

# From chaos in planetary motions to the HL-LHC design 

Rencontres du Vietnam, 2-12 August 2023

\author{

- <br> PSL 太
}


SORBONNE UNIVERSITÉ


## Is the Solar System Stable? Jürgen Moser (July 1928-december 1999)

In 1954 Kolmogorov indicated that, for certain mechanical systems, in some sense the "majority" of solutions are quasiperiodic. He indicated a possible method of solution but the actual proof was first provided by Arnold 8 years later, and, in a special case, by the author. In accordance with the modern usage this theory became known by the acronym KAM.

## Is the Solar System Stable? Jürgen Moser (July 1928-december 1999)

The mathematical theorems of KAM deal not only with the planetary system but also with general Hamiltonian systems.... One of these applications is the stability problem of proton accelerators, which since the 1950's have been built in every greater number and size.


Figure 4 a. Cross-section of the vacuum chamber at the position of the beam inflector, with indication of the stacking process

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- Questioned by Poincaré (I892)
-Positive answer by Arnold (KAM) (I963)


## Chaotic motion of the Solar System

Secular equations : 200 Ma : J. Laskar $(1989,1990)$ Direct integration : 100 Ma : J.G. Sussman \& J. Wisdom (1992)


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## Max eccentricity of Mercury WITH relativity direct equations (2501 sol) <br> ( 0.38 mm )



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## Origin of CHAOS



## Resonance overlap

Chirokov (1972)

$$
H=\frac{I^{2}}{2}+a \cos (\theta-t)+a \cos (\theta+t)
$$

$$
\Delta \omega=4 \sqrt{a}
$$







Fig. 3. (a-d) Examples of transition from libration around $0^{\circ}$ (a) to circulation (b), and from circulation (c) to libration around $180^{\circ}$ for the argument $2\left(\varpi_{4}^{-}-\varpi_{3}^{\dot{j}}\right)-\left(\Omega_{4}-\Omega_{3}\right)$. The quantity which is actually plotted in the complex plane is $z_{i 4} \exp i\left(2\left(\sigma_{4}^{*}-\omega_{i}\right)-\left(\Omega_{4}-\Omega_{3}\right)\right)(\mathrm{Eq} .(26))$.

$$
2\left(g_{4}-g_{3}\right)-\left(s_{4}-s_{3}\right)
$$

Origin of CHAOS
The origin of chaos in the Solar System through computer algebra
Federico Mogavero and Jacques Laskar

| Deg | 2 | 4 | 6 | 8 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| N monomials | 8 | 6304 | 188024 | 3394892 | 42817100 |


| $i$ |  | Fourier harmonic [ $\mathcal{F}_{i}$ ] | $\omega_{\text {hyp }}$ | $\tau_{\text {res }}$ | $\omega_{\text {ell }}$ | $\tau_{\text {libr }}$ | $\Delta \omega$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\star$ | $g_{3}-g_{4}-s_{3}+s_{4}$ | $0.31_{0.08}^{0.67}$ | 12\% | $0.65{ }_{0.26}^{1.56}$ | 18\% | $0.33_{0.09}^{0.53}$ |
| 2 | $\dagger$ | $g_{1}-g_{2}+s_{1}-s_{2}$ | $0.38_{0.12}^{0.79}$ | 19\% | $0.89_{0.26}^{1.34}$ | 26\% | $0.30_{0.15}^{0.061}$ |
| 3 | $\dagger$ | $g_{2}-g_{5}-2 s_{1}+2 s_{2}$ | $0.22_{0.07}^{0.33}$ | 23\% | $0.33_{0.16}^{0.45}$ | 56\% | $0.11_{0.04}^{0.22}$ |
| 4 | $\star$ | $2 g_{3}-2 g_{4}-s_{3}+s_{4}$ | $0.14{ }_{0.04}^{0.36}$ | 70\% | $0.43_{0.23}^{0.73}$ | 74\% | $0.08_{0.02}^{0.16}$ |
| 5 | $\dagger$ | $g_{1}-g_{5}-s_{1}+s_{2}$ | $0.08_{0.05}^{0.10}$ | 10\% | $0.22_{0.14}^{0.28}$ | 63\% | $0.070{ }_{0.06}^{0.18}$ |
| 6 |  | $g_{2}-g_{4}+s_{2}-s_{4}$ | $0.07{ }_{0.04}^{0.09}$ | 6\% | $0.07_{0.05}^{7.09}$ | 70\% | $0.07{ }_{0.03}^{0.10}$ |
| 7 |  | $g_{1}-2 g_{2}+g_{4}+s_{1}-2 s_{2}+s_{4}$ | $0.11_{0.10}^{0.13}$ | 5\% | $0.12_{0.11}^{0.13}$ | 69\% | $0.066_{0.05}^{0.07}$ |
| 8 |  | $g_{1}-g_{3}+s_{2}-s_{3}$ | $0.060_{0.03}^{0.09}$ | 17\% | $0.06{ }^{0.04}$ | 60\% | $0.060_{0.03}^{0.09}$ |
| 9 |  | $g_{1}+g_{3}-2 g_{4}+s_{2}-s_{3}$ | $0.08_{0.06}^{0.09}$ | 5\% | $0.08_{0.07}^{0.09}$ | 55\% | $0.05_{0.04}^{0.06}$ |
| 10 | $\star$ | $3 g_{3}-3 g_{4}-s_{3}+s_{4}$ | $0.11_{0.01}^{0.34}$ | 9\% | $0.14_{0.06}^{0.24}$ | 40\% | $0.05{ }_{0.01}^{0.14}$ |
| 11 |  | $g_{2}-g_{3}-s_{1}+2 s_{2}-s_{3}$ | $0.06{ }_{0.04}^{0.07}$ | 5\% | $0.06{ }_{0.05}^{0.07}$ | 50\% | $0.04{ }_{0.03}^{0.05}$ |
| 12 |  | $g_{1}-2 g_{3}+g_{4}+s_{2}-s_{4}$ | $0.066_{0.03}^{0.12}$ | 36\% | $0.06_{0.03}^{0.12}$ | 51\% | $0.04{ }^{0.02}$ |
| 13 |  | $2 g_{1}-g_{3}-g_{5}+s_{2}-s_{4}$ | $0.05_{0.04}^{0.06}$ | 5\% | $0.05_{0.04}^{0.06}$ | 52\% | $0.04{ }_{0.03}^{0.04}$ |



## ICARUS 88, 266-291 (1990)

The Chaotic Motion of the Solar System: A Numerical Estimate of the Size of the Chaotic Zones

J. LASKAR

## Measure of the chaotic diffusion in all directions in a <br> 32 dimensional <br> phase space

(Laskar, 1990)

## Non-quasiperiodic Motion

(Poincaré, Smale,...)

chaotic: non-quasiperiodic
Frequency MAP

## Frequency Maps

$$
H(I, \theta)=H_{0}(I)+\varepsilon H_{1}(I, \theta) \quad(I, \theta) \in \mathbb{B}^{n} \times \mathbb{T}^{n}
$$

$\varepsilon=0$

$$
\begin{array}{lc}
\dot{I}_{j}=0, & \dot{\theta}_{j}=\frac{\partial H_{0}(I)}{\partial I_{j}}=\nu_{j}(I) ; \\
I_{j}=I_{j 0} & \theta_{j}(t)=\theta_{j 0}+\nu_{j} t
\end{array}
$$

nondegenerate

$$
\left|\frac{\partial \nu(I)}{\partial I}\right|=\left|\frac{\partial^{2} H_{0}(I)}{\partial I^{2}}\right| \neq 0
$$

frequency map

$$
\begin{aligned}
F: \mathbb{B}^{n} & \longrightarrow \mathbb{R}^{n} \\
(I) & \longrightarrow(\nu)
\end{aligned}
$$

diffeomorphism on $F\left(\mathbb{B}^{n}\right)=\Omega$

## Frequency MAP

KAM theorem (Kolmogorov 1954, Arnold 1963, Moser 1963)

$$
\exists \Omega_{\varepsilon} \subset \Omega \quad \text { Cantor set } \quad|<k, \nu>|>\frac{\kappa_{\varepsilon}}{|k|^{p}}
$$

$(\nu) \in \Omega_{\varepsilon} \Rightarrow$ quasiperiodic solution

Pöschel (1982), there exists a diffeomorphism

$$
\begin{aligned}
\Psi: \mathbb{T}^{n} \times \Omega & \longrightarrow \mathbb{T}^{n} \times \mathbb{B}^{n} \\
(\varphi, \nu) & \longrightarrow(\theta, I)
\end{aligned}
$$

analytical $/ \varphi$ and $C^{\infty} / \nu$ on $\mathbb{T}^{n} \times \Omega_{\varepsilon}$ transforms the equations into

$$
\dot{\nu}_{j}=0, \quad \dot{\varphi}_{j}=\nu_{j} ;
$$

thus, if $(\nu) \in \Omega_{\varepsilon}$,

$$
z_{j}=I_{j} e^{i \theta_{j}},
$$

$$
z_{j}(t)=z_{j 0} e^{i \nu_{j} t}+\sum_{m} a_{m}(\nu) e^{i<m, \nu>t}
$$

## Frequency MAP

## Numerical Frequency Map

$\theta=\theta_{0}$


$$
\begin{aligned}
F_{\theta_{0}}^{T}: & \mathbb{R}^{n} \\
& \longrightarrow \mathbb{R}^{n} \\
& (I)
\end{aligned}>\left(\nu_{1}, \nu_{2}, \ldots, \nu_{n}\right)
$$

(Laskar+, 1992, 1993)

## Frequency MAP

$$
\theta=\theta_{0}
$$



$$
\begin{aligned}
F_{\theta_{0}}^{T}: \mathbb{R} \times \mathbb{R}^{n} & \longrightarrow \mathbb{R}^{n} \\
(\tau, I) & \longrightarrow(\nu)
\end{aligned}
$$

(Laskar+, 1992, 1993)

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Thus : on the set of KAM tori,

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$F_{\theta_{0}}^{T}$ is constant $/ \tau$

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\end{align*}
$$

Thus : on the set of KAM tori,
$F_{\theta_{0}}^{T}$ is constant $/ \tau$
$F_{\theta_{0}}^{T}$ is a smooth diffeomorphism



# PHYSICAL REVIEW <br> LETTERS 

17 MAY 1993
Typical ALS Sector

## Global Dynamics and Long-Time Stability in Hamiltonian Systems via Numerical Frequency Analysis

H. S. Dumas

Department of Mathematical Sciences, University of Cincinnati, Cincinnati, Ohio 45221-0025

## J. Laskar

Astronomie et Systèmes Dynamiques, Bureau des Longitudes, 77 av. Denfert-Rochereau, 75014 Paris, France
(Received 18 November 1992)


FIG. 1. Image in the frequency plane ("tune space") ( $f_{1}, f_{2}$ ) of the square sector $0 \leq I_{1}, I_{2} \leq 10^{-5}$, obtained using frequency analysis over 4052 turns.


FIG. 2. Transport rates in the frequency plane. The speed and location of local transport phenomena are visualized by plotting $f_{2}+0.00005 \log (r)$ against $f_{1}$ for the same data as in Fig. 1, where $r$ is the estimated local rate of transport (for $r$ above a given threshold).

## PHYSICAL REVIEW LETTERS

# Global Dynamics and Long-Time Stability in Hamiltonian Systems via Numerical Frequency Analysis 

H. S. Dumas

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## Example : Superbend Upgrade



## In three of the twelve ALS Sectors

- Replaced the central combined function dipole with a Superbend and two quadrupoles



## ALS : Ideal Lattice versus Calibrated Model



## Experimental Frequency Map at ALS

## Experiment

Numerical model




FIG. 4. Experimental frequency map for a previous setting of the ALS.

## CERN COURIER Jan/Feb 2001

# Mapping chaos in particle revolutions 

## Over the past decade the technique of frequency map analysis, developed to study astronomical systems, has shown its value in an increasing number of areas, including the analysis of particle orbits in accelerators.

At first glance, any close association between the planets of the solar system - huge masses of rock, liquid and gas gently guided by gravity through the vast emptiness of space - and the mad traffic of tightly bunched particles in a circular accelerator, crushed together by fierce radiofrequency and magnetic fields, could hardly seem less likely.

Nonetheless, the dynamics of planets moving through our solar system and particles moving in accelerators do share many similar features. Both demand an analysis of the evolution of a dynamic system over a very long time up to 1 billion revolutions for both the


Part of the ALS storage ring at the Lawrence Berkeley National Laboratory. Frequency mapping was first applied to measured rather than simulated electron trajectories at the ALS.
trajectories in a storage ring, at the Advanced Light Source (ALS) at the Lawrence Berkeley National Laboratory. The aim was to reveal the dynamics of an actual particle beam.

## Chaotic motion

The story of frequency map analysis began in 1989 when Jacques Laskar (Bureau des Longitudes, Paris) demonstrated that the motion of the solar system is chaotic (Laskar 1989). He showed that the separation between two orbits with similar initial conditions will diverge exponentially over time (e.g. the distance between the orbits will increase

## Project Director Dave Robin Announces ALS-U Project Beamlines

JANUARY 29, 2019

Over the past year, a process involving ALS and ALS-U staff, the ALS user community, and external advisory committees has been ongoing to select the insertion-device beamlines that will be built and upgraded within the scope of the ALS-U Project. These beamlines will join existing ALS beamlines to form the full complement of capabilities that will be available at the upgraded ALS in several years. I am delighted to inform you that the selection process is now complete and to announce the result.


## ALS-U will reach soft x-ray diffraction limit up to 1.5 keV



C. Steier /ALS


December 2021
Decision to launch a Technical Design Report

## Laurent S. Nadolski <br> Senior Accelerator Physicist Accelerators Coordinator Synchrotron SOLEIL, France

- Ph.D., (2001).
- "Application of Frequency Map Analysis to the Study of Beam Dynamics" (J. Laskar’s supervision).
- Former President of Accelerator Sect. of the French Physical Society
- SOLEIL II: nonlinear beam dynamics, robustness studies, storage ring coordination (collimation, radiation safety, machine protection interlock).


## Present SOLEIL II Timeline

Mid 2030
Expert
users

2035
Fully upgrade compatible beamlines

| 2016-2018 |
| :--- |
| Preparatory Phase |

ouncil authorization to produce the Conceptual Design Report (CDR)


Mid 2028 (18 mo
Dark-period

Expected Funding of the SOLEIL Upgrade


## SOLEIL II: a 4th Generation Synchrotron Light Source for the

 Science of Tomorrow
## An electron beam 40 times smaller and circular

## Photon beams at least 100 times brighter and more coherent in the X-ray range.

Obtaining a very compact layout:
Lifting of technological barriers, with the miniaturization of the vacuum chambers where the electrons circulate and of the magnets that guide them.

Storage ring: 354 m Booster: 157 m 29 beamlines optical luten


## Yannis Papaphilippou

PhD (1997) Application of the Frequency Map Analysis Method in Galactic Dynamics


Galactic model

PhD (1997) Application of the Frequency Map Analysis Method in Galactic Dynamics


Galactic model


The LHC $\underset{\substack{\text { Y. Papaphilifpou, } \\ \text { i999 }}}{ }$

## ABP@CERN organisation



## Goal of High-Luminosity LHC

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Prepare machine for operation beyond 2025 and up to 2035

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CERN

## Beam-Beam interaction

| Variable | Symbol | Value |
| :--- | :---: | :---: |
| Beam energy | $E$ | 7 TeV |
| Particle species | $\ldots$ | protons |
| Full crossing angle | $\theta_{c}$ | $300 \mu \mathrm{rad}$ |
| rms beam divergence | $\sigma_{x}^{\prime}$ | $31.7 \mu \mathrm{rad}$ |
| rms beam size | $\sigma_{x}$ | $15.9 \mu \mathrm{~m}$ |
| Normalized transv. |  |  |
| $\quad$ rms emittance | $\gamma \varepsilon$ | $3.75 \mu \mathrm{~m}$ |
| IP beta function | $\beta^{*}$ | 0.5 m |
| Bunch charge | $N_{b}$ | $\left(1 \times 10^{11}-2 \times 10^{12}\right)$ |
| Betatron tune | $Q_{0}$ | 0.31 |

- Long range beam-beam interaction represented by a 4D kick-map

$$
\begin{aligned}
& \Delta x=-n_{p a r} \frac{2 r_{p} N_{b}}{\gamma}[ \frac{x^{\prime}+\theta_{c}}{\theta_{t}^{2}}\left(1-e^{-\frac{\theta_{t}^{2}}{2 \theta_{x, y}^{2}}}\right) \\
&\left.-\frac{1}{\theta_{c}}\left(1-e^{-\frac{\theta_{c}^{2}}{2 \theta_{x, y}^{2}}}\right)\right] \\
& \Delta y=-n_{p a r} \frac{2 r_{p} N_{b}}{\gamma} \frac{y^{\prime}}{\theta_{t}^{2}}\left(1-e^{-\frac{\theta_{t}^{2}}{2 \theta_{x, y}^{2}}}\right)
\end{aligned}
$$

with

$$
\theta_{t} \equiv\left(\left(x^{\prime}+\theta_{c}\right)^{2}+y^{\prime 2}\right)^{1 / 2}
$$

## Wire compensation

- Current baring wire can improve DA by 1-2 $\boldsymbol{\sigma}$

Without correction


With correction


Tests in the LHC during 2017-2018

## Reduced crossing angle of $450 \mu \mathrm{rad} @ 15 \mathrm{~cm}$

S. Fartoukh, al., PRSTAB, 2015
$\rightarrow$ Nominal bunches with wire correction
$\rightarrow$ Nominal bunches without wire correction


## LHC: Power supply ripples (5D)

## $\square$ Scan of different ripple frequencies (50-900 Hz)


(CERN)
Y. Papaphilippou
S. Kostoglou, et al., PRAB 2021

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S. Kostoglou, et al., PRAB 2021

## Incoherent e-cloud effects at (HL-)LHC

- Analysis of experimental data from the LHC shows a slow beam degradation due to e-cloud both at injection and in collisions
- A development effort was launched to acquire the ability of simulating the effect of e-cloud forces within a symplectic tracking code over the required long timescales (10M turns). The development included:
- Theoretical framework
- Tricubic interpolator in sixtracklib tracking code to apply forces from a recorded pinch in a symplectic way
- Software infrastructure to simulate and condition the electron pinches and setup the simulation from the MAD-X description of the machine
- Presently capable of simulating $\mathbf{1 0} \mathbf{M}$ turns (15 minutes of beam time) by exploiting the computational power of GPUs

K. Paraschou, et al. 2023

Y. Papaphilippou

When you tackle difficult problems,
you need to invent new methods that may be of general use

## Some references

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