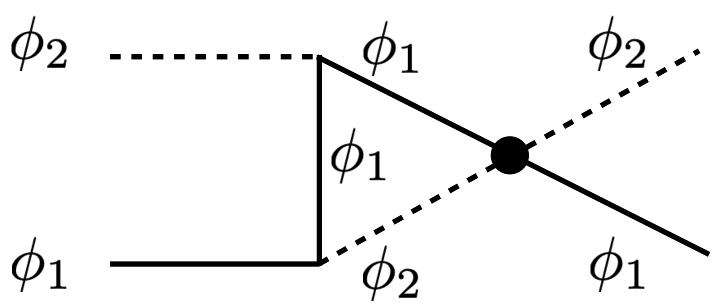
Self-resonant Dark Matter



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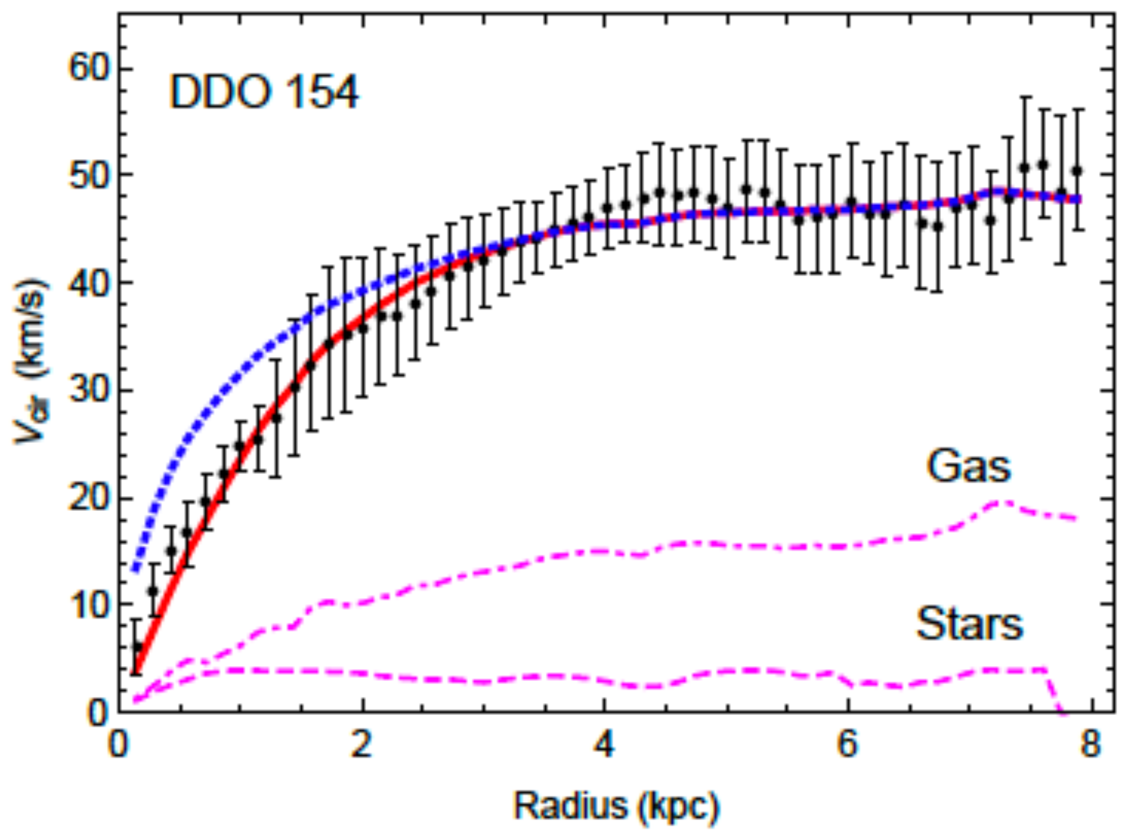
References: Seong-Sik Kim & Bin Zhu, JHEP 10 (2021) 239, JHEP 05 (2022) 148, Work in progress.

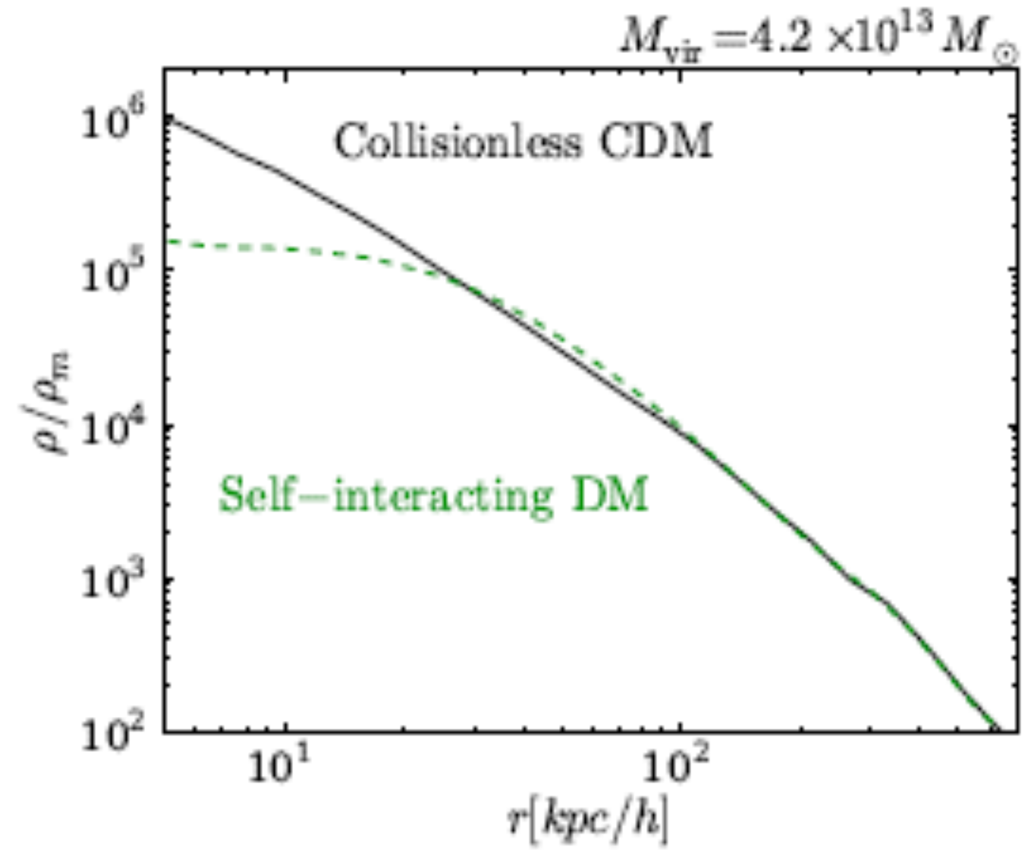
19th Recontres du Vietnam, Theory Meeting Experiment
ICISE centre, Quy Nhon, Vietnam, Jan 7, 2023

Core-Cusp problem



Galaxy rotation curves at <1kpc

[Spergel, Steinhardt, 2000; Tulin, Yu, 2017]



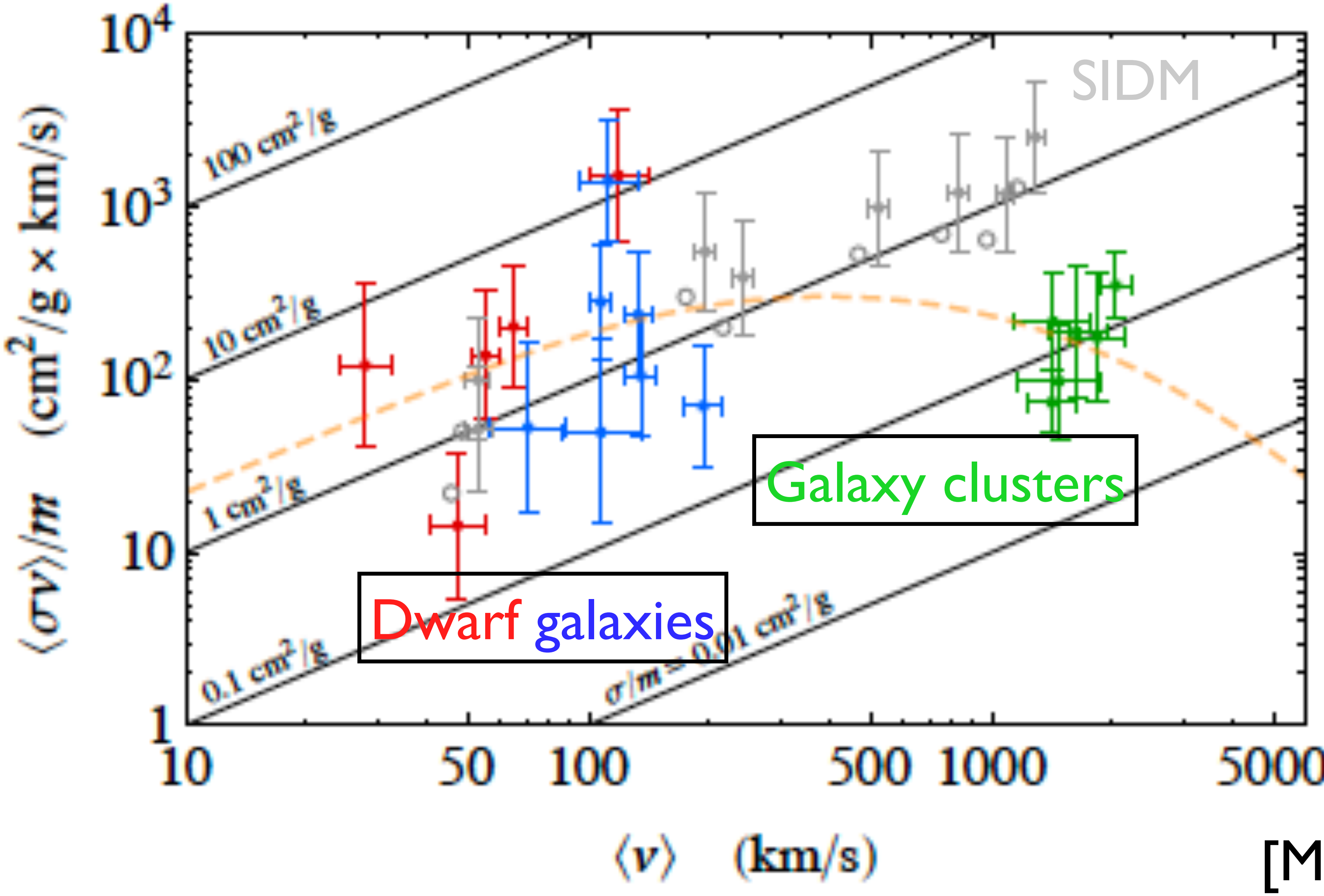
[Weinberg et al, 2013]

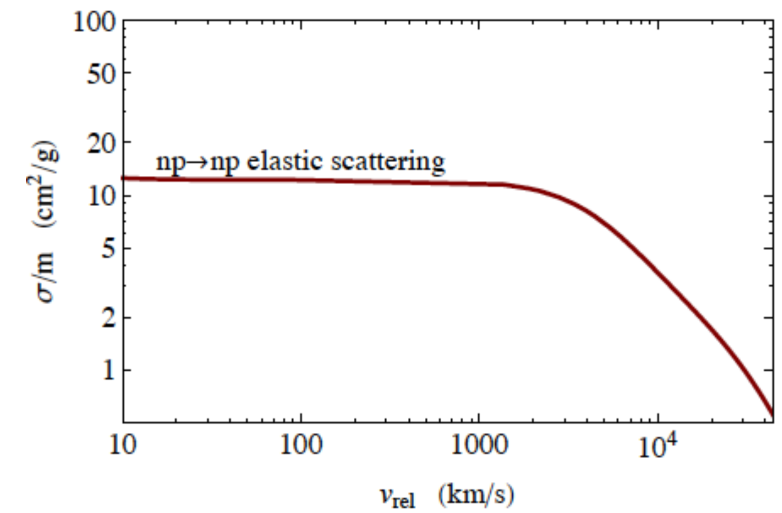
CDM N-body simulation ⟶ $v_{\rm cir} \sim \sqrt{r}, \quad \rho_{\rm dm} \sim r^{-1}$ "Cuspy"

Observed circular velocity ⟶ $v_{\rm cir} \sim r, \quad \rho_{\rm dm} \sim r^{0}$ "Cored"

⇒ Self-interaction of dark matter solves small-scale problems! $\frac{\sigma_{\rm self}}{m_{\rm DM}} \sim 0.1 - 10 {\rm cm}^2/{\rm g}$

Velocity-dependent SIDM





$\langle \sigma v \rangle$ =constant

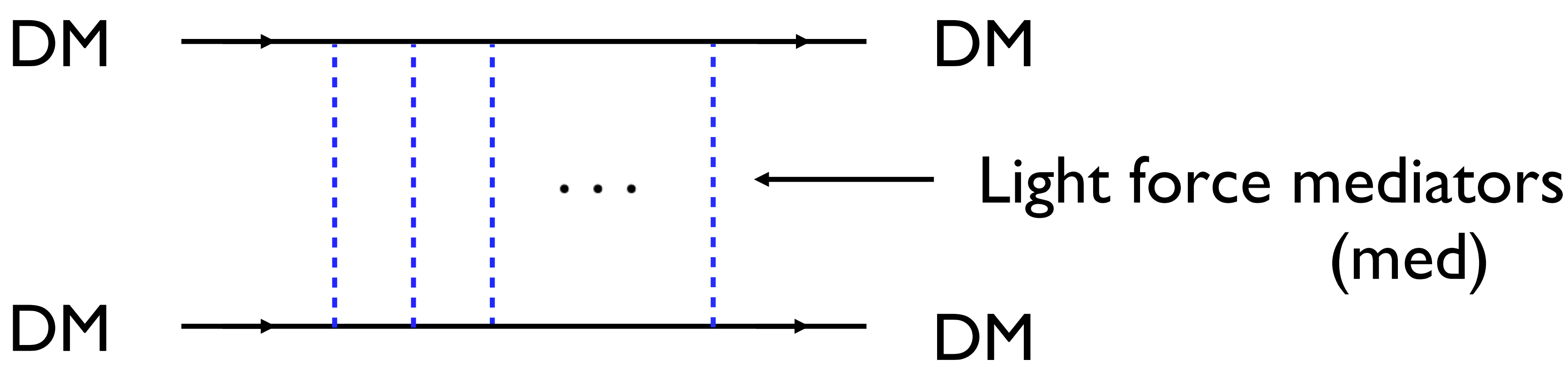
$$\Rightarrow \sigma \propto \frac{1}{\langle v \rangle}$$

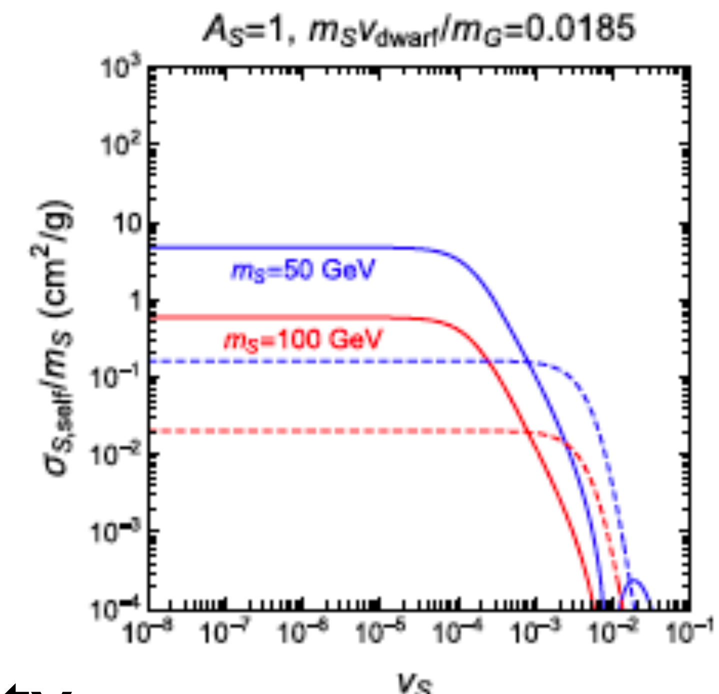
[M. Kaplinghat et al, 2015]

- Bullet cluster bound on DM self-scattering: $\sigma/m \lesssim 0.7\,\mathrm{cm}^2/\mathrm{g}$
- Velocity-dependent self-interaction for galaxy clusters.

T-channel forces in challenge

Light mediators for dark matter self-scattering

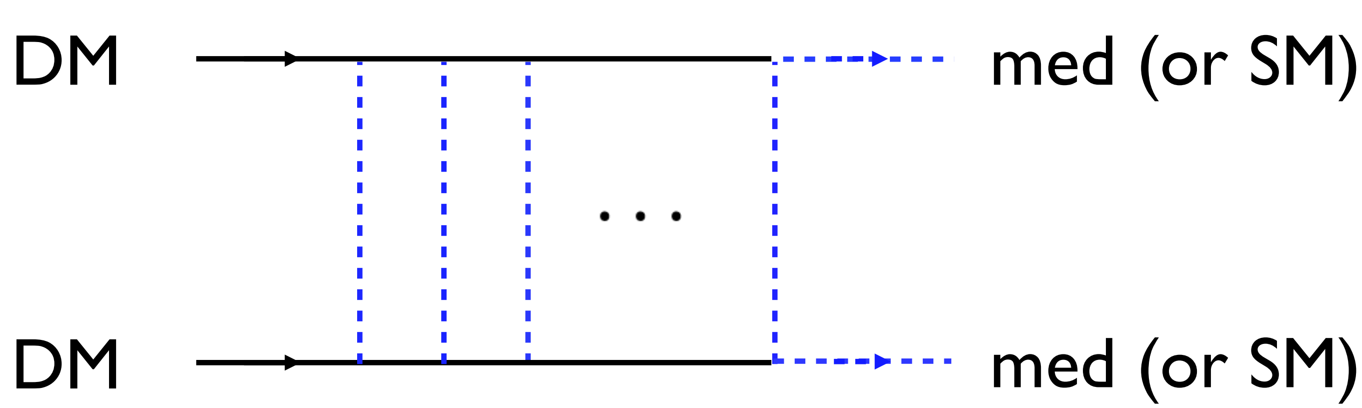




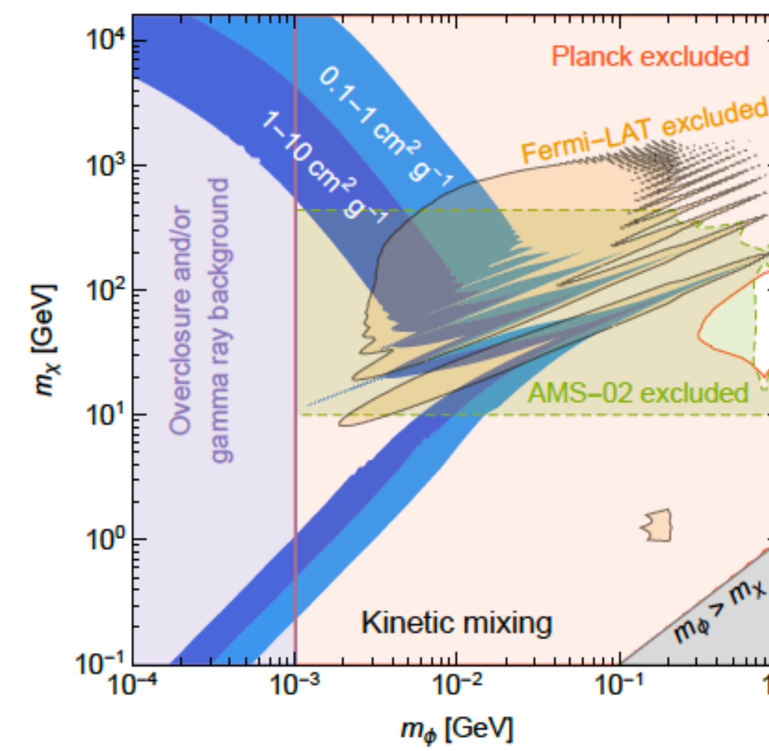
[Kang, HML, 2020]

Self-interaction enhanced at small velocity.

Sommerfeld effect enhances dark matter annihilations at small velocity.



Annihilation product almost ruled out by CMB and cosmic rays.



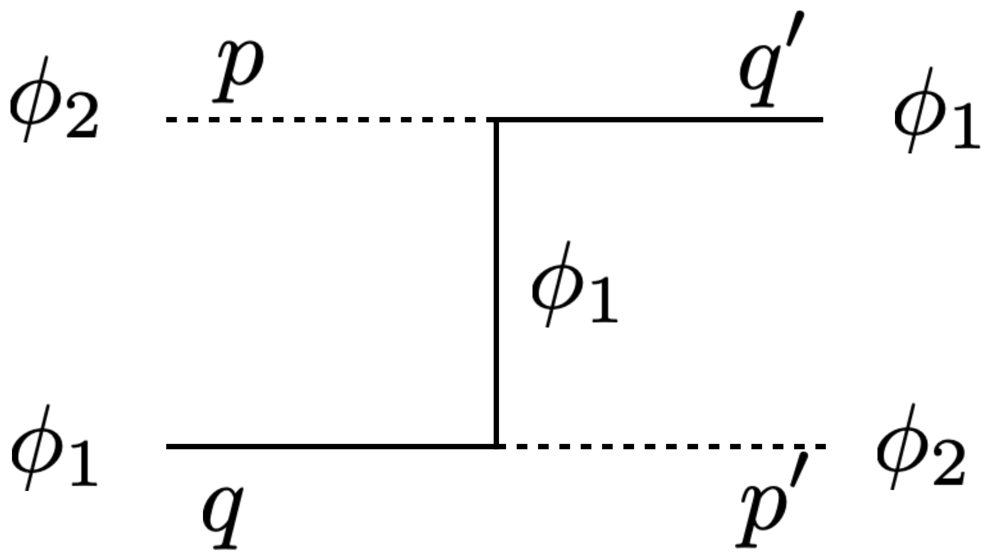
[Bringmann et al, 2016]

u-channel resonance

- Elastic 2→2 co-scattering with unequal masses

Self-coupling between dark matter: $\mathcal{L}_{\rm int} = -2g\, m_1 \phi_2 |\phi_1|^2$

[S. Kim, HML, B. Zhu, 2021, 2022]

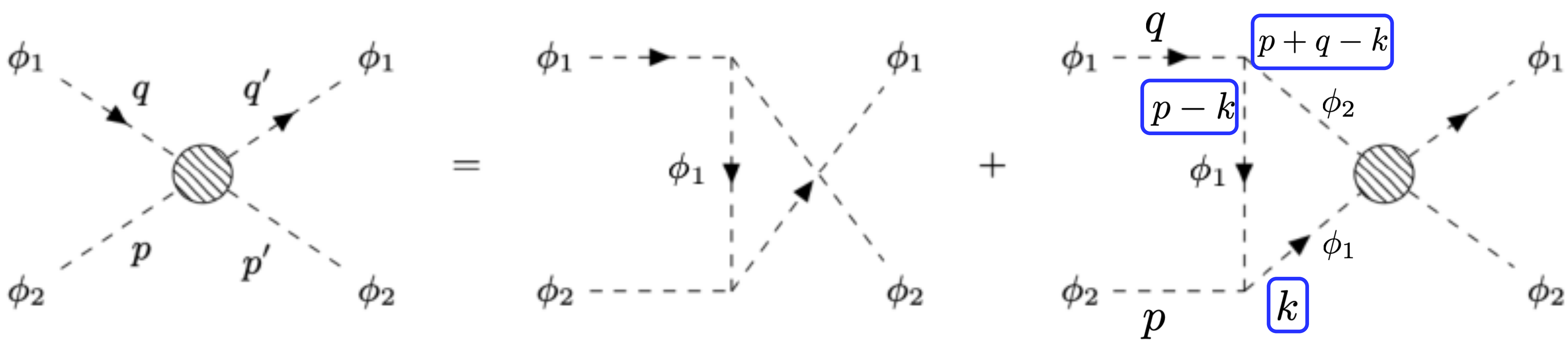


$$\tilde{\Gamma}_u(p,q;p',q') = \frac{4g^2 m_1^2}{|\vec{p}-\vec{q}'|^2 + m_1^2 - \omega^2}$$

$$\omega = p_0 - q_0' \approx m_2 - m_1 \neq 0$$

$m_2 = 2m_1$ "Effectively massless"

Resummation of u-channel ladder diagrams is needed.



$$i\Gamma(p,q;p',q') = i\tilde{\Gamma}(p,q;p',q') - \int \frac{d^4k}{(2\pi)^4}\tilde{\Gamma}(p,q;p+q-k,k)G_1(k)G_2(p+q-k)\Gamma(p+q-k,k;p',q')$$

Bethe-Salpeter equation

Bethe-Salpeter wave function for unequal masses:

$$\boxed{\chi(p,q;p',q') \equiv G_2(p)G_1(q)\Gamma(p,q;p',q') \equiv \chi(p,q)}$$

$$P = \frac{1}{2}(p+q), \quad Q = \mu\Big(\frac{p}{m_2} - \frac{q}{m_1}\Big): \ \chi(p,q) = \widetilde{\chi}(P,Q)$$

$$\widetilde{\psi}_{BS}(\vec{Q}) = \int \frac{dQ_0}{2\pi}\widetilde{\chi}(P,Q) \ ; \quad U \equiv \frac{4g^2m_1^2}{\Big(\sqrt{\frac{m_1}{m_2}}\vec{Q} + \sqrt{\frac{m_2}{m_1}}\vec{k}'\Big)^2 + m_2(2m_1 - m_2)}$$

$$\longrightarrow \quad \left(\frac{\vec{Q}^2}{2\mu} - E\right)\widetilde{\psi}_{\rm BS}(\vec{Q}) = \frac{1}{4m_1m_2}\int\frac{d^3k'}{(2\pi)^3}\,U\left(\left|\sqrt{\frac{m_1}{m_2}}\vec{Q} + \sqrt{\frac{m_2}{m_1}}\vec{k}'\right|\right)\widetilde{\psi}_{\rm BS}(\vec{k}')$$

Schroedinger-like equation:

$$\boxed{\left(-\frac{1}{2\mu}\nabla^2 - E\right)\psi_{BS}(\vec{x}) \ = \ -V(\vec{x})\psi_{BS}\Big(-\frac{m_2}{m_1}\vec{x}\Big)}$$

[S. Kim, HML, B. Zhu, 2021, 2022]

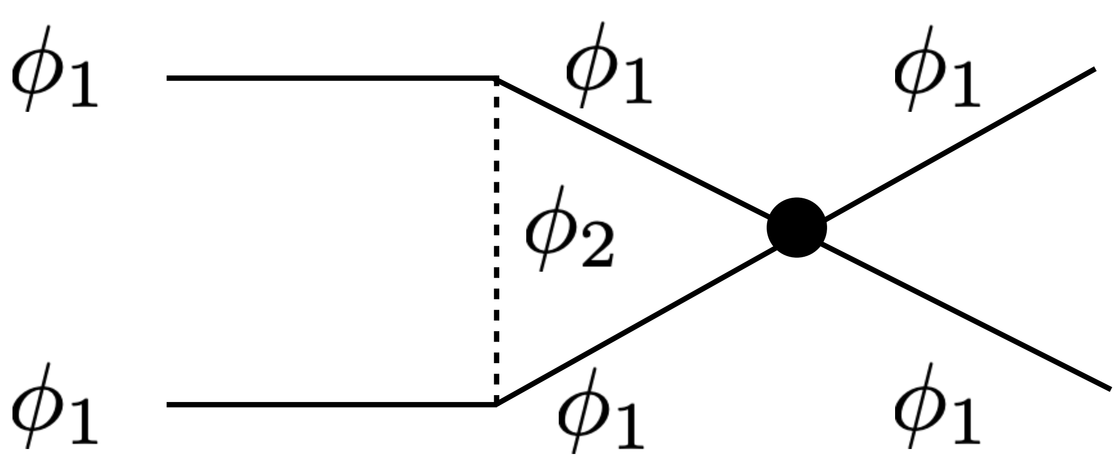
$$\begin{cases} V(\vec{x}) \ = -\dfrac{\alpha}{r}e^{-Mr} \\ \alpha \equiv \frac{g^2}{4\pi}, \quad M \equiv m_2\sqrt{2 - \dfrac{m_2}{m_1}} \end{cases} \quad \Longrightarrow \quad m_2 \lesssim 2m_1 : \quad M \ll m_2$$

Effective light mediator

t-channel vs u-channel

t-channel:

$$\tilde{\Gamma}_t = \frac{4g^2 m_1^2}{|\vec{p} - \vec{p'}|^2 + m_2^2}$$



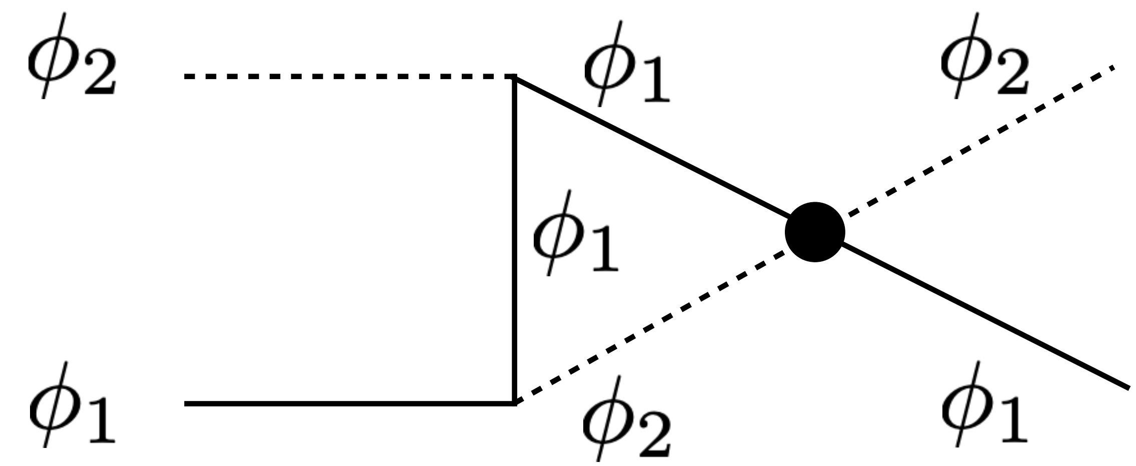
$$\chi(1,1') \sim \tilde{\Gamma}_t \chi(1,1')$$

$$\Big(\frac{\nabla^2}{2\mu} + E\Big)\psi(\vec{x}) = V(\vec{x})\psi(\vec{x})$$

No DM exchange in loops =>
No flip for BS wave-function

u-channel:

$$\tilde{\Gamma}_u = \frac{4g^2 m_1^2}{\left|\sqrt{\frac{m_1}{m_2}}\vec{p} - \sqrt{\frac{m_2}{m_1}}\vec{q'}\right|^2 + m_2(2m_1 - m_2)}$$



$$\chi(1,2) \sim \tilde{\Gamma}_u \chi(2,1)$$

$$\Big(\frac{\nabla^2}{2\mu} + E\Big)\psi(\vec{x}) = V(\vec{x})\psi\Big(-\frac{m_2}{m_1}\vec{x}\Big)$$

$\boxed{\text{DM exchange in loops => BS wave-function flips!}}$

Delayed interactions

BS w.f. in spherical coordinates:

$$\psi_{\rm BS}(\vec{x}) = R_l(r) Y_l^m(\theta,\phi) \longrightarrow \psi_{\rm BS}\left(-\frac{m_2}{m_1}\vec{x}\right) = \boxed{(-1)^l} R_l\left(\frac{m_2}{m_1} r\right) Y_l^m(\theta,\phi)$$

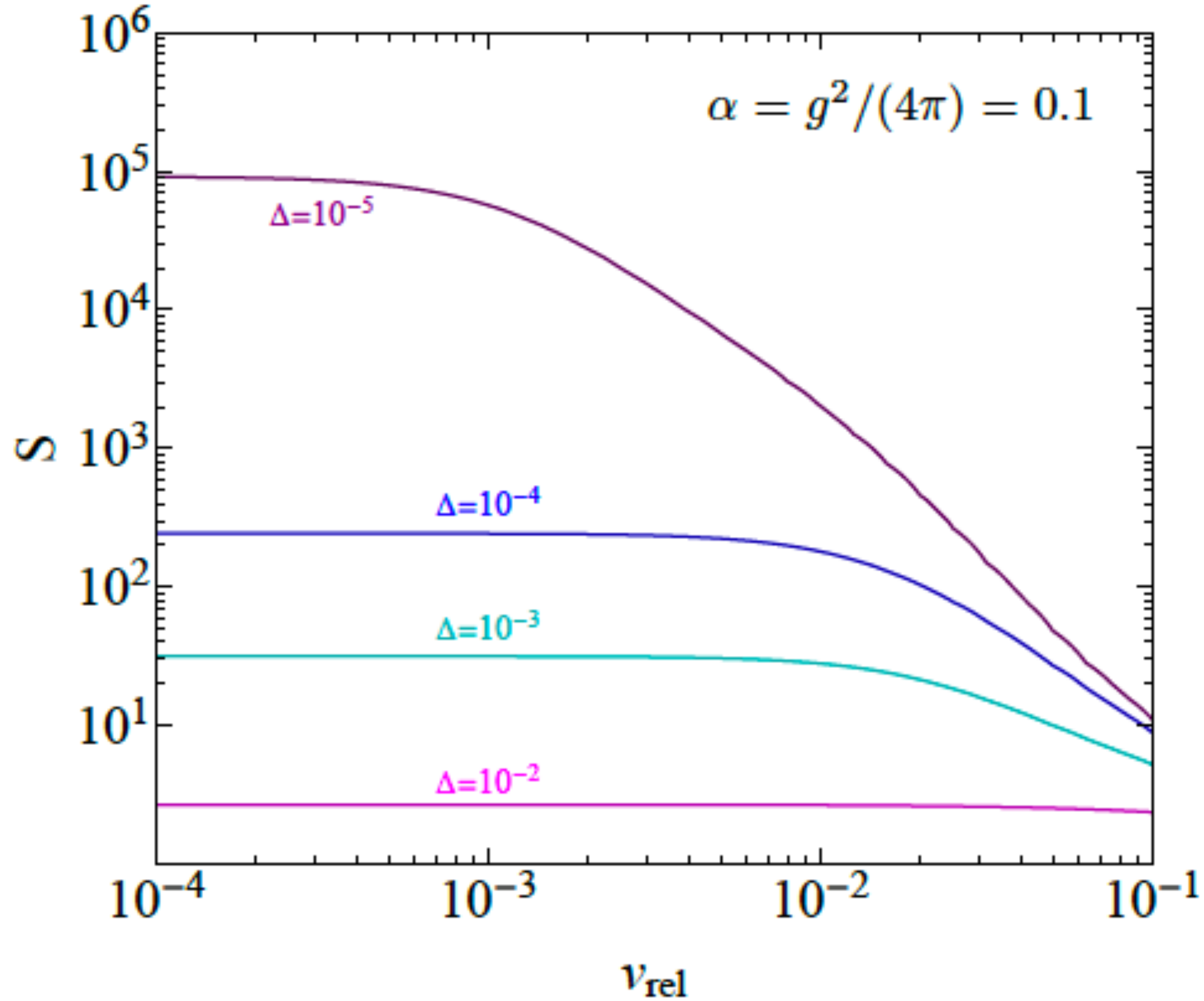
Radial equation : $R_l(x) = u_l(x)/x$, $a = \frac{2v_{\rm rel}}{\alpha}$, $b = \frac{m_2}{m_1}$ and $c = \frac{2M}{\mu\alpha}$

$$\Rightarrow \left(\frac{d^2}{dx^2} - \frac{l(l+1)}{x^2}\right) u_l(x) + \frac{4e^{-cx}}{b\,x} (-1)^l\, u_l(bx) + a^2\, u_l(x) = 0$$

- Potential flips: attractive for l=even ; repulsive for l=odd.
- Effective mediator mass: $M \equiv m_2 \sqrt{2 - \frac{m_2}{m_1}} \to 0, \quad m_2 \to 2m_1.$
- Delay differential eq:

$$x = e^{-\rho} \longrightarrow \tilde{u}_0''(\rho) + \tilde{u}_0'(\rho) + 2e^{-\rho}\,\tilde{u}_0(\underbrace{\rho - \ln 2}_{\text{delay term}}) + a^2 e^{-2\rho}\,\tilde{u}_0(\rho) = 0$$

Sommerfeld factor



Sommerfeld factor (s-wave):

$$S = \frac{|\psi_{\rm BS}(0)|^2}{|\psi_{\rm pert}(0)|^2} = A^2$$

Effective mediator mass

$$\longleftrightarrow \qquad \Delta = 1 - \frac{m_2}{2m_1} \geq 0$$

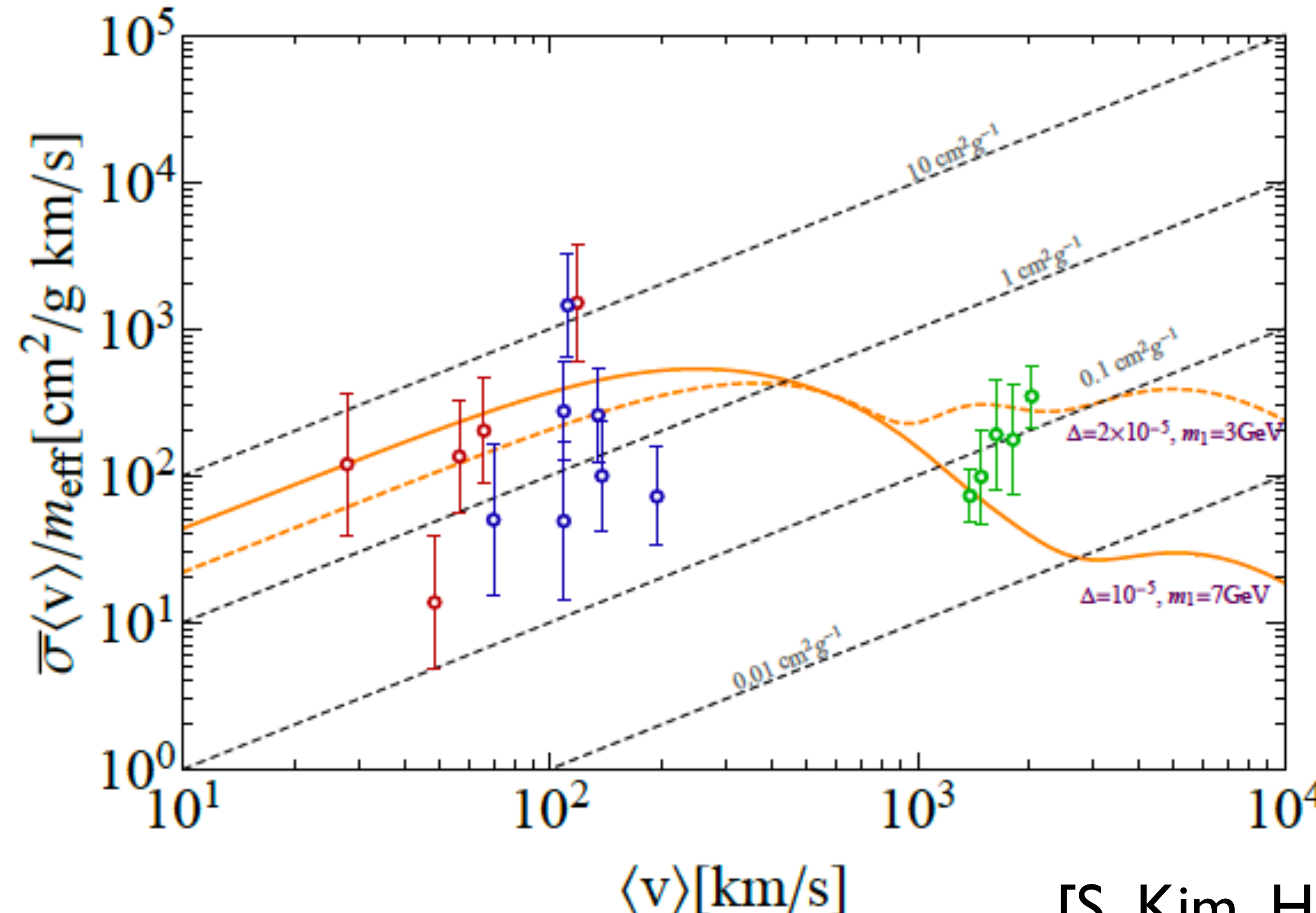
[S. Kim, HML, B. Zhu, 2021]

Boundary conditions (s-wave):

$$\tilde{u}_0(\rho) \longrightarrow \frac{1}{a}\sin(a\,e^{-\rho} + \delta_0), \qquad \rho \to -\infty, \qquad \text{“plane-wave”}$$

$$\tilde{u}_0(\rho) \longrightarrow A\,e^{-\rho}, \qquad \rho \to +\infty \qquad \text{“constant R”}$$

SIDM from co-scattering



$\alpha = g^2/(4\pi) = 0.1$

[S. Kim, HML, B. Zhu, 2021]

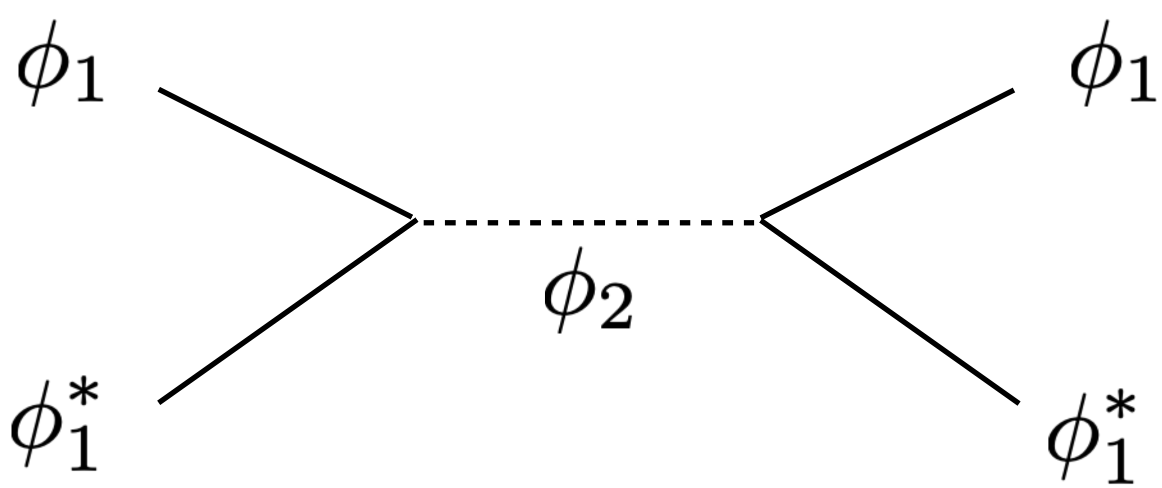
$\phi_1\phi_2 \to \phi_1\phi_2:\quad \sigma = \frac{4\pi}{k^2}\sin^2\delta_0$, total cross section

$\bar{\sigma} = \langle \sigma v_{\rm rel}^3\rangle/(24/\sqrt{\pi}v_0^3)$: energy-transfer average

Self-scattering for SRDM is velocity-dependent.

Other channels for SIDM

s-channel



$$\sigma^s = \frac{g^4 m_1^2}{4\pi(m_1^2(4+v_{\rm rel}^2) - m_2^2)^2}$$

$$\Delta = 1 - \frac{m_2}{2m_1} \geq 0$$

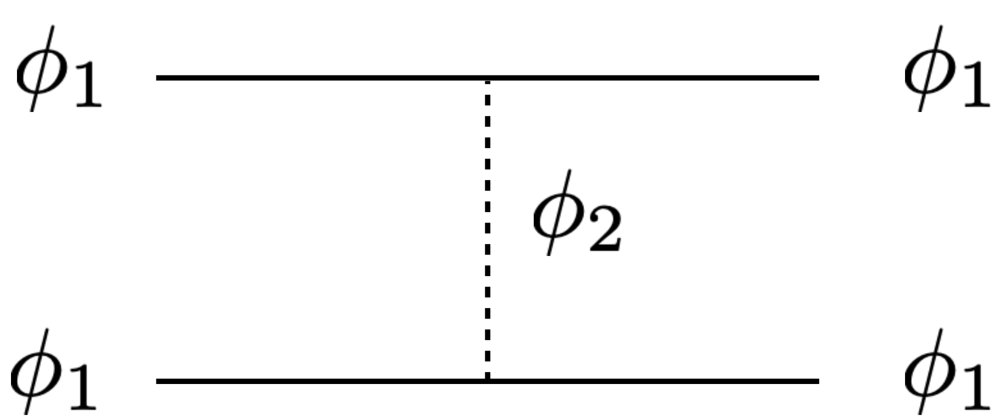
$$\longrightarrow \quad \sigma^s \simeq \frac{g^4}{4\pi m_1^2 (v_{\rm rel}^2 + 8\Delta)^2}$$

Additional enhancement for $v_{\rm rel}, \Delta \ll 1$

But, for very small Δ, cutoff to smaller value than for u-channel.

[S. Kim, HML, B. Zhu, in progress]

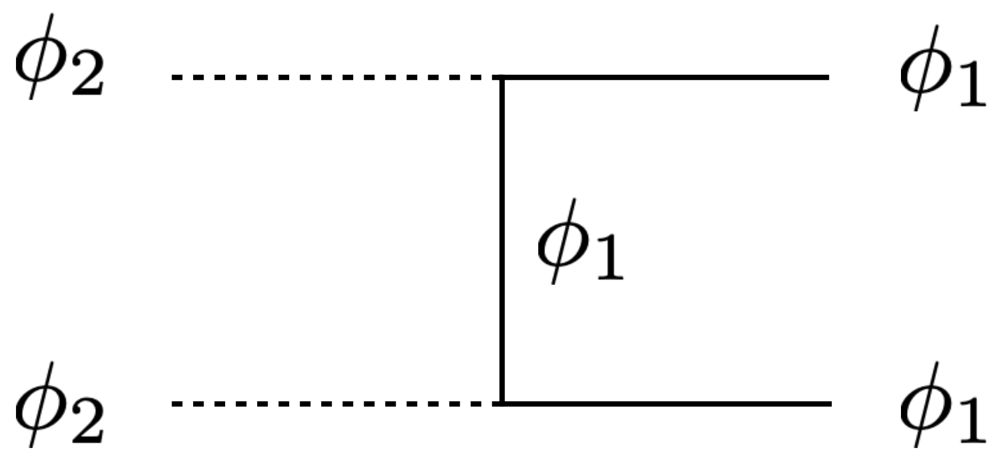
t-channel



$m_2 \approx 2m_1$

No light mediator:
no enhancement

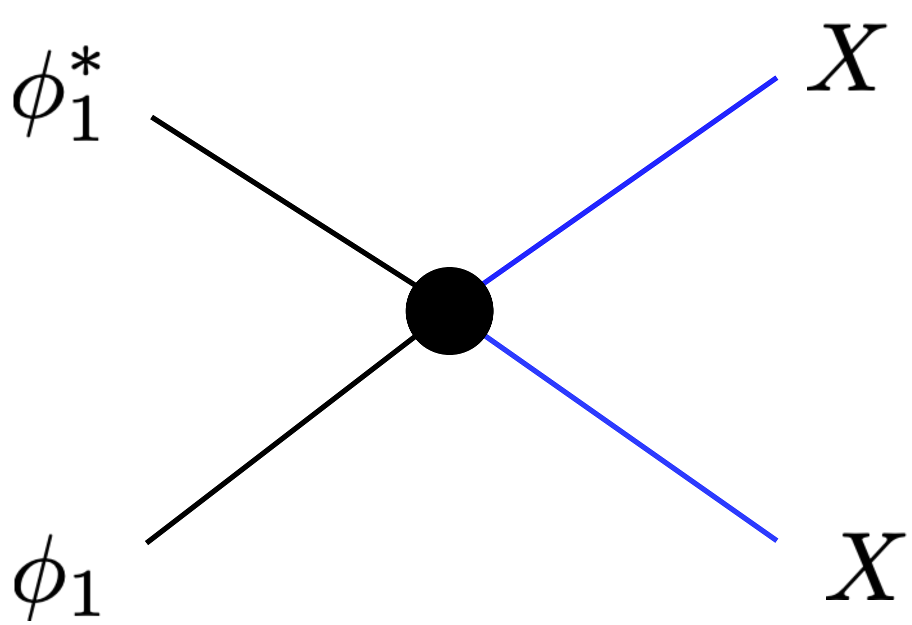
DM annihilation

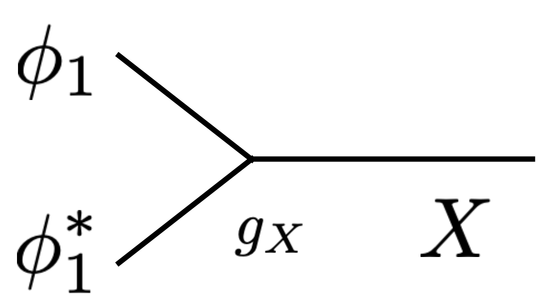


Heavier DM always annihilates into lighter DM, but no freeze-out.

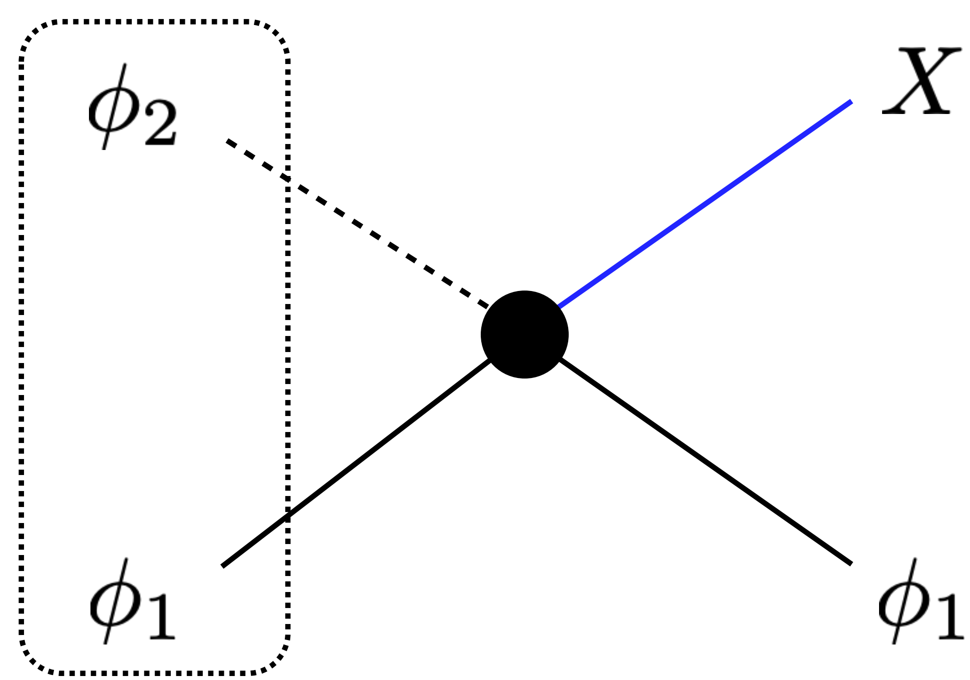
→ extra 2→2 annihilation

e.g. dark photon portal





BS w.f.

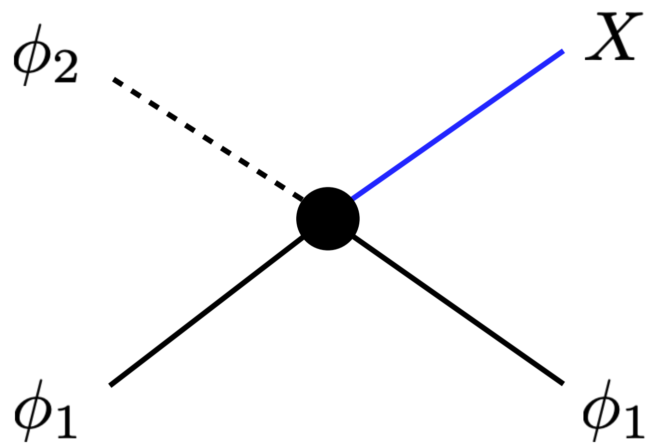


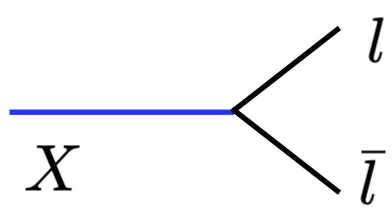
u-channel

Sommerfeld enhancement

Indirect detection

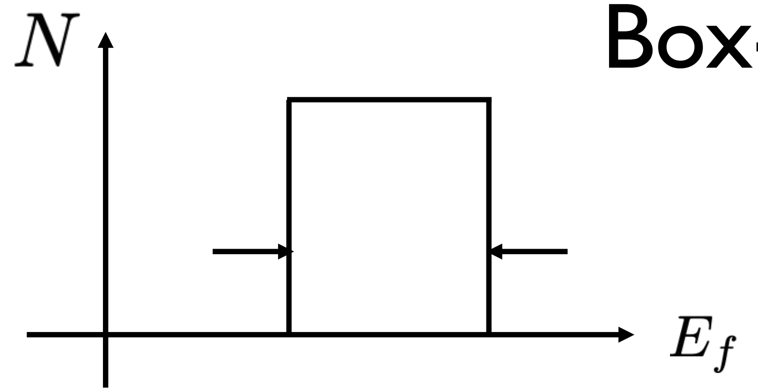
Cosmic ray (from semi-annihilation):





Leptons boosted in galactic center frame

$$E_f = \frac{1}{\gamma_X}\,\bar{E}_f\,(1 - v_X\ \cos\theta)^{-1}, \qquad \bar{E}_f = \frac{m_X}{2}$$



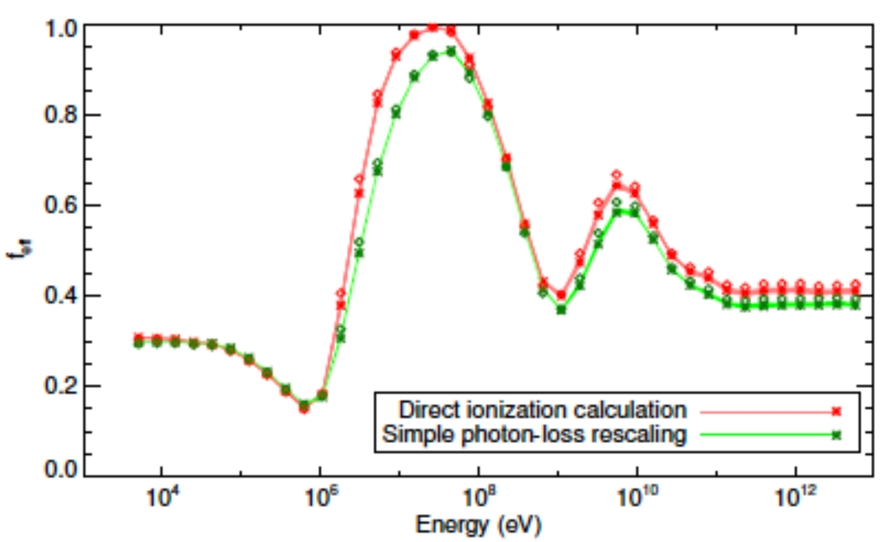
$$\Delta E_f = v_X \gamma_X\, m_X$$

Box-shaped positron energy
Fermi-LAT & AMD-02

cf. gamma-ray from
Fermi-LAT, HESS, etc.

CMB (from semi-annihilation):

[S. Kim, HML, B. Zhu, 2022]



[T. Slatyer, 2015]

Leptons injects energy to CMB photons

$$\langle\sigma v\rangle_{\phi_1\phi_2\to\phi_1 X} < 4\times 10^{-25}\,\mathrm{cm}^3/\mathrm{s}\left(\frac{f_{\rm eff}}{0.1}\right)^{-1}\cdot\frac{1}{r_1(1-r_1)}\cdot\left(\frac{m_2}{100\,\mathrm{GeV}}\right)$$

Efficient factor: $f_{\rm eff}(m_2) = \frac{\int_0^{m_2/2} dE_e\,E_e\,2 f_{\rm eff}^{e^+e^-}\,\frac{dN_e}{dE_e}}{m_2}, \qquad r_1 = \Omega_1/\Omega_{\rm DM}$

Benchmark models

CMB-consistent benchmark models [S. Kim, HML, B. Zhu, 2022]

	$m_2 \simeq 2m_1$ [GeV]	m_X [GeV]	$\alpha = \frac{g^2}{4\pi}$	$\alpha_X = \frac{g_X^2}{4\pi}$	$\langle\sigma v\rangle^0_{\phi_1\phi_2\to\phi_1 X}$ [cm^3/s]	$r_1 = \frac{\Omega_1}{\Omega_{\rm DM}}$	S_0	$\Delta = 1 - \frac{m_2}{2m_1}$	$\sigma_{\rm self}/m_{\rm eff}$ [cm^2/g]
B1	200	50	0.05	0.0045	9.9×10^{-27}	0.5	444.7	7.75×10^{-4}	0.014
B2	400	100	0.1	0.009	9.9×10^{-27}	0.5	889	10^{-4}	0.002
B3	26	5	0.00032	0.04	2.0×10^{-26}	0.005	1336	5×10^{-10}	0.003
B4	240	60	0.0032	0.03	2.9×10^{-27}	0.075	7379	$10^{-7.7}$	0.086

Consistency with CMB bound (& other indirect bounds)

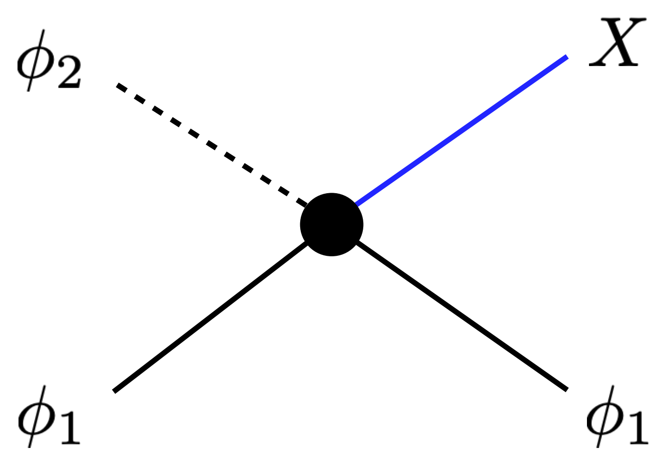
$\longrightarrow$ Either sizable mass splitting or small self-coupling.

Large u-channel resonances

$\longrightarrow$ B4 marginally solves the small-scale problems.

Direct detection

Boosted DM from semi-annihilation: [S. Kim, HML, B. Zhu, 2022]

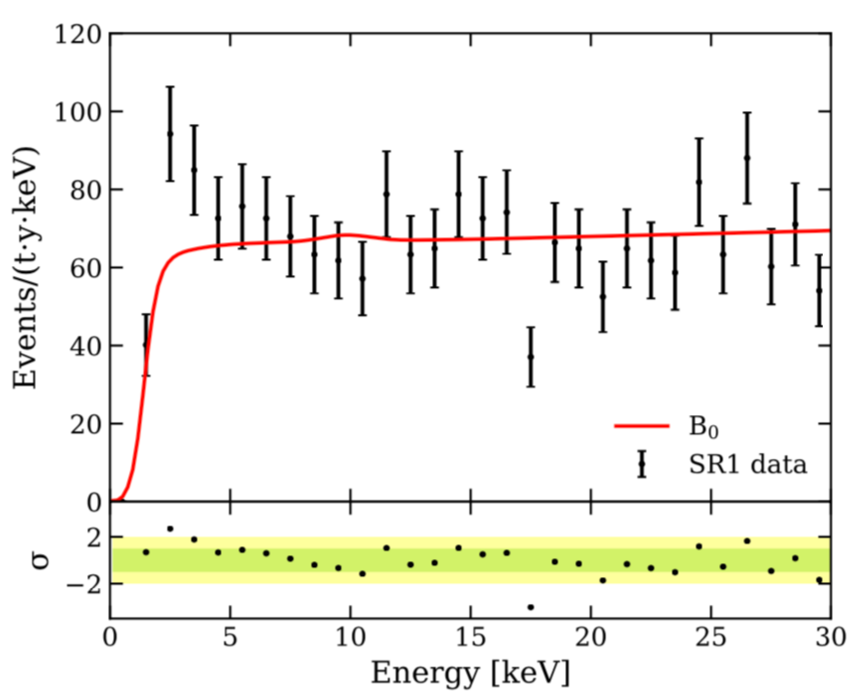


Gamma factor for boosted DM

$$\gamma_1 = (10m_1^2 - m_X^2)/(6m_1^2) \qquad m_2 \simeq 2m_1$$

DM flux from galactic center (NFW)

$$\Phi_1^{\rm G.C.} = 1.6\times 10^{-4}{\rm cm}^{-2}{\rm s}^{-1}\left(\frac{\langle\sigma v\rangle_{\phi_1\phi_2\to\phi_1 X}}{5\times 10^{-26}{\rm cm}^3/{\rm s}}\right)\left(\frac{(1\,{\rm GeV})^2}{m_1 m_2}\right) r_1(1-r_1)$$



XENON1T (old) excess

DM-electron scattering in XENON

$$\sigma_e = 10^{-33}\,{\rm cm}^2\left(\frac{10^{-1}{\rm cm}^{-2}{\rm s}^{-1}}{\Phi_1^{\rm G.C.}}\right)\left(\frac{N_{\rm sig}}{10}\right)$$

$$E_e = 2m_e v_1^2 = 3.6\,{\rm keV}, \quad m_1 = 2m_2 \sim 1\,{\rm GeV}\ {\rm and}\ r_1 \simeq \tfrac{1}{2}$$

$$\longrightarrow \quad m_X = 1.99729 m_1$$

XENON sensitive to u-channel resonance

EFT for SRDM

Candidates for co-scattering dark matter:

N

dark matter	scalar	pseudo-scalar	fermion	vector	axial-vector
scalar(ϕ)	$+4g^2m_\phi^2$	$+4g^2m_\phi^2$	$+2y_\chi^2 m_\chi(2m_\chi - m_\phi)$	NA	NA
pseudo-scalar(a)	–	–	$-2\lambda_\chi^2 m_\chi m_a$	NA	NA
fermion(χ)	–	–	NA	$-2g_{Z'}^2 m_\chi m_{Z'}$	$+2g_{A'}^2 m_\chi(2m_\chi - m_{A'})$
vector(Z')	NA	NA	–	$-6g_X^2 m_X(2m_X - m_{X_3})$	NA
axial-vector(A')	NA	NA	–	–	NA

$$\tilde{\Gamma}_u(p,q;p',q') = \frac{N}{\left(\sqrt{\frac{m_1}{m_2}}\vec{p} - \sqrt{\frac{m_2}{m_1}}\vec{q}'\right)^2 + m_2(2m_1 - m_2)}$$

[S. Kim, HML, B. Zhu, 2022]

Scalar & (pseudo)scalar DM: s-wave

fermion & pseudoscalar(or vector) DM: p-wave

Three-component scalar DM: s-wave $\mathcal{L}_{\rm int} = -2g\, m_1\phi_1\phi_2\phi_3^* + \text{h.c.}$

⟶ Effective mediator mass: $M = \sqrt{\frac{m_2}{m_1}}\sqrt{m_3^2 - (m_1 - m_2)^2}$

SRDM from extra dimension

- Self-interacting scalar field in 5D on S^1/Z_2 with radius R.

$$\mathcal{L}_{5D} = \sqrt{-g_5}\left(\frac{1}{2}(\partial_M\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{3}g\phi^3 - \frac{1}{4}\lambda\phi^4\right)$$

Kaluza-Klein expansion: $\phi(x,y) = \frac{1}{\sqrt{2\pi R}}\phi_0(x) + \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty}\cos\left(\frac{ny}{R}\right)\phi_n(x)$

KK masses: $m_n^2 = m_\phi^2 + \frac{n^2}{R^2}, \quad n = 0, 1, 2, \cdots, \quad m_0 = m_\phi,$

"u-channel resonance condition" $m_2 - 2m_1 \equiv -2m_1\delta \simeq -\frac{3}{4}m_0^2 R < 0$

Self-interactions for KK modes in 4D EFT:

[S. Kim, HML, B. Zhu, in progress]

$$\mathcal{L}_{\rm eff} = -g_{\rm eff}\, m_1\left(\frac{1}{6}\phi_0^3 + \phi_0\phi_1^2 + \phi_0\phi_2^2 + \phi_2\phi_1^2 + \cdots\right), \qquad g_{\rm eff} \equiv \frac{2g}{(2\pi R)^{3/2}}$$

<u>zero mode</u> ⟶ t-channel resonance, dark radiation

<u>1st, 2nd KK modes</u> ⟶ "Self-resonant dark matter"

Conclusions

- Generalized Sommerfeld effects on two-component dark matter were obtained a la Bethe-Salpeter formalism.

- Dark matter co-scattering undergoes a delayed interaction due to u-channel resonance, being enhanced without a light mediator.

- Effective models and microscopic models for u-channel resonances are shown; detectable by DM self-scattering, indirect and direct detection.