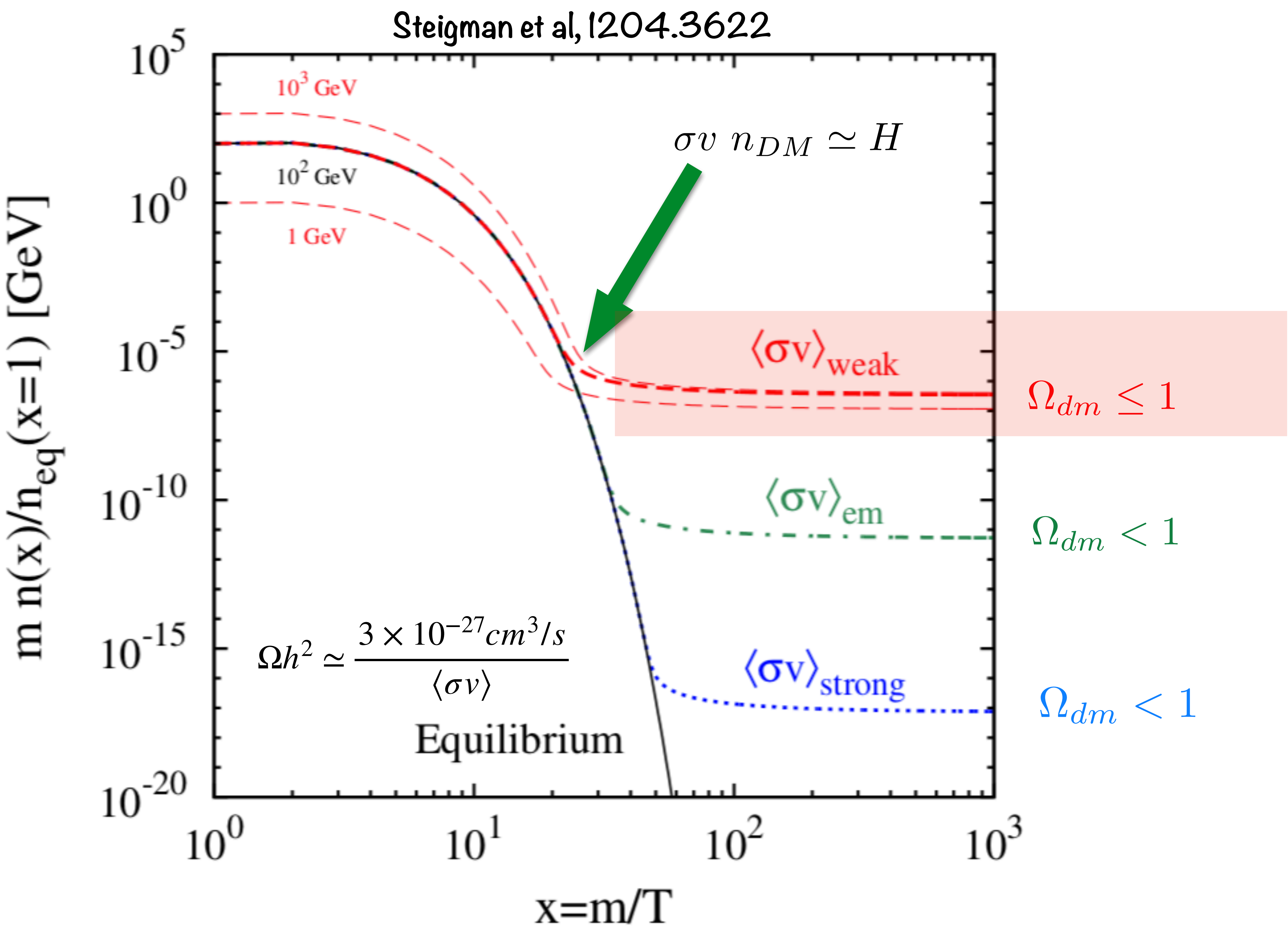
Relic density

$$\frac{dn}{dt} = -3Hn - \sigma v(n^2 - n_0^2)$$

$$\Omega h^2 \simeq \frac{3\times 10^{-27} cm^3/s}{\langle\sigma v\rangle}$$

Before WMAP/Planck



$$\sigma v \propto \frac{m_{DM}^2}{m_W^4} = 310^{-26}\mathrm{cm}^3/\mathrm{s}$$

DM must be heavy (Hut-Lee&Weinberg 77)

After Planck

Parameter	Plik best fit
$\Omega_b h^2$	0.022383
$\Omega_c h^2$	0.12011
$100\theta_{MC}$	1.040909
τ	0.0543
$\ln(10^{10} A_s)$	3.0448
n_s	0.96605
$\Omega_m h^2$	0.14314
H_0 [km s^{-1}Mpc^{-1}]	67.32
Ω_m	0.3158
Age [Gyr]	13.7971
σ_8	0.8120
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	0.8331
z_{re}	7.68
$100\theta_*$	1.041085
r_{drag} [Mpc]	147.049

Planck 2018

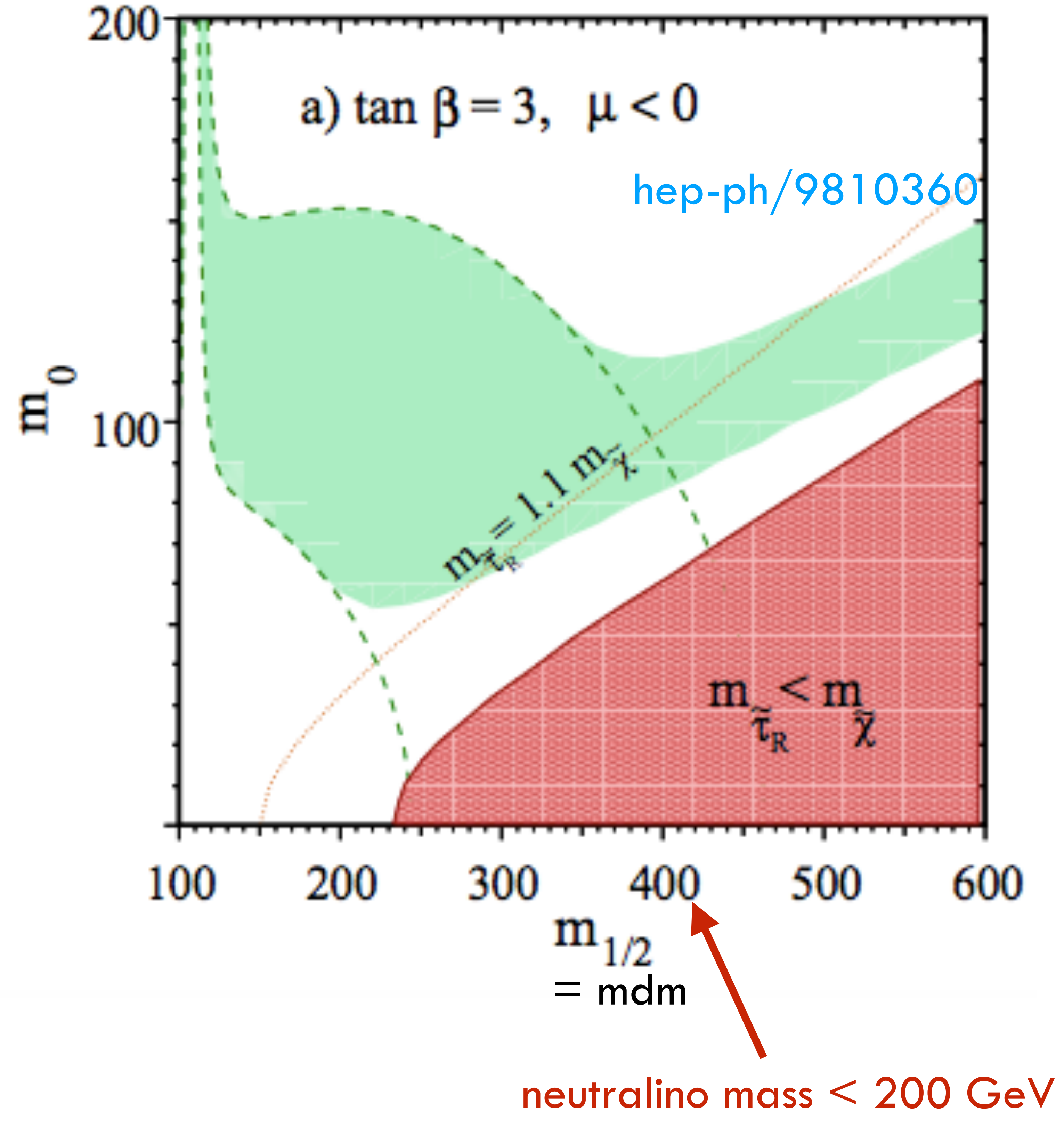
Planck's precision = precision on the cross section

Even though we did not specify a model...

Relic density: a SUSY example

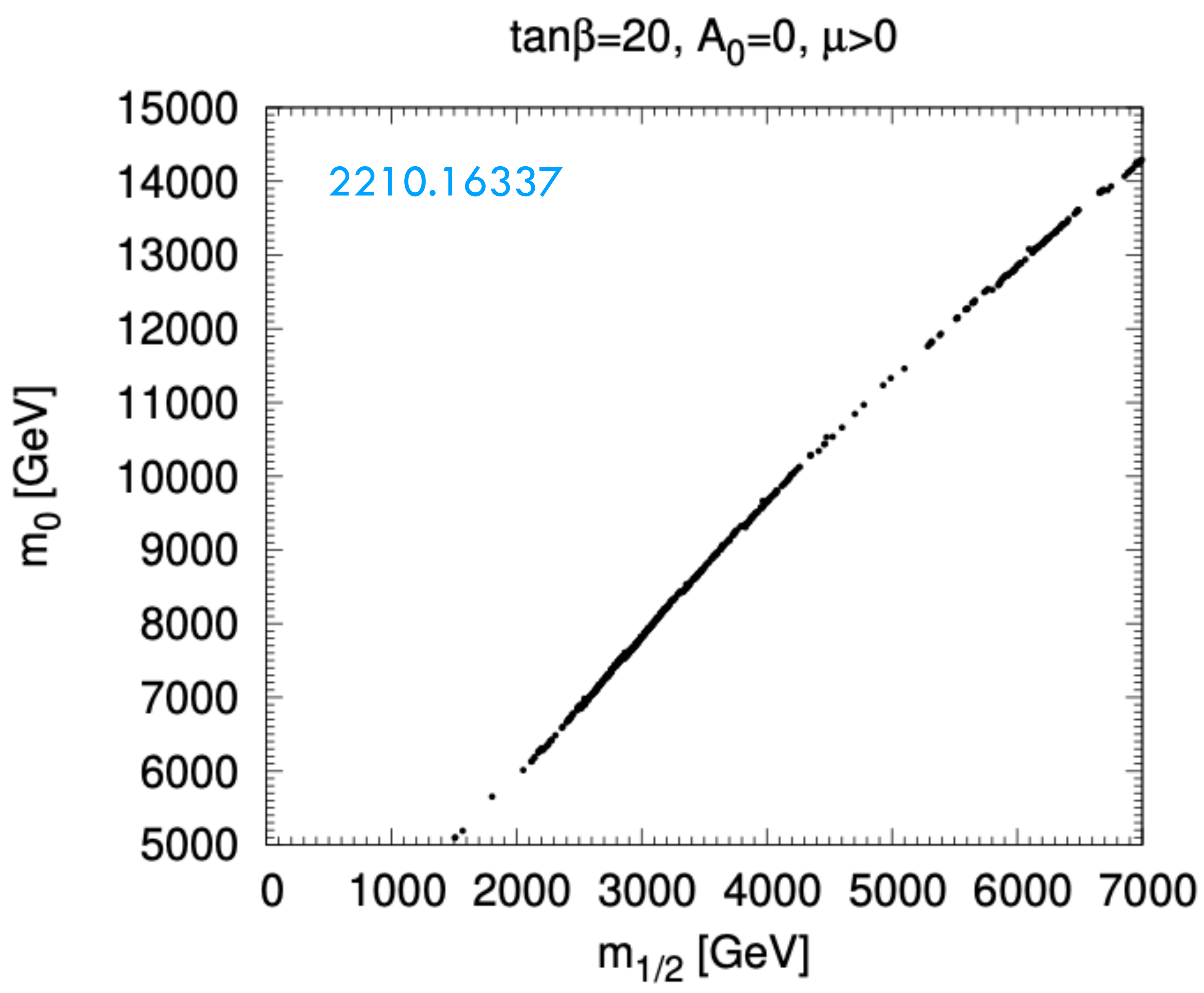
Before Planck

$$\sigma v \propto \frac{m_{dm}^2}{m_{\text{mediator}}^4}$$



After Planck

(Not same parameters)



SUSY parameter space is severely constrained.
Pushing theoreticians to consider mass degeneracies…

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The pros and cons of having a theory (SUSY)

Pros:

- Motivated by fundamental principles (at least partially)
- If found, then direct access to new fundamental laws of Nature
- For collider searches, it is ideal because it gives clues where to look

(And for theoreticians, where to hide their favourite candidates!)

Cons:

- Biased (in the worse case towards one very specific theory)
- Not necessary for direct nor indirect searches
- Depend on parameters which may be motivated at high energy but effective value is what counts...

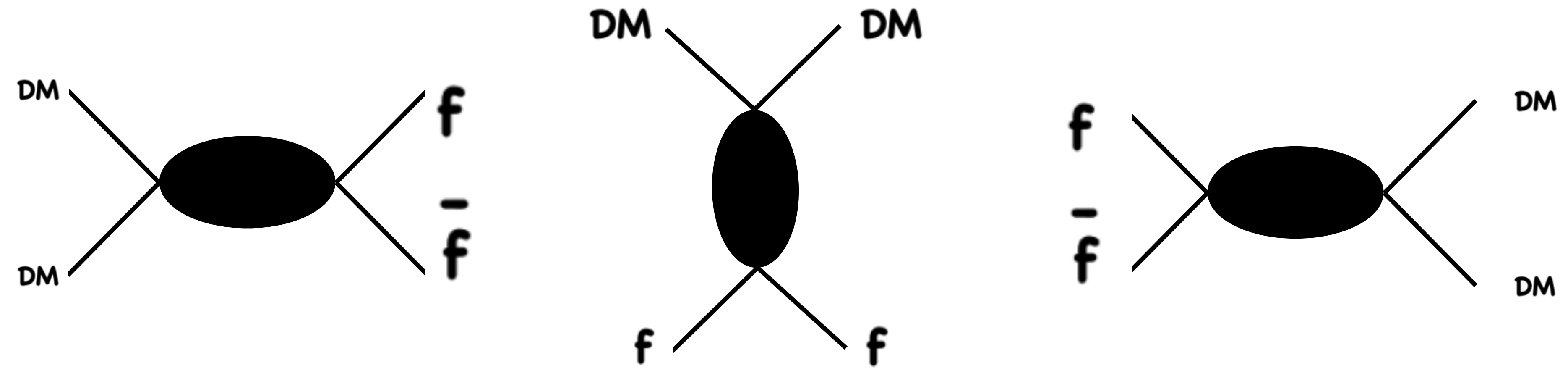
Perhaps no need for a theory after all

With neutrinos

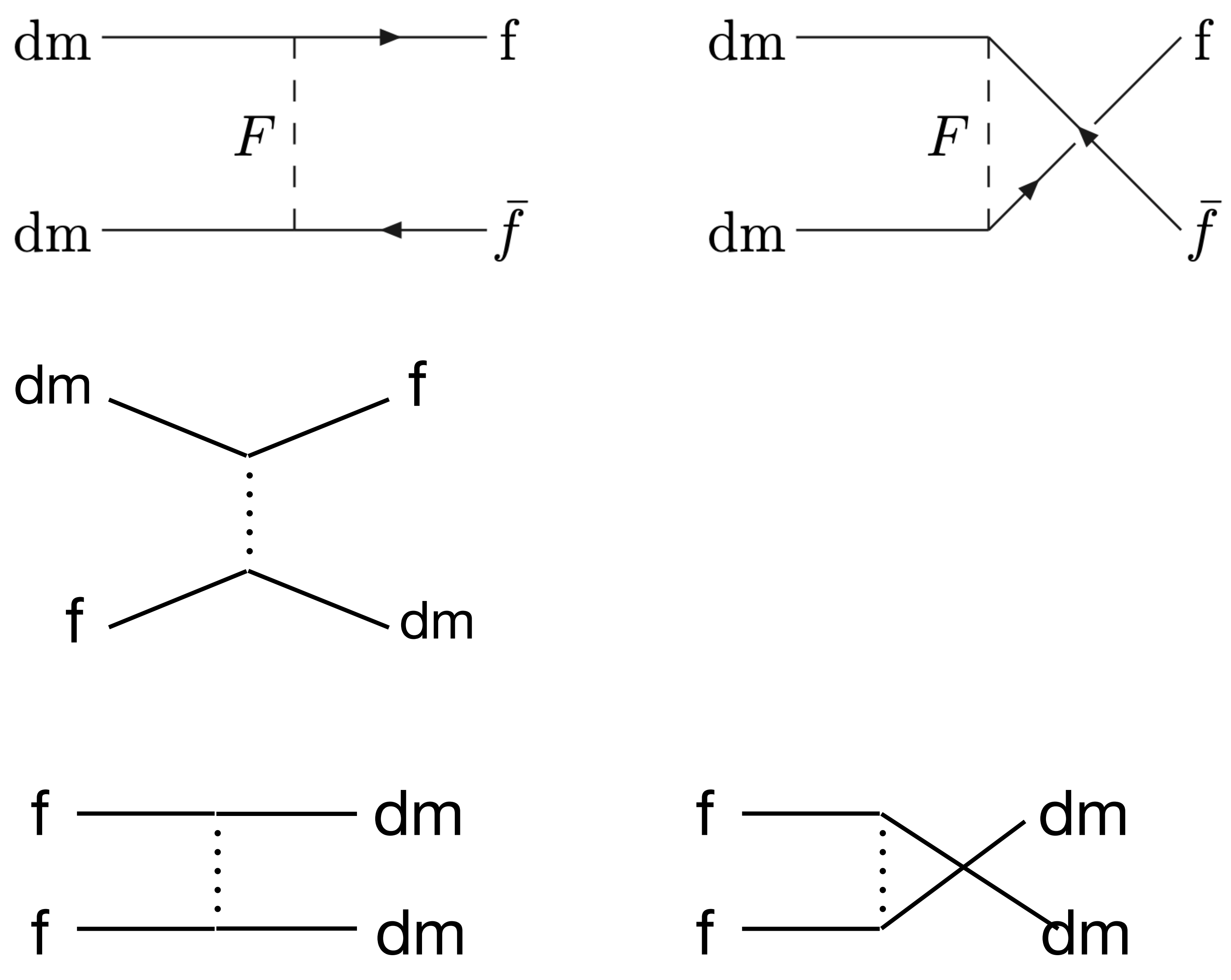
Scenario	Lagrangian ($\mathcal{L}_{\rm int}$)	$\sigma \mathbf{v}_{\rm r}$	$\sigma_{\rm el}$
Complex DM Dirac Mediator	$-g\,\chi\,\overline{N_{\rm R}}\,\nu_{\rm L}\;+\;{\rm h.c.}$	$\frac{g^4}{12\,\pi}\frac{m_{\rm DM}^2}{(m_{\rm DM}^2+m_{\rm N}^2)^2}\,v_{\rm CM}^2$	$\frac{g^4}{32\,\pi}\frac{m_{\rm DM}^2\,y^2}{(m_{\rm N}^2-m_{\rm DM}^2)^2}$
Real DM Dirac Mediator		$\frac{4\,g^4}{15\,\pi}\frac{m_{\rm DM}^6}{(m_{\rm DM}^2+m_{\rm N}^2)^4}\,v_{\rm CM}^4$	$\frac{g^4}{8\,\pi}\frac{m_{\rm DM}^6\,y^4}{(m_{\rm N}^2-m_{\rm DM}^2)^4}$
Complex DM Majorana Mediator		$\frac{g^4}{16\,\pi}\frac{m_{\rm N}^2}{(m_{\rm DM}^2+m_{\rm N}^2)^2}$	$\frac{g^4}{32\,\pi}\frac{m_{\rm DM}^2\,y^2}{(m_{\rm N}^2-m_{\rm DM}^2)^2}$
Real DM Majorana Mediator		$\frac{g^4}{4\,\pi}\frac{m_{\rm N}^2}{(m_{\rm DM}^2+m_{\rm N}^2)^2}$	$\frac{g^4}{8\,\pi}\frac{m_{\rm DM}^6\,y^4}{(m_{\rm N}^2-m_{\rm DM}^2)^4}$
Dirac DM Scalar Mediator	$-g\overline{\chi_{\rm R}}\nu_{\rm L}\phi\;+\;{\rm h.c.}$	$\frac{g^4}{32\,\pi}\frac{m_{\rm DM}^2}{(m_{\rm DM}^2+m_\phi^2)^2}$	$\frac{g^4}{32\,\pi}\frac{m_{\rm DM}^2\,y^2}{(m_{\rm DM}^2-m_\phi^2)^2}$
Majorana DM Scalar Mediator		$\frac{g^4}{12\,\pi}\frac{m_{\rm DM}^2}{(m_{\rm DM}^2+m_\phi^2)^2}\,v_{\rm CM}^2$	$\frac{g^4}{16\,\pi}\frac{m_{\rm DM}^2\,y^2}{(m_{\rm DM}^2-m_\phi^2)^2}$
Vector DM Dirac Mediator	$-g\overline{N_{\rm L}}\gamma^\mu\chi_\mu\nu_{\rm L}\;+\;{\rm h.c.}$	$\frac{2\,g^4}{9\,\pi}\frac{m_{\rm DM}^2}{(m_{\rm DM}^2+m_{\rm N}^2)^2}$	$\frac{g^4}{4\,\pi}\frac{m_{\rm DM}^2\,y^2}{(m_{\rm DM}^2-m_{\rm N}^2)^2}$
Vector DM Majorana Mediator		$\frac{g^4}{6\,\pi}\frac{m_{\rm N}^2}{(m_{\rm DM}^2+m_{\rm N}^2)^2}$	
Complex DM Vector mediator	$-g_\chi Z'^\mu((\partial_\mu\chi)\chi^\dagger-(\partial_\mu\chi)^\dagger\chi)$ $-g_\nu\overline{\nu_L}\gamma^\mu Z'_\mu\nu_{\rm L}$	$\frac{g_\chi^2\,g_\nu^2}{3\,\pi}\frac{m_{\rm DM}^2}{(4\,m_{\rm DM}^2-m_{\rm Z'}^2)^2}\,v_{\rm CM}^2$	$\frac{g_\chi^2\,g_\nu^2}{8\,\pi}\frac{m_{\rm DM}^2\,y^2}{m_{\rm Z'}^4}$
Dirac DM Vector Mediator	$-g_{\chi{\rm L}}\overline{\chi_{\rm L}}\gamma^\mu Z'_\mu\chi_{\rm L}\;-\;g_{\chi{\rm R}}\overline{\chi_{\rm R}}\gamma^\mu Z'_\mu\chi_{\rm R}$ $-g_\nu\overline{\nu_L}\gamma^\mu Z'_\mu\nu_{\rm L}$	$\frac{g_\chi^2\,g_\nu^2}{2\,\pi}\frac{m_{\rm DM}^2}{(4\,m_{\rm DM}^2-m_{\rm Z'}^2)^2}$	$\frac{g_\chi^2\,g_\nu^2}{8\,\pi}\frac{m_{\rm DM}^2\,y^2}{m_{\rm Z'}^4}$
Majorana DM Vector Mediator	$-\frac{g_\chi}{2}\bar\chi\gamma^\mu Z'_\mu\gamma^5\chi$ $-g_\nu\overline{\nu_L}\gamma^\mu Z'_\mu\nu_{\rm L}$	$\frac{g_\chi^2\,g_\nu^2}{12\,\pi}\frac{m_{\rm DM}^2}{(4\,m_{\rm DM}^2-m_{\rm Z'}^2)^2}\,v_{\rm CM}^2$	$\frac{3\,g_\chi^2\,g_\nu^2}{32\,\pi}\frac{m_{\rm DM}^2\,y^2}{m_{\rm Z'}^4}$
Vector DM Vector Mediator	$-g_\chi\frac{1}{2}\chi^\mu\partial_\mu\chi^\nu Z'_\nu\;+\;{\rm h.c.}$ $-g_\nu\overline{\nu_L}\gamma^\mu Z'_\mu\nu_{\rm L}$	$\frac{g_\chi^2\,g_\nu^2}{\pi}\frac{m_{\rm DM}^2}{(4\,m_{\rm DM}^2-m_{\rm Z'}^2)^2}\,v_{\rm CM}^2$	$\frac{g_\chi^2\,g_\nu^2}{8\,\pi}\frac{m_{\rm DM}^2\,y^2}{m_{\rm Z'}^4}$

Simplified models

hep-ph/0305261to combine searches



Give me a Lagrangian and then ..



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Constraints on (thermal) heavy Dark Matter

$\sigma v \simeq 310^{-26}\text{cm}^3/\text{s}$

$$\frac{\mathrm{d}^2\Phi\left(\langle\sigma v\rangle, J\right)}{\mathrm{d}E\,\mathrm{d}\Omega} = \frac{1}{4\pi}\frac{\langle\sigma v\rangle}{2m_\chi^2}\sum_f \mathrm{BR}_f \frac{\mathrm{d}N_f}{\mathrm{d}E} \times \frac{\mathrm{d}J}{\mathrm{d}\Omega}. \qquad \frac{\mathrm{d}J}{\mathrm{d}\Omega} = \int_{\mathrm{l.o.s.}} \rho^2_{\mathrm{DM}}(r(s,\theta))\,ds,$$

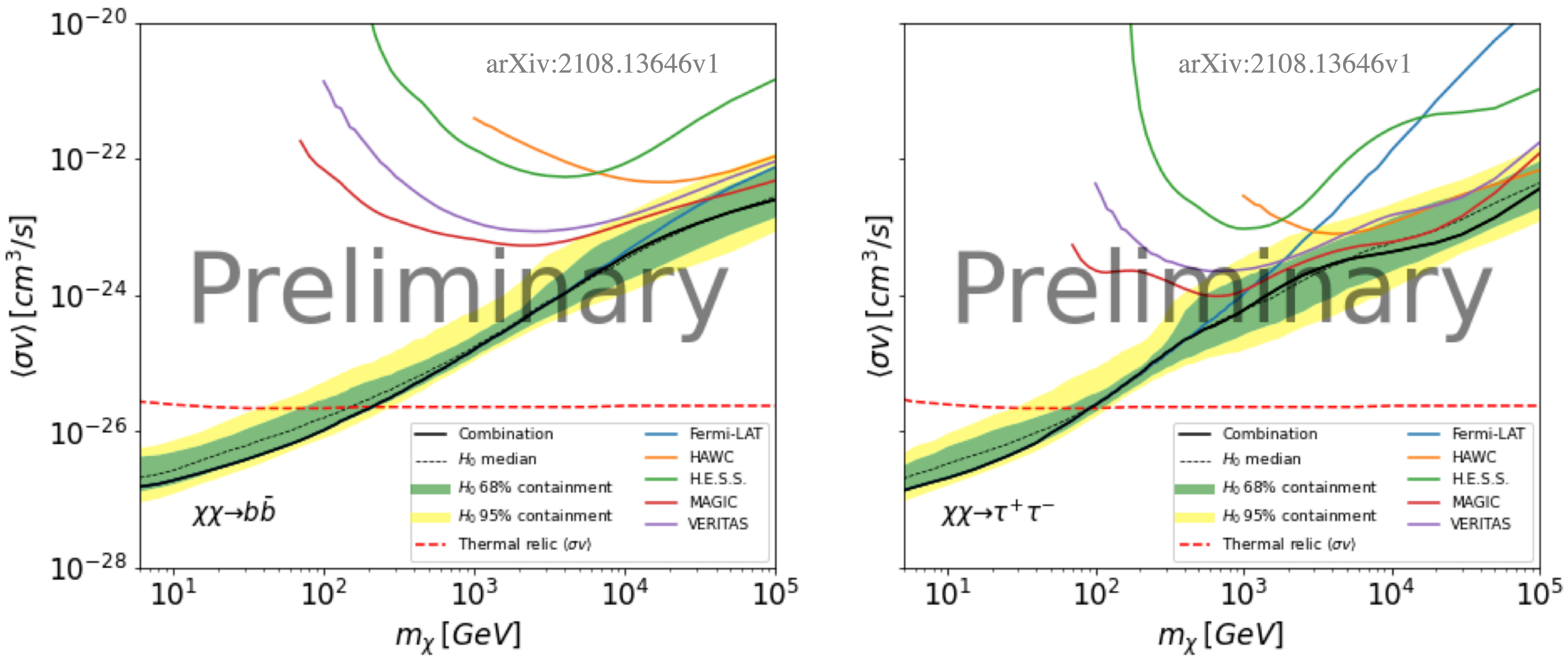
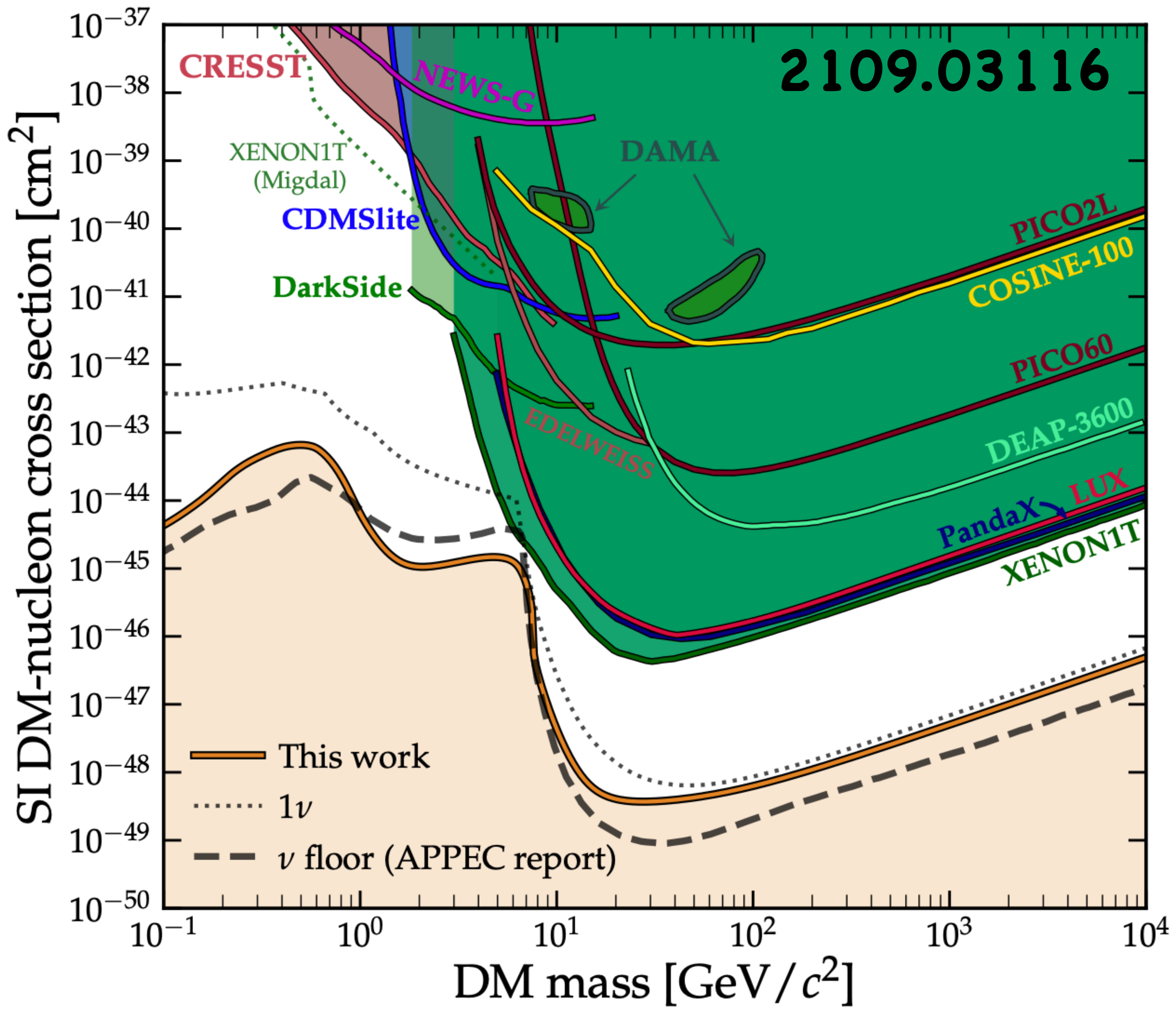


Figure 1: Upper limits at 95% confidence level on $\langle\sigma v\rangle$ as a function of the DM mass for the annihilation channels $b\bar{b}$ (left) and $\tau^+\tau^-$ (right), using the set of J factors from Ref. [8]. The black solid line represents the observed combined limit, the black dashed line is the median of the null hypothesis corresponding to the expected limit, while the green and yellow bands show the 68% and 95% containment bands. Combined upper limits for each individual detector are also indicated as solid, colored lines.



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The many assumptions in (thermal) heavy DM

- Thermal production
- One dominant DM particle
- Annihilations
- No decay
- Annihilations into SM particles
- In most scenarios annihilation into charged particles
- Symmetry in DM and anti DM number densities
- No modified gravity

Hut, Lee&Weinberg 77 $\sigma v \propto \frac{m_{DM}^2}{m_W^4}$ $\left(\sigma v \propto \frac{m_{dm}^2}{m_{\rm mediator}^4}\right)$ Can one get a different cross section?

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Towards (Thermal) light DM

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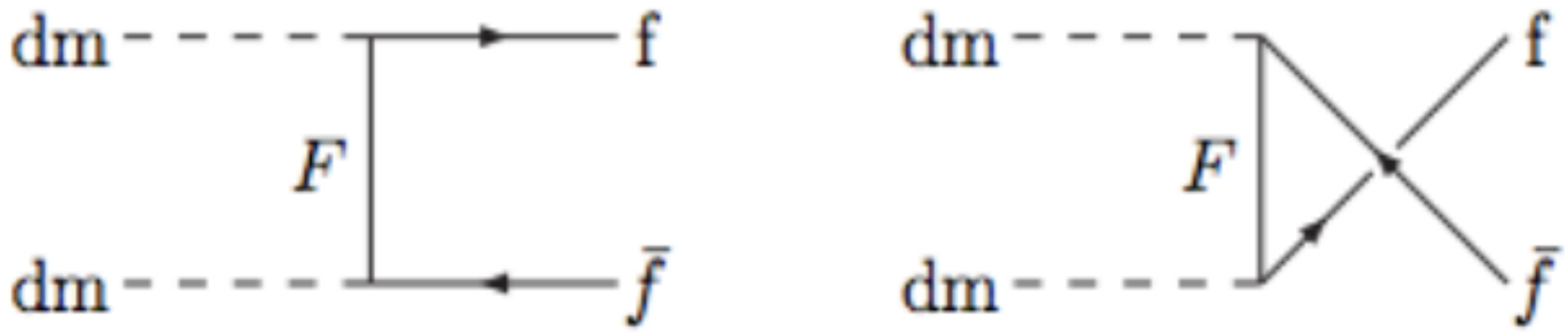
Alternative models of cross sections?

Generalisation of the sneutrino case

hep-ph/0305261

Found again in 0803.4196



$$\sigma v \propto \frac{1}{m_F^4} \left(\left(C_l^2 + C_r^2\right) m_f + 2 C_l \, C_r \, m_F \right)^2$$

$$\sigma v \propto \frac{C_l^2 C_r^2}{m_F^2}$$

Independent of DM mass!

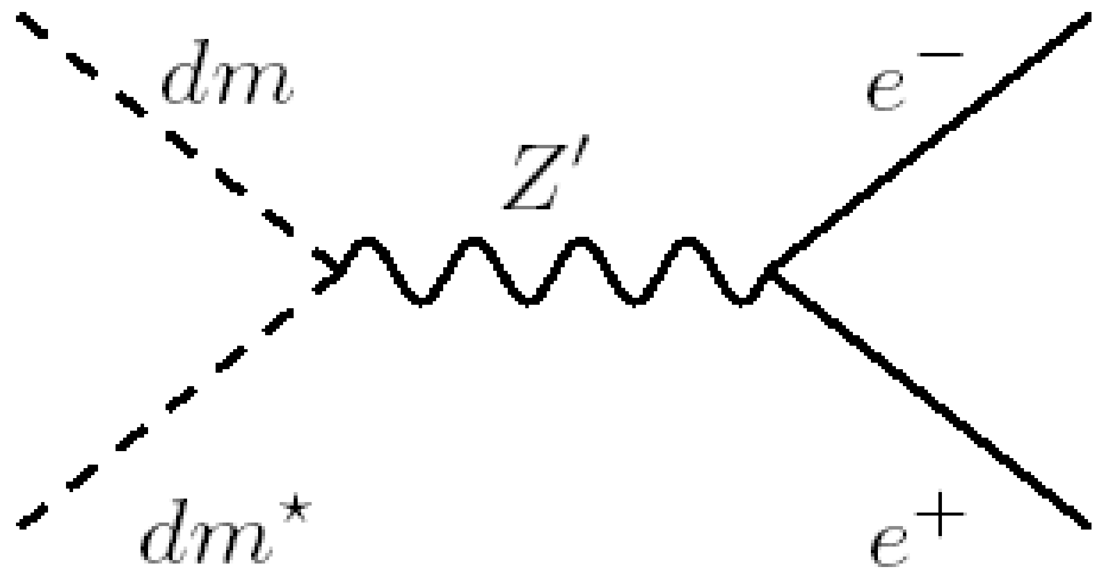
$= 3 10^{-26} \mathrm{cm}^3/\mathrm{s}$

DM can be lighter than a proton!

Alternative models of cross sections?

neutrino case with 2 twists

hep-ph/0305261



$$\sigma v \propto v^2 \, \frac{m_{\rm DM}^2}{m_{Z'}^4} \, g_{\rm DM}^2 \, g_e^2$$

Similar to $\sigma v \propto \frac{m_{dm}^2}{m_{\rm mediator}^4}$

But the mediator can be neutral

DM can be lighter than a proton!

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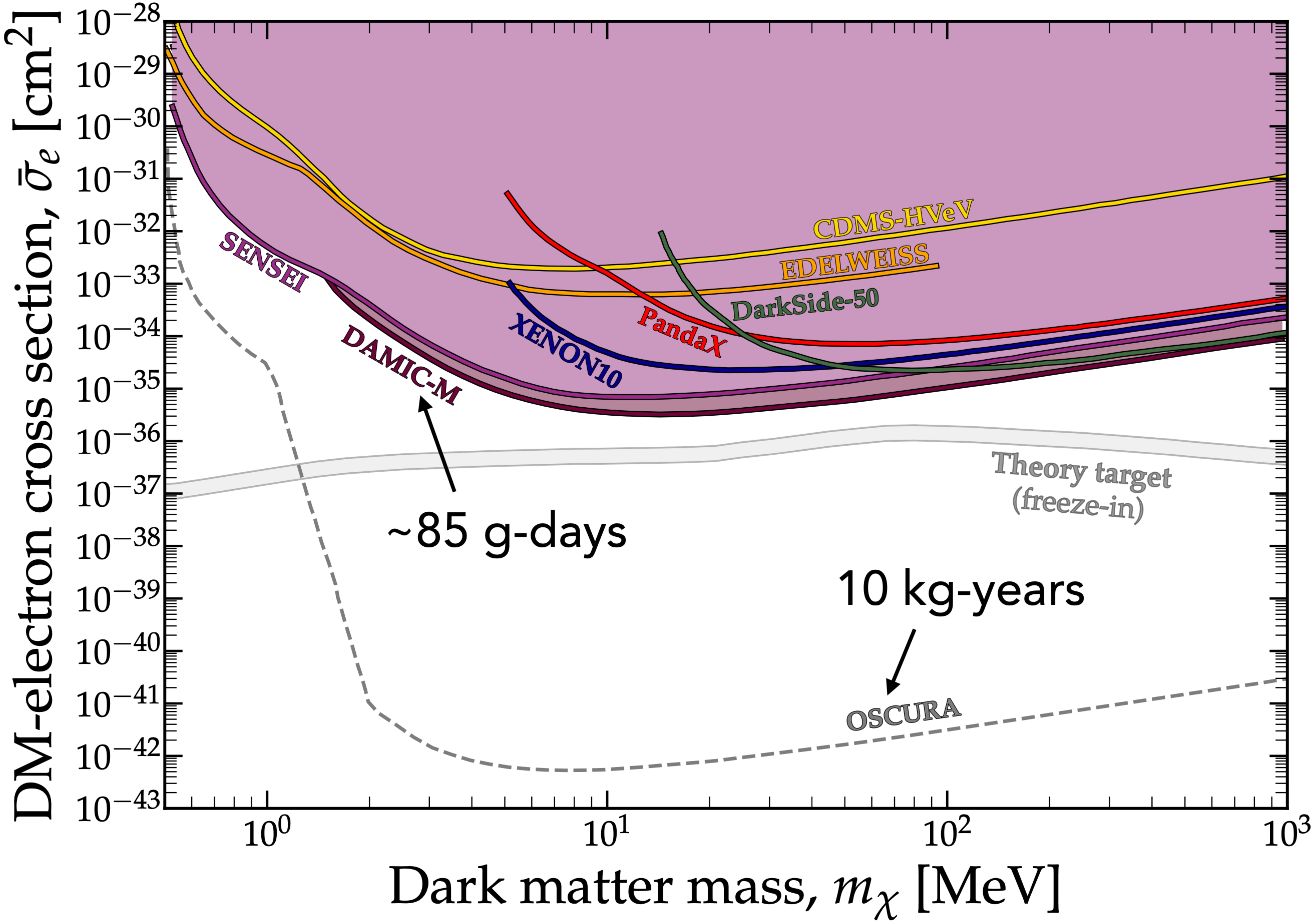
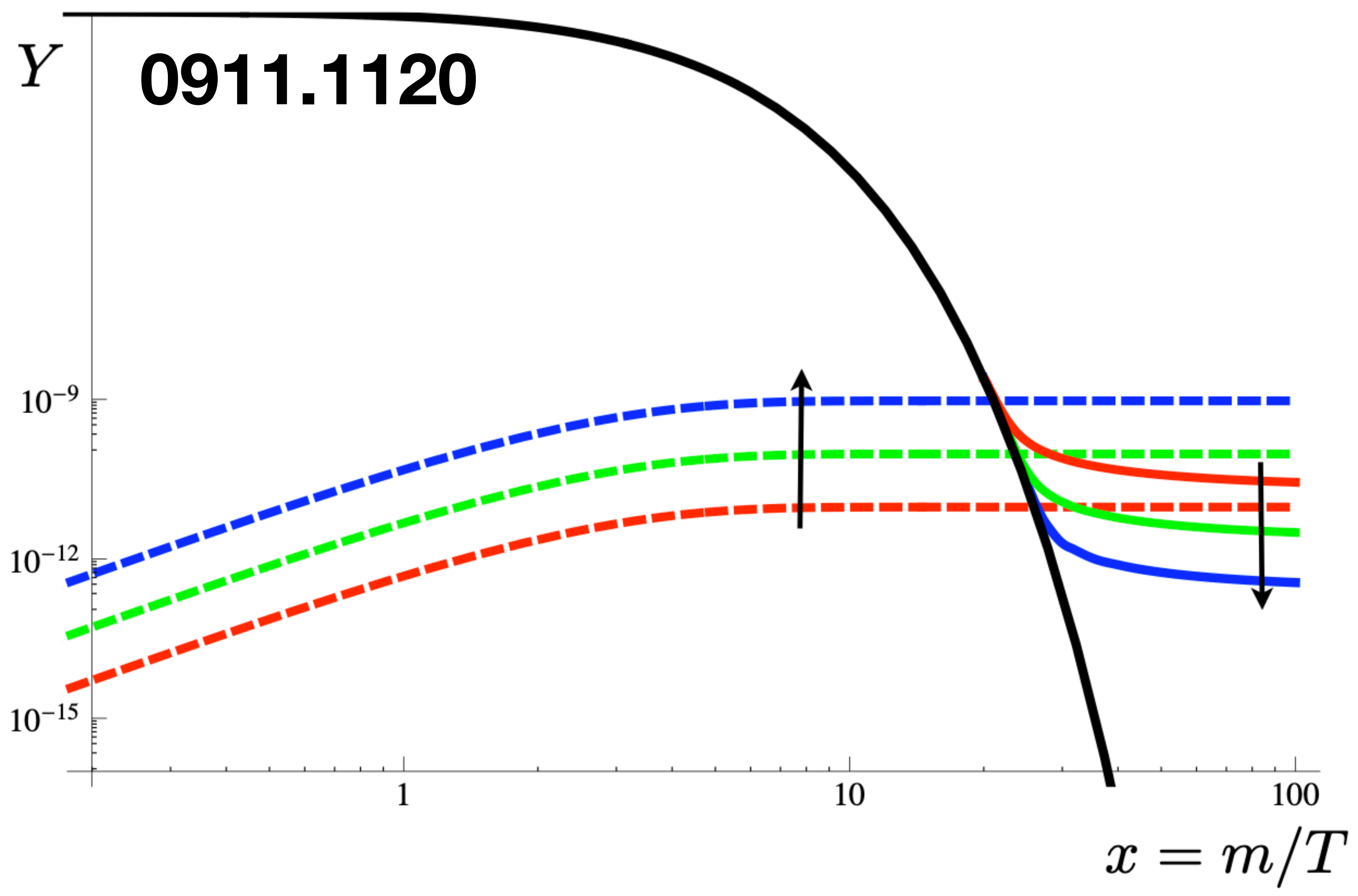
Constraints on light (thermal) MeV-GeV Dark Matter

(+ constraints on the mediators)



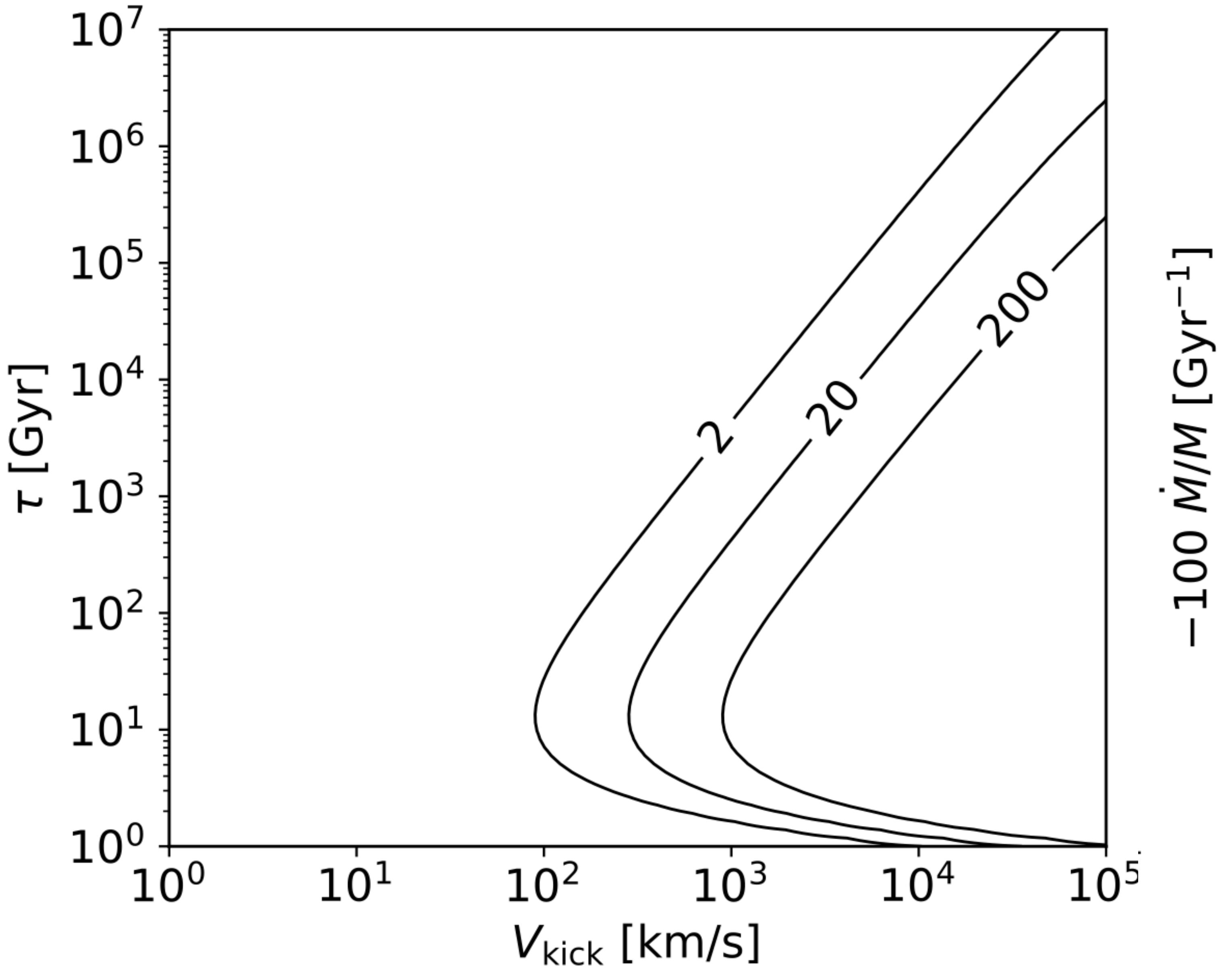
Abandoning the thermal assumption

Abandoning the relic density constraint



Should there be annihilations at all?

Asymmetric DM, annihilation into dark sectors, decaying DM, regeneration, cannibalism, sterile neutrinos etc



arXiv:2207.11739v1

Figure 6. Mass loss rate due to decay of dark matter for a dark matter halo as a function of $V_{\rm kick}$ and decay lifetime τ. Mass loss rate is shown for a sphere of radius 30 kpc and 13 Gyr after the formation of the halo. The solid lines are contours for mass loss rates per Gyr of 2%, 20% and 200%. The result is for an NFW halo with a virial mass of 0.8×10^{12} and concentration parameter $c = 20$, but approximated by a Plummer model following Abdelqader & Melia (2008).

+ implications on the number of sub-structures…

Towards (non thermal) very light candidates

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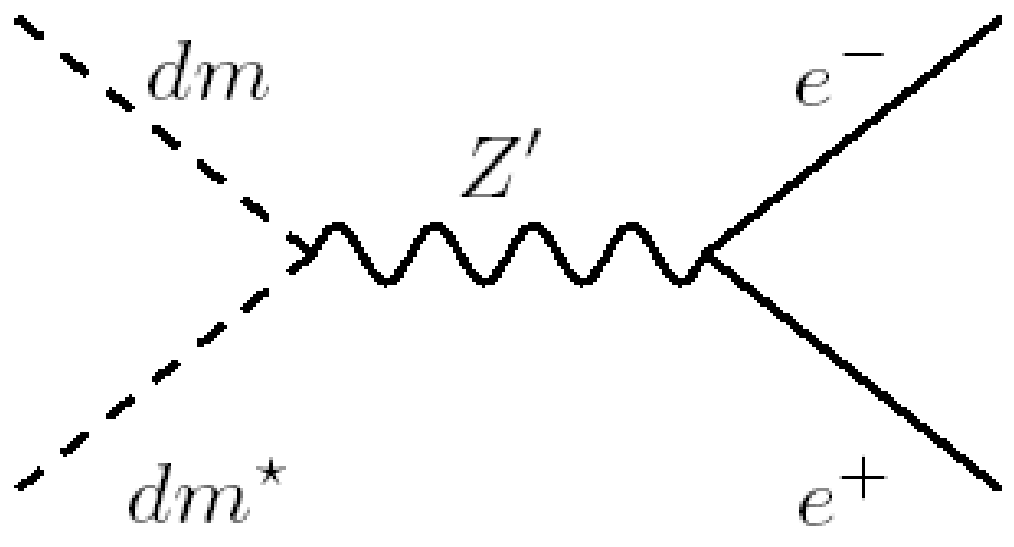
Very light candidates?

hep-ph/0305261



Z′ = vector and axial coupling

Like a photon
(Dark photon)

Like a Z boson

Massive or not massive

Massive but can be much lighter than a Z

Decaying or stable

Mediator or DM

Mixing angle with photons?

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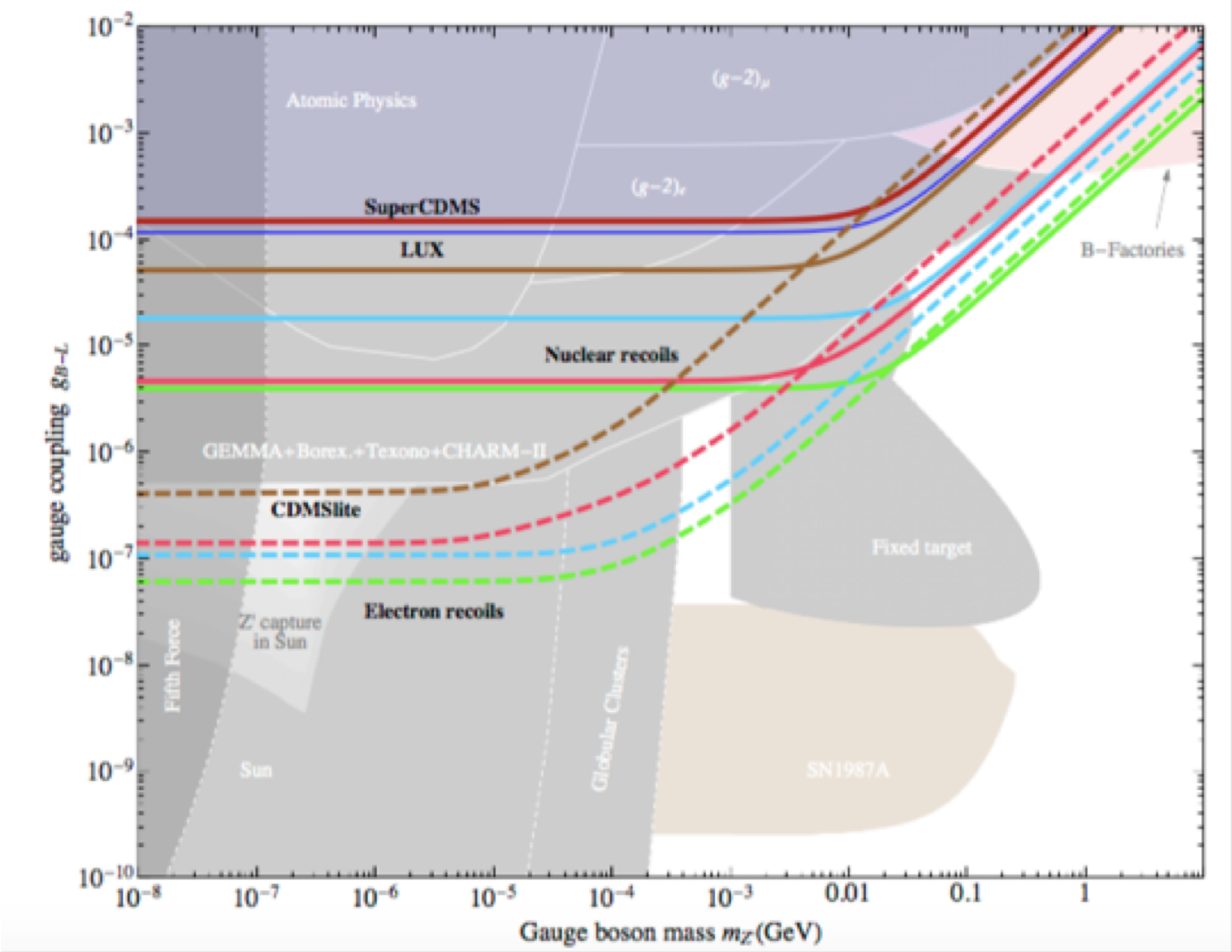
Constraints on light (dark) mediators



Towards (non thermal) ultra light candidates

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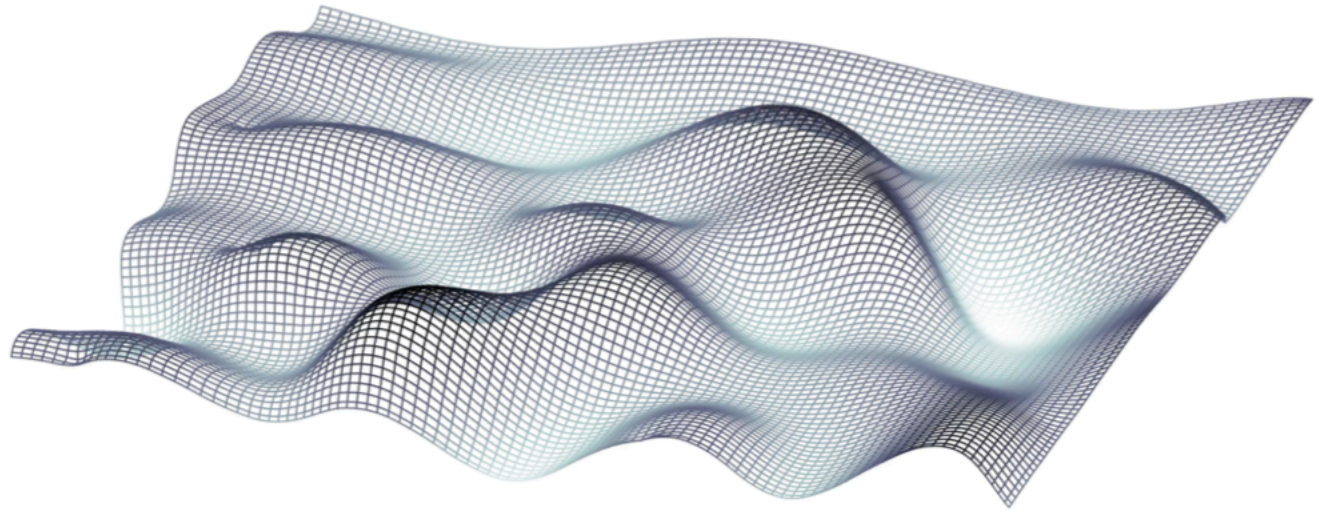
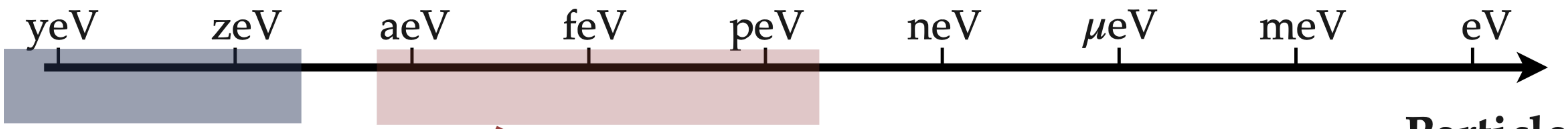
The path towards ultra light Dark Matter

Occupation number if boson dark matter

$$\mathcal{N} \approx (\rho_{\rm DM}/m) \times \lambda_{\rm dB}^3$$

$m = 1\,\mu\text{eV} \longrightarrow \mathcal{N} \approx 10^{32} \longrightarrow$ Macroscopically occupied ground state $\longrightarrow$ Classical field





"Fuzzy dark matter"
Wave-like effects on scales $\lambda_{dB} \gtrsim$ pc

"Black hole superradiance"
Build up of boson cloud around BHs sucks their spins away

Particle DM $\longrightarrow$

From Ciaran O'Hare slides

Slide courtesy C. O'Hare

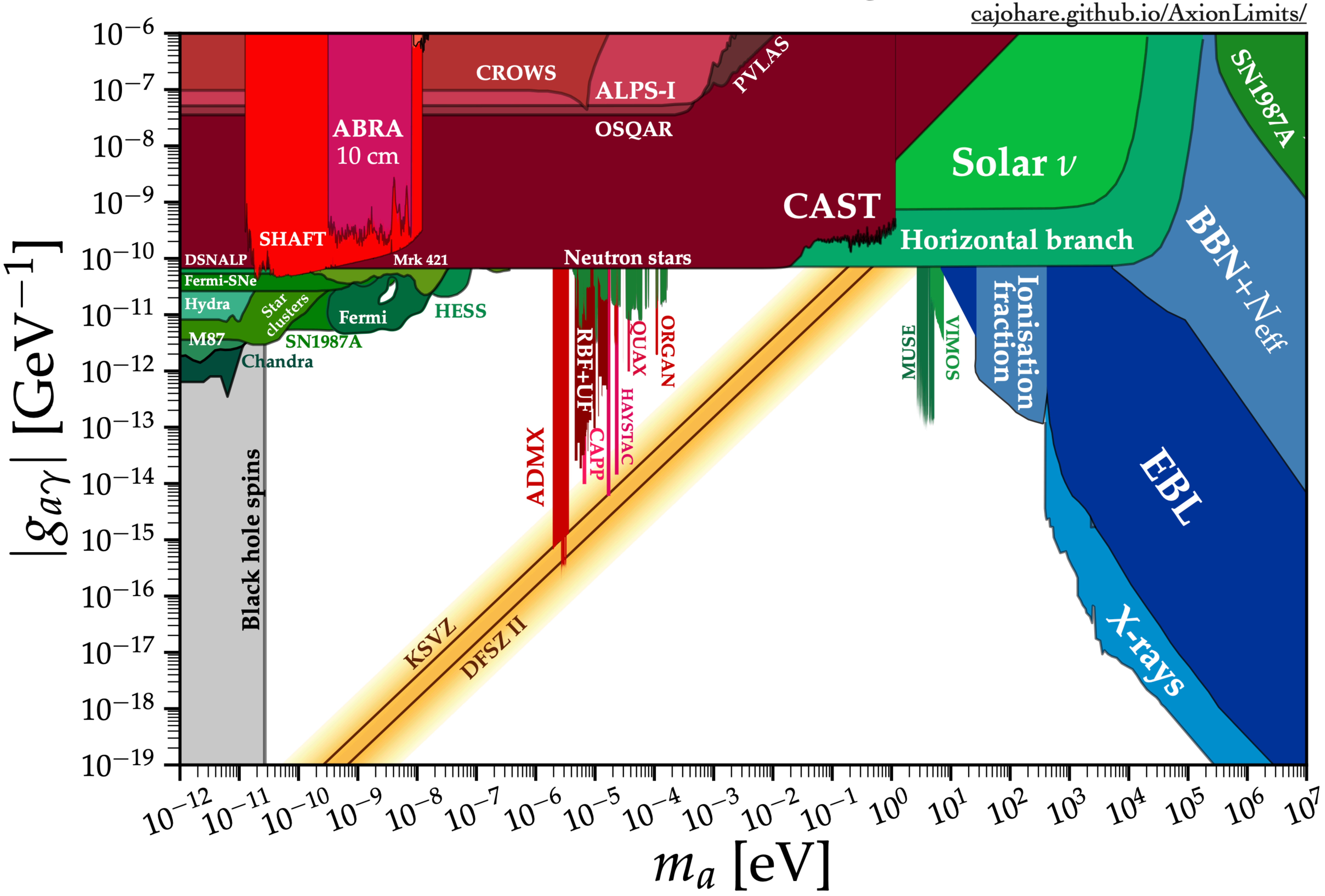
What can it couple to?

For more, see cajohare.github.io/AxionLimits/ → Now lists results from ~300 publications!

Slide courtesy C. O'Hare

The search for ultra light Dark Matter

The search for ultra light Dark Matter



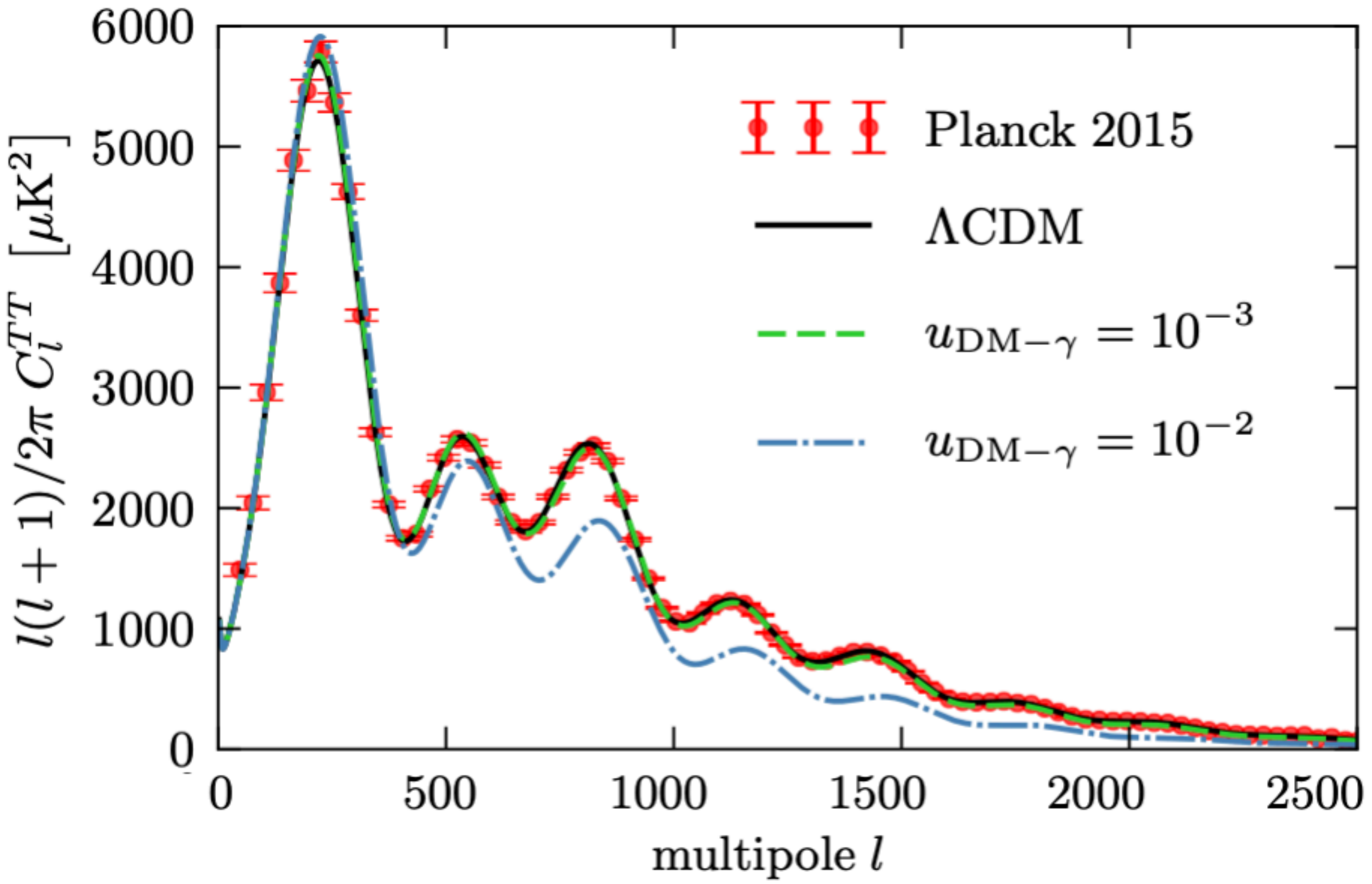
(New) ways to probe the DM interactions

Dark Matter interactions & structure formation

astro-ph/0012504, astro-ph/0112522, astro-ph/0205406, astro-ph/0410591

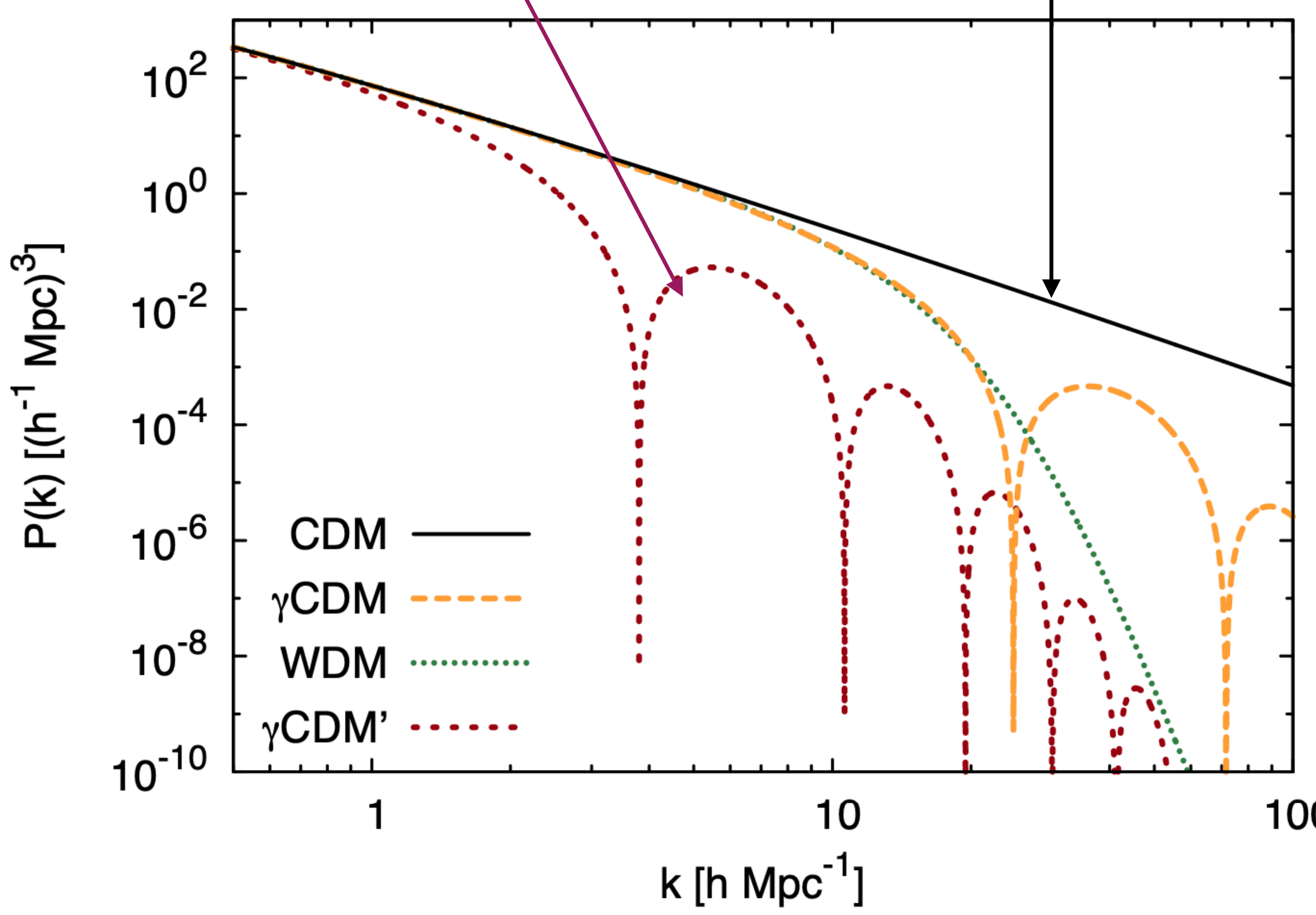
$$\dot\theta_b = k^2\psi - \mathcal{H}\theta_b + c_s^2 k^2 \delta_b - R^{-1}\dot\kappa(\theta_b - \theta_\gamma)$$
$$\dot\theta_\gamma = k^2\psi + k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) - \dot\kappa(\theta_\gamma - \theta_b) - \dot\mu(\theta_\gamma - \theta_{DM}) \,,$$
$$\dot\theta_{DM} = k^2\psi - \mathcal{H}\theta_{DM} - S^{-1}\dot\mu(\theta_{DM} - \theta_\gamma) \,.$$

DM-photon (require a ~weak interactions)
DM-neutrino (require a weak interactions)
DM-baryon (require large interactions)
DM self-interactions (require large interactions)

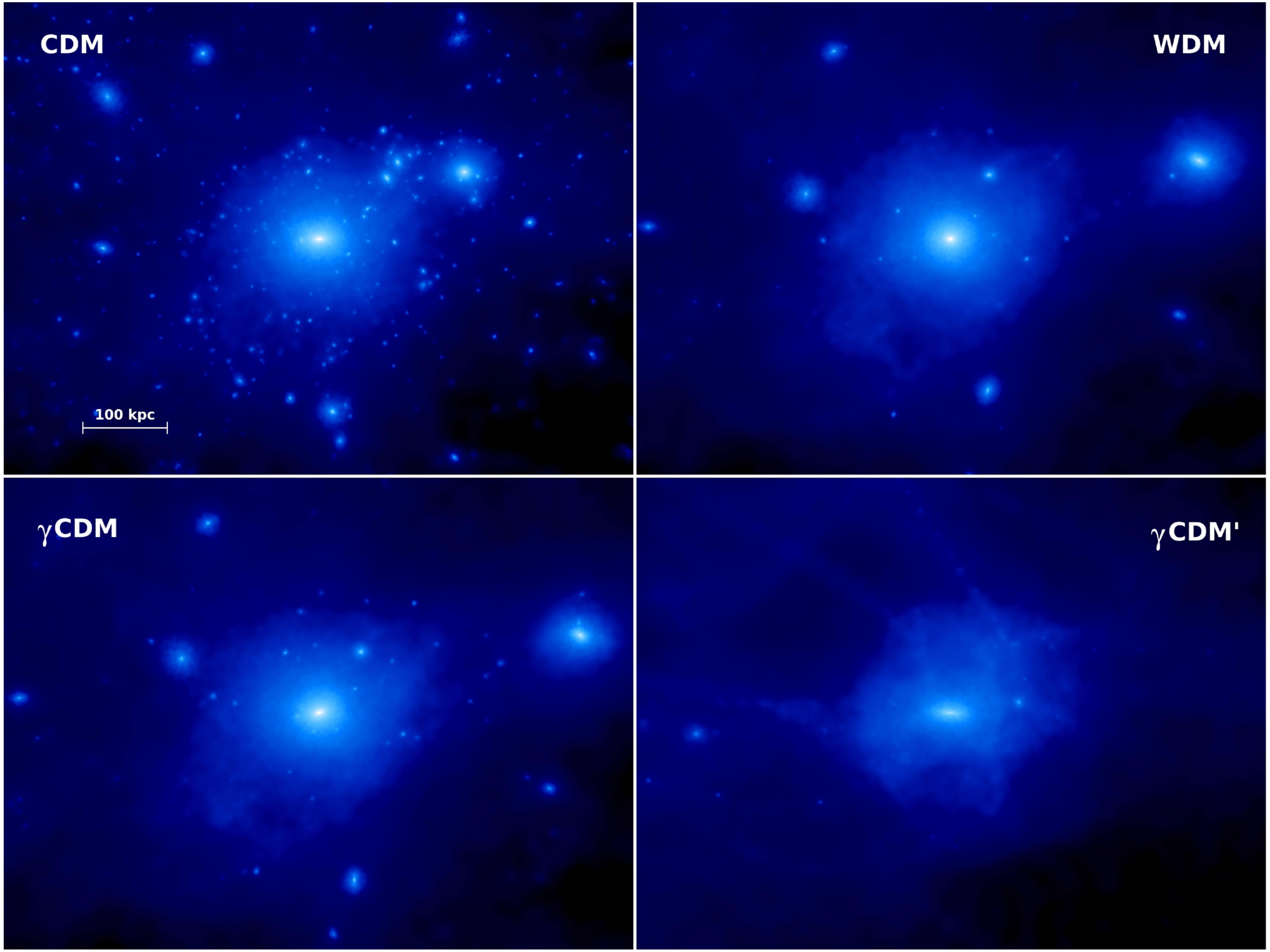


Other DM (with interaction) — dissipation

CDM (no interaction) — observed



Dark Matter interactions & structure formation

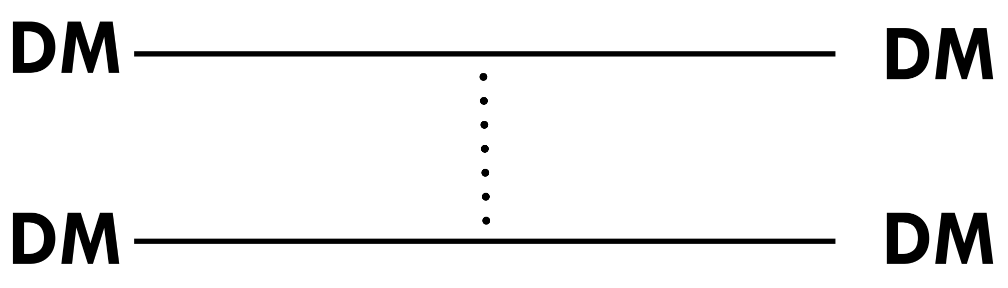


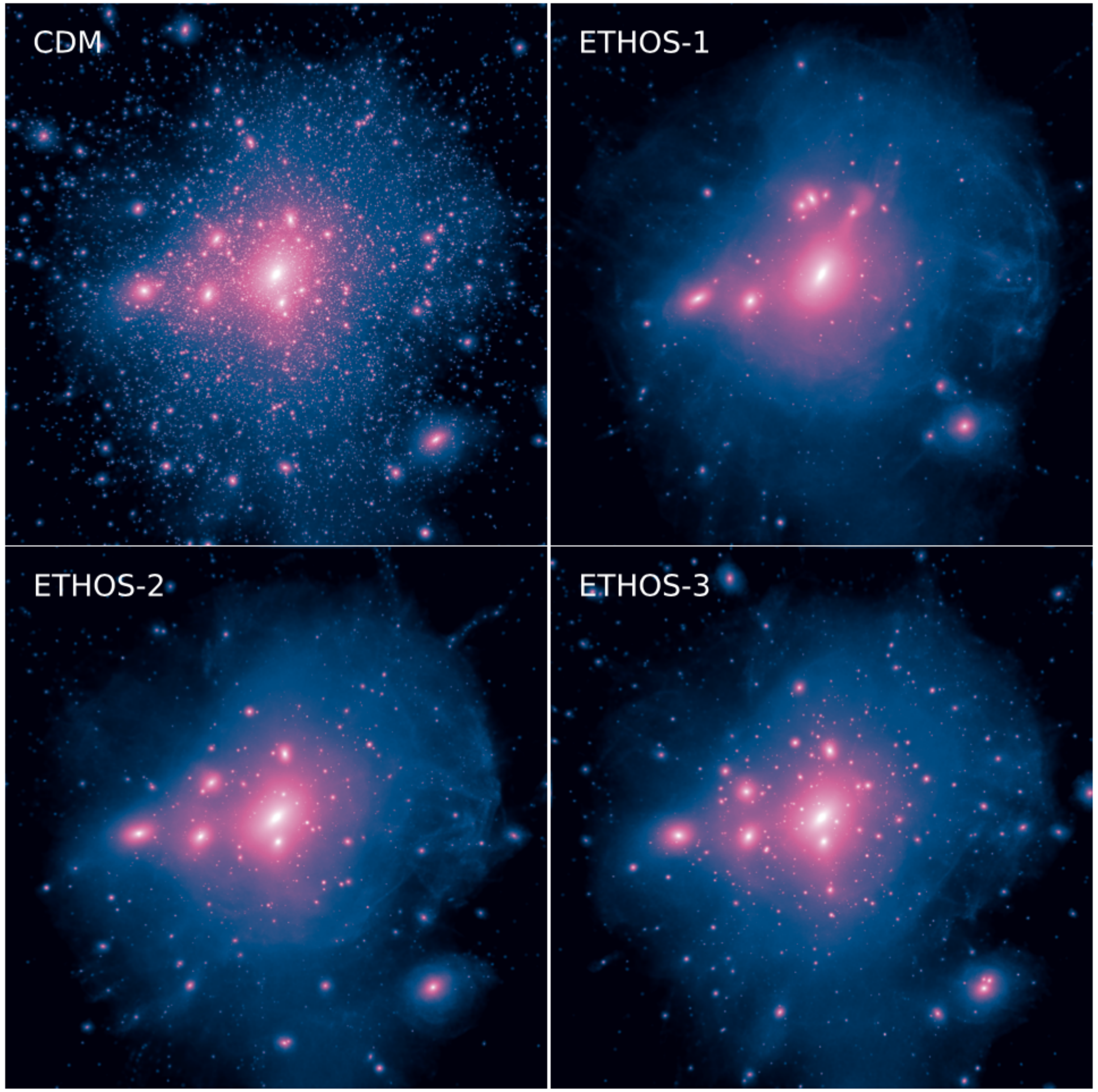
arXiv:1404.7012

http://www.youtube.com/watch?v=YhJHN6z_0ek

Dark matter self-interactions

DM-DR 1512.05349



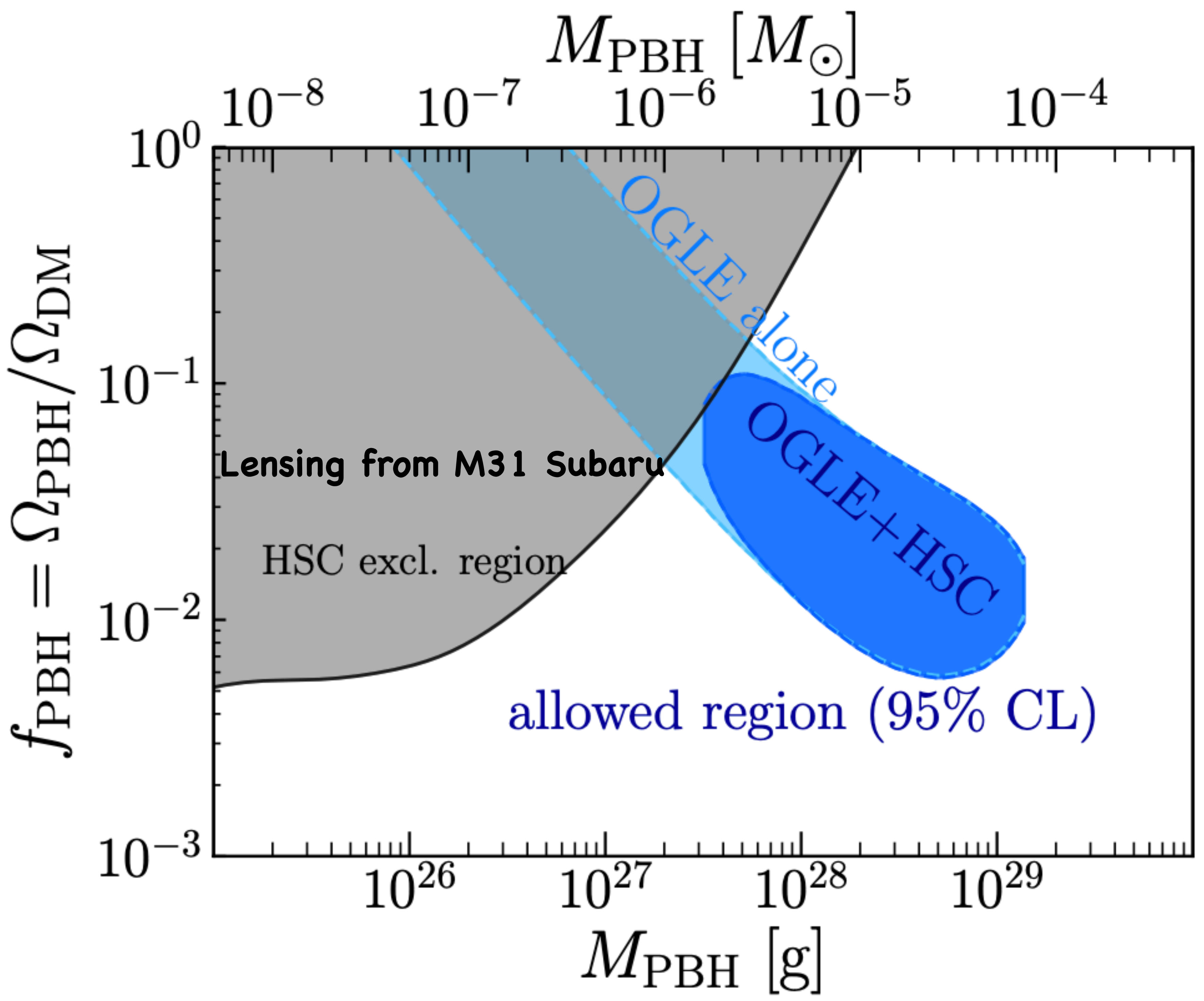


Alternatives to particle DM

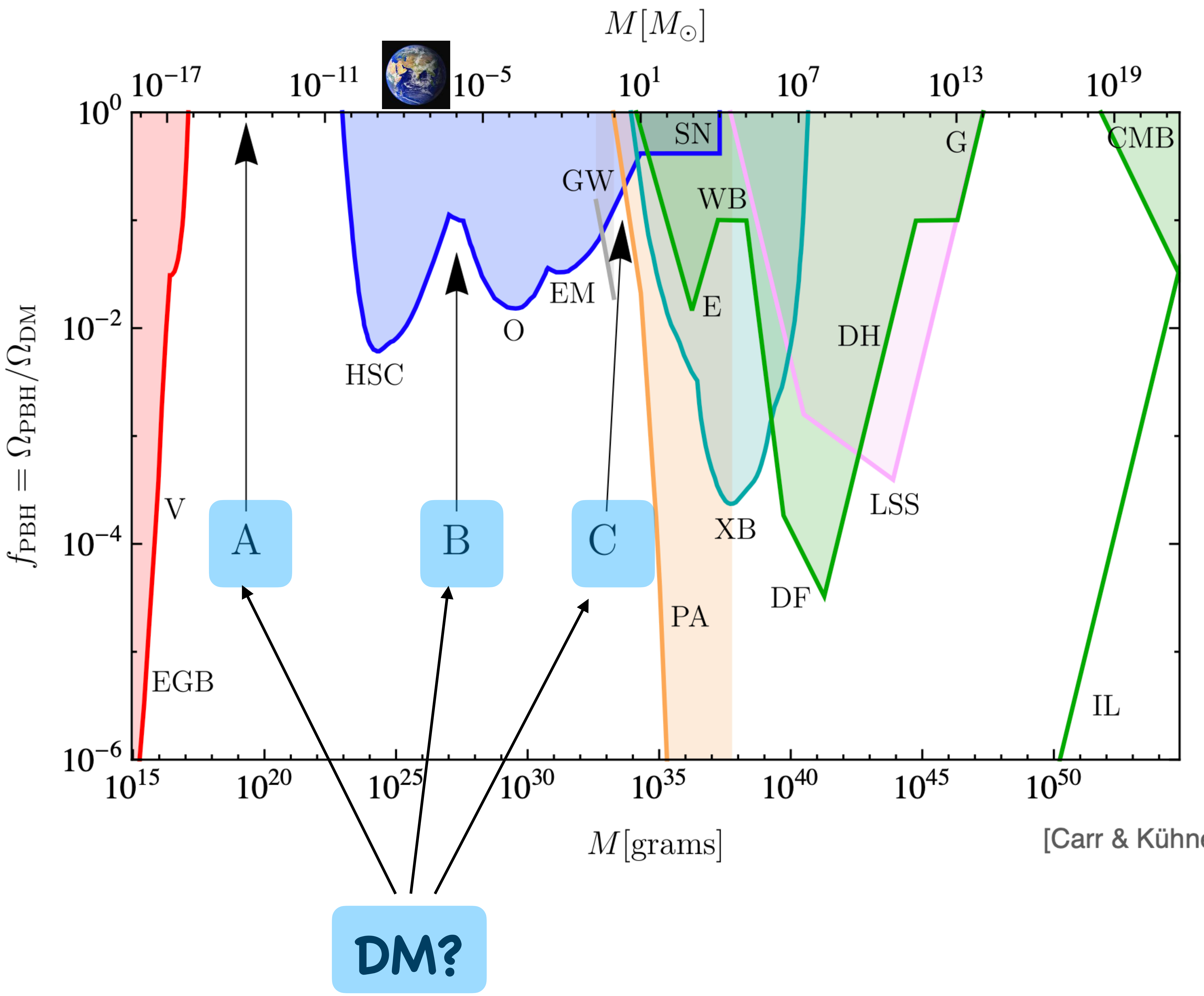
Primordial Black Holes

(See Kuhnel's talk at DSU2022 + papers)

OGLE detected events (0.1-0.3 days light curve timescale)
18/58 events consistent with 2-5 Msol PBH



[Niikura *et al.* 2019]



[Carr & Kühnel 2020]

Modifying Gravity

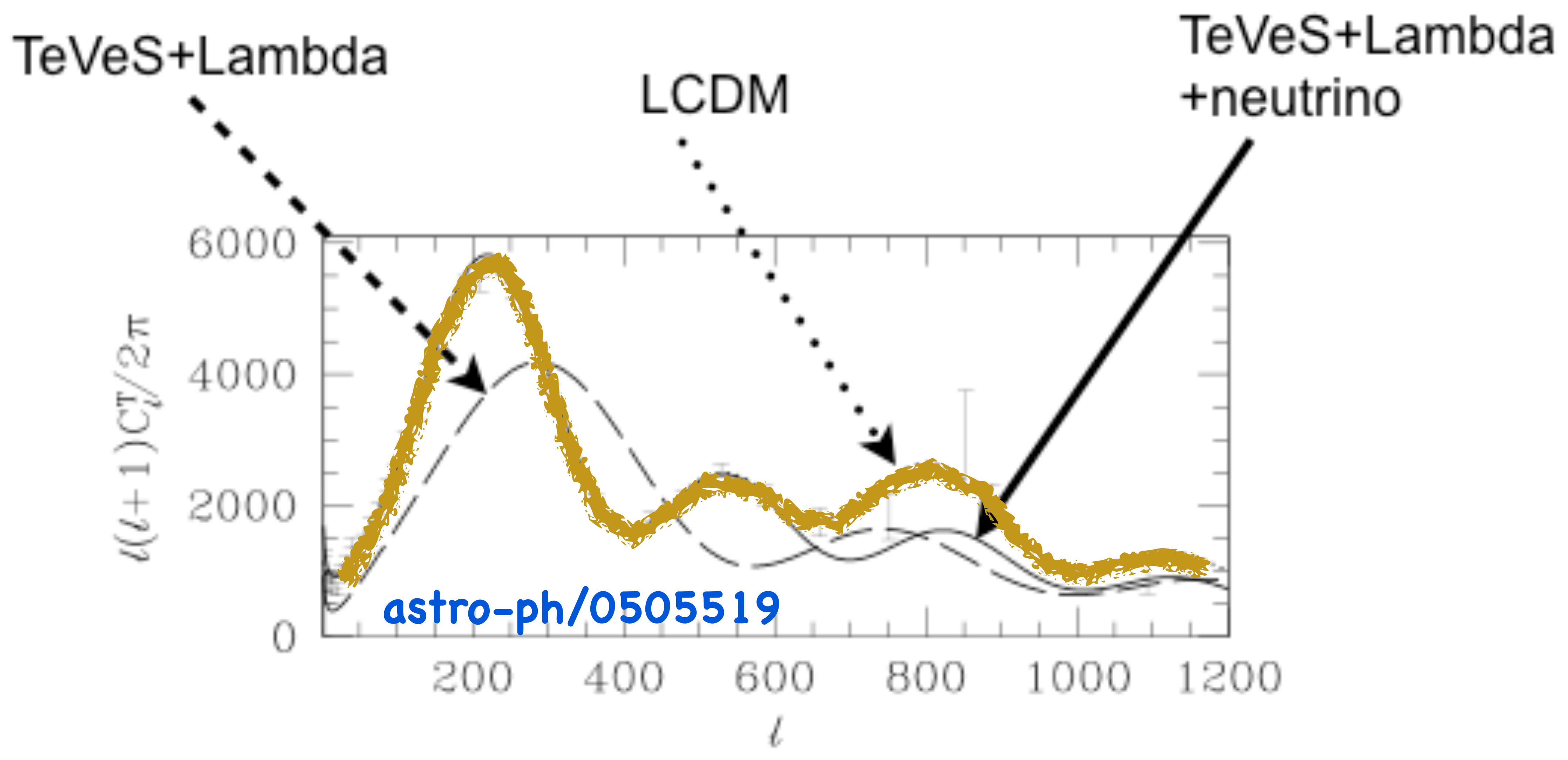
GR′ + SU(3)XSU(2)XU(1)

$$\mu\left(\frac{|\vec{a}|}{a_0}\right)\vec{a} = -\nabla\Phi$$

empirical

$\mu(x) = 1$ if x > 1 $\qquad \mu(x) \simeq x$ if x < 1

TEVES: astro-ph/0403694



arXiv:2007.00082v3 [astro-ph.CO] 14 Oct 2021

New Relativistic Theory for Modified Newtonian Dynamics

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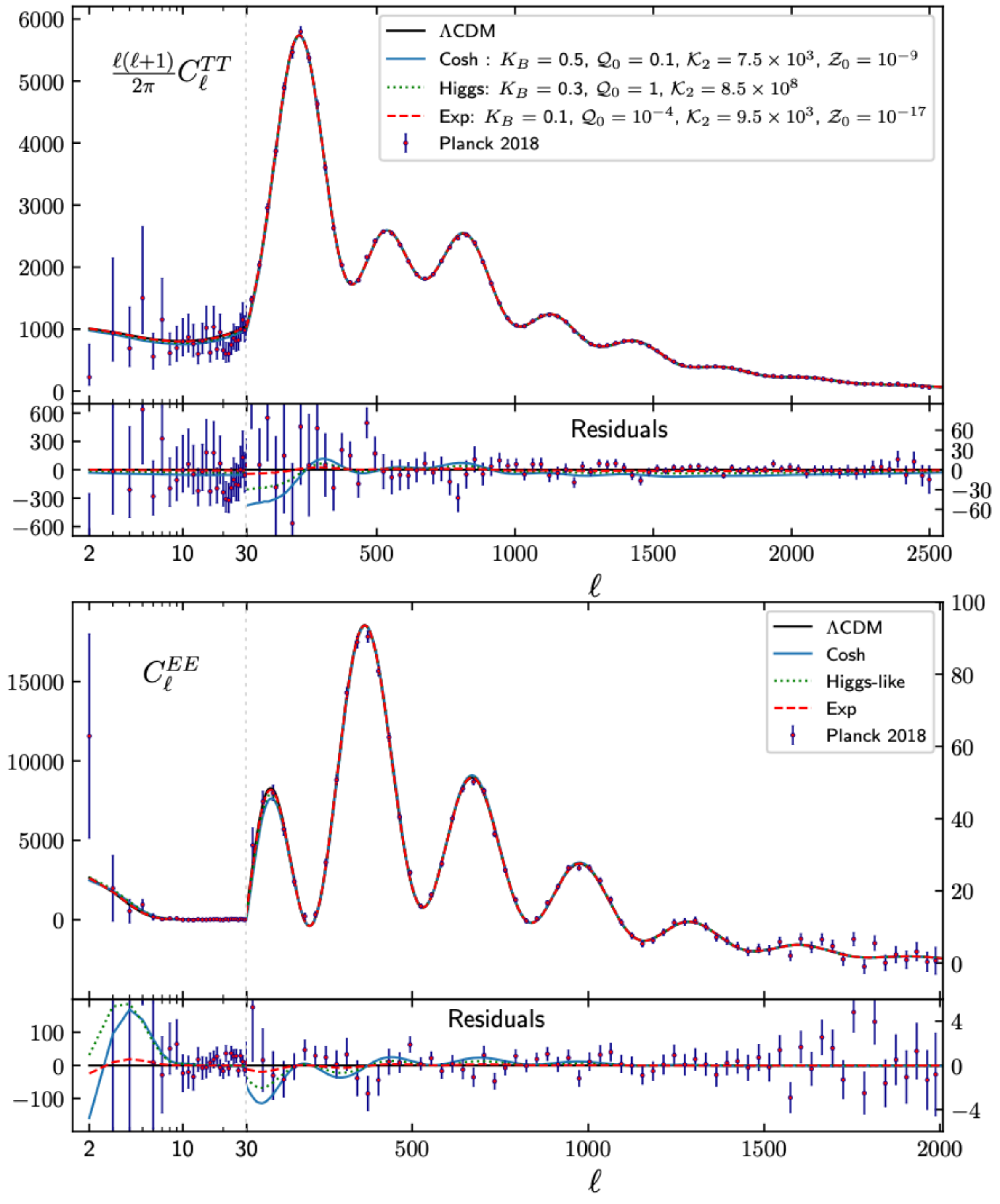
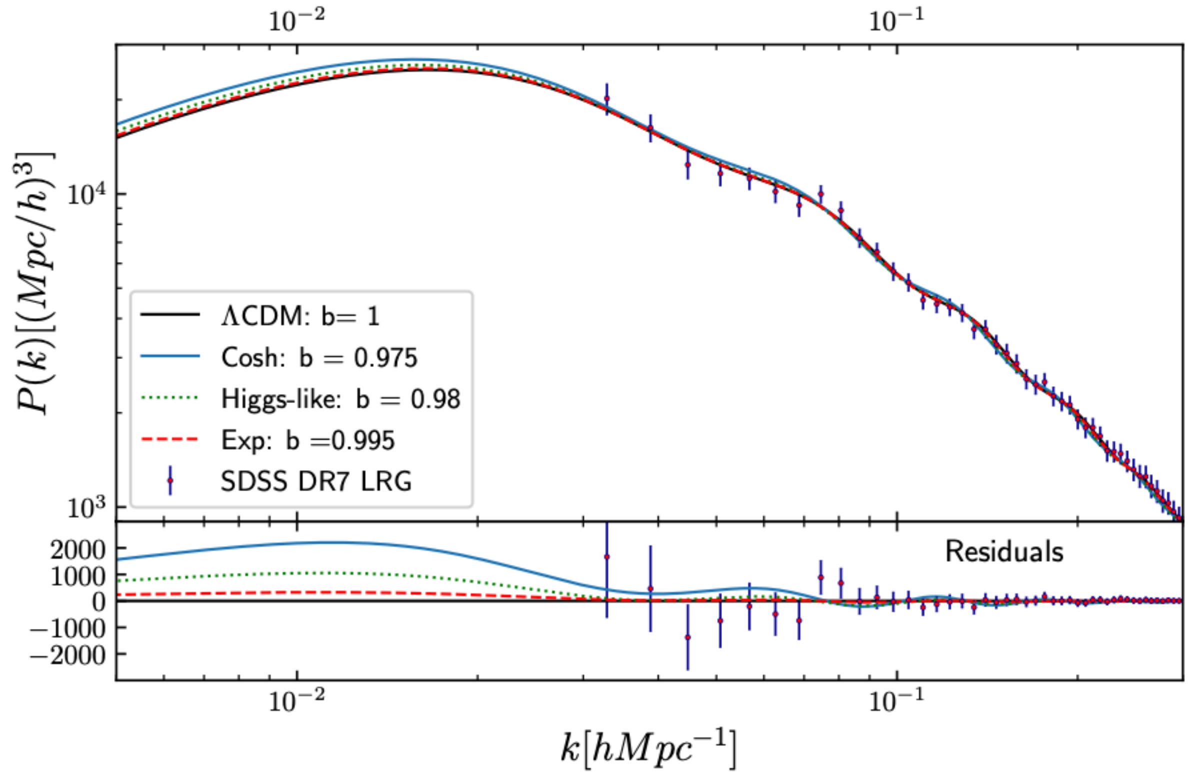
We propose a relativistic gravitational theory leading to modified Newtonian dynamics, a paradigm that explains the observed universal galactic acceleration scale and related phenomenology. We discuss phenomenological requirements leading to its construction and demonstrate its agreement with the observed cosmic microwave background and matter power spectra on linear cosmological scales. We show that its action expanded to second order is free of ghost instabilities and discuss its possible embedding in a more fundamental theory.



For a point source of mass M, the MOND-to-Newton transition occurs at $r_M \sim \sqrt{(G_N M/a_0)}$. A MOND force $\sim \sqrt{G_N M a_0}/r$ lends its way trivially to a Newtonian force $G_N M/r^2$ as $r \ll r_M$ but in the inner Solar System this is



Modifying Gravity

https://arxiv.org/pdf/2007.00082.pdf

$$S = \int d^4x \Big\{ -\frac{1}{2}\bar{\nabla}_\mu h \bar{\nabla}_\nu h^{\mu\nu} + \frac{1}{4}\bar{\nabla}_\rho h \bar{\nabla}^\rho h + \frac{1}{2}\bar{\nabla}_\mu h^{\mu\rho}\bar{\nabla}_\nu h^{\nu}{}_{\rho} - \frac{1}{4}\bar{\nabla}^\rho h^{\mu\nu}\bar{\nabla}_\rho h_{\mu\nu} K_B |\dot{\vec{A}} - \frac{1}{2}\vec{\nabla} h^{00}|^2 - 2K_B \vec{\nabla}_{[i}A_{j]}\vec{\nabla}^{[i}A^{j]}$$

$$+ (2 - K_B)\left[2(\dot{\vec{A}} - \frac{1}{2}\vec{\nabla} h^{00})\cdot(\vec{\nabla}\varphi + \mathcal{Q}_0\vec{A}) - (1+\lambda_s)|\vec{\nabla}\varphi + \mathcal{Q}_0\vec{A}|^2\right] + 2\mathcal{K}_2 \left|\dot{\varphi} + \frac{1}{2}\mathcal{Q}_0 h^{00}\right|^2 + \frac{1}{\tilde{M}_p^2} T_{\mu\nu}h^{\mu\nu}\Big\} \quad (13)$$

In preparation

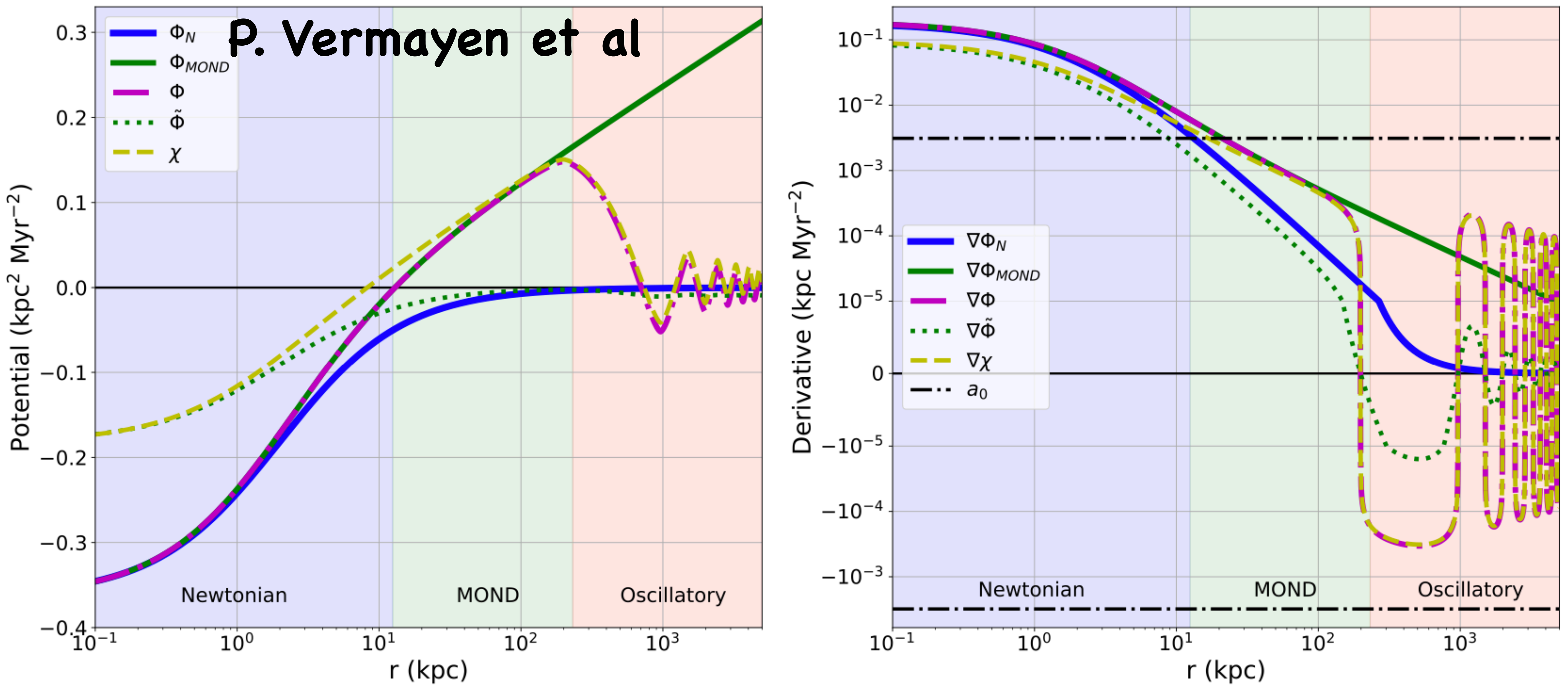


Figure 2. Solution of the field equations (left) and their gradients (right) for the Hernquist density profile and the fiducial model parameters with $(\lambda_s, \mu) = (1, 1\ \mathrm{Mpc}^{-1})$. The blue, green and red regions delineate the Newtonian, MOND and Oscillatory regions respectively. The yellow and green dashed lines are the auxiliary fields $\tilde{\Phi}$ and χ and the pink dotted-dashed line is the metric perturbation which is responsible for defining the trajectories of free falling particles. We have included the Newtonian (blue) and classical MOND (green) solutions for comparison. The break in the blue curve at $\nabla\Phi = 10^{-5}$ is not physical, but related to the symlog scaling that we use for the vertical axis of the right panel.

Conclusion & evolution of the field so far

(Thermal) heavy DM

(Thermal) light/MeV DM

(Non-thermal) ultra-light DM

Modified gravity???