The strong coupling: status & prospects

Windows on the Universe

30th Rencontres de Vietnam

Qui Nhon, 7th August 2023

David d'Enterria

STAND WITH UKRAINE

CERN

The strong coupling constant: State of the art and the decade ahead

(Snowmass-2021 White Paper)

arXiv:2203.08271 [hep-ph]

Introduction: QCD coupling α_s

- Determines strength of the strong interaction between quarks & gluons.
- Single free parameter of QCD in the $m_q = 0$ limit.
- Runs as inverse logarithm of energy scale Q^2 according to the RGE:

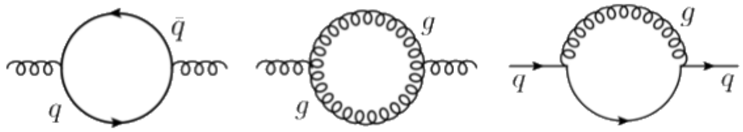
$$\frac{\partial \alpha_s}{\partial \log Q^2} = \beta(\alpha_s) = -\alpha_s^2(\beta_0 + \beta_1\alpha_s + \beta_2\alpha_s^2 + \mathcal{O}(\alpha_s^3))$$

- The solution at three-loop precision r

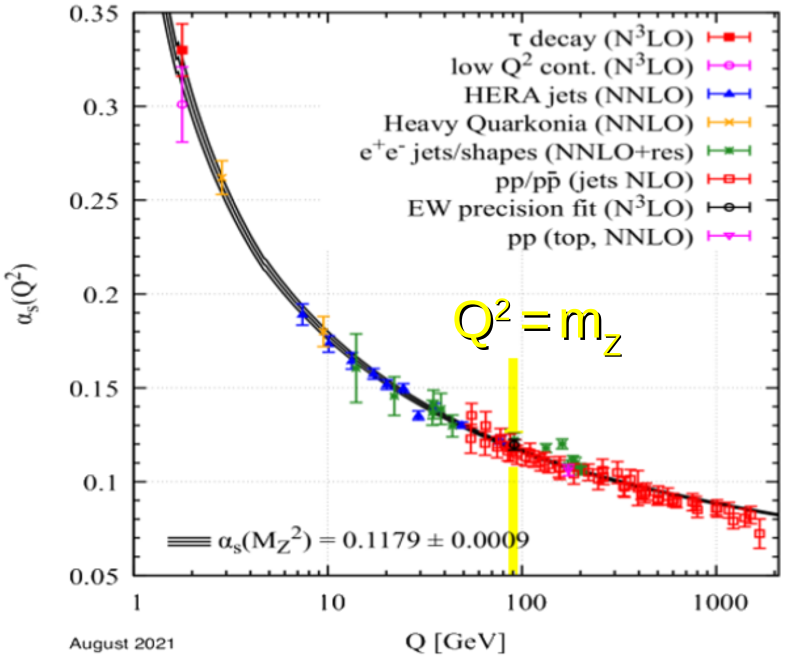
$$\frac{\alpha_s}{4\pi}(Q^2) = \frac{1}{\beta_0 x}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\log x}{x} + \frac{\beta_1^2}{\beta_0^4 x^2}\left(\log^2 x - \log x - 1 + \frac{\beta_2\beta_0}{\beta_1^2}\right)\right]; \quad x = \log\left(\frac{Q^2}{\Lambda^2}\right)$$

$\Lambda \approx 0.2$ GeV

Standard Model with n_f quark flavours:



$$\left.\begin{aligned}\beta_0 &= 11 - \frac{2}{3}n_f\\ \beta_1 &= 102 - \frac{38}{3}n_f\\ \beta_2 &= \frac{2857}{2} - \frac{5033}{18}n_f - \frac{325}{54}n_f^2\end{aligned}\right\}$$



Introduction: QCD coupling α_s

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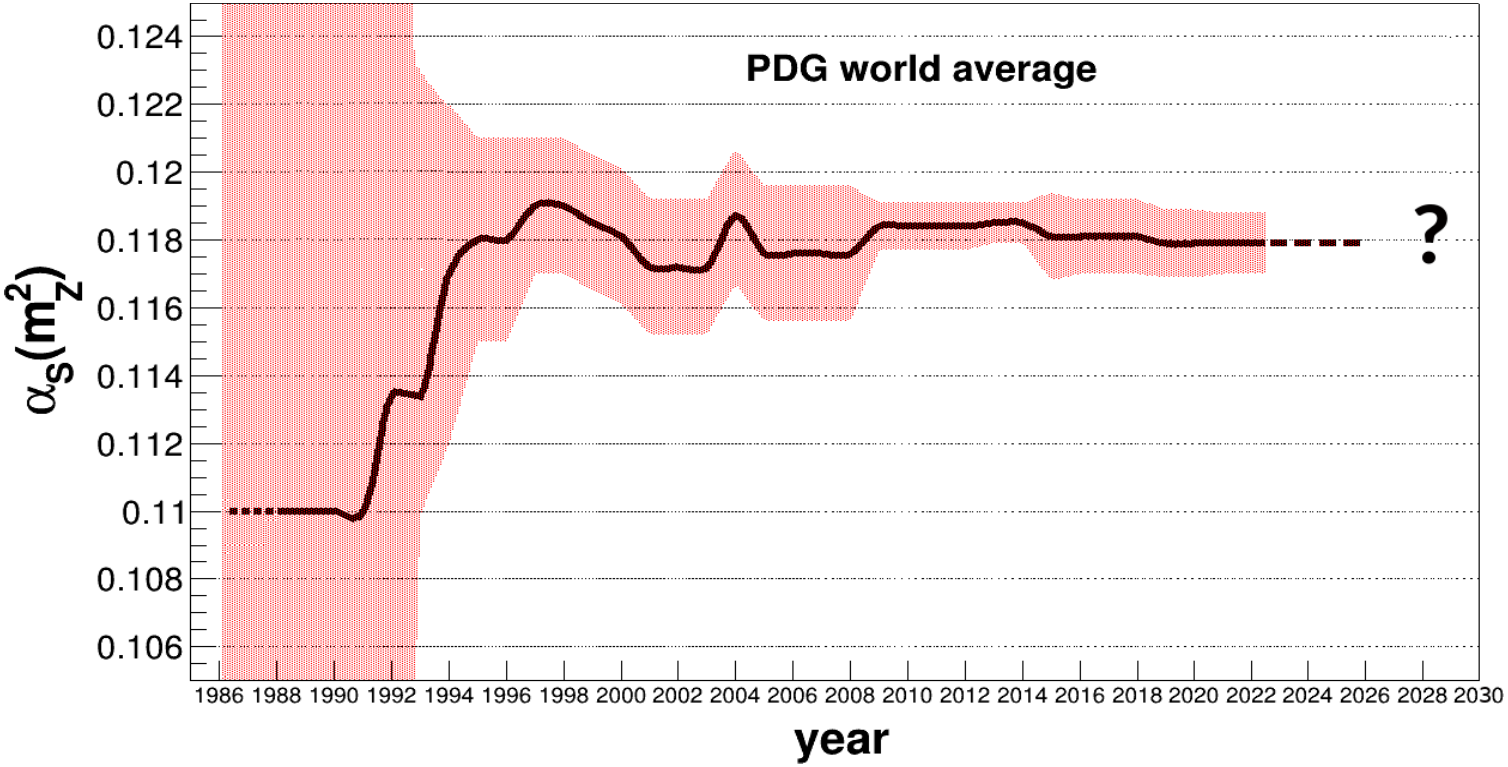
- Single free parameter of QCD in the $m_q = 0$ limit.
- Determined at a ref. scale ($Q=m_Z$), decreases as $\alpha_s \approx \ln(Q^2/\Lambda^2)^{-1}$, $\Lambda \approx 0.2$ GeV



➧ 1986: First PDG world-average with ~100% uncertainty.

➧ 2023: Where are we today, 37 yrs later? Where will we be in 10 years?

Motivation: QCD coupling α_s

- Determines strength of the strong interaction between quarks & gluons.
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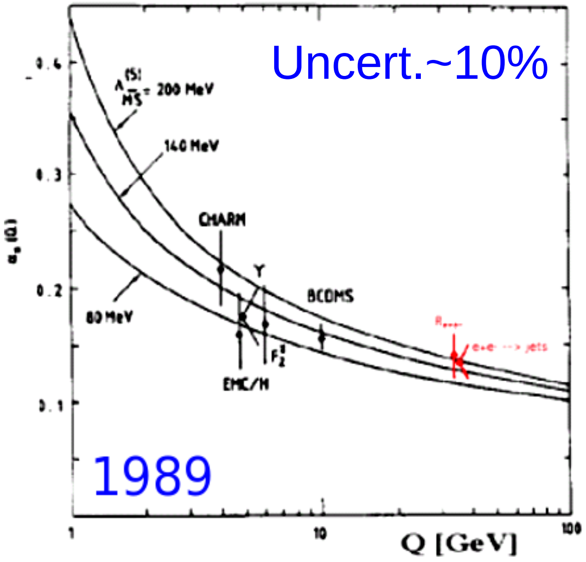
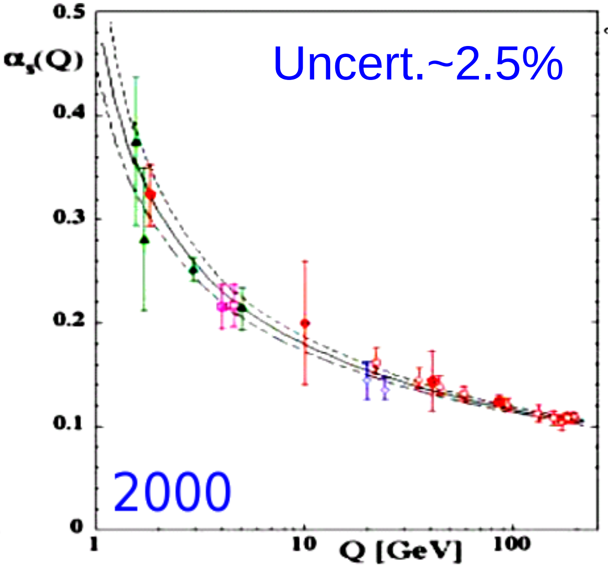


Uncert.~10%

1989

$\alpha_s(M_Z) = 0.110^{+0.006}_{-0.008}$ (NLO)

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989

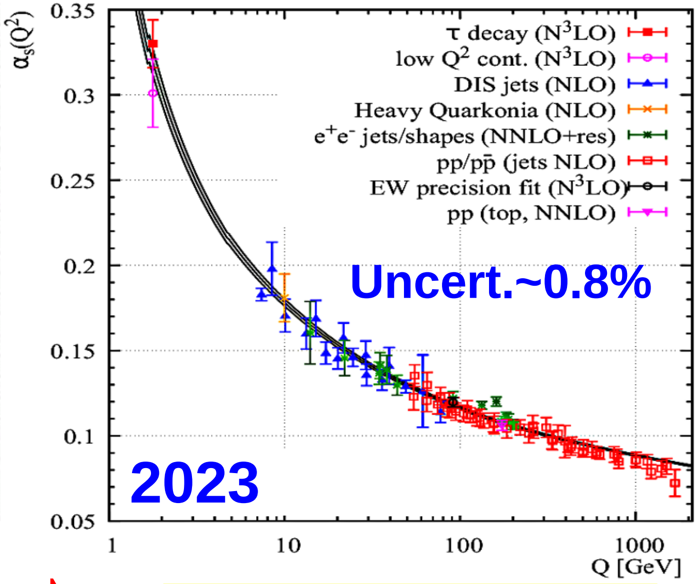


Uncert.~2.5%

2000

$\alpha_s(M_Z) = 0.1184 \pm 0.0031$ (NNLO)

S. B. , J. Phys. G 26, 2000



Uncert.~0.8%

2023

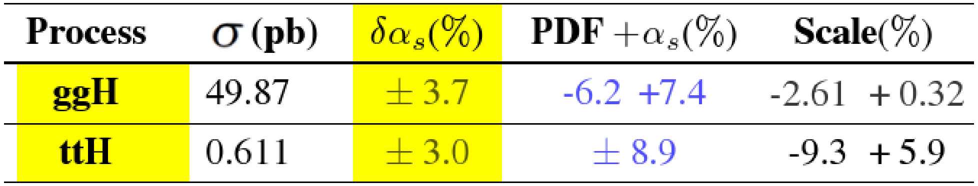
$\alpha_s(M_Z^2) = 0.1179 \pm 0.0010$

➧ Least precisely known of all interaction couplings !

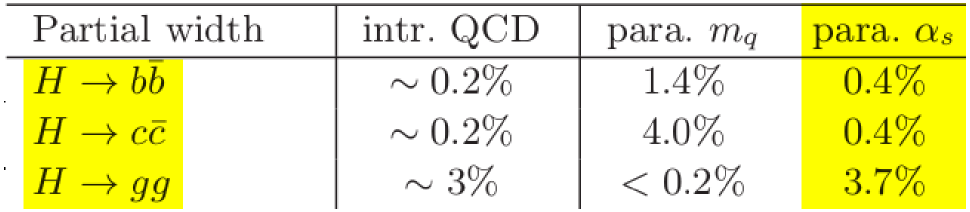
$\delta\alpha \sim 10^{-10} \ll \delta G_F \ll 10^{-7} \ll \delta G \sim 10^{-5} \ll \delta\alpha_s \sim 10^{-3}$

Motivation: α_s impact beyond QCD

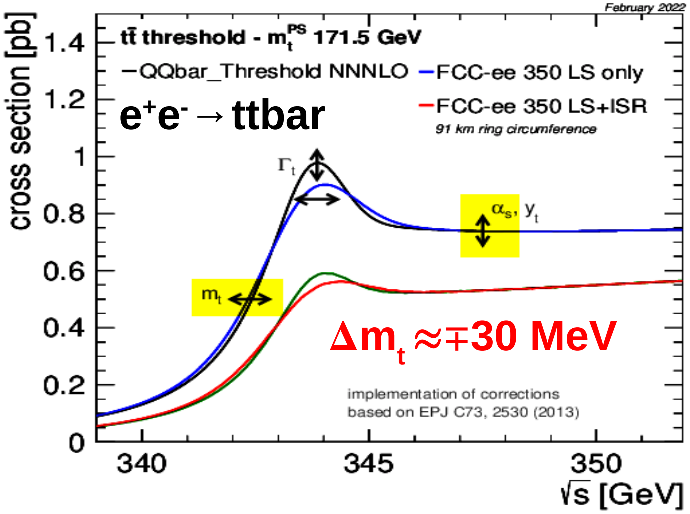
- Precision calculations of Higgs hadronic x-sections/decays, top mass, EWPO..



Process	σ (pb)	$\delta\alpha_s(\%)$	PDF $+\alpha_s(\%)$	Scale(%)
ggH	49.87	± 3.7	-6.2 +7.4	-2.61 +0.32
ttH	0.611	± 3.0	$\pm$ 8.9	-9.3 +5.9

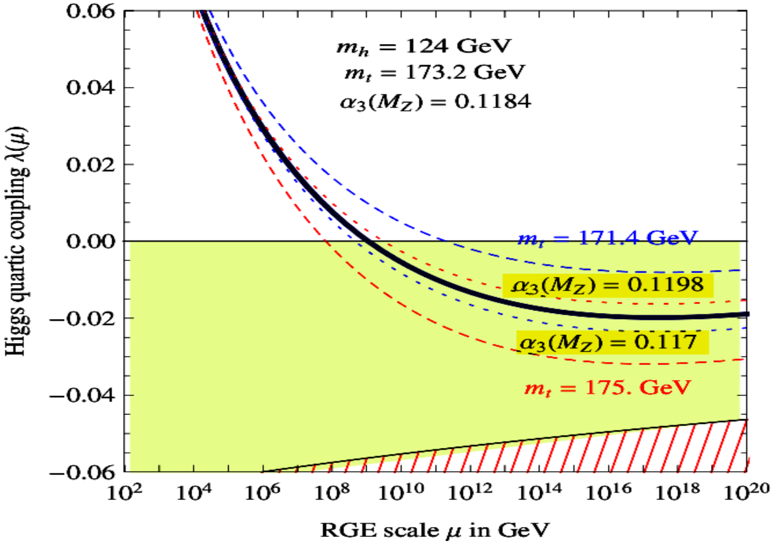


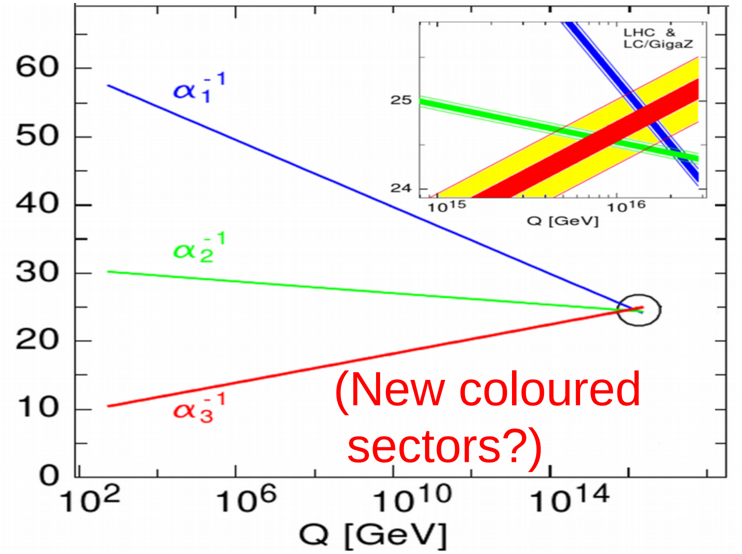
Partial width	intr. QCD	para. m_q	para. α_s
$H \to b\bar{b}$	$\sim 0.2\%$	1.4%	0.4%
$H \to c\bar{c}$	$\sim 0.2\%$	4.0%	0.4%
$H \to gg$	$\sim 3\%$	$< 0.2\%$	3.7%



$\Delta m_t \approx \mp 30$ MeV

- Impacts physics approaching Planck scale: EW vacuum stability, GUT



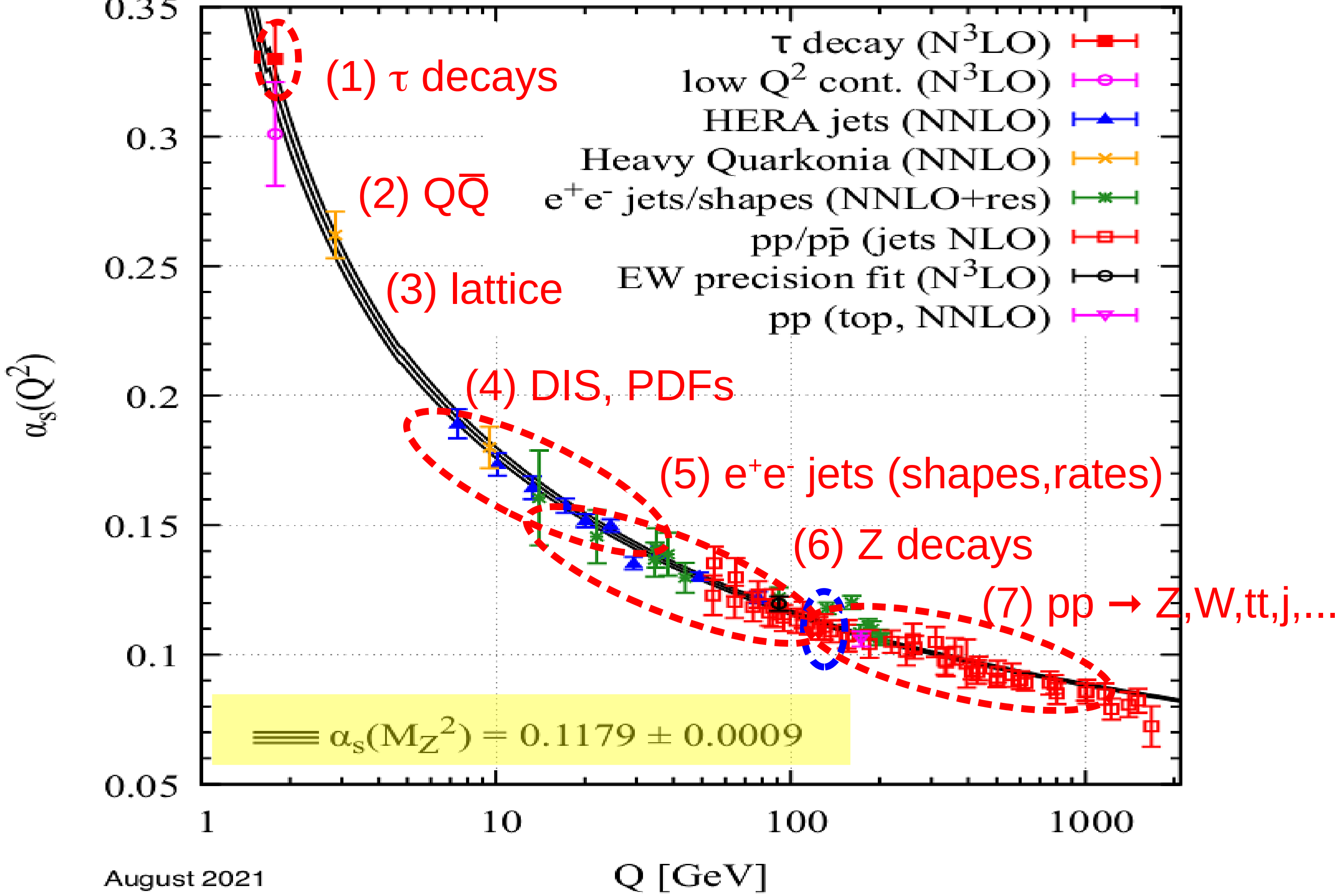


(New coloured sectors?)

World α_s determination (PDG 2023)

- Determined today by comparing 7 experimental observables to pQCD NNLO,N^{3}LO predictions, plus global average at the Z pole scale:

[Huston/Rabbertz/Zanderighi]



(1) τ decays

(2) $Q\bar{Q}$

(3) lattice

(4) DIS, PDFs

(5) e^+e^- jets (shapes,rates)

(6) Z decays

(7) pp → Z,W,tt,j,...

World-average α_s (PDG 2023)

- Average of pre-averages from 7 categories of observables:

$\alpha_s(M_z) = 0.1179 \pm 0.0009$ (±0.8%)

Hadronic tau decay (4 values):

$\alpha_s(M_z) = 0.1178 \pm 0.0019$ (±1.6%)

Quarkonia properties (4 values):

$\alpha_s(M_z) = 0.1181 \pm 0.037$ (±3.3%)

DIS & PDFs fits (6 values):

$\alpha_s(M_z) = 0.1162 \pm 0.0020$ (±1.7%)

$e^+e^- \rightarrow$ hadrons final states (10 values):

$\alpha_s(M_z) = 0.1171 \pm 0.0031$ (±2.6%)

Hadron collider measurements (5 values):

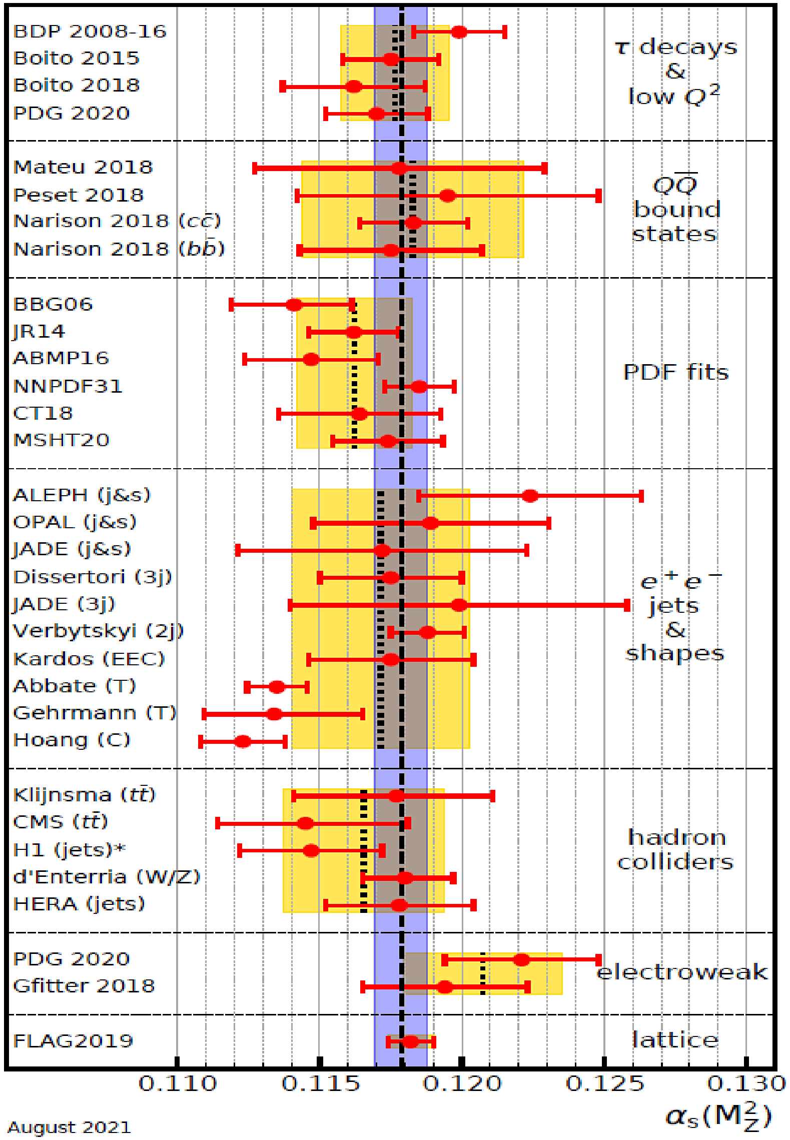
$\alpha_s(M_z) = 0.1165 \pm 0.0028$ (±2.4%)

Electroweak precision fits (2 values):

$\alpha_s(M_z) = 0.1208 \pm 0.0028$ (±2.3%)

Lattice QCD (1 FLAG value):

$\alpha_s(M_z) = 0.1182 \pm 0.0008$ (±0.7%)



(1) α_s from lattice QCD

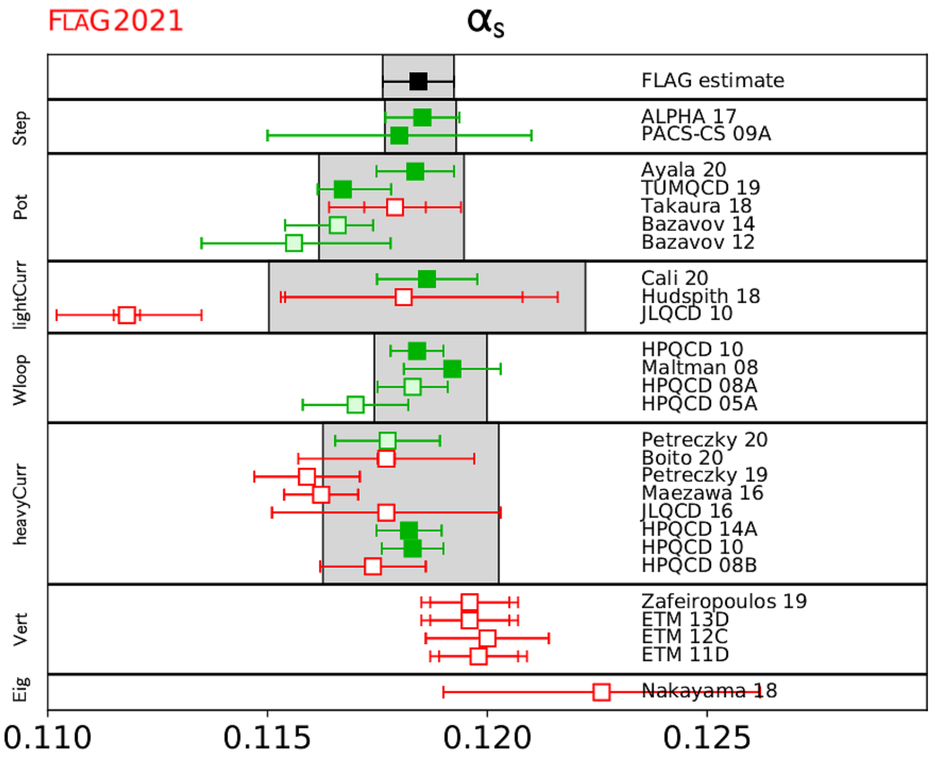
- Comparison of short-distance quantities (QCD static energy/force, light-q & heavy-Q currents, quarkonium,..) computed at $N^{2,3}LO$ in pQCD to lattice data with m_{had}, f_{had} fixed to exp. data:

[FLAG Collab. http://flag.unibe.ch]

$$K^{\mathrm{NP}} = K^{\mathrm{PT}} = \sum_{i=0}^{n} c_i \alpha_s^i$$

- Community-agreed (FLAG) criteria based on: renorm. scale, pQCD behaviour, continuum limit, peer-reviewed results.

- Current uncertainties driven mostly by pQCD truncation & matching, and continuum limit (lattice spacing & computing stats).



$\alpha_s = 0.1182 \pm 0.0008$ (±0.7%)

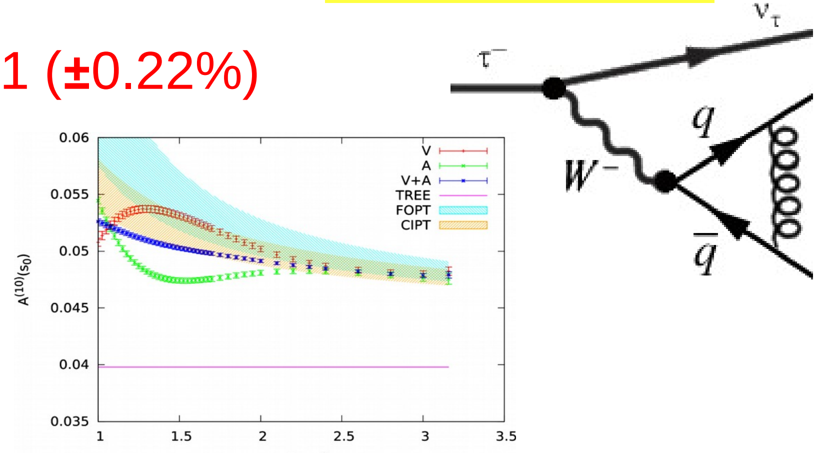
➧ Future prospects:

– Uncertainty in α_s halved with reduced latt. spacing, $N^{3,4}LO$ pQCD, active charm quark, extension of step-scaling method to more observables.

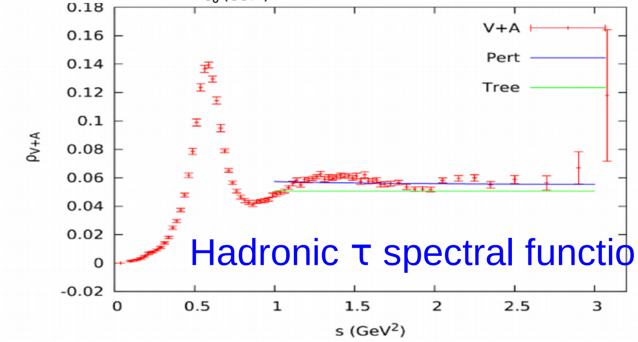
(2) α_s from hadronic τ-lepton decays

- Computed at N^3LO: $R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = S_{\rm EW} N_C (1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\rm np})$

- Experimentally: $R_{\tau,\rm exp} = 3.6355 \pm 0.0081$ (±0.22%)



- Uncertainty driven today by:
  - Differences in pQCD approaches. FOPT vs. CIPT OPE expansions:
  - Treatment of non-pQCD corrections (duality violations): Note: $(\Lambda/m_\tau)^2 \sim 2\%$



Hadronic τ spectral function

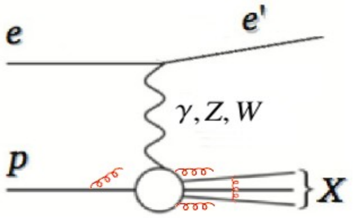
$\alpha_s = 0.1178 \pm 0.0019$ (±1.6%)

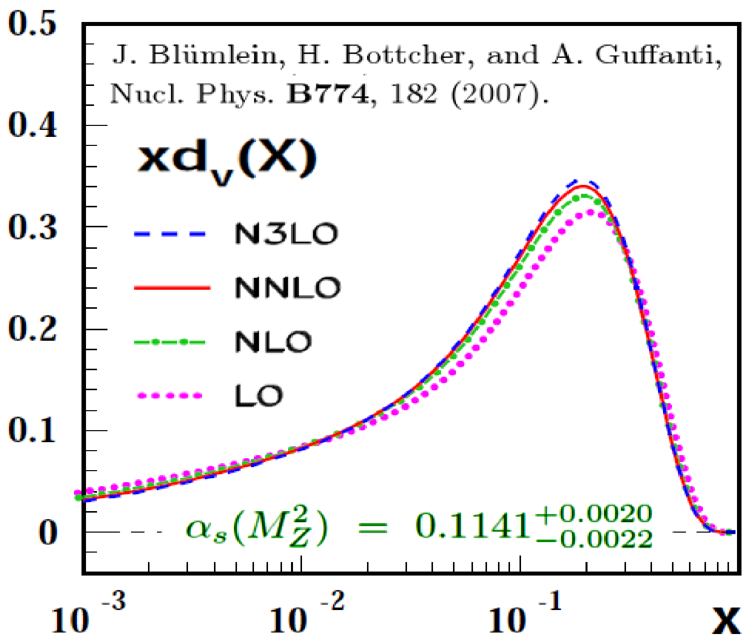
- Future prospects:
  - N^4LO calculations.
  - Reconciling FOPT vs CIPT results (IR renormalon-free gluon condensate)
  - Better spectral functions needed: BELLE-II, STCF. Longer future: $\mathcal{O}(10^{11})$ Z→ττ at FCC-ee(90) !

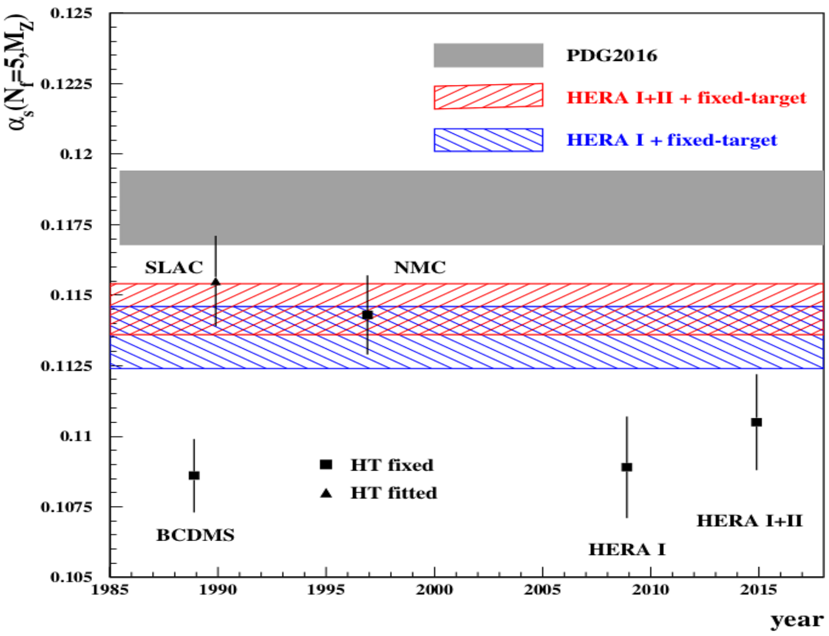
(3) α_s from DIS struct. functions & PDF fits

- N^{3}LO/NNLO analysis of (non)singlet struct. functions (BBG, JR14) (and NNLO global PDF AMBP fit) tend to give "lowish" $\alpha_s(M_z) \approx 0.1150$

$$F_2(x,Q^2) = x\sum_{n=0}^{\infty} \frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n} \sum_{i=q,g}\int_x^1 \frac{dz}{z}\, C_{2,i}^{(n)}(z,Q^2,\mu_R^2,\mu_F^2)\, f_{i/p}\left(\frac{x}{z},\mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$




J. Blümlein, H. Bottcher, and A. Guffanti, Nucl. Phys. **B774**, 182 (2007).

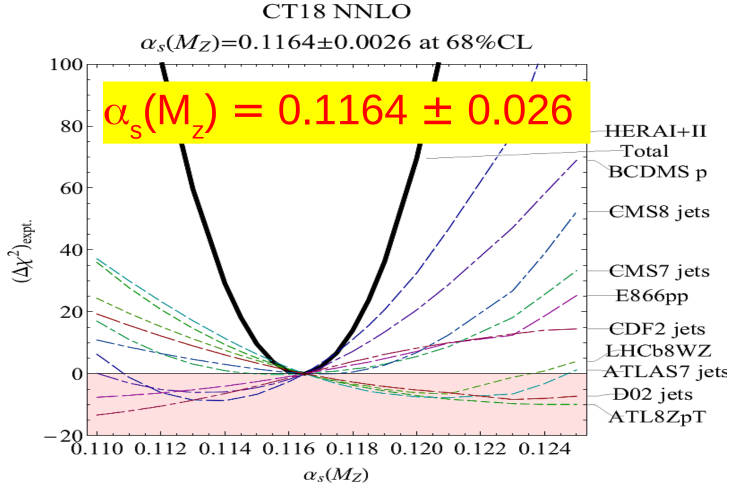


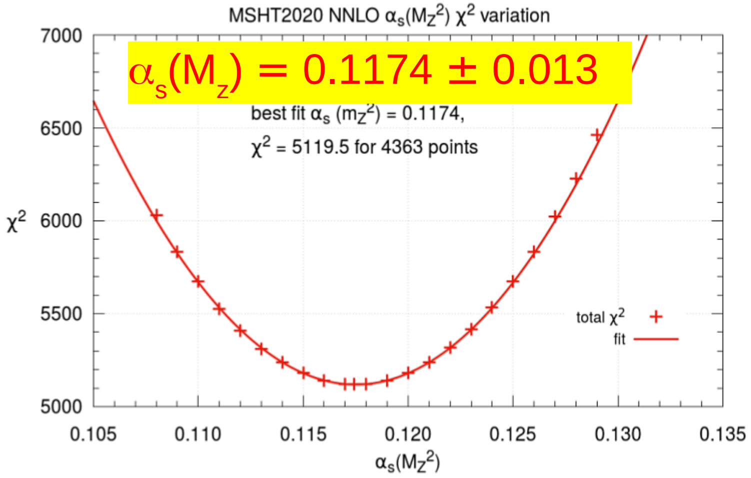
- Neglect of singlet contribs. for x>0.3 in NS fits? Size of higher-order corrs.?

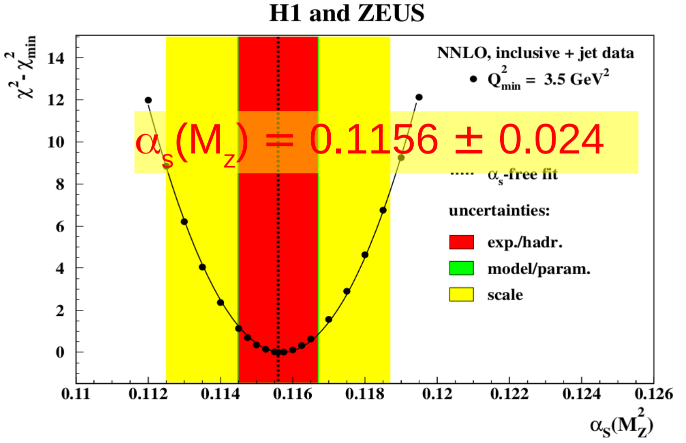
➧ Future: New high-precision $F_i(x,Q^2)$ & polarized $g_i(x,Q^2)$ at EIC.

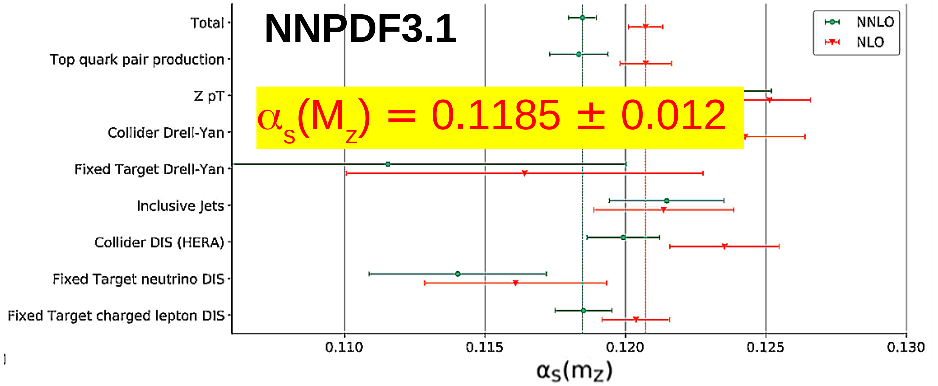
(3) α_s from DIS struct. functions & PDF fits

- NNLO global PDF+α_s fits: CT18, HERAPDF2.0+j, MSTH2020, NNPDF3.1









- DIS/FT (LHC) data tend to prefer lower (higher) values of $\alpha_s(M_z)$.
- Size of missing HO corrections? Global fits at N^{3}LO needed.

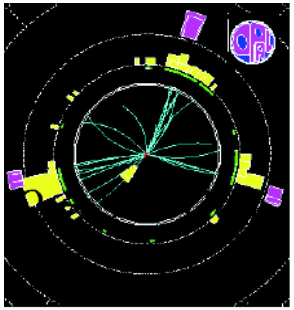
➧ Future: EIC, LHeC/FCC-eh (±0.2%?)

(4) α_s from e^+e^- event shapes & jet rates

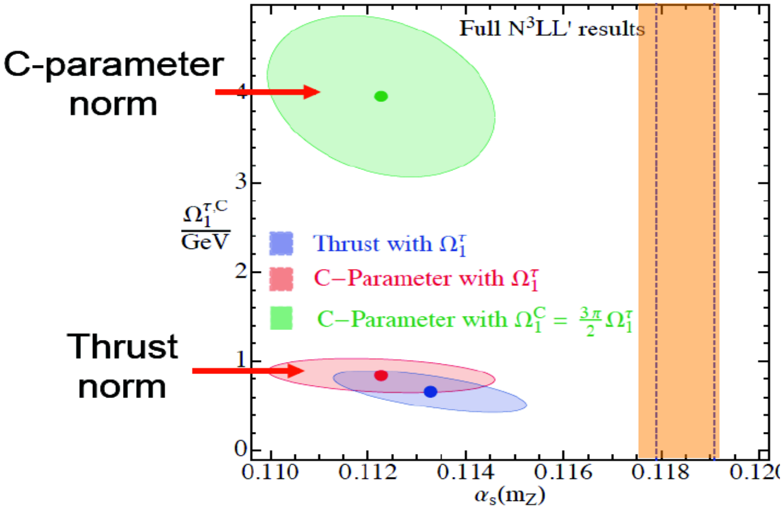
- Computed at $N^{2,3}LO+N^2LL$ accuracy.
- Experimentally (LEP):
  Thrust, C-parameter, jet shapes
  3-jet x-sections
- Results sensitive to non-pQCD (hadronization) corrections.
  - Accounted for via MCs or analytically:

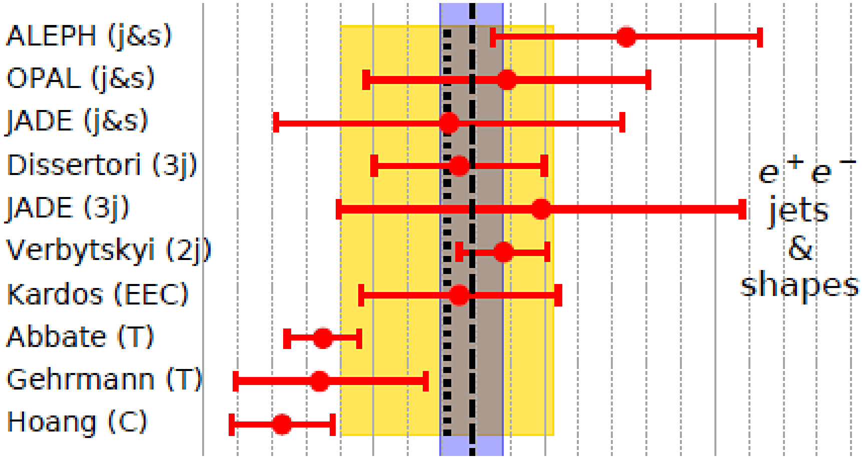
$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

$$C = \frac{3}{2}\frac{\sum_{i,j}|\vec{p}_i||\vec{p}_j|\sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$



OPAL 3 jet event





α_s = 0.1171 ± 0.0031 (±2.6%)

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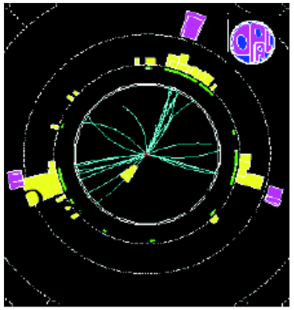
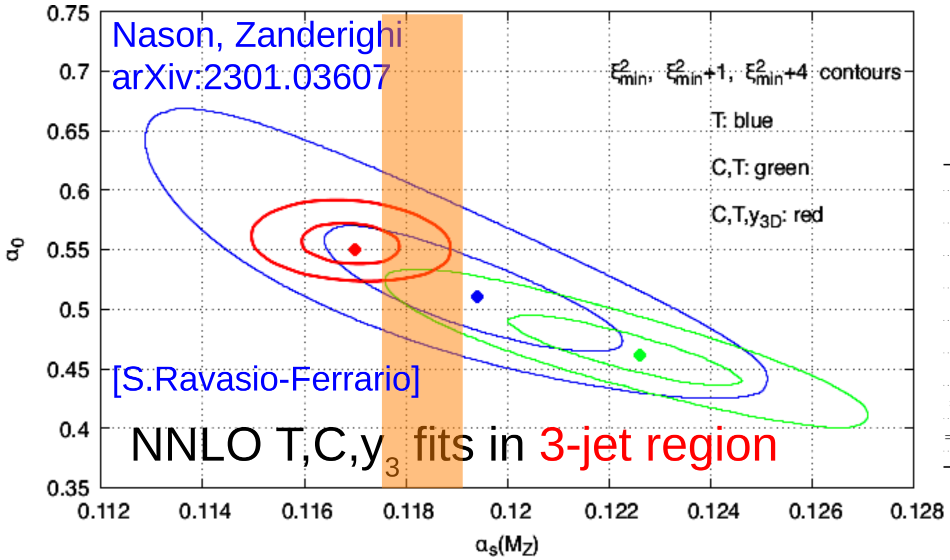


OPAL 3 jet event

- Results sensitive to non-pQCD (hadronization) corrections.
  - Improved evt-shape power-corrs:

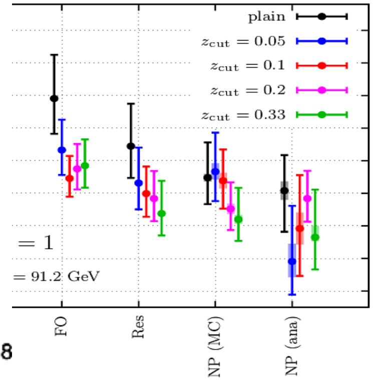


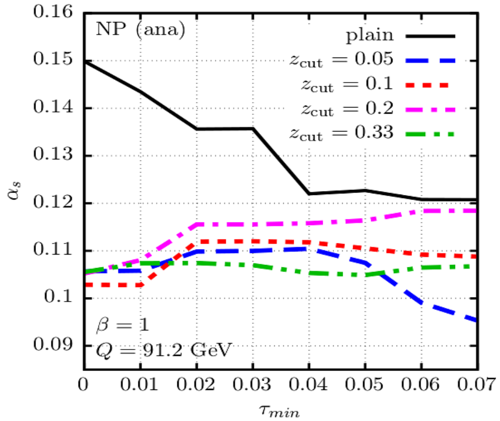
Nason, Zanderighi arXiv:2301.03607

[S.Ravasio-Ferrario]

NNLO T,C,y_3 fits in 3-jet region

- Modern jet substructure techniques: “Soft drop” can help reduce non-pQCD corrections for thrust:

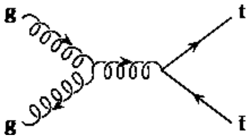


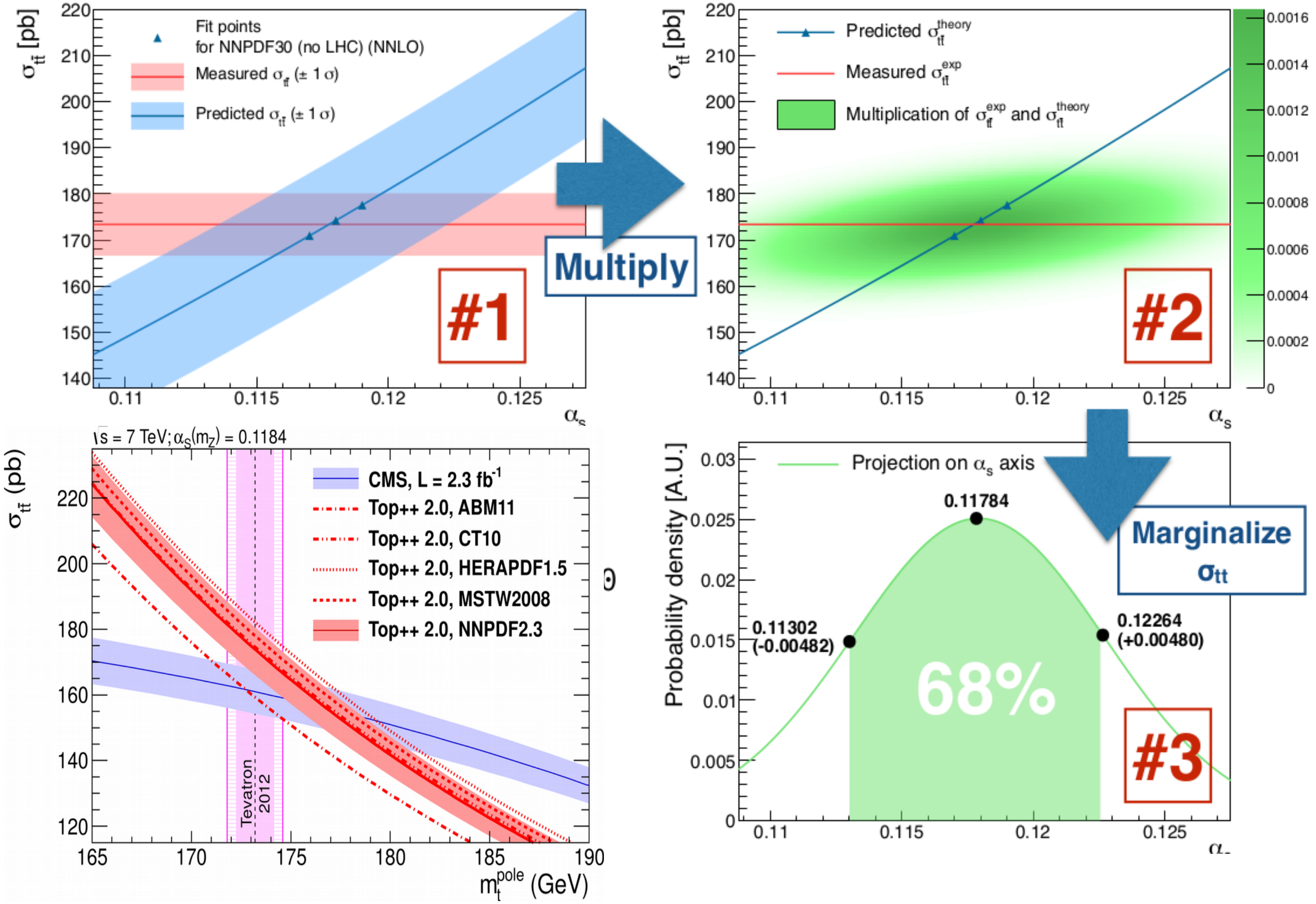


- Future:
  – Power-corrections for shapes, (N^{2,3}LL) resummation for rates. Grooming.
  – New e^+e^- data at lower-√s (Belle-II) & higher-√s (Higgs factories) needed.

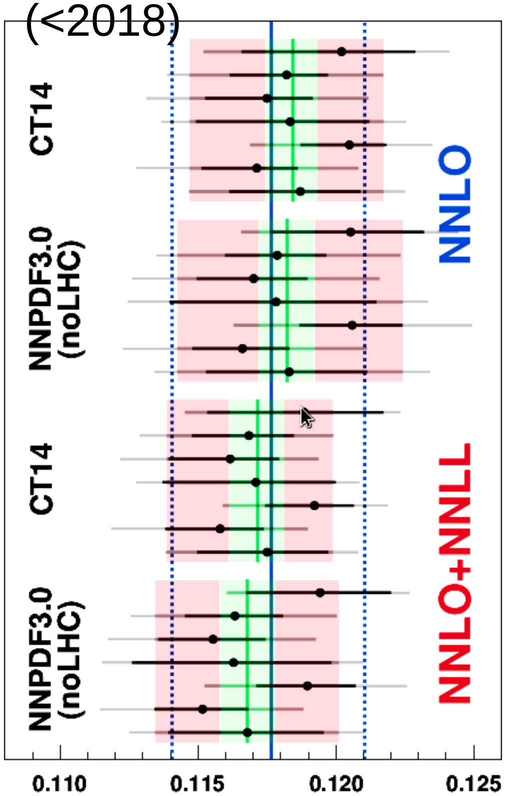
(5) α_s from hadron colliders: ttbar x-sections

- Top-pair inclusive x-sections available at NNLO
- Method: Compare σ(exp) to σ(NNLO) for diff. PDFs/α_s: Extract α_s via χ^2 minim.
  - Advantages: Direct LO sensitivity to α_s (via gg → ttbar)
  - Disadvantages: $\mathcal{O}$(5%) exp/th. uncertainties, dependence on m_{top}





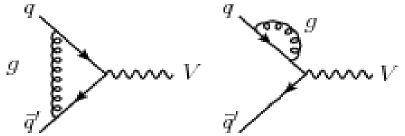
Analysis of 8 data sets (<2018)

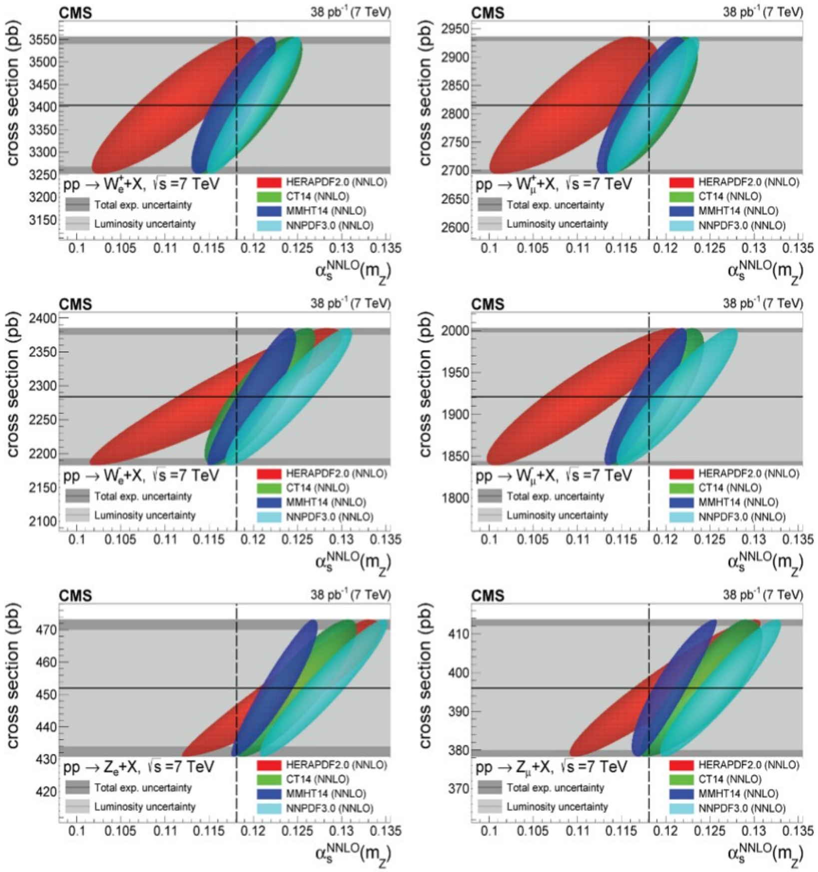


$$\alpha_s(m_Z) = 0.1177\,^{+0.0034}_{-0.0036}$$

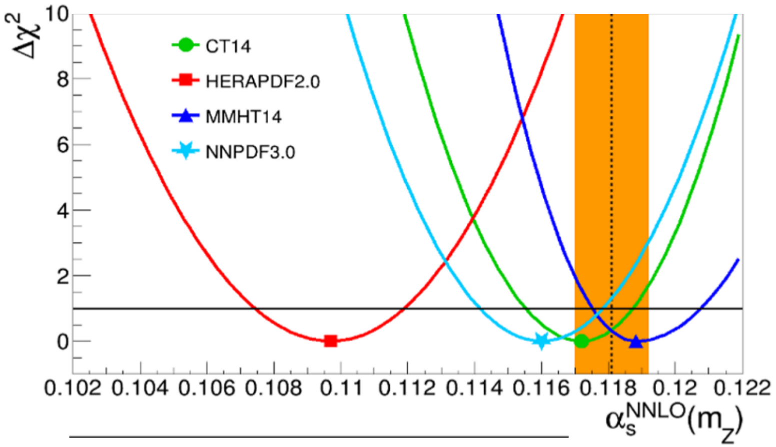
(5) α_s from hadron colliders: W, Z x-sections

- Inclusive W,Z boson x-sections available at $N^{2,3}LO$
- Method: Compare σ(exp) to σ(NNLO) for diff. PDFs/α_s: Extract α_s via χ^2 minim.
  - Advantages: $\mathcal{O}$(1–2%) exp/th. uncertainties
  - Disadvantages: No LO sensitivity to α_s (only via K_{NnLO}~1.3)





Combined fit of 28 LHC data sets



PDF	$\alpha_s(m_Z)$
CT14	$0.1172^{+0.0015}_{-0.0017}$
MMHT14	$0.1188^{+0.0019}_{-0.0013}$
HERAPDF2.0	$0.1097^{+0.0022}_{-0.0023}$
NNPDF3.0	0.1160 ± 0.0018

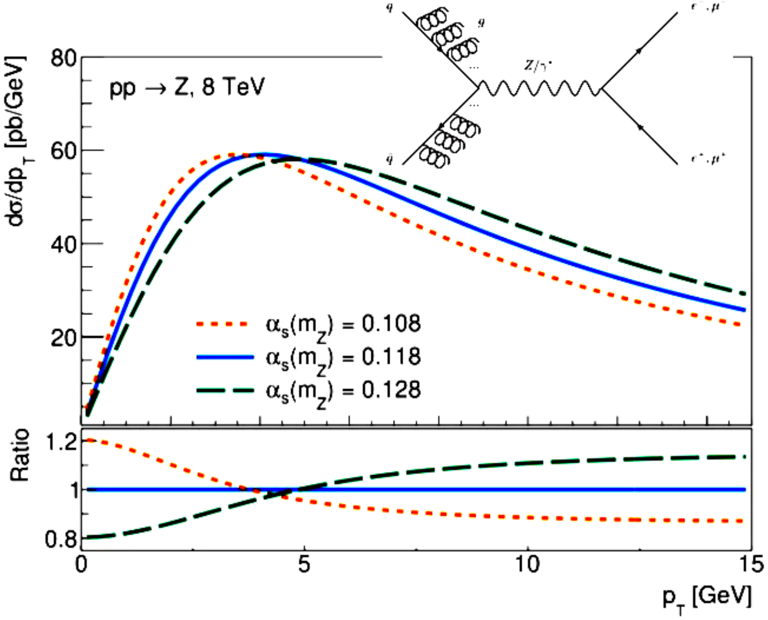
α_s = 0.1180 ± 0.0016 (±1.3%)

➧ Future: Incorporate σ(tt), σ(W,Z), σ(j), x-section ratios into global PDF+α_s fits.

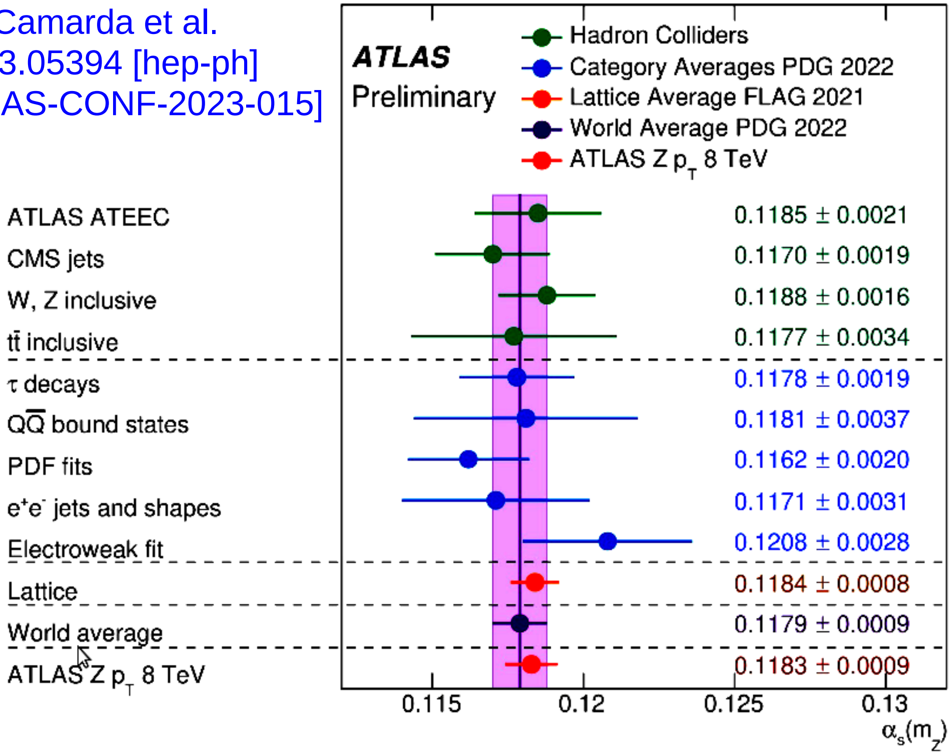
(5) α_s from hadron colliders: Z boson $d\sigma/dp_T$

NEW (not in PDG)

- Differential Z boson x-sections available at $N^{2,3}LO+N^{3,4}LL$
- Method: Compare $d\sigma/dp_T$(exp) to $d\sigma/dp_T(N^nLO+N^nLL)$ for diff. PDFs/α_s: Extract α_s
  - Advantages: $\mathcal{O}$(<1%) exp. uncertainties, direct sensitity to α_s (ISR gluon)
  - Disadvantages: Sensitivity to npQCD effects, resummation, HF PDFs



[S. Camarda et al. 2203.05394 [hep-ph]]
[ATLAS-CONF-2023-015]



- Extraction with "aggressive" ~0.8% uncertainty:
  - Just one N^3LO PDF fit so far with limited data
  - Gaussian npQCD model under good control?

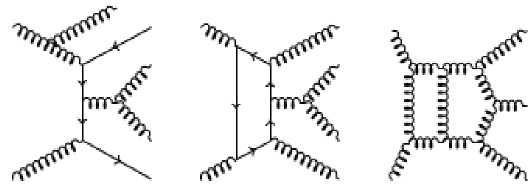
$\boxed{\alpha_s = 0.11828 +0.00084 -0.00088}$

(5) α_s from hadron colliders: EEC in multijets

NEW (not in PDG)

- Multijet transv. energy-energy corrs. available at NNLO
- Precise LHC measurements of dijet topologies up to $H_T \approx 4$ TeV

- Use multi-jets transverse energy–energy correlations (***TEEC***) and their associated azimuthal asymmetries (***ATEEC***) to perform the measurement



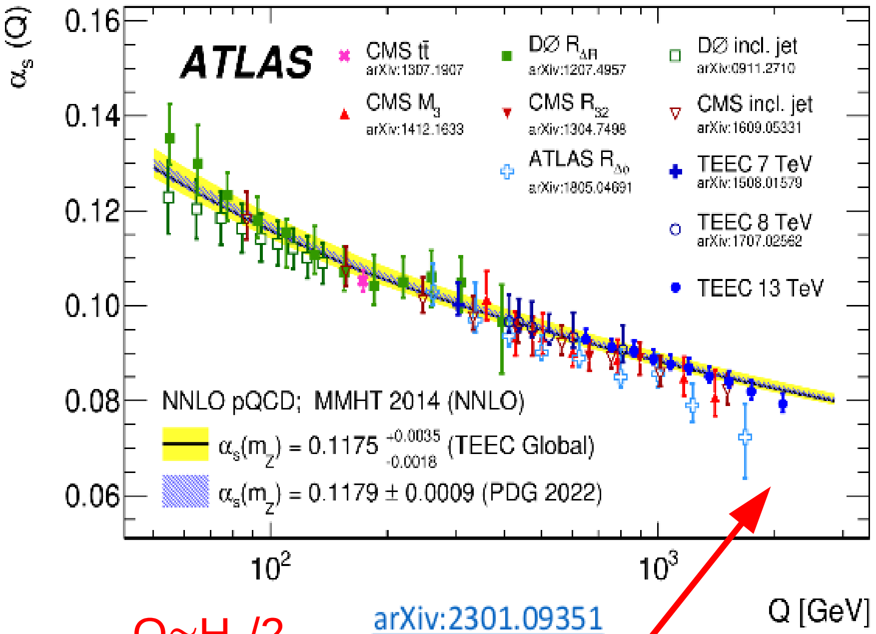
NNLO diagrams contributing to $gg \to$ jjj

- The TEEC function is defined as the transverse-energy-weighted distribution of the azimuthal differences between jet pairs in the final state

$$\frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi} = \frac{1}{N}\sum_{A=1}^{N}\sum_{ij}\frac{E_{\mathrm{T}i}^{A}E_{\mathrm{T}j}^{A}}{\left(\sum_k E_{\mathrm{T}k}^{A}\right)^2}\delta(\cos\phi - \cos\varphi_{ij})$$

Also use associated azimuthal asymmetries (ATEEC) to cancel uncertainties symmetric in cos(φ)

$$\frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi} = \frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi}\bigg|_{\phi} - \frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi}\bigg|_{\pi-\phi}$$



$Q \approx H_T/2$

arXiv:2301.09351

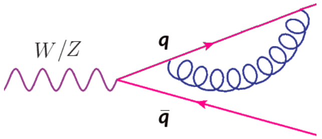
- Large theoretical improvement from NNLO correction to gg→3-jet production in pp collisions
  - reduction of theoretical uncertainty by a factor of 3

- RGE running of the strong coupling tested up to scales Q≈4 TeV

(6) α_s from Z boson hadronic decays

- Z-boson decays known at N^{3}LO, no NP uncerts. (but only ~4% sensitivity to α_s):

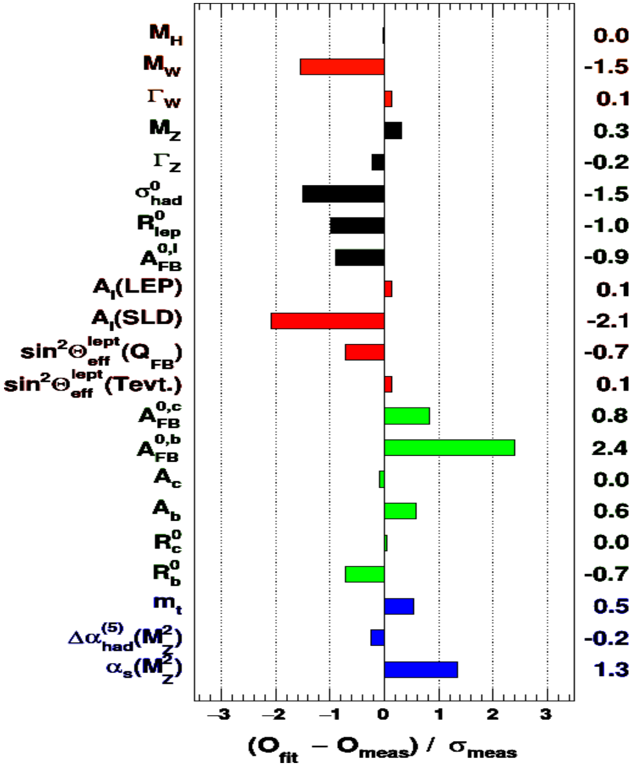
$$R_l^0 \equiv \frac{\Gamma(Z\to \mathrm{h})}{\Gamma(Z\to l)} = R_Z^{\mathrm{EW}} N_C \left(1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\mathrm{m}} + \delta_{\mathrm{np}}\right)$$

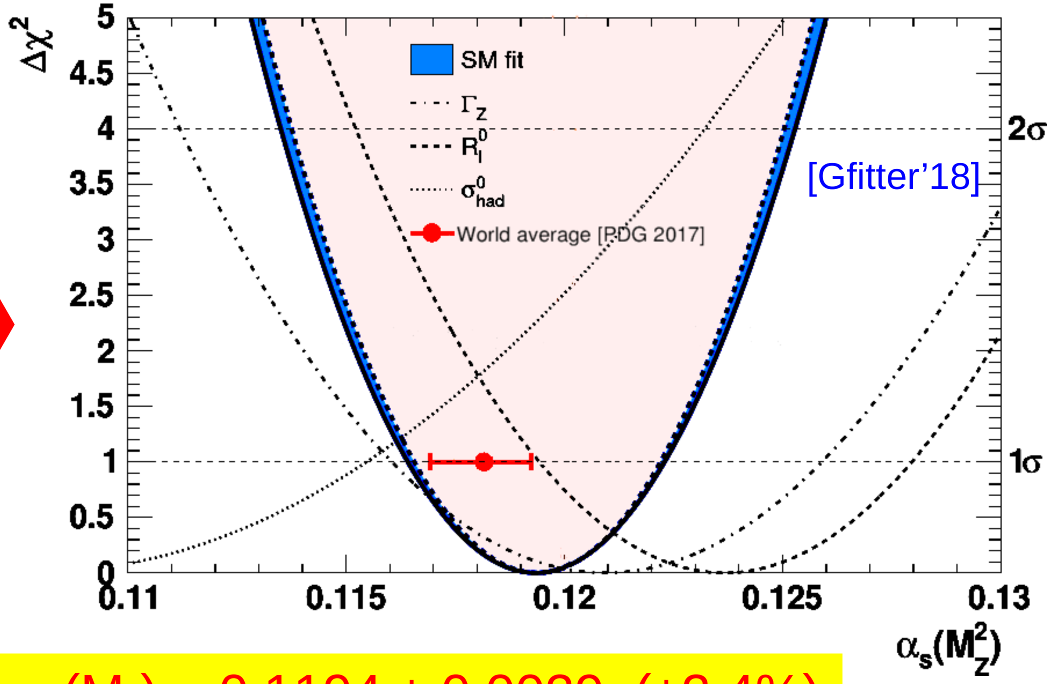


- Extraction from three Z-peak pseudo-observables (LEP, SLC):

Γ_Z = 2.4952±0.0023 GeV (±0.1%) → α_s (M_Z) = 0.1221 ± 0.0027 (±2.3%)

- Also from the global EW fit leaving α_s as single free parameter:



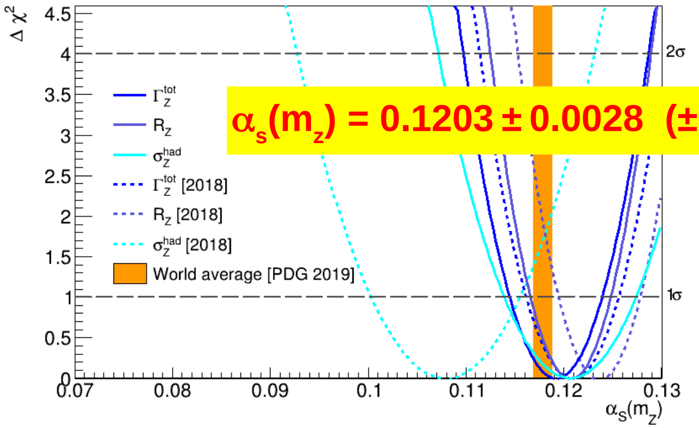


α_s (M_Z) = 0.1194 ± 0.0029 (±2.4%)

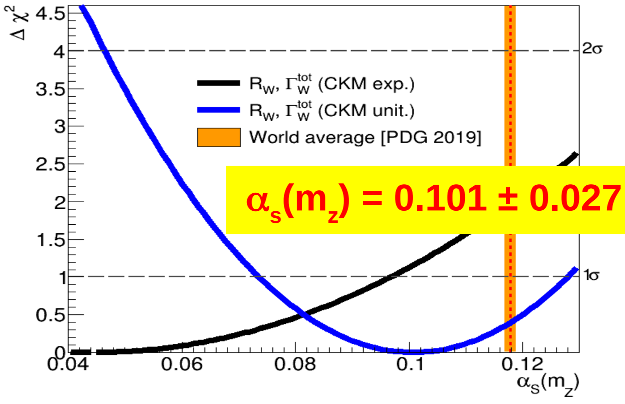
(6) α_s from EW bosons hadronic decays

- Updated Z,W-based $\alpha_s(m_z)$ extractions:

New fit with HO EW corrs. + corrected Z LEP data. New N^3LO fit to Γ_W, R_W

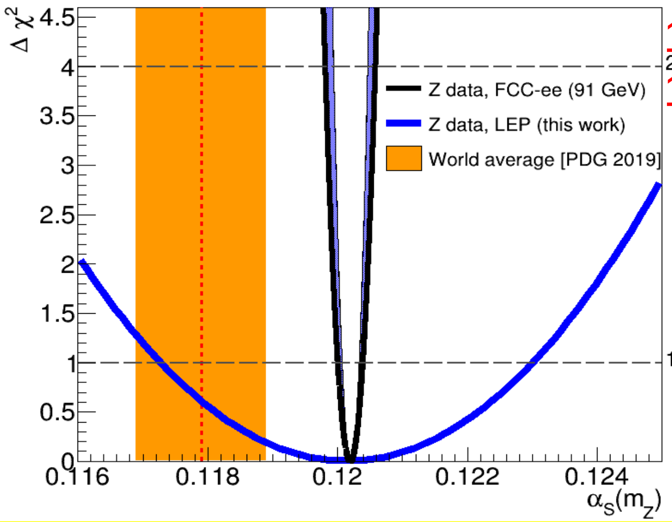


$\alpha_s(m_z) = 0.1203 \pm 0.0028$ (±2.3%)



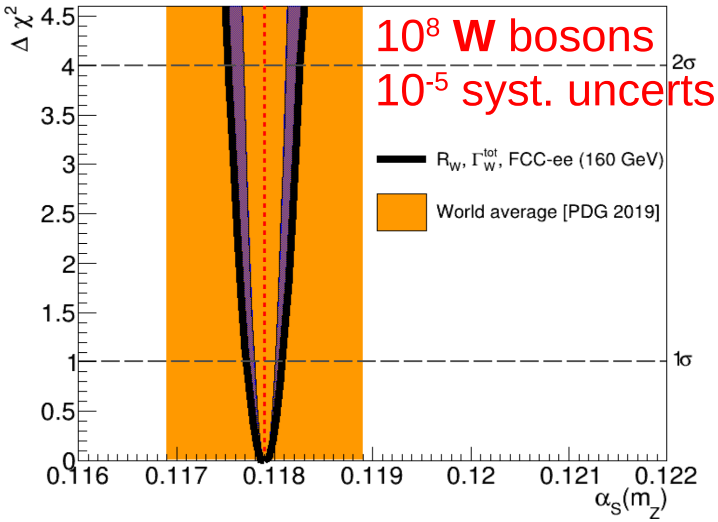
$\alpha_s(m_z) = 0.101 \pm 0.027$ (±27%)

- Future: Permille uncertainty possible only with a machine like FCC-e^+e^-



10^{12} **Z** bosons
10^{-5} syst. uncerts.

Strong SM
"stress test"

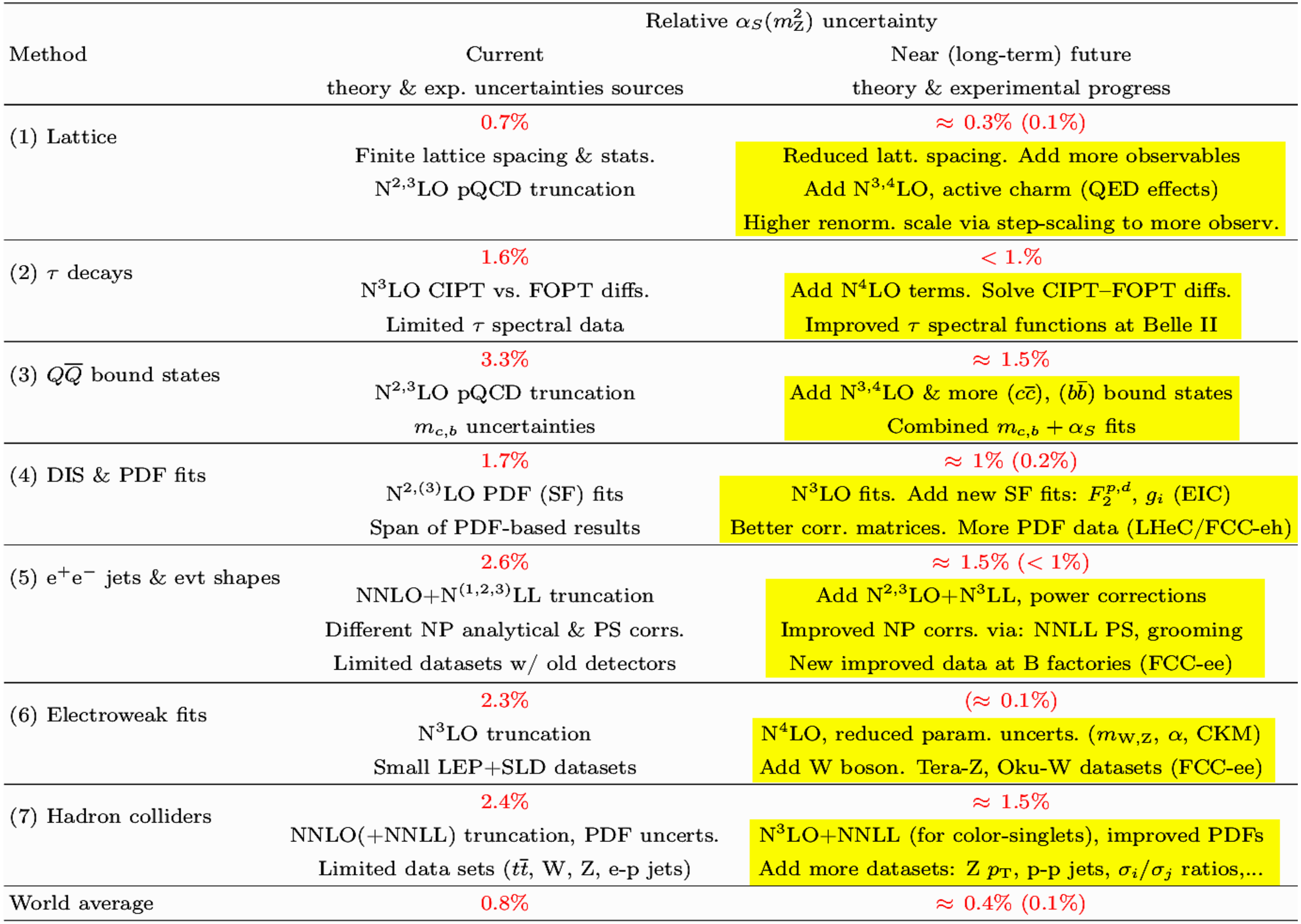


10^{8} **W** bosons
10^{-5} syst. uncerts.

$\alpha_s(m_z) = 0.12030 \pm 0.00028$ ±0.2% (tot), ±0.1% (exp)

$\alpha_s(m_z) = 0.11790 \pm 0.00023$ ±0.2% (tot), ±0.1% (exp)

Summary: Current & future α_s precision

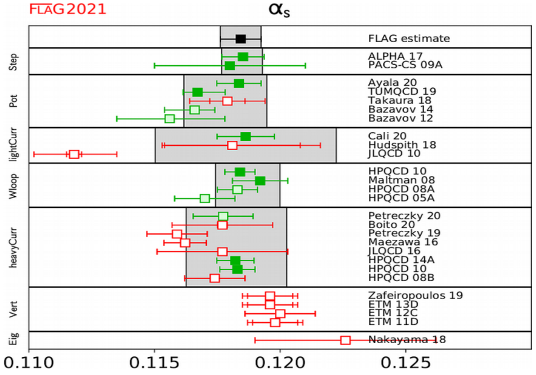


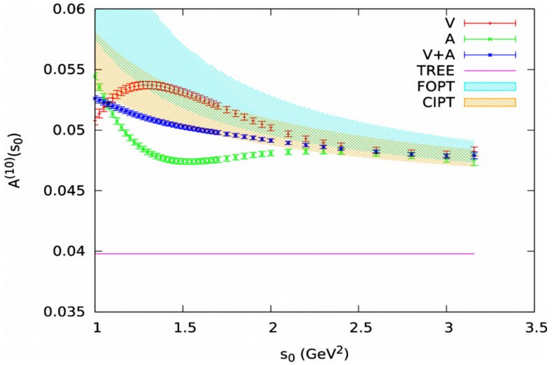
	Relative $\alpha_S(m_Z^2)$ uncertainty	
Method	Current theory & exp. uncertainties sources	Near (long-term) future theory & experimental progress
(1) Lattice	0.7% Finite lattice spacing & stats. $N^{2,3}$LO pQCD truncation	≈ 0.3% (0.1%) Reduced latt. spacing. Add more observables Add $N^{3,4}$LO, active charm (QED effects) Higher renorm. scale via step-scaling to more observ.
(2) τ decays	1.6% N^3LO CIPT vs. FOPT diffs. Limited τ spectral data	< 1.% Add N^4LO terms. Solve CIPT–FOPT diffs. Improved τ spectral functions at Belle II
(3) $Q\overline{Q}$ bound states	3.3% $N^{2,3}$LO pQCD truncation $m_{c,b}$ uncertainties	≈ 1.5% Add $N^{3,4}$LO & more $(c\bar{c})$, $(b\bar{b})$ bound states Combined $m_{c,b} + \alpha_S$ fits
(4) DIS & PDF fits	1.7% $N^{2,(3)}$LO PDF (SF) fits Span of PDF-based results	≈ 1% (0.2%) N^3LO fits. Add new SF fits: $F_2^{p,d}$, g_i (EIC) Better corr. matrices. More PDF data (LHeC/FCC-eh)
(5) e^+e^- jets & evt shapes	2.6% NNLO+$N^{(1,2,3)}$LL truncation Different NP analytical & PS corrs. Limited datasets w/ old detectors	≈ 1.5% (< 1%) Add $N^{2,3}$LO+N^3LL, power corrections Improved NP corrs. via: NNLL PS, grooming New improved data at B factories (FCC-ee)
(6) Electroweak fits	2.3% N^3LO truncation Small LEP+SLD datasets	(≈ 0.1%) N^4LO, reduced param. uncerts. ($m_{W,Z}$, α, CKM) Add W boson. Tera-Z, Oku-W datasets (FCC-ee)
(7) Hadron colliders	2.4% NNLO(+NNLL) truncation, PDF uncerts. Limited data sets ($t\bar{t}$, W, Z, e-p jets)	≈ 1.5% N^3LO+NNLL (for color-singlets), improved PDFs Add more datasets: Z p_T, p-p jets, σ_i/σ_j ratios,...
World average	0.8%	≈ 0.4% (0.1%)

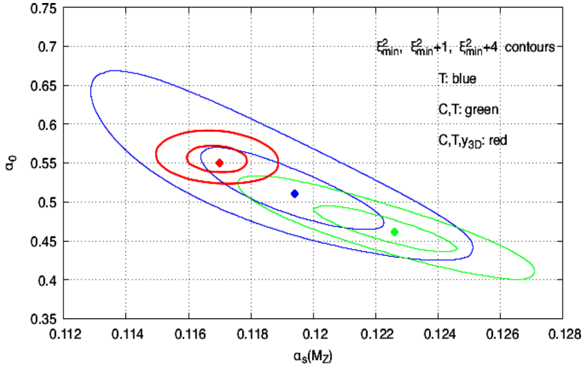
- Well-defined exp./th. path towards $\alpha_s(m_Z)$ permil precision in coming years

Summary: Present & future of the strong coupling

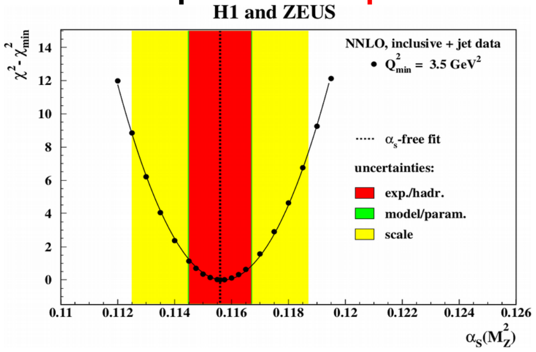
- Remarkable theoretical (N^nLO+N^nLL, lattQCD, non-pQCD) developments:

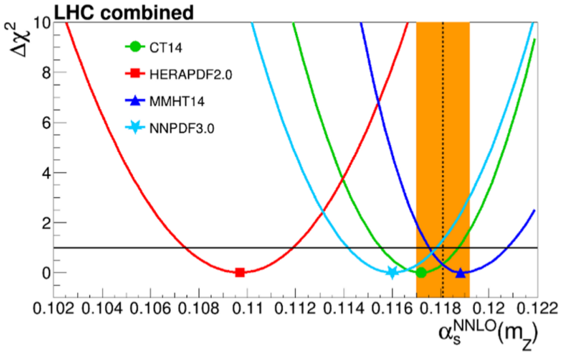


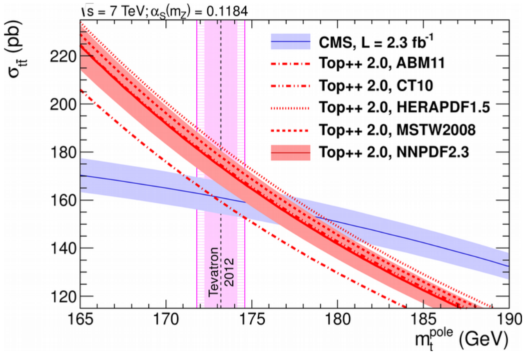


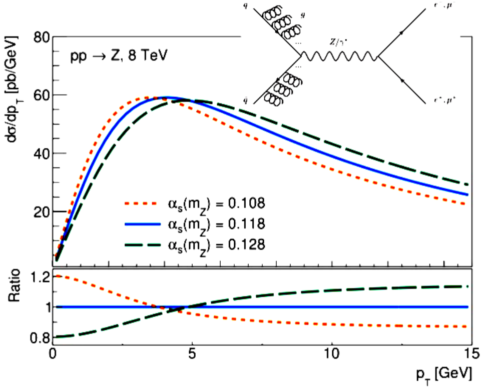


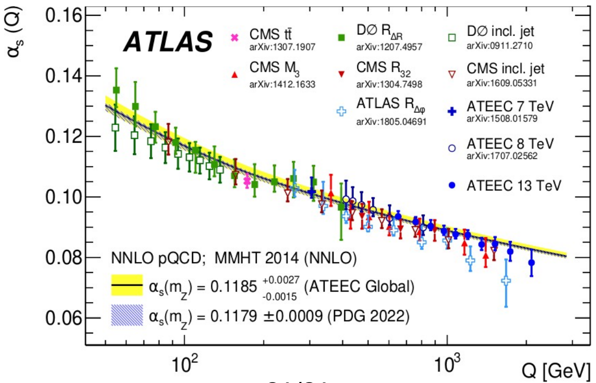
- Multiple new precision experimental measurements:

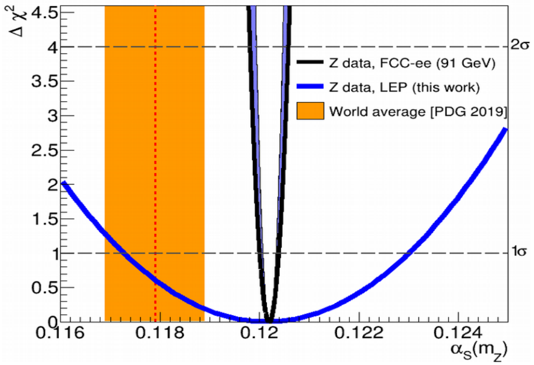












➧ Future: $\mathcal{O}$(0.1%)

Backup slides

Summary: α_s wish-list for the next ~10 years

■ Theoretical needs to approach $\mathcal{O}$(0.1%) precision:

(1) Lattice QCD: Sufficient dedicated computing resources & person-power to

- Develop pQCD N^{3,4}LO theory for observables in a finite space-time volume
- Extend higher renormalization scales via step-scaling to more observables

(2) Other phenomenology efforts:

- Completion of hadronic τ decay renormalon analysis
- Advanced power corrections for e^+e^- event shapes & resummation for jet rates
- NNLL accuracy parton showers matched to NNLO
- NNLO(+NNLL) MCs for complex final states in e^+e^-, e-p, p-p
- Differential NNLO predictions for LHC & HERA multi-jet observables,…

■ Experimental needs to approach $\mathcal{O}$(0.1%) precision:

(3) Extension of N^{2,3}LO hadron-collider- and/or PDF-based extractions via:

- LHC: Adding new precision observables & datasets (TEECC, Z pT, σ_i/σ_j ratios,...)
- LHC: Improved treatment of exp. correl. matrices uncertainties among measurements
- EIC: New DIS measurements (also LHeC/FCC-eh after it).
- FCC: Hadronic Z (and W) decays is the only non-lattice method known that can reach permille precision: Tera-Z machine needed.