CMB imprints of non-canonical anisotropic inflation

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Based on
EPJC**81**(2021)390
CQG**33**(2016)085009

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Contents

I. Motivations

- Cosmological principle: our universe is just simply homogeneous and isotropic on large scales as described by the spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime:

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right),$$

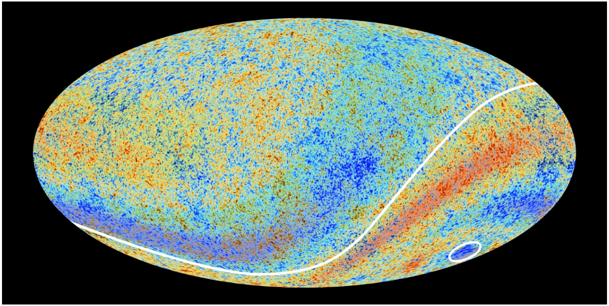
  $a(t)$ is the scale factor and t is the cosmic time.
- The cosmological principle has played as a basic assumption of all standard inflationary models.
- Confirming the validity of the cosmological principle is not straightforward.

How Isotropic is the Universe?

Daniela Saadeh, Stephen M. Feeney, Andrew Pontzen, Hiranya V. Peiris, and Jason D. McEwen
Phys. Rev. Lett. **117**, 131302 – Published 21 September 2016

Physics See Synopsis: Anisotropy Limits for the Universe

I. Motivations



Two CMB anomalous features, the hemispherical asymmetry and the Cold Spot, hinted by Planck's predecessor, NASA's WMAP, are confirmed in the new high precision data from Planck, both are not predicted by standard inflationary models which are basically based on the cosmological principle (Source: ESA and the Planck Collaboration).

There have been a number of mechanisms proposed to explain the origin of these anomalies. However, the physics behind the CMB anomalies have been still unknown up to now Schwarz, Copi, Huterer, & Starkman, CQG33(2016)184001.

I. Motivations

- If the cosmological principle was broken down in the early universe, would it still be unvalid in the late time universe ?
- Cosmic no-hair conjecture claims the late time universe should simply be homogeneous and isotropic, regardless of initial states, which might be inhomogeneous or/and anisotropic (a.k.a. spatial hairs) Gibbons & Hawking, PRD15(1977)2738; Hawking & Moss, PLB110(1982)35.
- This conjecture was firstly proven by Wald, PRD28(1983)2118, for the Bianchi spacetimes, which are homogeneous but anisotropic, using energy conditions approach. Many follow-up papers have also been proposed but a complete proof to this conjecture has remained unknown.

→ *if the cosmic no-hair conjecture was valid then the late time universe would obey the cosmological principle.*

S. W. Hawking, G. W. Gibbons, and I. G. Moss (Source: Internet).

I. Motivations

Recently, there have been some recent observational studies claiming that the current universe might be anisotropic, in contrast to the prediction of the cosmic no-hair conjecture !?

A&A 631, L13 (2019)

Letter to the Editor

Evidence for anisotropy of cosmic acceleration*

Jacques Colin[1], Roya Mohayaee[1], Mohamed Rameez[2] and Subir Sarkar[3]

PAPER

Does Hubble tension signal a breakdown in FLRW cosmology?

C Krishnan[1], R Mohayaee[2], E Ó Colgáin[6,3,4], M M Sheikh-Jabbari[5] and L Yin[3,4]

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Classical and Quantum Gravity, Volume 38, Number 18

Hints of FLRW breakdown from supernovae

Chethan Krishnan, Roya Mohayaee, Eoin Ó Colgáin, M. M. Sheikh-Jabbari, and Lu Yin

Phys. Rev. D **105**, 063514 – Published 11 March 2022

II. KSW anisotropic inflation model

- It seems to be the first valid counterexample to the cosmic no-hair conjecture Kanno, Soda & Watanabe, PRL102(2009)191302, JCAP12(2010)024

$$S_{\text{KSW}} = \int d^4x\sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right],$$

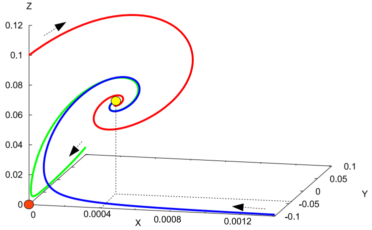
- The last term is a supergravity-motivated one with $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ the field strength of the electromagnetic field A_μ.
- The non-constant $f(\phi)$ breaks down the conformal invariance of $A_\mu \to$ the so-called conformal-violating Maxwell theory proposed to explain the origin of primordial magnetic fields Ratra, AJ391(1992)L1 [c.f. Tina Kahniashvili's talk].
- Homogeneous but anisotropic Bianchi type I metric:

$$ds^2 = -dt^2 + \exp[2\alpha(t) - 4\sigma(t)]\,dx^2 + \exp[2\alpha(t) + 2\sigma(t)]\left(dy^2 + dz^2\right).$$

- $\sigma(t)$ a deviation from the isotropy determined by $\alpha(t)$, i.e., $\sigma(t) \ll \alpha(t)$.
- Vector and scalar fields: $A_\mu = (0, A_x(t), 0, 0)$ and $\phi = \phi(t)$.

II. KSW anisotropic inflation model

- This model admits stable and attractive Bianchi type I inflationary solutions $\rightarrow$ violates the Hawking cosmic no-hair conjecture



Attractor behavior of anisotropic fixed point, which is equivalent to anisotropic power-law solution [taken from JCAP12(2010)024].

- CMB imprints of the KSW model have been investigated by Soda and his colleagues Watanabe, Kanno & Soda, MNRAS412(2011)L83; PTP123(2010)1041; Ohashi, Soda, &Tsujikawa, JCAP12(2013)009 as well as by other people Dulaney & Gresham, PRD81(2010)103532; Gumrukcuoglu, Himmetoglu, & Peloso, PRD81(2010)063528; Bartolo, Matarrese, & Peloso, PRD87(2013)023504
- CMB imprints of an extension of the KSW model, in which the inflaton is complex, have also been investigated Chen, Emami, Firouzjahi, & Wang, JCAP08(2014)027

III. Non-canonical extensions of the KSW model

- A general action of non-canonical extensions of the KSW model:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + P(\phi, X) - \frac{1}{4}f^2(\phi)\,F_{\mu\nu}F^{\mu\nu}\right],$$

where $P(\phi, X)$ is an arbitrary function of scalar field ϕ and its kinetic $X \equiv -\partial^\mu\phi\partial_\mu\phi/2$, which was firstly introduced in the so-called *k*-inflation Armendariz-Picon, Damour, & Mukhanov, PLB458(1999)209.

- A number of counterexamples have been found in non-canonical extensions of the KSW models Do & Kao, PRD84(2011)123009; Ohashi, Soda & Tsujikawa, PRD88(2013)103517; Do & Kao, CQG33(2016)085009; Do, EPJC81(2021)77.

$\rightarrow$ CMB imprints of non-canonical anisotropic inflation ?

III. CMB imprints: Background metric

- Due the smallness of anisotropy deviation, we can take a good approximation, in which the background metric is the spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) metric rather than the Bianchi type I metric for simplicity, c.f. Watanabe, Kanno & Soda, MNRAS412(2011)L83; Ohashi, Soda, &Tsujikawa, JCAP12(2013)009; Chen, Emami, Firouzjahi, & Wang, JCAP08(2014)027.

- The non-vanishing vector field could, however, lead to some significant CMB imprints → Let's see how

III. CMB imprints: Scalar perturbations

- Since the statistical isotropy of CMB is broken, the scalar power spectrum is modified as Ackerman, Carroll, & Wise, PRD75(2007)083502

$$\mathcal{P}_k^{\zeta(0)} \to \mathcal{P}_{k,\mathrm{ani}}^{\zeta} = \mathcal{P}_k^{\zeta(0)} \left(1 + g_* \cos^2\theta\right).$$

  - g_* characterizes the deviation from the spatial isotropy, i.e., $|g_*| < 1$.
  - θ is the angle between the comoving wave number **k** with the *privileged* direction **V** close to the ecliptic poles.
  - $\mathcal{P}_k^{\zeta(0)}$, the isotropic scalar power spectrum ($g_* = 0$), for non-canonical scalar field is defined as Armendariz-Picon, Damour, & Mukhanov, PLB458(1999)219

$$\mathcal{P}_k^{\zeta(0)} = \mathcal{P}_{k,\mathrm{nc}}^{\zeta(0)} = \frac{1}{8\pi^2 M_p^2} \frac{H^2}{c_s \epsilon}\bigg|_{c_s^* k_* = a_* H_*}$$

  where $c_s^2 \equiv \partial_X p / \partial_X \rho \leq 1$ is the speed of sound of scalar perturbation and $\epsilon \equiv -\dot{H}/H^2 \ll 1$ is the slow-roll parameter.

III. CMB imprints: Scalar perturbations

- Observational constraints of g_*:
  - $g_* = 0.29 \pm 0.031$ at 9σ using the 5-year WMAP data Groeneboom, Ackerman, Wehus, & Eriksen, AJ722(2010)452.
  - $g_* = 0.002 \pm 0.016$ at 68% CL using the Planck 2013 data Kim & Komatsu, PRD88(2013)011301(R).
  - $|g_*| < 0.072$ at 95% CL using the 9-year WMAP data Ramazanov & G. Rubtsov, PRD89(2014)043517.
  - $-0.041 < g_* < 0.036$ at 95% CL using the Planck 2015 data Ramazanov, Rubtsov, Thorsrud, & Urban, JCAP03(2017)039.
  - $-0.09 < g_* < 0.08$ at 95% CL using the LSS surveys data Sugiyama, Shiraishi, & Okumura, MNRAS473(2018)2737.
- Our goal: Calculate the corresponding g_* for non-canonical anisotropic inflation using the standard Bunch-Davies (BD) vacuum state for the non-canonical scalar field Chen, Huang, Kachru, & Shiu, JCAP01(2007)002

$$\zeta^{(0)}_{nc}(k,\eta) = \frac{H}{2\sqrt{c_s\epsilon}M_p k^{3/2}}\left(1 + ic_s k\eta\right) e^{-ic_s k\eta}.$$

III. CMB imprints: Scalar perturbations

- The full power spectrum in the Heisenberg interaction picture for the scalar perturbation, up to the second order, is given by

$$\langle 0|\hat{\zeta}_{\rm nc}(k,\eta)\hat{\zeta}_{\rm nc}(k',\eta)|0\rangle$$
$$\simeq \frac{2\pi^2}{k^3}\delta^3(\mathbf{k}+\mathbf{k}')\mathcal{P}^{\zeta(0)}_{k,\rm nc} + \frac{2\pi^2}{k^3}\delta^3(\mathbf{k}+\mathbf{k}')\frac{c_s^4 E_x^2 N_{c_s k}^2}{\pi^2\epsilon^2 M_p^4}\sin^2\theta,$$

with $N_{c_s k} \simeq 60$ the e-fold number and $E_x \equiv (f/a^2)A_x^{(0)'}$. This implies

- $\mathcal{P}^{\zeta}_{k,\rm nc} = \mathcal{P}^{\zeta(0)}_{k,\rm nc}\left(1 + \frac{8c_s^5 E_x^2 N_{c_s k}^2}{\epsilon M_p^2 H^2}\sin^2\theta\right) \simeq \mathcal{P}^{\zeta(0)}_{k,\rm nc}\left(1 - \frac{8c_s^5 E_x^2 N_{c_s k}^2}{\epsilon M_p^2 H^2}\cos^2\theta\right)$
- $g_* = -c_s^5\frac{8E_x^2 N_{c_s k}^2}{\epsilon M_p^2 H^2} = c_s^5 g_*^0 < 0$, where $g_*^0 = -\frac{8E_x^2 N_{c_s k}^2}{\epsilon M_p^2 H^2} < 0$ for canonical anisotropic inflation $\rightarrow |g_*| \ll |g_*^0|$ if $c_s^2 \ll 1$.
- Scalar spectral index:
$$n_s - 1 \equiv \left.\frac{d\ln\mathcal{P}^{\zeta}_{k,\rm nc}}{d\ln k}\right|_{c_s^* k_* = a_* H_*} \simeq -2\epsilon - \tilde{\eta} - s + \left(\frac{2}{N_{c_s k}} - 5s\right)\frac{2g_*}{3-2g_*}.$$

III. CMB imprints: Tensor perturbations

- The full tensor power spectrum for the non-canonical scalar field is given by

$$\langle 0|\hat{h}_{ij}(\mathbf{k})\hat{h}_{ij}(\mathbf{k}')|0\rangle = \frac{2\pi^2}{k^3}\delta^3(\mathbf{k}+\mathbf{k}')\left(\mathcal{P}^{h(0)}_{k,\mathrm{nc}} + \frac{4E_x^2 N_k^2}{\pi^2 M_p^4}\sin^2\theta\right),$$

  which implies $\mathcal{P}^{h}_{k,\mathrm{nc}} \simeq \mathcal{P}^{h(0)}_{k,\mathrm{nc}}\left(1 - \frac{\epsilon g_*^0}{4}\sin^2\theta\right)$ → similar to that of canonical scalar field.
- Here, $\mathcal{P}^{h(0)}_{k,\mathrm{nc}}(\mathbf{k}) = \frac{2}{\pi^2}\frac{H^2}{M_p^2}\Big|_{k_*=a_*H_*} = 16c_s\epsilon\mathcal{P}^{\zeta(0)}_{k,\mathrm{nc}}(\mathbf{k})$ is the isotropic tensor power spectrum for non-canonical scalar field Armendariz-Picon, Damour, & Mukhanov, PLB458(1999)219.
- Tensor spectral index: $n_t \equiv \frac{d\ln\mathcal{P}^{h}_{k,\mathrm{nc}}}{d\ln k}\Big|_{k_*=a_*H_*} \simeq -2\epsilon.$

III. CMB imprints: Tensor-to-scalar ratio

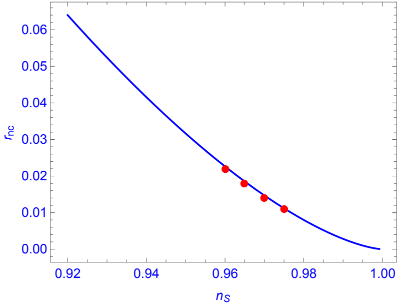
- For non-canonical isotropic inflation: $r_{\rm nc}^{iso} = 16 c_s \epsilon$ Armendariz-Picon, Damour, & Mukhanov, PLB458(1999)219.
- The full tensor-to-scalar ratio for non-canonical anisotropic inflation:

$$r_{\rm nc} \equiv \frac{\mathcal{P}^{h}_{k,\rm nc}}{\mathcal{P}^{\zeta}_{k,\rm nc}} = 16 c_s \epsilon \frac{1 - \frac{1}{4}\epsilon g_*^0 \sin^2\theta}{1 - c_s^5 g_*^0 \sin^2\theta} \simeq 16 c_s \epsilon \frac{6 - \epsilon g_*^0}{6 - 4 c_s^5 g_*^0},$$

with the average value of $\sin^2\theta$ as $\langle \sin^2\theta \rangle = 2/3$ Ohashi, Soda, &Tsujikawa, JCAP12(2013)009.

III. CMB imprints: tensor-to-scalar ratio

Example: Anisotropic power-law k-inflation EPJC**81**(2021)77



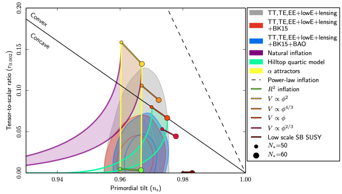


Fig. 8. Marginalized joint 68 % and 95 % CL regions for n_s and r at $k = 0.002\,\mathrm{Mpc}^{-1}$ from *Planck* alone and in combination with BK15 or BK15+BAO data, compared to the theoretical predictions of selected inflationary models. Note that the marginalized joint 68 % and 95 % CL regions assume $dn_s/d\ln k = 0$.

(Left) The $n_s - r_{\rm nc}$ diagram for the anisotropic power-law inflation with $g_*^0 = -0.03$ and $10^{-2} \leq c_s \leq 10^{-1}$. Four red points have the corresponding values such as $(n_s, r_{\rm nc}) \simeq (0.96, 0.022)$ for $c_s \simeq 0.07$, $(0.965, 0.018)$ for $c_s \simeq 0.066$, $(0.97, 0.014)$ for $c_s \simeq 0.06$, and $(0.975, 0.011)$ for $c_s \simeq 0.056$. (Right) The Planck observational data in comparison with the prediction of tensor-to-scalar ratio of some leading inflationary models [taken from A&A641(2020)A10].

III. CMB imprints: Correlations

- The correlators of CMB observables take the following form Watanabe, Kanno & Soda, MNRAS412(2011)L83; Chen, Emami, Firouzjahi, & Wang, JCAP08(2014)027

$$C^{X_1 X_2}_{l_1 l_2 m_1 m_2} \equiv \langle a^{X_1}_{l_1 m_1} a^{X_2 *}_{l_2 m_2} \rangle$$
$$= 4\pi \int \frac{dk}{k} \Delta^{i_1 X_1}_{l_1}(k) \Delta^{i_2 X_2}_{l_2}(k) \int \left[{}_{i_1}Y^*_{l_1 m_1}(\theta,\phi) {}_{i_2}Y_{l_2 m_2}(\theta,\phi) \right] P^{i_1,i_2}(k,\theta,\phi) d\Omega,$$

  where Δ is the transfer function, while X represents the temperature anisotropy ($X_i = T$), the E-mode ($X_i = E$), or the B-mode ($X_i = B$). In addition, ${}_iY_{lm}(\theta,\phi)$ is the spin-i-weighted spherical harmonics.
- The power spectra of helicity bases, P^{i_1,i_2}, are given by Watanabe, Kanno & Soda, MNRAS412(2011)L83
  - Scalar perturbations: $P^{0,0} = \mathcal{P}_\zeta$.
  - Cross-correlations: $P^{0,\pm 2} = P^{\pm 2,0} = \frac{1}{\sqrt{2}} P^{0,+} \equiv \frac{1}{\sqrt{2}} \mathcal{P}_{\zeta h_+}$.
  - Tensor perturbations: $P^{\pm 2,\pm 2} = \frac{1}{2}\left(P^{++} + P^{\times\times}\right) = \frac{1}{2}\left(\mathcal{P}_{h_+} + \mathcal{P}_{h_\times}\right) \equiv \mathcal{P}^{\rm unp}_h$.
  - Linear polarization: $P^{\pm 2,\mp 2} = \frac{1}{2}\left(P^{++} - P^{\times\times}\right) = \frac{1}{2}\left(\mathcal{P}_{h_+} - \mathcal{P}_{h_\times}\right) \equiv \mathcal{P}^{\rm pol}_h$.
- Note: For isotropic inflation, $P^{i_1,i_2} \sim \delta_{i_1 i_2} \to \mathcal{P}_{\zeta h_+} = 0$ and $\mathcal{P}^{\rm pol}_h = 0$.
  $\to$ The non-vanishing $\mathcal{P}_{\zeta h_+}$ and $\mathcal{P}^{\rm pol}_h$ should be a smoking gun for the anisotropic inflation.

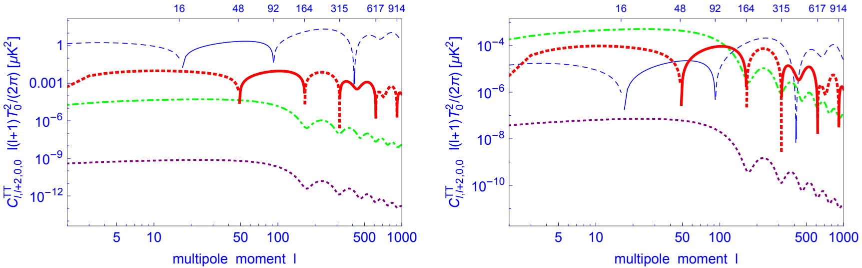
III. CMB imprints: Numerical results

- Choose $g_*^0 = -0.03 < 0$ according to the mentioned observational constraints.
- Choose $r_{nc} = 0.03$ according to the joint analysis on primordial gravitational waves of BICEP2 and Keck array using the Planck, WMAP, and new BICEP2/Keck observations through the 2015 season, which provides the upper bound $r \leq 0.07$ at 95 % CL. [c.f. John Kovac's talk, $r > 0.003$ at 5σ @ CMB-S4].
- Use the Cosmic Linear Anisotropy Solving System (CLASS) package Lesgourgues, arXiv:1104.2932 with the latest observational data of Planck 2018 to calculate the transfer function Δ.

CLASS

the Cosmic Linear Anisotropy Solving System

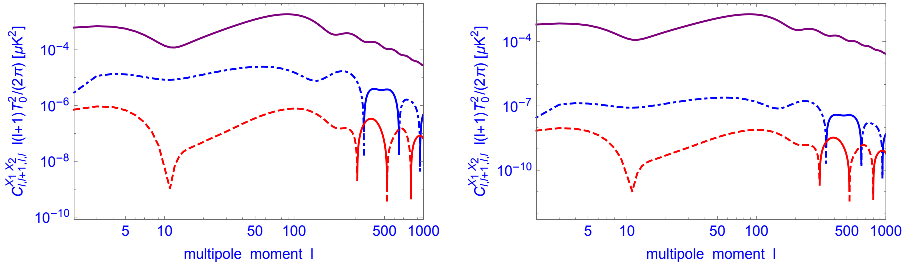
III. CMB imprints: Numerical results



The magnitude of the TT spectra $C^{TT}_{l,l+2,0,0}$ induced by the anisotropy in the scalar perturbations (the upper blue dashed-solid curve); by the anisotropy in the tensor perturbations (the green dotted-dashed curve); by the cross-correlations (the thicker red dotted-solid curve); and by the linear polarization (the bottom purple dotted curve) are shown respectively. The left hand side and right hand side figures correspond to $c_s = 1$ and $c_s = 0.1$, respectively.

The left figure implies that the TT spectrum associated with the scalar perturbations dominates over the others for the canonical scalar field. However, this will not be true for the non-canonical scalar field as shown in the right figure since $c_s < 1$.

IIII. CMB imprints: Numerical results



The magnitude of TB spectrum $C^{TB}_{l,l+1,l,l}$ (the middle blue dashed-solid curve) and the magnitude of EB spectrum $C^{EB}_{l,l+1,l,l}$ (the bottom red dotted-solid curve), both induced by the cross-correlations, are shown in comparison with the magnitude of the isotropic BB spectrum $C^{BB}_{l,l,0,0}$ (the upper purple dotted curve). The left hand side and right hand side figures correspond to $c_s = 1$ and $c_s = 0.1$, respectively.

IV. Conclusions

- The validity of cosmological principle should be (re-)considered more seriously due to the emergence of many exotic observations, which seem to be beyond its predictions.
- The cosmic no-hair conjecture is extensively violated in the KSW model as well as in its non-canonical extensions.
- The non-canonical extensions of KSW could provide a viable tensor-to-scalar ratio, c.f., EPJC**81**(2021)77.
- The TT spectra induced by the tensor perturbations as well as by the linear polarization will increase when the speed of sound decreases.
- The CMB imprints of anisotropic inflation might be detected by more sensitive detectors built in the near future (expectation).

Thank you all for your attention !