# The y-type polarised kinetic Sunyaev-Zeldovich effect - Pairwise & cross-pairwise estimator, E and B modes


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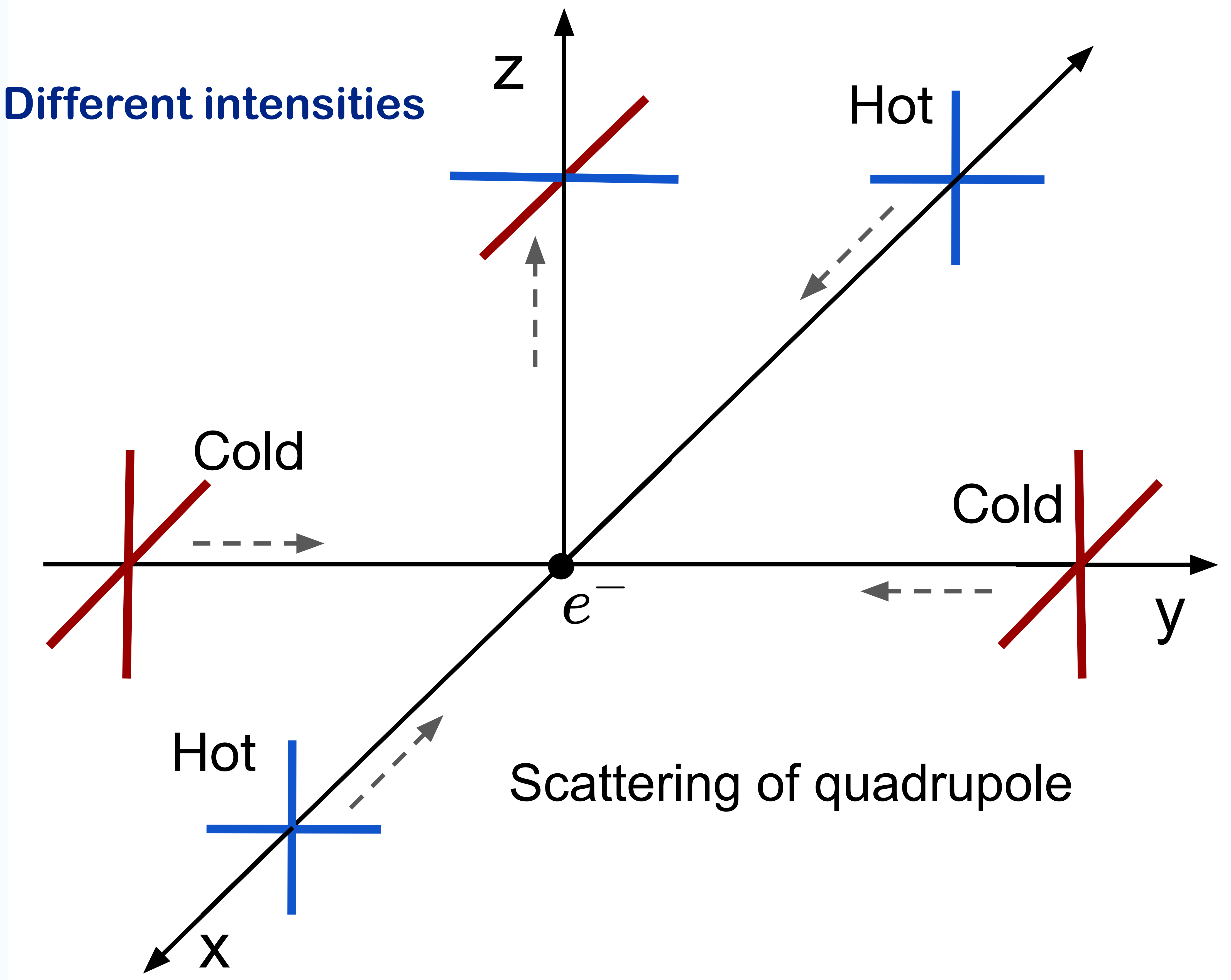

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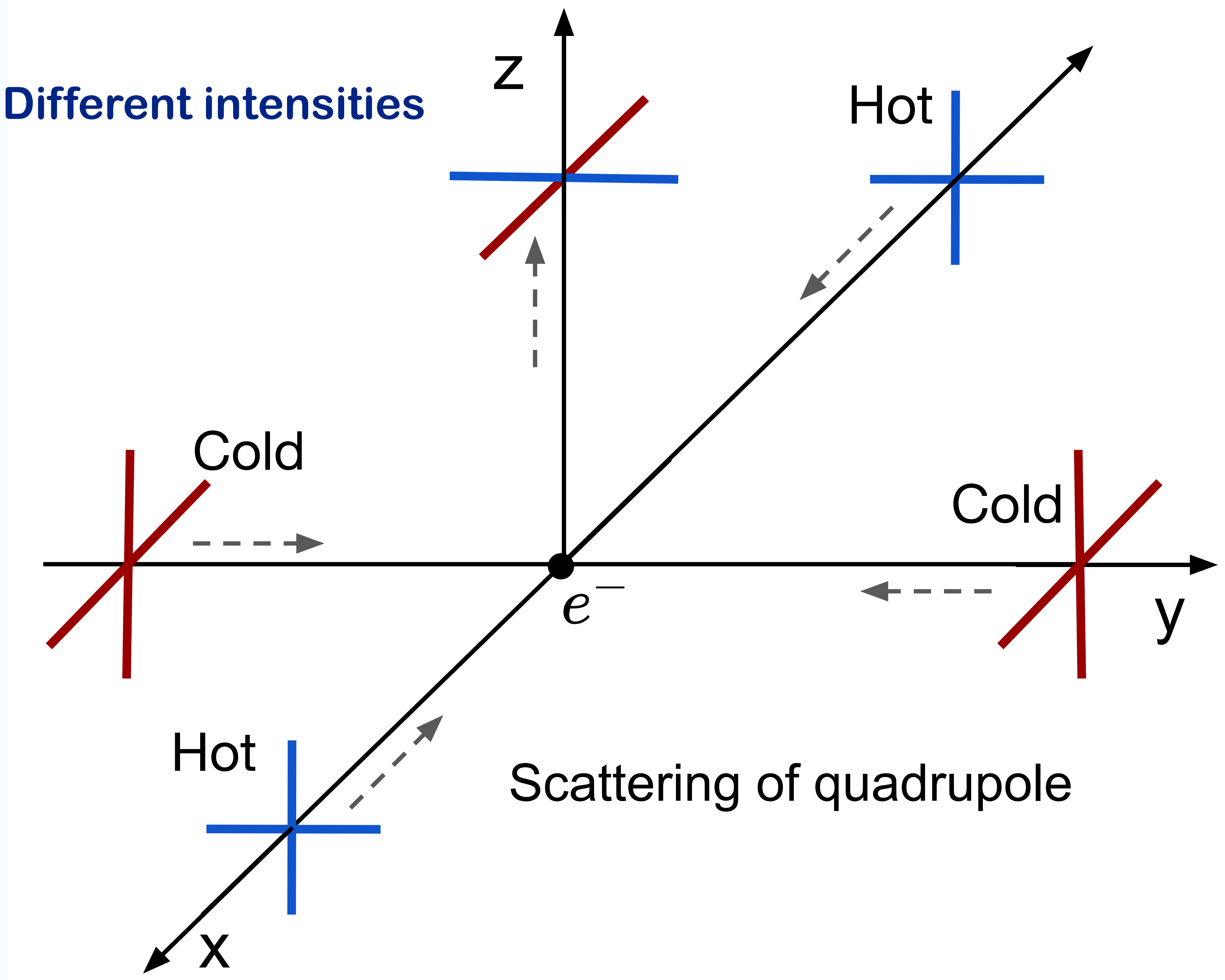

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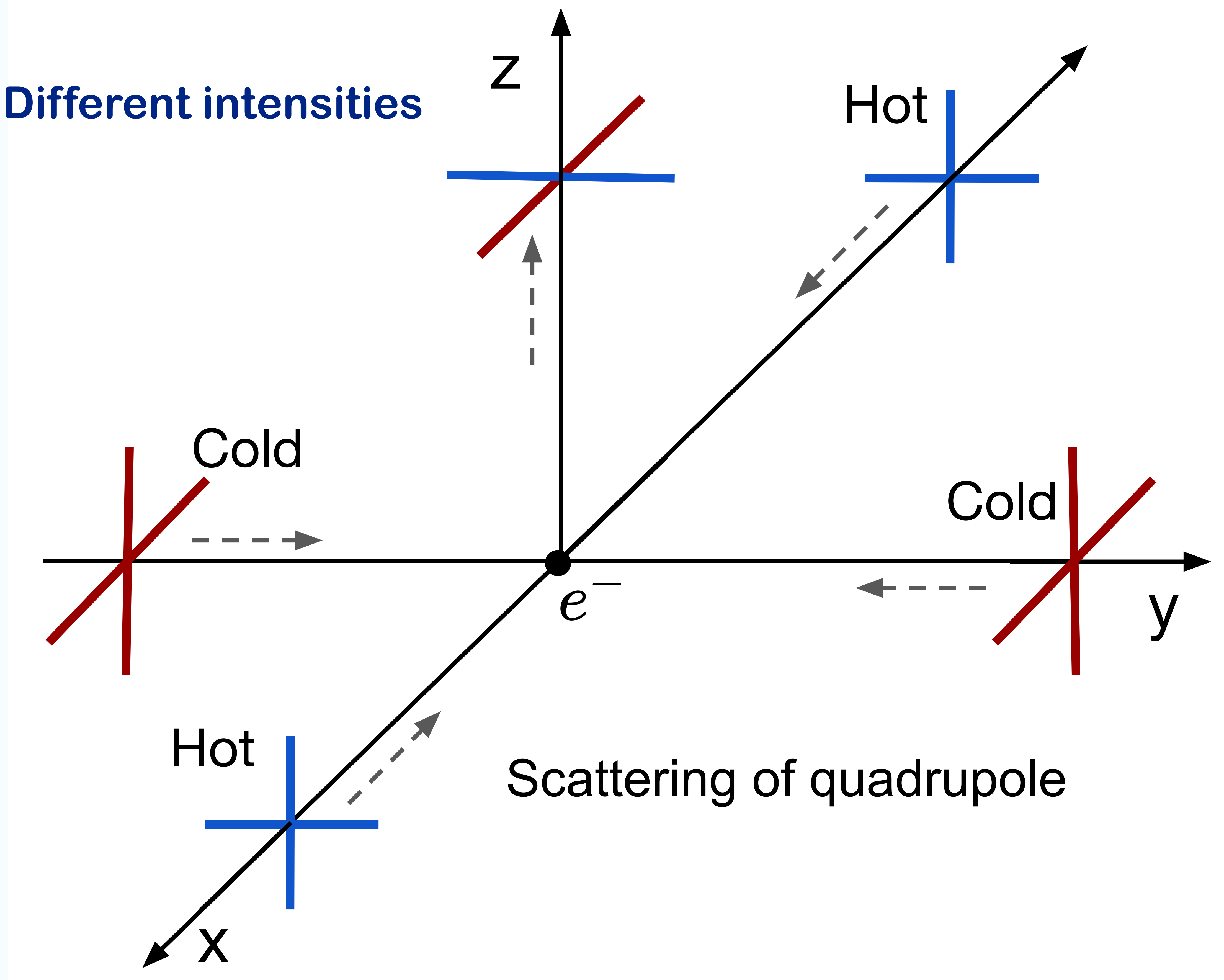
- Clusters have peculiar velocities; CMB is not isotropic in electron's rest frame - In particular a quadrupole is present.
- Thomson scattering of this CMB quadrupole by the free electrons produces polarisation.
- Scattered spectrum not only has a differential blackbody but also a y-type distortion.

$$\left(\frac{\delta I}{I}\right)\Bigg|_{\text{(quadrupolar)}} = 2\left(\mathbf{v}\cdot\hat{\mathbf{n}}'\right)^2 g(x) + \frac{1}{2}y(x)\left(\mathbf{v}\cdot\hat{\mathbf{n}}'\right)^2$$

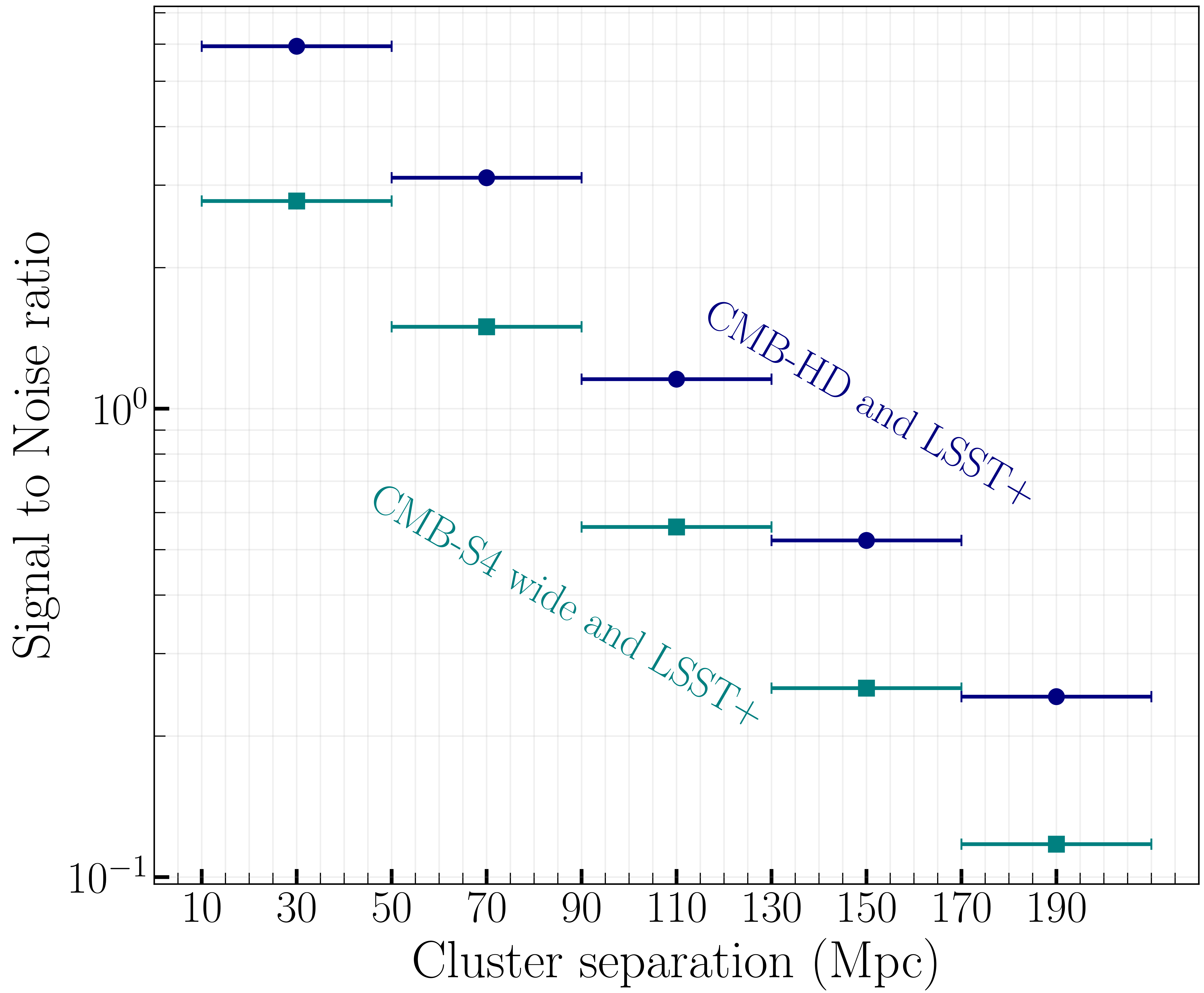
$$x = \frac{h\nu}{k_B T_0}$$

$$y(x) = \frac{xe^x}{(e^x-1)}\left(x\frac{e^x+1}{e^x-1} - 4\right)$$



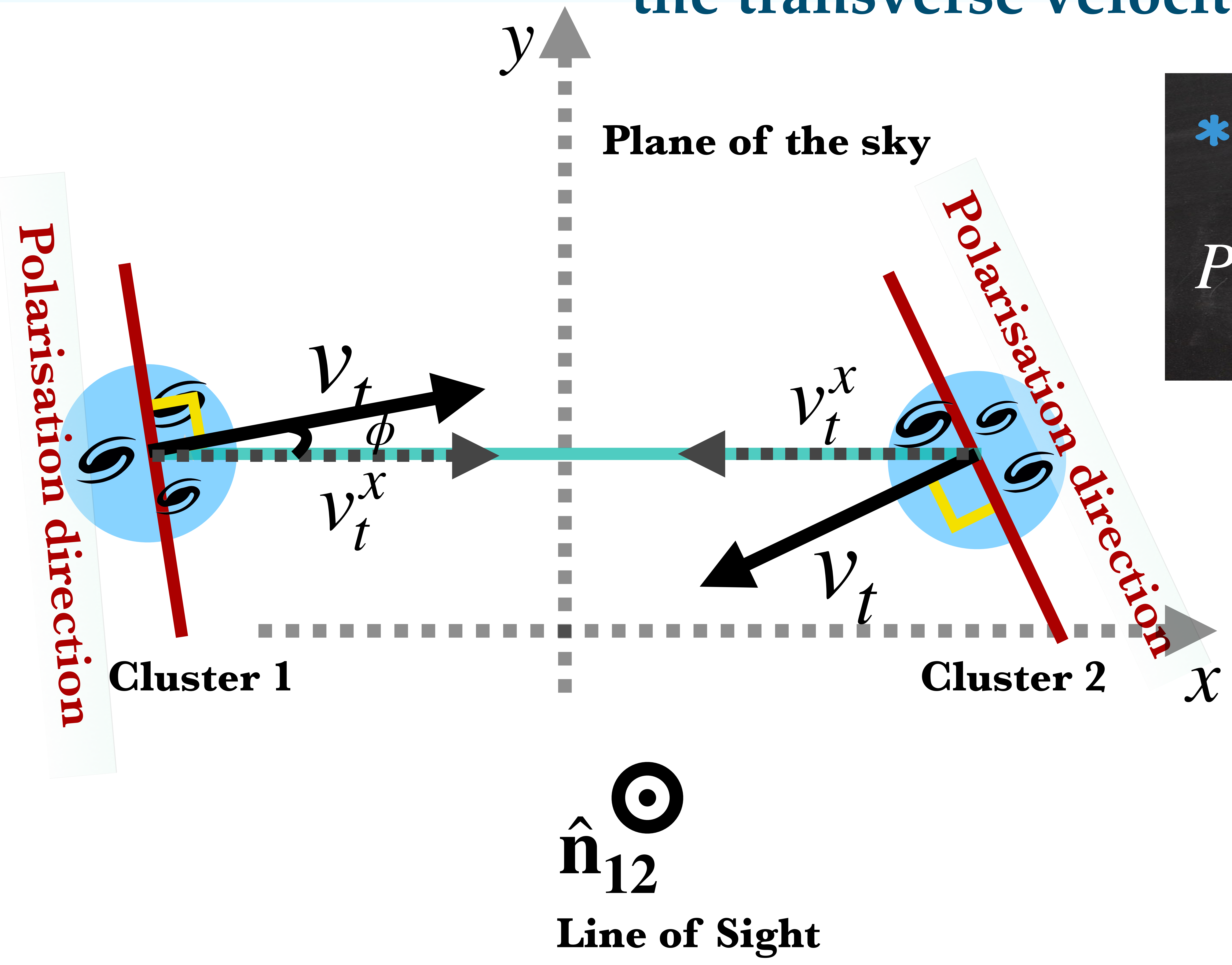

- Predicted by Sunyaev and Zeldovich in 1980.
- Previous works: Renaux-Petel et al. 2013 Hotinli et al. 2022

# A new pairwise-framework to detect the pkSZ effect




arXiv: 2308.01370

# Polarisation direction is always perpendicular to the transverse velocity direction




* The polarisation field : $(\mathcal{Q} \pm i\mathcal{U})(\hat{\mathbf{n}}) \equiv P_{\pm}(\hat{\mathbf{n}})$

$$P_{+}(\hat{\mathbf{n}} \equiv \hat{\mathbf{z}}) = -\frac{1}{10}\tau_{\rm eff}\, v_t^2(\mathbf{x})\, e^{-2i\phi}$$

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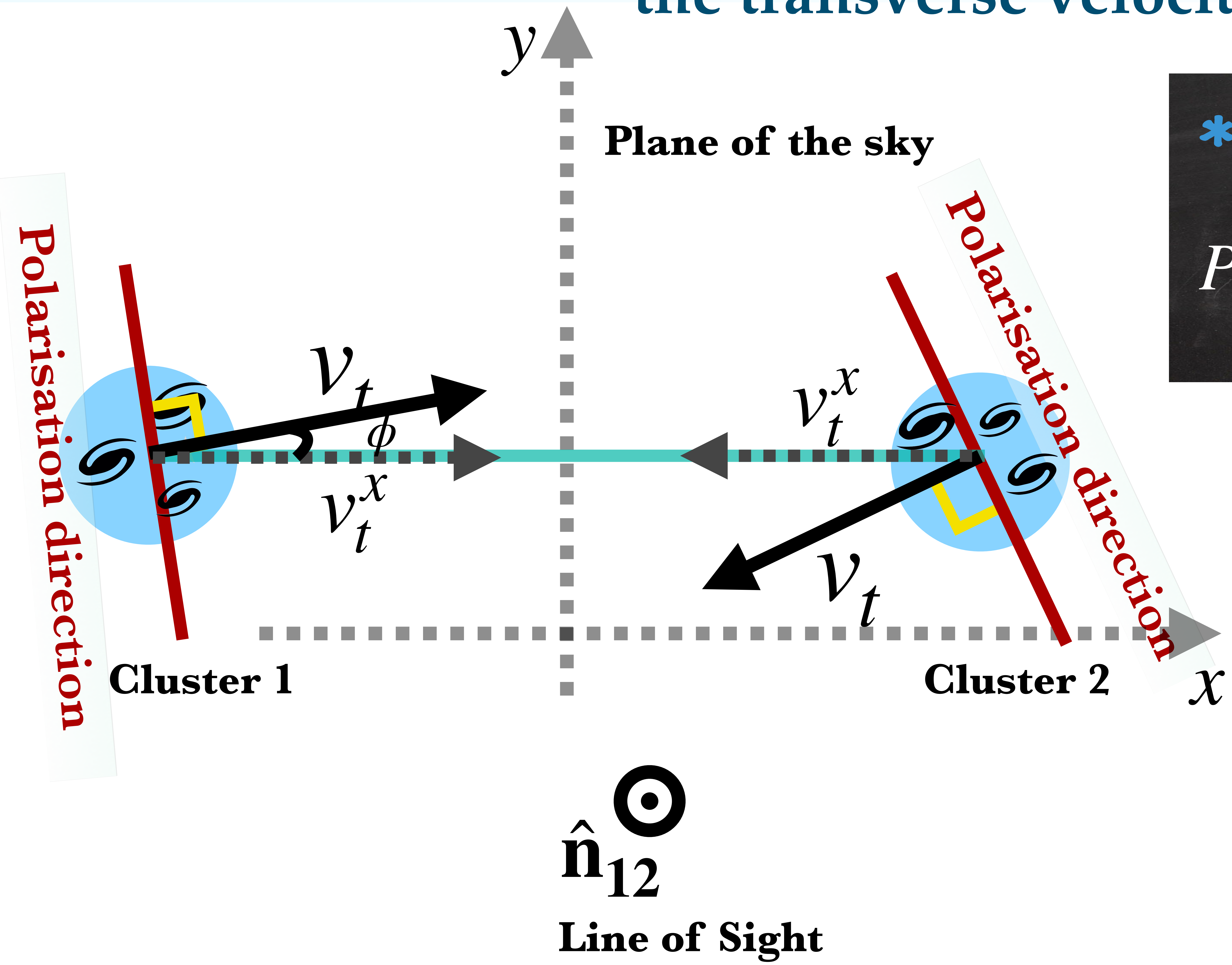


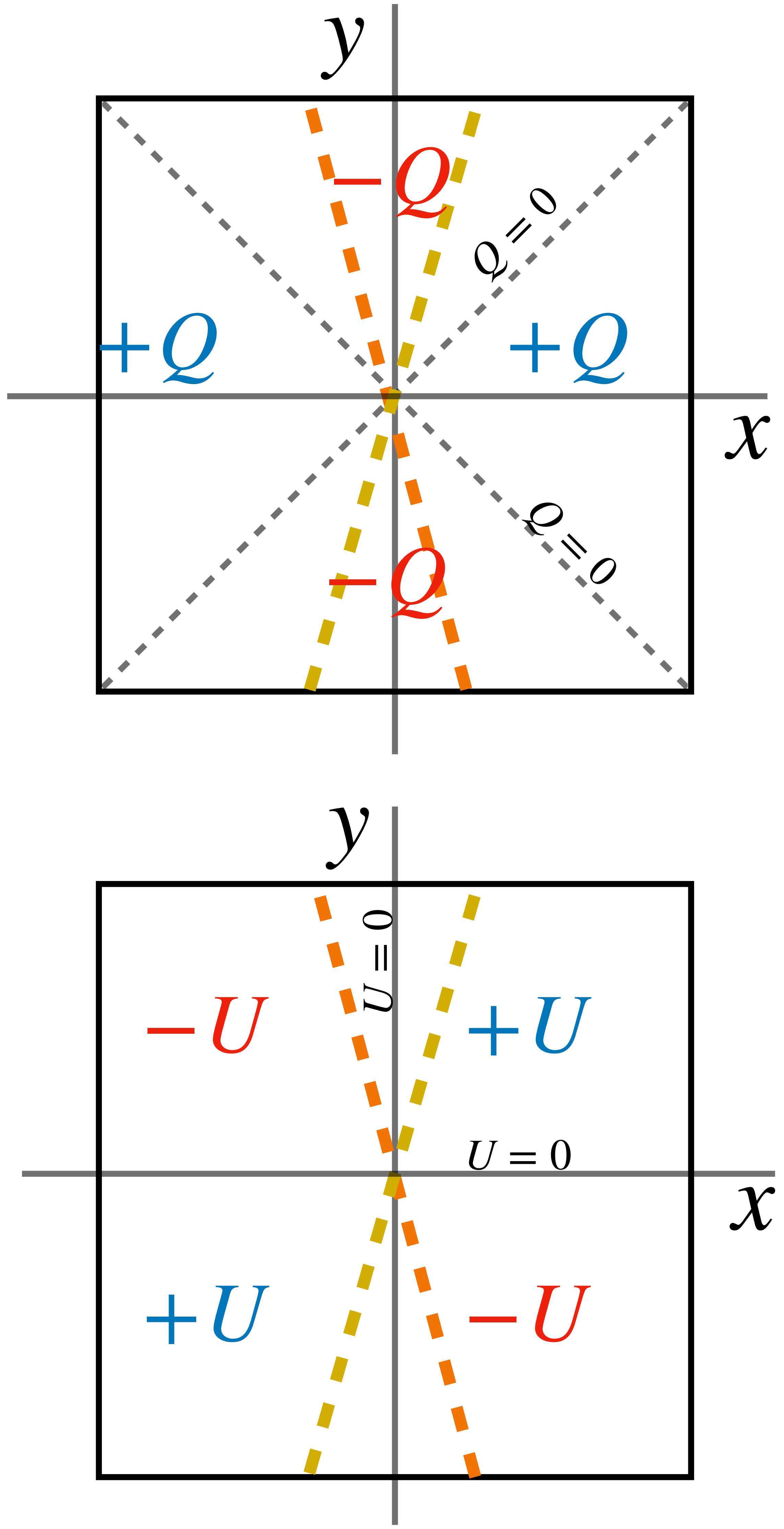

- The polarisation field : $\left(\mathcal{Q} \pm i\mathcal{U}\right)\left(\hat{\mathbf{n}}\right) \equiv P_{\pm}\left(\hat{\mathbf{n}}\right)$

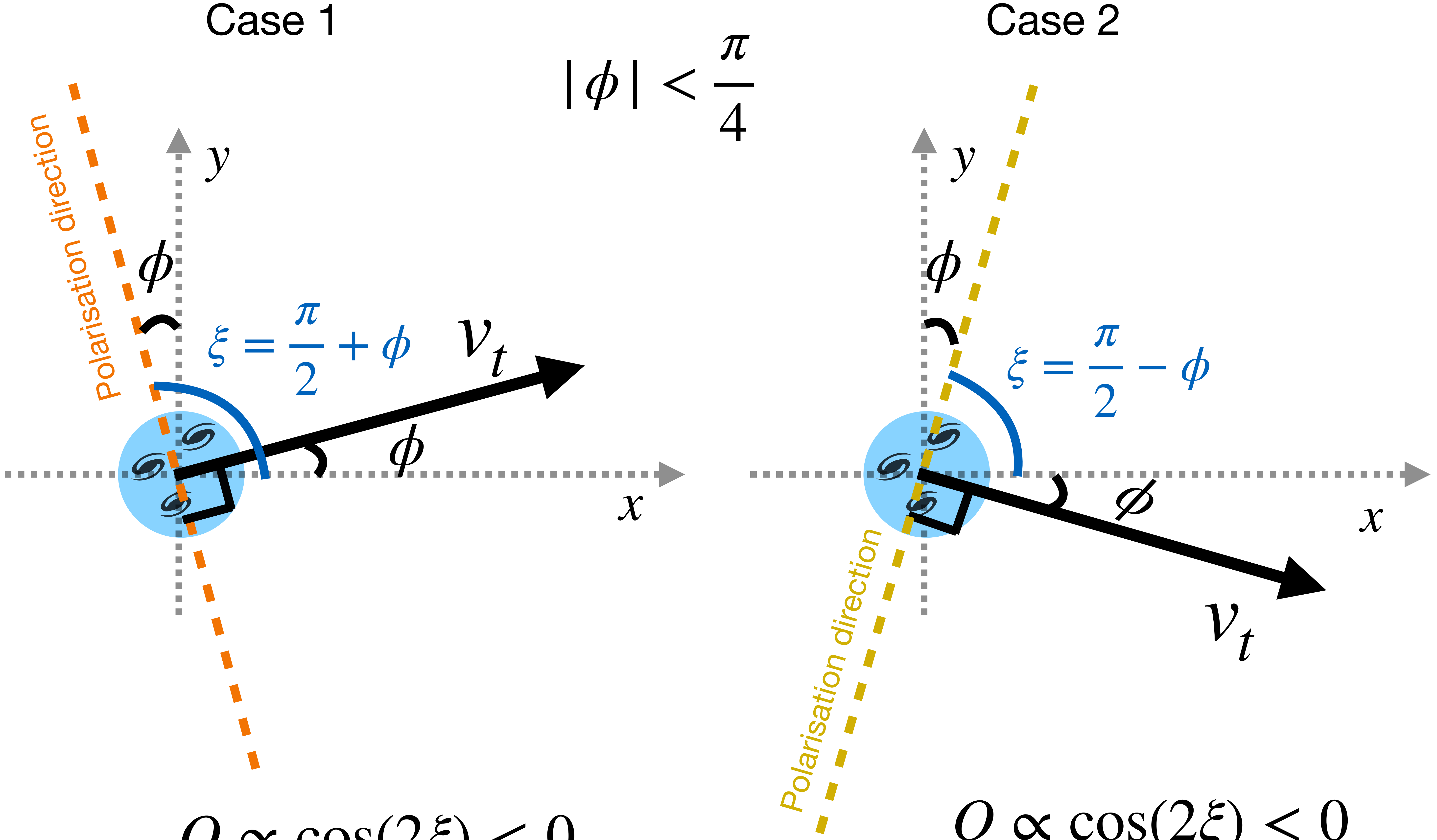
$$P_+\left(\hat{\mathbf{n}} \equiv \hat{\mathbf{z}}\right) = -\frac{1}{10}\,\tau_{\rm eff}\, v_t^2(\mathbf{x})\, e^{-2i\phi}$$

- Clusters which are close to each other will have peculiar velocities ~ towards each other

- Averaging over many clusters which are at a fixed separation will generate a net non-zero polarisation signal.

# Coherent addition of the Q parameter gives a net non-zero polarisation signal.




$Q \propto \cos(2\xi) < 0$

$U \propto \sin(2\xi) < 0$

$Q \propto \cos(2\xi) < 0$

$U \propto \sin(2\xi) > 0$

# Theoretical formalism of the pairwise estimator

$$\hat{P}_{\text{pairwise}}(x) = \sum_{\text{i}} w_i \left(P_{i1+} + P_{i2+}\right)\Bigg|_{\substack{\text{separation along} \\ \text{x - axis}}}$$

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$$\left\langle \hat{P}_{\text{pairwise}}(x) \right\rangle \propto \left\langle \left(P_{i1+} + P_{i2+}\right) \right\rangle = P_{\text{pairwise}}(\mathbf{x}, \hat{\mathbf{n}}_{\mathbf{12}} \,|\, m, \chi)$$

# Pairwise signal - dependent cosmological and astrophysical parameters

$$\hat{P}_{\text{pairwise}}(x) = \sum_{\text{i}} w_i \left(P_{i1+} + P_{i2+}\right)\Big|_{\text{separation along x-axis}}$$

$$\left\langle \hat{P}_{\text{pairwise}}(x) \right\rangle \propto \left\langle (P_{i1+} + P_{i2+}) \right\rangle \doteq P_{\text{pairwise}}(\mathbf{x}, \hat{\mathbf{n}}_{\mathbf{12}} \,|\, m, \chi)$$

Thomson Optical depth of clusters

$$P_{\text{pairwise}}(\mathbf{x}, \hat{\mathbf{n}}_{\mathbf{12}} \,|\, m, \chi) = \left[ \frac{\sqrt{\pi}}{10\pi^5} D^4 (Hfa)^2 \, \tau_{\text{eff}} \, Y_{2-2}(\hat{\mathbf{x}}; \hat{\mathbf{n}}_{\mathbf{12}}) \sum_{L_1, L_2} \sum_{q=0}^{2} \sum_{l} i^{(L_1+L_2)} (-1)^{(L_1+1)} (2L_1+1)(2L_2+1)(2l+1) \right.$$

$$\frac{q!}{(q-l)!!(q+l+1)!!} \begin{pmatrix} 1 & L_1 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & L_2 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L_1 & L_2 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & L_2 & L_1 \\ l & 1 & 1 \end{pmatrix}$$

$$\left. \int dk_1 dk_2 \, k_1^2 k_2^2 \, G_q(k_1, k_2, b_1, b_2) j_{L_1}(k_1 x) \, j_{L_2}(k_2 x) P(k_1) P(k_2) \right] \left[ 1 + \frac{D^2 \, b_1^2}{2\pi^2} \int dk \, k^2 j_0(kx) P(k) \right]^{-1}$$

Cosmological parameters

Cluster bias factors

Linear matter power spectrum

$$P_{\text{pairwise}}(\mathbf{x}, \hat{\mathbf{n}}_{12} \,|\, m, \chi) = A \left[(D^2 H f a)^2\right](\chi)\, \tau_{\text{eff}}(m, \chi) \left[ \frac{b_1(m,\chi)\, C_1(x) + b_2(m,\chi)\, C_2(x)}{1 + D^2(\chi)\, b_1^2(m,\chi)\, C_3(x)} \right] Y_{2-2}(\hat{\mathbf{x}}; \hat{\mathbf{n}}_{12})$$

Denoting the orientation of the cluster pair wrt LOS

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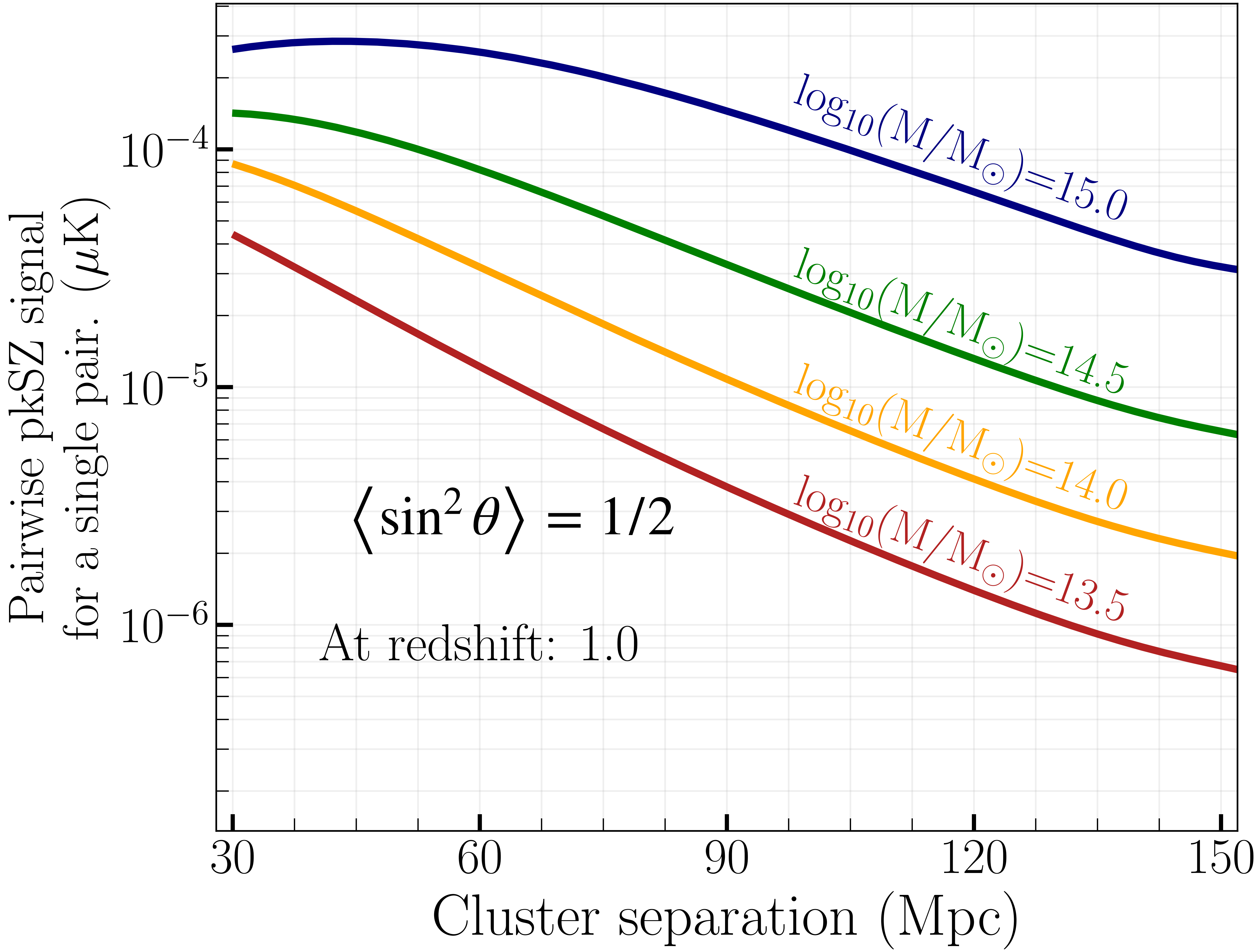
**Smaller mass cluster have lower optical depth, thus lower polarisation signal.**



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## Optimal estimator

$$w_i = \frac{1}{\sum_i m_i \sin^2\theta_i} \, m_i \sin^2\theta_i$$

$$\hat{P}_{\text{pairwise}}(x) = \sum_{i} w_i \left(P_{i1+} + P_{i2+}\right)\Big|_{\substack{\text{separation along}\\ \text{x - axis}}} = \sum_{i} w_i\, P_{\text{pairwise}}\left(x \,|\, m_i, \chi_i, \theta_i\right)$$

Optimal estimator

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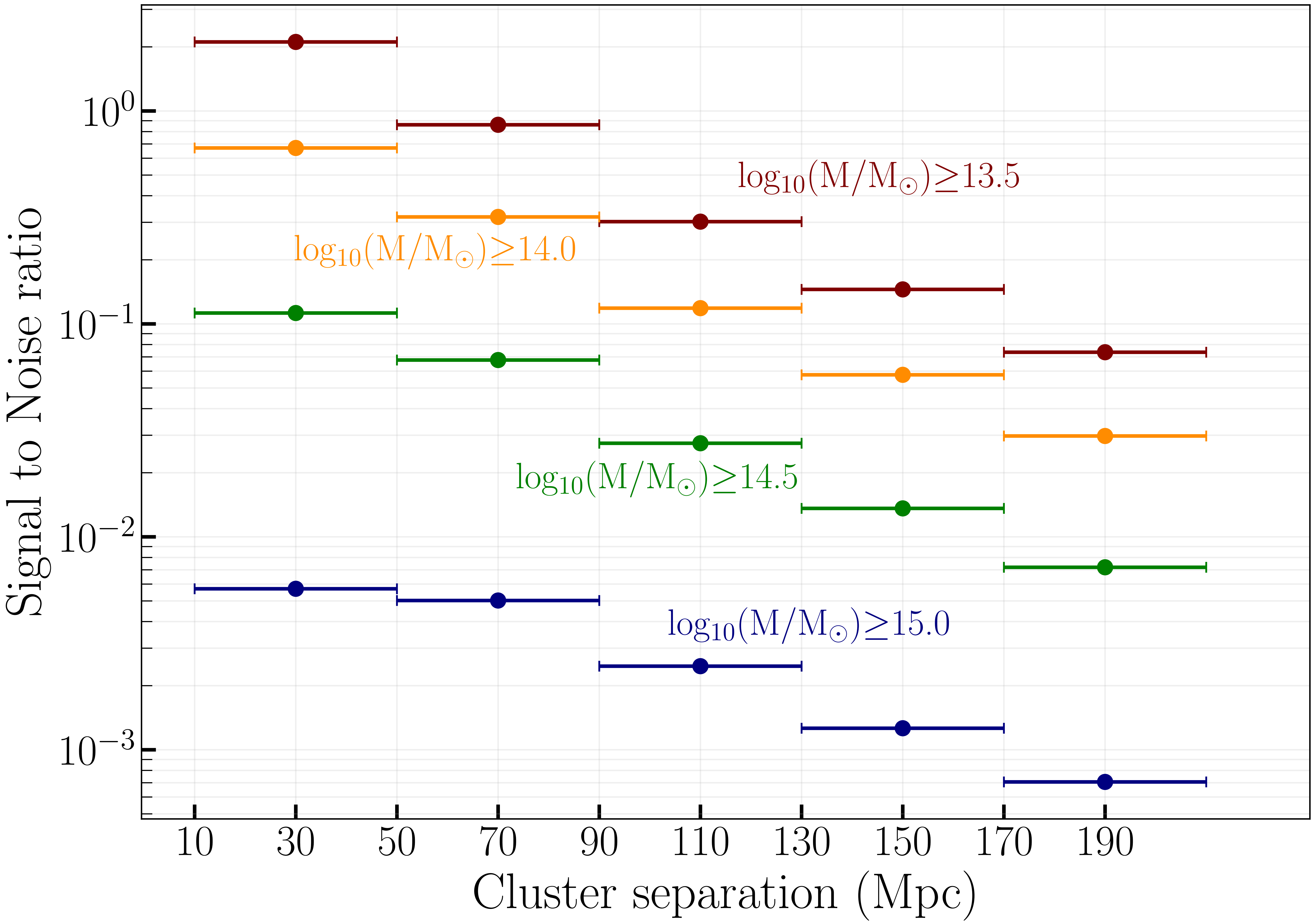
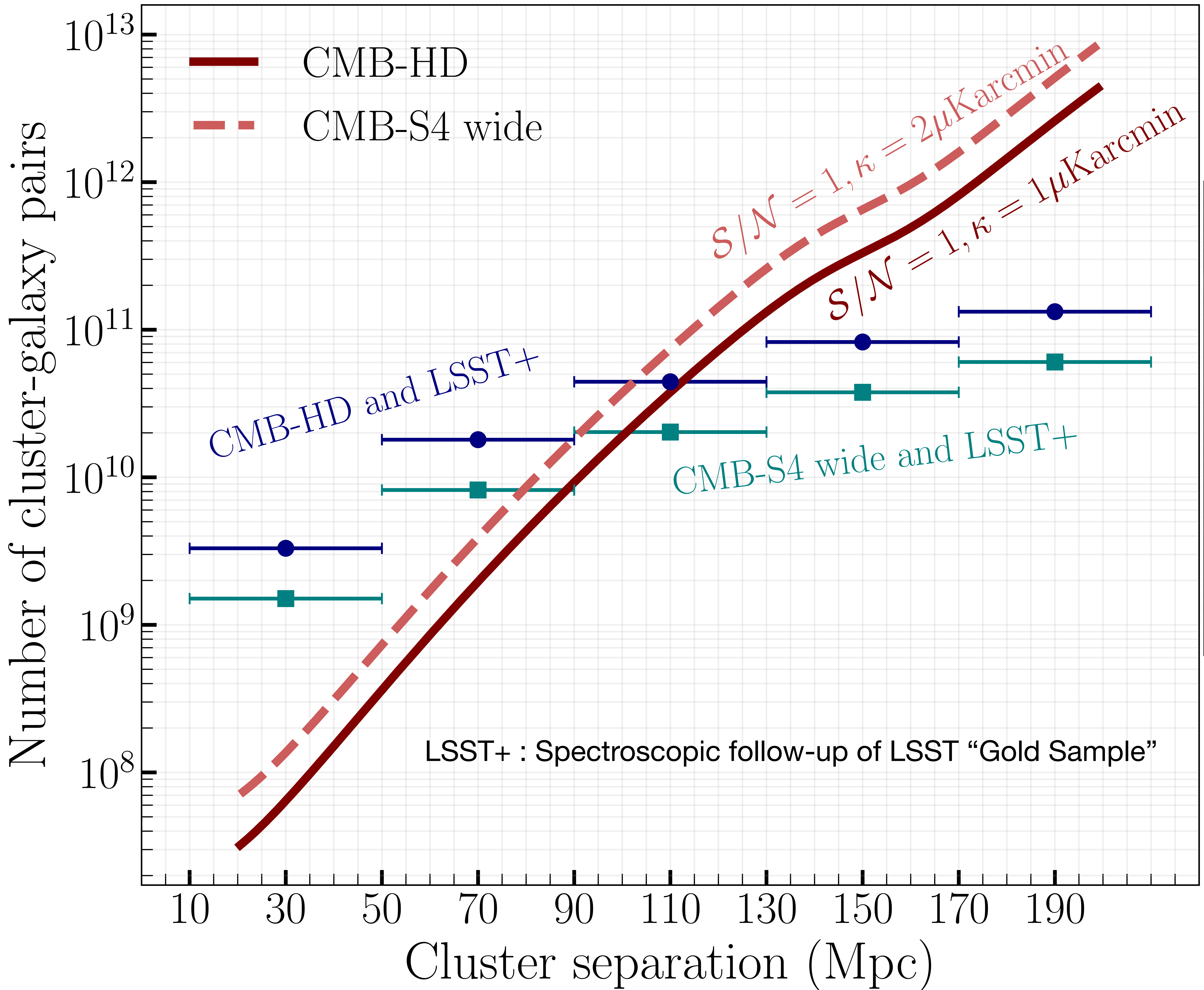
**Detecting all clusters with**

$\log_{10}\left(M_{\text{min}}^{500c}/M_{\odot}\right) \geq 13.5$

**will enable a detection of the pairwise pkSZ effect .**



# Cross-pairwise pkSZ effect




* In general, we can pair up a cluster with any other indicator of the large-scale gravitational potential around the cluster.

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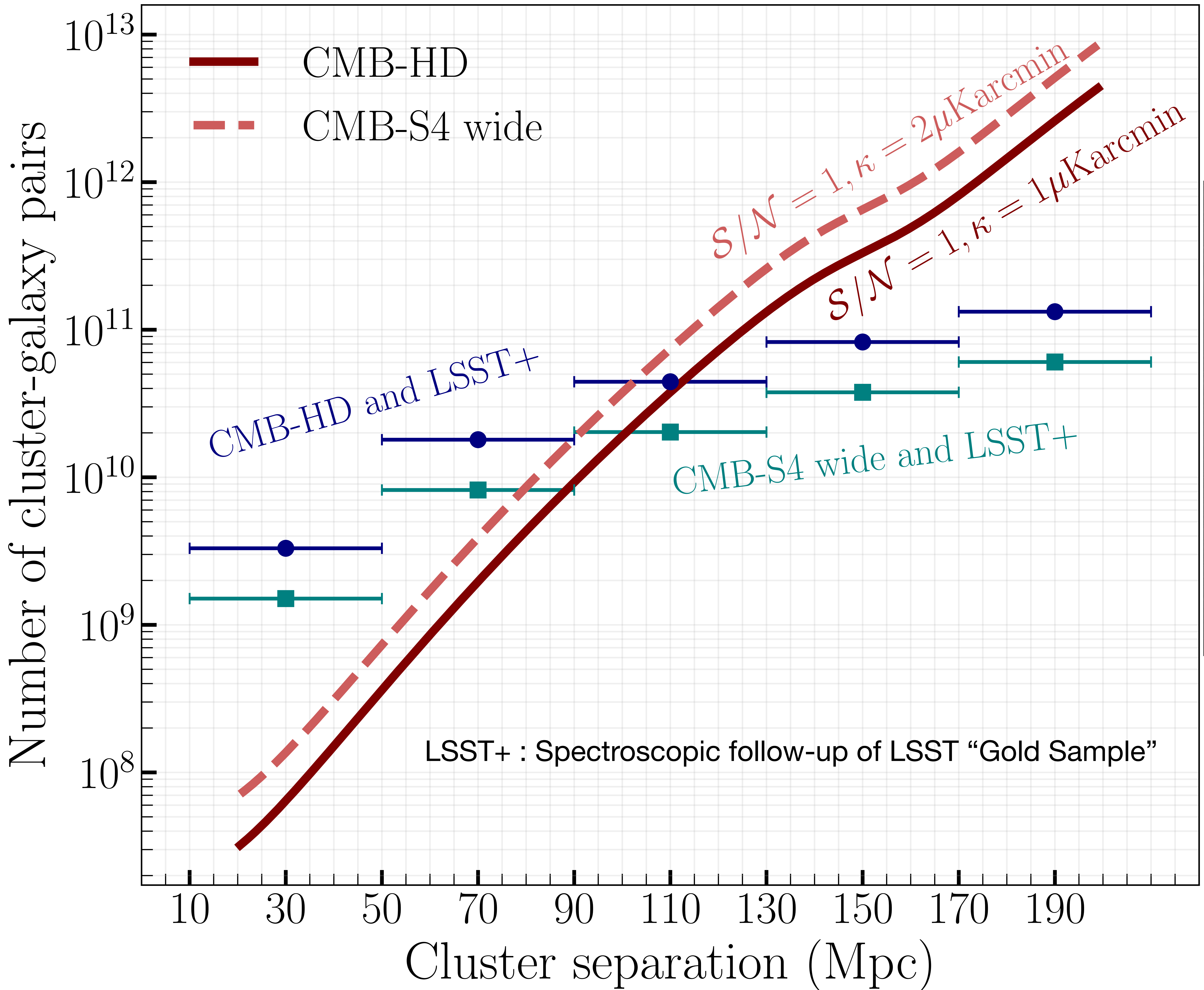


- In general, we can pair up a cluster with any other indicator of the large-scale gravitational potential around the cluster.
- Clusters being rare - noisy tracers, better to use galaxies.

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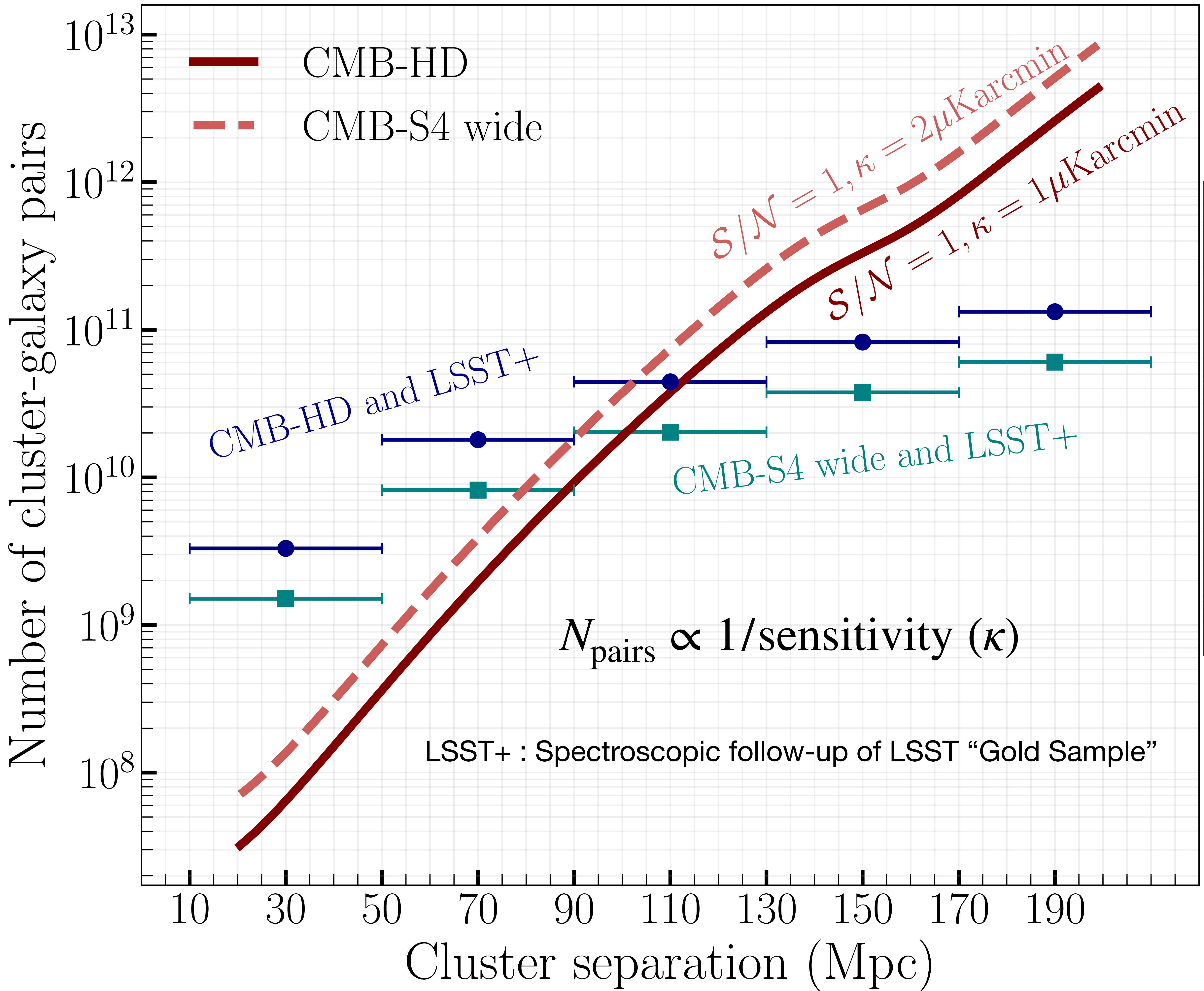
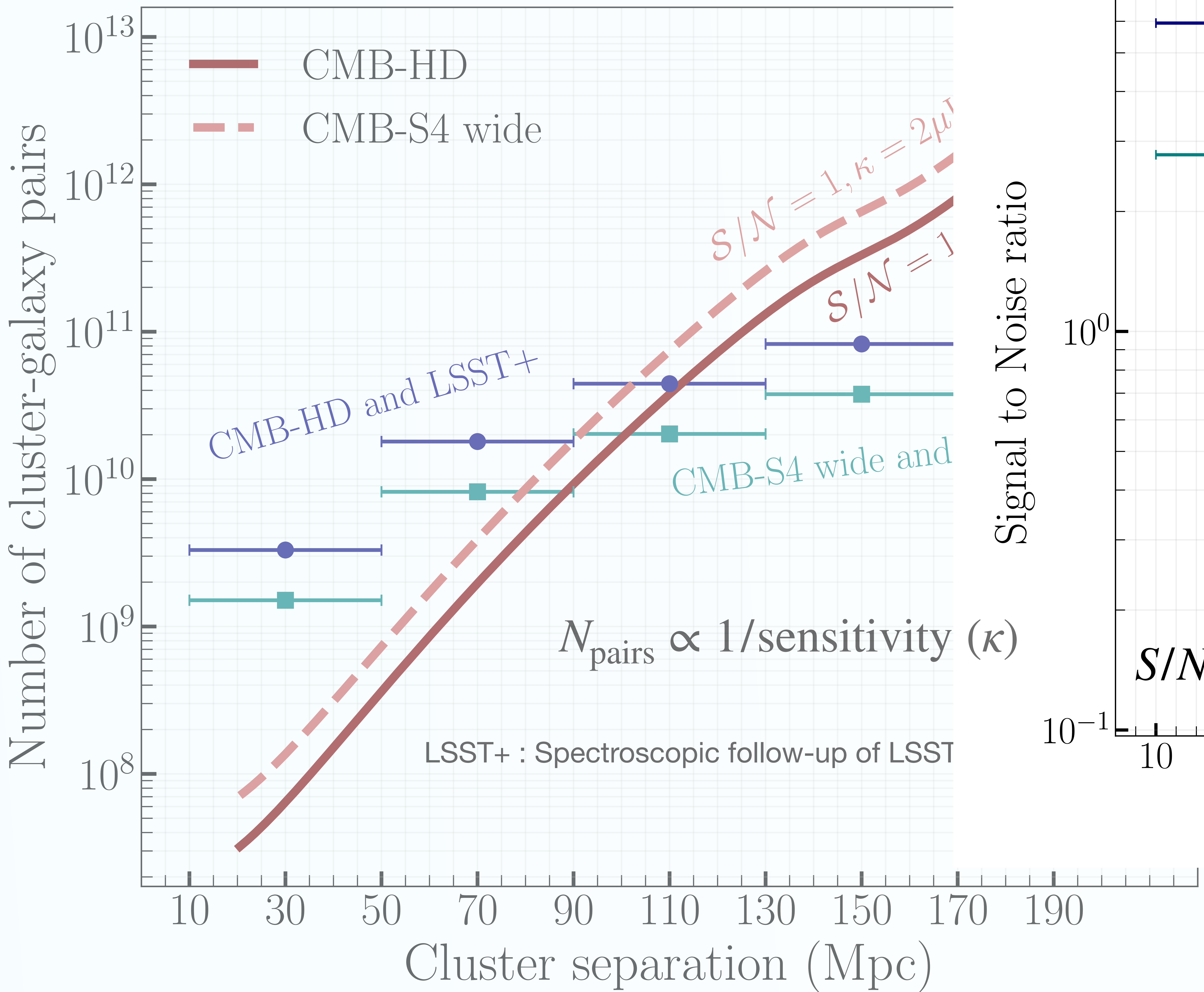
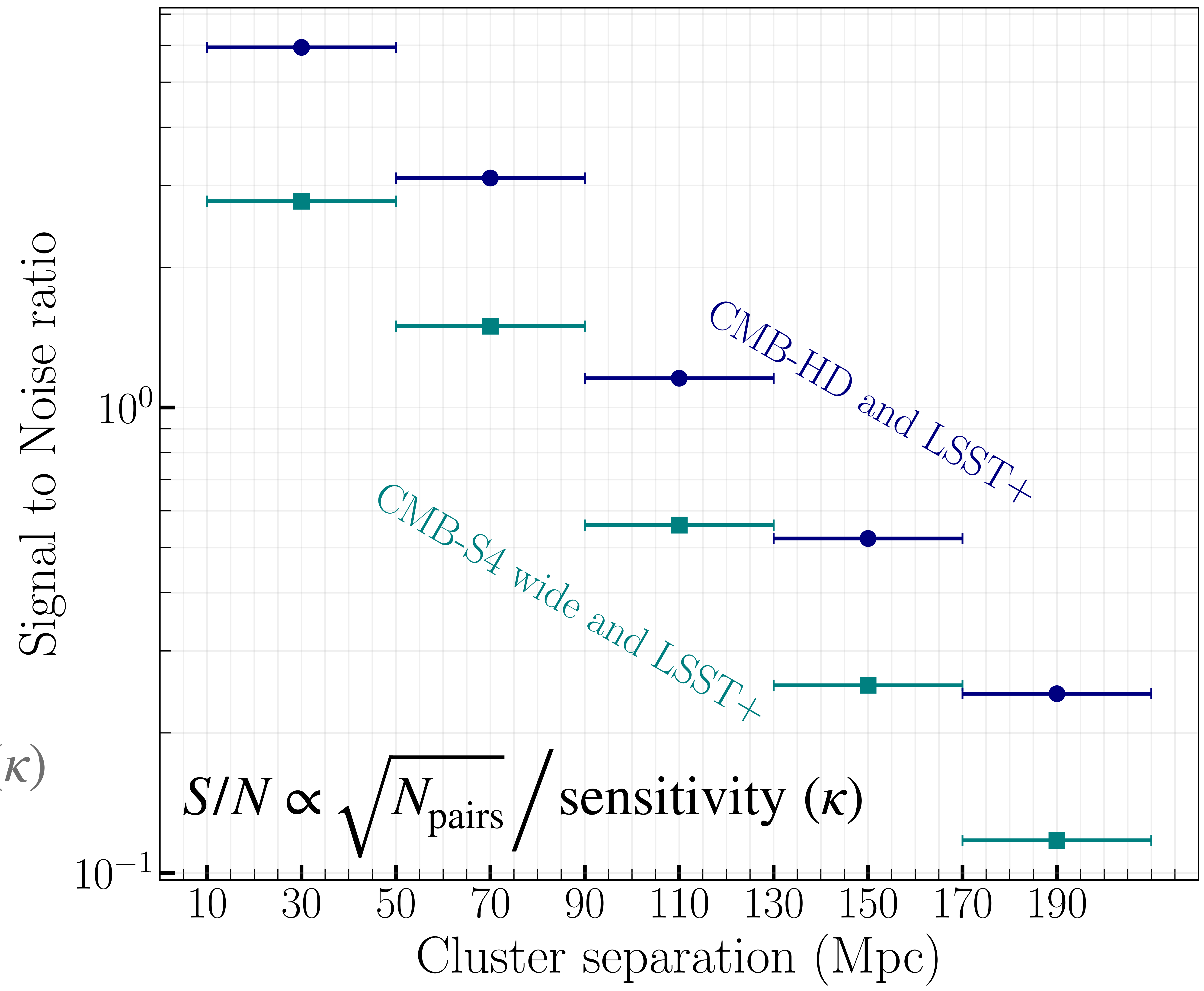



- In general, we can pair up a cluster with any other indicator of the large-scale gravitational potential around the cluster.
- Clusters being rare - noisy tracers, better to use galaxies.
- Polarisation signal from galaxies is negligible, but they are more numerous.

# Cross-pairwise pkSZ effect

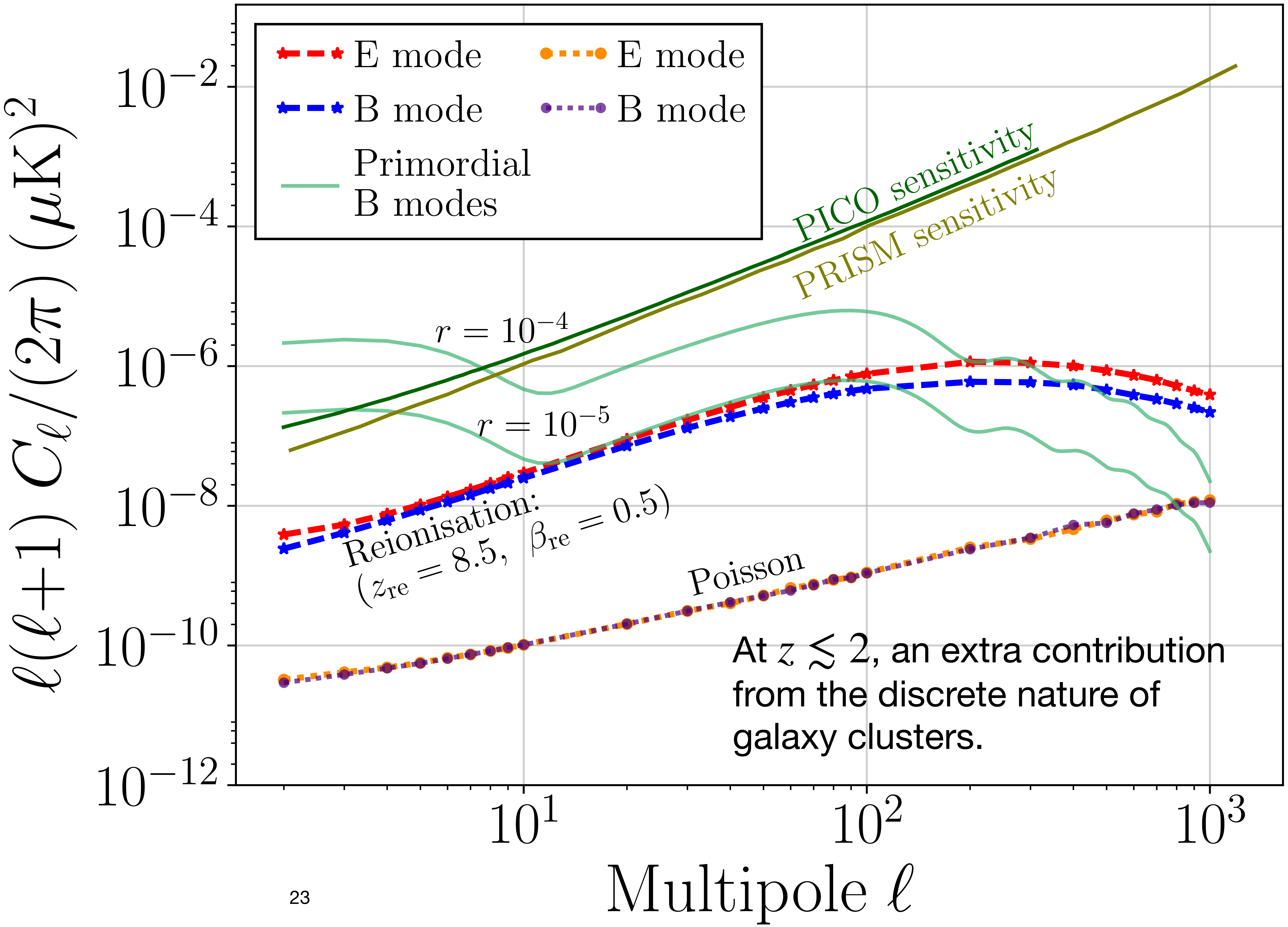


# Beating the cosmic variance with pkSZ effect

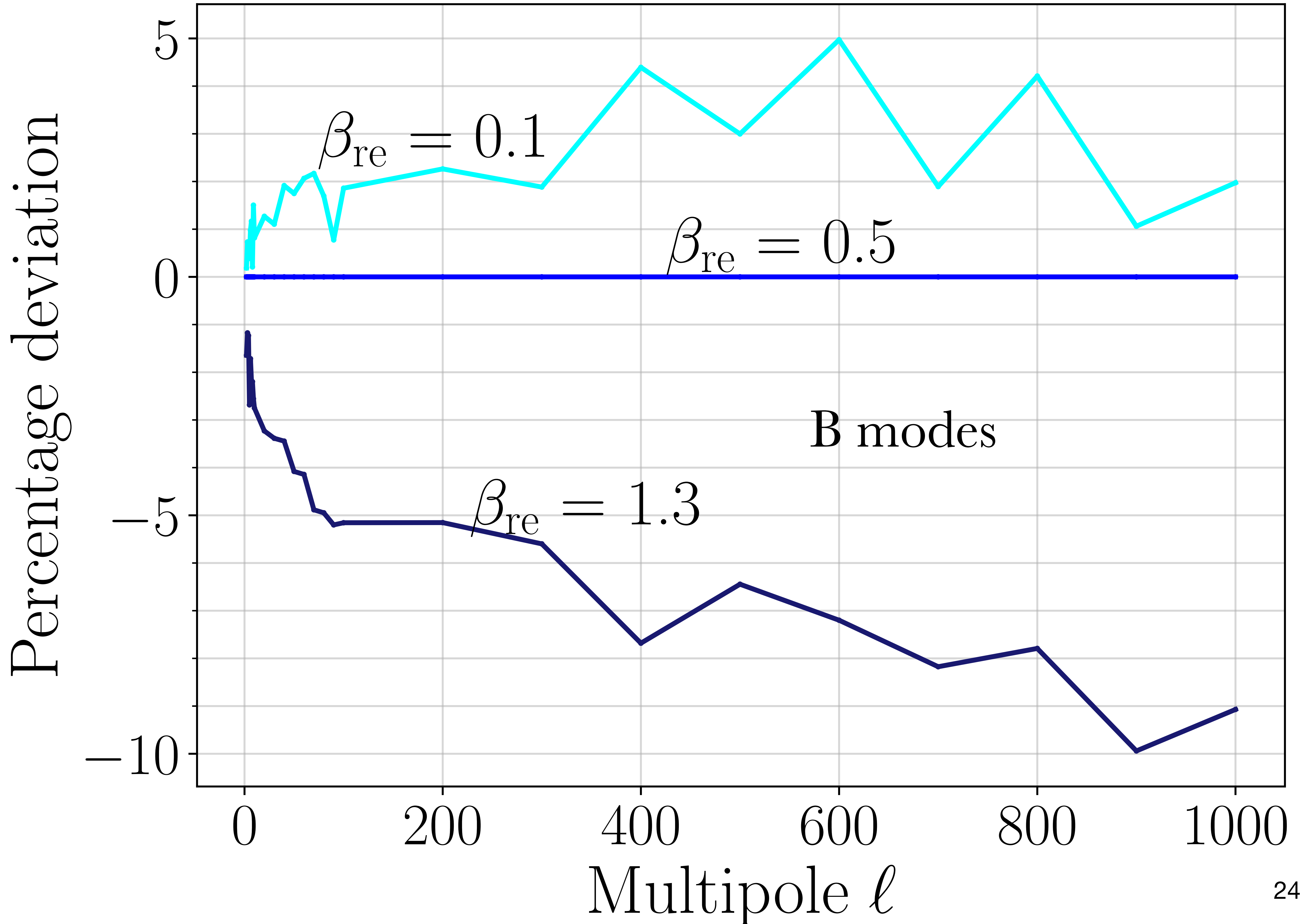
arXiv: 2208.02270 JCAP10(2022)056

- Sensitive to the reionisation history - how fast reionisation happens.
- A way to beat the cosmic variance of primary CMB anisotropies.

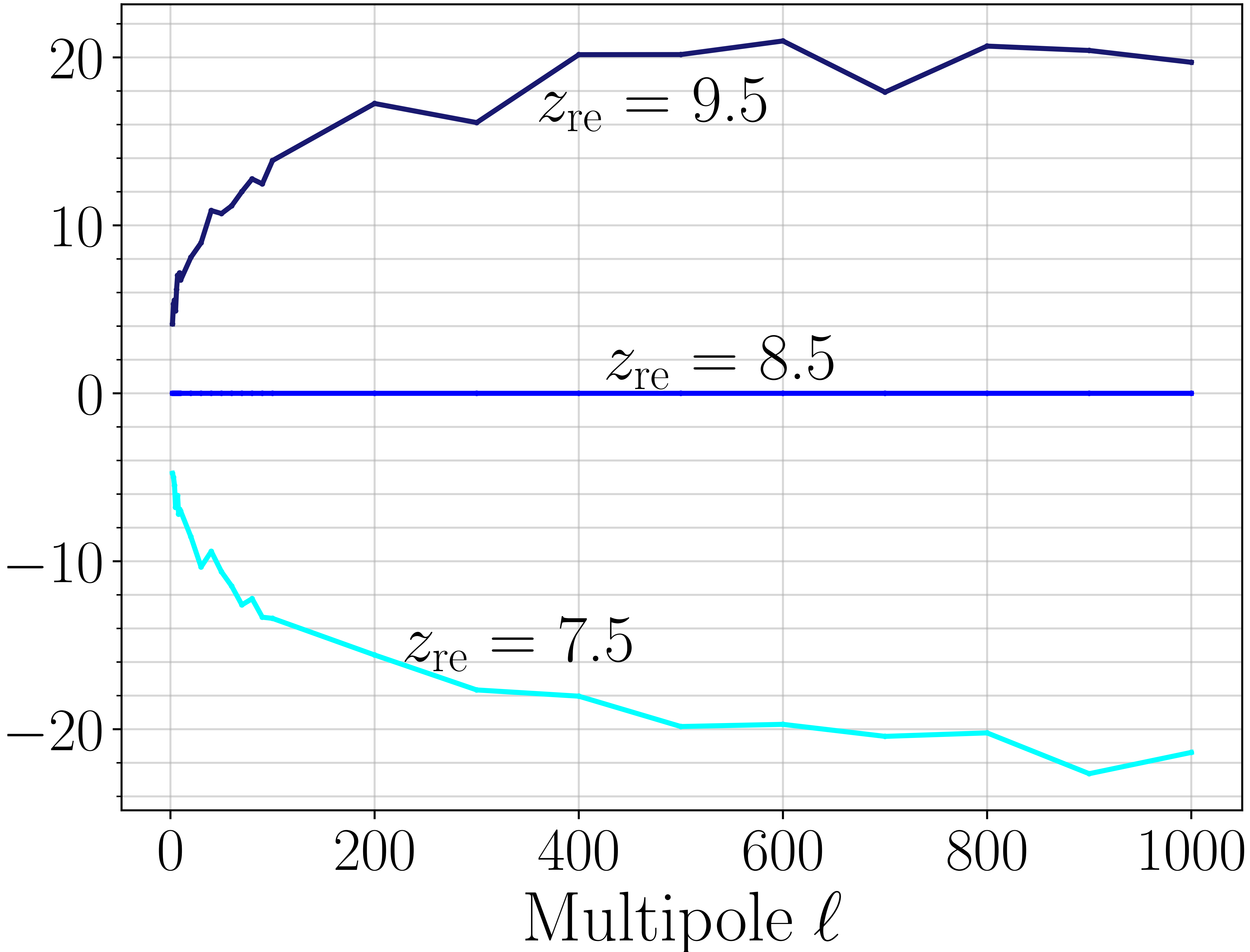


# The pkSZ effect is sensitive to both optical depth and reionisation history

**Varying width of reionisation**



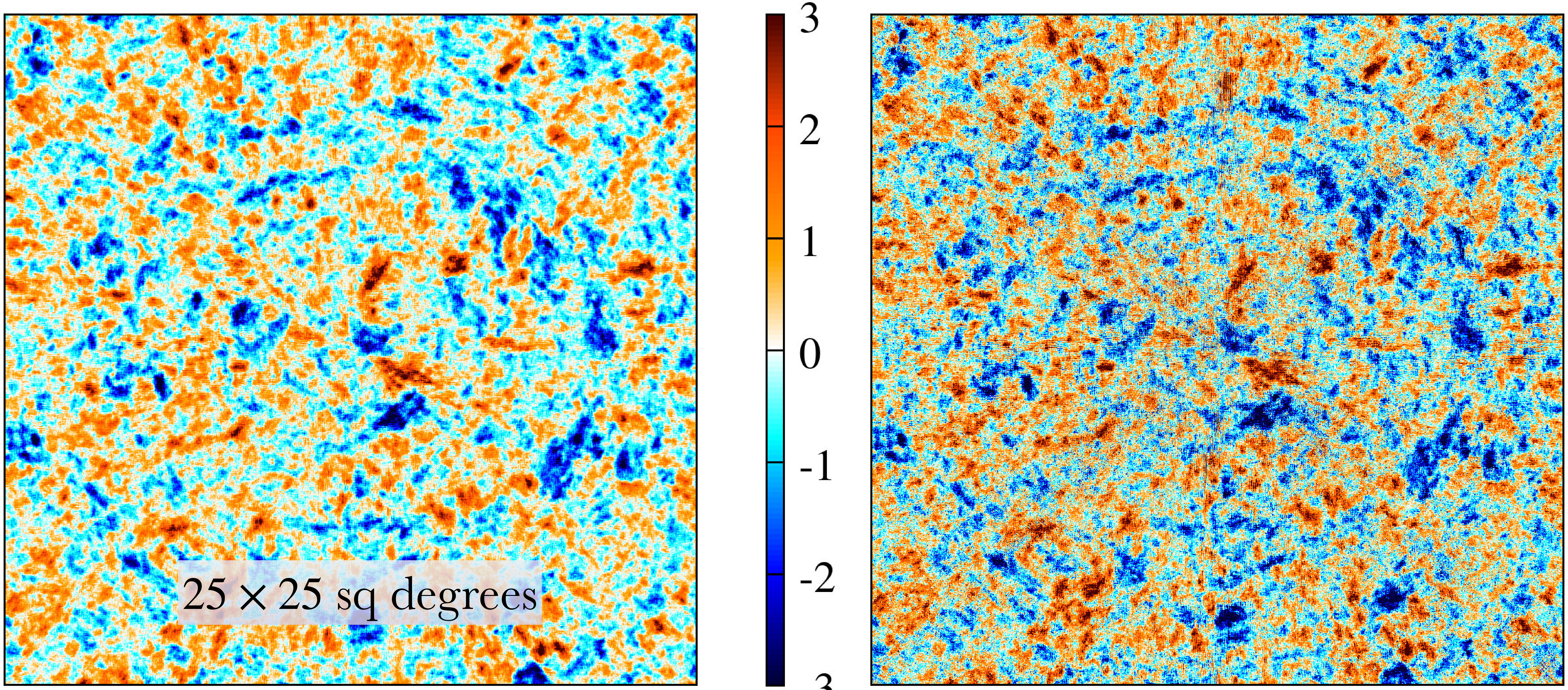

**Varying central redshift of reionisation**



# Patchy reionisation creates small scale anisotropies

Preliminary

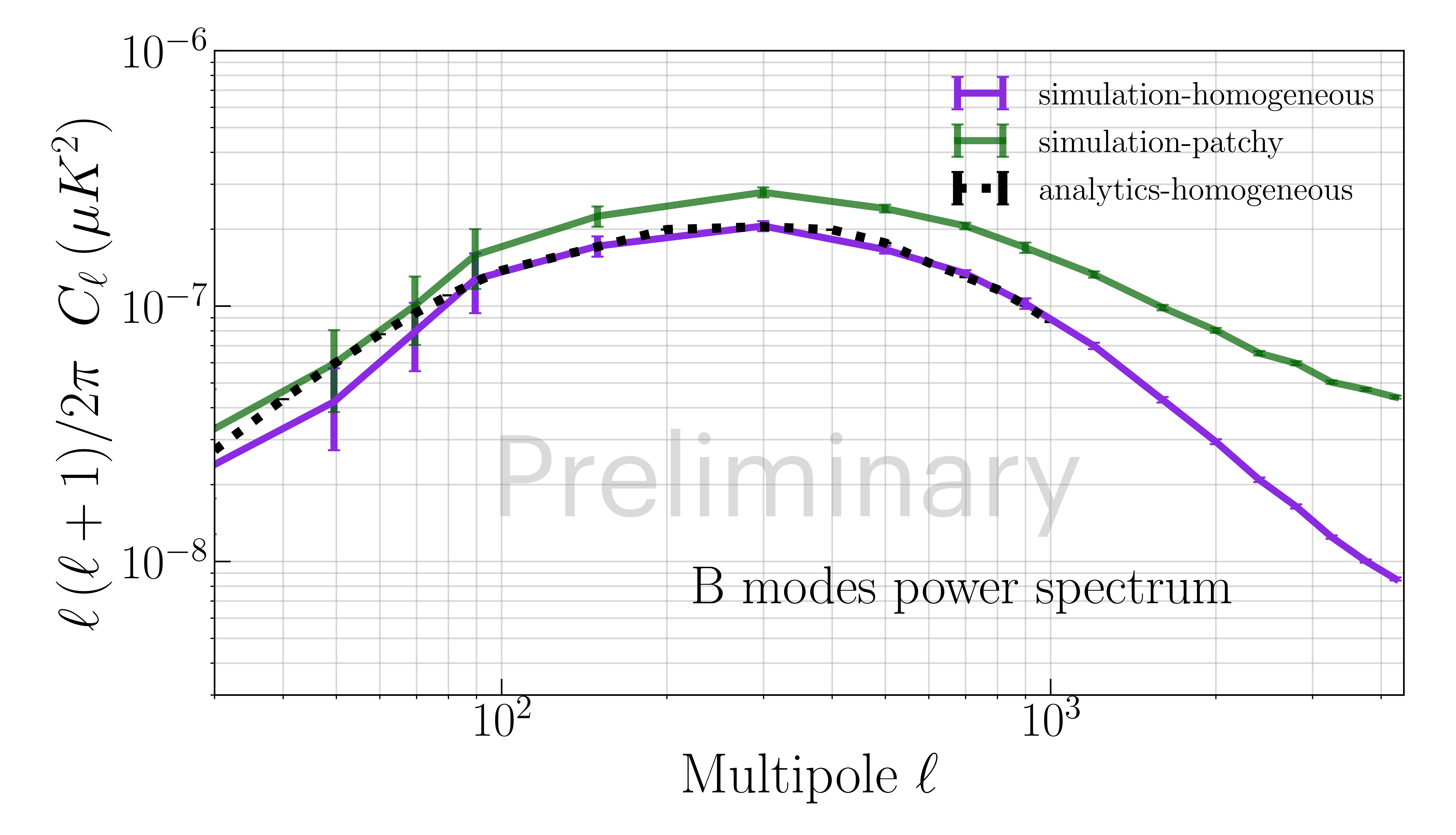
* Using 21cmFast, Healpix and PolSpice




non-patchy Q

patchy Q

# Patchy reionisation enhances the power spectrum



# Concluding Remarks

* This polarisation signal with y-type distortion exists within the Standard Cosmological model of the Universe
* The cross-pairing clusters from CMB-S4 with galaxies from large overlapping spectroscopic survey can provide a way to detect the signal.
* Free from the cosmic variance of the primary CMB polarisation signal and lensing B modes.
* Primary CMB anisotropies are sensitive to only the total reionisation optical depth but the pkSZ effect is sensitive to the Reionisation history.

## Thank You