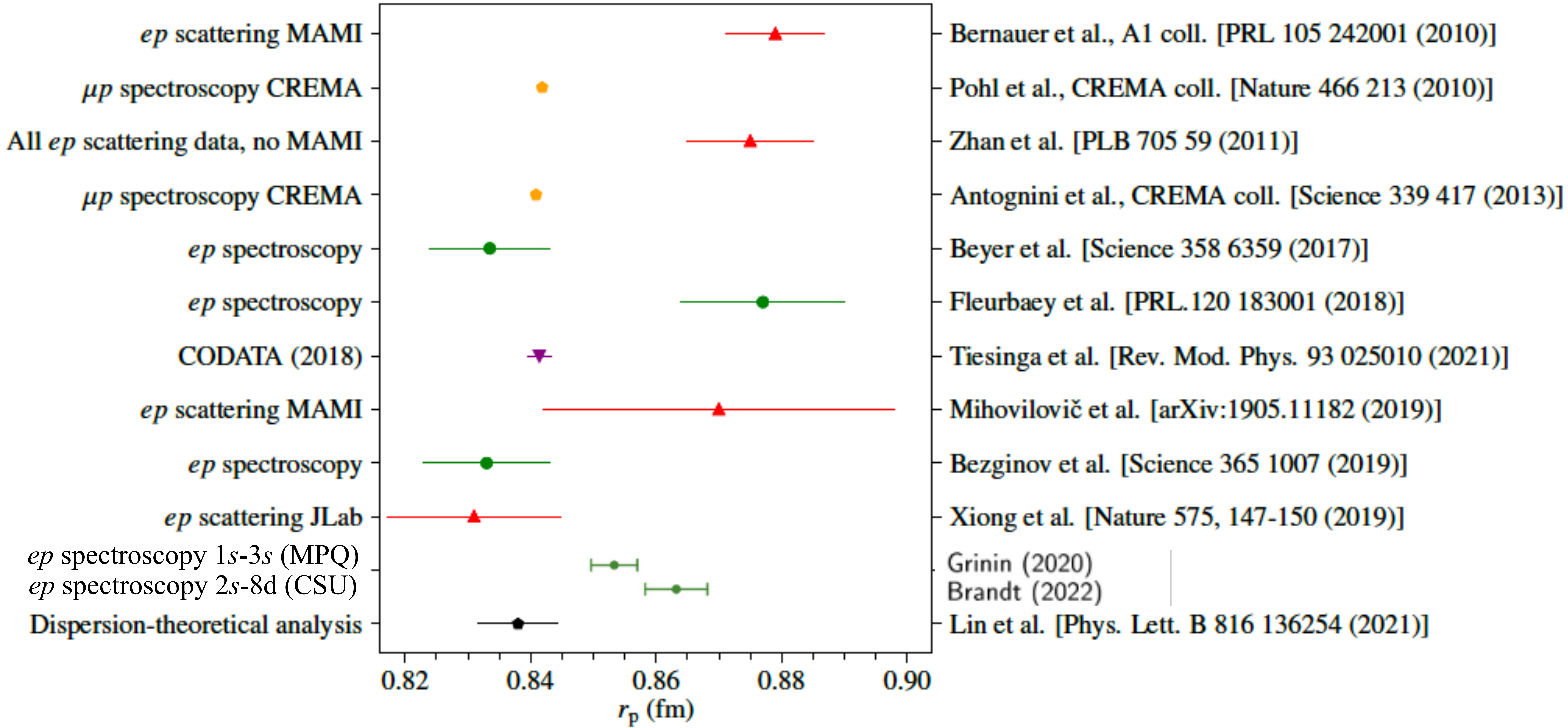
AMBER
Apparatus for Meson and Baryon
Experimental Research

# Measuring Hadron Charge Radii with AMBER

Stephan Paul
Technical University Munich and Max-Planck-Institute for Physics Munich

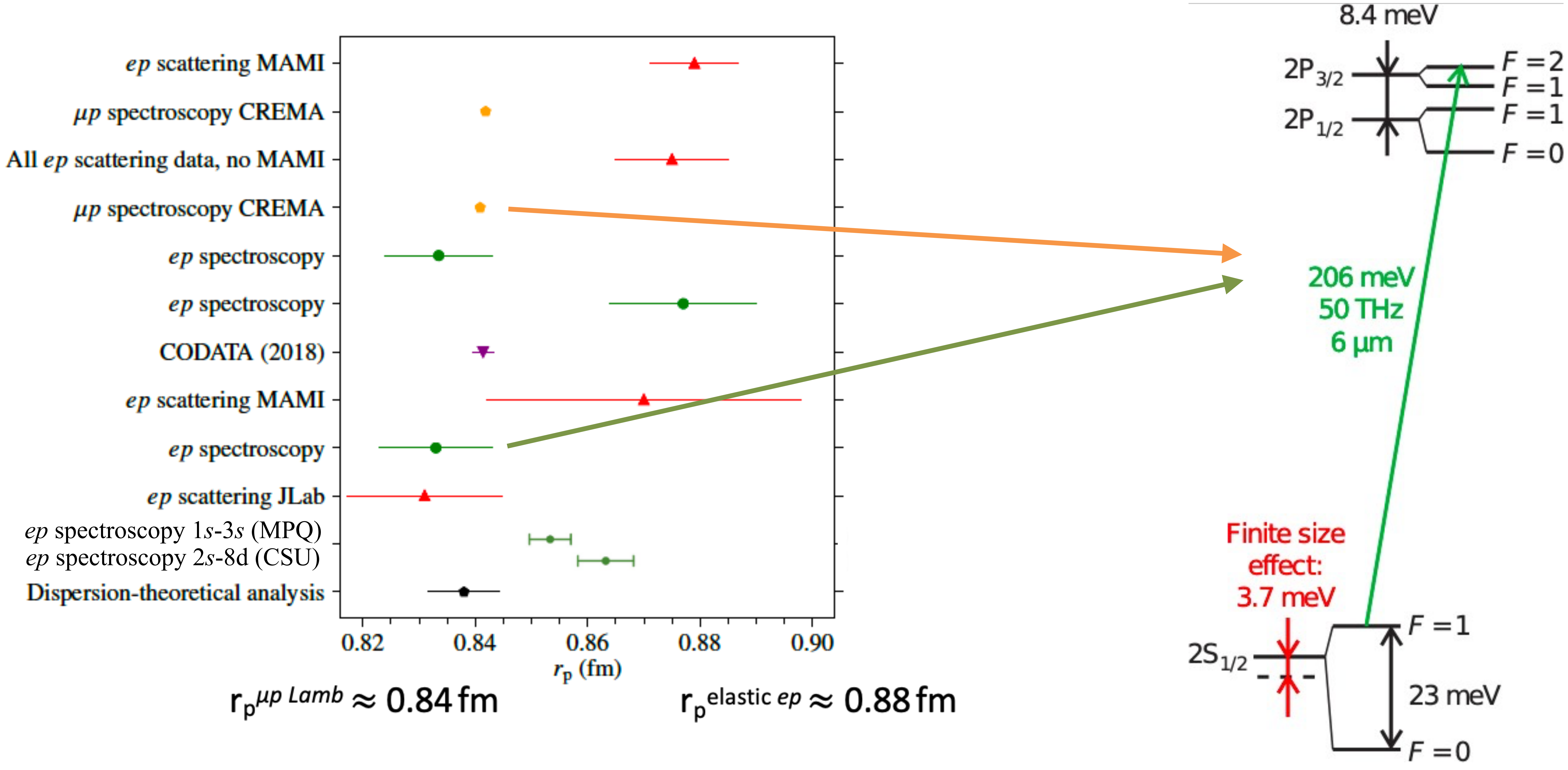
August 2023
Quy Nhon - Vietnam

# Proton Radius Measurements



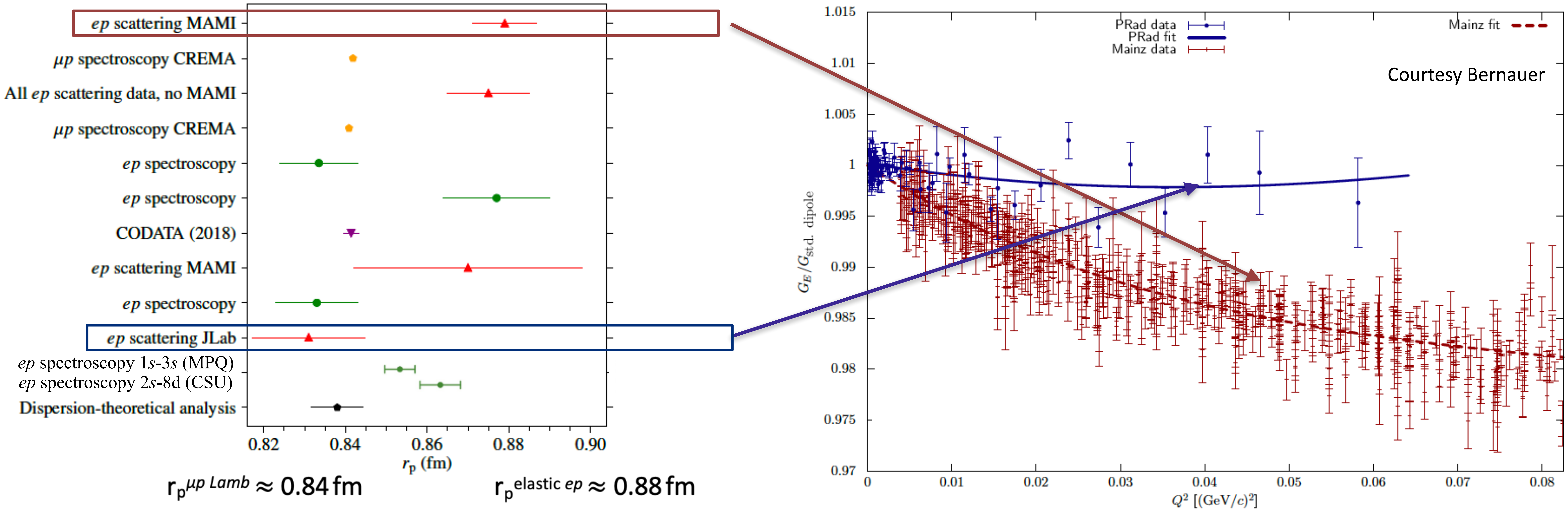
$r_p^{\mu p\ Lamb} \approx 0.84\,\text{fm}$ $r_p^{elastic\ ep} \approx 0.88\,\text{fm}$

# Proton Radius Measurements



$r_p^{\mu p\ Lamb} \approx 0.84\,\text{fm}$ $r_p^{\text{elastic}\ ep} \approx 0.88\,\text{fm}$

# Proton Radius Measurements



Courtesy Bernauer

$r_p^{\mu p\ Lamb} \approx 0.84$ fm

$r_p^{\text{elastic } ep} \approx 0.88$ fm

# Alternative techniques

- MUSE: low energy $\mu$ and e beams of both polarities
- COMPASS: high energy $\mu$ beams of both polarities ( x 500 beam energy of MUSE!!)
  - beam energy irrelevant.. $Q^2$ is important variable (see details later)
  - COMPASS has demonstrated excellent $Q^2$ resolution with Primakoff reactions
  - Coulomb peak from $\pi A$ scattering $\pi + Z \rightarrow \pi + \gamma + Z_{recoil}$ - $\Delta Q^2 \approx 5 \times 10^{-4} (GeV/c)^2$
  - well performing spectrometer and well understood apparatus

……………….

# Proposal of a New Measurement

$$< r_p^2 > = -6\hbar^2 \cdot \frac{dG_E(Q^2)}{dQ^2}$$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} R \left(\varepsilon G_E^2 + \tau G_M^2\right)$$

J. C. Bernauer

AMBER
Apparatus for Meson and Baryon
Experimental Research

# Proposal of a New Measurement

$$< r_p^2 > = -6\hbar^2 \cdot \frac{dG_E(Q^2)}{dQ^2}$$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} R \left(\varepsilon G_E^2 + \tau G_M^2\right)$$

- Measure close to $Q^2 \rightarrow 0$
  → suppress influences from higher order terms (fit)
  → high-energy $\mathcal{O}$(10 - 100 GeV) — Cross-section $\propto (G_E^P(Q^2))^2$
- Sufficient $Q^2$ range to determine radius:
  → Aimed precision better 1 %
  → Aimed $Q^2$-range: 0.001 - 0.04 $(\mathrm{GeV}/c)^2$

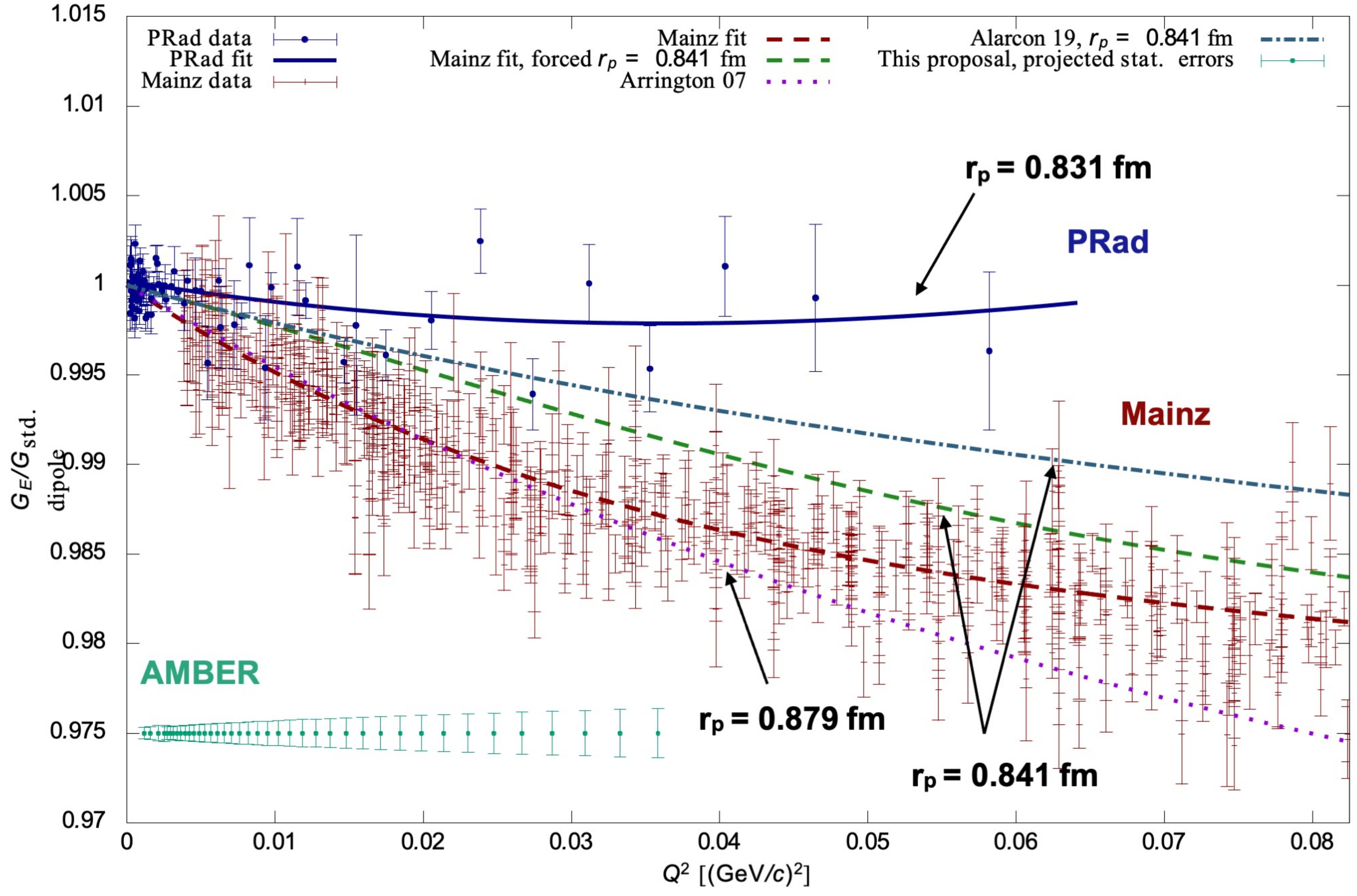
J. C. Bernauer

# Proposal of a New Measurement

$$< r_p^2 > = -6\hbar^2 \cdot \frac{dG_E(Q^2)}{dQ^2}$$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} R \left(\varepsilon G_E^2 + \tau G_M^2\right)$$

- Measure close to $Q^2 \rightarrow 0$
  → suppress influences from higher order terms (fit)
  → high-energy $\mathcal{O}(10 - 100\text{ GeV})$ — Cross-section $\propto (G_E^P(Q^2))$
- Sufficient $Q^2$ range to determine radius:
  → Aimed precision better 1 %
  → Aimed $Q^2$-range: 0.001 - 0.04 $(\text{GeV}/c)^2$
- Below $Q^2 = 0.001\text{ GeV}^2/c^2$:
  → Deviation from point-like proton level of $\mathcal{O}(10^{-3})$
  → systematic effects e.g. $Q^2$ resolution
- Above $Q^2 = 0.04\ (\text{GeV}/c)^2$
  → Non-linearity of the cross section
  → Predominant source of uncertainty



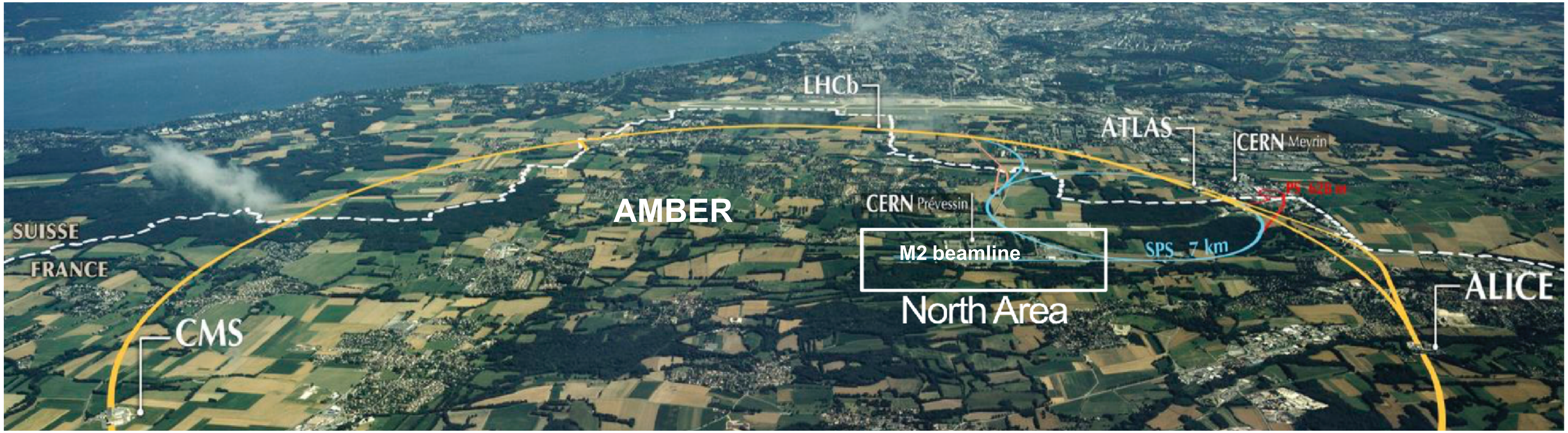
J. C. Bernauer

J. C. Bernauer

# Beamline for High-Energy Muon Beams

## M2 beamline at *CERN's SPS*

North Area of *CERN* :  M2 beamline provides a unique high-intensity muon beam



- Muon momenta up to 200 GeV/*c*  - flux up to $10^7$ μ/s
- PRM: beam momentum of 100 GeV/*c* and 2 MHz beam rate

- *AMBER* as successor at *COMPASS* location starting 2023 with the first full PRM pilot run in 10/2023

→ broad physics program: PRM, Drell-Yan, Anti-Proton Cross-Section, use RF separated beams (plan)

AMBER
Apparatus for Meson and Baryon Experimental Research

# The AMBER μP measurement

Choose scattering of high energy muons of gaseous hydrogen

👍 high energy muons have little multiple scattering - good measurement of scattering angle

👍 high energy muons do not radiate (little)

👎 muon energy loss very small - basically no useable information from muon momentum

⇒ need to measure recoil proton

👎 low energy recoil protons carry information about $Q^2$

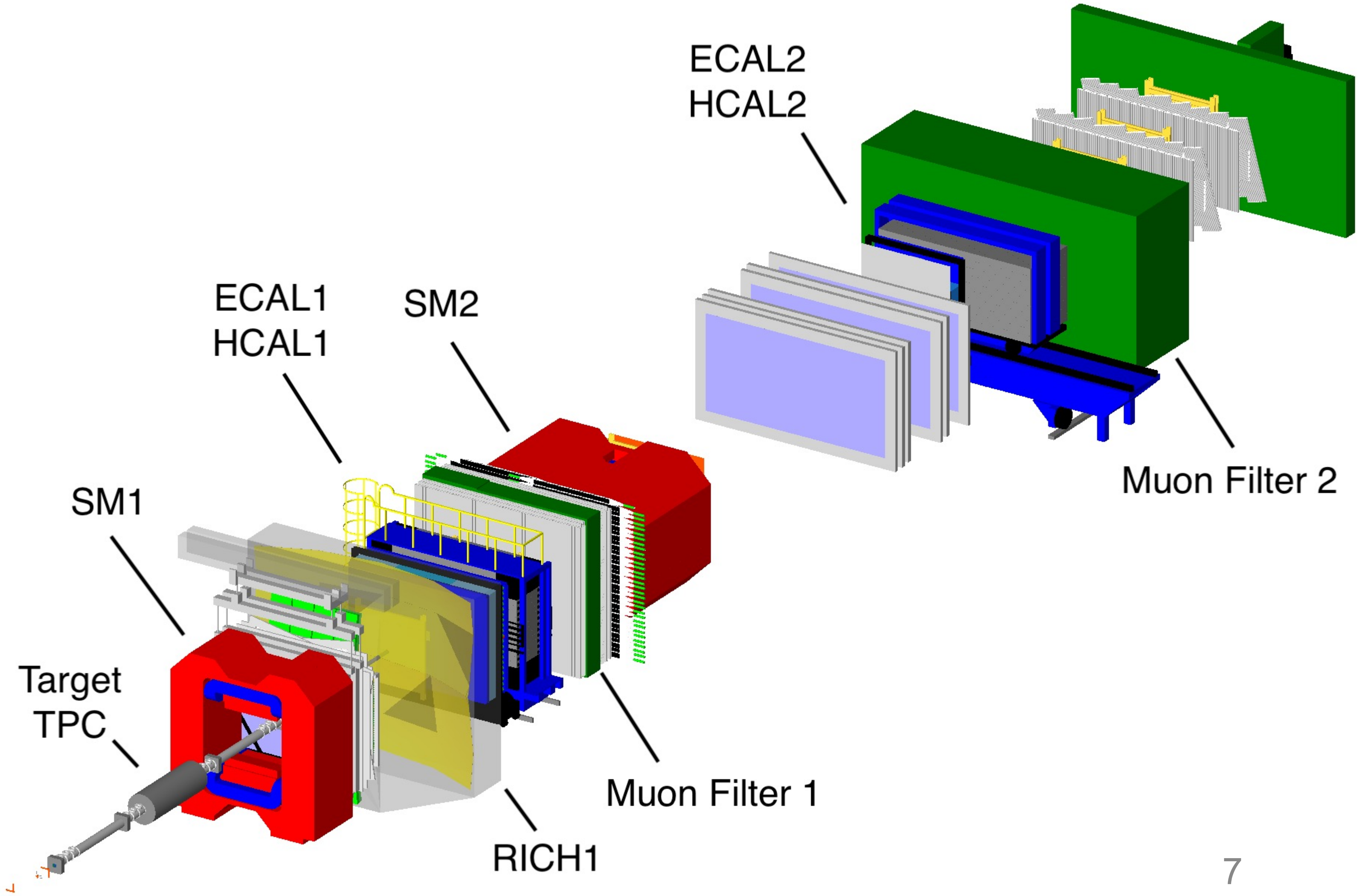
⇒ measure their energy via an active target

👍 keep the advantages and circumvent the disadvantage by excellent instrumentation

Apparatus for Meson and Baryon Experimental Research

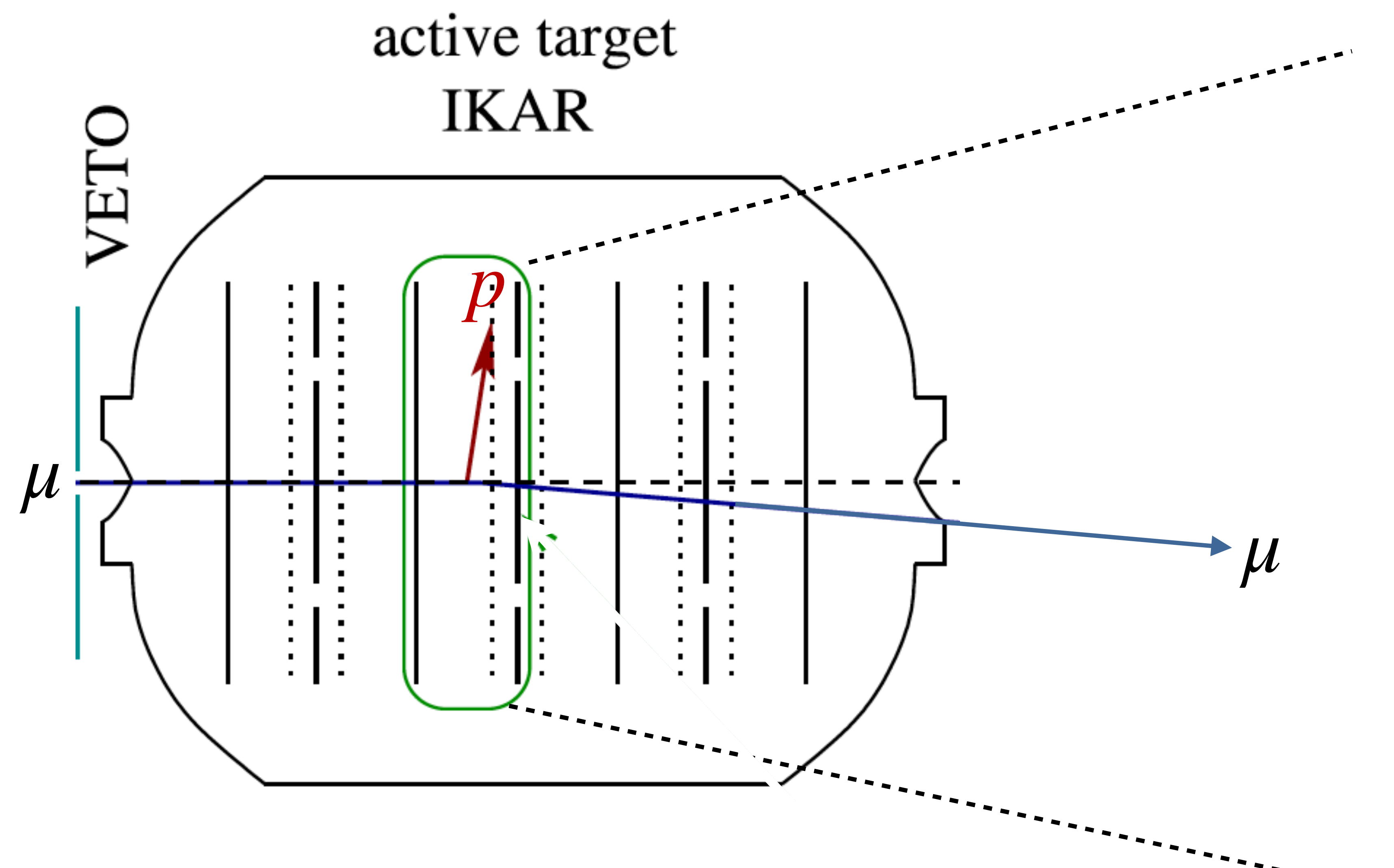
# Proton Radius from Muon-Proton Elastic Scattering at High Energy

- 100 GeV muon beam
- Active-target TPC with high-pressure $H_2$
- goal: 70 million elastic scattering events in the range $10^{-3} < Q^2 < 4 \cdot 10^{-2}\ (GeV/c)^2$
- Precision on the proton radius ~0.01 fm

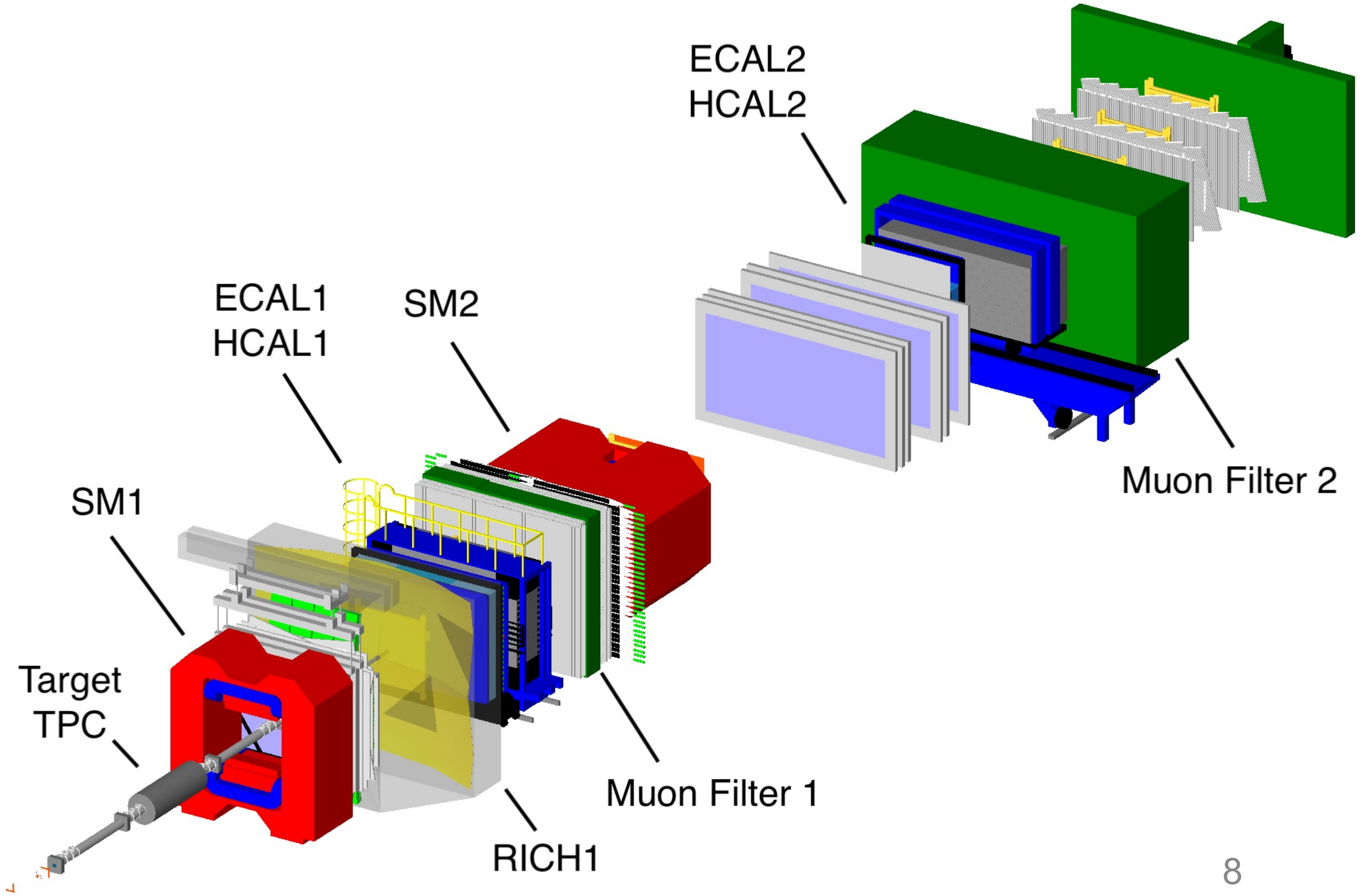


# Proton Radius from Muon-Proton Elastic Scattering at High Energy

- 100 GeV muon beam
- Active-target TPC with high-pressure $H_2$
- goal: 70 million elastic scattering events in the range $10^{-3} < Q^2 < 4 \cdot 10^{-2}\ (GeV/c)^2$
- Precision on the proton radius ~0.01 fm

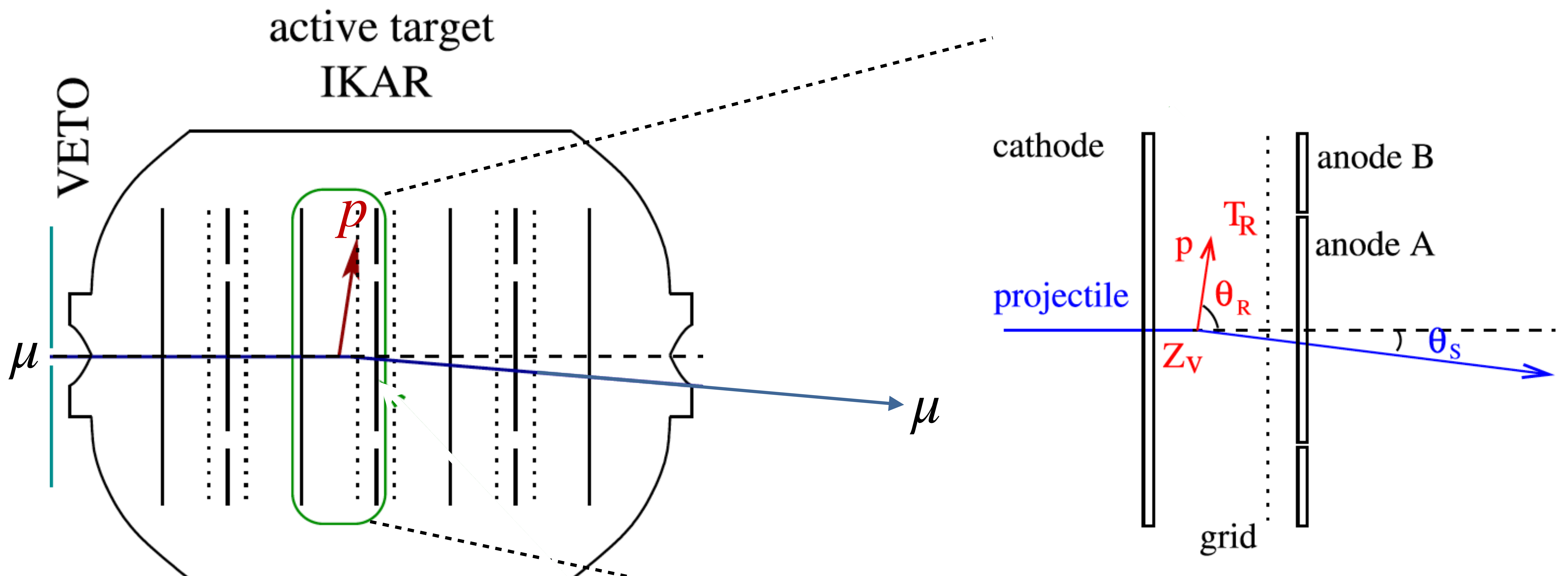


High Pressure Hydrogen - 4-20 bar



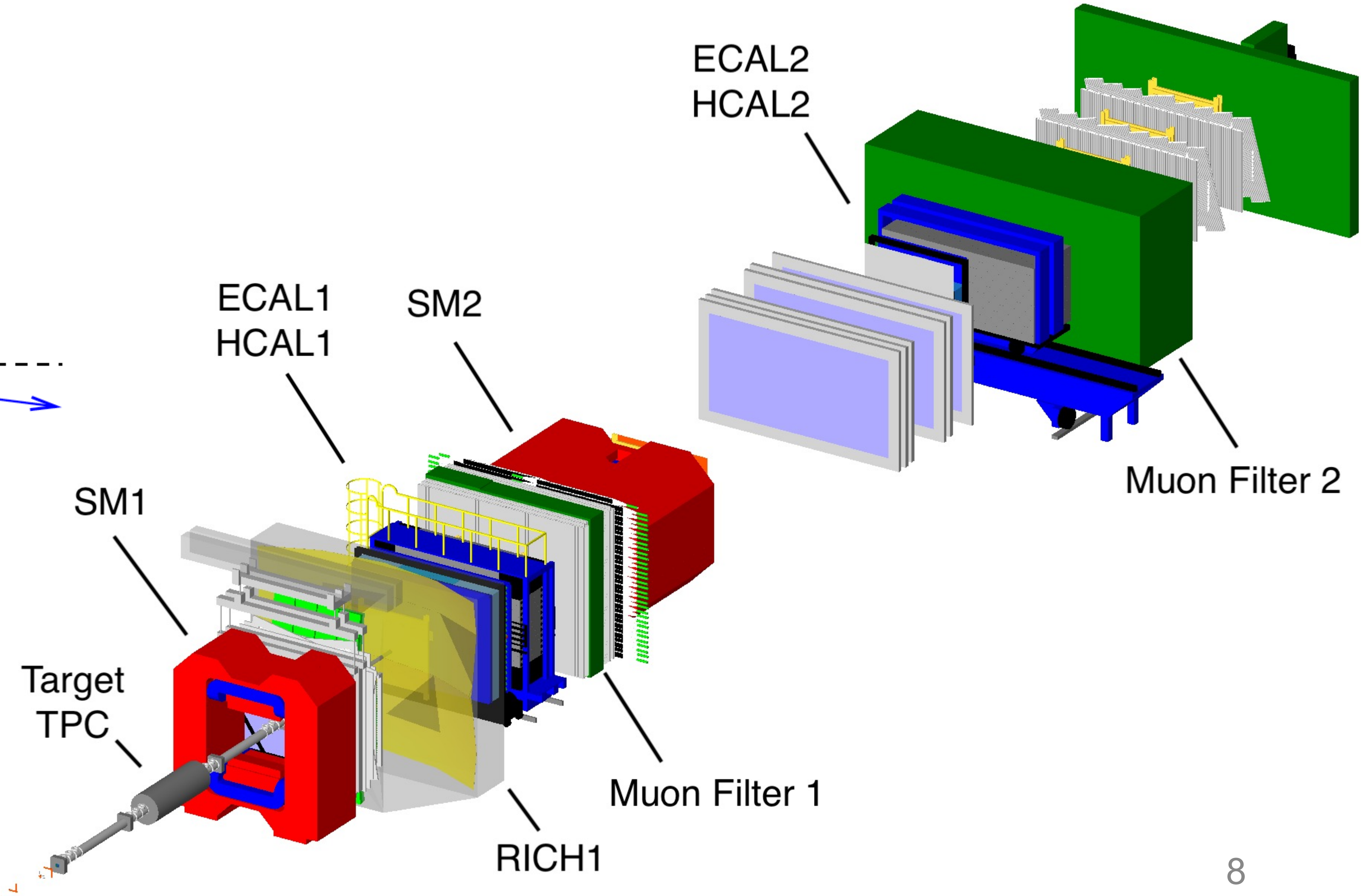
# Proton Radius from Muon-Proton Elastic Scattering at High Energy

- 100 GeV muon beam
- Active-target TPC with high-pressure $H_2$
- goal: 70 million elastic scattering events in the range $10^{-3} < Q^2 < 4 \cdot 10^{-2}\ (GeV/c)^2$
- Precision on the proton radius ~0.01 fm



High Pressure Hydrogen - 4-20 bar

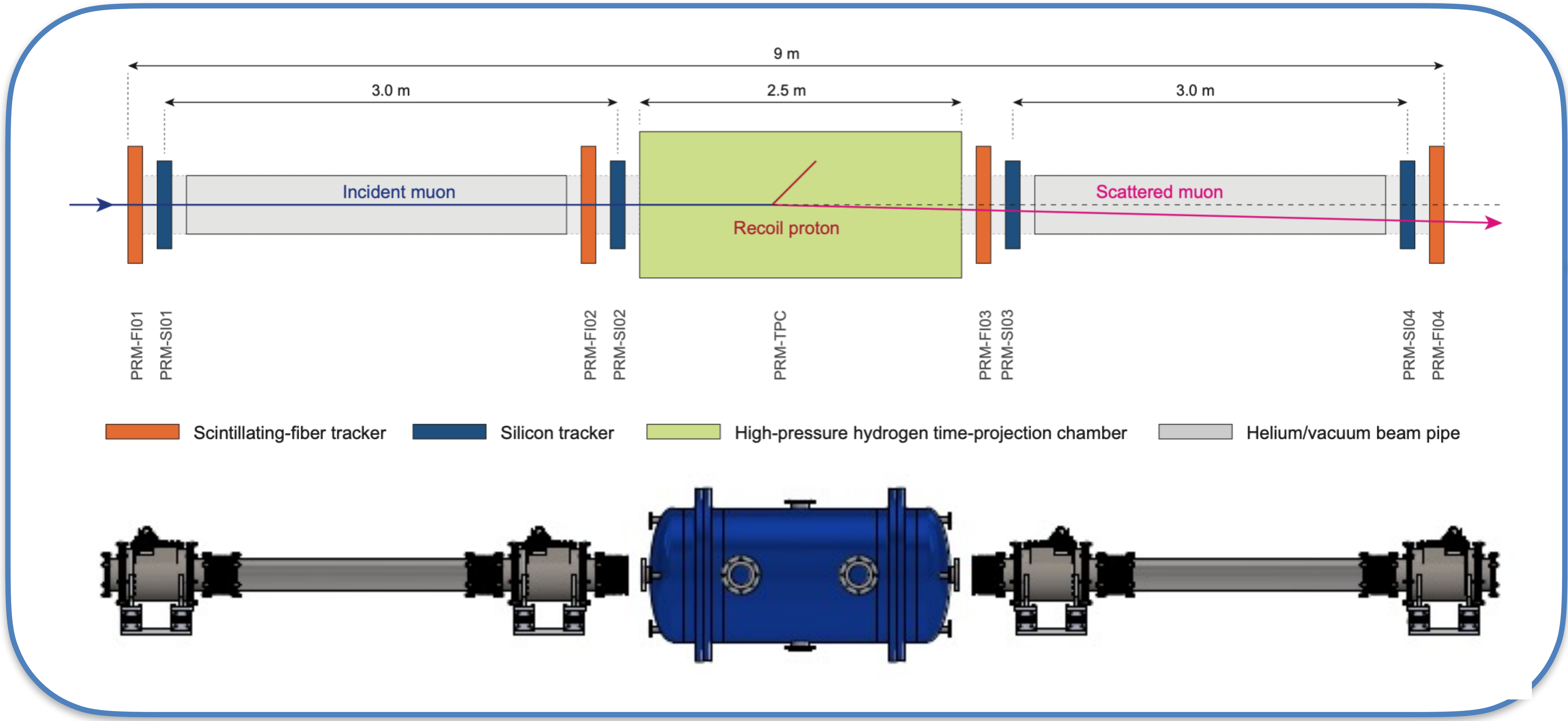
ionisation **TPC**

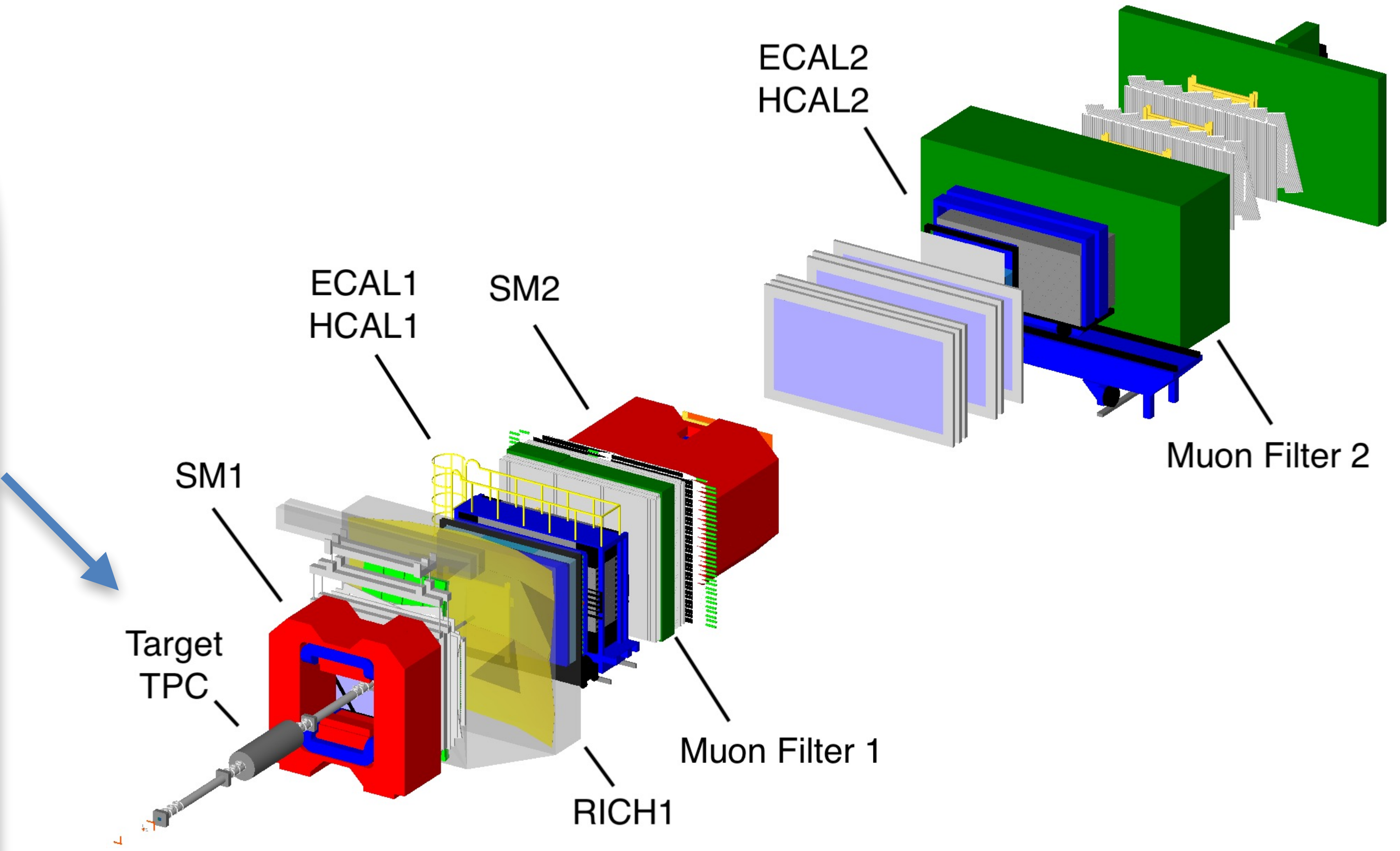


Apparatus for Meson and Baryon
Experimental Research

# Proton Radius from
# Muon-Proton Elastic Scattering at High Energy

- 100 GeV muon beam
- Active-target TPC with high-pressure $H_2$
- goal: 70 million elastic scattering events in the range $10^{-3} < Q^2 < 4 \cdot 10^{-2}\ (GeV/c)^2$
- Precision on the proton radius ~0.01 fm



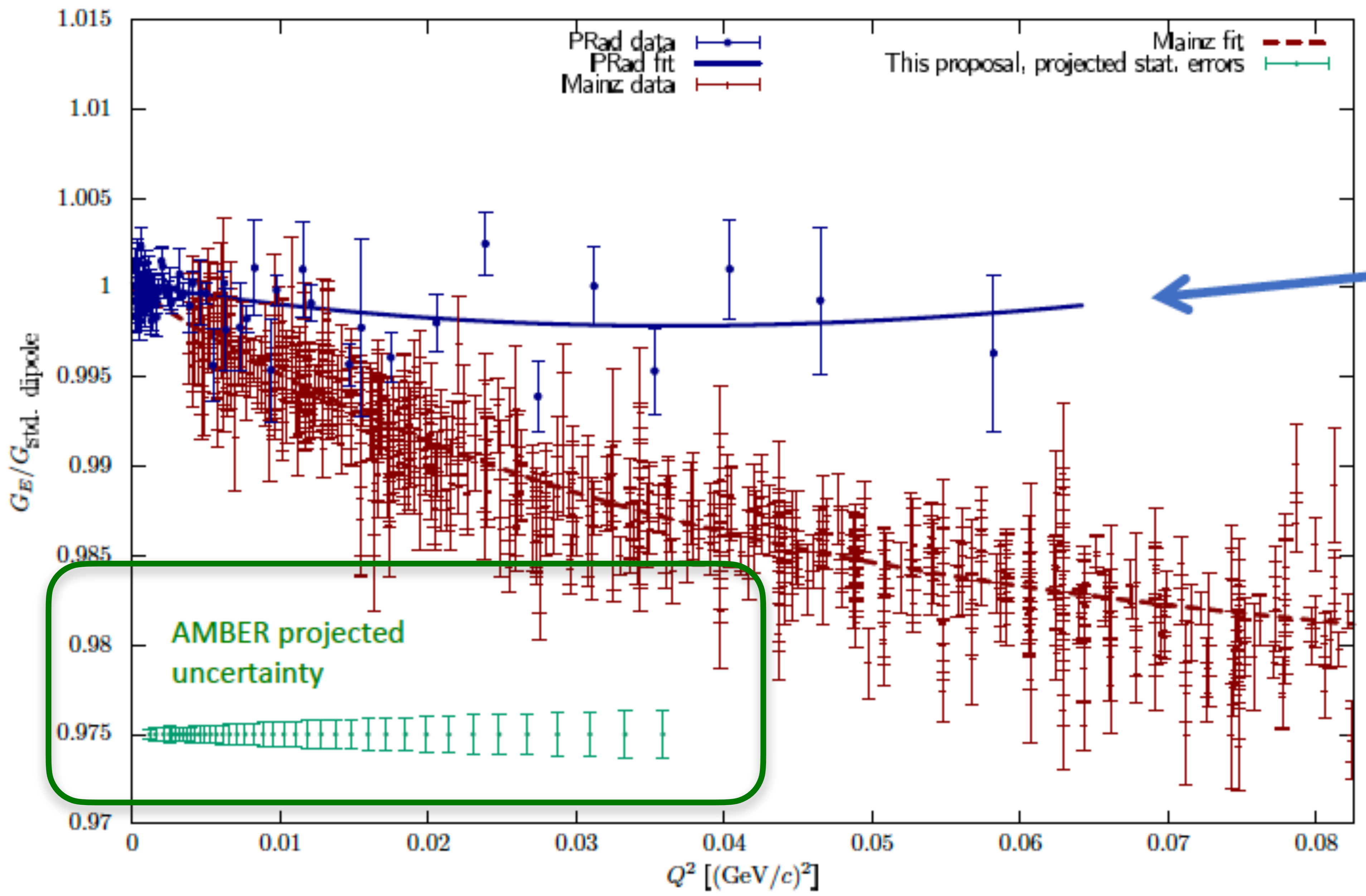


# Summary and Outlook

## High-energy elastic muon-proton scattering — PRM@*AMBER*

## Ongoing Preparations - promising developments

- New approach - elastic muon-protons scattering at $E_\mu = 100$ GeV
  - → Redundant measurement to control systematic effects
  - → Radiative corrections (factor 5-10) smaller compared to electron-proton scattering
  - → Additional dataset to contribute to a solution of the puzzle



Proton Radius Experiment at Jefferson Lab

PRoton Radius

MAMI
MAINZER MIKROTRON

*figure: J. Bernauer*

### Time schedule

→ New detector systems with novel triggerless DAQ — many beam tests (2019-2023)

→ Physics physics runs foreseen 2024 - 2025

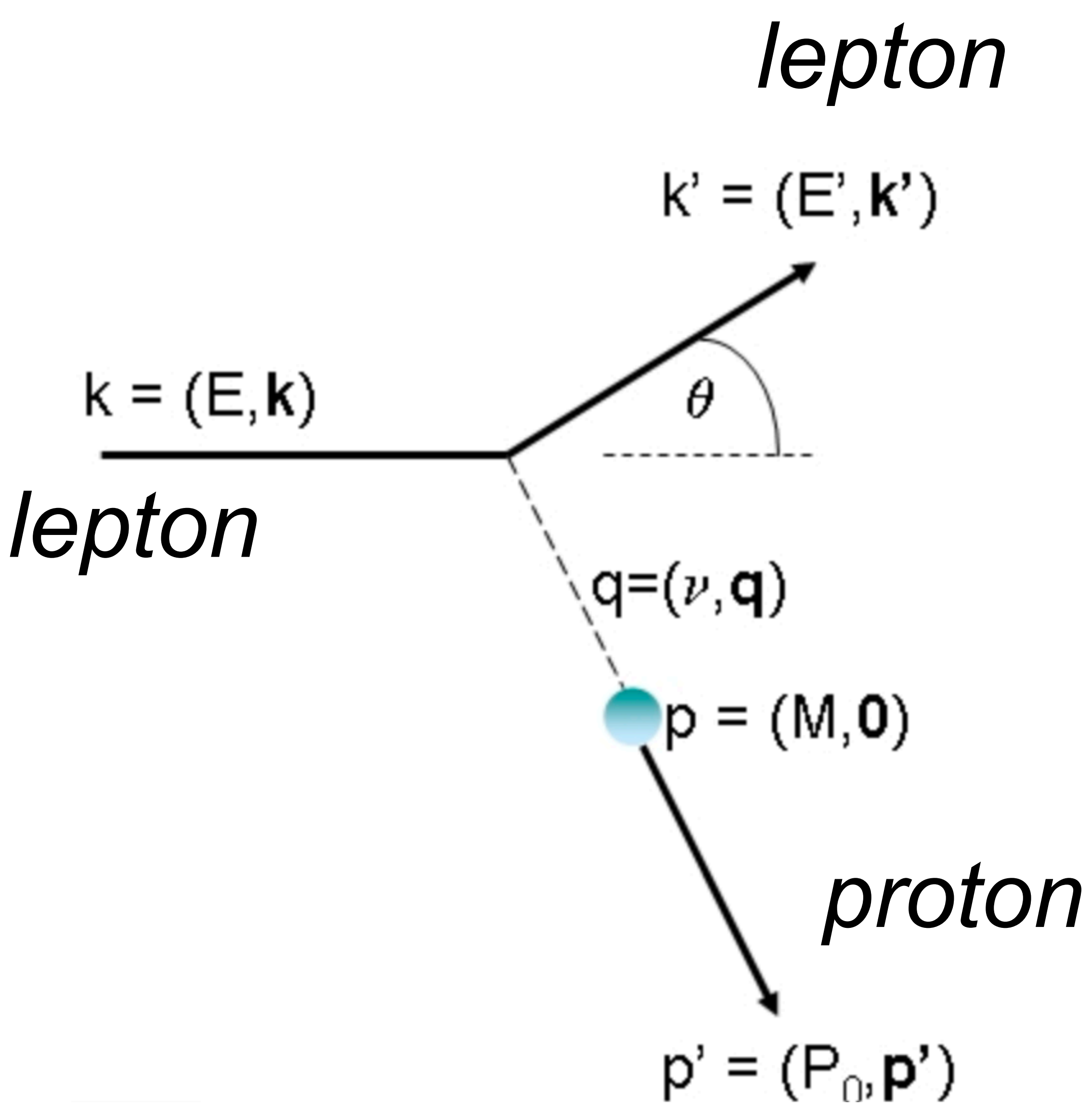
# Hadron Charge Radii
# Through Elastic Hadron-Lepton Scattering at low $Q^2$

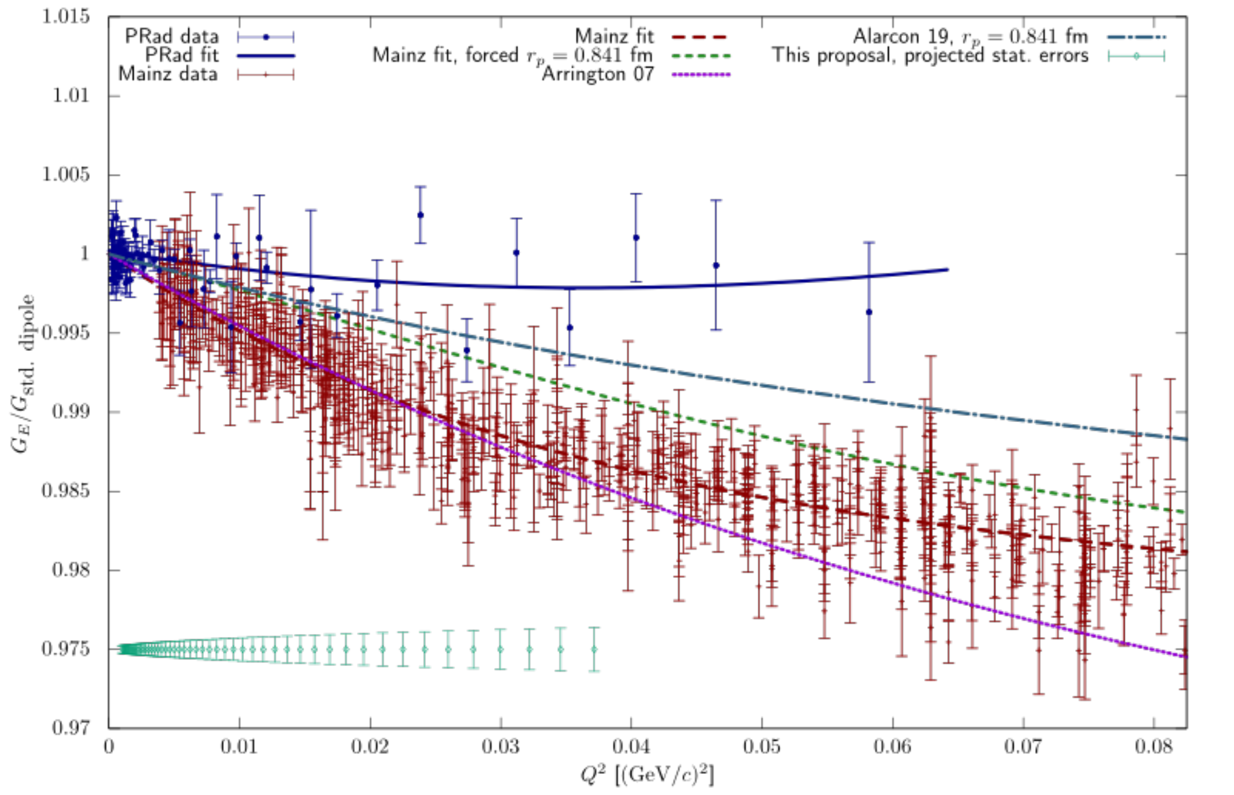
Protons in hydrogen target (or other stable nuclei):
Measurement via elastic electron or muon scattering
Cross section:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} R \left(\varepsilon G_E^2 + \tau G_M^2\right)$$

Charge radius from the slope of $G_E$

$$\langle r_E^2 \rangle = -6\hbar^2 \left.\frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2}\right|_{Q^2\to 0}$$





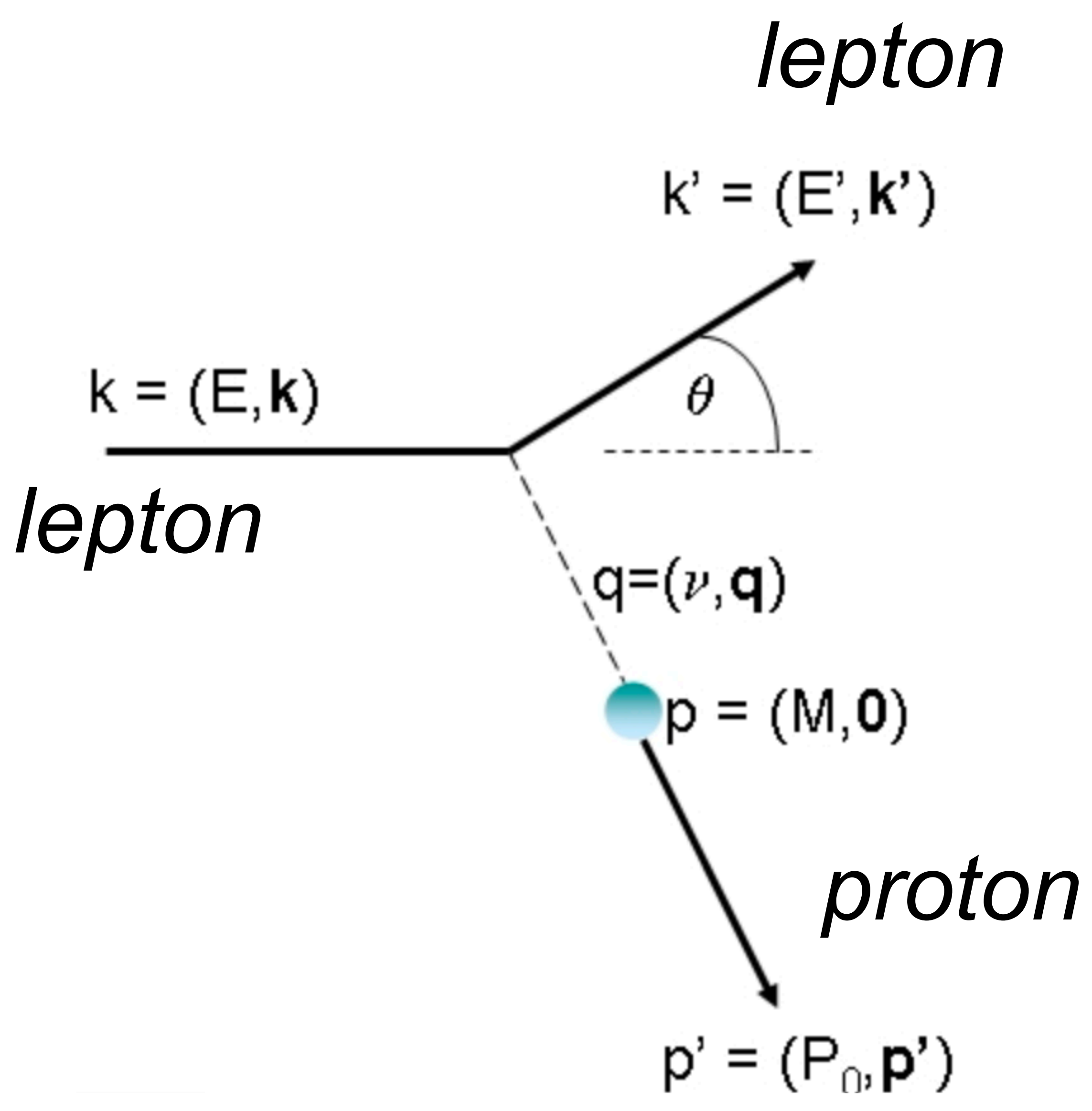
# Hadron Charge Radii Through Elastic Hadron-Lepton Scattering at low $Q^2$

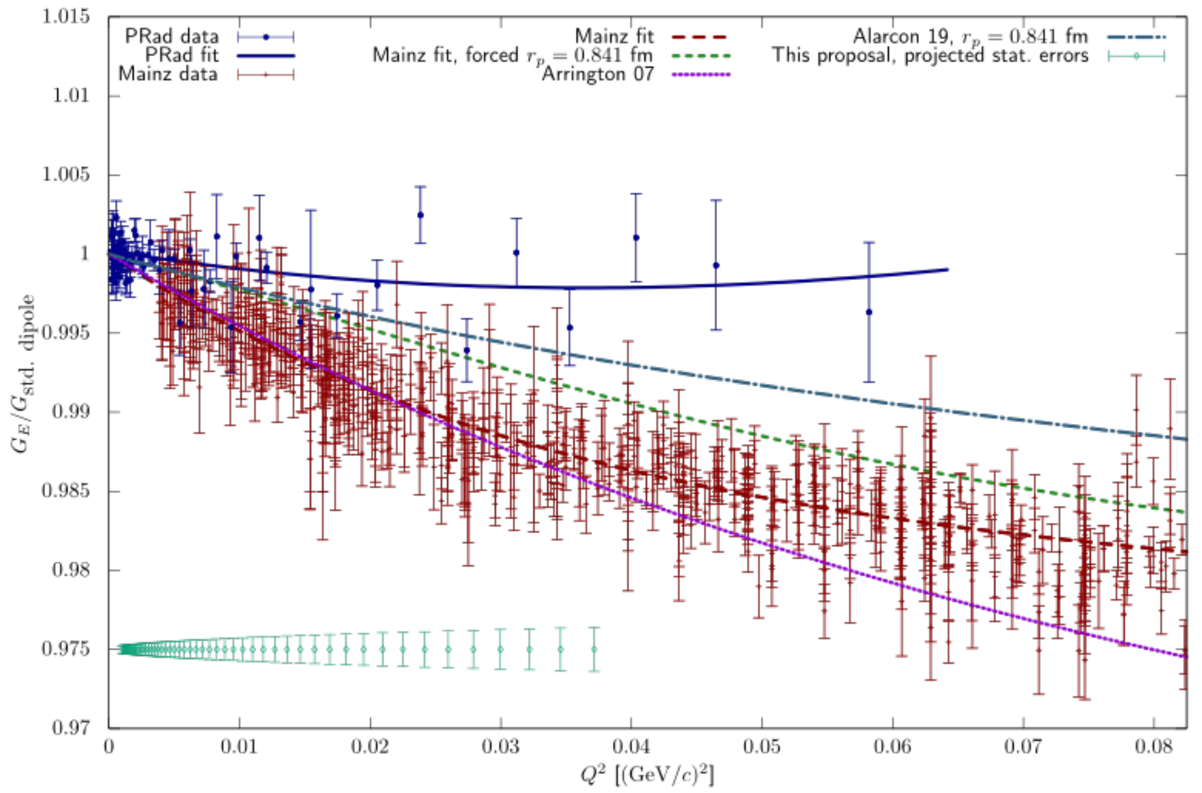
Protons in hydrogen target (or other stable nuclei):

Measurement via elastic electron or muon scattering

Cross section:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} R \left(\varepsilon G_E^2 + \tau G_M^2\right)$$

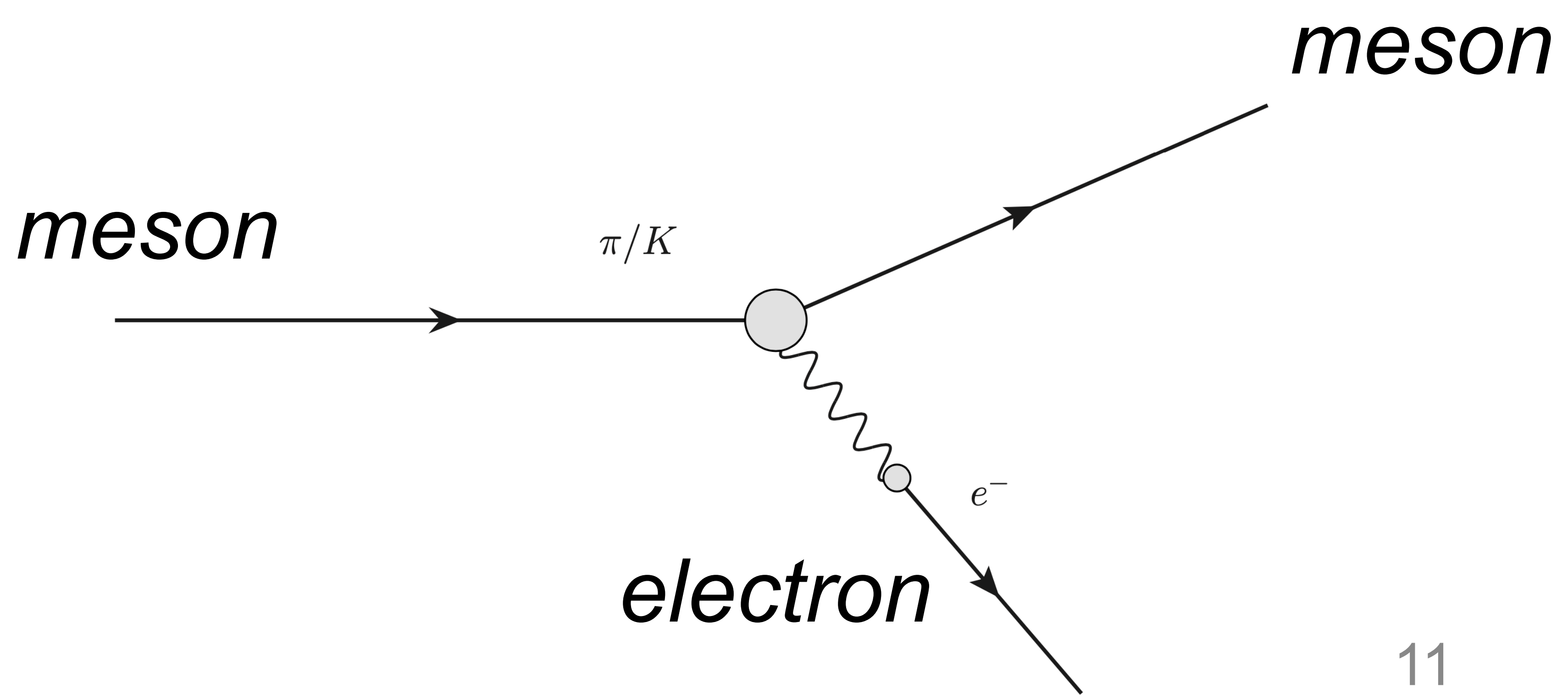




Charge radius from the slope of $G_E$

$$\langle r_E^2 \rangle = -6\hbar^2 \left.\frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2}\right|_{Q^2 \to 0}$$

For unstable particles, electron scattering can only be realised in *inverse kinematics*



# Hadron Radius Measurements

From: EPJC 8 (**1999**) 59, The WA89 Collaboration (measurement of $\Sigma^-$ charge radius)    updated 21.6.2022

Measured $\langle r^2_{ch} \rangle$ in $fm^2$ of various hadrons

|  | Experiment |  | experiment<br>year |
|---|---|---|---|
| p | $\approx 0.84 - 0.87$ |  | 2023 |
| $\overline{p}$ |  | unmeasured |  |
| n | $-0.1101 \pm 0.0086$ |  | 2021 |
| $\Sigma^-$ | $0.61 \pm 0.12 \pm 0.09$ |  | 2001 |
| $\pi^-$ | $0.439 \pm 0.008$ [5] |  | 1986 |
| $K^-$ | $0.34 \pm 0.02$ [6] |  | 1986 |
| $K^0_L$ | $-0.077 \pm 0.007 \pm 0.011$ | $K^0_L \rightarrow \pi^-\pi^+e^+e^-$ | 1998 |

*comparatively good accuracies (pion radius ~2%) stem from assuming a theoretical shape of the form factor*

# Measuring Hadron Charge Radii in Inverse Kinematics

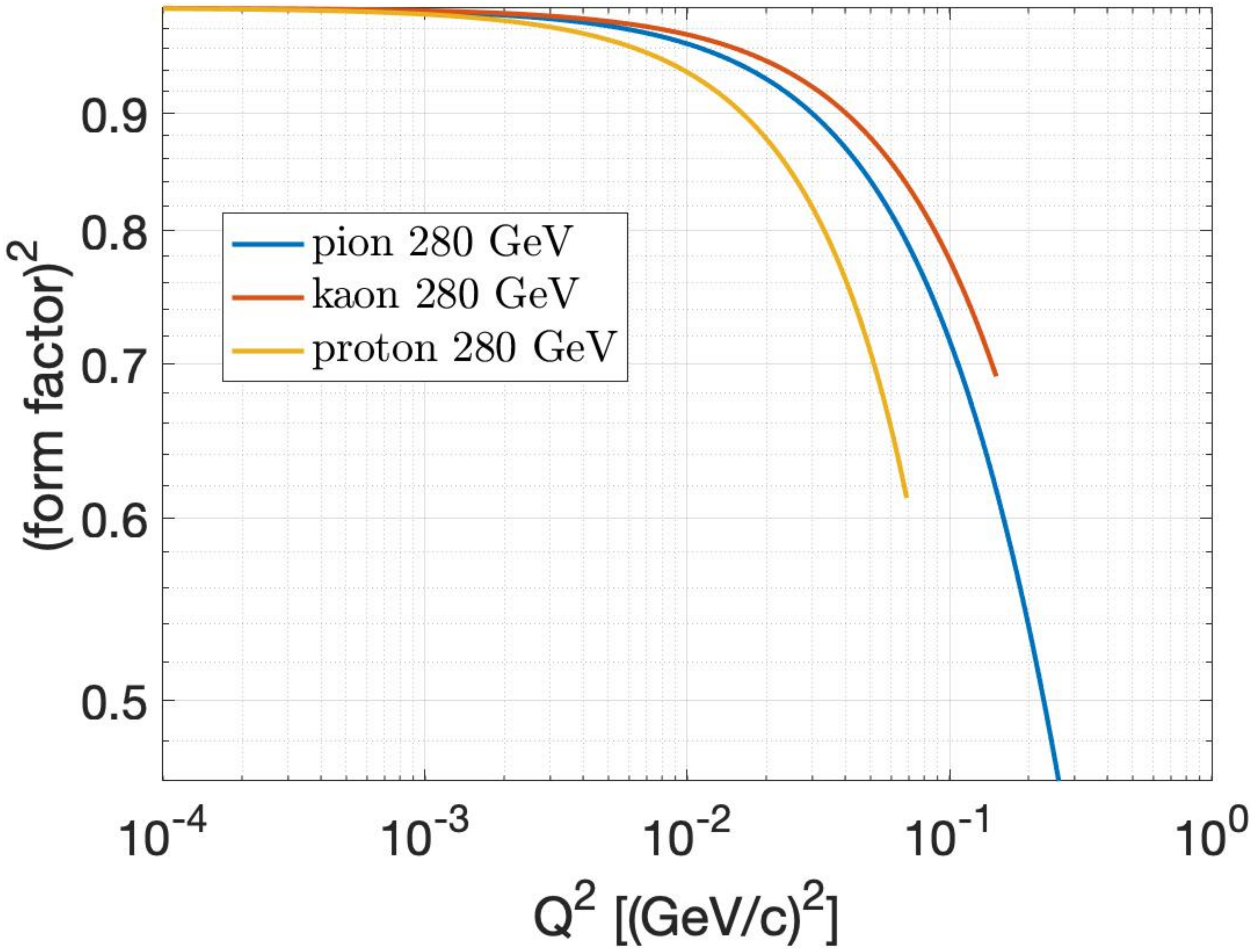
Why using inverse kinematics ?

- with no stable meson target existing - use stable lepton target
  - hadron is beam particle —> reaction in inverse kinematics

- kinematic range experimentally „unreachable“
  - make use of „easily“ measurable quantities to address „difficult regime“ (mostly low $Q^2$)
- electron initially at rest —> no initial external Bremsstrahlung
- final electron is accelerated —> external Bremsstrahlung for outgoing electron
  - impact on particle momentum
  - Impact on particle trajectory
- internal Bremsstrahlung effects independent of reference system (vertex corrections)

# What is the role of $Q^2_{max}$

- large values of Q²: higher sensitivity to charge distribution —> $< r_E^2 >$
- small values of Q²: smaller extrapolation uncertainties to Q² = 0 and $\frac{dF(Q^2)}{dQ^2}|_{Q^2=0}$

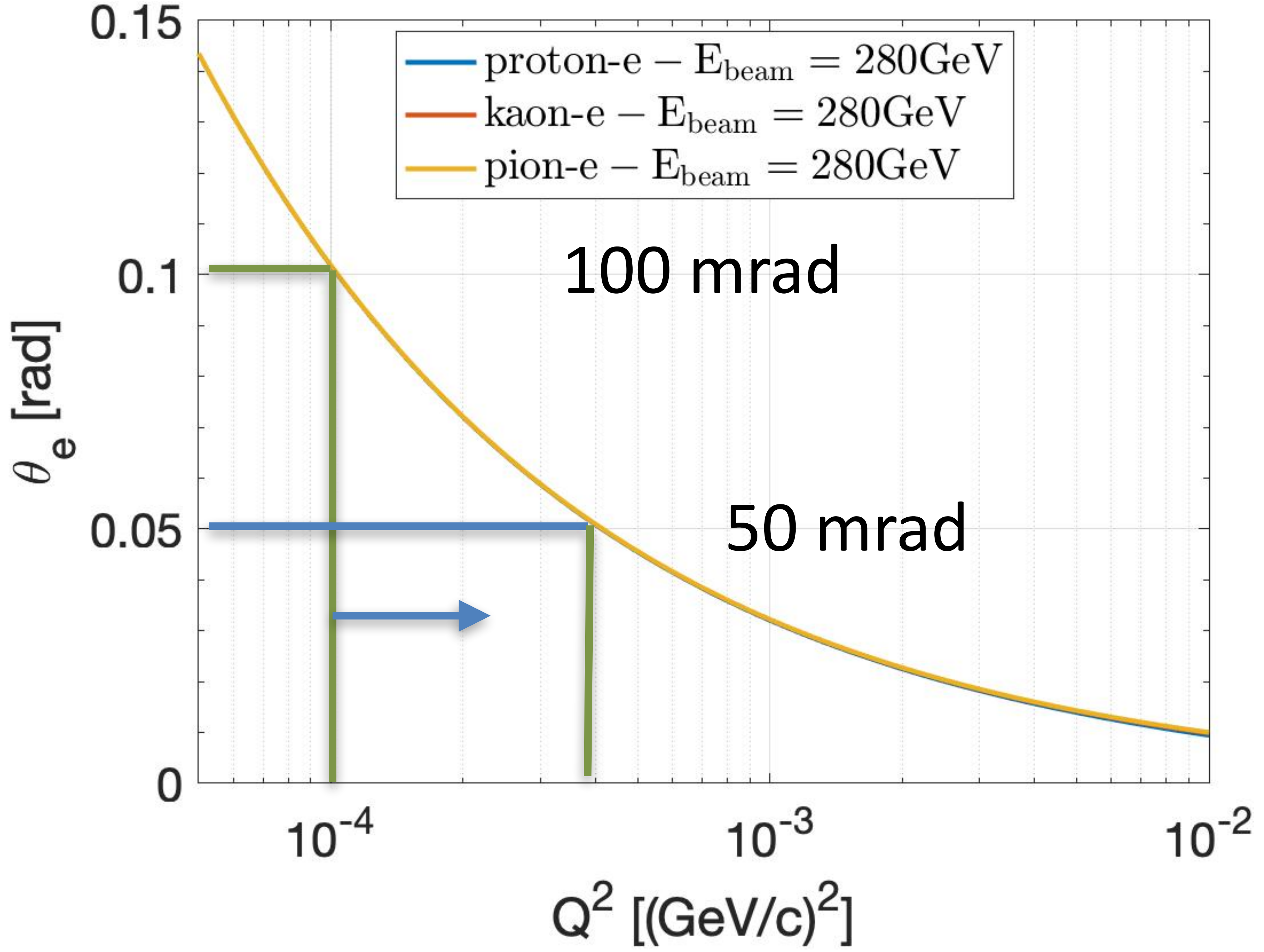


| Beam | $E_{\text{beam}}$ [GeV] | $Q^2_{max}$ [GeV²] | Relative charge-radius effect on σ(Q²) |
|---|---|---|---|
| π | 280 | 0,268 | ~54% |
| K | 280 | 0,15 | ~30% |
| K | 80 | 0,021 | ~5% |
| K | 50 | 0,009 | ~2-3% |
| p | 280 | 0,070 | ~28% |

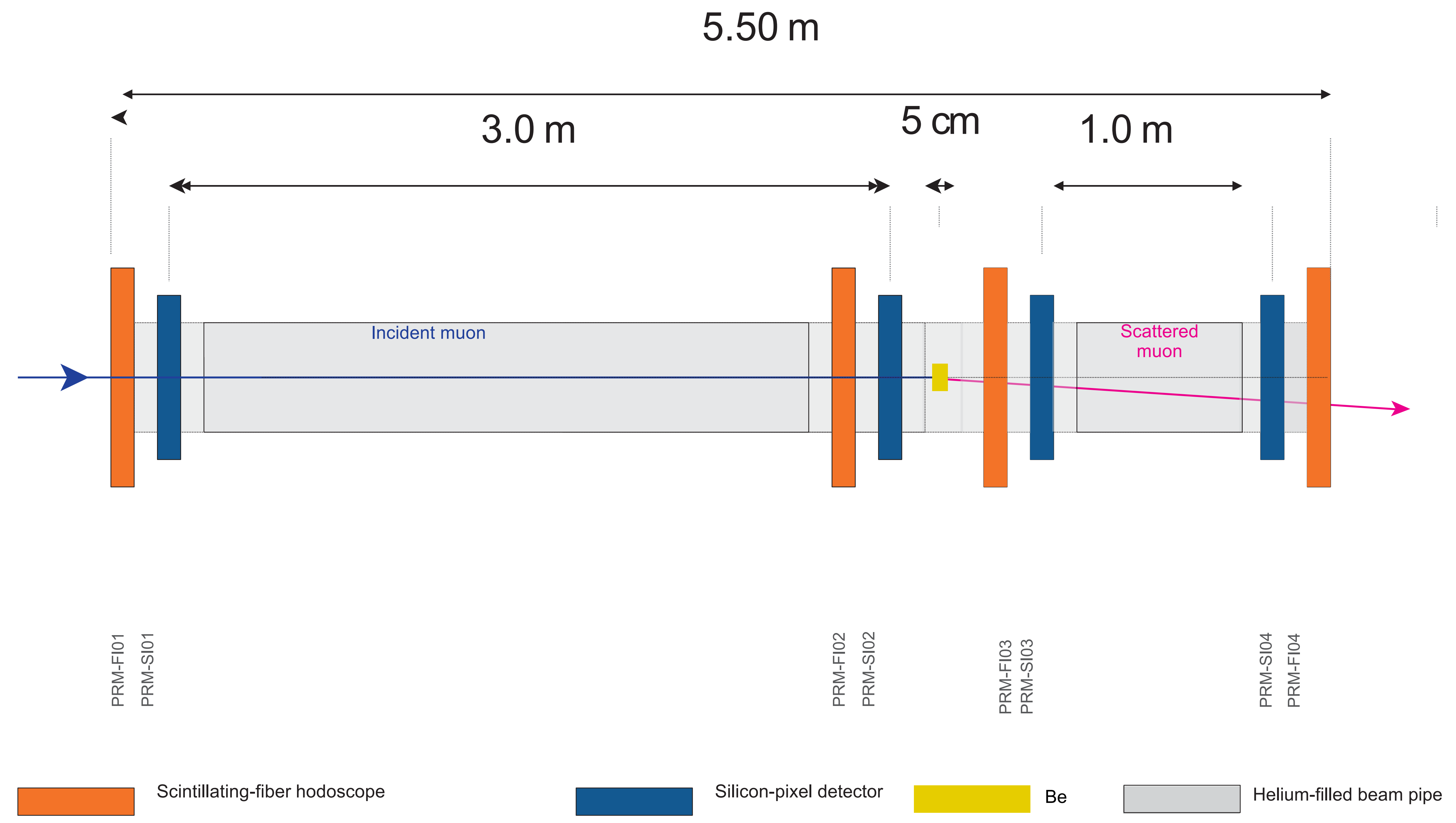
# Setup for solid target

- solid target (e.g. 1-25 mm Be) offers large acceptance for outgoing electron
- compress set-up
- Q² via three independent measurements - $\theta_e$ , $\theta_p$ , $p'_{hadron}$

$e^-$acceptance effect



little dependence on $E_{inc}$

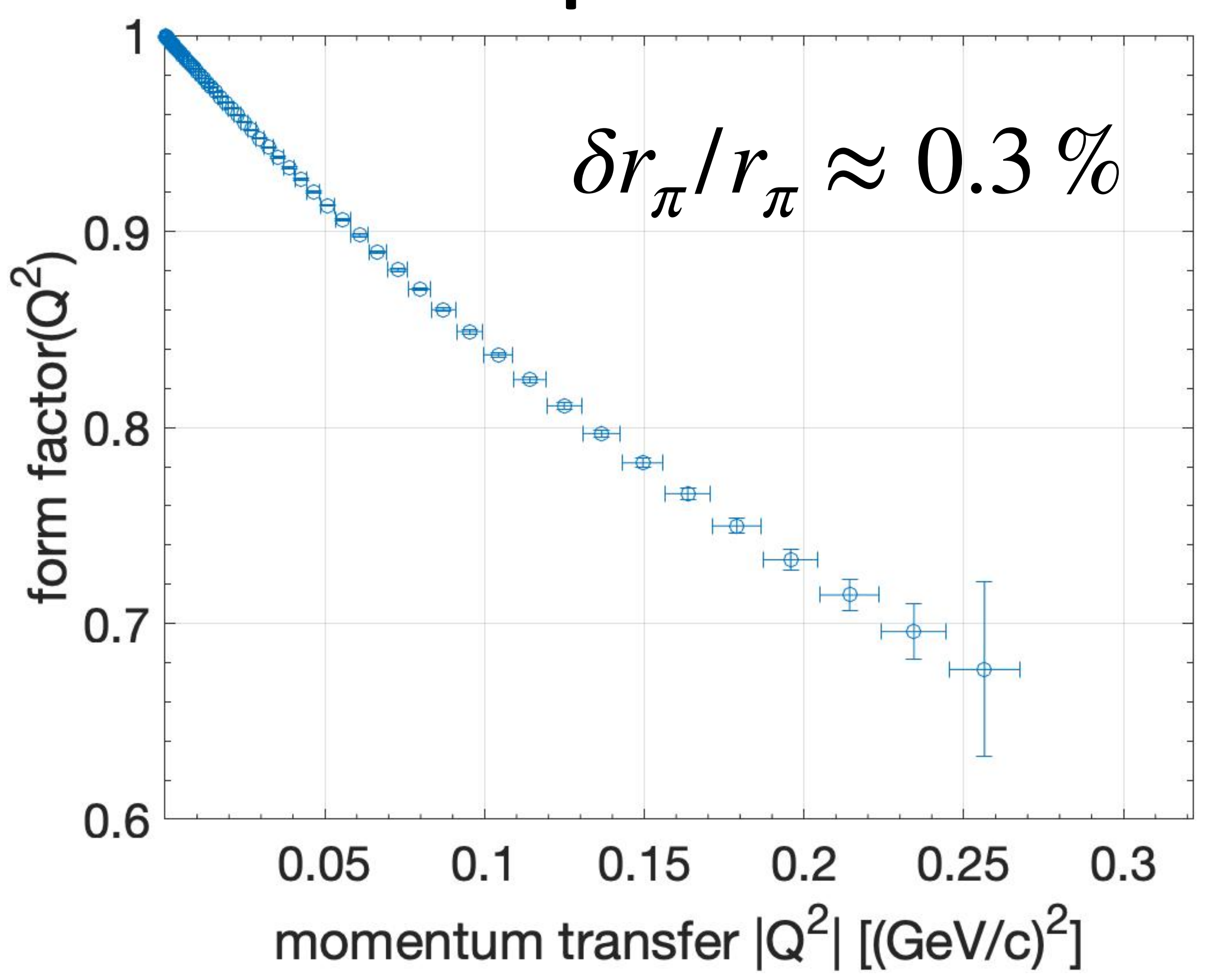


compact set-up

# Simulate Results for Kaons and Pions

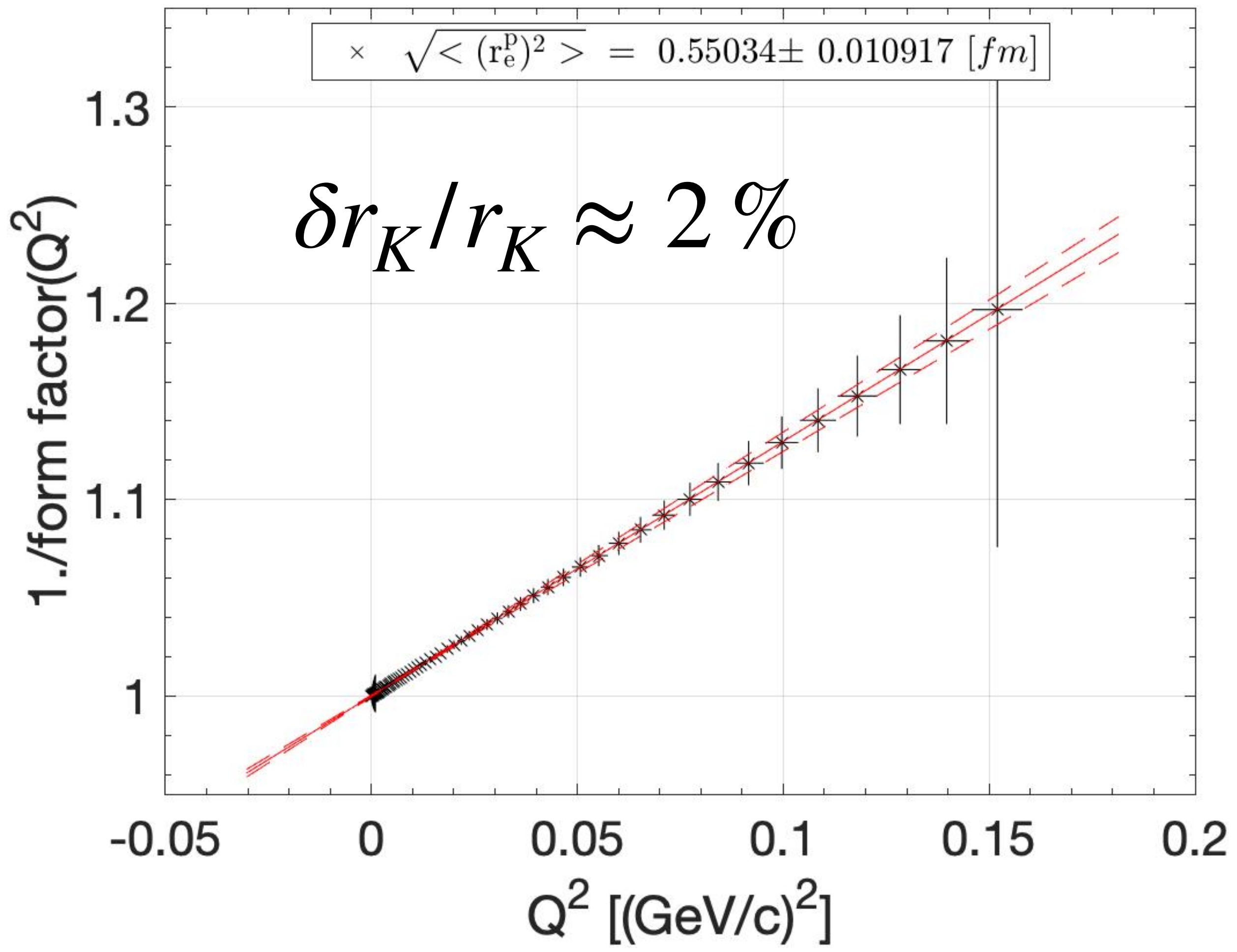
- Assume 30 days of beam time (100% efficiency) - use pole description for FF

pion



$\delta r_\pi / r_\pi \approx 0.3\,\%$

kaon



$\delta r_K / r_K \approx 2\,\%$

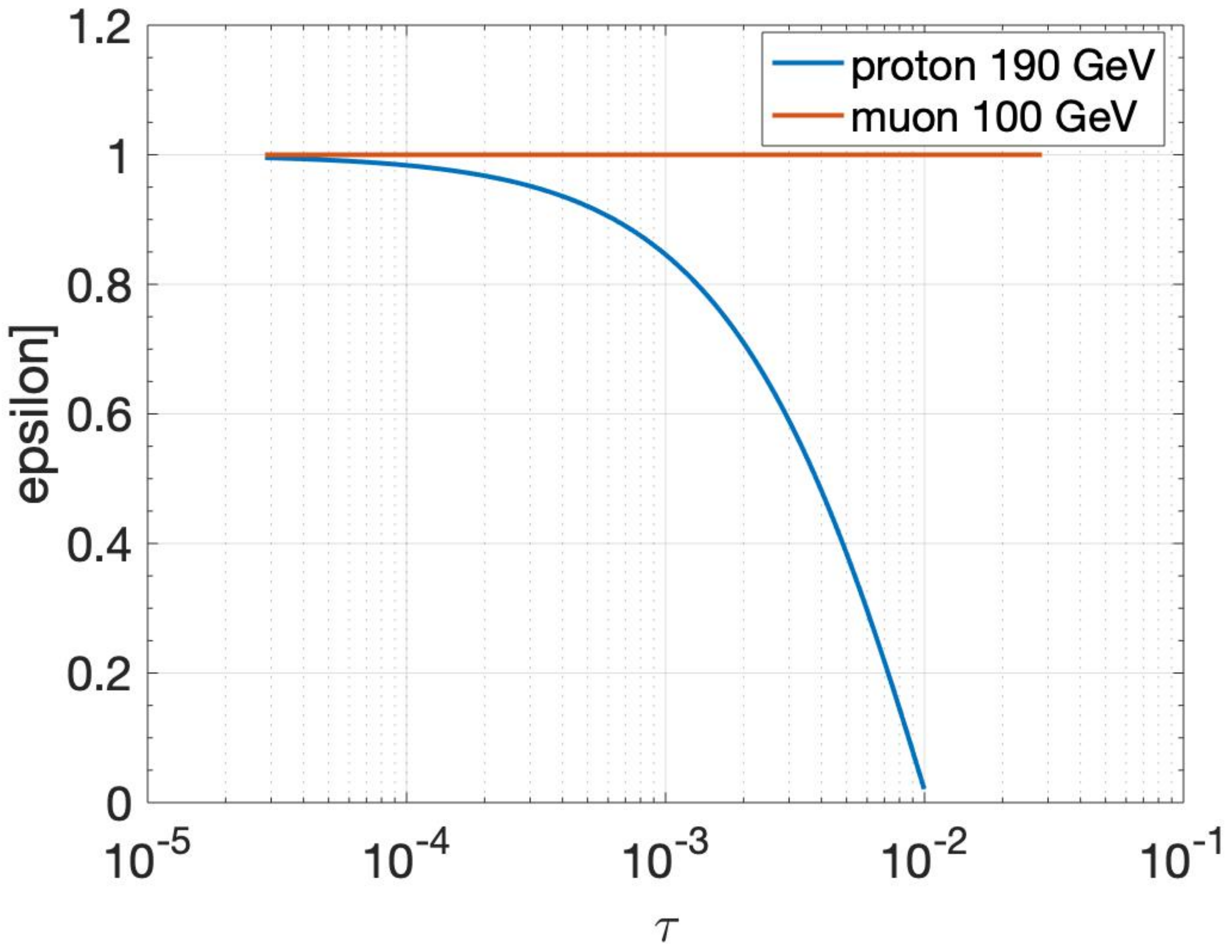
# Nucleons in Inverse Kinematics

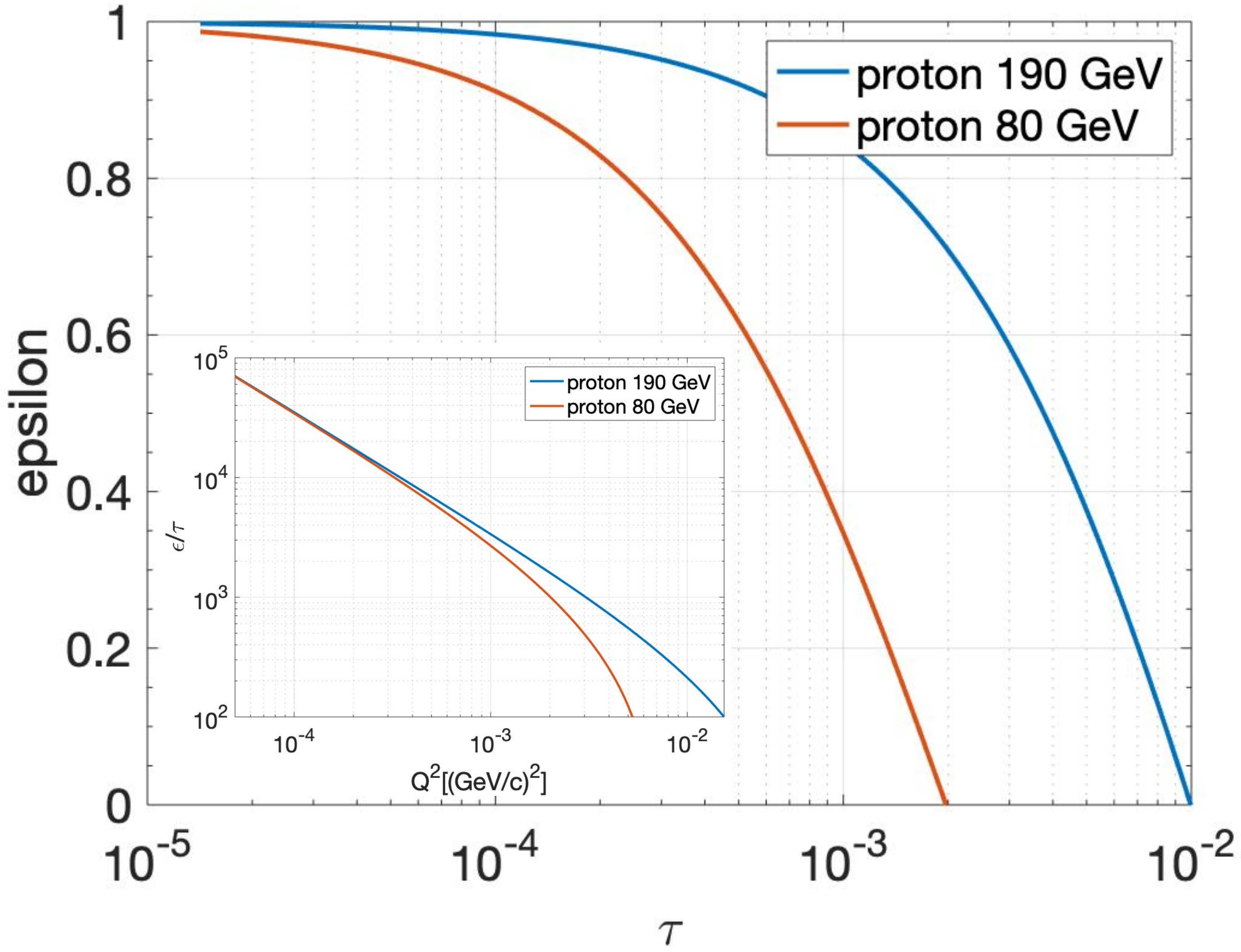
$\epsilon$ : photon polarization
$\tau$ : reduced $Q^2$
R:  normalization

Inverse kinematics allows easy way to access difficult *ep* kinematics

- kinematic variables R, ε, τ $\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} R\left(\epsilon \cdot G_E^2 + \tau \cdot G_M^2\right)$
- access Rosenbluth technique through variation of $p_{\text{beam}}$

$$\sigma_R = \left(\frac{d\sigma}{d\Omega}\right)_{exp} / \left(\frac{d\sigma}{d\Omega}\right)_M \frac{\epsilon(1+\tau)}{\tau} = \boxed{\frac{\epsilon}{\tau} G_E^2} + \boxed{G_M^2}$$





high energy muon scattering:
little sensitivity to $G_M^2(Q^2)$

use different nucleon beam
momenta to access $G_M^2(Q^2)$

# $G_M^p(Q^2)$

- Rosenbluth separation allows for extract $G_M^p(Q^2)$ at low Q$^2$ !
- presently - knowledge data only for $Q^2 > 0.02(GeV/c)^2$ (Mainz data)
- Inverse kinematics could add kinematically $0.004 > Q^2 > 0.04(GeV/c)^2$
- first measurement in this kinematic range for this quantity !
- equivalent incoming electron energies: 30-105 MeV

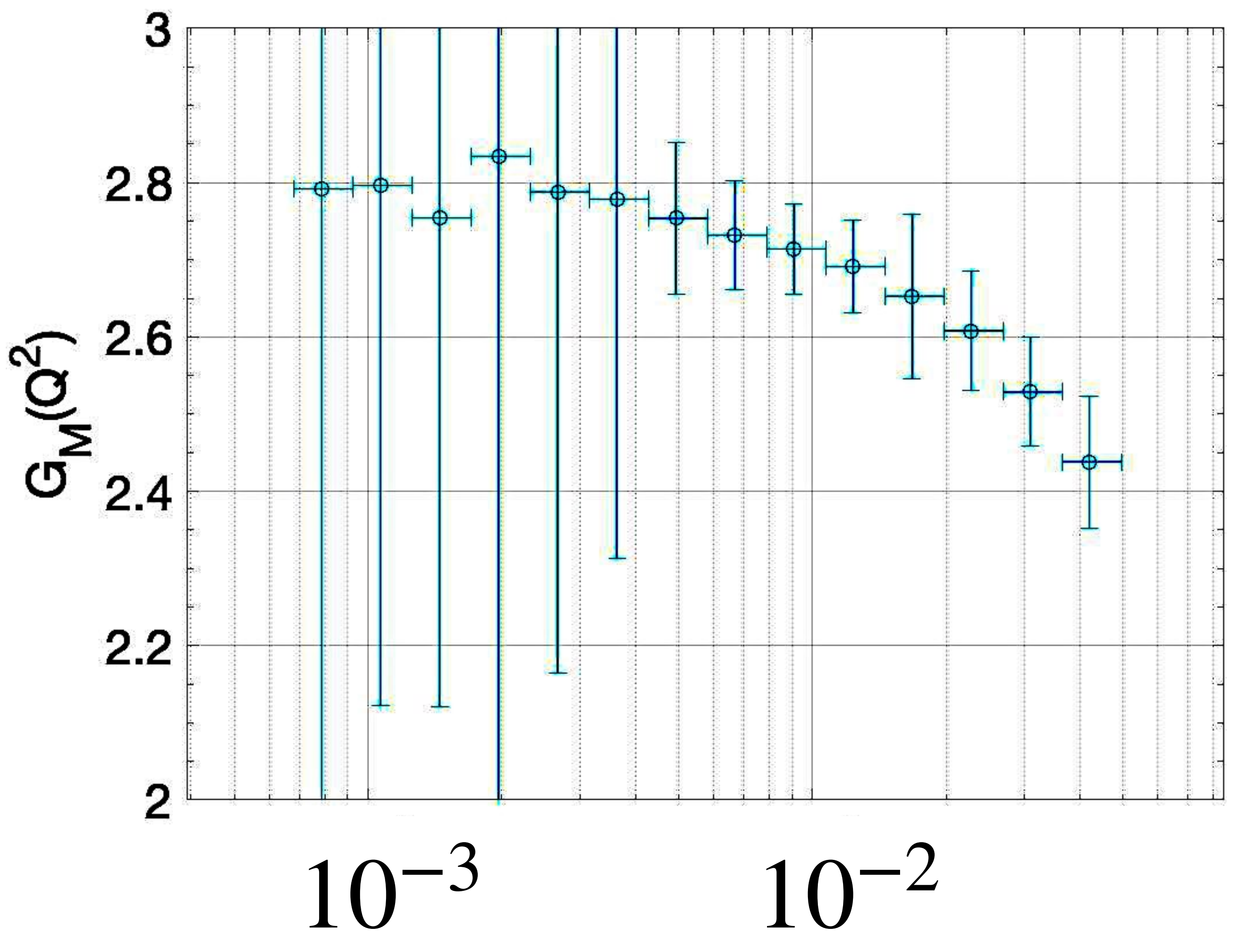
# Extraction of $G_M^p(Q^2)$ and $G_E^p(Q^2)$

use 10 different settings (energy/target thickness) - assume 130 days of beam time (100% efficiency)

perform Rosenbluth separation and fit $\sigma_R$ versus $\epsilon$

$$\sigma_R = \left(\frac{d\sigma}{d\Omega}\right)_{exp} / \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}$$

- error bars depend on fitting method (very preliminary)



$$G_M^p(Q^2)$$

$$G_E^p(Q^2)$$

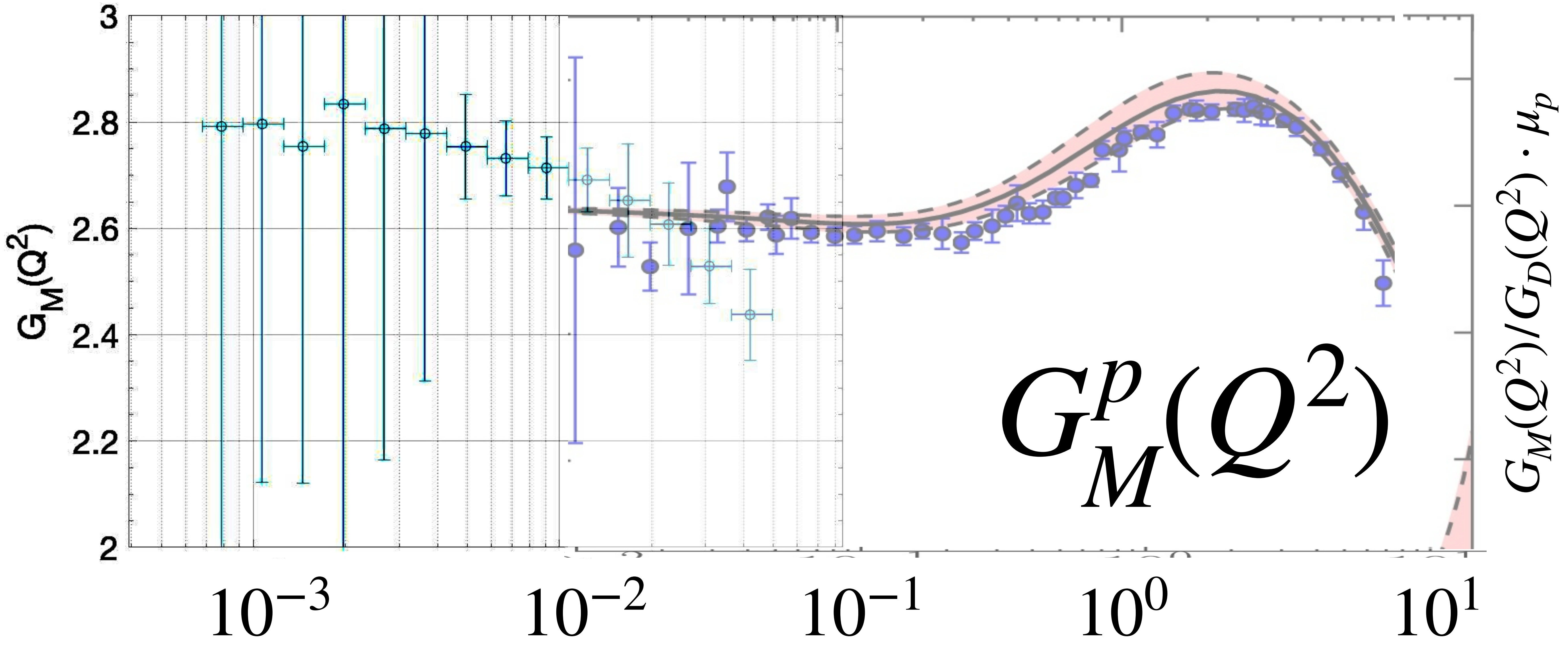
# Extraction of $G_M^p(Q^2)$ and $G_E^p(Q^2)$

use 10 different settings (energy/target thickness) - assume 130 days of beam time (100% efficiency)

perform Rosenbluth separation and fit $\sigma_R$ versus $\epsilon$

$$\sigma_R = \left(\frac{d\sigma}{d\Omega}\right)_{exp} / \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}$$

- error bars depend on fitting method (very preliminary)



$G_E^p(Q^2)$

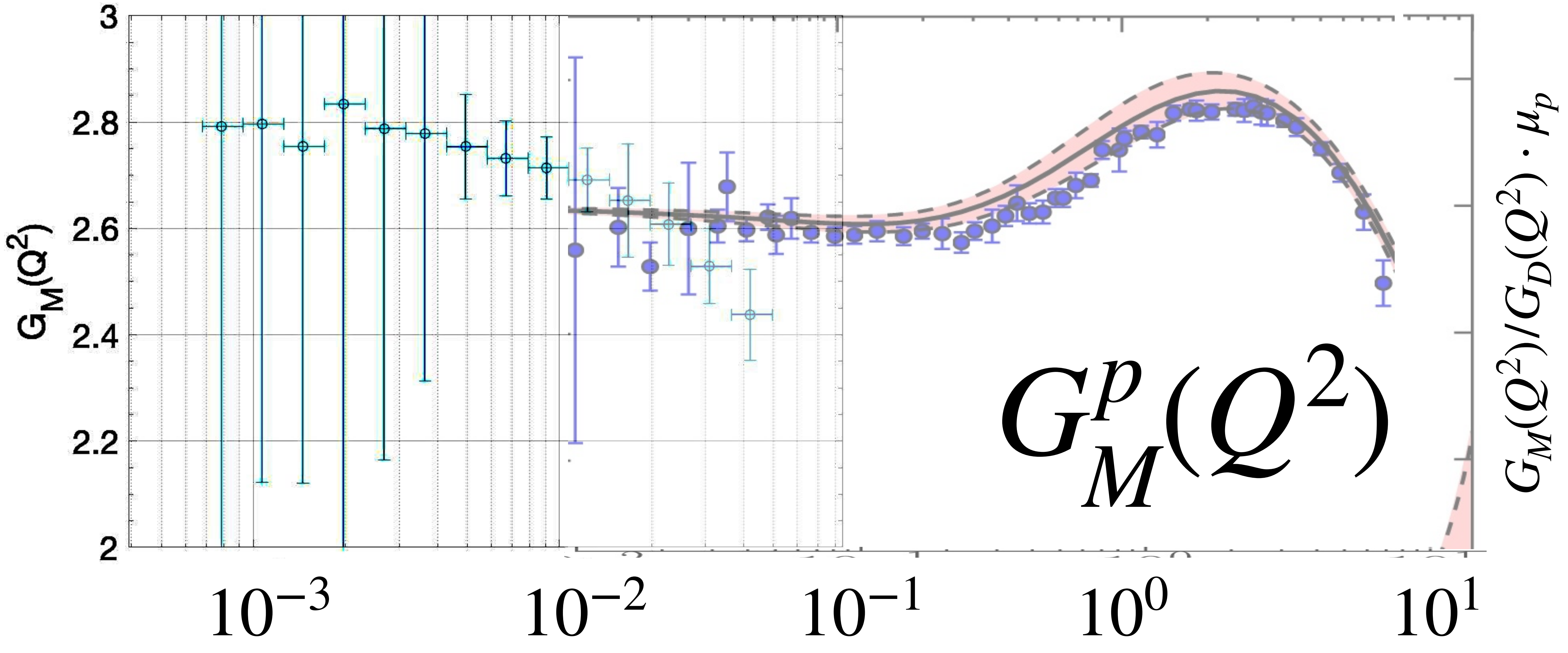
# Extraction of $G_M^p(Q^2)$ and $G_E^p(Q^2)$

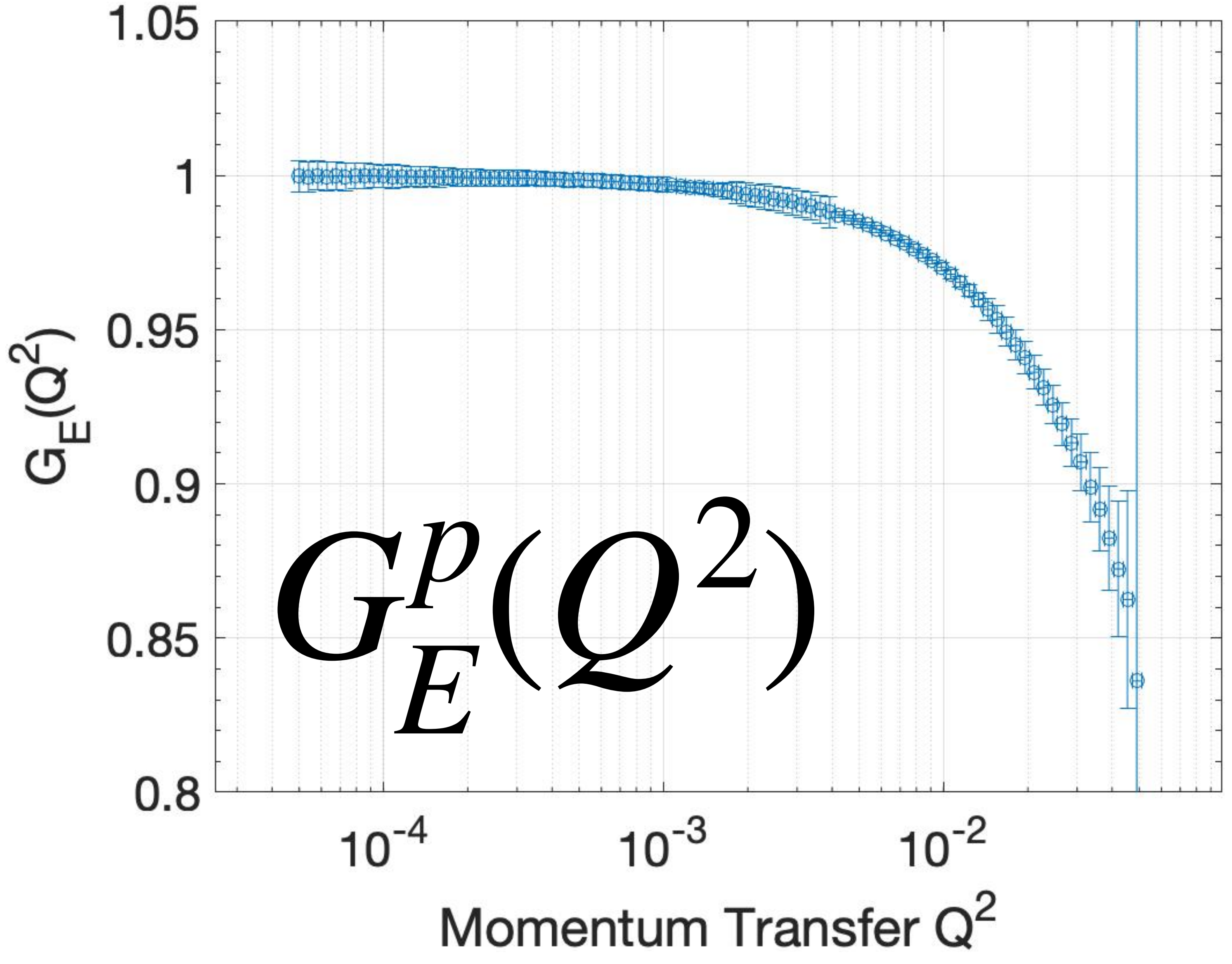
use 10 different settings (energy/target thickness) - assume 130 days of beam time (100% efficiency)

perform Rosenbluth separation and fit $\sigma_R$ versus $\epsilon$

$$\sigma_R = \left(\frac{d\sigma}{d\Omega}\right)_{exp} / \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}$$

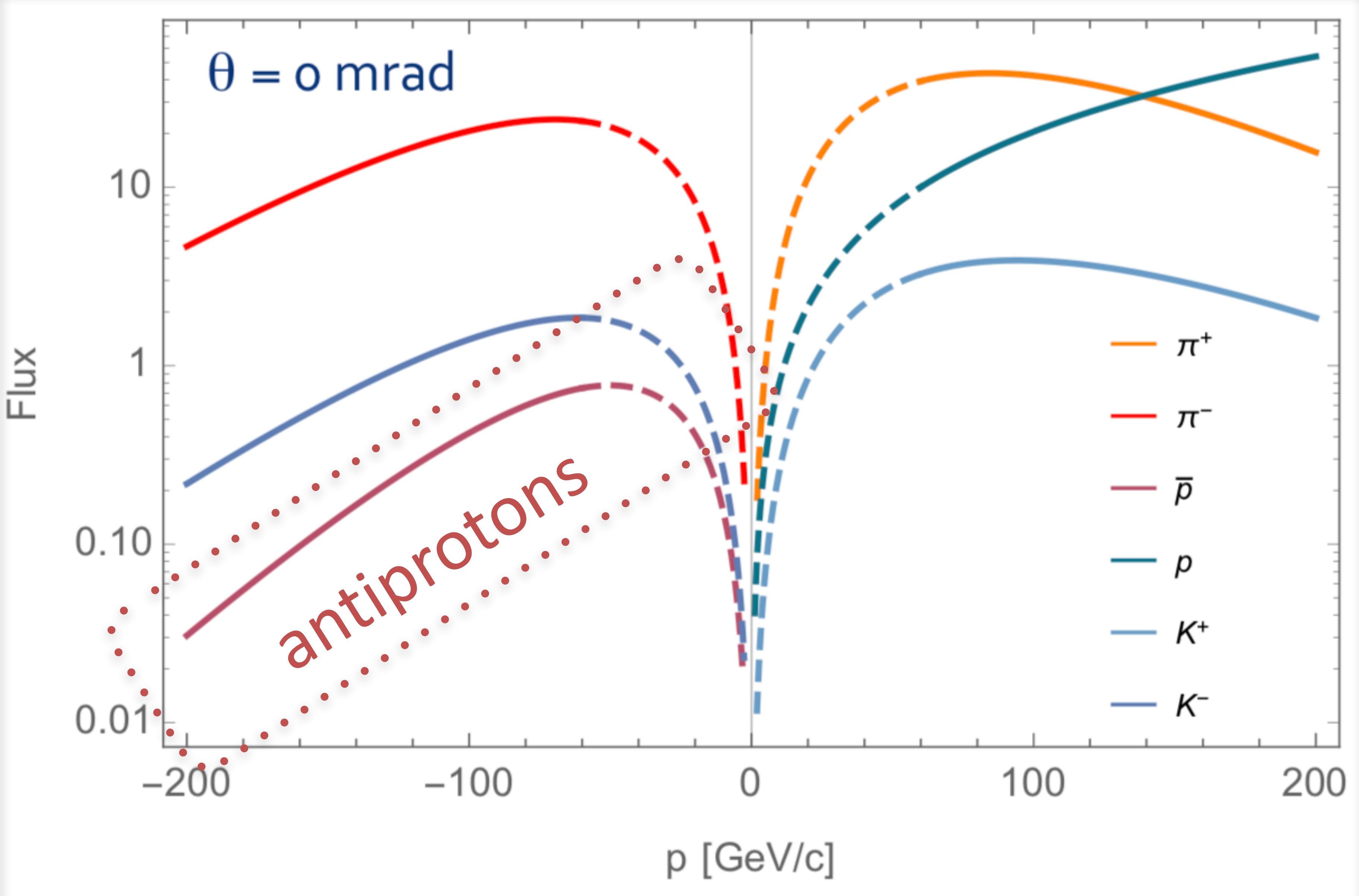
- error bars depend on fitting method (very preliminary)



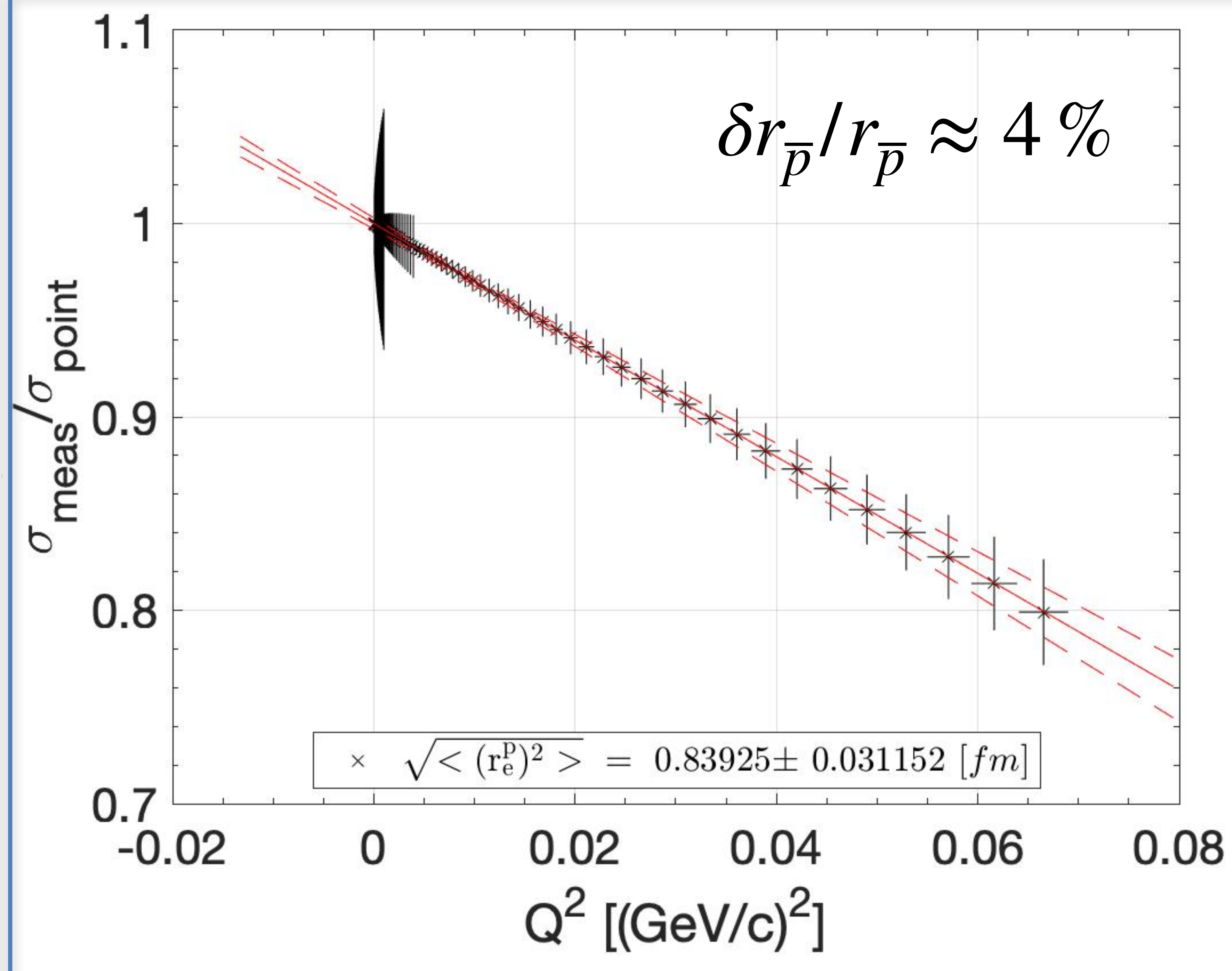


# Charge Radius of Antiprotons

- Use data taking mode with pions - assume 30 days (no variation of $E_{in}^{beam}$)
- use energy dependent fraction of $\overline{p}$ in pion beam



M2 beam @ CERN SPS

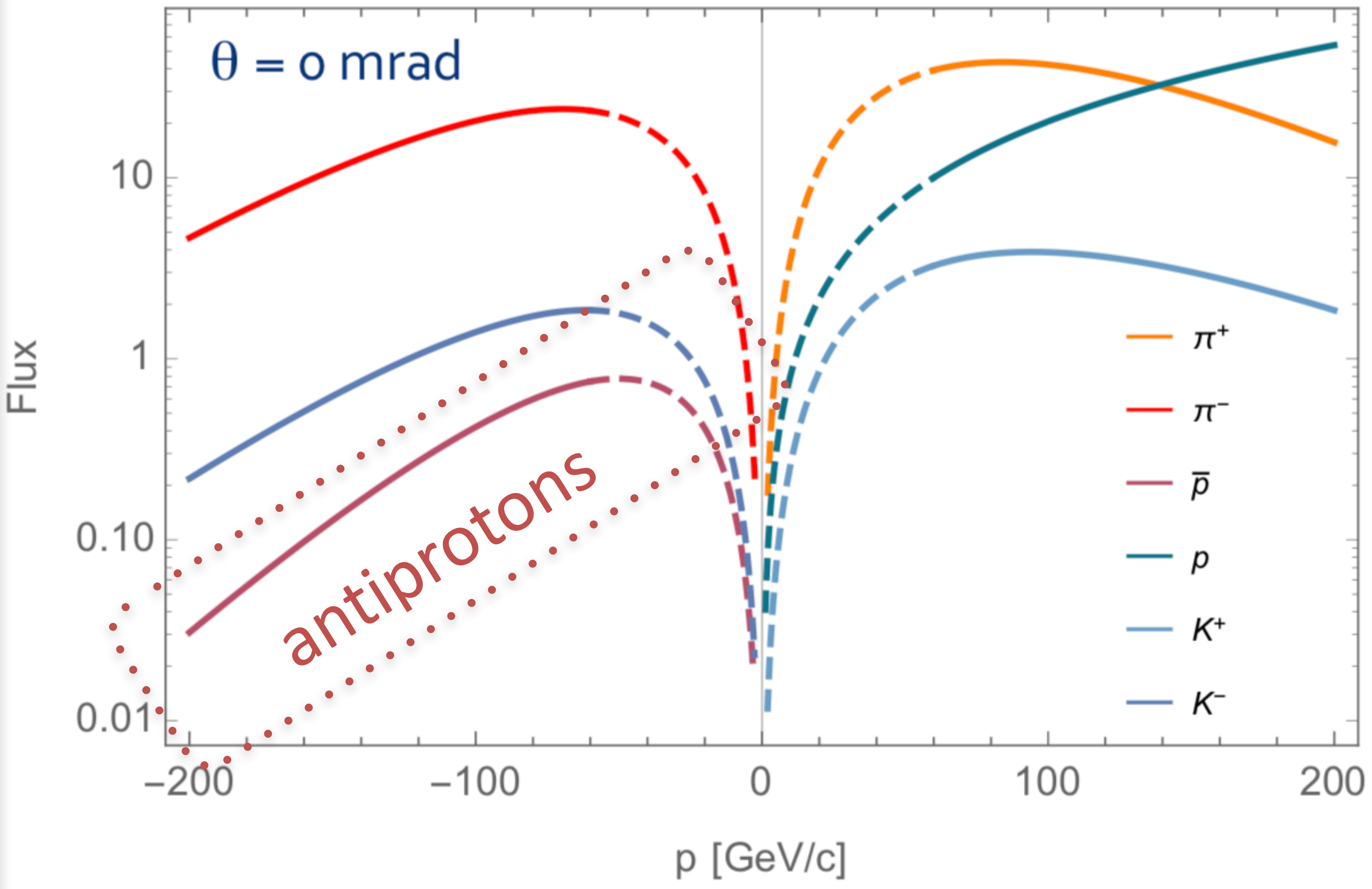


parasitic running with pion

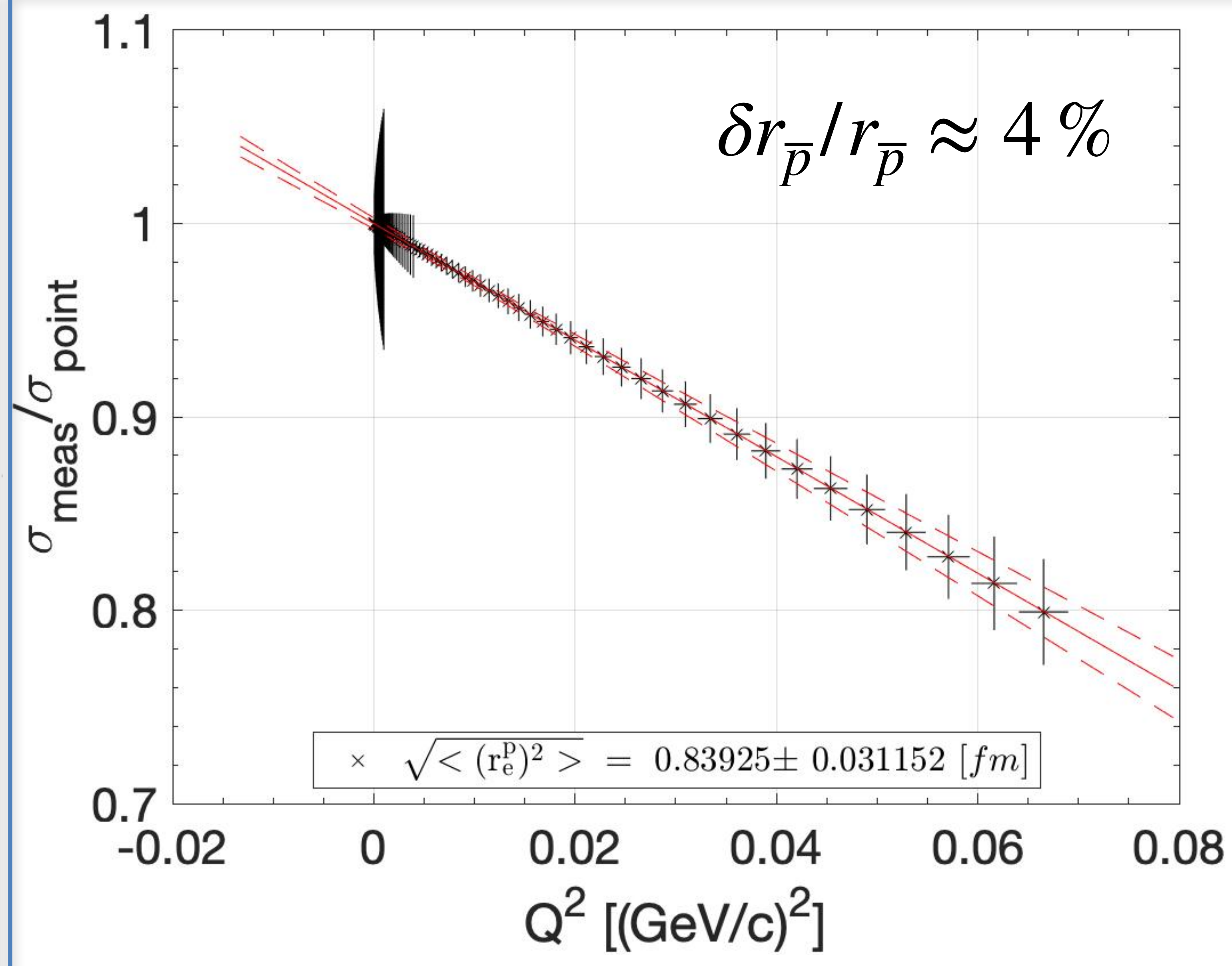
# Charge Radius of Antiprotons

- Use data taking mode with pions - assume 30 days (no variation of $E_{in}^{beam}$)
- use energy dependent fraction of $\overline{p}$ in pion beam
- perform Rosenbluth separation and fit $\sigma_R$ versus $\epsilon$ and obtain $G_E^{\overline{p}}(Q^2)$

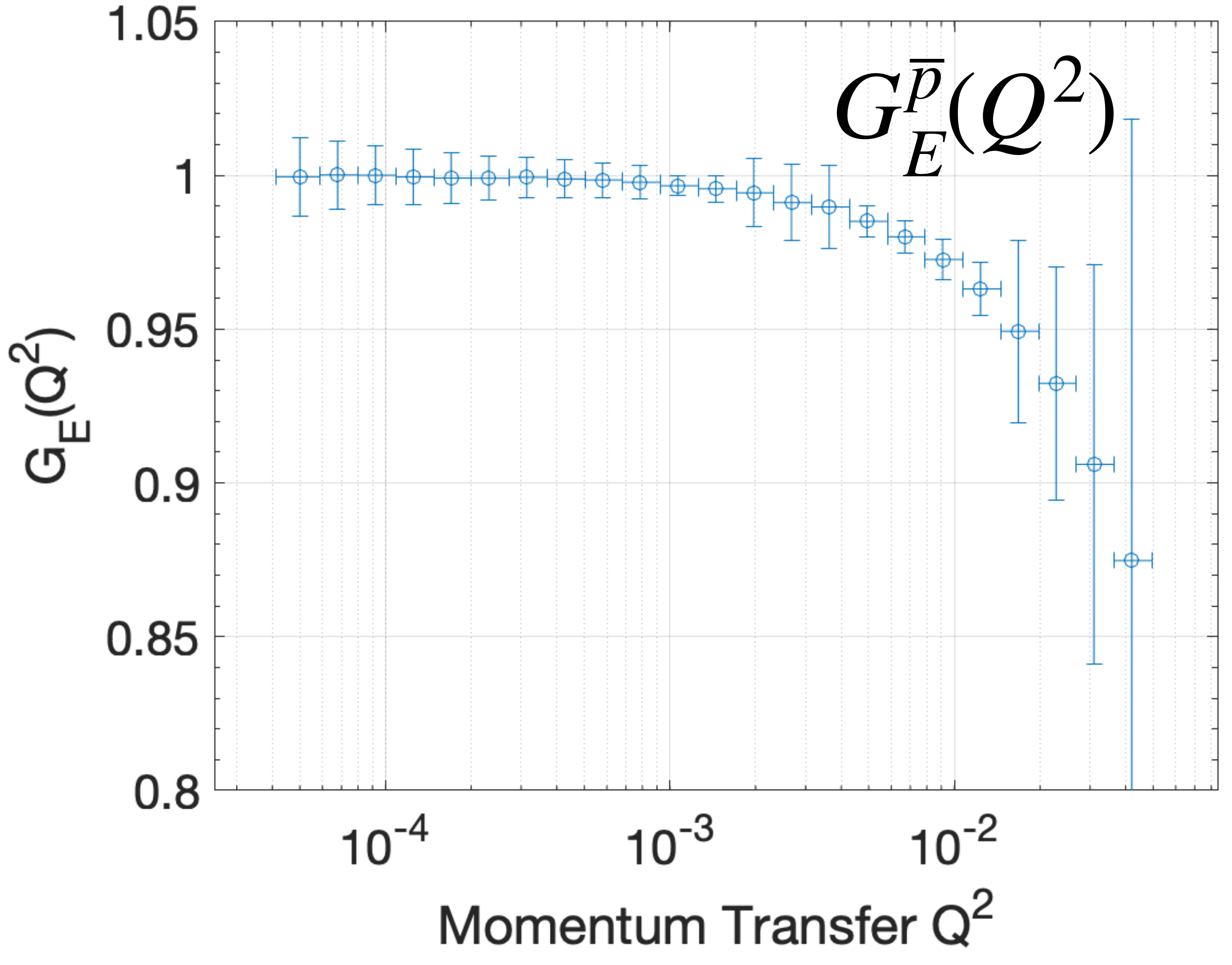
$$\sigma_R = \left(\frac{d\sigma}{d\Omega}\right)_{exp} / \left(\frac{d\sigma}{d\Omega}\right)_M$$



M2 beam @ CERN SPS



parasitic running with pion



dedicated running like protons

# Summary Inverse Kinematics

- Meson radii are of key interest in understanding their inner structure and the emergence of hadron mass

- pions : data of previous experiments can be challenged (statistics !! + systematics)

- kaons : significant increase of the form factor knowledge in the range $10^{-4} < Q^2 < 0.15\ [(GeV/c)^2]$ (factor 10)

- large $Q^2$ range possible (in particular down to very small $Q^2$)
  accessible $Q^2$ range determined by detection requirements for outgoing electron

- Proton inverse kinematics allows low $Q^2$ kinematics and Rosenbluth separation $G_M^p(Q^2)$
- Antiprotons: First ever measurements of form factors (incl. Rosenbluth separation)