Quantum Fields in Condensed Matter

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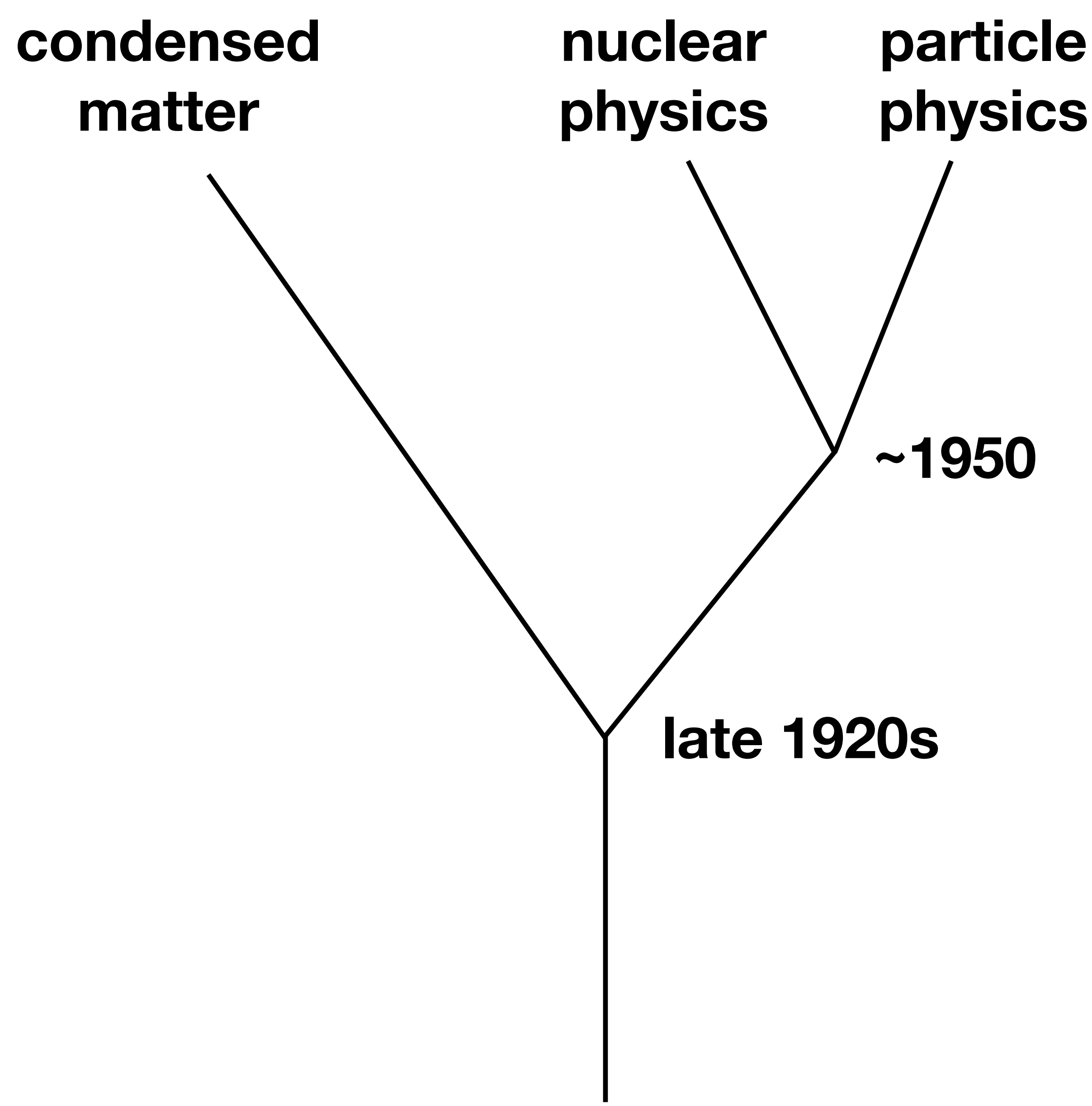
Windows to the Universe, Quy Nhon, 2023

Particle physics and CMP: an early split



Wolfgang Pauli

'Ich mag diese Physik des festen Körpers nicht . . . zwar habe ich damit angefangen' (*I don't like this solid state physics . . . I initiated it though*)

The foregoing quotation summarizes Pauli's attitude towards solid state physics. As a matter of fact it was one of the few subjects in physics—gas discharges and the theory of the chemical bond were others—towards which he showed a profound dislike. Of course Pauli was mainly occupied with really fundamental problems but this might only explain his lack of interest not his antipathy. Applied science for instance he treated with humorous condescension and he could be thoroughly amused when according to his own saying he had not understood one single word of a lecture on advanced design of thermionic valves or some similar subject. What irritated him in solid state physics was the lack of mathematical rigour and logical completeness and the introduction of vaguely defined models that are tantamount to unverifiable mathematical hypotheses and are only of value when used in conjunction with an intelligent classification of a mass of experimental data. On the other hand he admired Onsager's work

H. Casimir "Pauli and Theory of the Solid State"

- Solid-state physics has overcome this stereotype
- Modern Condensed Matter Physics emphasizes universal, emergent laws
- Effectiveness of quantum field theory in condensed matter physics
- Observed phenomena CMP improve theoretical understanding QFTs in nonperturbative regime

Low-energy universality

- What makes condensed matter physics systematic is the universality of low-energy phenomena
- Many condensed matter systems behave in the same way, independent of microscopic details
- Often, the low-energy regime is the regime of most physical interest

Example 1: Nambu-Goldstone Boson

- Spontaneous symmetry breaking: NGBs, Lagrangian fixed by symmetries

- Example: phonon in solid

  - displacement $\vec{u}(x)$, symmetry $\vec{u}(x) \to \vec{u}(x) + \vec{c}$

  - Effective Lagrangian:
    $\mathscr{L} = (\partial_t \vec{u})^2 - (\partial_i u_j)^2 - (\partial u)^3 + \cdots$

  - Cubic interaction: restricted by shift symmetry

NGBs in magnets

- Ferromagnets and antiferromagnets: spontaneous-symmetry breaking $O(3) \to O(2)$

- Antiferromagnet: $\mathscr{L} = \sum_{a=1}^{2} (\partial_t S^a)^2 - v^2(\partial_i S^a)^2$

  two NGBs with dispersion relation $\omega = vk$

- Ferromagnets: $\mathscr{L} = iS_1\partial_t S_2 - (\partial_i S_a)^2$

  ONE NGB with dispersion relation $\omega \sim k^2$

Counting of NGBs

- Usually Goldstone theorem is thought to imply

  # of NGBs = # of broken generators

  This is true in Lorentz invariant theories

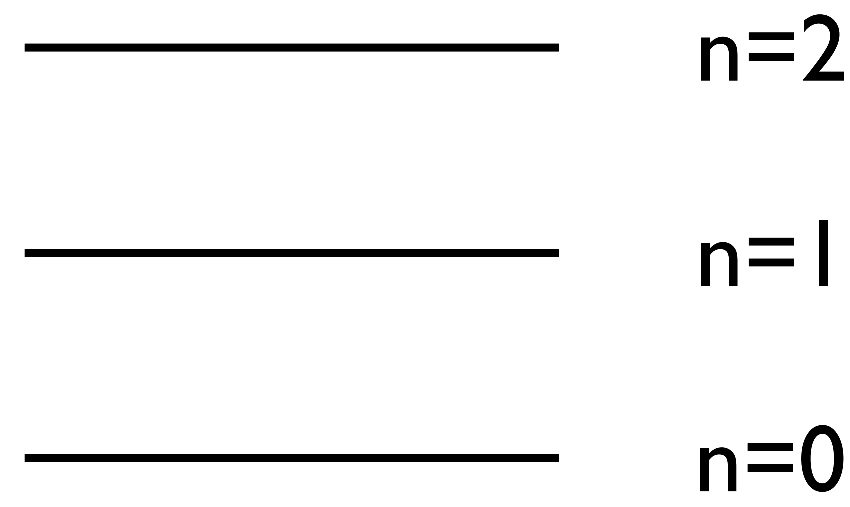
- In Lorentz-noninvariant theories:

$$N_{\text{linear NGB}} + 2N_{\text{quadratic NGB}} = N_{\text{brokgen generators}}$$

Example 2: Quantum Hall physics and duality

- Fractional Quantum Hall systems are the ultimate strongly coupled system in CMP
- Here we see one of the most sophisticated use of QFT
- Set up: electrons on the lowest Landau level
- The electrons filled a fraction ν of the states on the LLL

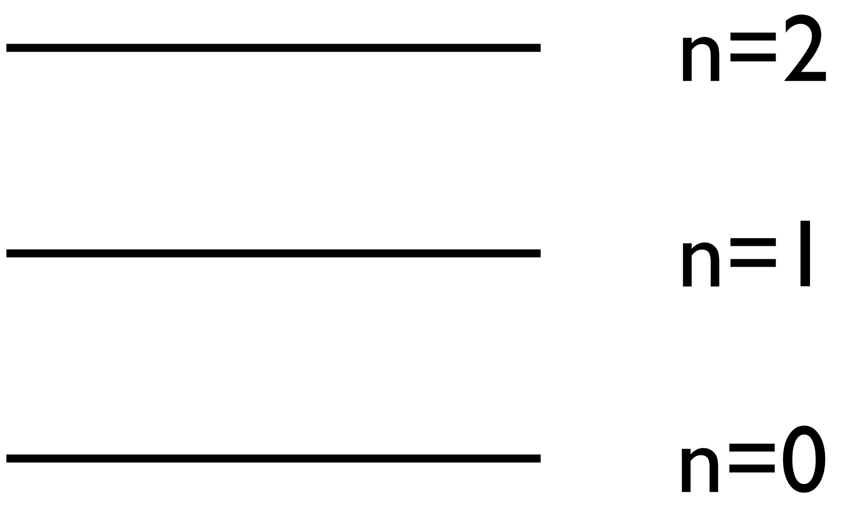
The fractional quantum Hall effect (FQHE)



The fractional quantum Hall effect (FQHE)

filling factor

$$\nu = \frac{\text{number of electrons}}{\text{degeneracy of a LL}}$$



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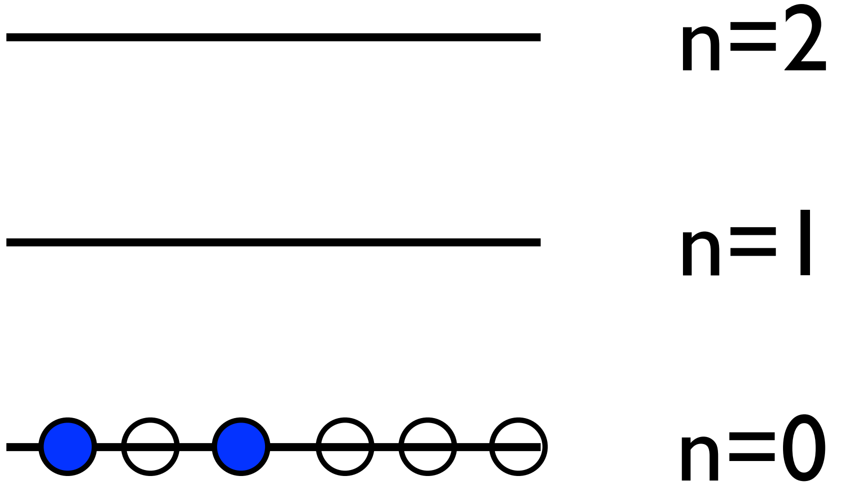
$$\nu < 1$$



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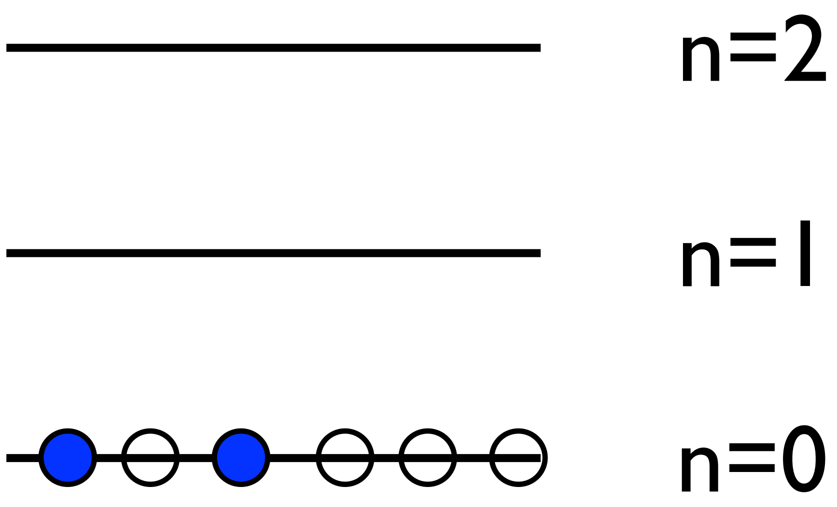

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- without interaction: large ground-state degeneracy

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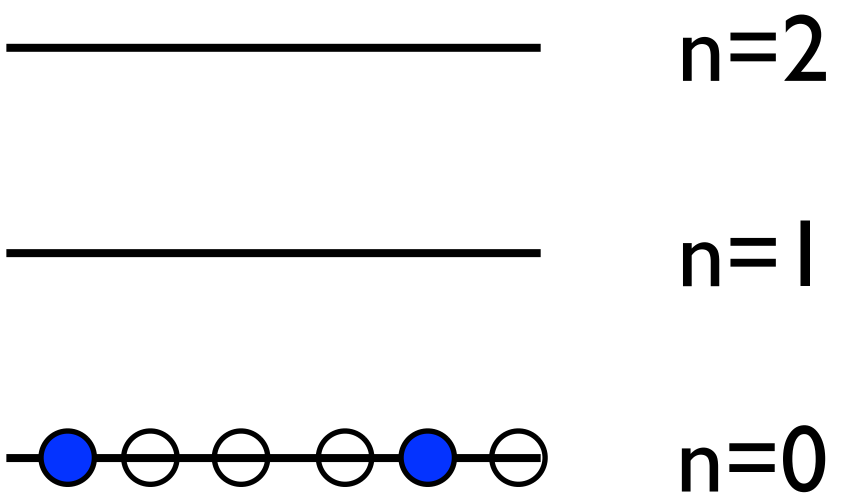

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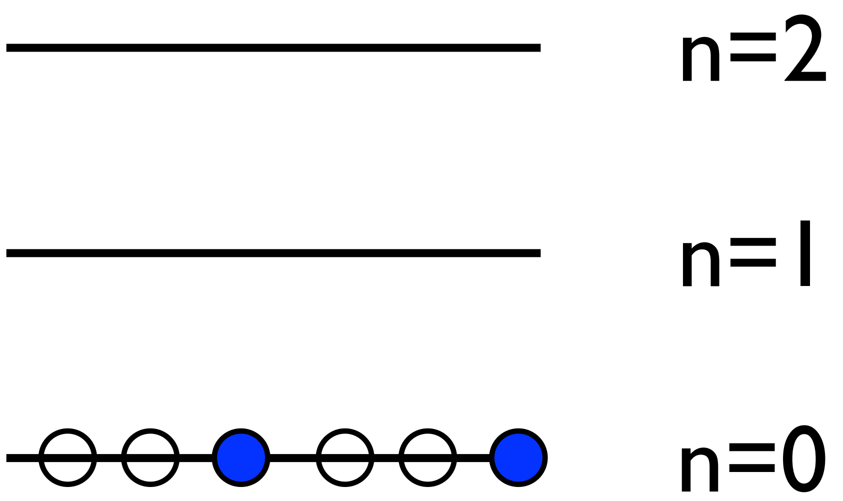

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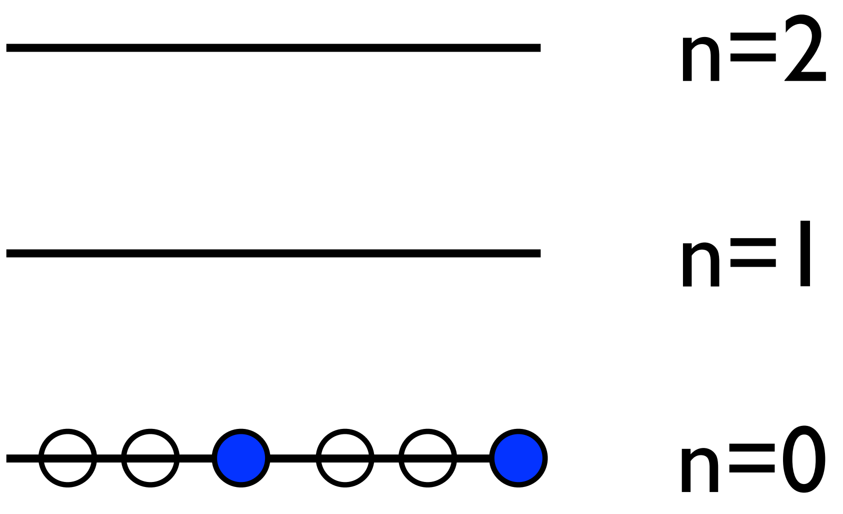

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- Interactions are essential for determining the ground state

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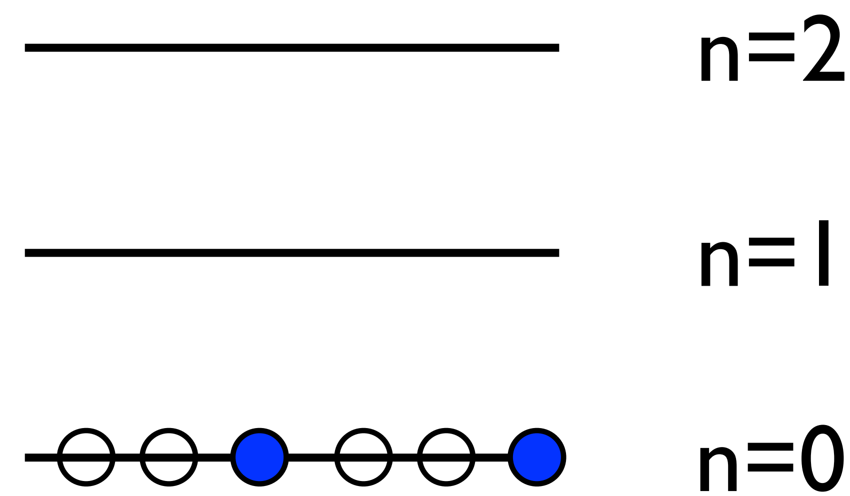

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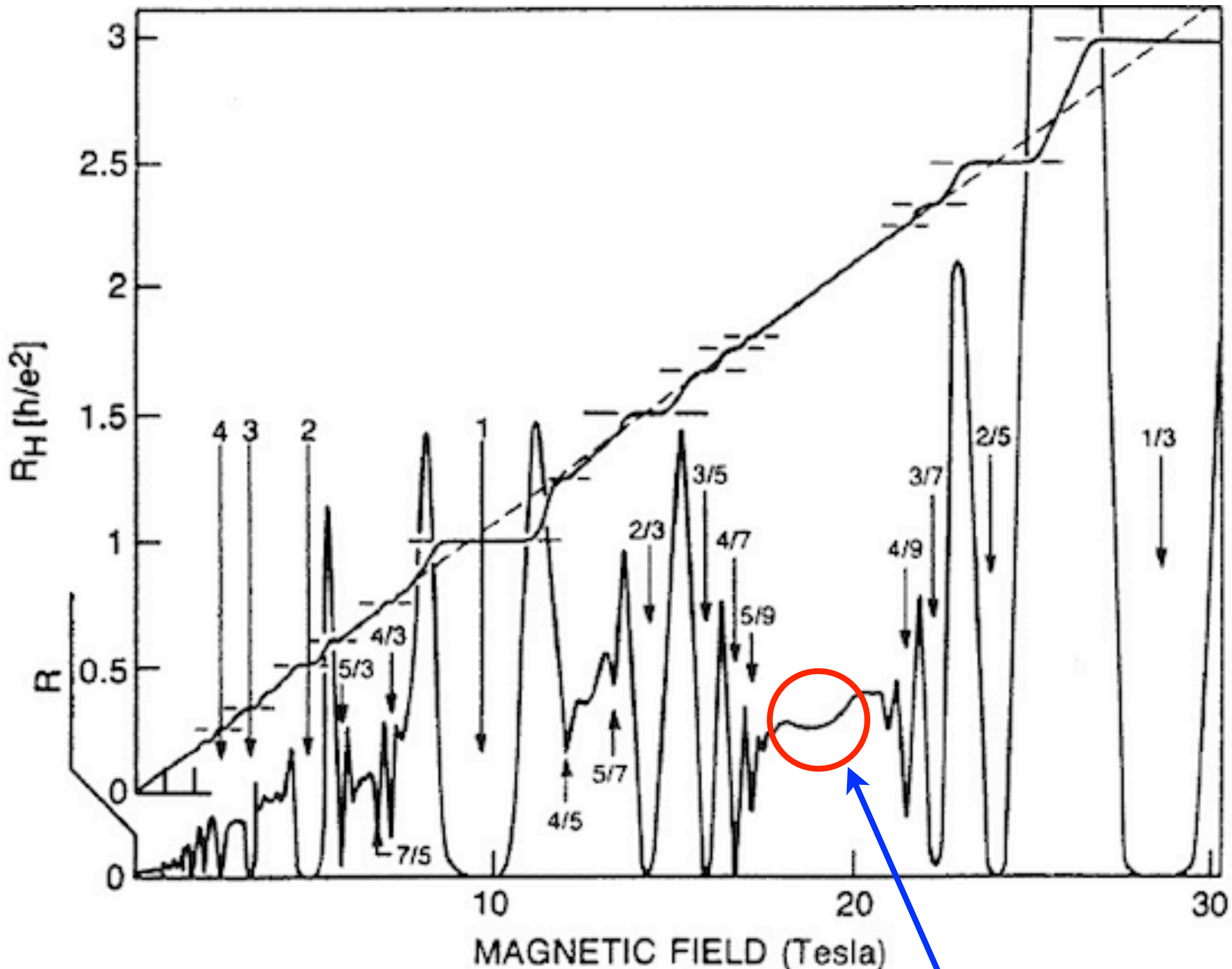
$$K \ll V$$



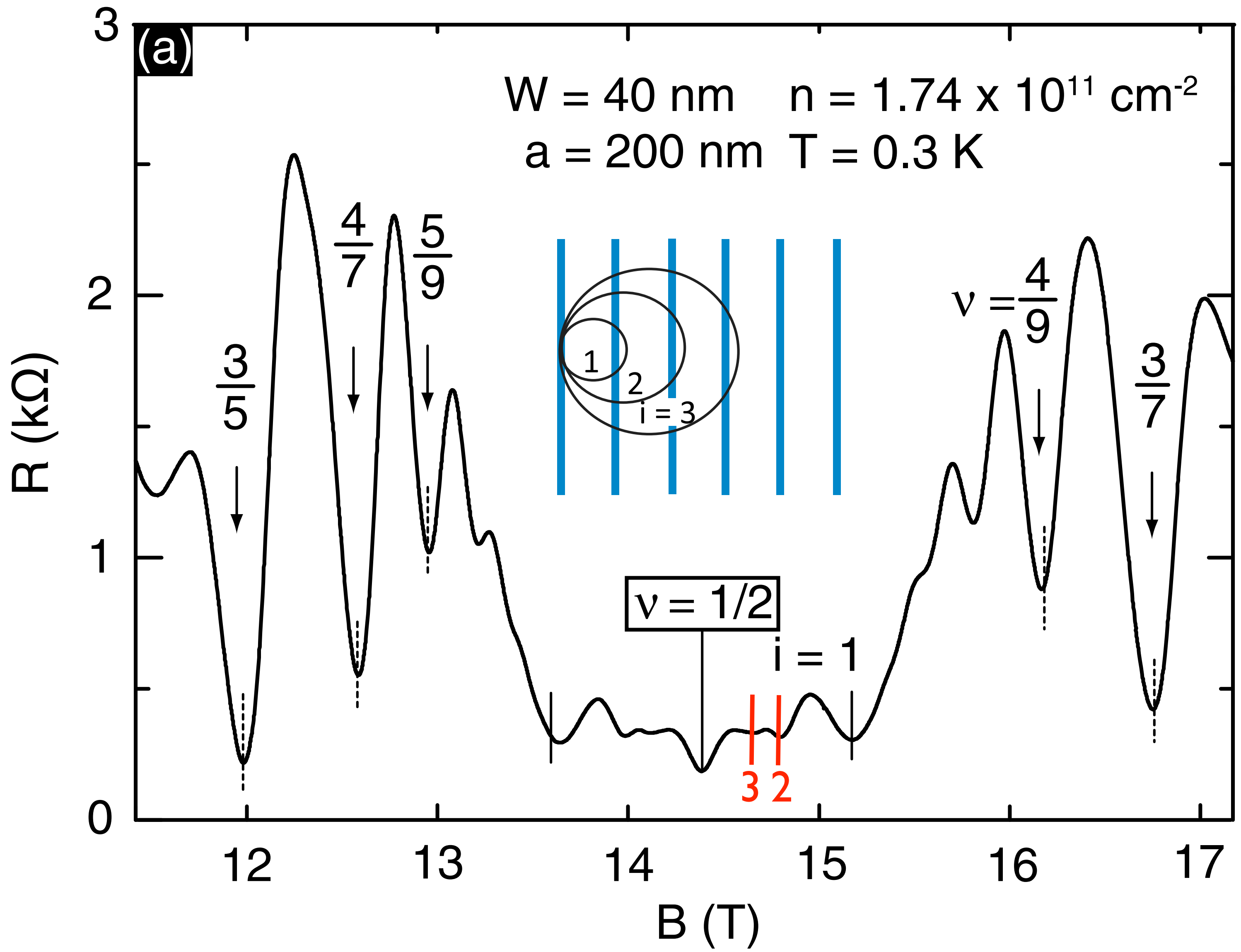
Half-filled Landau level

$$\nu = \frac{1}{2}$$

The half-filled Landau level

- Experiments show that the half-filled Landau level is a gapless state
- also show that the low-energy degree of freedom does not carry charge (move in straight line in B field)
- What is the nature of thes low-energy degrees of freedom?

At half filling, a quasiparticle appears which moves in straight line



(Kamburov et al, 2014)

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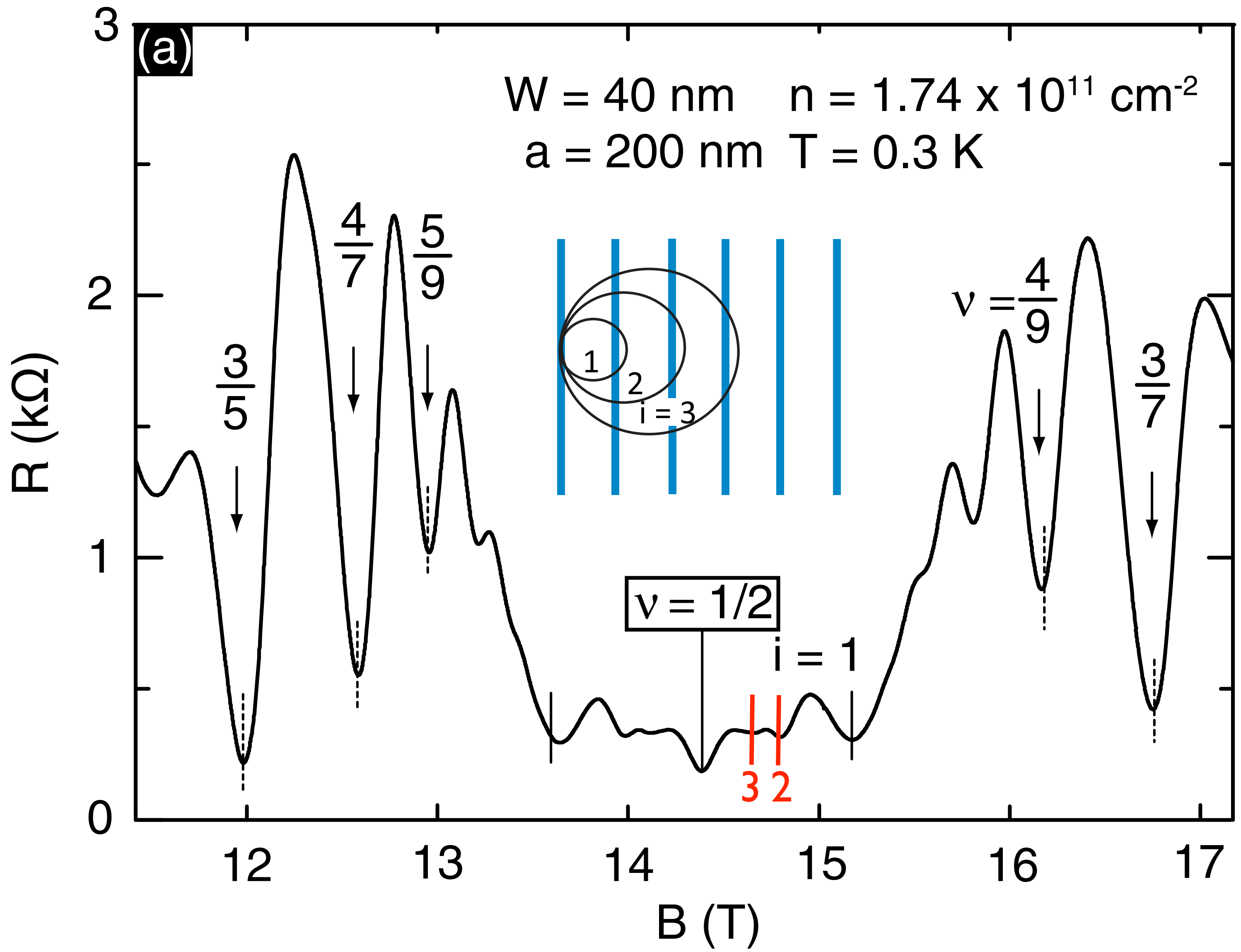

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What is the nature of this quasiparticle?

Quasiparticle

- Quasiparticle is a central notion in CMP
- Example: Landau's quasiparticle in a Fermi liquid (e.g., electron gas in a metal)
    - Landau's quasipartIce = dressed electron of QED

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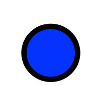

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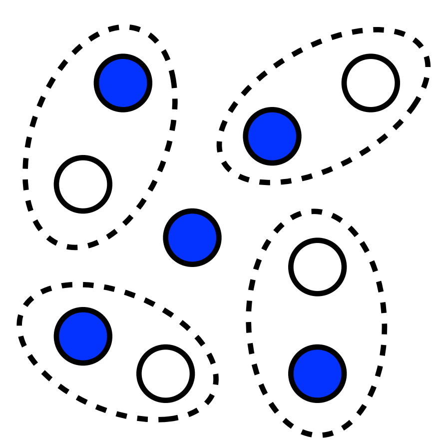

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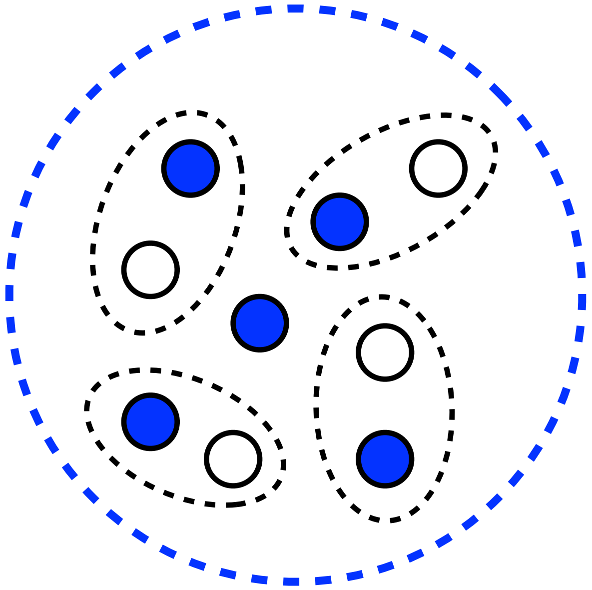

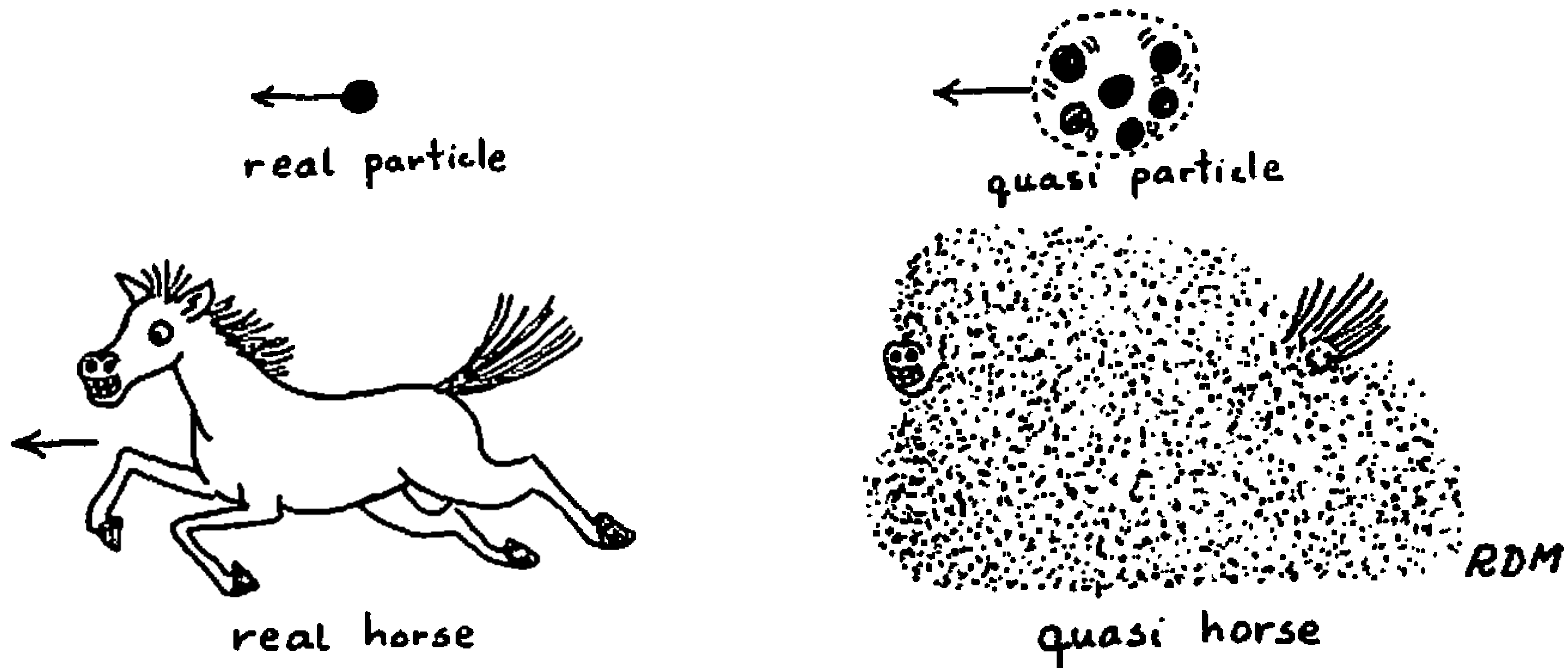

Fig. 0.4 ***Quasi Particle Concept***

from R. Mattuck, A Guide to Feynman Diagrams in the Many-Body Problem

- Landau’s quasiparticle can have a mass drastically different from the bare mass of the electron

- But two things it cannot compromise: charge and spin

    - charge: it cannot be the quasiparticle of the half-filled Landau level: the latter can move in straight line!

Duality in 2+1D QFT

- The problem of understanding the nature of the half-filled Landau level and its quasiparticle catalyzed new progress in QFT

- “Web of duality” between 2+1D quantum field theories

- Duality: equivalence between two QFTs whose Lagrangians look very differently, but have the same IR behavior

- Most of these dualities remain unproved conjectures, but with strong arguments supporting their validity

Fermionic particle-vortex duality

DTS; Metlitski, Vishwanath; Wang, Senthil 2015

free fermion = "QED" in 2+1 D

physical EM field

Electron: $$\mathcal{L} = i\bar{\psi}_e\gamma^\mu(\partial_\mu - iA_\mu)\psi_e$$

Quasiparticle: $$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda$$

emergent U(1) gauge field

Quasiparticle not coupled to the physical electromagnetic field but to an emergent U(1) gauge field

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- The example of half-filled Landau level illustrates a nontrivial interaction between theory and experiment:
  - a puzzle about the behavior of a physical system in the lab spurs a development in formal quantum field theory

The “seed duality”

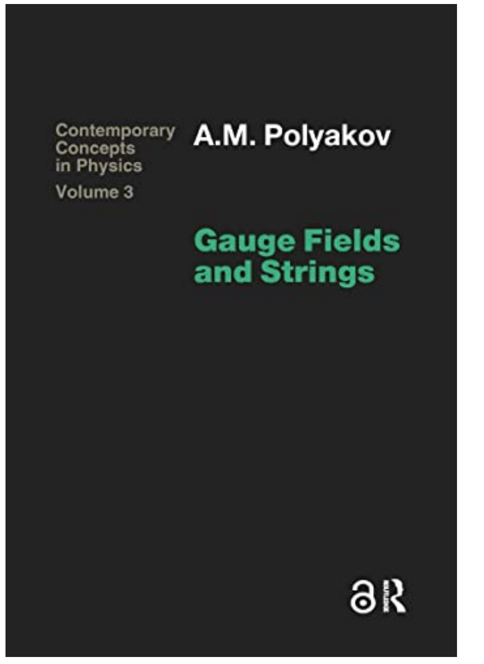
fermion = boson + flux

$$\mathcal{L} = L[\psi, A] - \frac{1}{2}\frac{1}{4\pi}AdA \qquad \mathcal{L} = L[\phi, a] + \frac{1}{4\pi}ada + \frac{1}{2\pi}Ada$$

From this duality, a whole “web” of new dualities is derived

Karch, Tong; Seiberg, Senthil, Wang, Witten

Universal language



- “We have no better way of describing elementary particles than field theory”
- Many phases of condensed matter are described by quantum field theory
- CMP phenomena drive a better understanding of QFTs in the nonperturbative regime
- A large chunk of CMP is to understand the content of the low-energy effective field theorie (and also possible phases beyond QFT)