Gravitational Positivity in Electroweak Sector

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Based on K. Aoki, TQL, T. Noumi, J. Tokuda, *Phys.Rev.Lett.* 127 (2021) 9, 091602
arXiv: 2104.09682 [hep-th]
and K. Aoki, TQL, T. Noumi, J. Tokuda, TBA

Introduction

- When does General Relativity meet Standard Model? Are they independent?
- Swampland Program [Cumrun Vafa ’05]: Which low-energy EFT are consistent with non-perturbative quantum gravity (UV completed) considerations?
- (Gravitational) Positivity Bounds: UV completion conditions put positivity constraints on EFT’s (Wilson) coefficients.
- EFT’s Cut-off scale + Parameters constraints: Derived from Positivity Bounds.
- In this talk, we study $\gamma\gamma \to \gamma\gamma, HH \to HH, \gamma H \to \gamma H$ processes in the SM coupled to GR, a low-energy EFT of Quantum Gravity.

Contents

UV & IR Physics

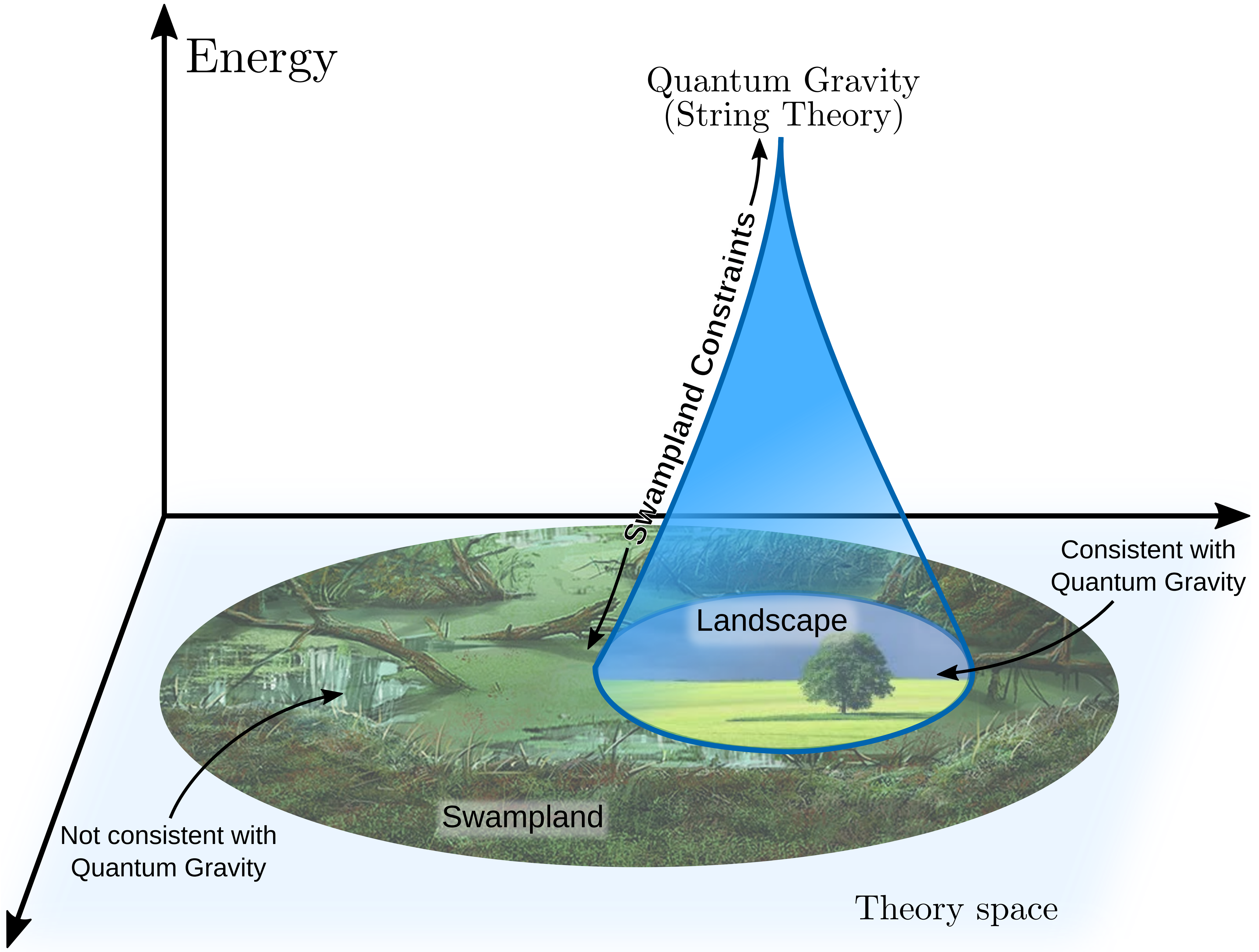


Fig. 1: The Swampland and Landscape of EFTs.
[arXiv: 2102.01111 [hep-th]](https://arxiv.org/abs/2102.01111)

Quantum Gravity

Top down

General Principles: Dualities, Symmetries to write down EFTs' operators

UV completion conditions:

- Lorentz inv.
- Unitary
- Locality
- Causality

→ Positivity Constraints to EFTs' parameters

Bottom up

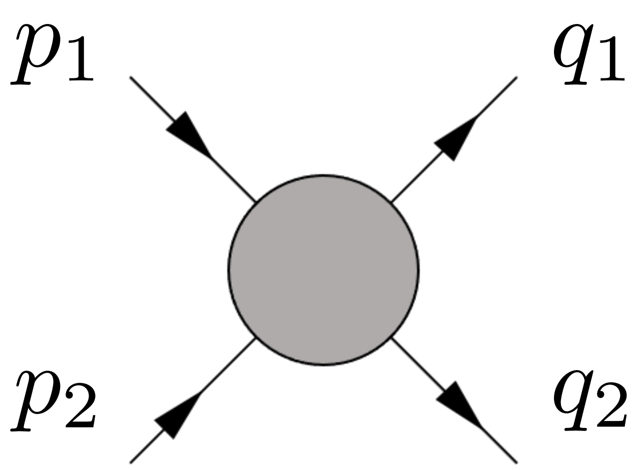
Effective Field Theories (EFTs)

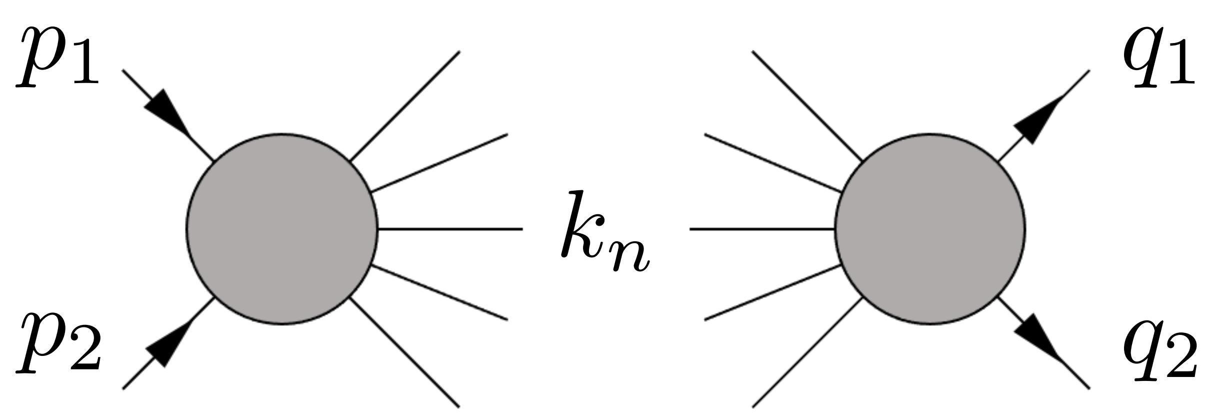
Positivity Bound: S-matrix properties (1)

Lorentz invariance

Mandelstam variables: $A(p_1, p_2, q_1, q_2) \to A(s,t), \text{with Mandelstam var.} \begin{cases} s = -(p_1+p_2)^2 = E_{\text{CM}}^2, \\ t = -(p_1-p_2)^2 = -\frac{s-4m^2}{2}(1-\cos\theta), \\ u = -(p_1-q_2)^2 = 4m^2 - s - t. \end{cases}$

Unitarity





Optical Theorem:

$$\begin{cases} S^\dagger S = 1 \\ S = 1 + iM \end{cases} \Rightarrow M - M^\dagger = iM^\dagger M. \quad \text{Im } M(p_1p_2 \to q_1q_2) = \frac{1}{2}\sum_n \int d\tilde{\Pi}_n M^*(p_1p_2 \to k_n) M(q_1q_2 \to k_n)$$

$$= \sqrt{s(s-4m^2)\sigma_t} > 0.$$

Locality

Froissart-Martin Bound: $\lim_{|s|\to\infty} \left| \frac{A(s,t)}{s^2} \right| = 0 \text{ with } t \neq m^2, t < 4m^2.$

⟶ Scattering Amplitude is Polynomially bounded.

Causality

Crossing symmetry: $A(s,t)$ is invariant under $s \leftrightarrow u, t \leftrightarrow u, s \leftrightarrow t.$

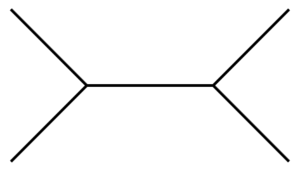
Analiticity: Analytical continuation to complex plane, integral relations and singularity conditions.

Positivity Bound: S-matrix properties (2)

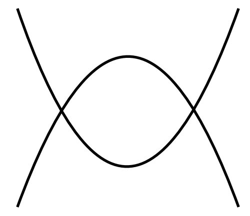
Analiticity arXiv:1605.06111 [hep-th] arXiv:1702.06134 [hep-th]

Fixed t, $u = 4m^2 - s - t.$

Poles from intermediary particles:



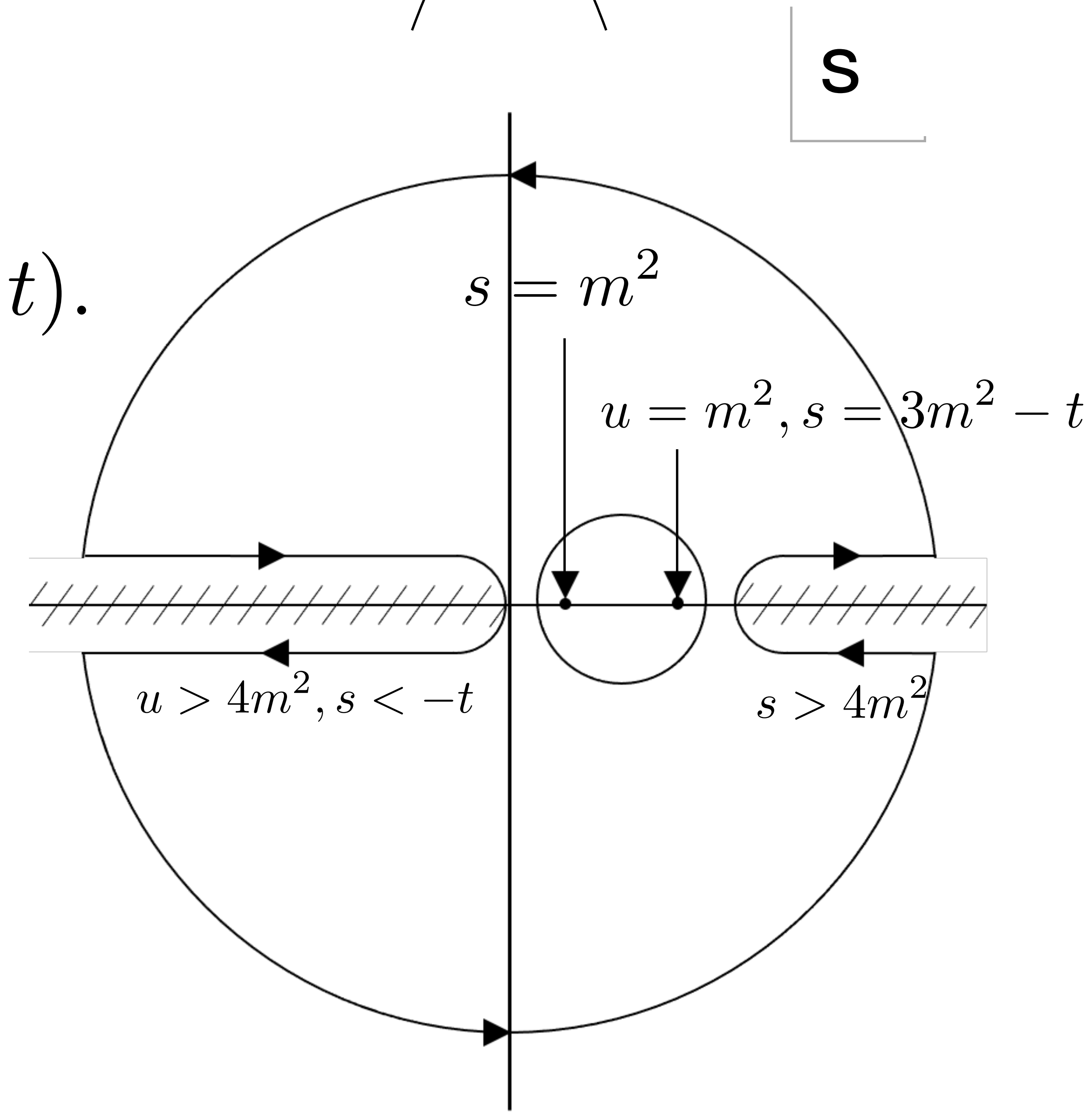
Branch-cuts from loops:



Dispersion relation

$$A(s,t) = \frac{1}{2\pi i}\oint_C d\widetilde{s}\frac{A(\widetilde{s},t)}{\widetilde{s}-s}$$

$$= \frac{\lambda}{m^2-s} + \frac{\lambda}{m^2-u} + \int_{C\pm\infty} \mathrm{d}\widetilde{s}\frac{A(\widetilde{s},t)}{\widetilde{s}-s} + \frac{1}{\pi}\int_{4m^2}^{\infty}\mathrm{d}\mu\left[\frac{1}{\mu-s}+\frac{1}{\mu-u}\right]\mathrm{Im}A(\mu,t).$$



Identity
$$\frac{1}{\mu-s} = \frac{(s-\mu_p)^2}{(\mu-\mu_p)^2}\frac{1}{\mu-s} + 2\frac{(s-\mu_p)}{(\mu-\mu_p)^2} + \frac{(\mu-s)}{(\mu-\mu_p)^2}.$$

Twice subtracted Dispersion relation

$$A(s,t) = \frac{1}{2\pi i}\oint_C d\widetilde{s}\frac{A(\widetilde{s},t)}{\widetilde{s}-s}$$

$$= a(t) + \frac{\lambda}{m^2-s} + \frac{\lambda}{m^2-u} + \frac{1}{\pi}\int_{4m^2}^{\infty}\mathrm{d}\mu\left[\frac{(s-\mu_p)^2}{(\mu-\mu_p)^2(\mu-s)}+\frac{(s-\mu_p)^2}{(\mu-\mu_p)^2(\mu-s)}\right]\mathrm{Im}A(\mu,t).$$

Positivity Bound:
Subtracted Scattering Amplitude & Improved Positivity Bound

Substracted Scattering Amplitude

$$B(s,t) = A(s,t) - \frac{\lambda}{m^2 - s} - \frac{\lambda}{m^2 - u} - \frac{\lambda}{m^2 - t}. \qquad \widetilde{B}(v,t) := B(s,t)\big|_{s=v+2m^2-t/2}.$$

$$B^{2N,M}(t) = \frac{1}{M!}\partial_v^{2N}\partial_t^{M}\widetilde{B}(v,t)\big|_{v=0}.$$

$$B^{2N,0}(t) = \frac{(2N)!2}{\pi}\int_{4m^2}^{\infty} \mathrm{d}\mu \frac{\mathrm{Im}\, A(\mu,t)}{(\mu - 2m^2 + \frac{t}{2})^{2N+1}} > 0, \quad \text{with no t derivative (in forward limit, } t \to 0).$$

Improved positivity

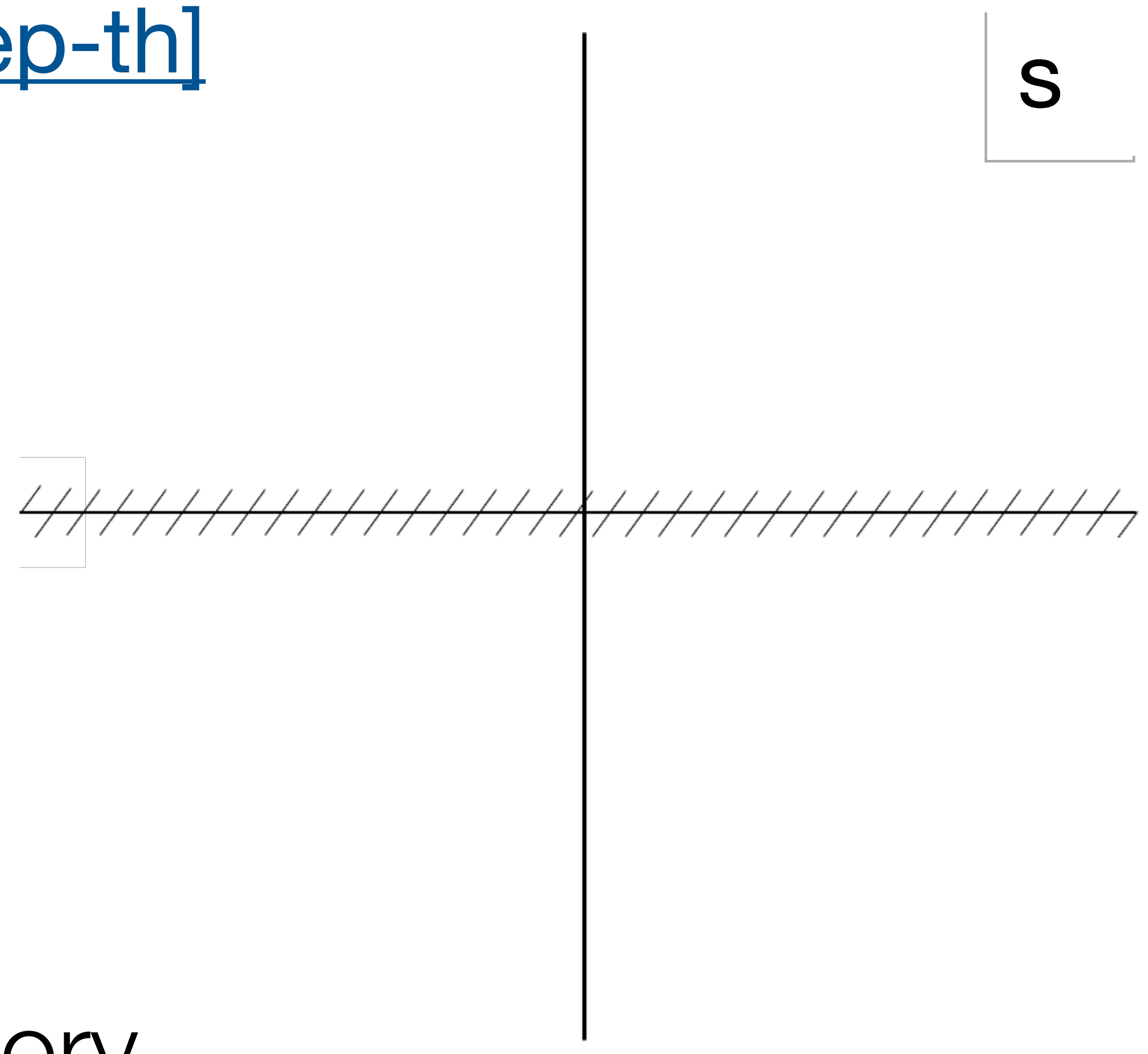
$$B^{2N,0}_{\Lambda_{EFT}}(t) := \frac{(2N)!2}{\pi}\left(\int_{4m^2}^{\infty} - \int_{4m^2}^{\Lambda^2_{EFT}}\right)\mathrm{d}\mu \frac{\mathrm{Im}\, A(\mu,t)}{(\mu - 2m^2 + \frac{t}{2})^{2N+1}} > 0$$

$$= \frac{(2N)!2}{\pi}\int_{\Lambda^2_{EFT}}^{\infty} \mathrm{d}\mu \frac{\mathrm{Im}\, A(\mu,t)}{(\mu - 2m^2 + \frac{t}{2})^{2N+1}} > 0.$$

(Subtract the known part from unknown parts to enhance the positivity).

Gravitational Positivity Bound

Adding massless Graviton causes some troubles arXiv:2007.15009 [hep-th]



Analytical structure

Issue: Massless loop create a branch cut along x-axis,

disconnecting the 2 half-planes.

Solution: The theory UV completed at Planck scale. In linearised theory,

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{\sqrt{32\pi}}{M_{\mathrm{Pl}}} h_{\mu\nu}.$$

→ Graviton loop is supressed at UV completion of gravity, so we can neglect it.

Gravitational Positivity Bound

Boundedness arXiv:2007.15009 [hep-th]

Issue: Encounter a non-gapped system: with massless particles → cannot use Froissart bound.

$$B^{(2)}(\Lambda,t) - \frac{8}{M_{\rm Pl}^2 t} = \frac{4}{\pi}\int_{\Lambda^2}^{\infty} \mathrm{d}s' \, \frac{\mathrm{Im}\,\mathcal{M}(s'+i\epsilon,t)}{(s'+(t/2))^3} \, .$$

with similar bound, $\lim_{|s|\to\infty}\left|\frac{M(s,t<0)}{s^2}\right| = 0.$

Solution: We assume the Regge behavior,

$\alpha > 0$

$$\mathrm{Im}\,\mathcal{M}(s,t) = f(t)\left(\frac{s}{M_s^2}\right)^{2+\alpha' t+\alpha'' t^2+\cdots} + \cdots$$

with Single scaling,

$$|(\partial_t f/f)_{t=0}|, |\alpha''/\alpha'|, \alpha' \lesssim \mathcal{O}(M_s^{-2}).$$

$$\longrightarrow \quad B^{(2)}(\Lambda) := B^{(2)}(\Lambda,0) > -\mathcal{O}(M_{\rm Pl}^{-2}M_s^{-2})\,,$$

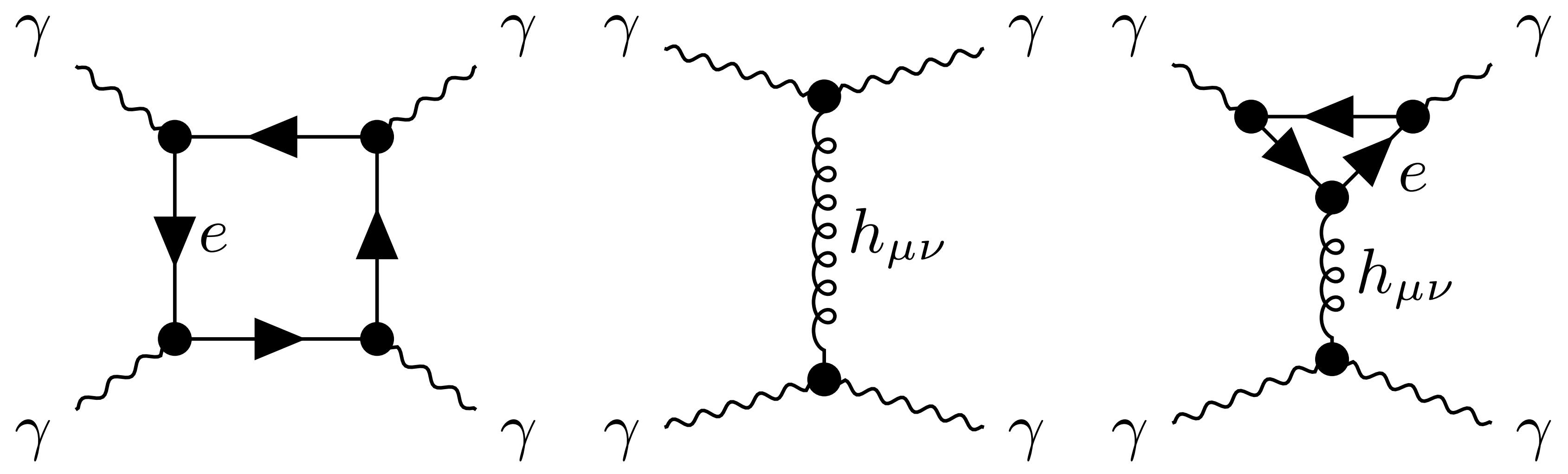
Small amount of negativity is still allowed, R.H.S. is suppressed by not only $M_{\rm Pl}^{-2}$ but also M_s^{-2} which is small enough to provide the constraints on the SM amplitudes with gravity.

$\equiv \mathcal{M}_i(s'+i\epsilon, t=0).$

⟶ At forward scattering limit, substracted amplitude reads $B_i^{(2)}(\Lambda) = \partial_s^2 \mathcal{A}(s)\big|_{2m^2} - \frac{4}{\pi}\int_{\Lambda^2}^{\infty}\mathrm{d}s' \, \frac{\mathrm{Im}\,\mathcal{A}_i(s'+i\epsilon)}{(s'-4m^2)^3}\,,$

Positivity QED (with GR) from $\gamma\gamma \to \gamma\gamma$

Leading QED contribution and GR

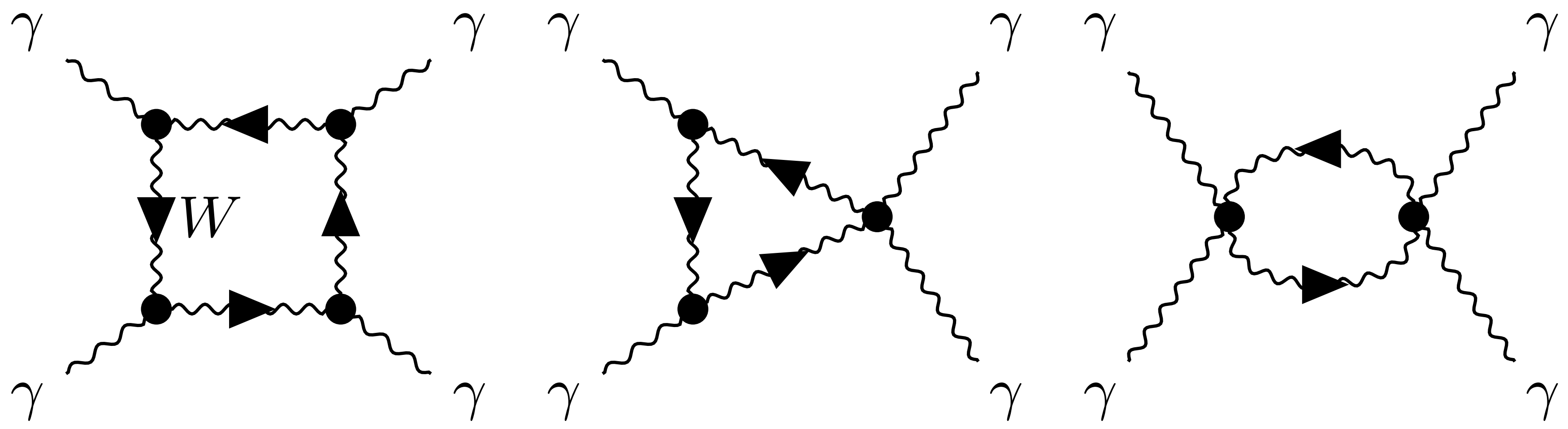


For GR, tree-level contribution canceled, only 1-loop level contribute

$$B^{(2)}_{\text{QED}} \approx \frac{64\alpha^2}{\Lambda^4}\left(\ln\frac{\Lambda}{m_e} - \frac{1}{4}\right).$$

$$B^{(2)}_{\text{GR}} \approx -\frac{22\alpha}{45\pi m_e^2 M_{\text{Pl}}^2}.$$

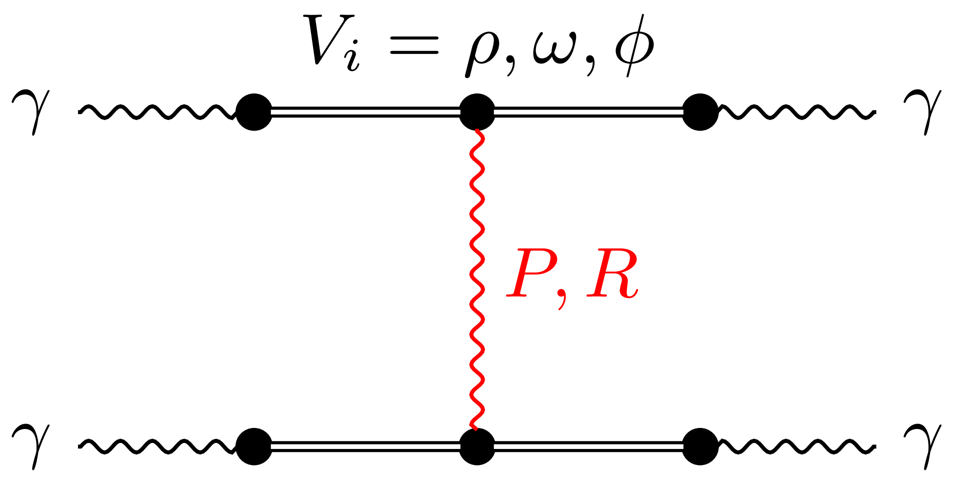
EW contributions



$$B^{(2)}_{\text{Weak}} \approx \frac{128\alpha^2}{m_W^2\Lambda^2}.$$

$$\Lambda_{\text{EW}} =\simeq 3.8 \times 10^{13}\text{GeV}.$$

QCD contributions



$$\Lambda_{\text{SM}} \simeq 3 \times 10^{16}\text{GeV}.$$

$$\Lambda_{\text{SM}} \simeq \begin{cases} 2 \times 10^{15}\text{GeV} & \text{(linear growth)}, \\ 1 \times 10^{17}\text{GeV} & \text{(Froissart type)}. \end{cases}$$

Positivity SM + GR from $\gamma\gamma \to \gamma\gamma$

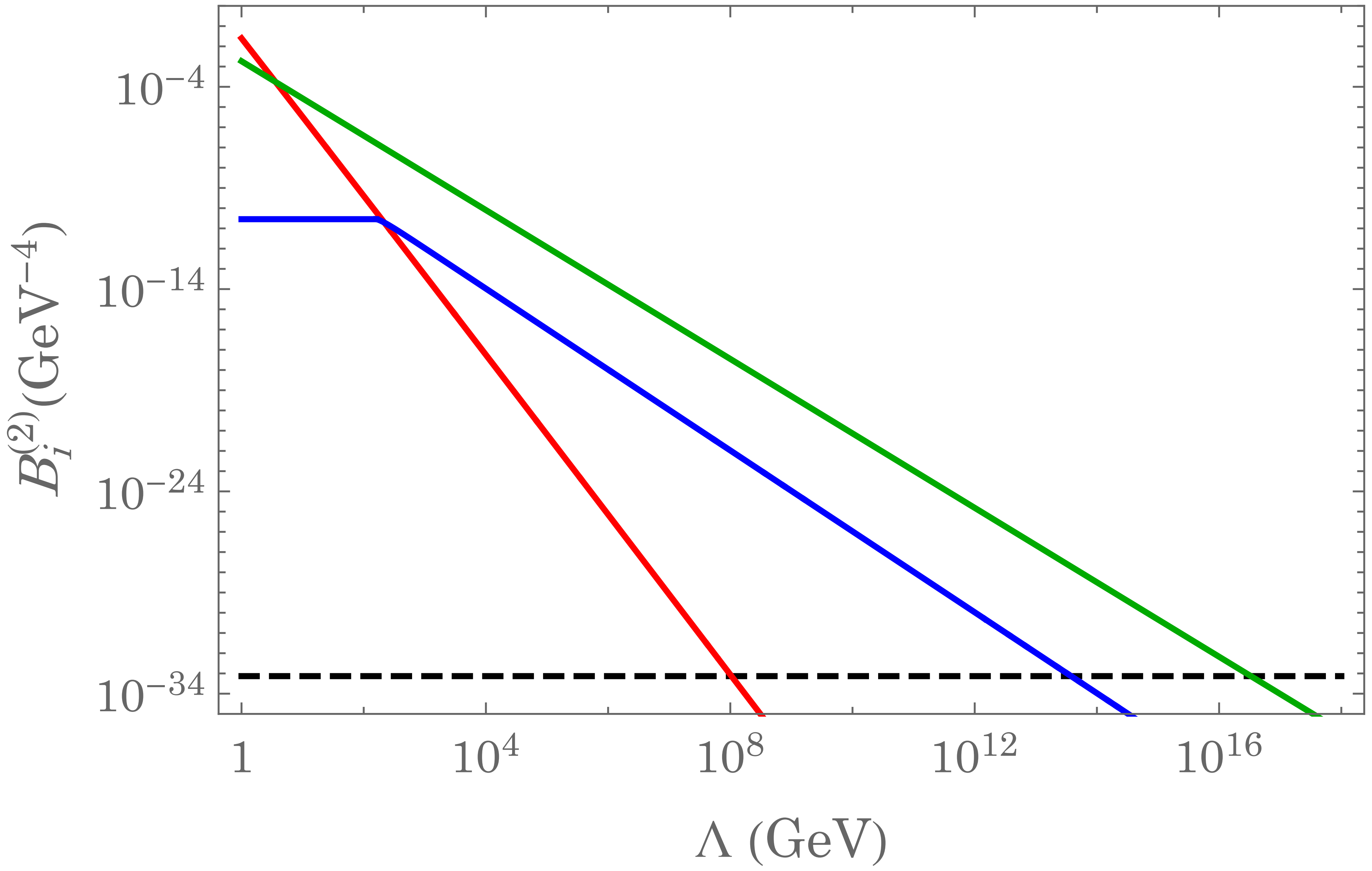


Fig. 2: The Λ dependence of $B_i^{(2)}$ where i = QED (red), Weak (blue), and QCD (green), and the black dashed line represents $-B^{(2)}_{\mathrm{GR}}$.

Unlike QED and EW, QCD is strongly coupled, hence, sensitive inclusion of new charged particles at UV

$$B^{(2)}_{\mathrm{QED}} \approx \frac{64\alpha^2}{\Lambda^4}\left(\ln\frac{\Lambda}{m_e} - \frac{1}{4}\right). \quad B^{(2)}_{\mathrm{Weak}} \approx \frac{128\alpha^2}{m_W^2\Lambda^2}.$$

$$B^{(2)}_{\mathrm{QCD}} \approx \frac{34.5989\alpha^2}{\Lambda^{1.84}}. \quad B^{(2)}_{\mathrm{GR}} \approx -\frac{22\alpha}{45\pi m_e^2 M_{\mathrm{Pl}}^2}.$$

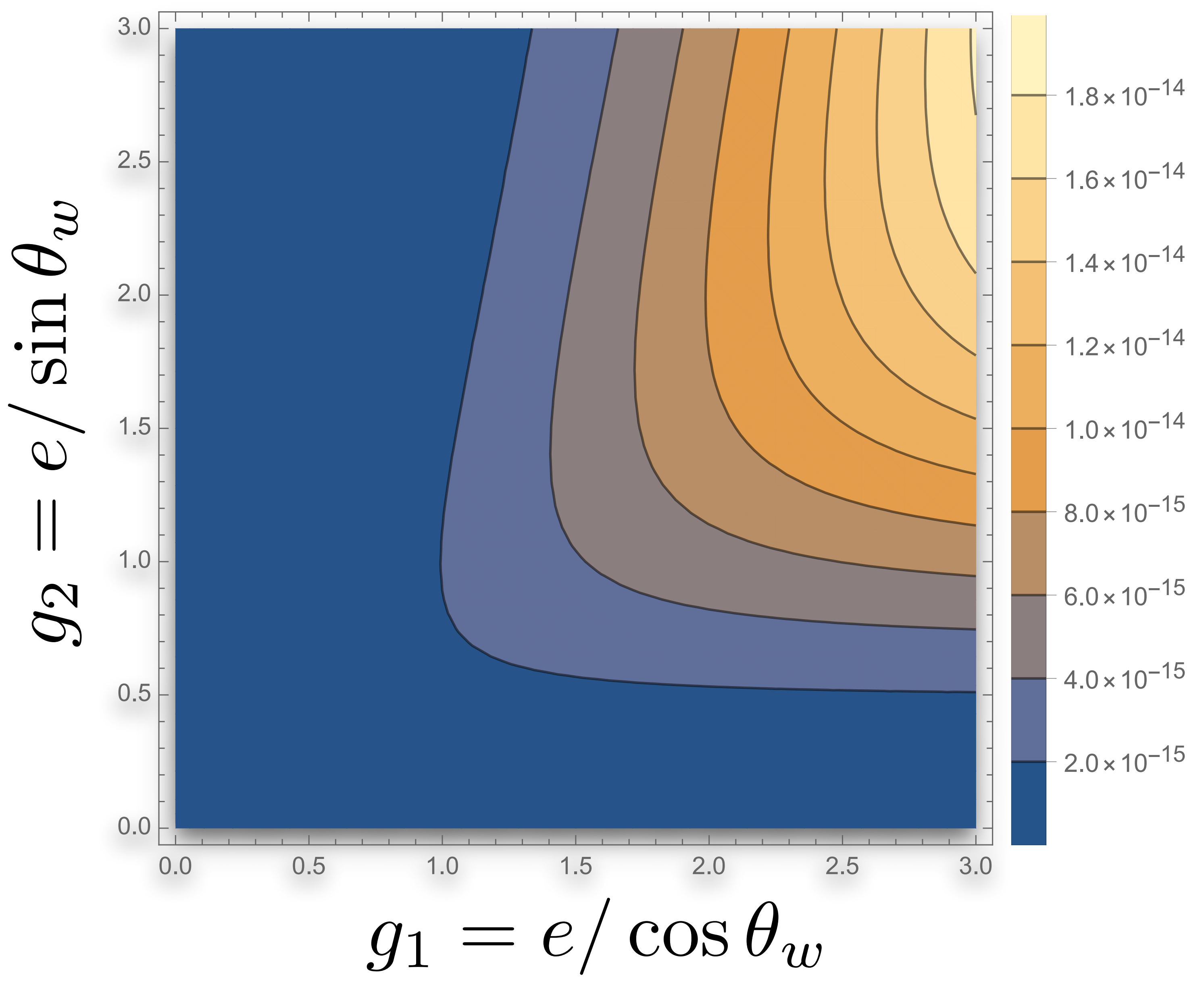
The intersection between the solid line and the dashed line determines the cutoff Λ_i. Also,

$$\Lambda_{\mathrm{EW}} = \sqrt{\frac{2880\pi\alpha}{11}\frac{m_e M_{\mathrm{Pl}}}{m_W}} \Rightarrow y_e \sin\theta_{\mathrm{W}} = \sqrt{\frac{11}{1440}}\frac{\Lambda_{\mathrm{EW}}}{M_{\mathrm{Pl}}}.$$

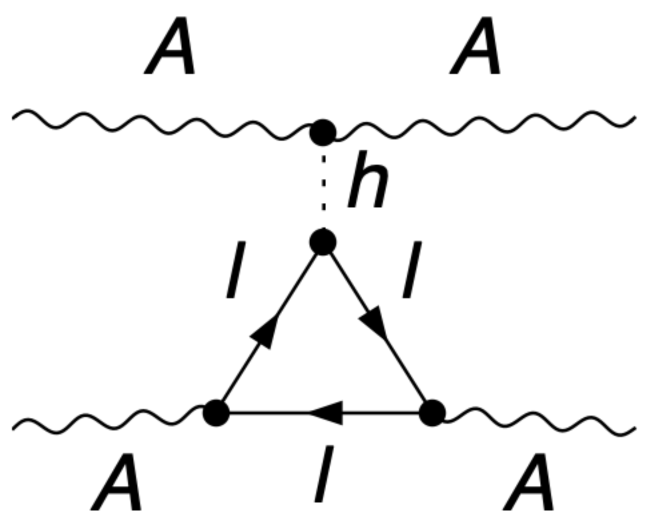
Existence of new Swampland conditions on the coupling strengths?

Effects of anomalous threshold to gauge couplings $m_H = 2m_{\text{loop}}$

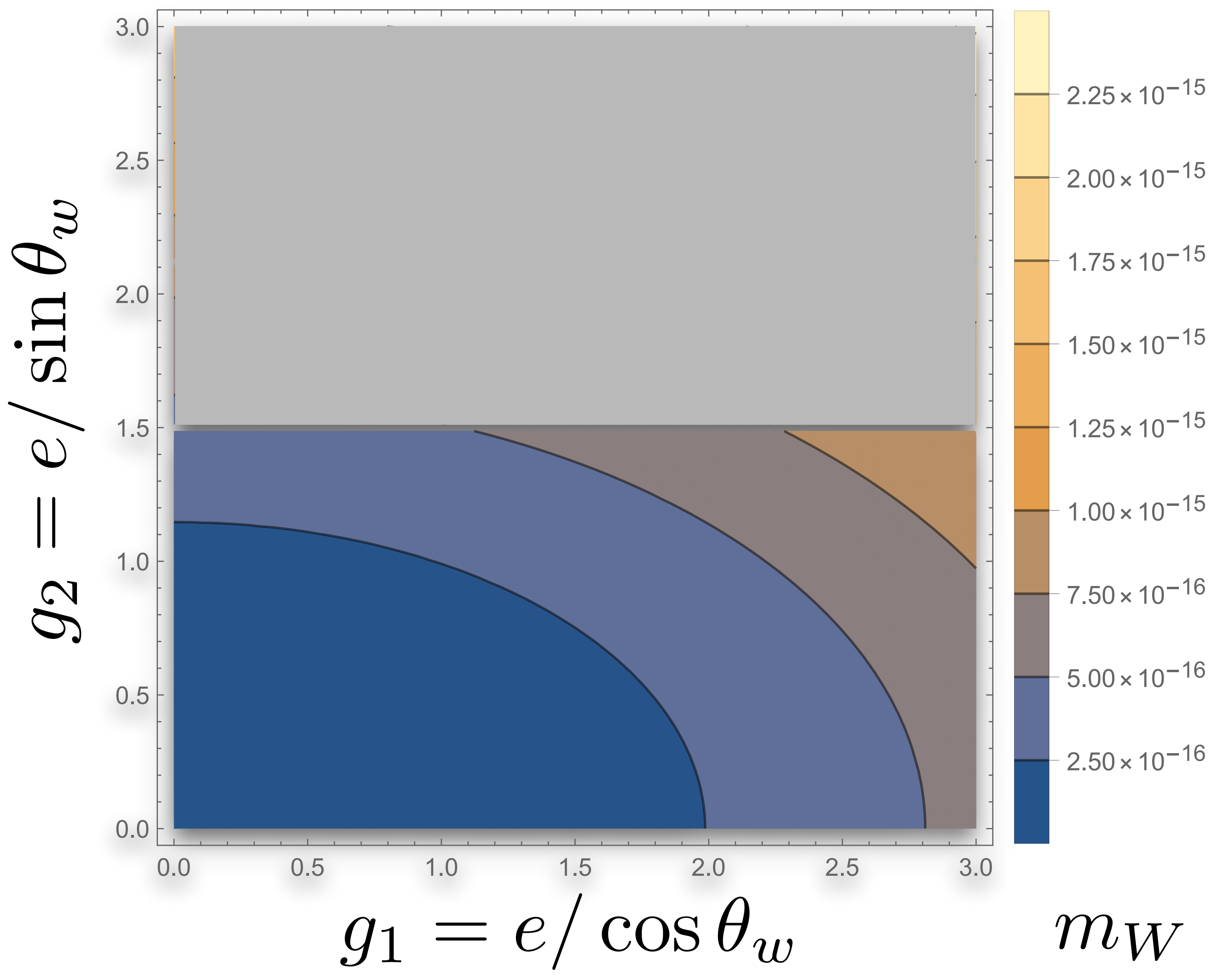
SM + GR



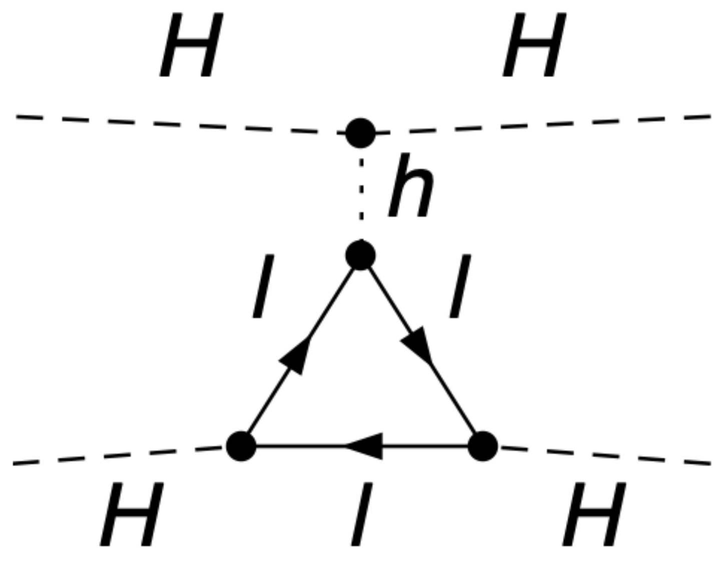
$\gamma\gamma \to \gamma\gamma$



non-SM + GR

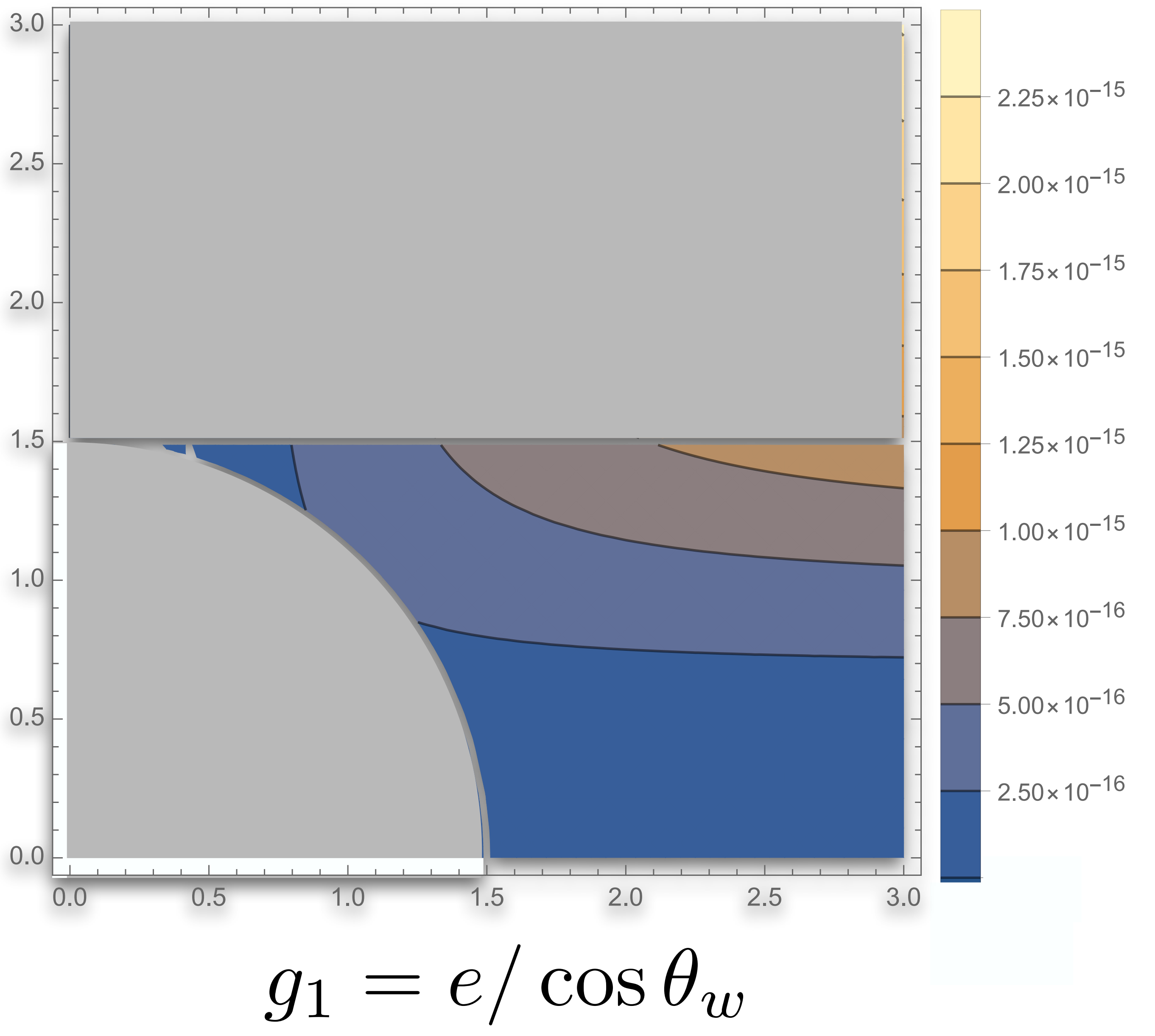


$HH \to HH$

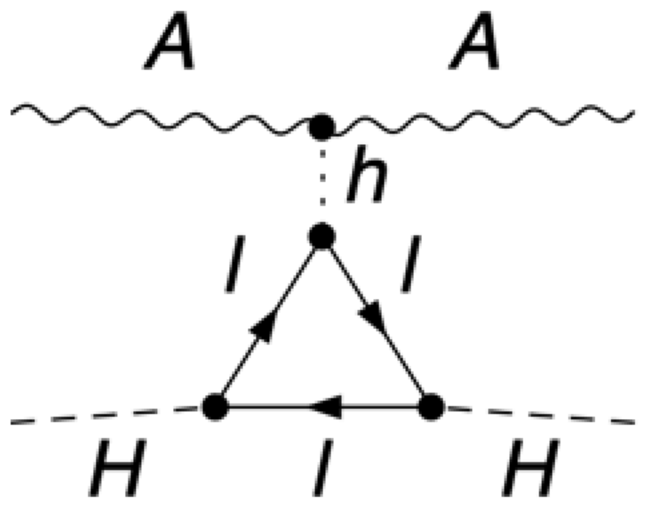


$$m_W = \frac{v g_2}{2}$$

non-SM + GR



$\gamma H \to \gamma H$



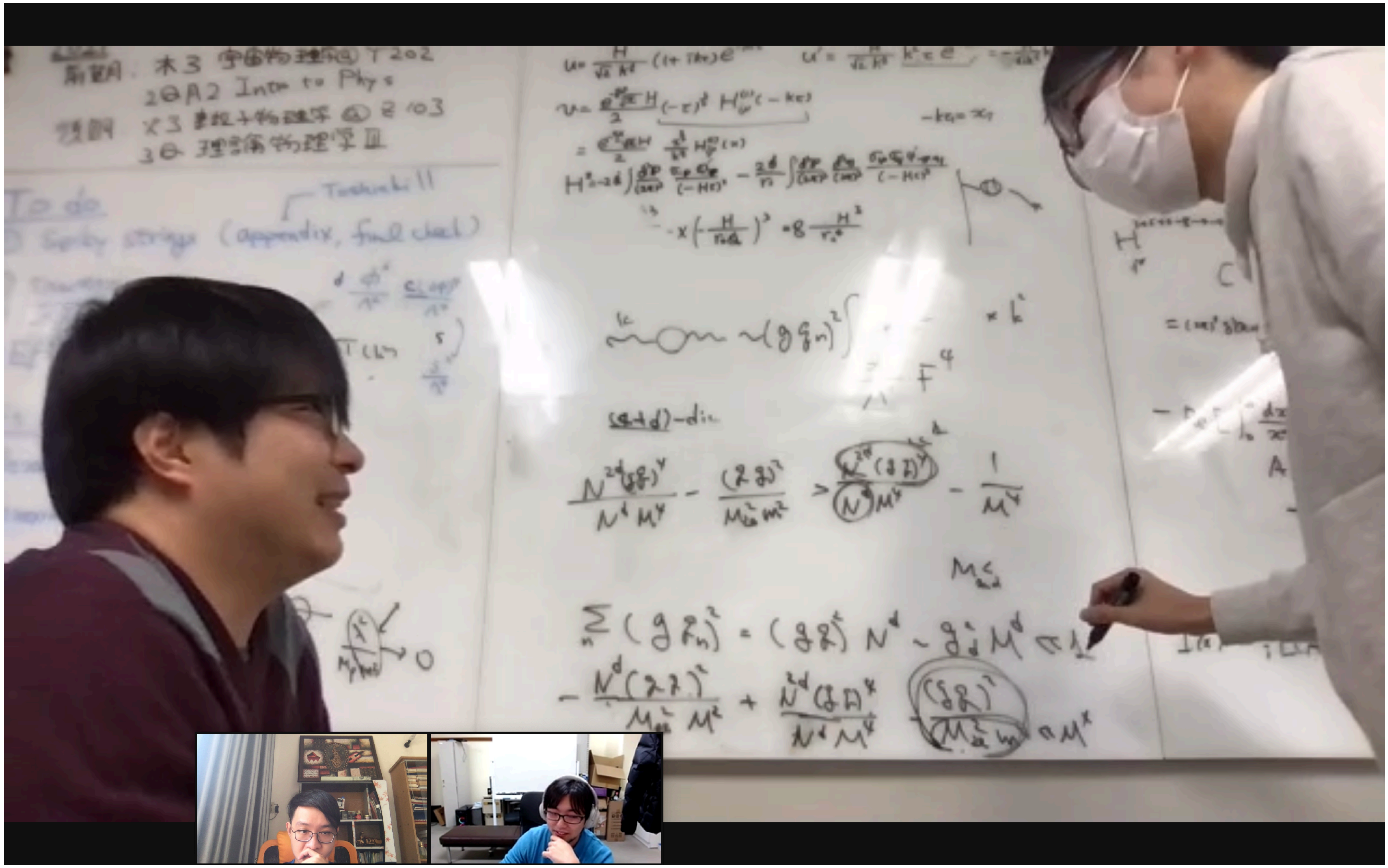
Conclusions & Outlooks

- **Conclusions:**
  - Gravitational positivity bounds of light-by-light scattering yields cut-off scale of order 10^{16} GeV.
  - More insight to Swampland program (correlations between gauge couplings as given by the Electro-Weak bounds).
  - Pomeron plays an important role & may have some phenomenological implications.
- **Outlooks:**
  - Anomalous threshold of SM + GR consistent with UV completion?
  - Find implications from Dark Matter models.

Conclusions & Outlooks

- **Conclusions:**
    - Gravitational positivity bounds of light-by-light scattering yields cut-off scale of order 10^{16} GeV.
    - More insight to Swampland program (correlations between gauge couplings as given by the Electro-Weak bounds).
    - Weakly coupled charged particles up to spin-1 do not help to push up the cutoff scale, hence BSM Physics at E >> GeV will be irrelevant.
    - Pomeron plays an important role & may have some phenomenological implications.
- **Outlooks:**
    - Anomalous threshold of SM + GR consistent with UV completion?
    - Find implications from Dark Matter models.

THANK YOU FOR YOUR LISTENING!



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