(Y. Gominet, IMCCE)

Jacques Laskar Observatoire de Paris

From chaos in planetary motions to the HL-LHC design



Rencontres du Vietnam, 2-12 August 2023











CMS-LHC

Jürgen Moser (July 1928 - december 1999)

In 1954 Kolmogorov indicated that, for certain mechanical systems, in some sense the "majority" of solutions are quasiperiodic. He indicated a possible method of solution but the actual proof was first provided by Arnold 8 years later, and, in a special case, by the author. In accordance with the modern usage this theory became known by the acronym KAM.



volume 1 (1978), 65-71

Jürgen Moser (July 1928 - december 1999)

The mathematical theorems of KAM deal not only with the planetary system but also with general Hamiltonian systems.... One of these applications is the stability problem of proton accelerators, which since the 1950's have been built in every greater number and size.



Figure 4 a. Cross-section of the vacuum chamber at the position of the beam inflector, with indication of the stacking process



volume 1 (1978), 65-71

Question raised since Newton (1704)

•Question raised since Newton (1704)

•Positive answer by Lagrange and Laplace (1772-76)

•Question raised since Newton (1704)

•Positive answer by Lagrange and Laplace (1772-76)

•Questioned by Poincaré (1892)

•Question raised since Newton (1704)

- •Positive answer by Lagrange and Laplace (1772-76)
- •Questioned by Poincaré (1892)
- •Positive answer by Arnold (KAM) (1963)

Chaotic motion of the Solar System Secular equations : 200 Ma : J. Laskar (1989, 1990) Direct integration : 100 Ma : J.G. Sussman & J. Wisdom (1992)











Origin of CHAOS



Resonance overlap

Chirokov (1972)

$$H = \frac{I^2}{2} + a\cos(\theta - t) + a\cos(\theta + t)$$
$$\Delta\omega = 4\sqrt{a}$$





a=0.05





FIG. 3. (a-d) Examples of transition from libration around 0° (a) to circulation (b), and from circulation (c) to libration around 180° for the argument $2(\varpi_4^{\circ} - \varpi_3^{\circ}) - (\Omega_4^{\circ} - \Omega_3^{\circ})$. The quantity which is actually plotted in the complex plane is $z_{04} \exp i(2(\varpi_4^{\circ} - \varpi_3^{\circ}) - (\Omega_4^{\circ} - \Omega_3^{\circ}))$ (Eq. (26)).

$$2(g_4 - g_3) - (s_4 - s_3)$$

(Laskar, 1990)

Origin of CHAOS

Astronomy Astrophysics

Letter to the Editor

(9 May 2022)

The origin of chaos in the Solar System through computer algebra

Federico Mogavero and Jacques Laskar

Deg	2	4	6	8	10
N monomials	8	6304	188024	3 394 892	42 817 100

i		Fourier harmonic $[\mathcal{F}_i]$	$\omega_{ m hyp}$	$ au_{ m res}$	$\omega_{ m ell}$	$ au_{ m libr}$	$\Delta \omega$
1	*	$g_3 - g_4 - s_3 + s_4$	$0.31^{0.67}_{0.08}$	12%	$0.65^{1.56}_{0.26}$	18%	$0.33_{0.09}^{0.53}$
2	+	$g_1 - g_2 + s_1 - s_2$	$0.38^{0.79}_{0.12}$	19%	$0.89^{1.34}_{0.26}$	26%	$0.30^{0.61}_{0.15}$
3	+	$g_2 - g_5 - 2s_1 + 2s_2$	$0.22^{0.33}_{0.07}$	23%	$0.33_{0.16}^{0.45}$	56%	$0.11_{\scriptstyle 0.04}^{\scriptstyle 0.22}$
4	\star	$2g_3 - 2g_4 - s_3 + s_4$	$0.14^{0.36}_{0.04}$	70%	$0.43^{0.73}_{0.23}$	74%	$0.08^{0.16}_{0.02}$
5		$g_1 - g_5 - s_1 + s_2$	$0.08^{0.10}_{0.05}$	10%	$0.22^{0.28}_{0.14}$	63%	$0.07^{\scriptscriptstyle 0.18}_{\scriptscriptstyle 0.06}$
6		$g_2 - g_4 + s_2 - s_4$	$0.07^{0.09}_{0.04}$	6%	$0.07^{0.09}_{0.05}$	70%	$0.07^{\scriptscriptstyle 0.10}_{\scriptscriptstyle 0.03}$
7		$g_1 - 2g_2 + g_4 + s_1 - 2s_2 + s_4$	$0.11_{\scriptstyle 0.10}^{\scriptstyle 0.13}$	5%	$0.12^{0.13}_{0.11}$	69%	$0.06^{\scriptscriptstyle 0.07}_{\scriptscriptstyle 0.05}$
8		$g_1 - g_3 + s_2 - s_3$	$0.06^{0.09}_{0.03}$	17%	$0.06^{0.10}_{0.04}$	60%	$0.06^{0.09}_{0.03}$
9		$g_1 + g_3 - 2g_4 + s_2 - s_3$	$0.08^{0.09}_{0.06}$	5%	$0.08^{0.09}_{0.07}$	55%	$0.05^{\scriptscriptstyle 0.06}_{\scriptscriptstyle 0.04}$
10	\star	$3g_3 - 3g_4 - s_3 + s_4$	$0.11_{0.01}^{0.34}$	9%	$0.14^{0.24}_{0.06}$	40%	$0.05^{\scriptscriptstyle 0.14}_{\scriptscriptstyle 0.01}$
11		$g_2 - g_3 - s_1 + 2s_2 - s_3$	$0.06^{0.07}_{0.04}$	5%	$0.06^{0.07}_{0.05}$	50%	$0.04^{0.05}_{0.03}$
12		$g_1 - 2g_3 + g_4 + s_2 - s_4$	$0.06^{0.12}_{0.03}$	36%	$0.06^{0.12}_{0.03}$	51%	$0.04^{0.08}_{0.02}$
13		$2g_1 - g_3 - g_5 + s_2 - s_4$	$0.05^{0.06}_{0.04}$	5%	$0.05^{0.06}_{0.04}$	52%	$0.04^{0.04}_{0.03}$
1 /		. 0		\mathbf{a}	0.05	FCM	0.02004



Mogavero & Laskar, 2021

Origin of CHAOS

$$\theta_{2:1} = 2(g_4 - g_3) - (s_4 - s_3)$$
$$\theta_{3:2} = (g_4 - g_3) - (s_4 - s_3)$$
$$\theta_{1:1} = (g_4 - g_3) - (s_4 - s_3)$$



Laskar, 200?



ICARUS 88, 266–291 (1990)

The Chaotic Motion of the Solar System: A Numerical Estimate of the Size of the Chaotic Zones

J. LASKAR

Measure of the chaotic diffusion in all directions in a 32 dimensional phase space

(Laskar, 1990)



Non-quasiperiodic Motion

(Poincaré, Smale,...)







chaotic : non-quasiperiodic



Frequency Maps

$$H(I,\theta) = H_0(I) + \varepsilon H_1(I,\theta) \qquad (I,\theta) \in \mathbb{B}^n \times \mathbb{T}^n$$

$\varepsilon = 0$

$$\dot{I}_{j} = 0 , \quad \dot{\theta}_{j} = \frac{\partial H_{0}(I)}{\partial I_{j}} = \nu_{j}(I) ;$$
$$I_{j} = I_{j0} \qquad \theta_{j}(t) = \theta_{j0} + \nu_{j}t$$

nondegenerate

$$\left|\frac{\partial\nu(I)}{\partial I}\right| = \left|\frac{\partial^2 H_0(I)}{\partial I^2}\right| \neq 0$$

frequency map

$$F: \mathbb{B}^n \longrightarrow \mathbb{R}^n$$
$$(I) \longrightarrow (\nu)$$

diffeomorphism on $F(\mathbb{B}^n) = \Omega$

Frequency MAP

$$\varepsilon \neq 0$$

KAM theorem (Kolmogorov 1954, Arnold 1963, Moser 1963)

Frequency MAP

$$\exists \ \Omega_{\varepsilon} \subset \Omega$$
 Cantor set $|\langle k, \nu \rangle| > \frac{\kappa_{\varepsilon}}{|k|^p}$

 $(\nu) \in \Omega_{\varepsilon} \Rightarrow$ quasiperiodic solution

Pöschel (1982), there exists a diffeomorphism

$$\Psi : \mathbb{T}^n imes \Omega \longrightarrow \mathbb{T}^n imes \mathbb{B}^n$$

 $(\varphi, \nu) \longrightarrow (\theta, I)$

analytical $/\varphi$ and C^{∞}/ν on $\mathbb{T}^n \times \Omega_{\varepsilon}$ transforms the equations into

$$\dot{
u}_j = 0$$
 , $\dot{\varphi}_j =
u_j$;

thus, if $(\nu) \in \Omega_{\varepsilon}$, $z_j = I_j e^{i\theta_j}$, $z_j(t) = z_{j0} e^{i\nu_j t} + \sum_m a_m(\nu) e^{i < m, \nu > t}$



$$F_{\theta_0}^T : \mathbb{R}^n \longrightarrow \mathbb{R}^n$$
$$(I) \longrightarrow (\nu_1, \nu_2, \dots, \nu_n)$$



 $F_{\theta_0}^T : \mathbb{R} \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ $(\tau, I) \longrightarrow (\nu)$

Frequency MAP



Frequency MAP

Thus : on the set of KAM tori,



$$F_{\theta_0}^T : \mathbb{R} \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$$
$$(\tau, I) \longrightarrow (\nu)$$

Frequency MAP

Thus : on the set of KAM tori, $F_{\theta_0}^T$ is constant / au



Thus : on the set of KAM tori, $F_{\theta_0}^T$ is constant $/\tau$

 \int_0^{r} is a smooth diffeomorphism

Frequency MAP







Global Dynamics and Long-Time Stability in Hamiltonian Systems via Numerical Frequency Analysis

H. S. Dumas

Department of Mathematical Sciences, University of Cincinnati, Cincinnati, Ohio 45221-0025

J. Laskar Astronomie et Systèmes Dynamiques, Bureau des Longitudes, 77 av. Denfert-Rochereau, 75014 Paris, France (Received 18 November 1992)



FIG. 1. Image in the frequency plane ("tune space") (f_1, f_2) of the square sector $0 \le I_1, I_2 \le 10^{-5}$, obtained using frequency analysis over 4052 turns.



FIG. 2. Transport rates in the frequency plane. The speed and location of local transport phenomena are visualized by plotting $f_2 + 0.00005 \log(r)$ against f_1 for the same data as in Fig. 1, where r is the estimated local rate of transport (for r above a given threshold).



Global Dynamics and Long-Time Stability in Hamiltonian Systems via Numerical Frequency Analysis

H. S. Dumas

Department of Mathematical Sciences, University of Cincinnati, Cincinnati, Ohio 45221-0025

J. Laskar

Astronomie et Systèmes Dynamiques, Bureau des Longitudes, 77 av. Denfert-Rochereau, 75014 Paris, France (Received 18 November 1992)



FIG. 1. Image in the frequency plane ("tune space") (f_1, f_2) of the square sector $0 \le I_1, I_2 \le 10^{-5}$, obtained using frequency analysis over 4052 turns.



FIG. 2. Transport rates in the frequency plane. The speed and location of local transport phenomena are visualized by plotting $f_2 + 0.00005 \log(r)$ against f_1 for the same data as in Fig. 1, where r is the estimated local rate of transport (for r above a given threshold).



In three of the twelve ALS Sectors Replaced the central combined function dipole with a Superbend and two quadrupoles



Robin, Steir, Nadolski, Laskar

FMAP_Workshop - April 1, 2004

ALS : Ideal Lattice versus Calibrated Model



rrrr

Robin, Steir, Nadolski, Laskar

FMAP_Workshop - April 1, 2004

Experimental Frequency Map at ALS

Experiment

Numerical model



Robin, Steier, Laskar, Nadolski, PRL, 2000

VOLUME 85, NUMBER 3

PHYSICAL REV



FIG. 4. Experimental frequency map for a previous setting of the ALS.

Robin, Steier, Laskar, Nadolski, PRL, 2000

CERN COURIER Jan/Feb 2001

FREQUENCY MAP ANALYSIS

Mapping chaos in particle revolutions

Over the past decade the technique of frequency map analysis, developed to study astronomical systems, has shown its value in an increasing number of areas, including the analysis of particle orbits in accelerators.

At first glance, any close association between the planets of the solar system – huge masses of rock, liquid and gas gently guided by gravity through the vast emptiness of space – and the mad traffic of tightly bunched particles in a circular accelerator, crushed together by fierce radiofrequency and magnetic fields, could hardly seem less likely.

Nonetheless, the dynamics of planets moving through our solar system and particles moving in accelerators do share many similar features. Both demand an analysis of the evolution of a dynamic system over a very long time – up to 1 billion revolutions for both the



Part of the ALS storage ring at the Lawrence Berkeley National Laboratory. Frequency mapping was first applied to measured rather than simulated electron trajectories at the ALS. trajectories in a storage ring, at the Advanced Light Source (ALS) at the Lawrence Berkeley National Laboratory. The aim was to reveal the dynamics of an actual particle beam.

Chaotic motion

The story of frequency map analysis began in 1989 when Jacques Laskar (Bureau des Longitudes, Paris) demonstrated that the motion of the solar system is chaotic (Laskar 1989). He showed that the separation between two orbits with similar initial conditions will diverge exponentially over time (e.g. the distance between the orbits will increase

Project Director Dave Robin Announces ALS-U Project Beamlines

JANUARY 29, 2019

Over the past year, a process involving ALS and ALS-U staff, the ALS user community, and external advisory committees has been ongoing to select the insertion-device beamlines that will be built and upgraded within the scope of the ALS-U Project. These beamlines will join existing ALS beamlines to form the full complement of capabilities that will be available at the upgraded ALS in several years. I am delighted to inform you that the selection process is now complete and to announce the result.

BERKELEY LAB



ALS-U will reach soft x-ray diffraction limit up to 1.5 keV

ALS-U





C. Steier /D. Robin





Laurent S. Nadolski Senior Accelerator Physicist Accelerators Coordinator Synchrotron SOLEIL, France

- Ph.D., (2001).
- "Application of Frequency Map Analysis to the Study of Beam Dynamics" (J. Laskar's supervision).
- Former President of Accelerator Sect. of the French Physical Society
- SOLEIL II: nonlinear beam dynamics, robustness studies, storage ring coordination (collimation, radiation safety, machine protection interlock).

Present SOLEIL II Timeline





SOLEIL II: a 4th GENERATION SYNCHROTRON LIGHT SOURCE for the **Science of Tomorrow**

An electron beam 40 times smaller and circular Photon beams at least 100 times brighter and more coherent in the X-ray range.

Design Report

Obtaining a very compact layout: Lifting of technological barriers, with the miniaturization of the vacuum chambers where the electrons circulate and of the magnets that guide them.



CERN Accelerating Science

Yannis Papaphilippou

PhD (1997) Application of the Frequency Map Analysis Method in Galactic Dynamics





Galactic model

Yannis Papaphilippou

The LHC

Y. Papaphilippou, 1999



PhD (1997) Application of the Frequency Map Analysis Method in Galactic Dynamics



Galactic model

ABP *a* **CERN** organisation





Y. Papaphilippou

Prepare machine for operation beyond 2025 and up to 2035



Y. Papaphilippou

Prepare machine for operation beyond 2025 and up to 2035

Devise beam parameters and operation scenarios for enabling at total integrated luminosity of **3000 fb**⁻¹



Prepare machine for operation beyond 2025 and up to 2035

Devise beam parameters and operation scenarios for enabling at total integrated luminosity of **3000 fb**⁻¹

→ 10 times the luminosity reach over first 10 years of LHC operation



Y. Papaphilippou

Prepare machine for operation beyond 2025 and up to 2035

Devise beam parameters and operation scenarios for enabling at total integrated luminosity of **3000 fb**⁻¹

→ 10 times the luminosity reach over first 10 years of LHC operation





Beam-Beam interaction

Variable	Symbol	Value		
Beam energy	E	7 TeV		
Particle species		protons		
Full crossing angle	$ heta_c$	$300 \ \mu rad$		
rms beam divergence	σ'_r	31.7 μ rad		
rms beam size	σ_x	15.9 μm		
Normalized transv.				
rms emittance	γε	3.75 µm		
IP beta function	β^*	0.5 m		
Bunch charge	N_b	$(1 \times 10^{11} - 2 \times 10^{12})$		
Betatron tune	Q_0	0.31		

 Long range beam-beam interaction represented by a 4D kick-map

PACMAN bunch head-on collision Δy long-range collisions with



$$\theta_t \equiv \left((x' + \theta_c)^2 + {y'}^2 \right)^{1/2}$$



Y. Papaphilippou

Y. Papaphilippou

Wire compensation



LHC: Power supply ripples (5D)

□ Scan of different ripple frequencies (50-900 Hz)



LHC: Power supply ripples (5D)

□ Scan of different ripple frequencies (50-900 Hz)



LHC: Power supply ripples (5D)

□ Scan of different ripple frequencies (50-900 Hz)



Incoherent e-cloud effects at (HL-)LHC

- Analysis of experimental data from the LHC shows a slow beam degradation due to e-cloud both at injection and in collisions
- A development effort was launched to acquire the ability of simulating the effect of e-cloud forces within a symplectic tracking code over the required long timescales (10M turns). The development included:
 - Theoretical framework
 - Tricubic interpolator in sixtracklib tracking code to apply forces from a recorded pinch in a symplectic way
 - Software infrastructure to simulate and condition the electron pinches and setup the simulation from the MAD-X description of the machine
- Presently capable of simulating 10 M turns (15 minutes of beam time) by exploiting the computational power of GPUs



When you tackle difficult problems, you need to invent new methods that may be of general use

Some references

The chaotic motion of the solar system: A numerical estimate of the size of the chaotic zones J Laskar Icarus 88 (2), 266-291

Frequency analysis for multi-dimensional systems. Global dynamics and diffusion

J Laskar Physica D: Nonlinear Phenomena 67 (1-3), 257-281

The measure of chaos by the numerical analysis of the fundamental frequencies. Application to the standard mapping

J Laskar, C Froeschlé, A Celletti Physica D: Nonlinear Phenomena 56 (2-3), 253-269

Existence of collisional trajectories of Mercury, Mars and Venus with the Earth

J Laskar, M Gastineau Nature 459 (7248), 817-819

Frequency map analysis and particle accelerators

J Laskar Proceedings of the 2003 Particle Accelerator Conference 1, 378-382

Measuring and optimizing the momentum aperture in a particle accelerator

C Steier, D Robin, L Nadolski, W Decking, Y Wu, J Laskar Physical Review E 65 (5), 056506

Review of single particle dynamics for third generation light sources through frequency map analysis

L Nadolski, J Laskar Physical Review Special Topics-Accelerators and Beams 6 (11), 114801

Global dynamics and long-time stability in Hamiltonian systems via numerical frequency analysis

HS Dumas, J Laskar Physical review letters 70 (20), 2975

Understanding the nonlinear beam dynamics of the advanced light source

D Robin, J Laskar AIP Conference Proceedings 391 (1), 15-20