中山大學天琴中心
TIANQIN CENTER FOR GRAVITATIONAL PHYSICS, SYSU

Dynamics of electroweak phase transition at lepton colliders and gravitational wave experiments

Fa Peng Huang
Sun Yat-sen University

Siyu Jiang, **FPH**, Xiao Wang, Phys.Rev.D 107 (2023) no.9, 095005 [arXiv:2211.13142]
Siyu Jiang, **FPH,** Chong Sheng Li, arXiv:2305.02218
Siyu Jiang, Aidi Yang, Jiucheng Ma, **FPH**, arXiv:2306.17827

30th Anniversary of the Rencontres du Vietnam Windows on the Universe
@ICISE, Quy Nhon, Vietnam, 2023.08.08

Outline

I. Research motivation for electroweak transition dynamics (EWPT)

II. Bubble wall velocity in inert double model(IDM)

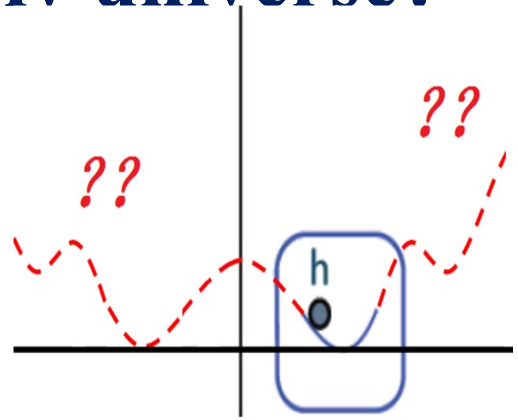
III. Dynamical dark matter (DM) by phase transition dynamics

IV. Summary and outlook

I. Motivation: Higgs potential

What is the shape of Higgs potential now and in the early universe?

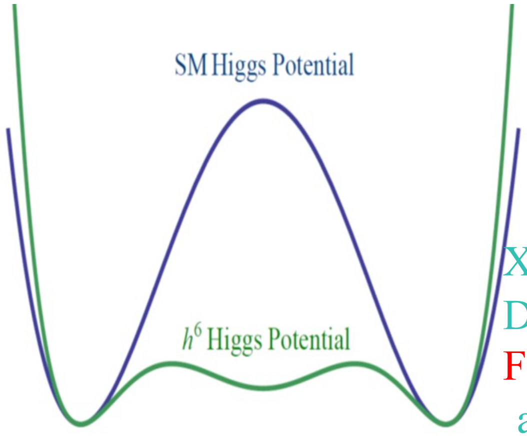
Current data tell us nothing but the quadratic oscillation around 246 GeV with 125 GeV mass.



$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4} h^4$$

or $$V(h) = \frac{1}{2}\mu^2 h^2 - \frac{\lambda}{4} h^4 + \frac{1}{\Lambda^2} h^6$$

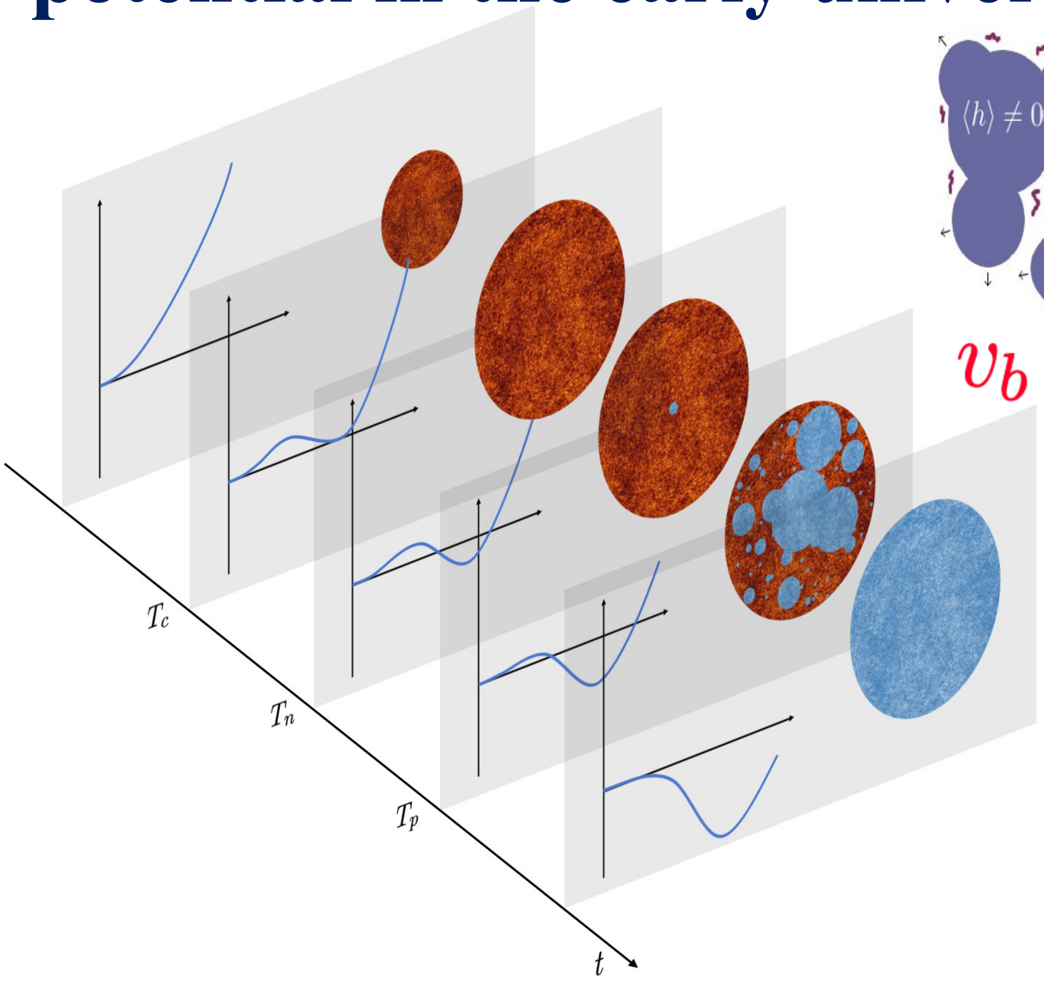
Produce a SFOPT, large deviation of Higgs trilinear coupling and GW

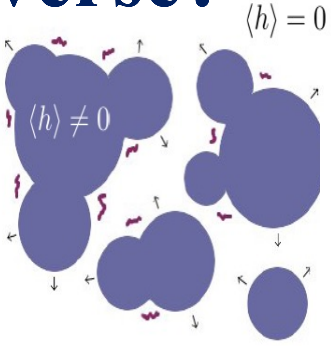


Xinmin Zhang Phys.Rev. D47 (1993) 3065-3067;C. Grojean, G. Servant, J. Well PRD71(2005)036001
D.J.H. Chung, A. J. Long, Lian-tao Wang Phys.Rev. D87(2013) 023509
FPH, et.al, Phys.Rev.D94(2016)no.4,041702 ; **FPH**, et.al, Phys.Rev.D93 (2016) no.10,103515
arXiv:1511.06495, Nima Arkani-Hamed et. al.; PreCDR of CEPC; arXiv: 1811.10545,CDR of CEPC

EWPT in the early universe

What is the shape of Higgs potential in the early universe?





v_b

Calculate the finite-temperature effective potential using the thermal field theory: free energy density.

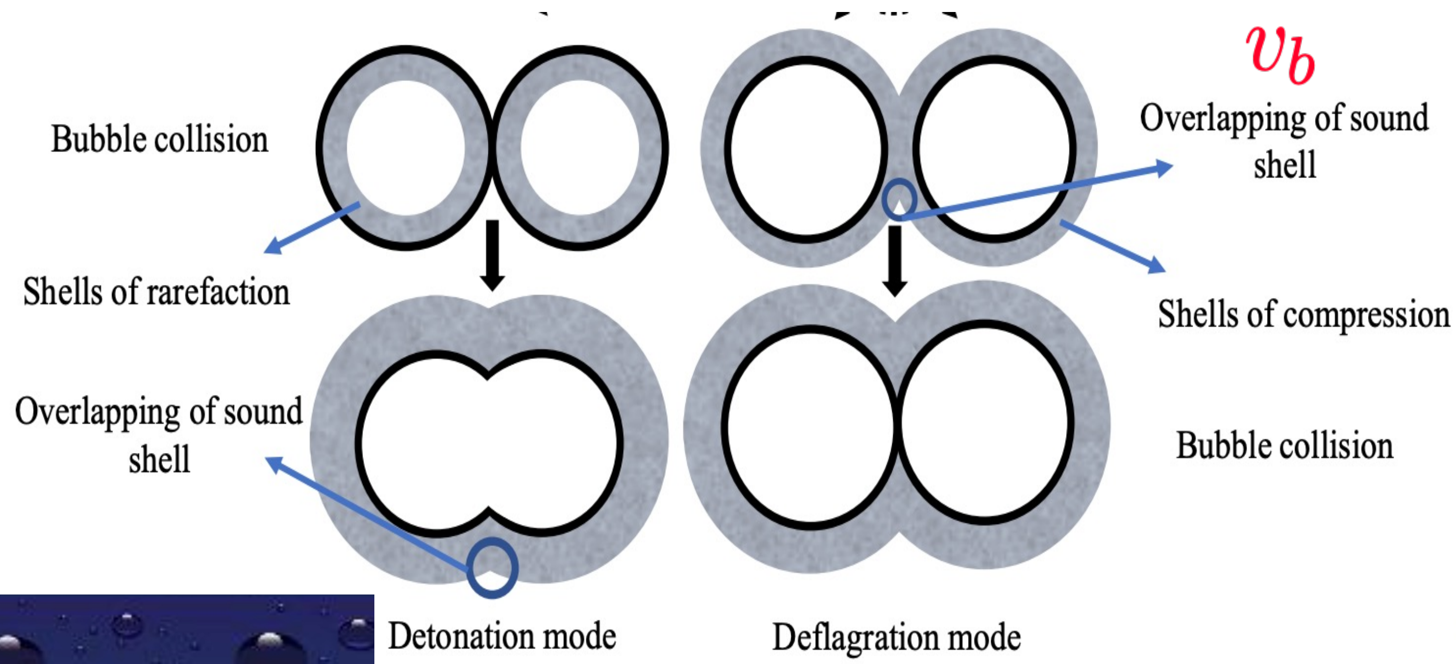
$$V_{\rm eff}^{(1)}(\bar{\phi}) = \sum_i n_i \left[\int \frac{{\rm d}^D p}{(2\pi)^D} \ln\left(p^2 + m_i^2(\bar{\phi})\right) + J_{\rm B,F}\left(\frac{m_i^2(\bar{\phi})}{T^2}\right) \right]$$

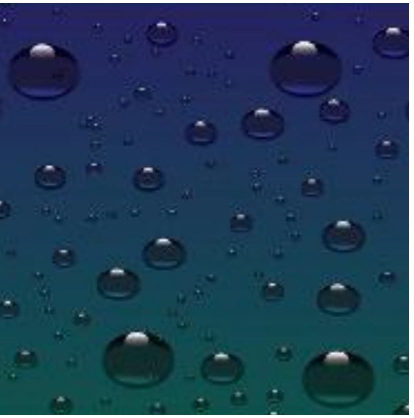
$$S(T) = \int d^4x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 + V_{\rm eff}\,(\phi, T) \right]$$

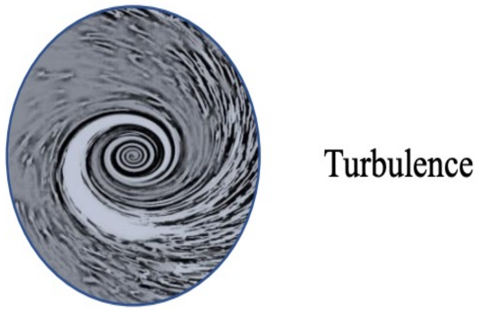
$$\Gamma = \Gamma_0 e^{-S(T)}$$

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

Phase transition GW in a nutshell

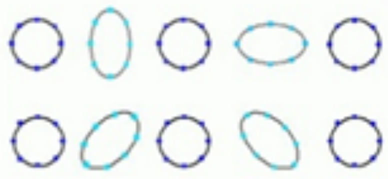






$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$h_{ij} \simeq \frac{2G}{c^4 r} \ddot{Q}_{ij}^{TT}(t - r/c)$$



E. Witten, Phys. Rev. D 30, 272 (1984)
C. J. Hogan, Phys. Lett. B 133, 172 (1983);
M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994))

EW phase transition GW becomes more interesting and realistic after the discovery of Higgs by LHC and GW by LIGO.

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

Phase transition GW in a nutshell

$$\ddot{h}_{ij}(\mathbf{x},t) + 3\,H\,\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}h_{ij}(\mathbf{x},t) = 16\pi G\,\Pi_{ij}(\mathbf{x},t)$$

Possible sources of tensor anisotropic stress in the early universe

- Scalar field gradients $\Pi_{ij} \sim [\partial_i\phi\partial_j\phi]^{TT}$
- Bulk fluid motion $\Pi_{ij} \sim [\gamma^2(\rho+p)v_i v_j]^{TT}$
- Gauge fields $\Pi_{ij} \sim [-E_iE_j - B_iB_j]^{TT}$
- Second order scalar perturbations, Π_{ij} from a combination of $\partial_i\Psi, \partial_i\Phi$
- … arXiv:1801.04268

Higgs cosmology

What is the role of Higgs in the early universe ?

After the Higgs discovery it becomes a realistic portal to study the fundamental physics and its deep connections to cosmology

LISA/TianQin in synergy with CEPC helps to explore the early universe around 100 GeV, DM and baryogenesis.

Collider signals at loop level@CEPC/LHC

Correlation and complementary test with gravitational wave (GW)

Higgs physics (potential), Z-pole physics, TeV new physics

EWPT dynamics and GW: Precise calculation

Baryon asymmetry of our Universe (EW baryogenesis)

Early universe; symmetry breaking at high energy

New DM formation mechanism;
New approach to explore DM

Phase transition Dynamics

Precise predictions on the phase transition dynamics and its GW signals

GW detection favor lager v_b
EW baryogenesis favor smaller v_b
Dynamical DM is sensitive to v_b

Finite-temperature effective potential

$V_{eff}(\phi, T)$

(1). **Daisy resummation problem: Pawani scheme vs. Arnold scheme**
(2). **Gauge dependence problem: see Michael J. Ramsey-Musolf's works**
(3). **No perturbative calculations: lattice calculations and dim-reduction method: by D. Weir, Michael J. Ramsey-Musolf et.al**

Bubble wall velocity

v_b

Energy budget

κ

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang, arXiv:2007.10343,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith, arXiv:2009.14295v2
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Siyu Jiang, **FPH**, xiao wang, Phys.Rev.D 107 (2023) 9, 095005

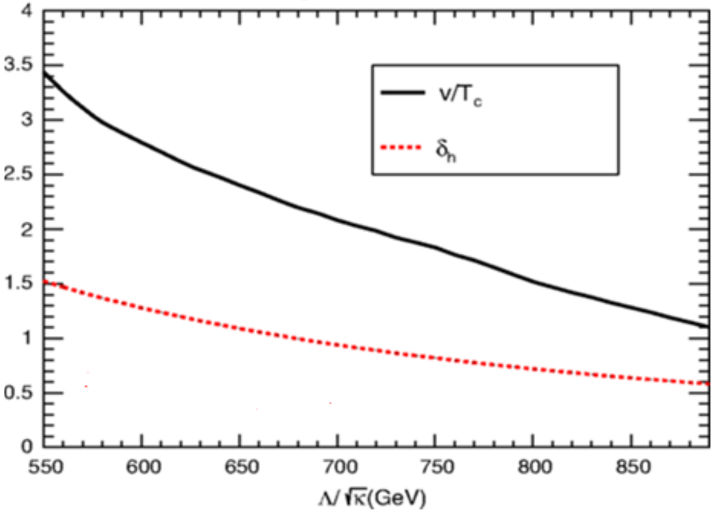
F. Giese, T. Konstandin, K. Schmitz and J. van de Vis,arXiv:2010.09744
Xiao Wang, **FPH** and Xinmin Zhang, Phys.Rev.D 103 (2021) 10, 103520
Xiao Wang, Chi Tian, **FPH**, arXiv: 2301.12328

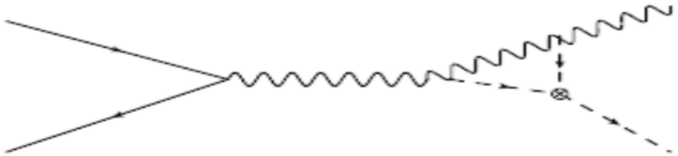
Higgs potential

Correlate collider and GW: double test on Higgs potential from particle to wave

SFOPT leads to obvious deviation of the tri-linear Higgs coupling

$$\mathcal{L}_{hhh} = -\frac{1}{3!}(1+\delta_h)A_h h^3$$



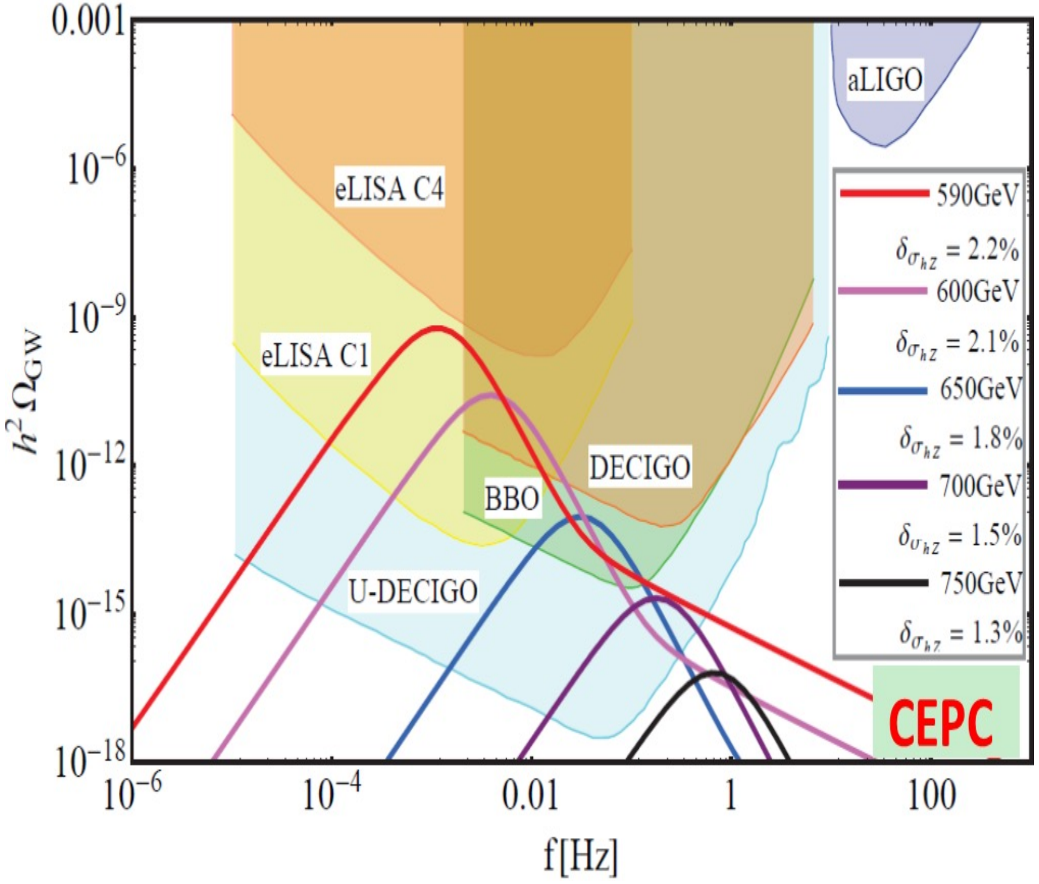


At one-loop level, deviation of the tri-linear Higgs coupling

$$\delta_h \in (0.6, 1.5)$$

The Circular Electron Positron Collider (CEPC), ILC, FCC-ee can precisely test this scenario by precise measurements of the hZ cross section ($e^- e^+ \longrightarrow hZ$).
SM NNLO hZ cross section recently by Lilin Yang, et al 2016, Yu Jia et at 2016

$$\delta_\sigma = \frac{\sigma_{hz,\delta_h \neq 0}}{\sigma_{hz,SM}} - 1$$



FPH, et.al, Phys.Rev.D94(2016)no.4,041702 ; FPH, et.al, Phys.Rev.D93 (2016) no.10,103515

Higgs potential

SM EFT

$$\mathcal{L} \supset -\mu^2|H|^2 - \lambda|H|^4 + c_6|H|^6$$

$$+ c_T \mathcal{O}_T + c_{WW}\mathcal{O}_{WW} + \text{other dimension-six operators}$$

$$\delta_{\sigma(hZ)} \approx (0.26c_{WW} + 0.01c_{BB} + 0.04c_{WB} - 0.06c_H - 0.04c_T + 0.74c_L^{(3)\ell} + 0.28c_{LL}^{(3)\ell} + 1.03c_L^\ell - 0.76c_R^e) \times 1\ \text{TeV}^2 + 0.016\delta_h,$$

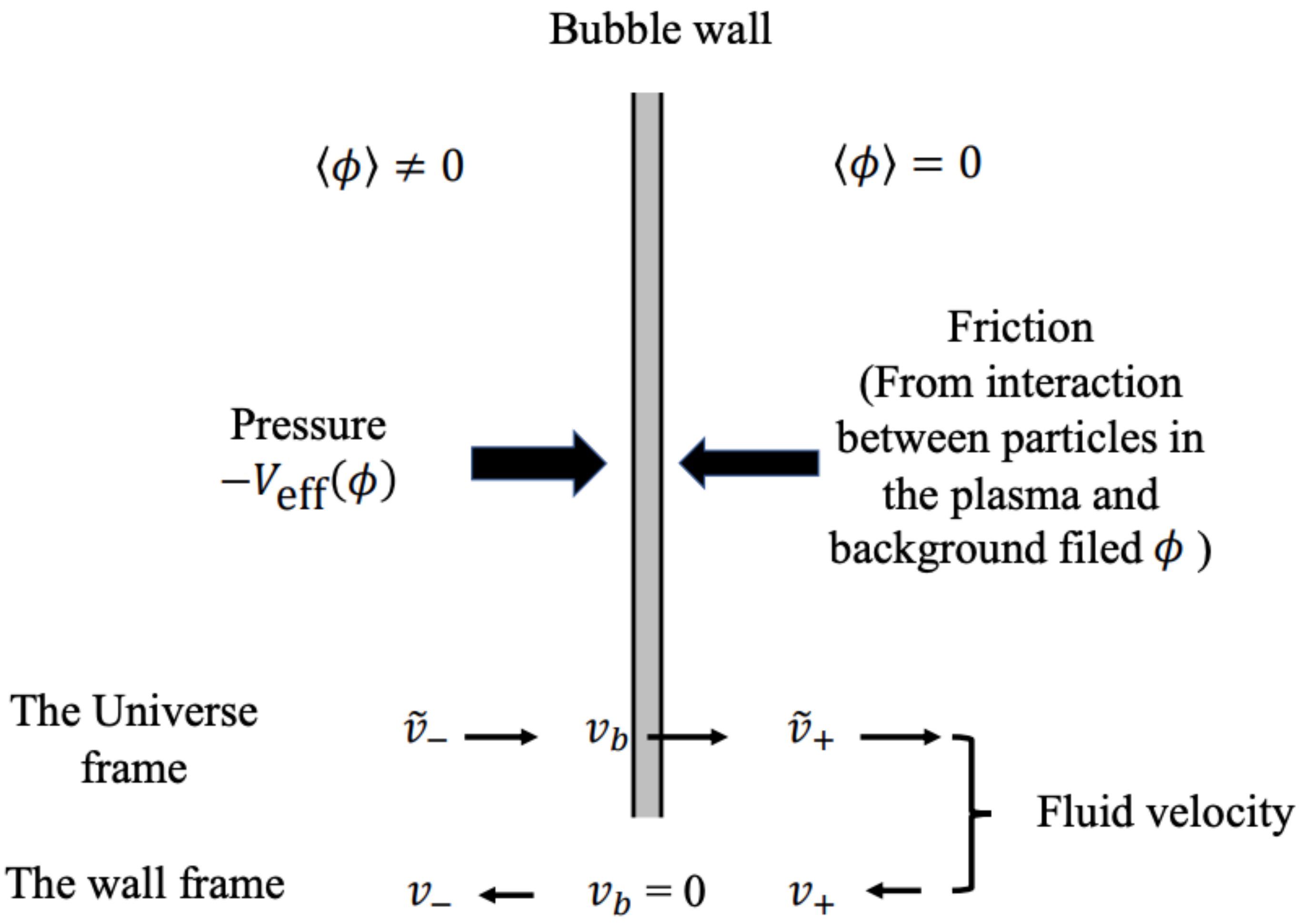
SFOPT produces large modification of trilinear Higgs coupling δ_h

c_6 dominates the hZ cross section deviation

Taking a general study of the scalar extended models and the composite Higgs model as examples, we find that the Higgs sextic scenario still works well after considering all the dim-6 operators and the precise measurements.

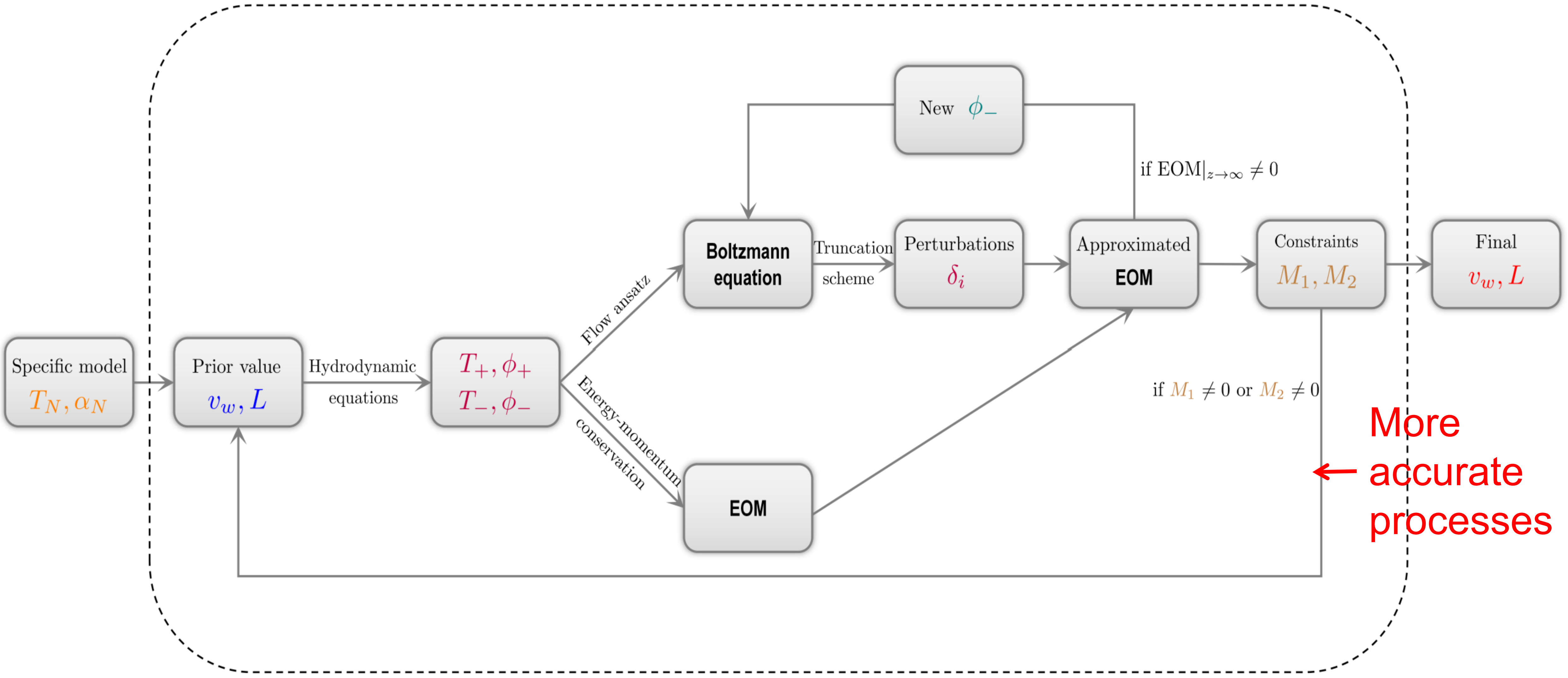
Qing-Hong Cao, FPH, Ke-Pan Xie, Xinmin Zhang, arXiv:1708.0473.

II.Bubble wall velocity in IDM



Siyu Jiang, **FPH**, Xiao Wang, Phys.Rev.D 107 (2023) no.9, 095005 [arXiv:2211.13142]

Bubble wall velocity in IDM



Bubble wall velocity in IDM

Inert doublet model(IDM):

$$V_0 = \mu_1^2|\Phi|^2 + \mu_2^2|\eta|^2 + \frac{1}{2}\lambda_1|\Phi|^4 + \frac{1}{2}\lambda_2|\eta|^4 + \lambda_3|\Phi|^2|\eta|^2 + \lambda_4|\Phi^\dagger\eta|^2 + \frac{1}{2}\{\lambda_5(\Phi^\dagger\eta)^2 + \mathrm{H.c.}\}\ ,$$

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(h + v + iG^0) \end{pmatrix},\ \eta = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix},$$

$$V_{\mathrm{CW}}(\phi, T = 0) = \sum_i \frac{n_i}{64\pi^2}\left[m_i^4(\phi)\left(\ln\frac{m_i^2(\phi)}{\overline{m}_i^2} - \frac{3}{2}\right) + 2\overline{m}_i^2 m_i^2(\phi)\right],$$

$$V_{\mathrm{T}}(\phi, T > 0) = \sum n_i \frac{T^4}{2\pi^2} I_b\left(\frac{M_i^2}{T^2}\right),$$

$$V_{\mathrm{eff}}(\phi, T) = V_0(\phi) + V_{\mathrm{CW}}(\phi) + V_{\mathrm{T}}(\phi, T)\ .$$

Bubble wall velocity in IDM

Benchmark parameters to satisfy the full DM density, DM direct search, collider search, and the condition for a strong FOPT.

$v = 246$ GeV $\quad \overline{m}_h = 125$ GeV, $\lambda_2 = 0.2.$

$$\overline{m}_H^2 = \mu_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5)v^2,$$
$$\overline{m}_A^2 = \mu_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_5)v^2,$$
$$\overline{m}_{H^\pm}^2 = \mu_2^2 + \frac{1}{2}\lambda_3 v^2 \ .$$

$$\lambda_L = (\lambda_3+\lambda_4+\lambda_5)/2$$

	$\overline{m}_H$ [GeV]	$\overline{m}_A = \overline{m}_{H^\pm}$ [GeV]	λ_L	T_c [GeV]	T_N [GeV]
Benchmark A	62.66	300	0.0015	118.3	117.1
Benchmark B	65.00	300	0.0015	118.6	117.5
Benchmark C	63.00	295	0.0015	119.4	118.4

TABLE I. Three sets of benchmark model parameters.

Bubble wall velocity in IDM

The EOM of Higgs filed (order parameter field)

Energy momentum conservation of scaler-plasma system in WKB approximation

$$\Box\phi + \frac{\partial V_0(\phi)}{\partial\phi} + \sum \frac{dm^2}{d\phi}\int \frac{d^3p}{(2\pi)^3 2E} f(p,x) = 0 \,,$$

Zero-temperature part of effective potential

$$f \equiv f_0 + \delta f .$$

$$\Box\phi + \frac{\partial V_{\rm eff}(\phi,T)}{\partial\phi} + \sum \frac{dm^2}{d\phi}\int \frac{d^3p}{(2\pi)^3 2E} \delta f(p,x) = 0 \,,$$

Full thermal effective potential

Friction term

Deviation from equilibrium

Bubble wall velocity in IDM

The deviation distribution part for each massive particle is crucial

With WKB approximation $p \gg 1/L_w$, we can describe it with Boltzmann equation

$$\frac{d}{dt} f = \left(\frac{\partial}{\partial t} + \dot{z} \frac{\partial}{\partial z} + \dot{p_z} \frac{\partial}{\partial p_z} \right) f = -C[f] \ ,$$

- For thermal equilibrium situation, the collision term is zero, hence the non-equilibrium part can be derived from this equation

To solve the Boltzmann equation

- *Appropriate form for the distribution function (flow ansatz)*
- *Specific truncation scheme*
- *Proper treatment of collision term*

Bubble wall velocity in IDM

Evolution of the perturbations

source term

$$\hat{A}\delta' + \Gamma\delta = \Sigma,$$

collision term

$$\delta = (\mu_t, \delta T_t, T\delta v_t, \mu_W, \delta T_W, T\delta v_W, \mu_A, \delta T_A, T\delta v_A),$$

New particles introduced by IDM

$$\Sigma = \frac{v_w}{2T}\left(c_1^t (m_t^2)', c_2^t (m_t^2)', 0, c_1^W (m_W^2)', c_2^W (m_W^2)', 0, c_1^A (m_A^2)', c_2^A (m_A^2)', 0\right),$$

$$\hat{A} = \begin{pmatrix} \hat{A}_t & 0 & 0 \\ 0 & \hat{A}_W & 0 \\ 0 & 0 & \hat{A}_A \end{pmatrix}, \quad \text{where} \quad \hat{A}_i = \begin{pmatrix} v_w c_2^i & v_w c_3^i & \frac{1}{3} c_3^i \\ v_w c_3^i & v_w c_4^i & \frac{1}{3} c_4^i \\ \frac{1}{3} c_3^i & \frac{1}{3} c_4^i & \frac{1}{3} v_w c_4^i \end{pmatrix},$$

Bubble wall velocity in IDM

Collision terms

$$\Gamma_{\mu1,t} \simeq \left(5.0 \times 10^{-4} g_s^4 + 5.8 \times 10^{-4} g_s^2 y_t^2\right) T \ ,$$

$$\Gamma_{T1,t} \simeq \Gamma_{\mu2,t} \simeq \left(1.1 \times 10^{-3} g_s^4 + 1.3 \times 10^{-3} g_s^2 y_t^2\right) T \ ,$$

$$\Gamma_{T2,t} \simeq \left(1.1 \times 10^{-2} g_s^4 + 4.0 \times 10^{-3} g_s^2 y_t^2\right) T \ ,$$

$$\Gamma_{v,t} \simeq \left(2.0 \times 10^{-2} g_s^4 + 1.8 \times 10^{-3} g_s^2 y_t^2\right) T \ ,$$

$$\Gamma_{\mu1,W} \simeq \left(2.3 \times 10^{-3} g_s^2 g_w^2 + 2.0 \times 10^{-3} g_w^4\right) T \ ,$$

$$\Gamma_{T1,W} \simeq \Gamma_{\mu2,W} \simeq \left(4.7 \times 10^{-3} g_s^2 g_w^2 + 4.1 \times 10^{-3} g_w^4\right) T$$

$$\Gamma_{T2,W} \simeq \left(1.5 \times 10^{-2} g_s^2 g_w^2 + 1.5 \times 10^{-2} g_w^4\right) T \ ,$$

$$\Gamma_{v,W} \simeq \left(5.7 \times 10^{-2} g_s^2 g_w^2 + 1.5 \times 10^{-2} g_w^4\right) T \ ,$$

$$\Gamma_{\mu1,A} \simeq 1.0 \times 10^{-2} \lambda_3^4 T \ ,$$

$$\Gamma_{T1,A} \simeq \Gamma_{\mu2,A} \simeq 4.9 \times 10^{-3} \lambda_3^4 T \ ,$$

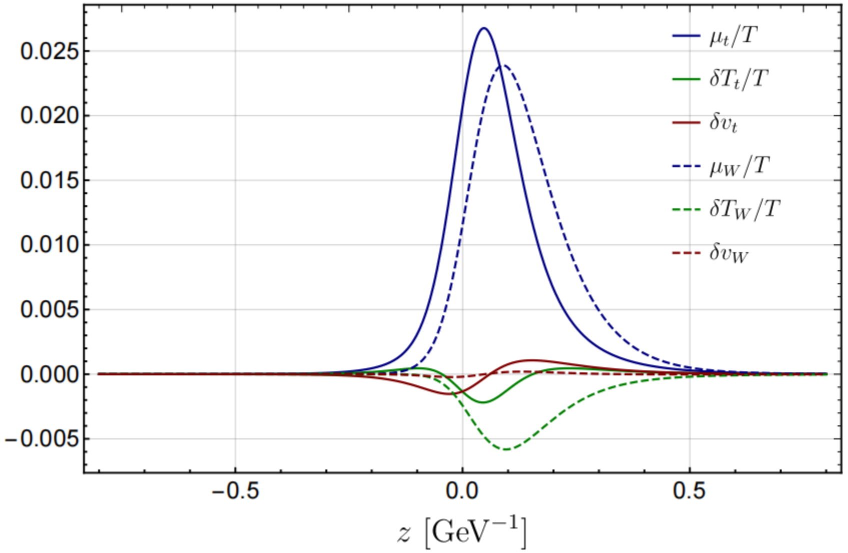
$$\Gamma_{T2,A} \simeq 5.1 \times 10^{-3} \lambda_3^4 T \ ,$$

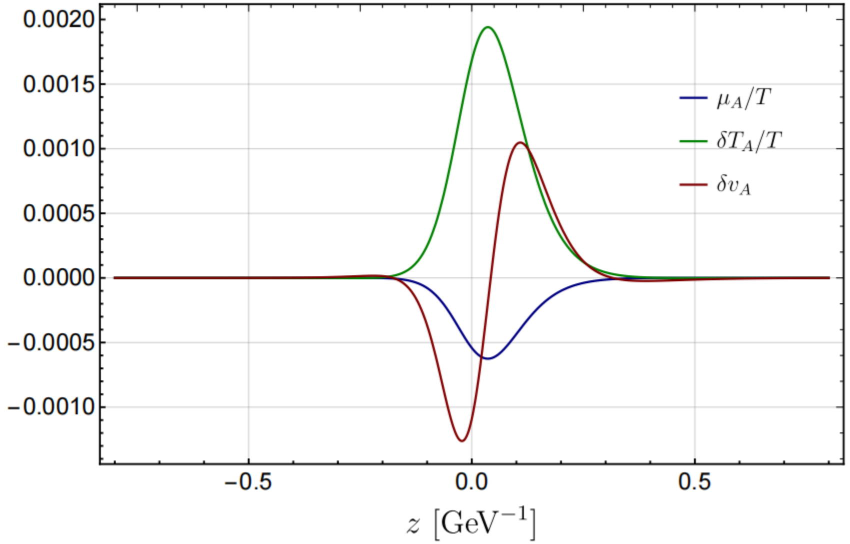
$$\Gamma_{v,A} \simeq 1.8 \times 10^{-3} \lambda_3^4 T \ .$$

Bubble wall velocity in IDM

Green Function Method

$$\delta_i(z) = \chi_{ij} \int_{-\infty}^{\infty} dy G_j(z,y)[\chi^{-1}\hat{A}^{-1}\Sigma(y)]_j \ ,$$

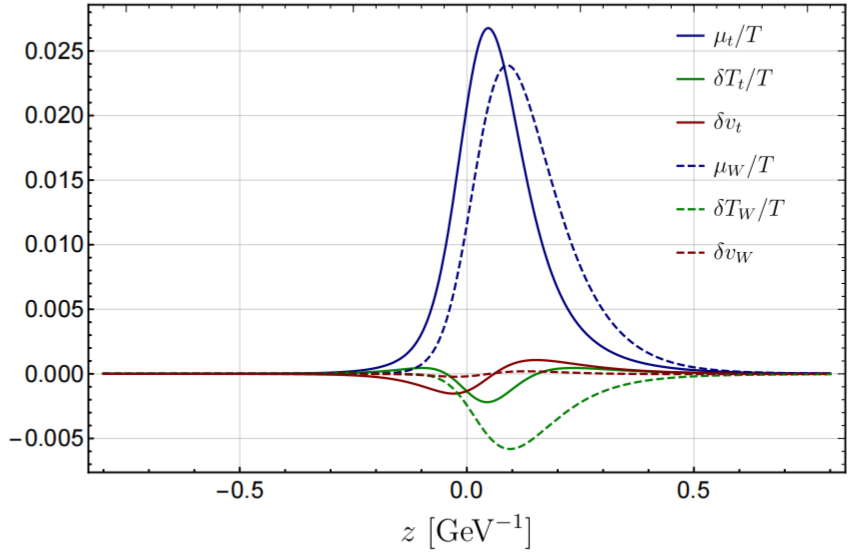


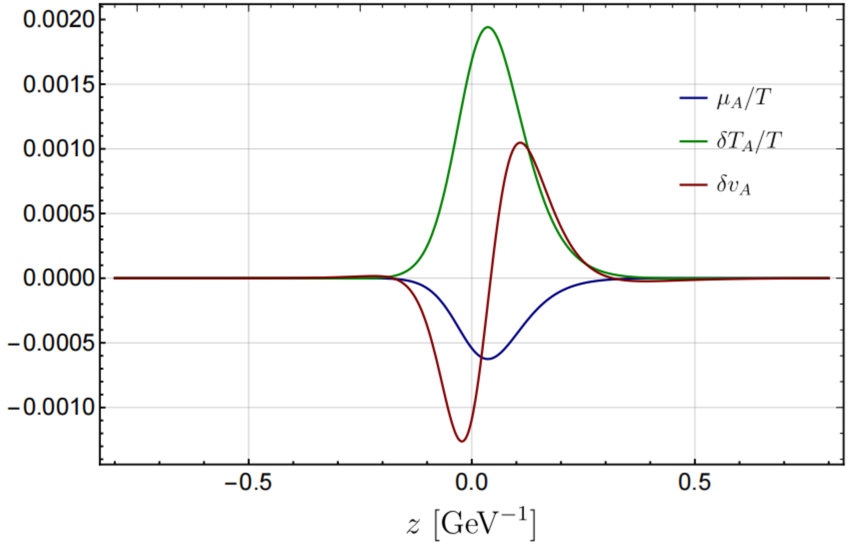


Bubble wall velocity in IDM

Green Function Method

$$\delta_i(z) = \chi_{ij} \int_{-\infty}^{\infty} dy G_j(z,y)[\chi^{-1}\hat{A}^{-1}\Sigma(y)]_j \ ,$$



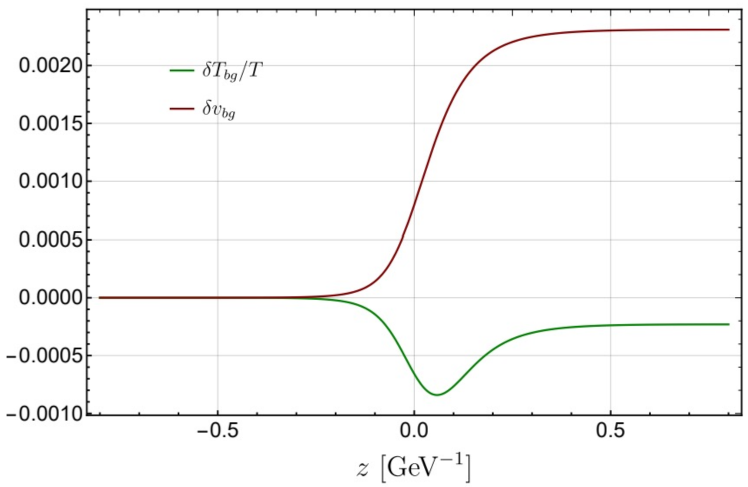


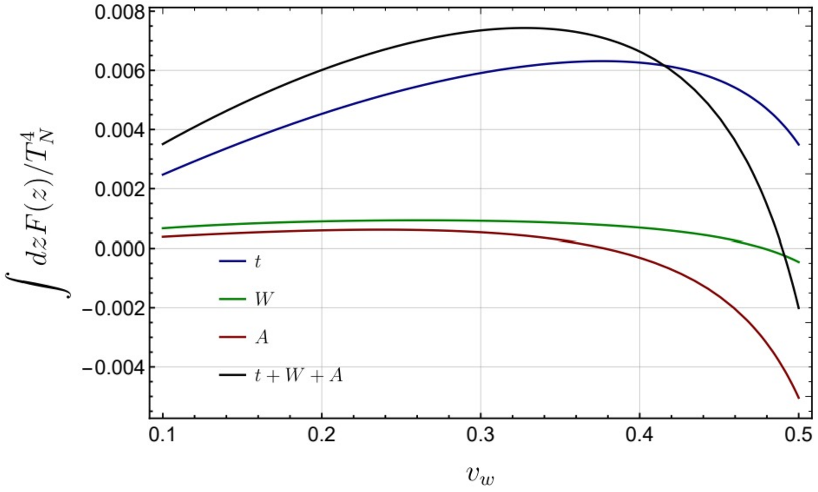
Bubble wall velocity in IDM

Perturbations for background

$$\tilde{c}_4\left(v_w \delta T'_{bg} + \frac{\delta v'_{bg}}{3} T\right) = N_t(\mu_t \Gamma_{\mu 2,t} + \delta T_t \Gamma_{T2,t}) + \sum_{\text{bosons}} N_b(\mu_b \Gamma_{\mu 2,b} + \delta T_b \Gamma_{T2,b}) \ ,$$

$$\frac{\tilde{c}_4}{3}(\delta T'_{bg} + v_w T \delta v'_{bg}) = N_t T \delta v_t \Gamma_{v,t} + \sum_{\text{bosons}} N_b T \delta v_b \Gamma_{v,b} \ , \quad \mu_{bg} = 0 \ ,$$



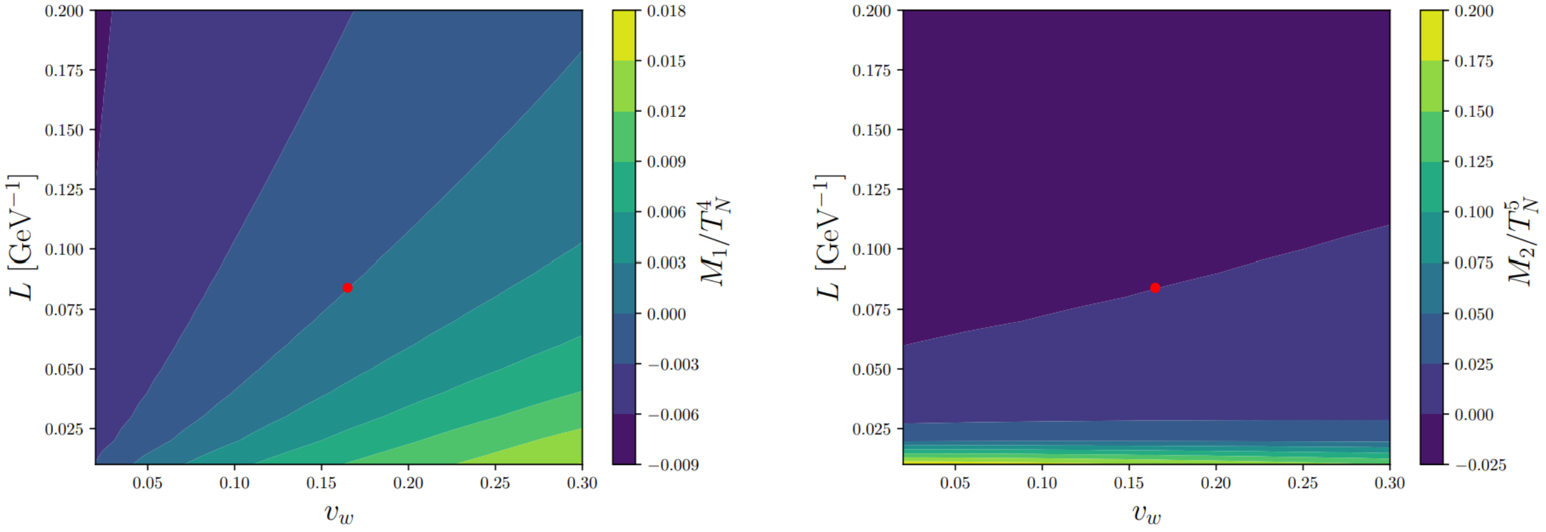


Bubble wall velocity in IDM

$$S_{\text{EOM}} \equiv \left(1-v_w^2\right)\phi'' + \frac{\partial V_{\text{eff}}\left(\phi, T_+\right)}{\partial \phi} + \frac{N_t T_+}{2}\frac{dm_t^2}{d\phi} \times \left(c_1^t \mu_t + c_2^t\left(\delta T_t + \delta T_{bg}\right)\right)$$

$$+ \sum_b \frac{N_b T_+}{2}\frac{dm_b^2}{d\phi}\left(c_1^b \mu_b + c_2^b\left(\delta T_b + \delta T_{bg}\right)\right) = 0 \ ,$$

$$M_1 = \int S_{\text{EOM}}\phi' dz = 0, \quad M_2 = \int S_{\text{EOM}}(2\phi - \phi_-)\phi' dz = 0 \ .$$



Bubble wall velocity in IDM

	T_c [GeV]	T_N [GeV]	v_w	L [GeV^{-1}]
Benchmark A	118.3	117.1	0.165	0.084
Benchmark B	118.6	117.5	0.164	0.085
Benchmark C	119.4	118.4	0.164	0.088

In the allowed parameter space, the bubble wall velocity varies slightly around 0.165.

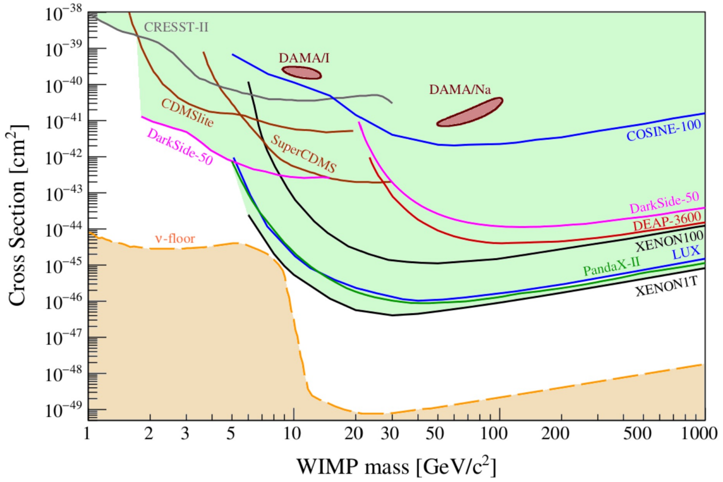
Siyu Jiang, **FPH**, Xiao Wang, Phys.Rev.D 107 (2023) no.9, 095005 [arXiv:2211.13142]

Detailed discussions on GW signals and lepton collider simulation at one-loop level, see our previous works:

FPH, Jiang-Hao Yu, Phys.Rev. D98 (2018) no.9, 095022

Yan Wang, Chong Sheng Li, and **FPH**, P*hys.Rev.D* 104 (2021) 5, 053004;

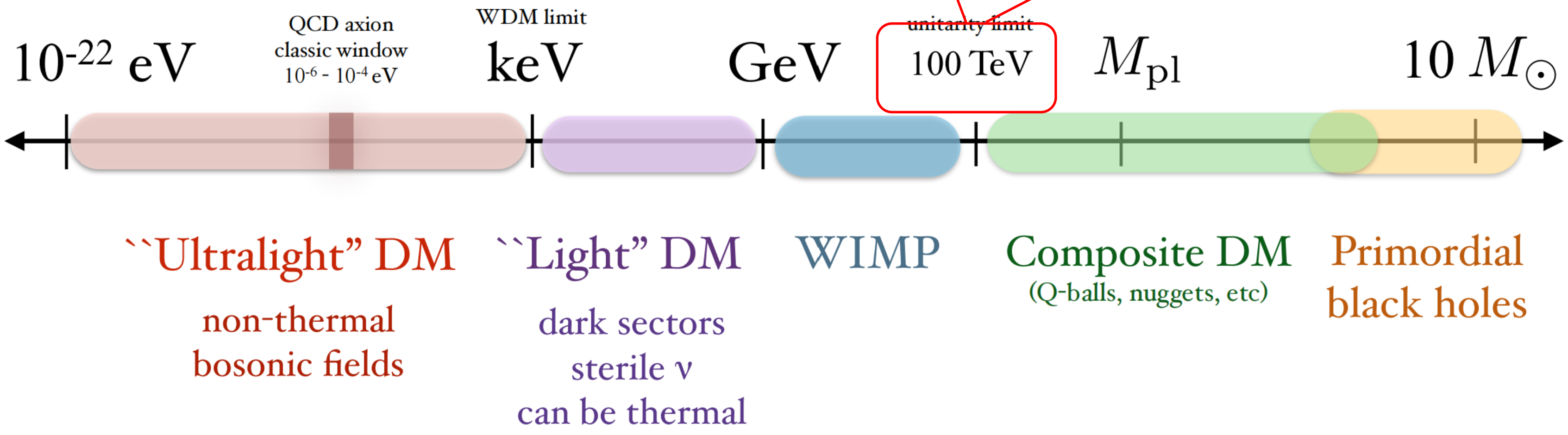
III. Dynamical DM by phase transition dynamics



Unitary bound

$$\sum_i \frac{1}{S_i} \int d\Pi_n |\mathcal{M}_{f\to i}|^2 \leq 2\mathrm{Im}\mathcal{M}(f \to f),$$

$$\sigma_l \leq 4\pi(2l+1)/|\vec{k}|^2$$



III. Dynamical DM by phase transition dynamics

Phase transition dynamics in the early universe provides new DM production scenario in the early universe.

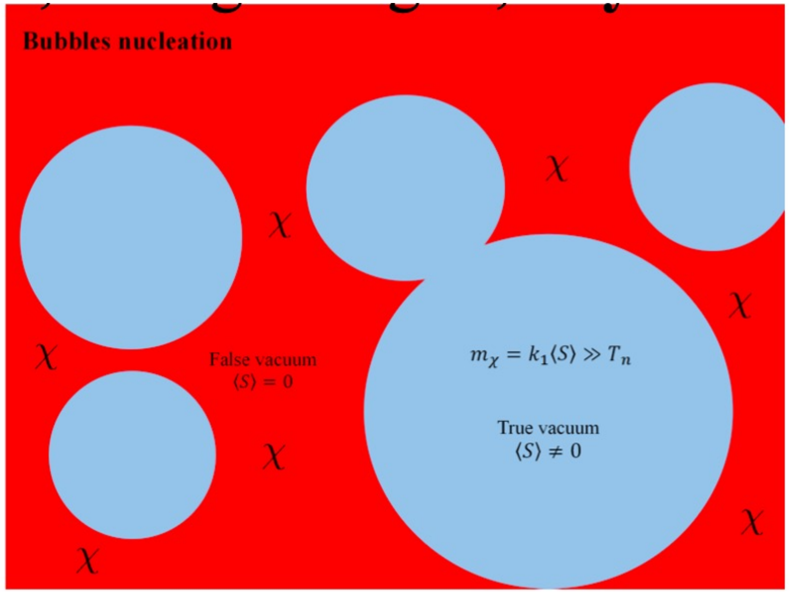
The cosmic phase transition with Q-balls production can explain baryogenesis and DM simultaneously, where constraints on DM mass and reverse dilution are significantly relaxed. We study how to probe this scenario by GW signals and collider signals at QCD NLO.

FPH, Chong Sheng Li, Phys. Rev. D96 (2017) no.9, 095028

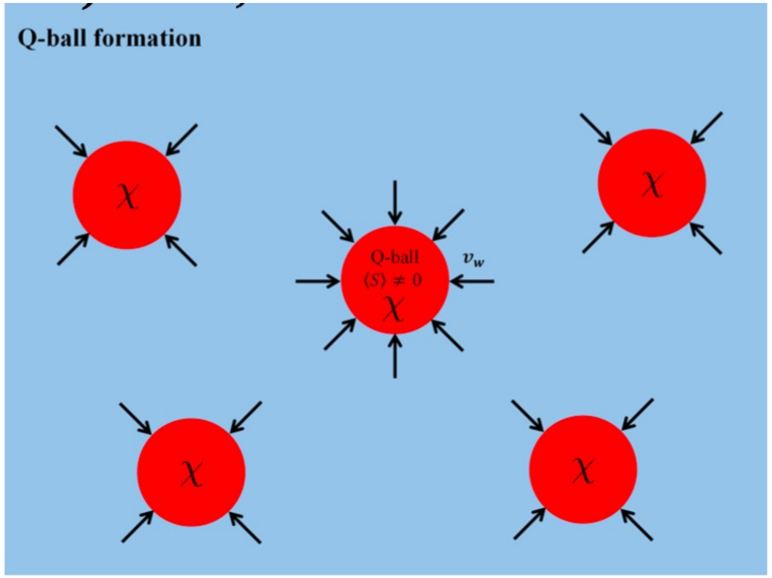
$$\rho_{\rm DM}^4 v_b^{3/4} = 73.5(2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$

III. Dynamical DM by phase transition dynamics

FOPT naturally correlates DM, baryogenesis, particle collider and GW signals.



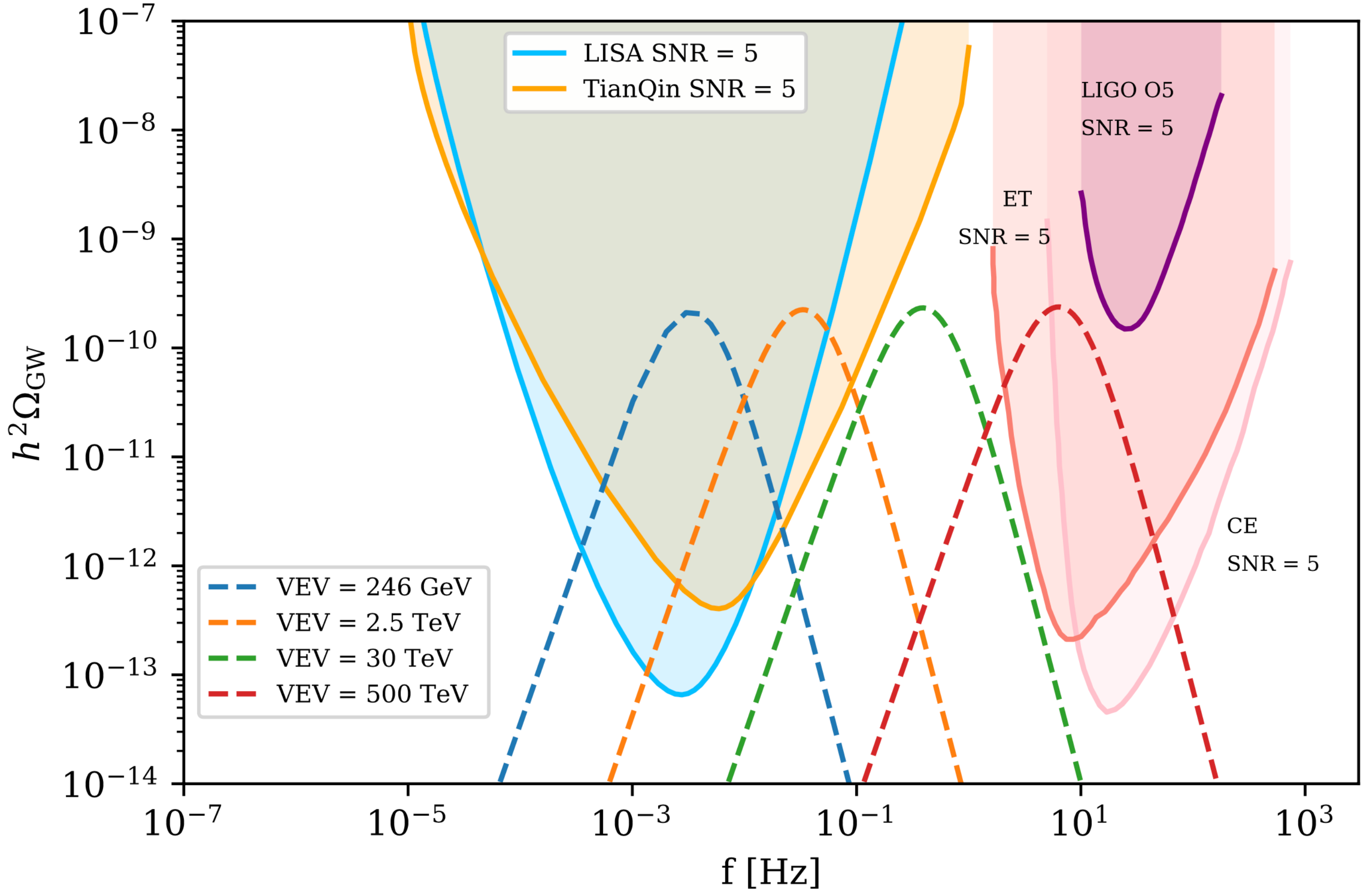
(a) Bubble nucleation: χ particles trapped in the false vacuum due to Boltzmann suppression



(b) Q-ball formation: After the formation of Q-balls, they should be squeezed by the true vacuum

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028

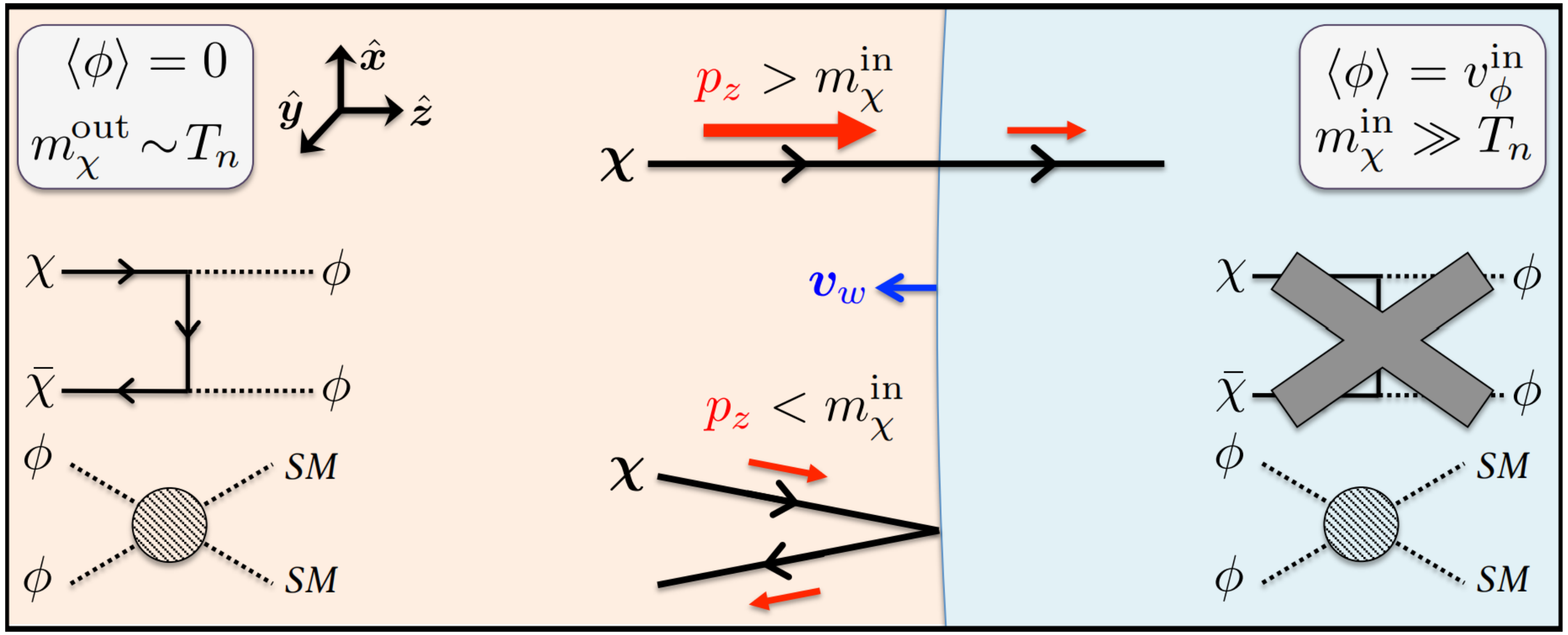
III. Hydrodynamic effects on filtered DM



working in progress

Hydrodynamic effects on filtered DM

In the opposite direction, Filtered DM proposed by Michael J. Baker, Joachim Kopp, Andrew J. Long, Phys.Rev.Lett. 125 (2020) 15, 151102



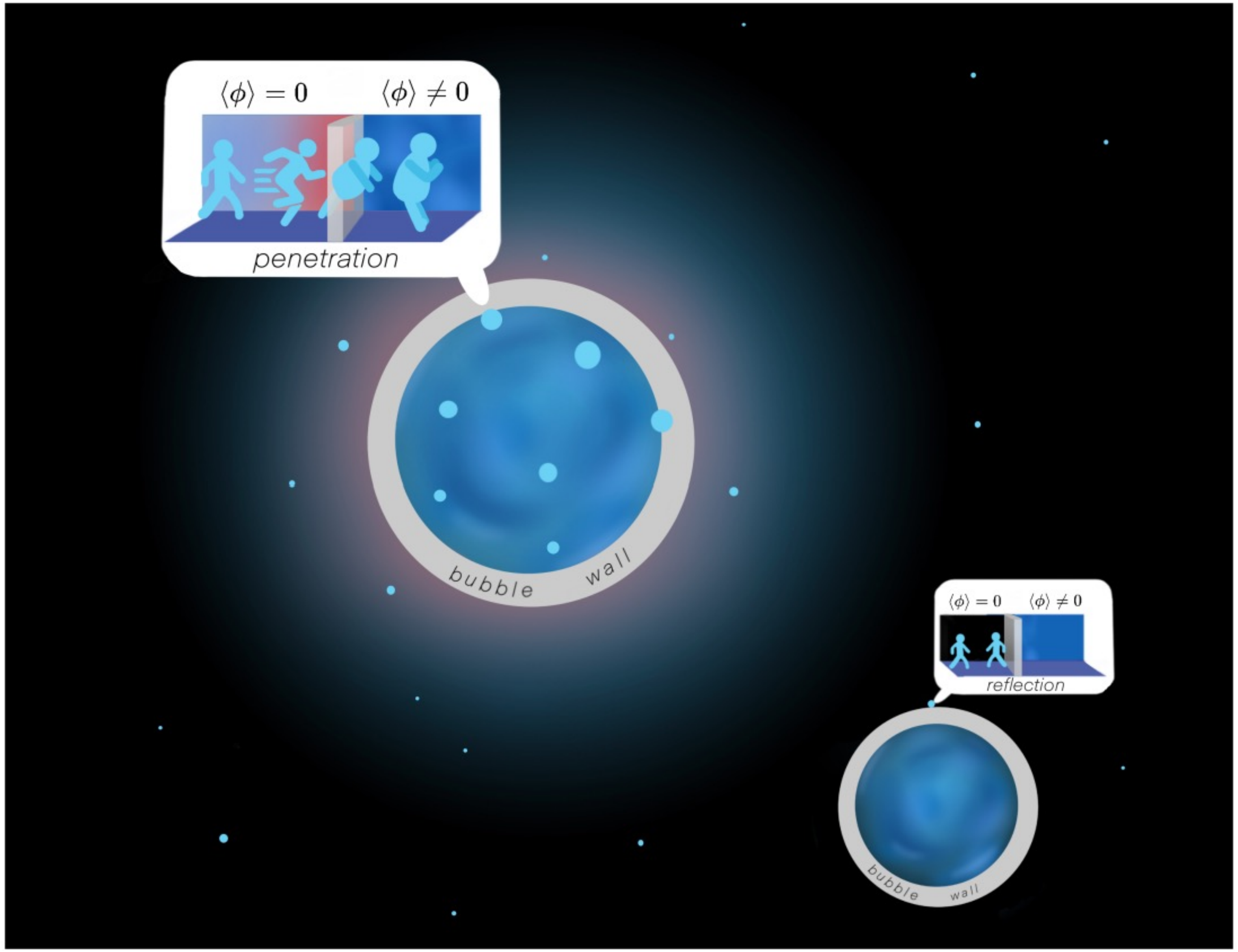
$$\mathscr{L} = -y_\chi \bar{\chi}\Phi\chi - V(\Phi) - \kappa\Phi^\dagger\Phi H^\dagger H + h.c.\ ,$$

$$\Omega_{\rm DM}h^2 \approx 0.17\left(\frac{T_n}{\rm TeV}\right)\left(\frac{m_\chi^\infty}{30T_n}\right)^{-\frac{5}{2}}\exp\left(-\frac{m_\chi^\infty}{30T_n}\right)$$

Hydrodynamic effects on filtered DM

Phase transition dynamics play an essential role in the filtered DM mechanism.

Siyu Jiang, FPH, Chong Sheng Li, arXiv:2305.02218



Hydrodynamic effects on filtered DM

outside the bubble wall:

$$f_\chi^{\rm eq} = \frac{1}{e^{\tilde{\gamma}_{\rm pl}\left(\sqrt{(p^w)^2+m_0^2}-\tilde{v}_{\rm pl}p_z^w\right)/T} \mp 1} ,$$

$$J_\chi^w = g_\chi \int \frac{d^3p^w}{(2\pi)^3}\frac{p_z^w}{E^w} f_\chi^{\rm eq}\Theta\left(p_z^w - \sqrt{\Delta m^2}\right) , \qquad n_\chi^{\rm in} = \frac{J_\chi^w}{\gamma_w v_w} .$$

$$n_\chi^{\rm in} \simeq \frac{g_\chi T^3}{\gamma_w v_w}\left(\frac{\tilde{\gamma}_{\rm pl}\left(1-\tilde{v}_{\rm pl}\right)m_\chi^{\rm in}/T+1}{4\pi^2\tilde{\gamma}_{\rm pl}^3\left(1-\tilde{v}_{\rm pl}\right)^2}\right) e^{-\frac{\tilde{\gamma}_{\rm pl}\left(1-\tilde{v}_{\rm pl}\right)m_\chi^{\rm in}}{T}} ,$$

original works:

$$\boxed{\tilde{v}_{\rm pl} = v_w, \quad T = T' = T_n \quad \rightarrow \quad \Omega_{\rm DM}^{(0)}h^2 \ [2] .}$$

our works:

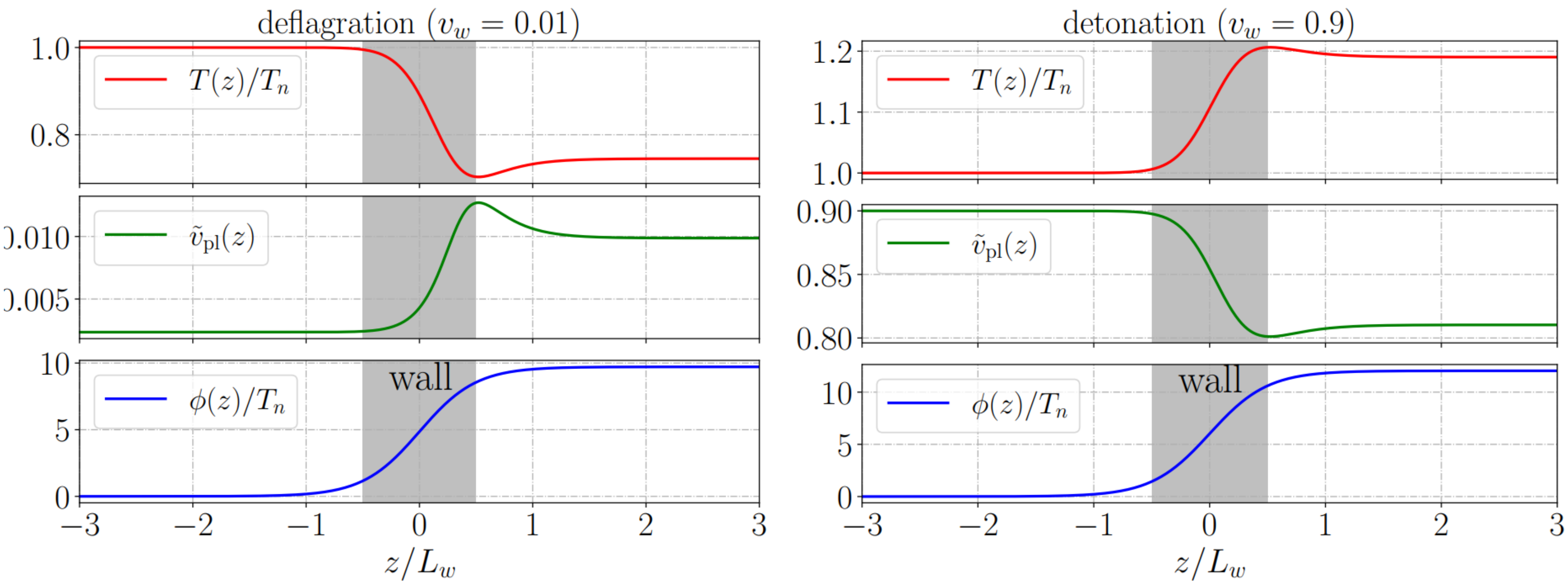
$$\boxed{\tilde{v}_{\rm pl} = \tilde{v}_+, \quad T = T_+, \quad T' = T_- \qquad \text{(this work with hydrodynamic effects)} .}$$

$$n_\chi^{\rm in} \simeq \frac{g_\chi T_+^3}{\gamma_w v_w}\left(\frac{\tilde{\gamma}_+\left(1-\tilde{v}_+\right)m_\chi^{\rm in}/T_+ +1}{4\pi^2\tilde{\gamma}_+^3\left(1-\tilde{v}_+\right)^2}\right) e^{-\frac{\tilde{\gamma}_+\left(1-\tilde{v}_+\right)m_\chi^{\rm in}}{T_+}}$$

Hydrodynamic effects on filtered DM

bubble wall with non-zero width

$$\phi(z) = \frac{\phi(T_-)}{2}\left(1 + \tanh\frac{2z}{L_w}\right) .$$



Hydrodynamic effects on filtered DM

Numerical method: solving Boltamann equations

$$f_\chi = \mathcal{A}(z,p_z)\, f^{\text{eq}}_{\chi,+} = \mathcal{A}(z,p_z)\exp\left(-\frac{\tilde{\gamma}_+(E-\tilde{v}_+p_z)}{T_+}\right) ,$$

$$\mathbf{L}[f_\chi] = \mathbf{C}[f_\chi]$$

$$p_z^2 + m_\chi(z)^2 = \text{const.} \longrightarrow \mathbf{L}[f_\chi] = \frac{p_z}{E}\frac{\partial f_\chi}{\partial z} - \frac{m_\chi}{E}\frac{\partial m_\chi}{\partial z}\frac{\partial f_\chi}{\partial p_z} .$$

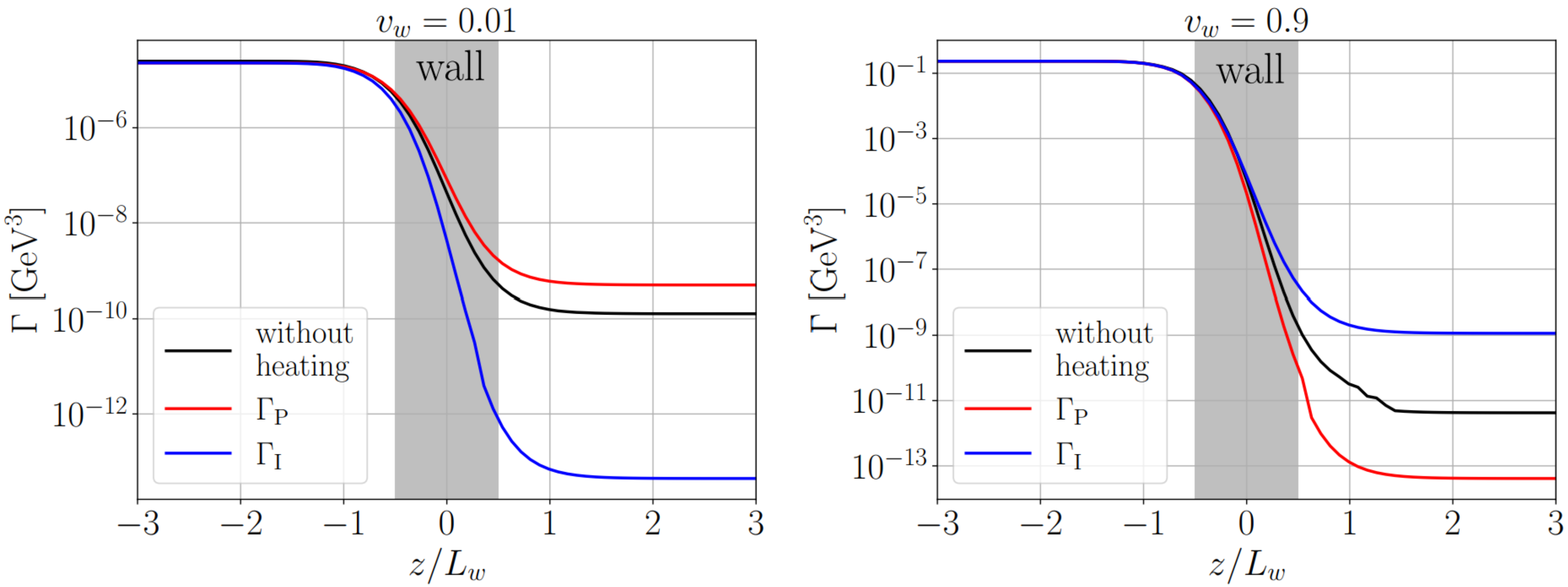
$$g_\chi \int \frac{dp_x dp_y}{(2\pi)^2}\mathbf{L}[f_\chi] \approx$$
$$\left[\left(\frac{p_z}{m_\chi}\frac{\partial}{\partial z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\partial}{\partial p_z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\tilde{\gamma}_+\tilde{v}_+}{T_+}\right)\mathcal{A}(z,p_z)\right]\frac{g_\chi m_\chi T_+}{2\pi\tilde{\gamma}_+}e^{\tilde{\gamma}_+\left(\tilde{v}_+p_z - \sqrt{m_\chi^2+p_z^2}\right)/T_+} .$$

$$\mathbf{C}[f_\chi] = -\sum_{\text{spins}} d\Pi_{q^p} d\Pi_{k^p} d\Pi_{l^p} \frac{(2\pi)^4}{2E_p^p}\delta^{(4)}\left(p^p + q^p - k^p - l^p\right)|\mathcal{M}|^2\mathcal{P}[f_\chi]$$

$$\mathcal{P}[f_\chi] = f_{\chi_p} f_{\bar{\chi}_q}\left(1 \pm f_{\phi_k}\right)\left(1 \pm f_{\phi_l}\right) - f_{\phi_k} f_{\phi_l}\left(1 \pm f_{\chi_p}\right)\left(1 \pm f_{\bar{\chi}_q}\right)$$

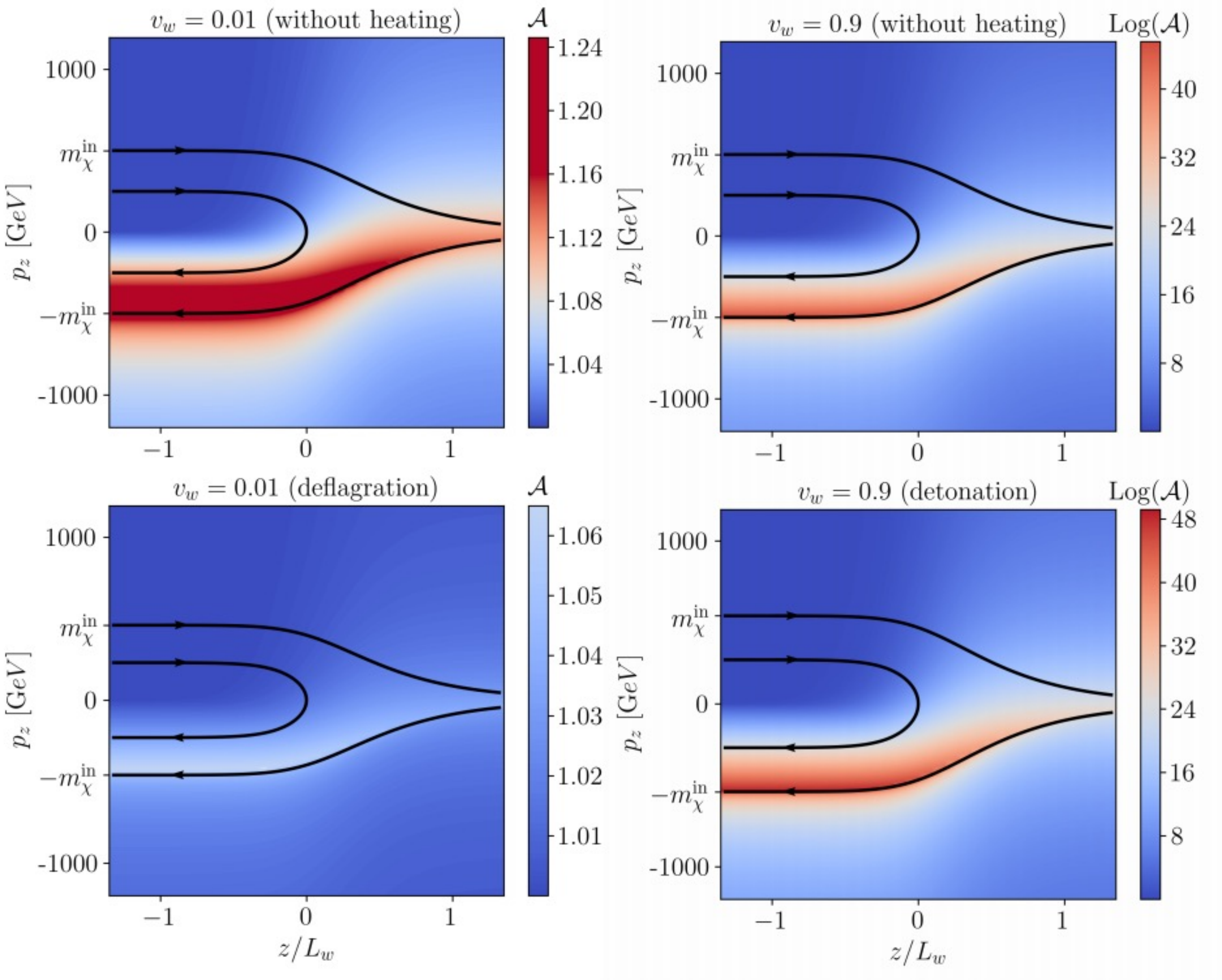
Hydrodynamic effects on filtered DM

$$
\begin{aligned}
g_\chi \int \frac{dp_x dp_y}{(2\pi)^2} \mathbf{C}\left[f_\chi\right] &= -g_\chi g_{\bar\chi} \int \frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}} d\Pi_{q^{\mathcal{P}}} 4F\sigma_{\chi\bar\chi\to\phi\phi}\left[f_{\chi_p} f^{\rm eq}_{\bar\chi_q,+} - f^{\rm eq}_{\chi_p} f^{\rm eq}_{\bar\chi_q}\right] \\
&= -g_\chi g_{\bar\chi} \int \frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}} d\Pi_{q^{\mathcal{P}}} 4F\sigma_{\chi\bar\chi\to\phi\phi}\left[\mathcal{A} f^{\rm eq}_{\chi_p,+} f^{\rm eq}_{\bar\chi_q,+} - f^{\rm eq}_{\chi_p} f^{\rm eq}_{\bar\chi_q}\right] \\
&\equiv \Gamma_{\rm P}(z,p_z)\mathcal{A}\left(z,p_z\right) - \Gamma_{\rm I}(z,p_z)\ ,
\end{aligned}
$$



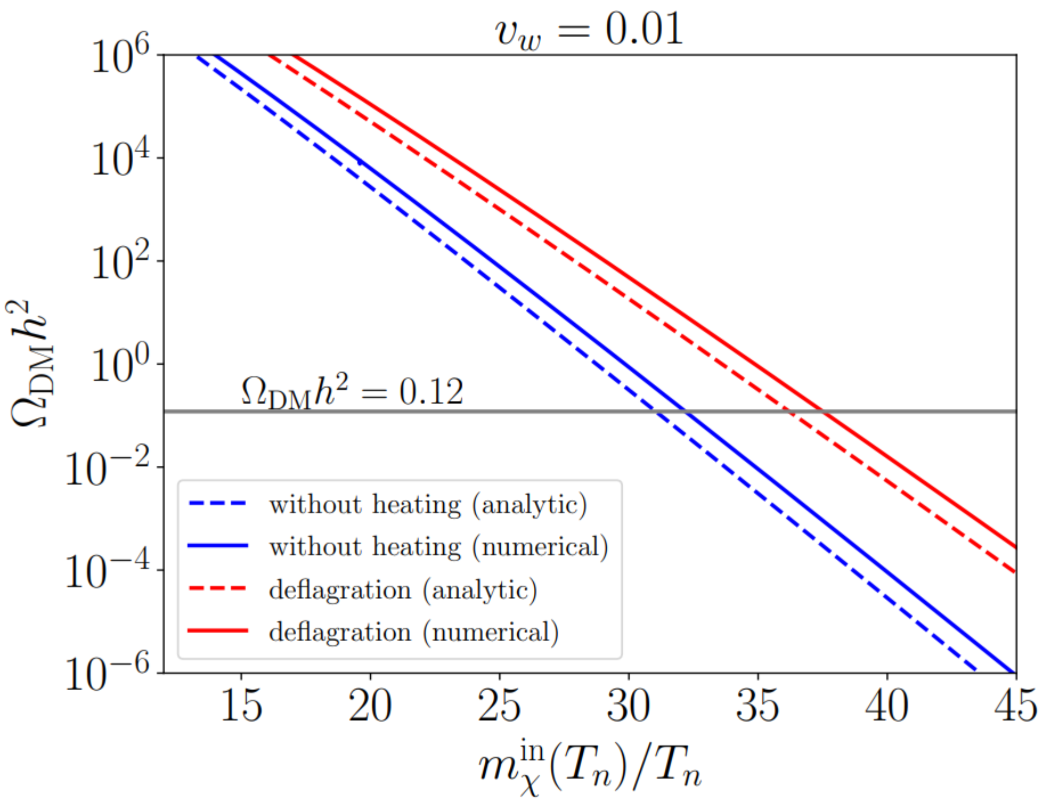
Hydrodynamic effects on filtered DM

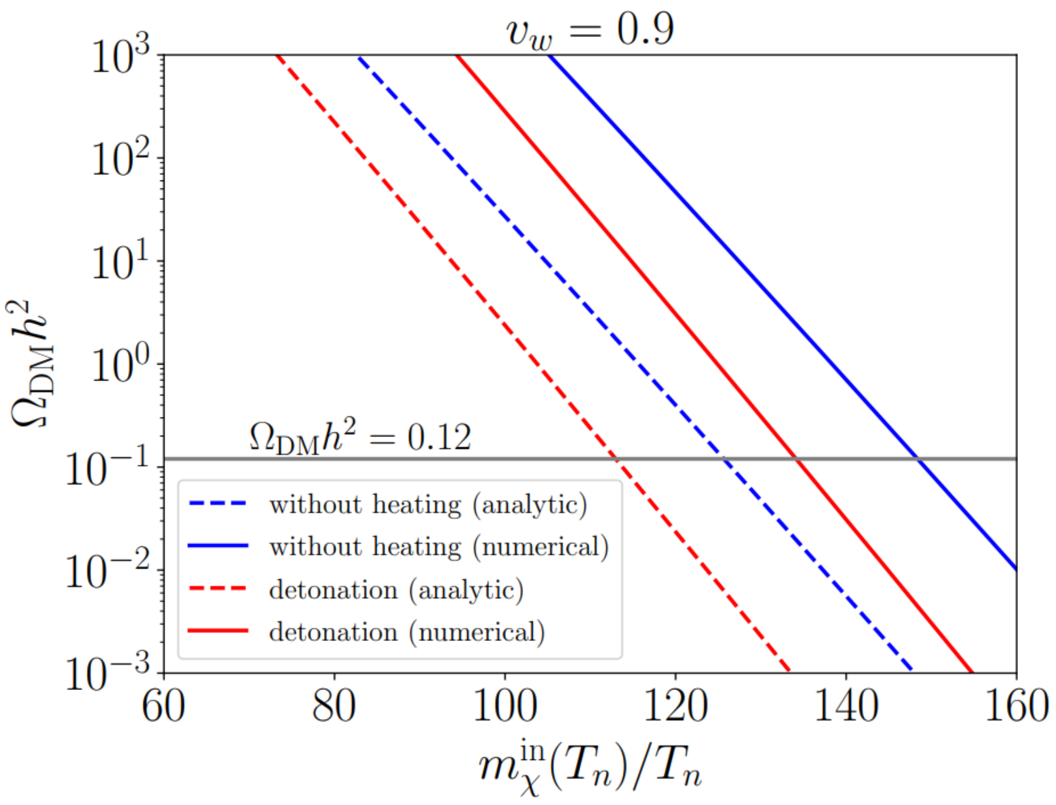
$$f_\chi = \mathcal{A}(z^w, p_z^w)\exp\left(-\frac{E^p}{T}\right) .$$



Hydrodynamic effects on filtered DM

$$n_\chi^{\text{in}} = \frac{T_+}{\gamma_w \tilde{\gamma}_+} \int_0^\infty \frac{dp_z}{(2\pi)^2} \mathcal{A}(z \gg L_w, p_z) \exp\left[\tilde{\gamma}_+ \left(\tilde{v}_+ p_z - \sqrt{p_z^2 + (m_\chi^{\text{in}})^2}\right)/T_+\right] \left(\sqrt{p_z^2 + (m_\chi^{\text{in}})^2} + \frac{T_+}{\tilde{\gamma}_+}\right)$$





Hydrodynamic effects on filtered DM

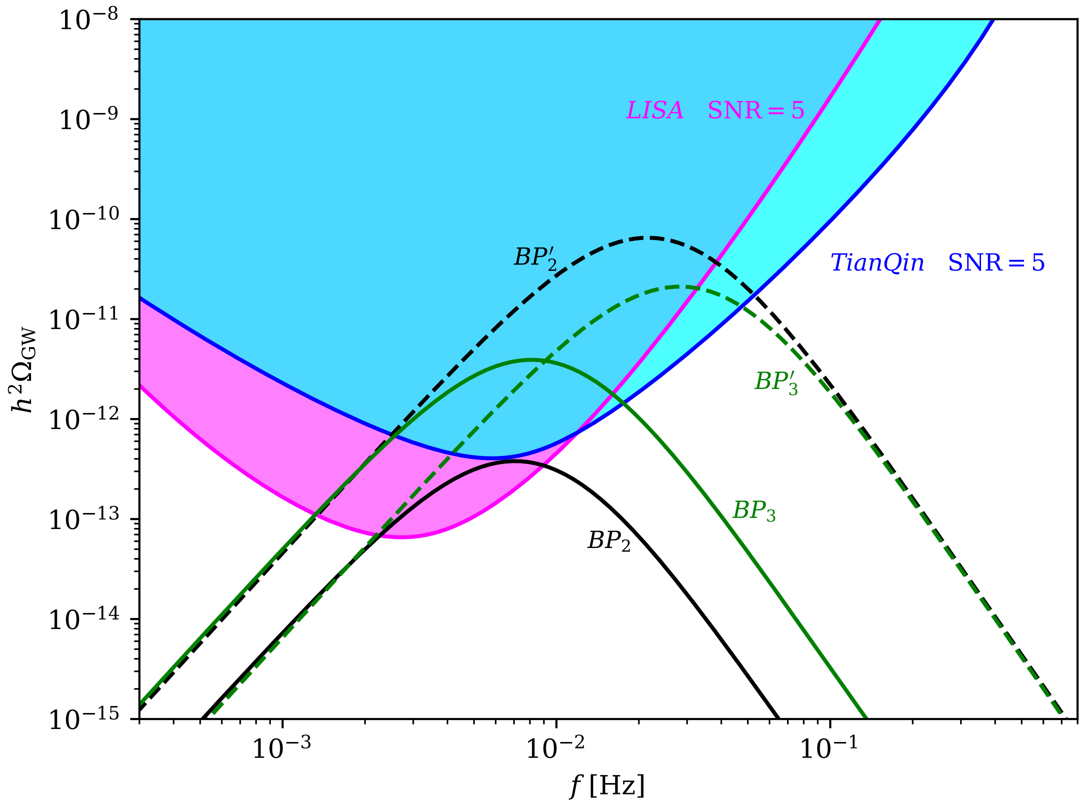
Ratio of DM relic density with and without hydrodynamic effects

$v_w = 0.01$

	analytic		numerical	
	$m_\chi^{in}(T_n)/T_n$	$\Omega_{DM}^{(hy)}h^2/\Omega_{DM}^{(0)}h^2$	$m_\chi^{in}(T_n)/T_n$	$\Omega_{DM}^{(hy)}h^2/\Omega_{DM}^{(0)}h^2$
BP_1	31	66	32	71
BP_2	31.1	7.9	32.2	8.1
BP_3	30.8	778.8	31.9	858.5
BP_4	$\star$	$\star$	$\star$	$\star$

$v_w = 0.9$

	analytic		numerical	
	$m_\chi^{in}(T_n)/T_n$	$\Omega_{DM}^{(hy)}h^2/\Omega_{DM}^{(0)}h^2$	$m_\chi^{in}(T_n)/T_n$	$\Omega_{DM}^{(hy)}h^2/\Omega_{DM}^{(0)}h^2$
BP_1	125.3	1/19	147.8	1/27
BP_2	125.9	1/7	148.7	1/9
BP_3	124.6	1/10	147.3	1/12
BP_4	123.8	$1/(1.2\times10^{13})$	146.5	$1/(2.2\times10^{15})$



Siyu Jiang, FPH, Chong Sheng Li, arXiv:2305.02218

Summary and outlook

- **The correlation between GW and collider signals at CEPC can make complementary test on the Higgs nature, baryogenesis, dark matter and the cosmic evolution history at 100 GeV.**

- **More precise study on the phase transition dynamics are essential to understand the role of Higgs in the the early universe: reliable resummation, non-perturbation, bubble wall velocity, energy budget…**

Thanks! Comments and collaborations are welcome!

Email:huangfp8@sysu.edu.cn