Dark matter **subhalo boost** factor

[based on 2007.10392, 2203.16440, and 2203.16491]

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in collaboration with
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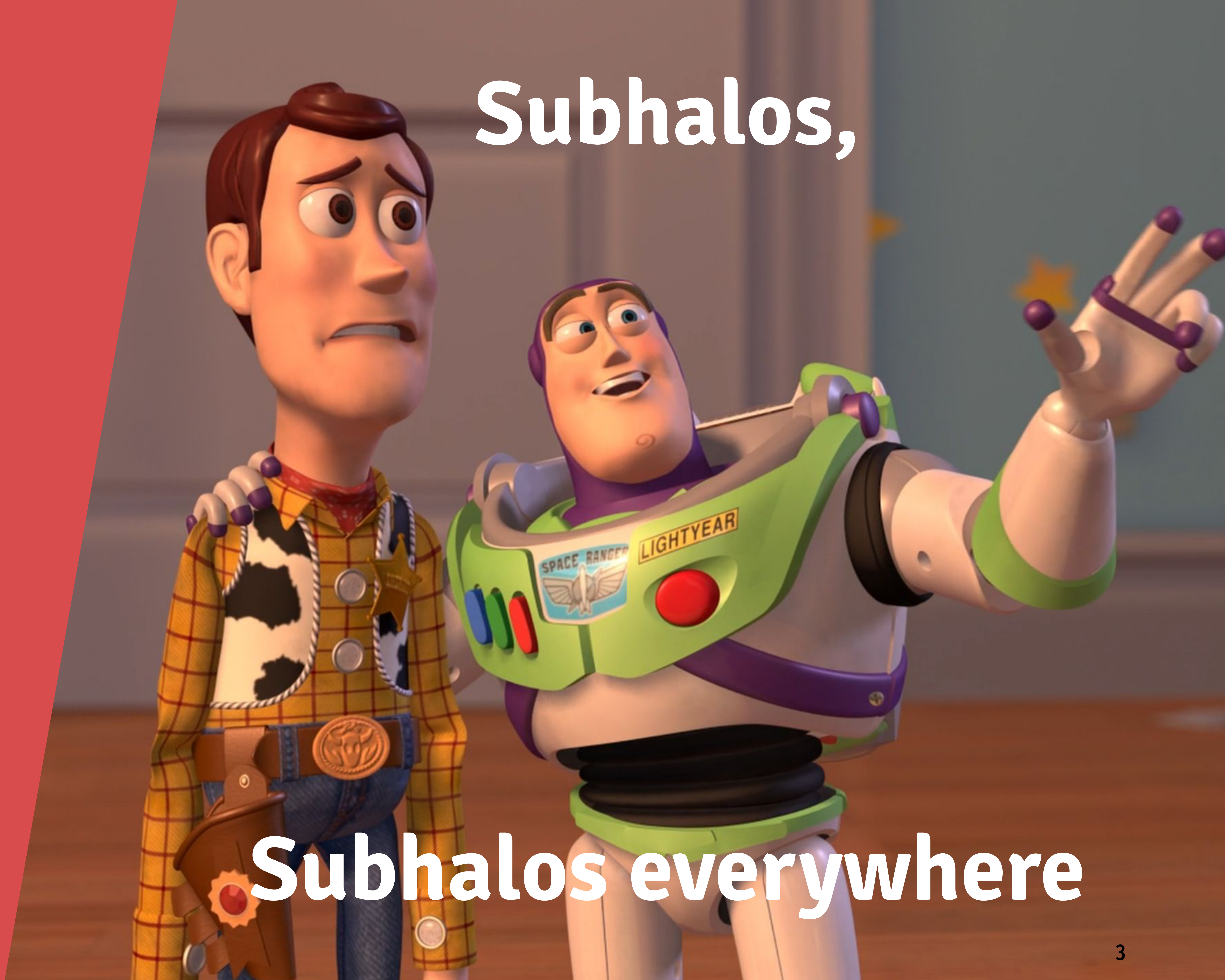
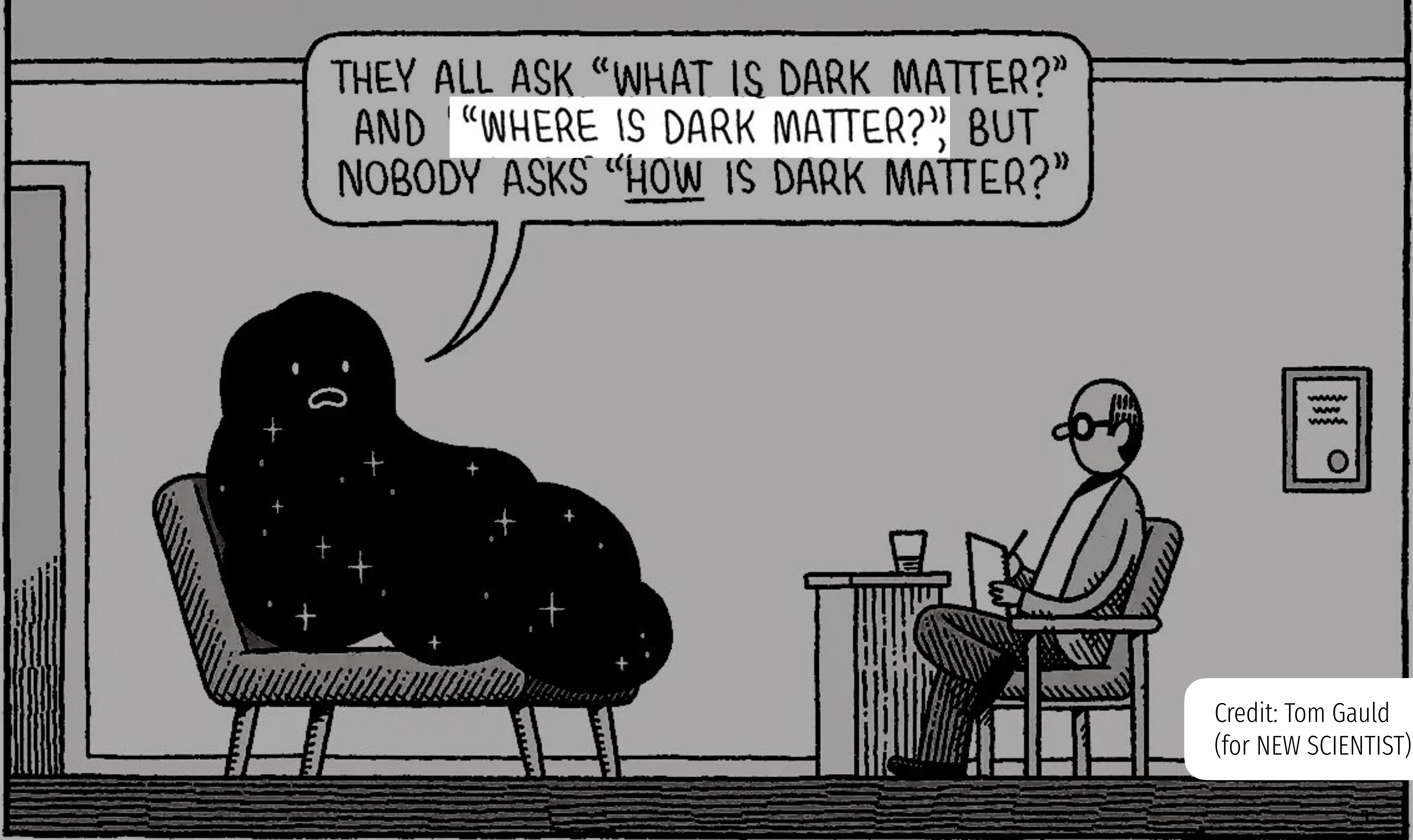
Credit: Tom Gauld
(for NEW SCIENTIST)

INTRODUCTION

WHAT IS THE SUBHALO BOOST FACTOR?



Let us first discuss DM structuring



Credit: Tom Gauld
(for NEW SCIENTIST)

The story starts from
the initial perturbations
of the matter density field...

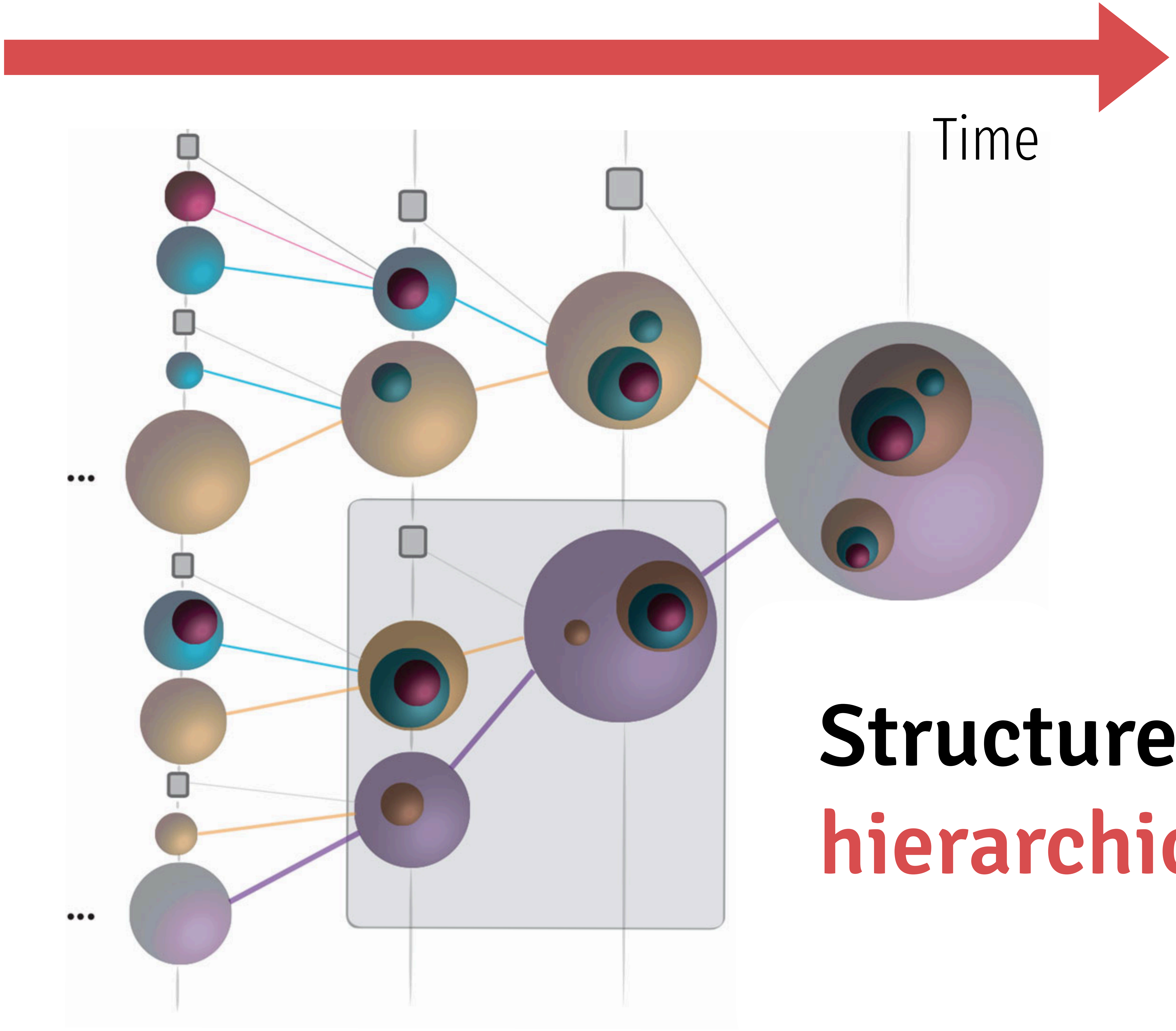
... to their growth ...

... and their collapse into halos (virialisation)

In the CDM paradigm,
the Universe is populated
by ~~aliens~~ dark matter halos

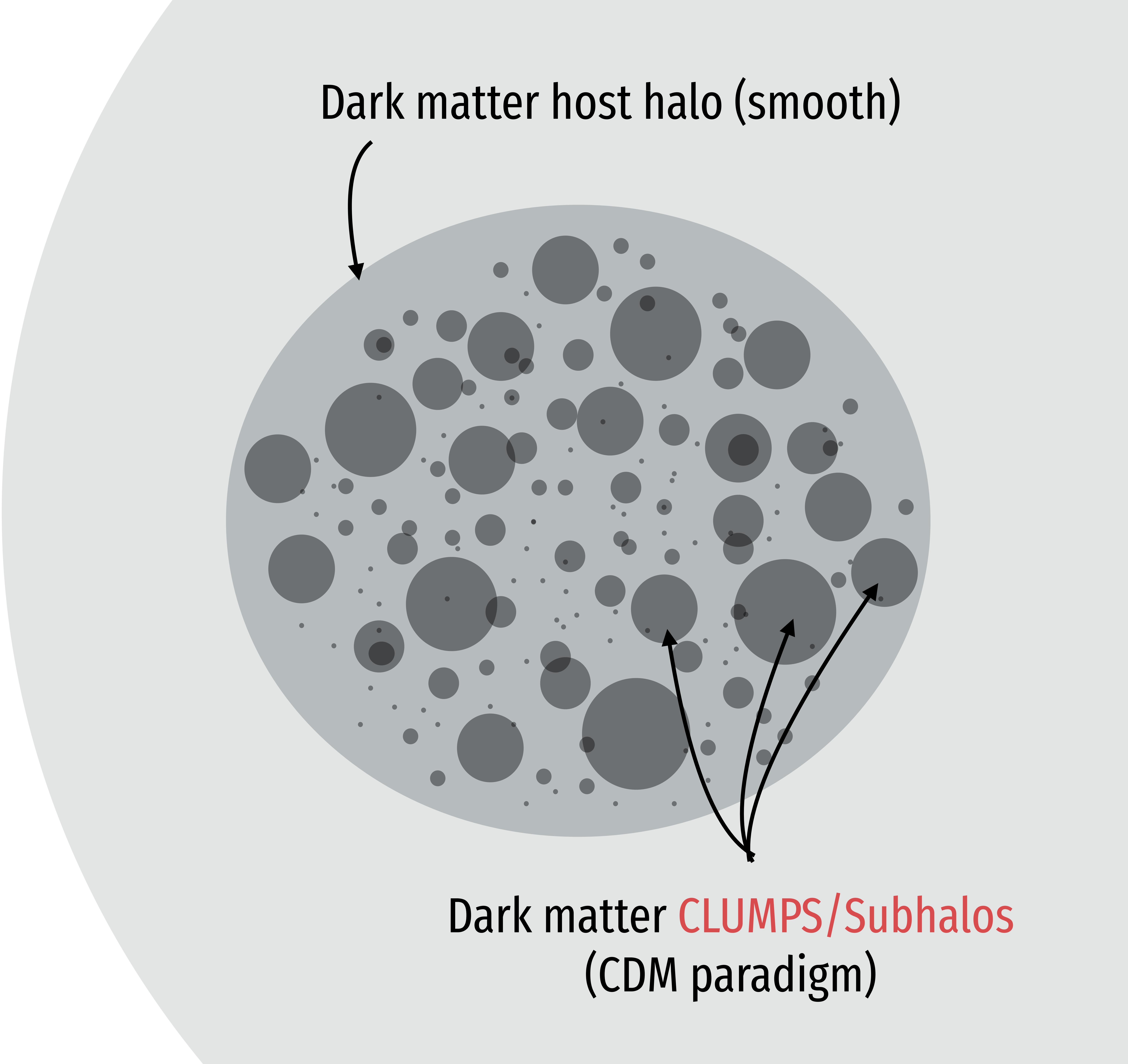
What about DM subhalos?

[figure from Jiang+14]



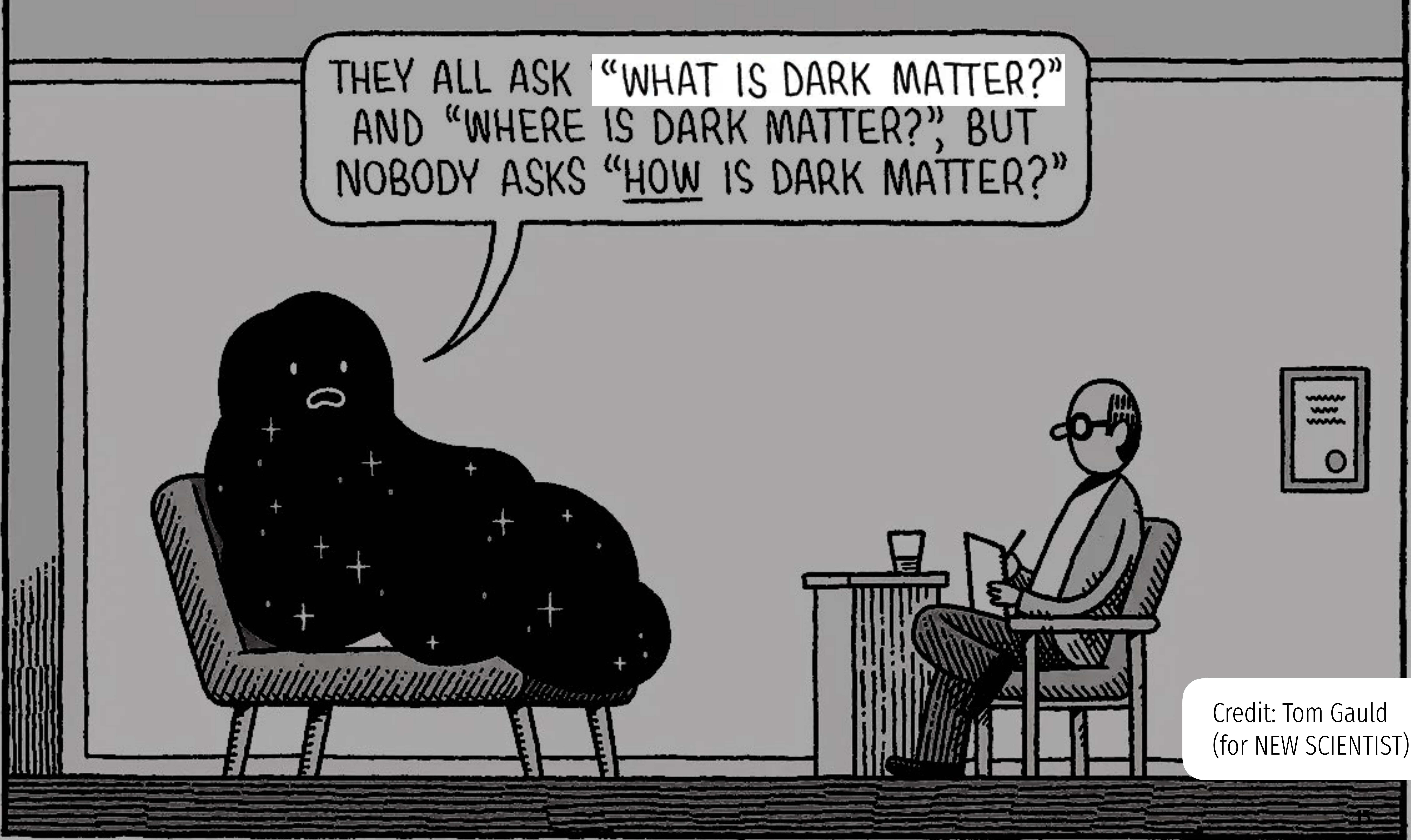
Structures form hierarchically

Halos are clumpy



A boost?! Boost of what?

Let us now discuss DM annihilation



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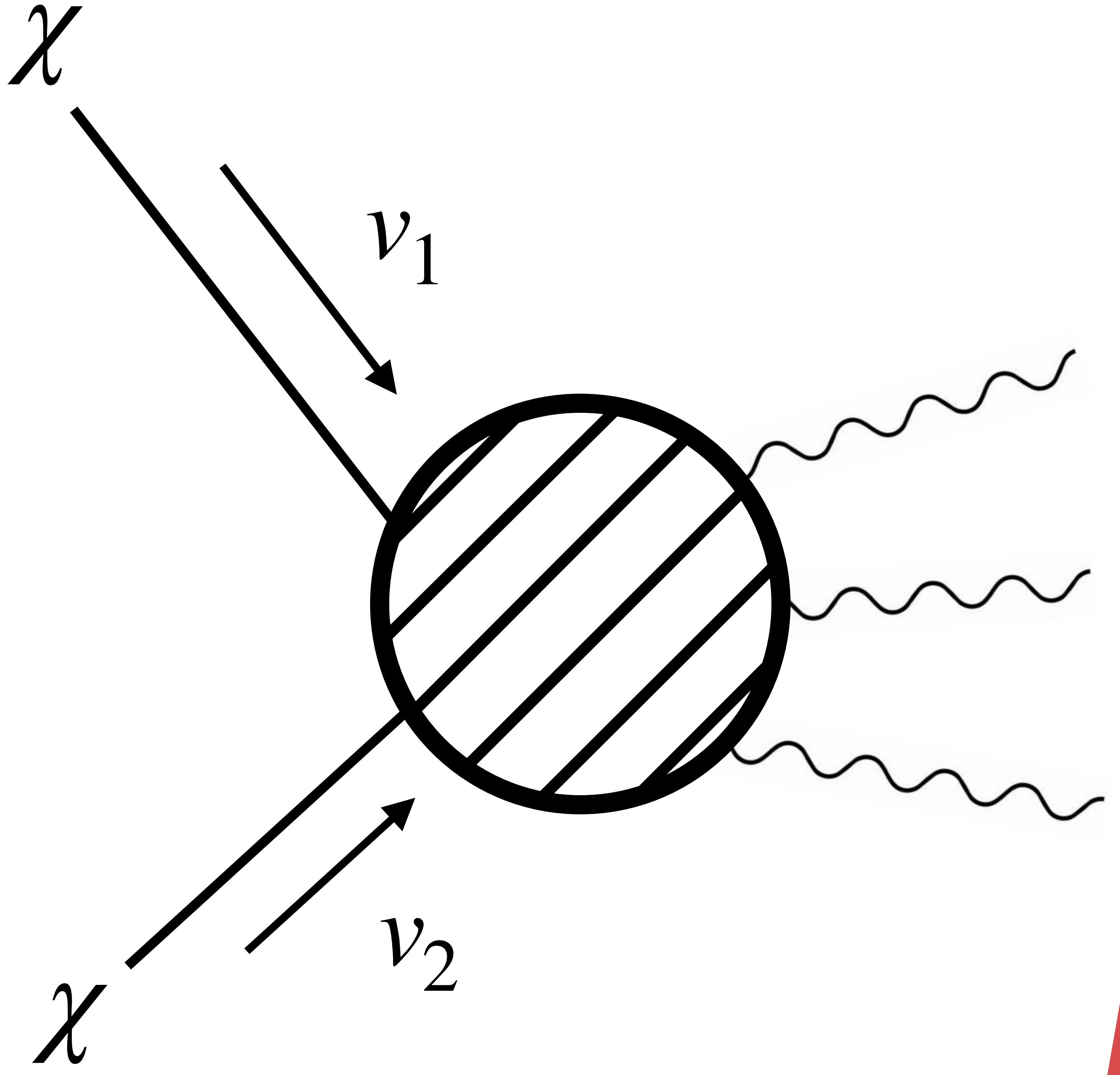
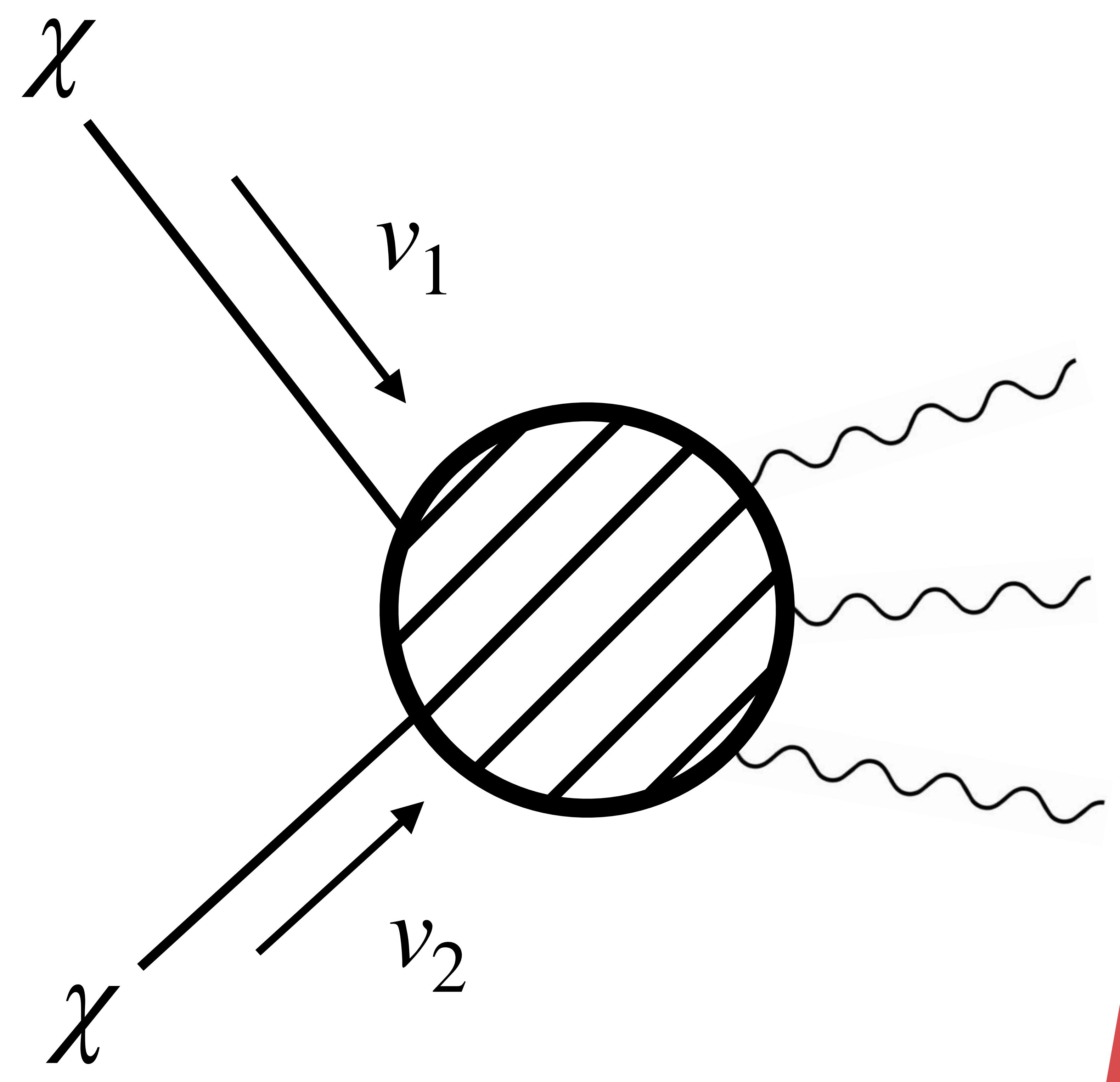


Write the annihilation cross-section in terms of the partial-wave expansion

$$\sigma_{\text{ann}} v_{\text{rel}} = \sigma_0 + \sigma_1 v_{\text{rel}}^2 + \mathcal{O}\left(v_{\text{rel}}^4\right)$$



Write the annihilation cross-section in terms of the partial-wave expansion

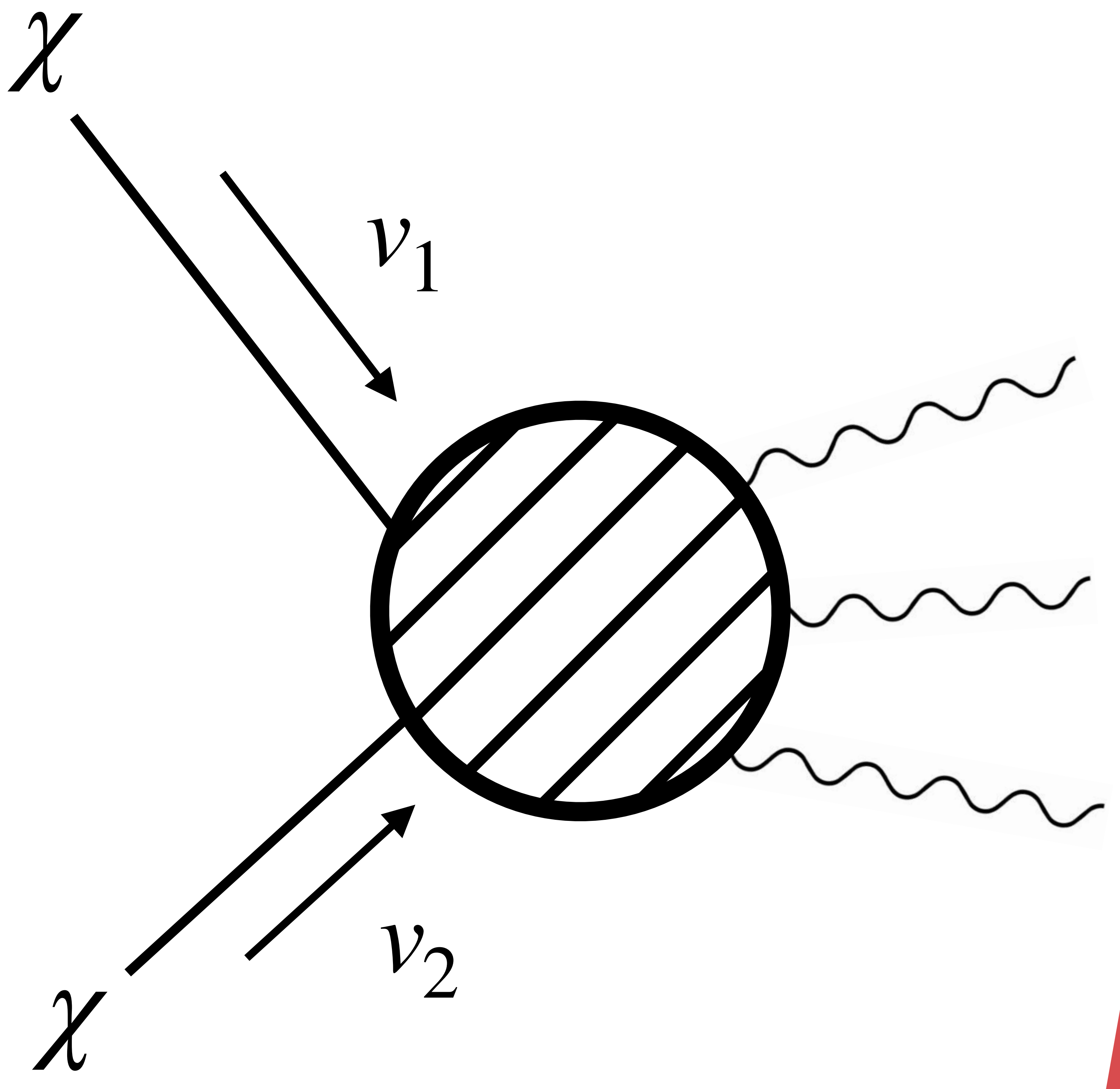
$$\sigma_{\mathrm{ann}} v_{\mathrm{rel}} = \sigma_0 + \sigma_1 v_{\mathrm{rel}}^2 + \mathcal{O}\left(v_{\mathrm{rel}}^4\right)$$

σ_0: s-wave

Most studied case

Strong constraints from indirect searches

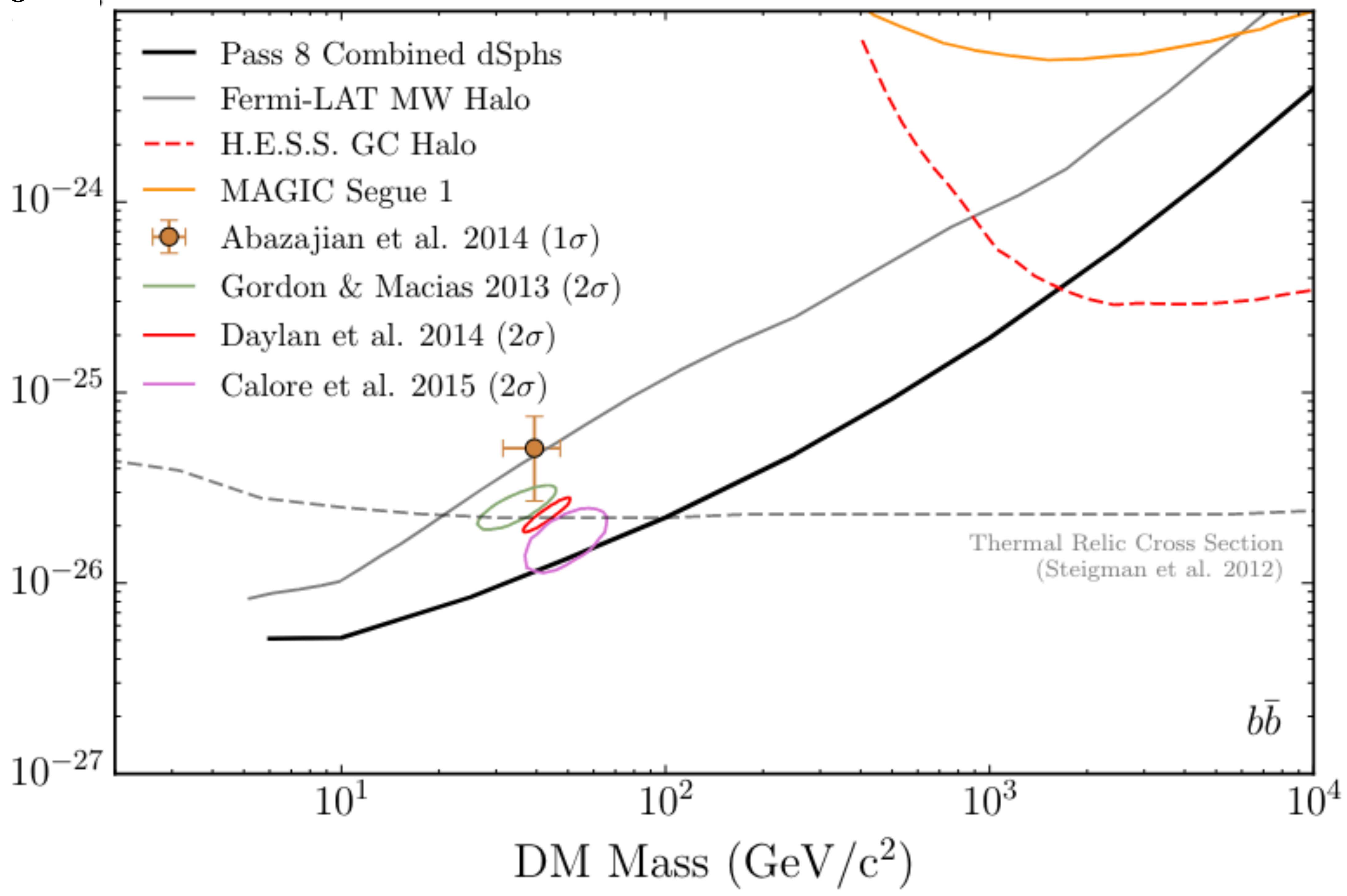
Thermal cross-section excluded m_χ < 10-100 GeV

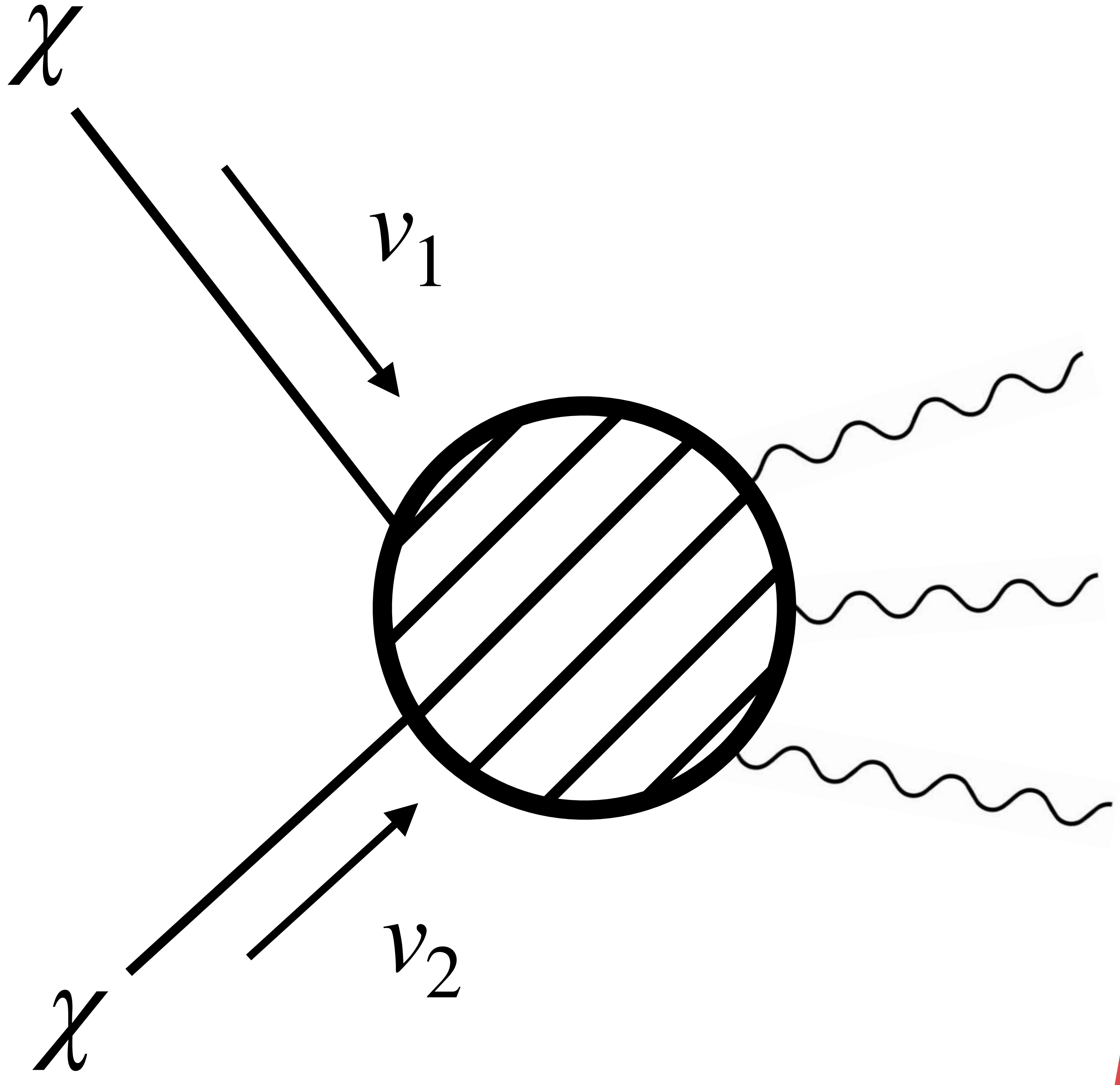


Write the annihilation cross-section in terms of the partial-wave expansion

$\sigma_0\ [10^{26}\ \mathrm{cm}^3 \cdot \mathrm{s}^{-1}]$

[Fermi-LAT 2015]





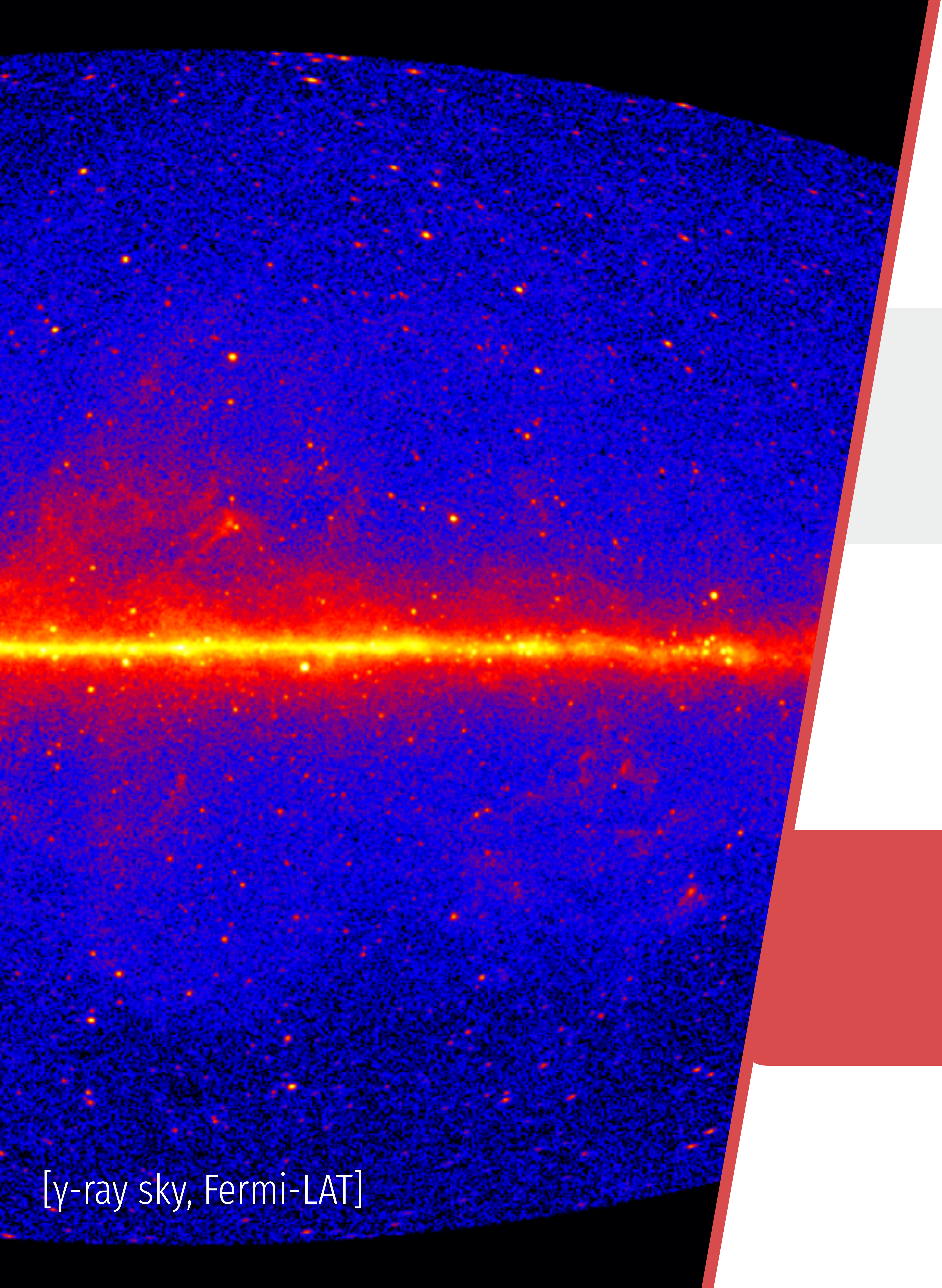
Write the annihilation cross-section in terms of the partial-wave expansion

$$\sigma_{\mathrm{ann}} v_{\mathrm{rel}} = \sigma_0 + \sigma_1 v_{\mathrm{rel}}^2 + \mathcal{O}\left(v_{\mathrm{rel}}^4\right)$$

$\sigma_1 v^2$: p-wave

Suppressed indirect signal
Thermal cross-section unconstrained
Not exotic (fermions with scalar couplings)

and in particular
(for the example)
gamma rays



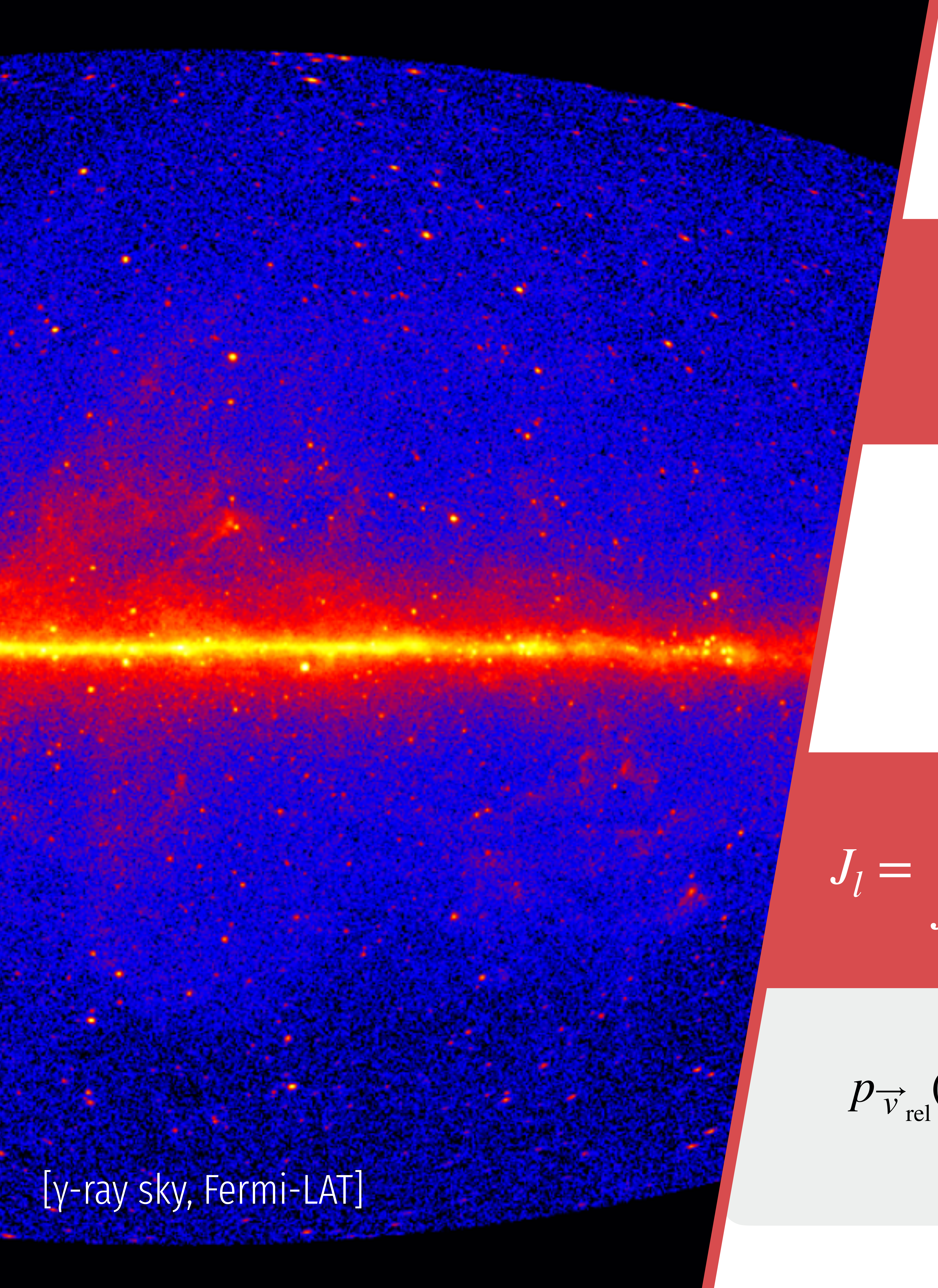
Let us generalize

$$\sigma_{\text{ann}} v_{\text{rel}} = \sum_l \sigma_l \mathcal{S}_l(v_{\text{rel}}) \qquad \mathcal{S}_l(v) = v^{2l}$$

(for partial waves)

the associated γ-ray flux

$$\frac{\mathrm{d}\phi}{\mathrm{d}E} = \frac{1}{8\pi m_\chi^2} \frac{\mathrm{d}N_\gamma}{\mathrm{d}E} \sum_l \sigma_l J_l$$


[γ-ray sky, Fermi-LAT]

Here we have the J-factor:

$$\frac{\mathrm{d}\phi}{\mathrm{d}E} = \frac{1}{8\pi m_\chi^2}\frac{\mathrm{d}N_\gamma}{\mathrm{d}E}\sum_l \sigma_l J_l$$

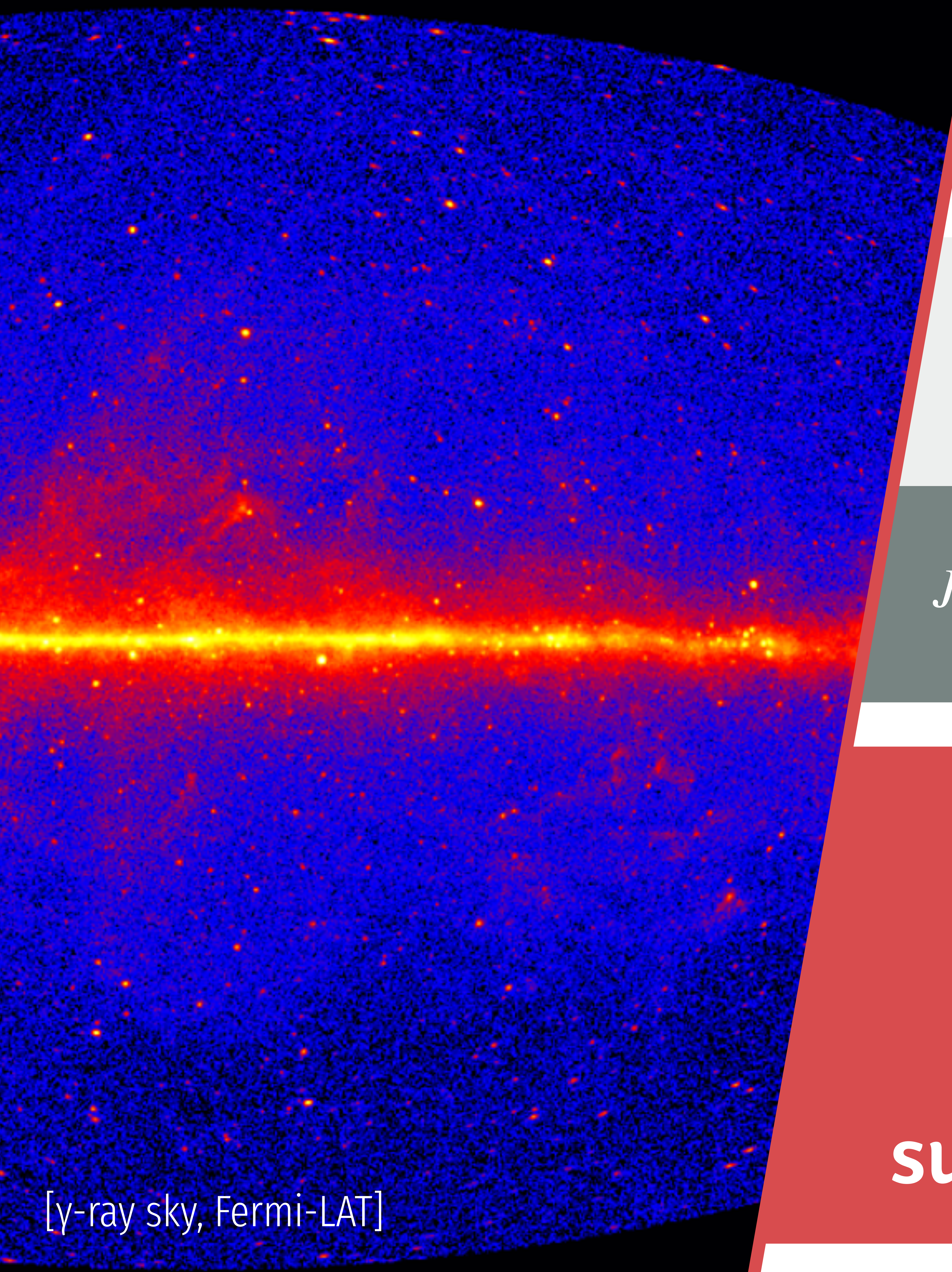
as a measure of the « luminosity »

(along a line of sight in a given solid angle, with weights from velocity factors)

$$J_l = \int \mathrm{d}s\mathrm{d}\Omega\, \rho_\chi^2(\vec{r}(s,\vec{\theta}))\left[\int_{\mathbb{R}^3} p_{\vec{v}_{\rm rel}}(\vec{r}(s,\vec{\theta}),\vec{v}_{\rm rel})\mathcal{S}_l(v_{\rm rel})\mathrm{d}^3\vec{v}_{\rm rel}\right]$$

$$p_{\vec{v}_{\rm rel}}(\vec{r},\vec{v}_{\rm rel}) \equiv \frac{1}{\rho_\chi^2(r)}\int f\left(\vec{r},\vec{v}_c - \frac{\vec{v}_{\rm rel}}{2}\right) f\left(\vec{r},\vec{v}_c + \frac{\vec{v}_{\rm rel}}{2}\right)\mathrm{d}^3\vec{v}_c$$

(distribution of relative velocity)



[γ-ray sky, Fermi-LAT]

(example) s-wave case:

$$\frac{\mathrm{d}\phi}{\mathrm{d}E} = \frac{\sigma_0}{8\pi m_\chi^2}\frac{\mathrm{d}N_\gamma}{\mathrm{d}E}J_0$$

$$J_0 = \int \mathrm{d}s\mathrm{d}\Omega\, \rho_\chi^2(\vec{r}(s,\vec{\theta}))\left[\int_{\mathbb{R}^3} p_{\vec{v}_{\mathrm{rel}}}(\vec{r}(s,\vec{\theta}),\vec{v}_{\mathrm{rel}})\mathrm{d}^3\vec{v}_{\mathrm{rel}}\right]$$

$$J_0 = \int \mathrm{d}s\mathrm{d}\Omega\, \rho_\chi^2(\vec{r}(s,\vec{\theta})) \propto \frac{m}{4\pi D^2}$$

Squared density: the presence of subhalos enhances/modifies the signal

$$\text{boost factor} = \frac{\text{signal with clumpiness (true signal)}}{\text{signal if DM was smoothly distributed}}$$

The boost factor is encoded in the J-factors/luminosity

$$\text{boost factor} = \frac{J_l^{\text{smooth part}} + J_l^{\text{clumps}}}{J_l^{\text{all smooth}}}$$

I will talk more about this astrophysical examples ...

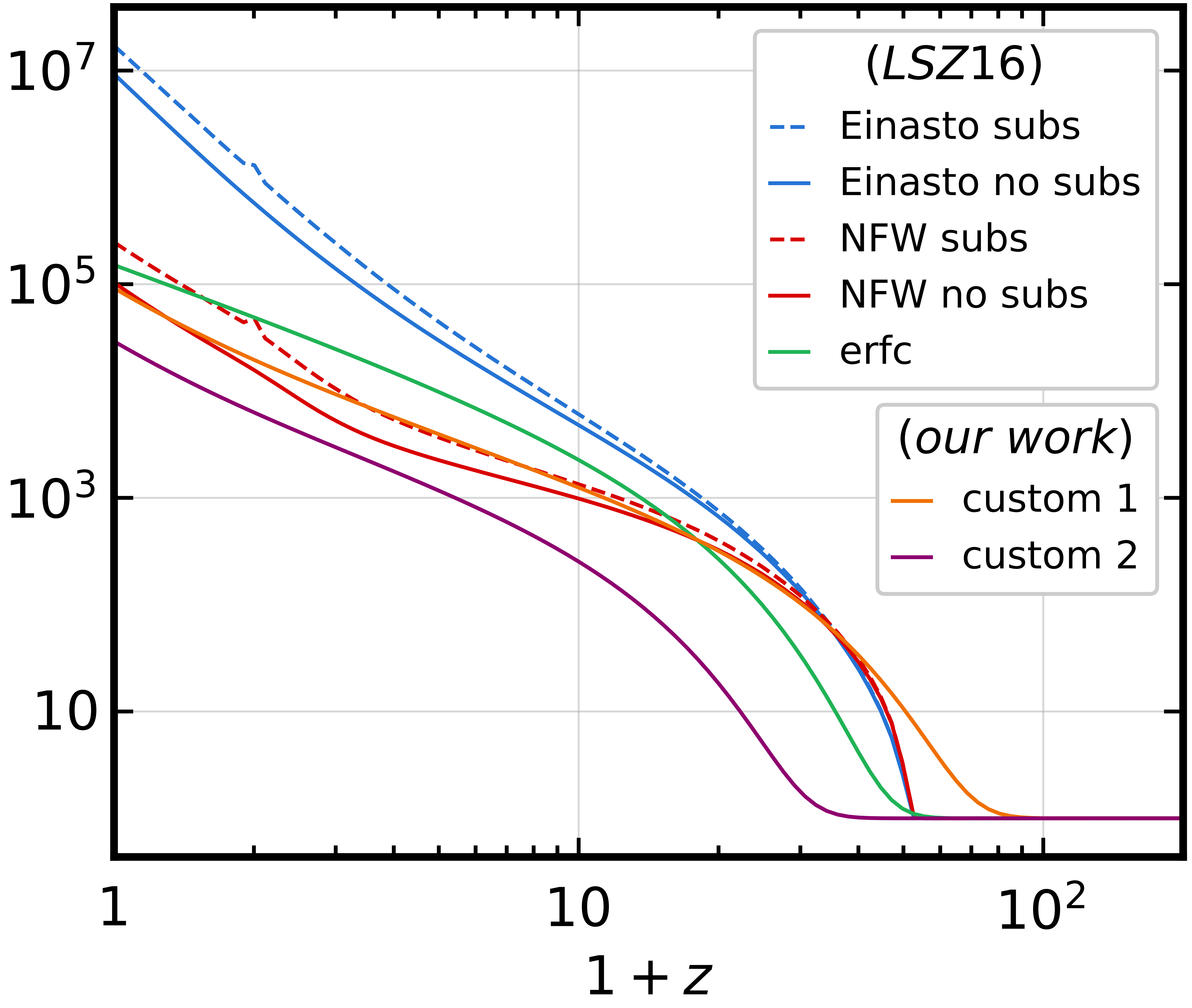
... but the boost may also be important in cosmology (e.g. for the 21cm)

Boost from halos
but
without subhalos

Boost from halos
and
without subhalos

Enhancement due to subhalos

Cosmological boost (s-wave)



[from Liu+16]

I.

A generic analytical subhalo model

II.

Sommerfeld enhanced and subhalo boosted J-factors

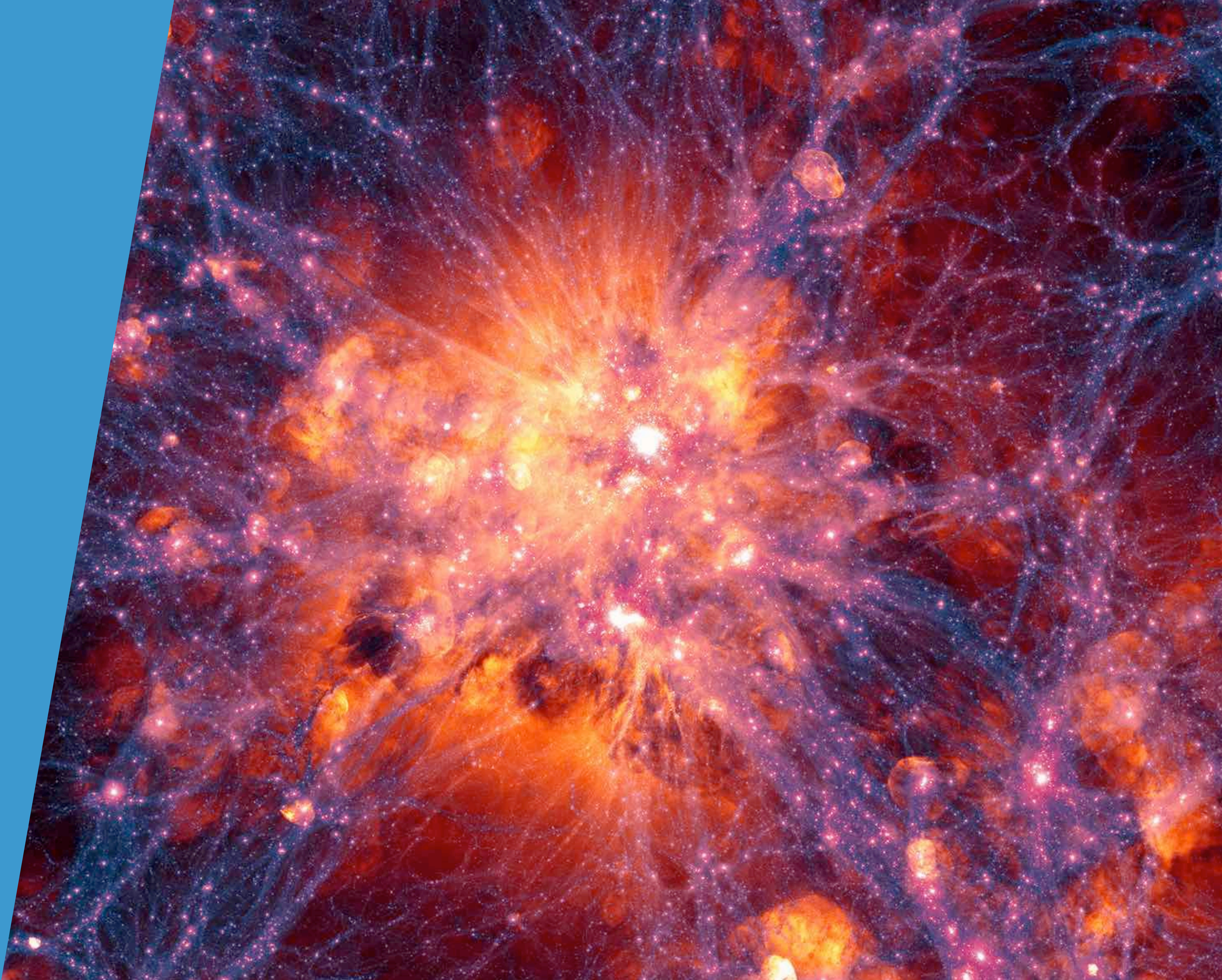
III.

Subhalos as point sources in the Milky Way

Part 1:

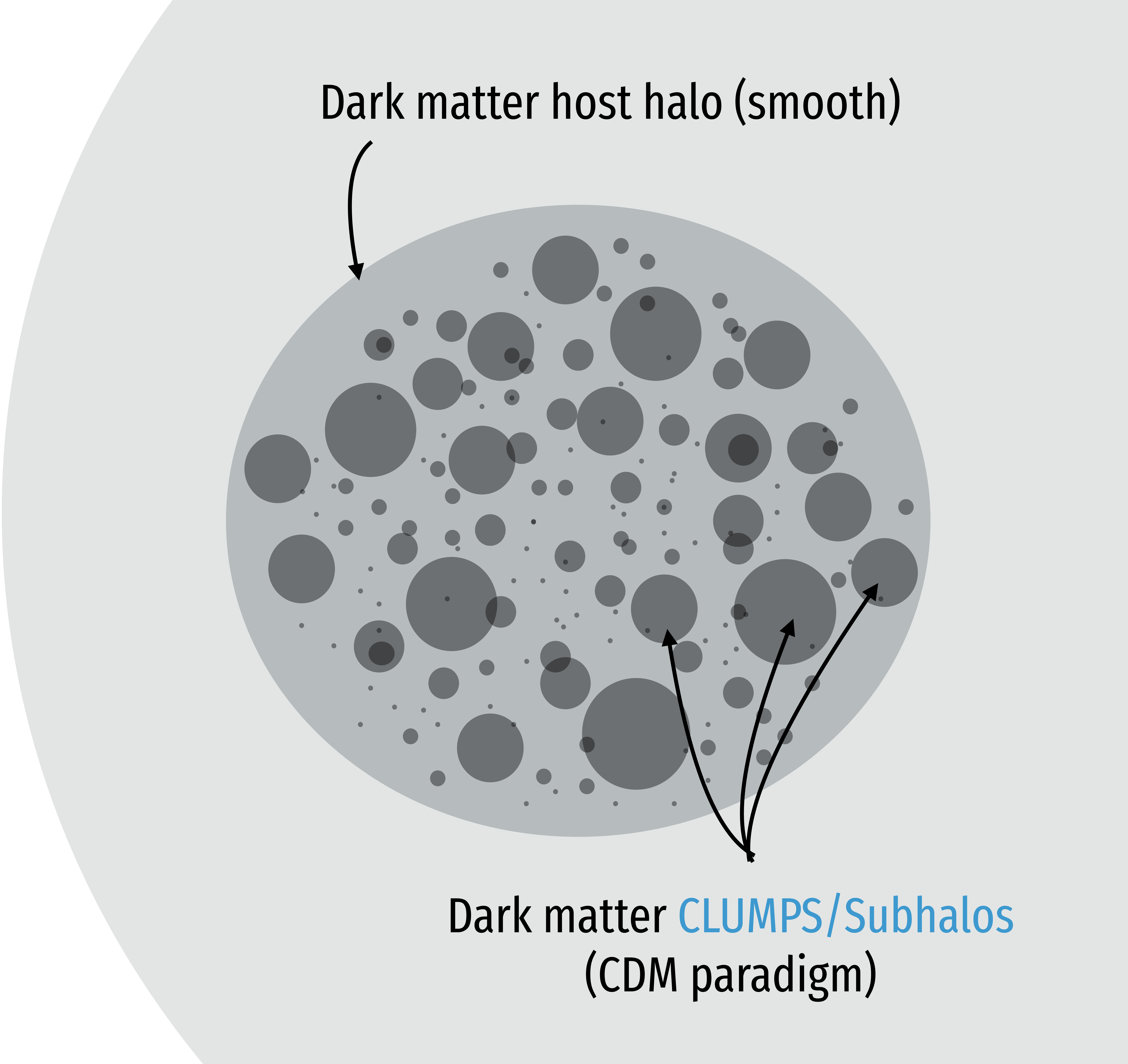
A generic analytical subhalo model

[1610.02233, 2201.09788]



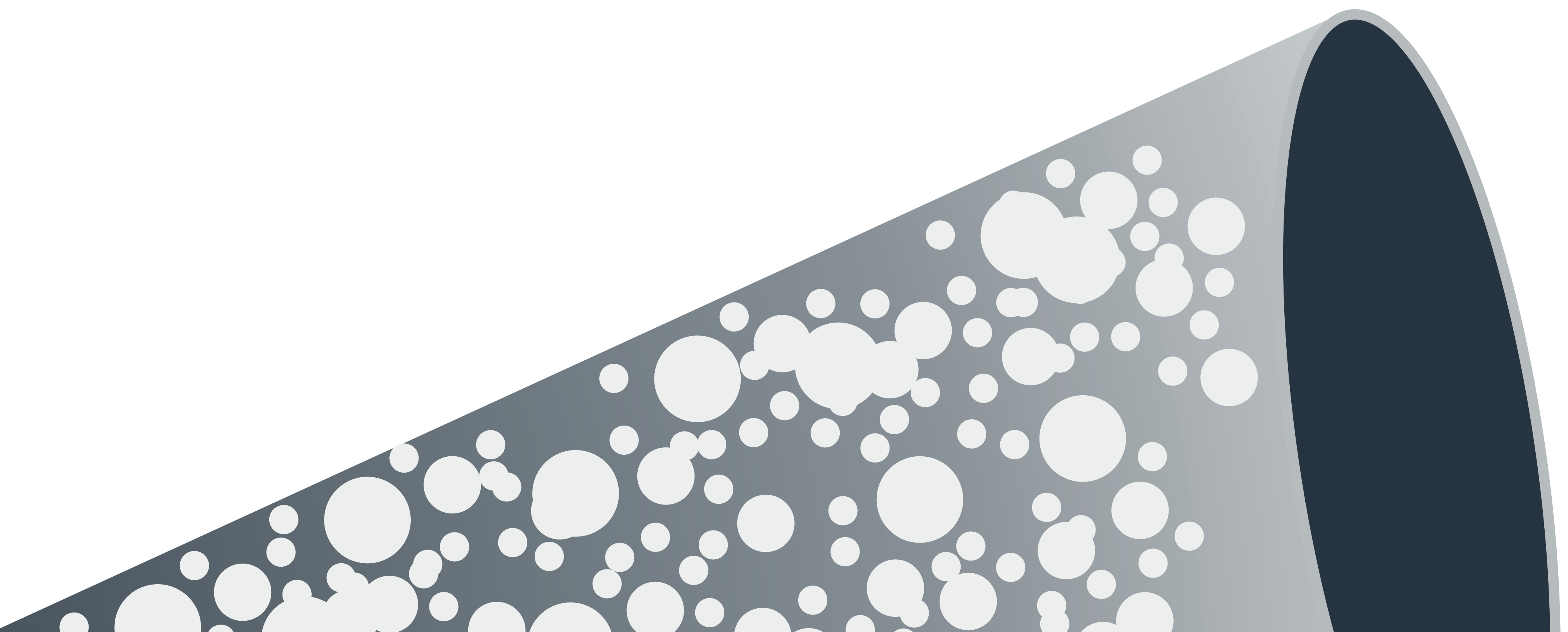
Consider a host halo

$$\rho_\chi = \rho_{\text{smooth}} + \sum_{i=1}^{N_{\text{sub}}} \rho_i$$



Subhalo contribution to the J-factors (in a given volume)

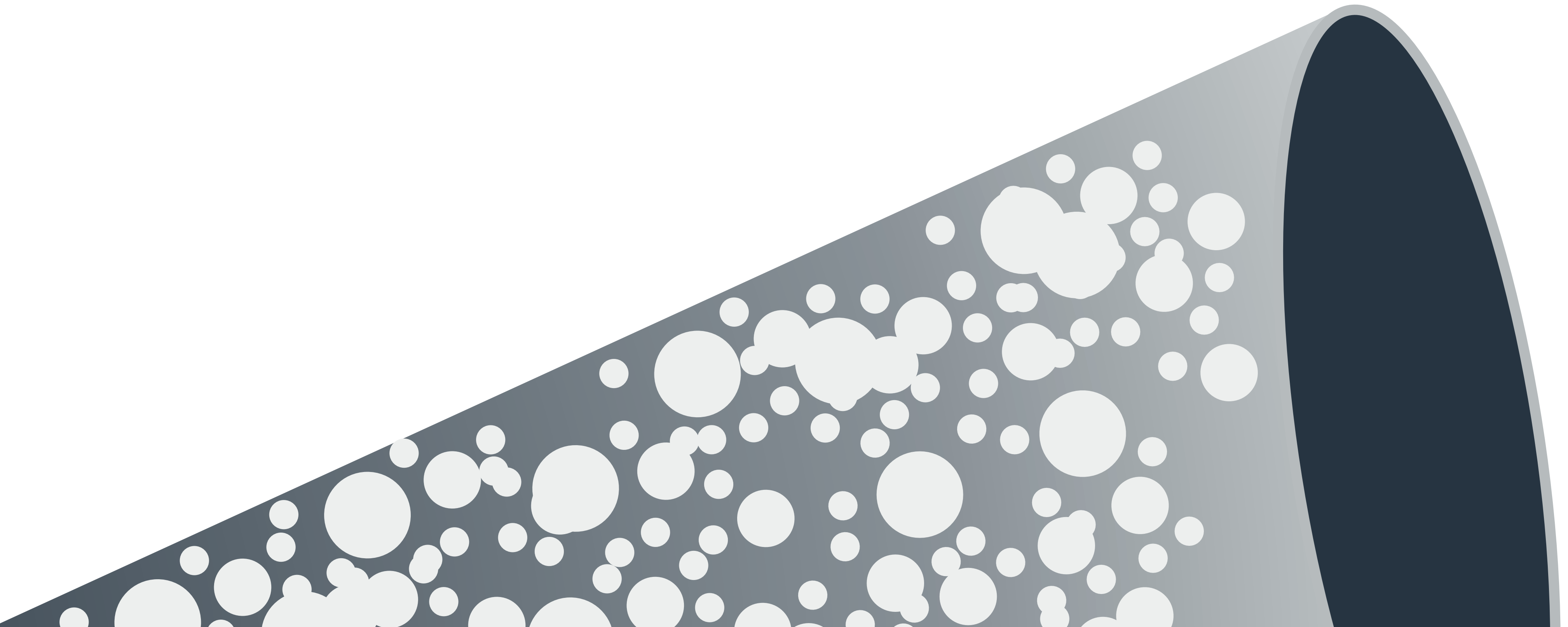
$$J_l^{\mathrm{sub}} \equiv \frac{1}{D^2} \int_{\mathscr{V}} \left. \frac{\partial^2 n}{\partial m_{\mathrm{t}} \partial c} \right|_{\mathrm{sub}} (m_{\mathrm{t}}, c, \overrightarrow{R}) \mathscr{L}_l(m_{\mathrm{t}}, c) \mathrm{d}m_{\mathrm{t}} \mathrm{d}c \mathrm{d}^3 \overrightarrow{R}$$



Subhalo contribution to the J-factors (in a given volume)

$$J_l^{\mathrm{sub}} \equiv \frac{1}{D^2}\int_{\mathscr{V}} \left.\frac{\partial^2 n}{\partial m_{\mathrm{t}}\partial c}\right|_{\mathrm{sub}}(m_{\mathrm{t}}, c, \vec{R})\,\mathscr{L}_l(m_{\mathrm{t}}, c)\,\mathrm{d}m_{\mathrm{t}}\mathrm{d}c\mathrm{d}^3\vec{R}$$

Luminosity of a single subhalo



Subhalo contribution to the J-factors (in a given volume)

$$J_l^{\rm sub} \equiv \frac{1}{D^2}\int_{\mathscr{V}} \left.\frac{\partial^2 n}{\partial m_{\rm t}\partial c}\right|_{\rm sub}(m_{\rm t}, c, \vec{R})\,\mathscr{L}_l(m_{\rm t}, c)\,{\rm d}m_{\rm t}{\rm d}c{\rm d}^3\vec{R}$$

Luminosity of a single subhalo

« Luminosity » of a subhalo (of mass m_t and concentration c)

$$\mathscr{L}_l(m_{\rm t}, c) = \int \rho^2_{\rm sub}(\vec{r}) \int f^{\rm sub}_{\vec{v}_{\rm rel}}(\vec{r}, \vec{v}_{\rm rel})\,\mathcal{S}_l(v_{\rm rel})\,{\rm d}^3\vec{v}_{\rm rel}\,{\rm d}^3\vec{r}$$

$$\simeq \int \rho^2_{\rm sub}(\vec{r})\,\mathcal{S}_l[\langle v_{\rm rel}\rangle_n(\vec{r})]\,{\rm d}^3\vec{r}$$

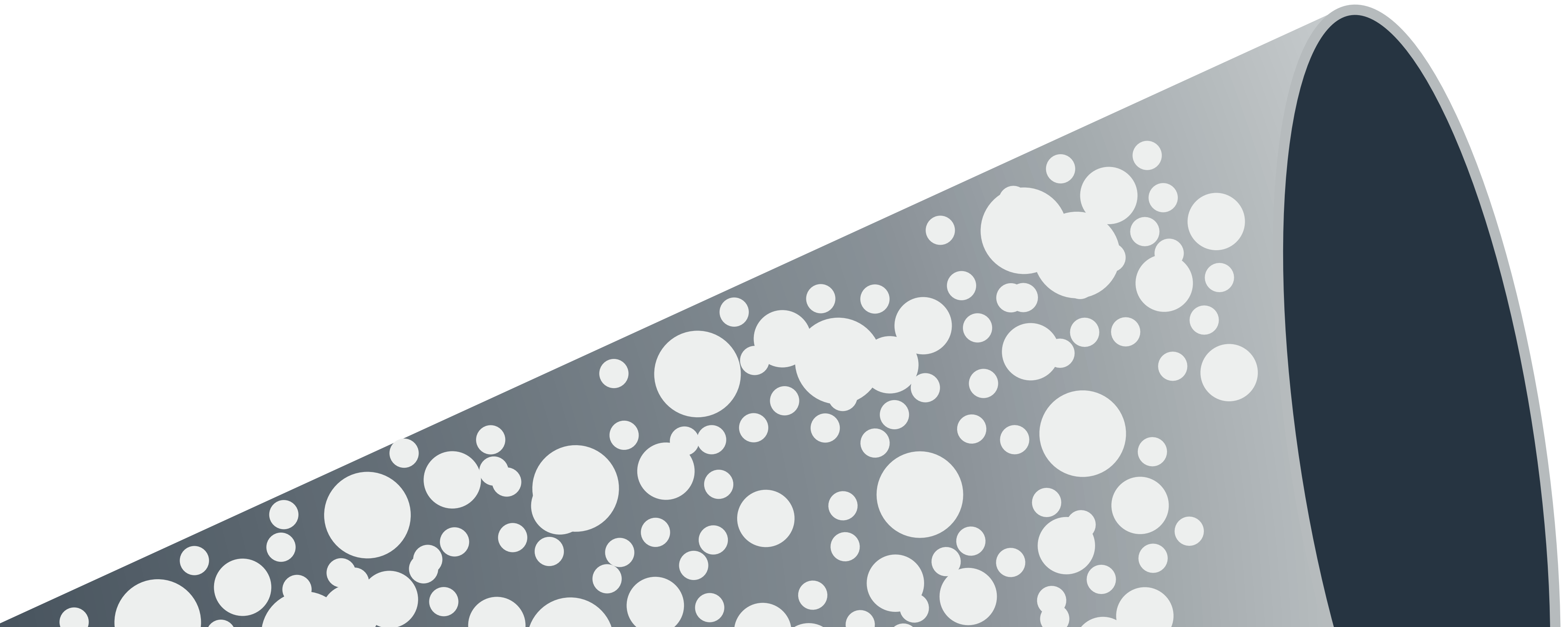
with

$$\langle v_{\rm rel}\rangle_n(\vec{r}) = \left[\int p^{\rm sub}_{\vec{v}_{\rm rel}}(\vec{r}, \vec{v}_{\rm rel})\,v^n_{\rm rel}\right]^{1/n}$$

Subhalo contribution to the J-factors (in a given volume)

$$J_l^{\mathrm{sub}} \equiv \frac{1}{D^2}\int_{\mathscr{V}} \left.\frac{\partial^2 n}{\partial m_{\mathrm{t}}\partial c}\right|_{\mathrm{sub}} (m_{\mathrm{t}}, c, \overrightarrow{R})\,\mathscr{L}_l(m_{\mathrm{t}}, c)\,\mathrm{d}m_{\mathrm{t}}\mathrm{d}c\mathrm{d}^3\overrightarrow{R}$$

subhalo distribution



How to describe the subhalo population?

[**GF, Stref and Lavalle 2022**, Stref+17,
Benson+12, Bartels+15,
Hiroshima+18, Hiroshima+22,
Zavala+14,
Van den Bosch+05,
Peñarrubia+05, ...]

with analytical models

Number of CDM subhalos in the targets >> 10^5

Use a statistical description of the subhalos

A recipe from
[Stref and Lavalle 2017]
[GF, Stref and Lavalle 2022]

How to describe the subhalo population?

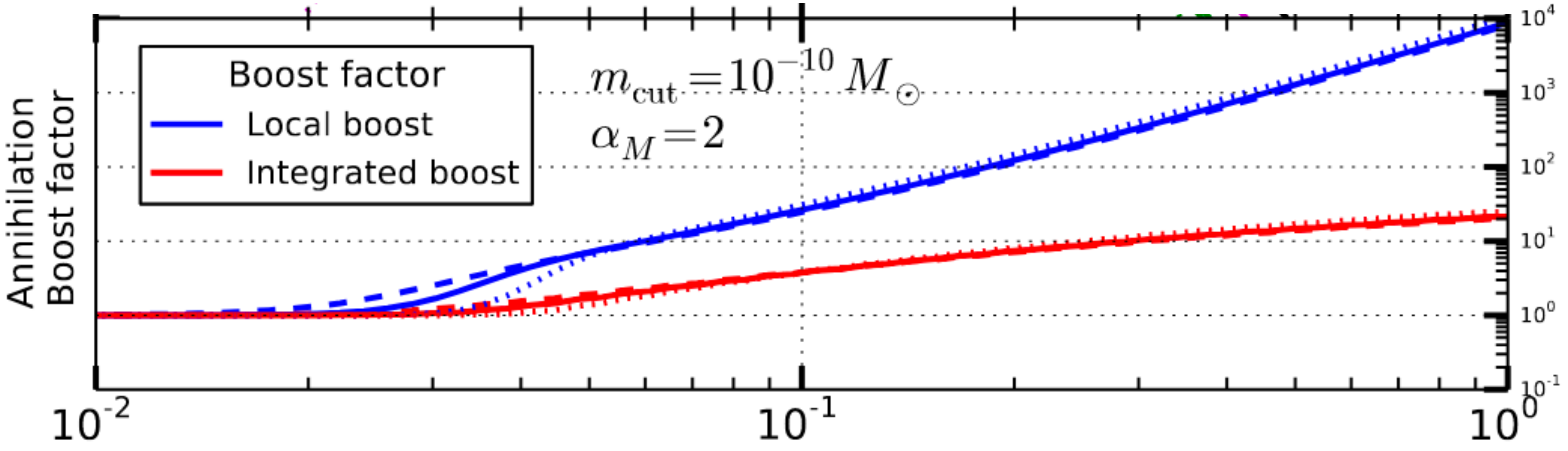
[GF, Stref and Lavalle 2022, Stref+17, Benson+12, Bartels+15, Hiroshima+18, Hiroshima+22, Zavala+14, Van den Bosch+05, Peñarrubia+05, ...]

in two lines:

- 1. Cosmological subhalo distribution
- 2. Dynamical effects in the host
  (Make subhalo shrink over time)

First application of the model:

(*s*-wave) [Stref and Lavalle 2017]



Distance from MW center (normalized)

Boost factor in the Milky Way

Building an analytical model for a subhalo population: the recipe

which, in equation, gives:

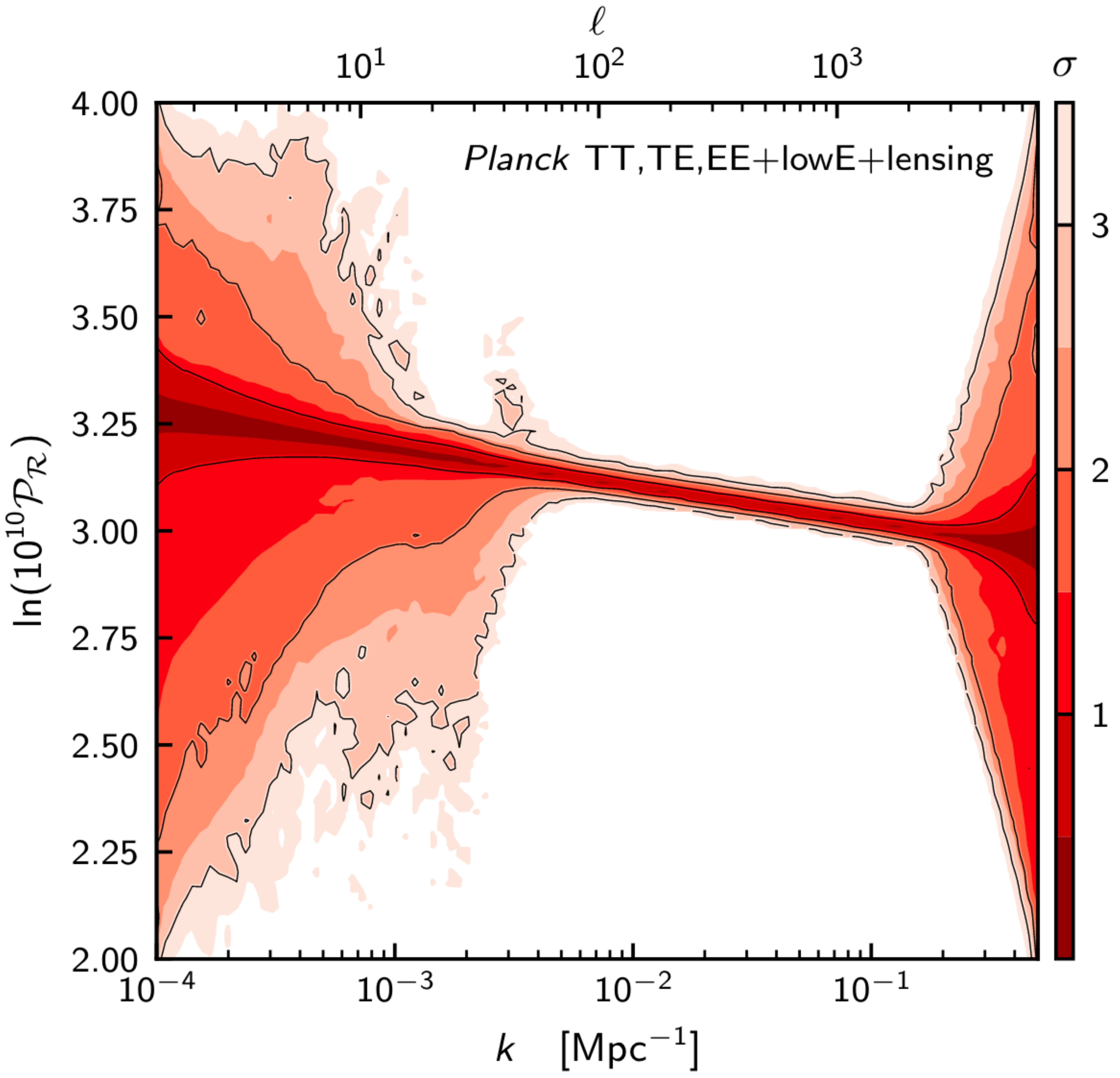
$$\left.\frac{\partial^2 n}{\partial m_{\rm t}\partial c}\right|_{\rm f} = \int \left.\frac{\partial^2 n}{\partial m \partial c}\right|_{\rm i} \Theta\left(\frac{r_{\rm t}(m, c, \vec{R}, z)}{r_{\rm s}(m, c, z)} - \epsilon_t\right) \delta[m_{\rm t} - m_{\rm t}^{\star}(m, c, \vec{R}, z)]{\rm d}m$$

Tidal disruption/stripping terms

Initial (cosmological) distribution

$$\left.\frac{\partial^2 n}{\partial m \partial c}\right|_{i} = \frac{{\rm d}N_{\rm sub}}{{\rm d}m}(m \mid M_{\rm host}, z) p_{\vec{R}}(\vec{R}) p_c(c \mid m)$$

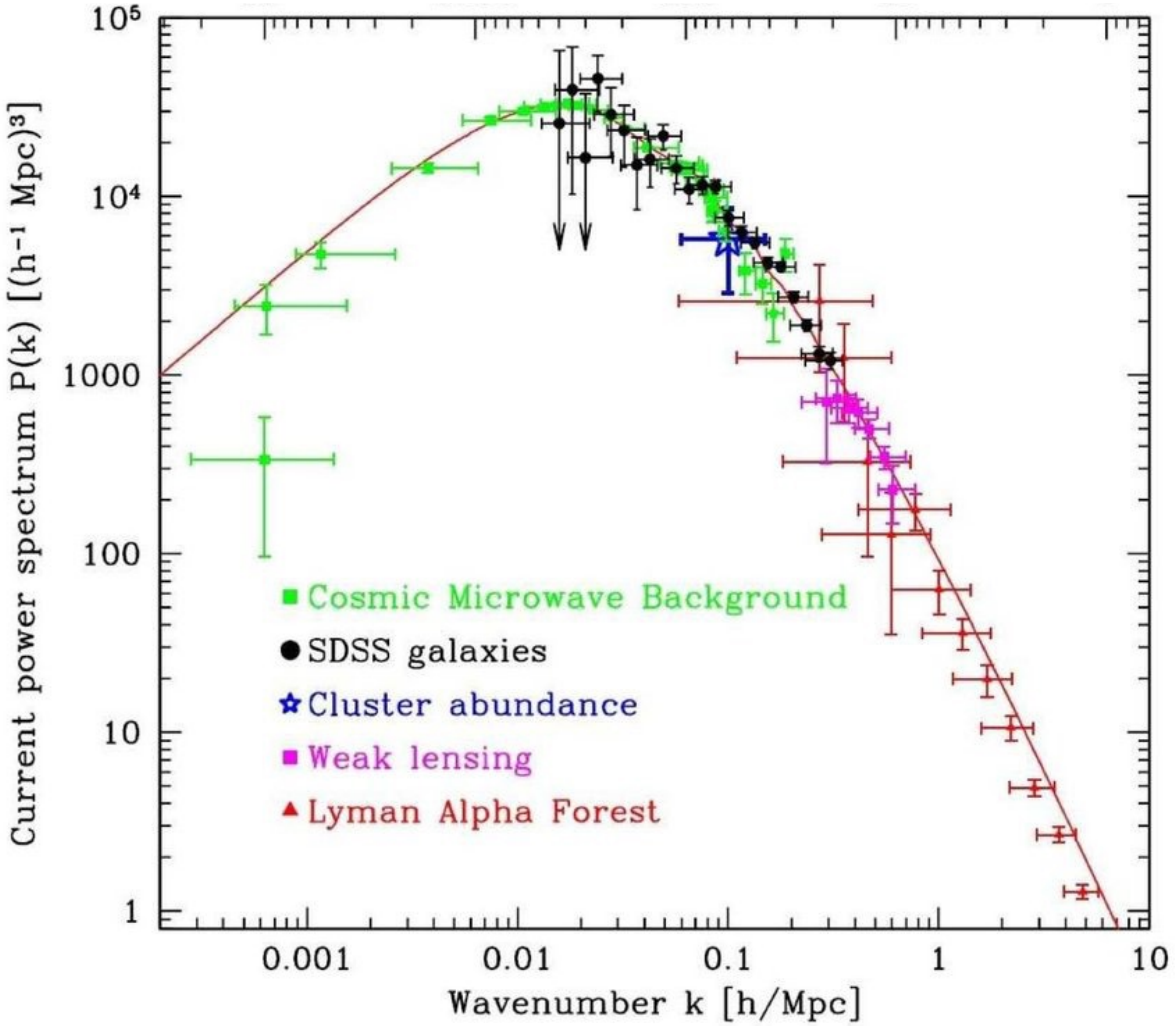
The cosmological subhalo mass function



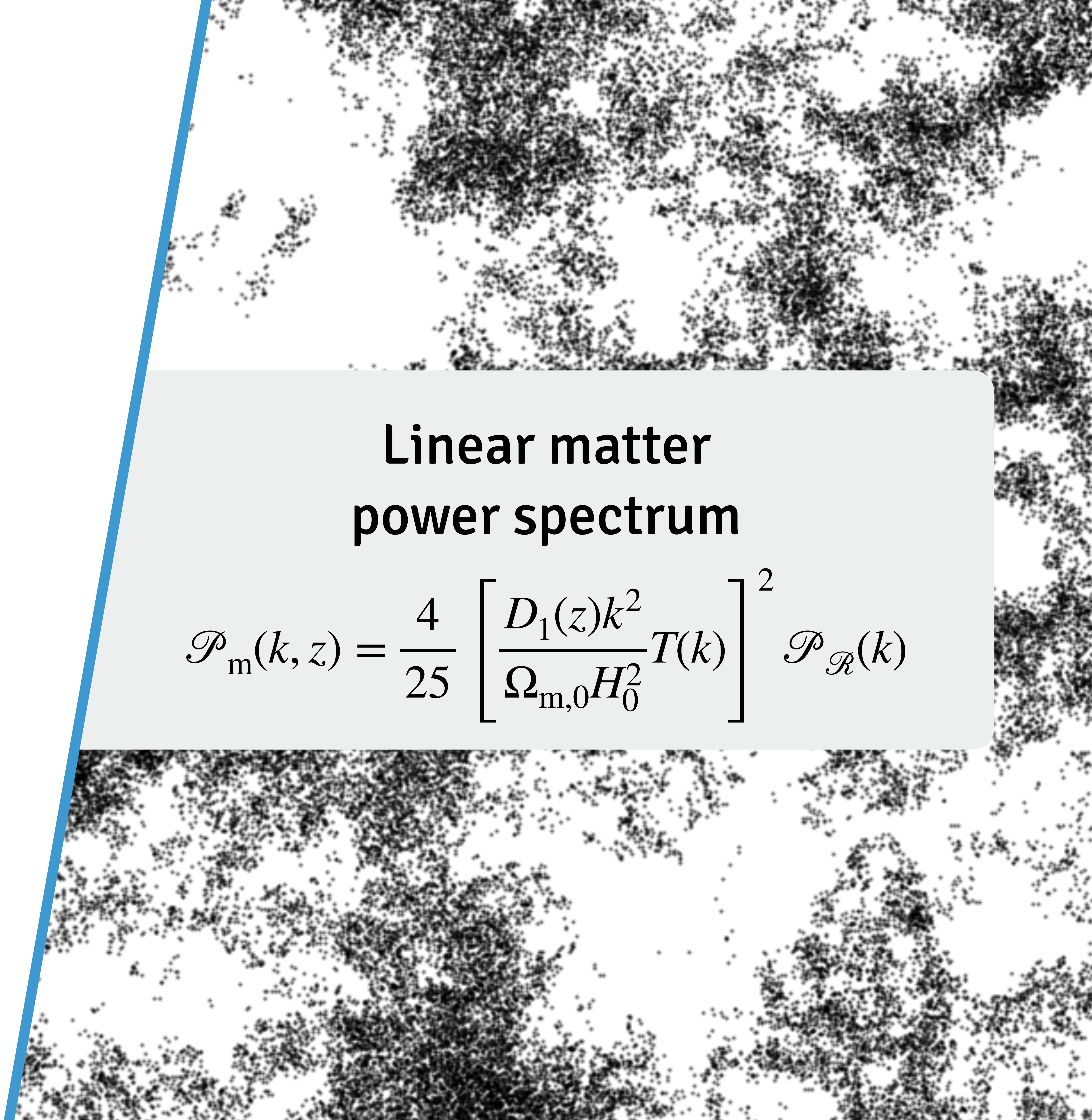
[Planck18]

Primordial power spectrum

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{A}_s \left(\frac{k}{k_0}\right)^{n_s-1}$$

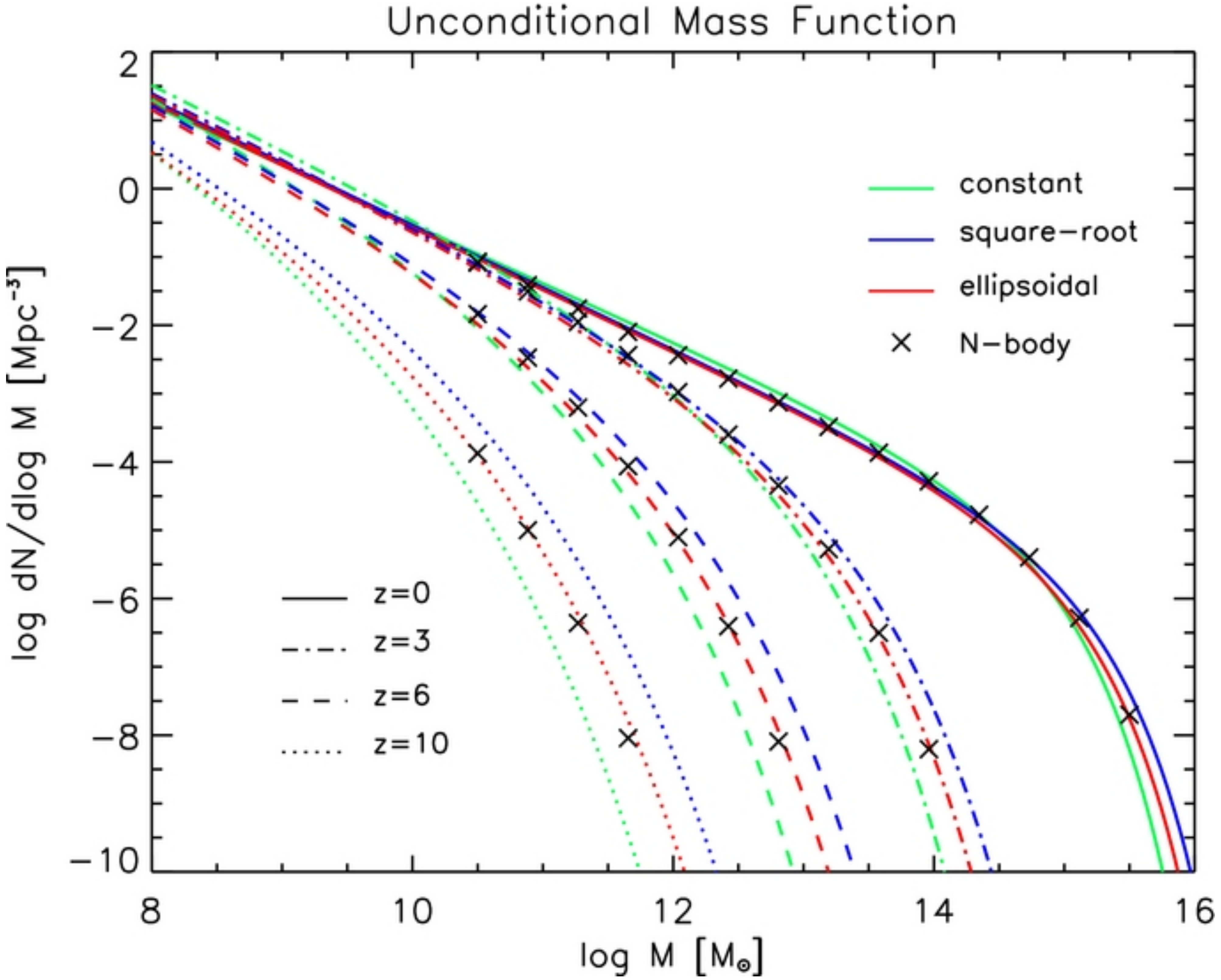


[Tegmark+04]

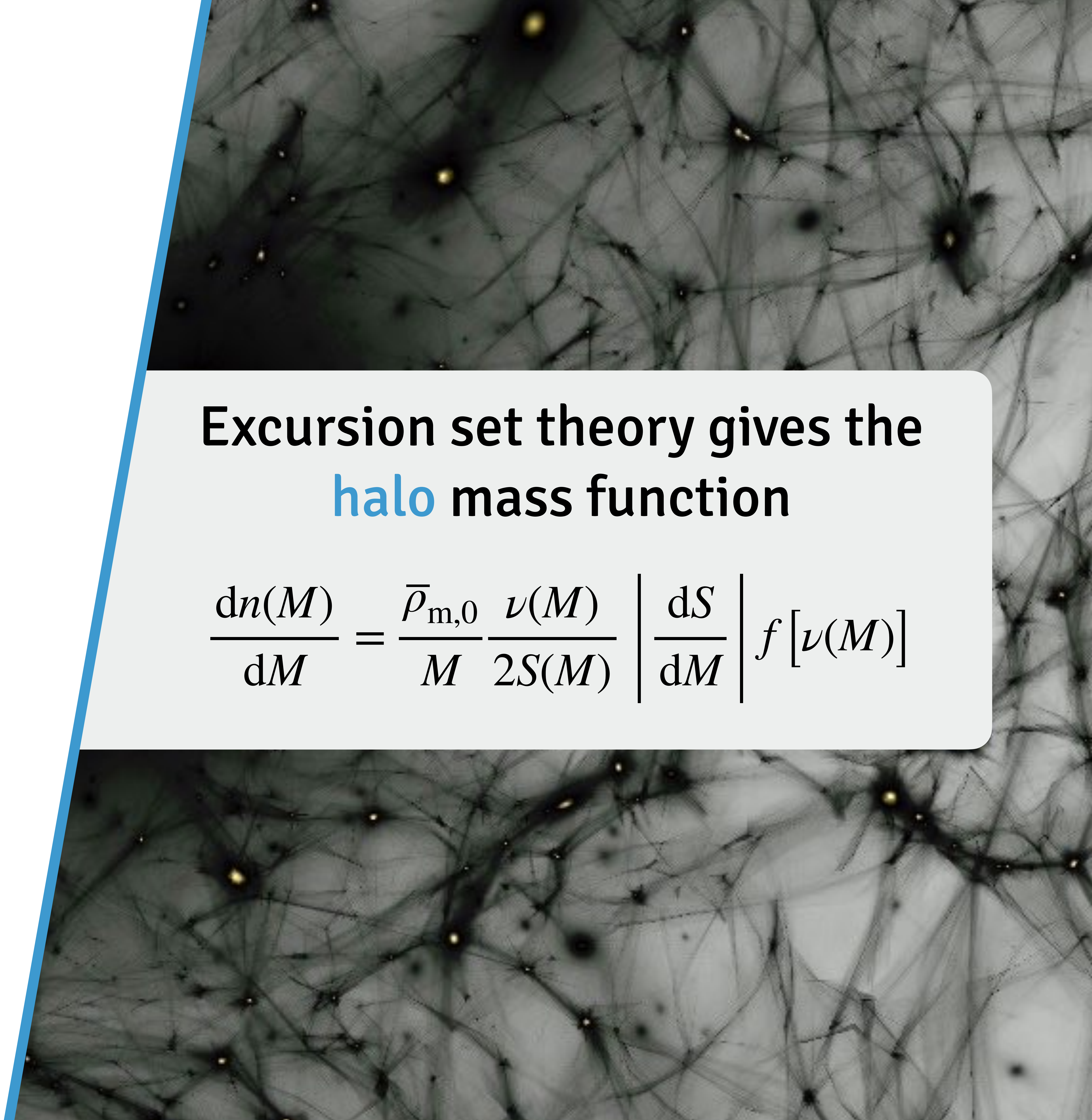


Linear matter power spectrum

$$\mathscr{P}_{\mathrm{m}}(k,z) = \frac{4}{25}\left[\frac{D_1(z)k^2}{\Omega_{\mathrm{m},0}H_0^2}T(k)\right]^2 \mathscr{P}_{\mathscr{R}}(k)$$

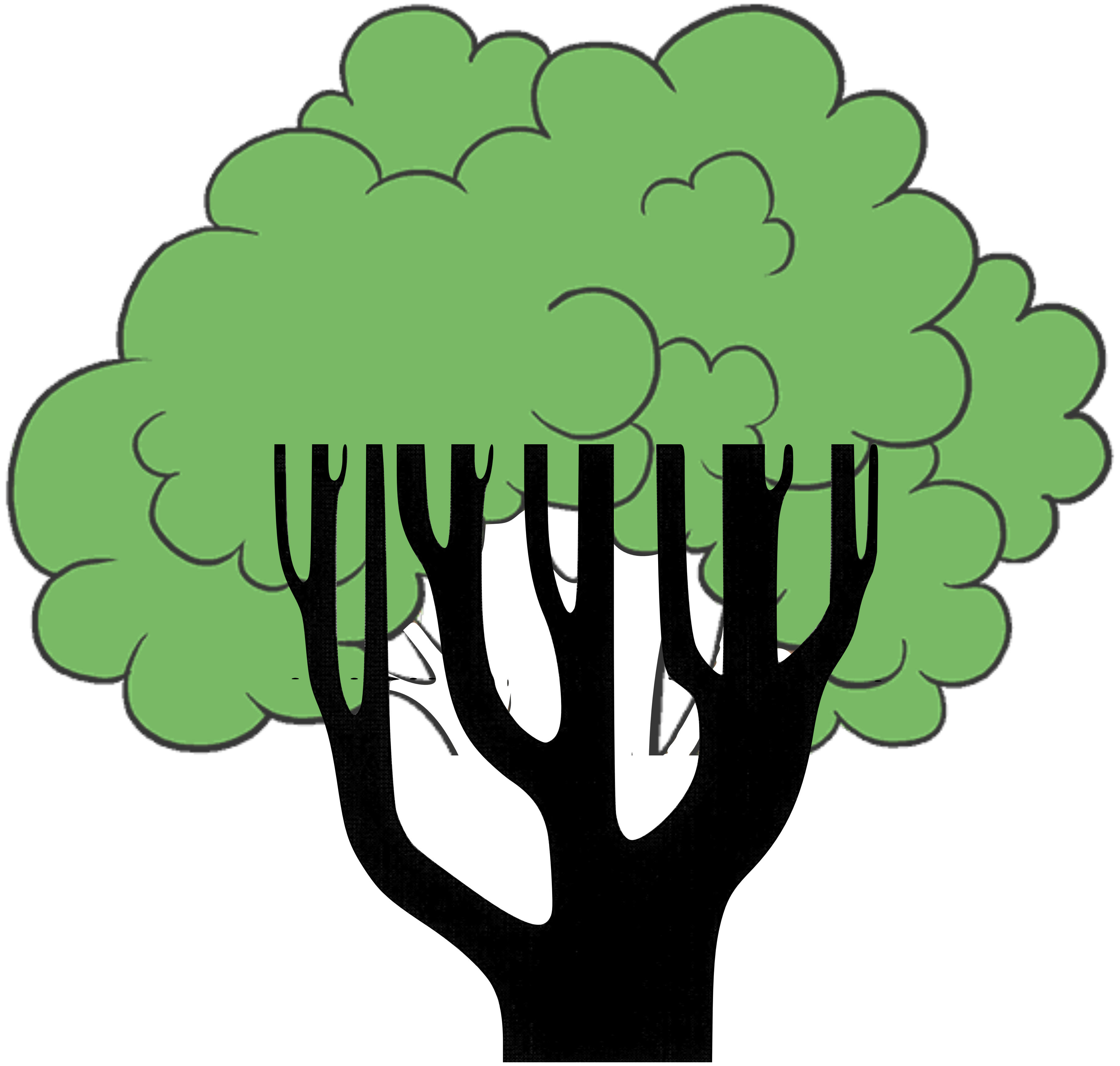


[Lapi+13]



Excursion set theory gives the halo mass function

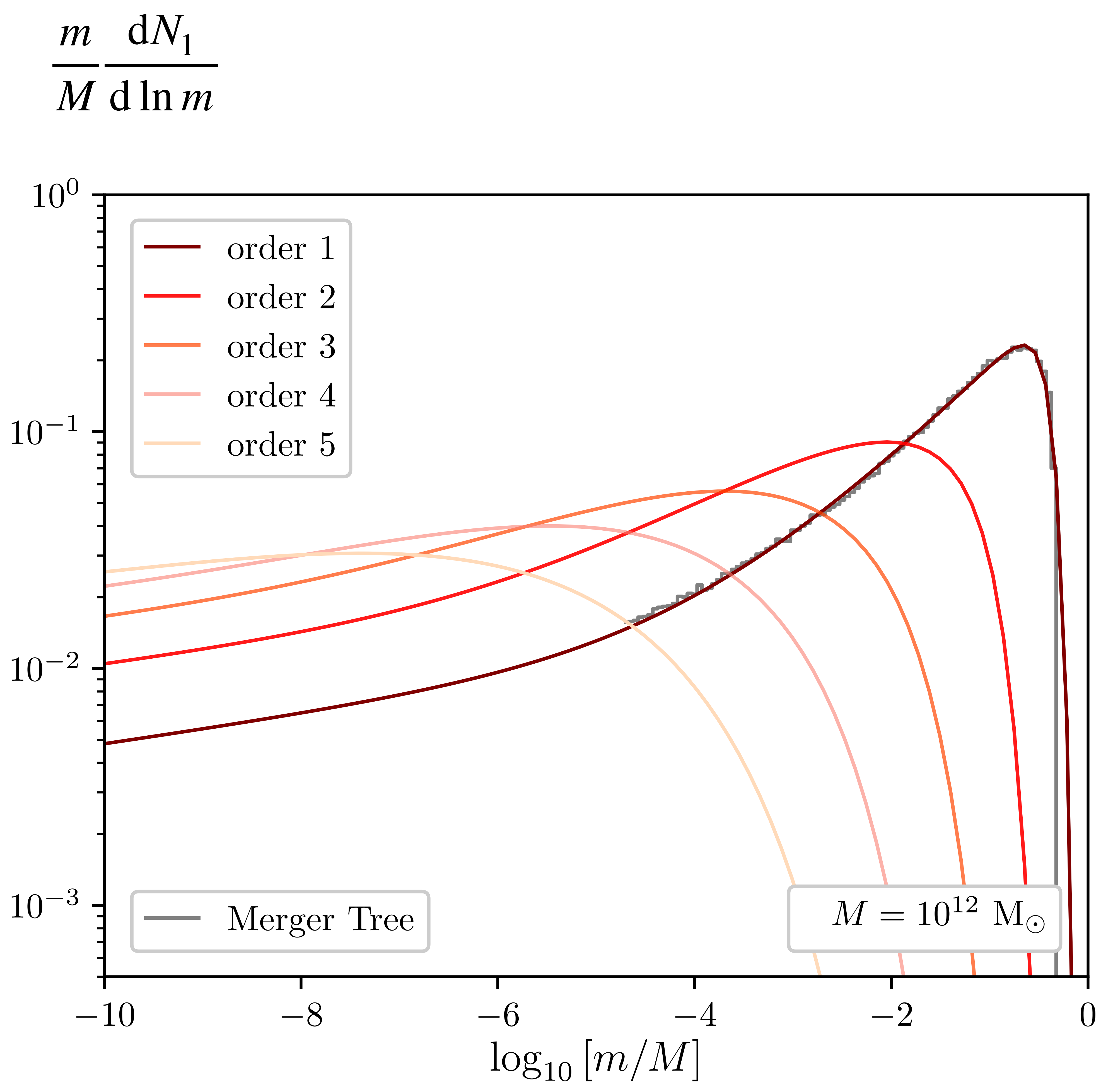
$$\frac{\mathrm{d}n(M)}{\mathrm{d}M} = \frac{\bar{\rho}_{\mathrm{m},0}}{M}\frac{\nu(M)}{2S(M)}\left|\frac{\mathrm{d}S}{\mathrm{d}M}\right| f\left[\nu(M)\right]$$



[image from Lacey+93]

Excursion set formalism + merger tree algorithms give the « cosmological » subhalo mass function

$$\frac{\mathrm{d}n(m\,|\,M)}{\mathrm{d}m}$$



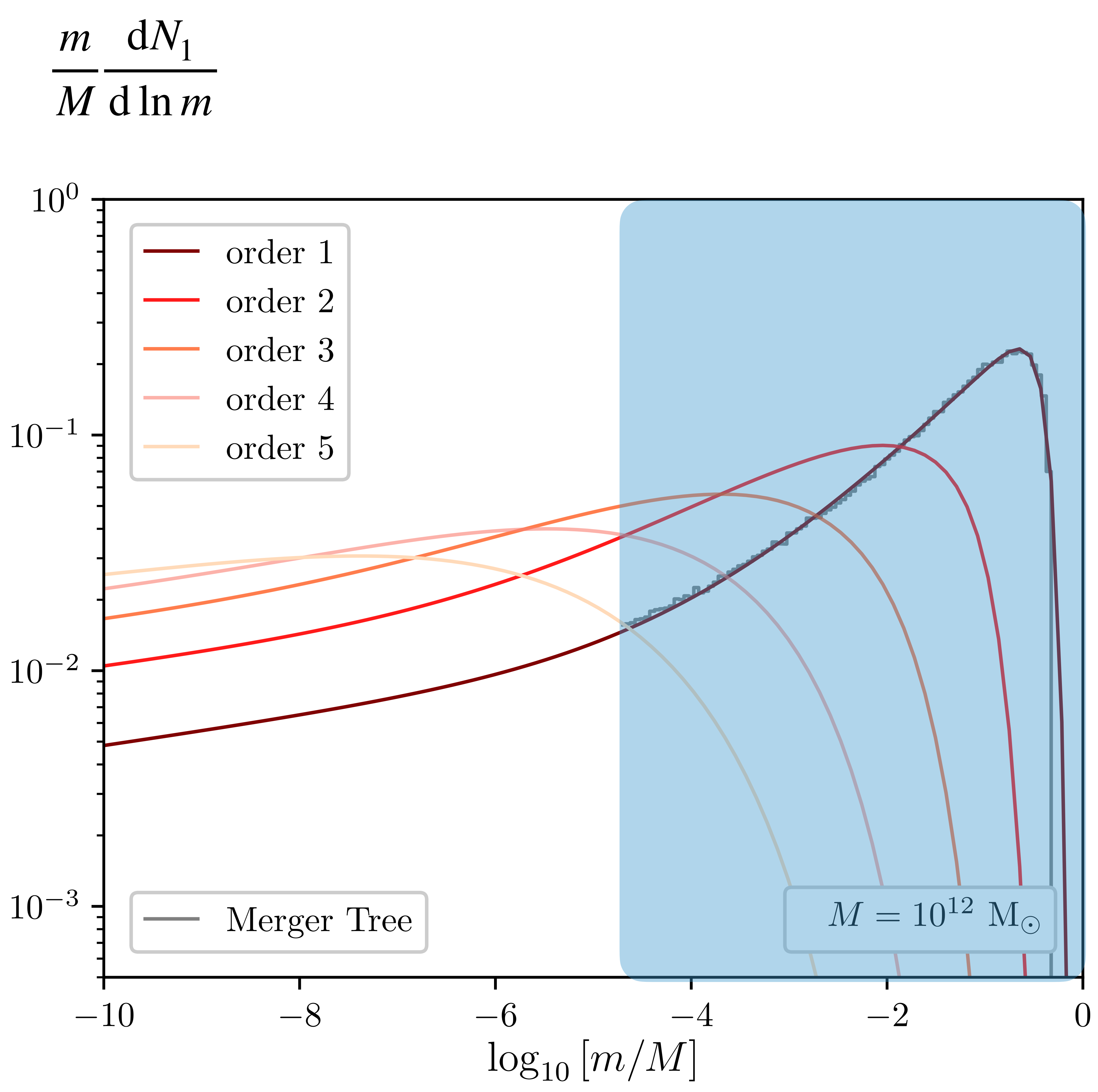
[GF and Lavalle (in prep.),
see also e.g., Jiang+14, Giocolo+08]

We fit the subhalo mass function at z=0

Run the Cole+00 algorithm gives the mass function at large mass

Fit with the function

$$f(m,M) = \frac{1}{m}\left[\sum_{i=1,2}\gamma_i\left(\frac{m}{M}\right)^{-\alpha_i}\right]\exp\left\{-\beta\left(\frac{m}{M}\right)^{\zeta}\right\}$$



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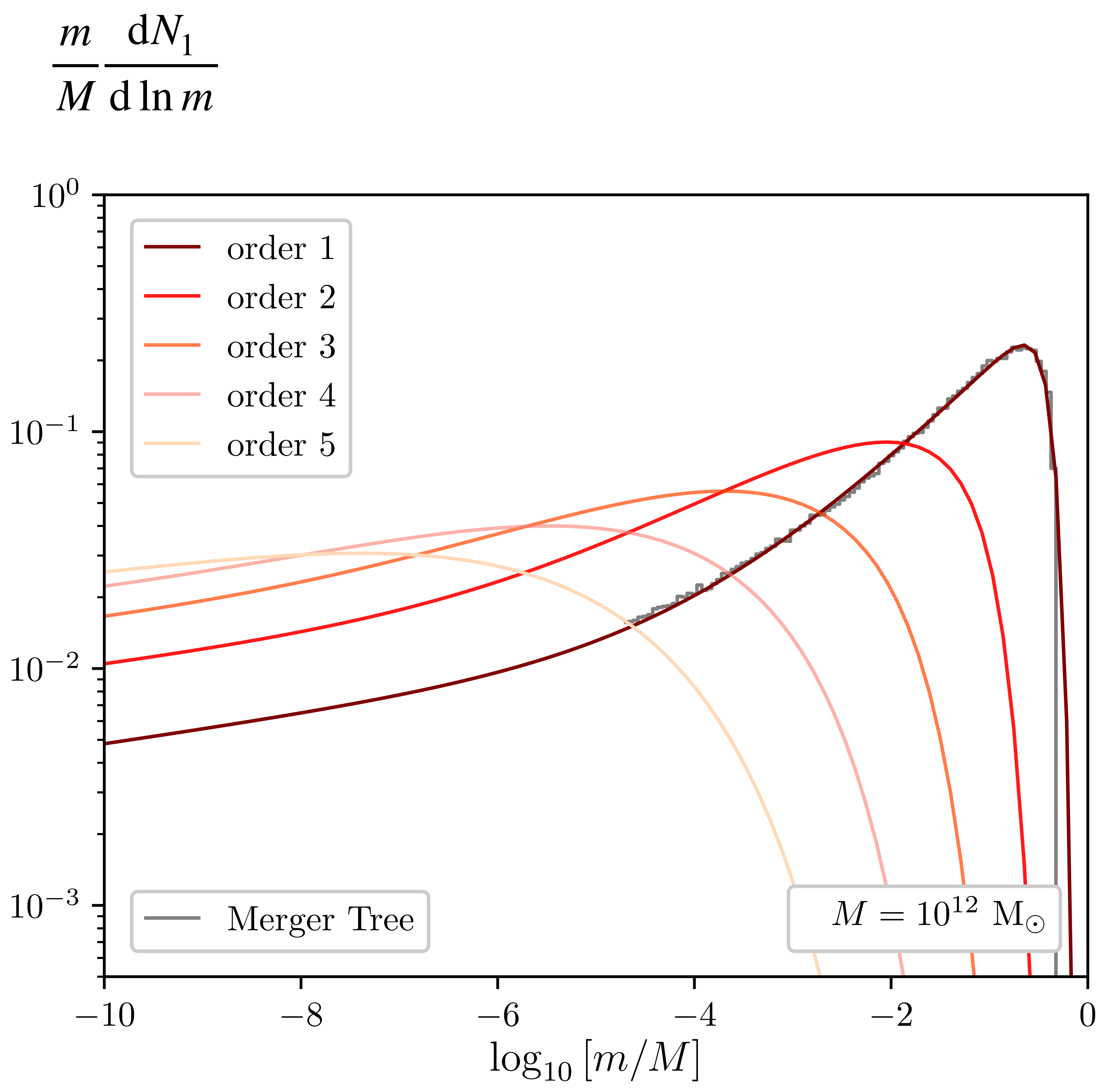
We fit the subhalo mass function at z=0

But ....

mass function at small mass inferred only from the behaviour at large mass



[GF and Lavalle (in prep.),
see also e.g., Jiang+14, Giocolo+08]

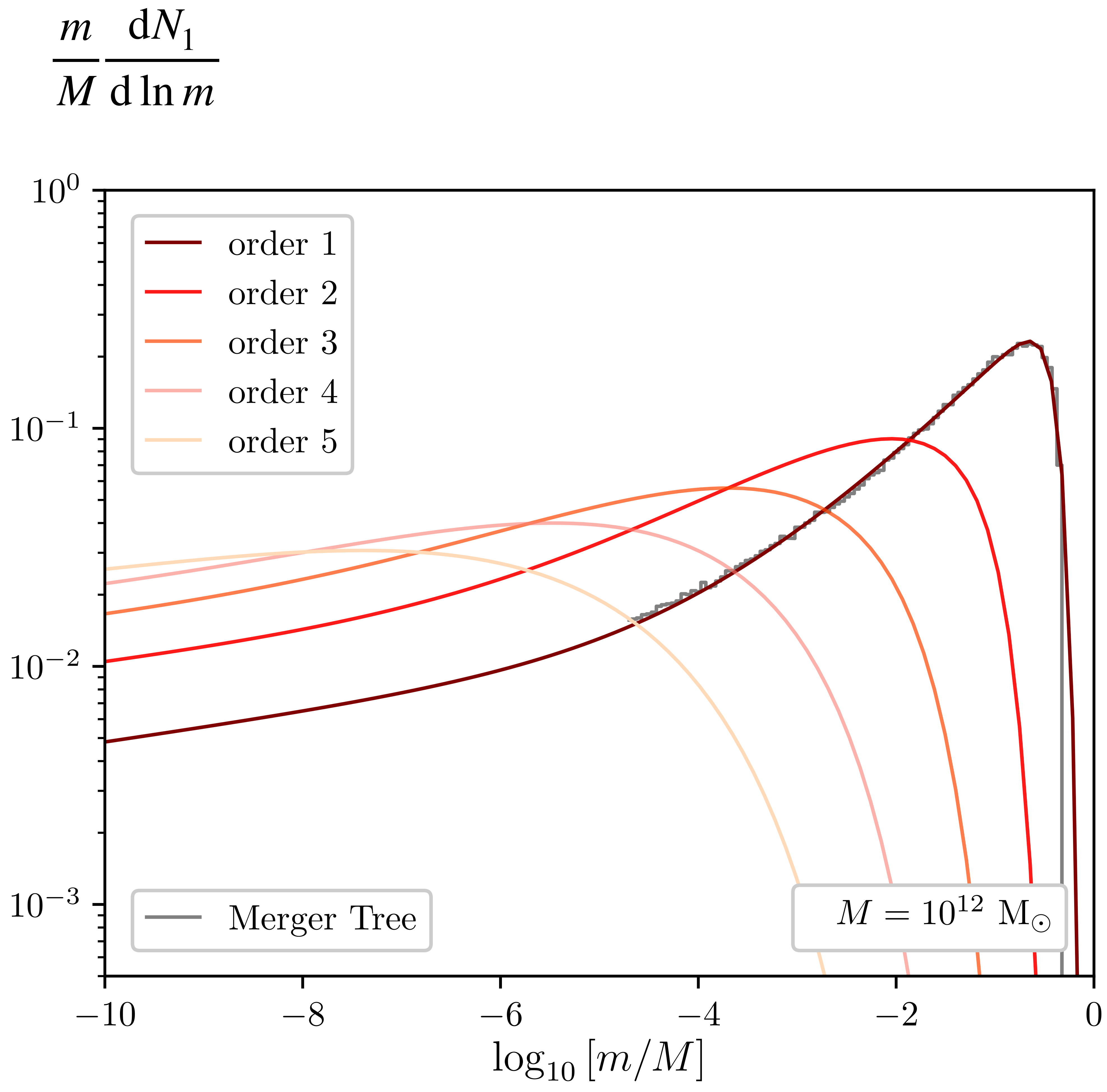
We fit the subhalo mass function at z=0

Introduce a specific fitting procedure

Constrain the fit with the condition:

$$\frac{1}{M}\int_0^M m\frac{dN_1}{dm}dm = 1$$

The host halo is entirely made of subhalos (fractal picture)



[GF and Lavalle (in prep.),
see also e.g., Jiang+14, Giocolo+08]

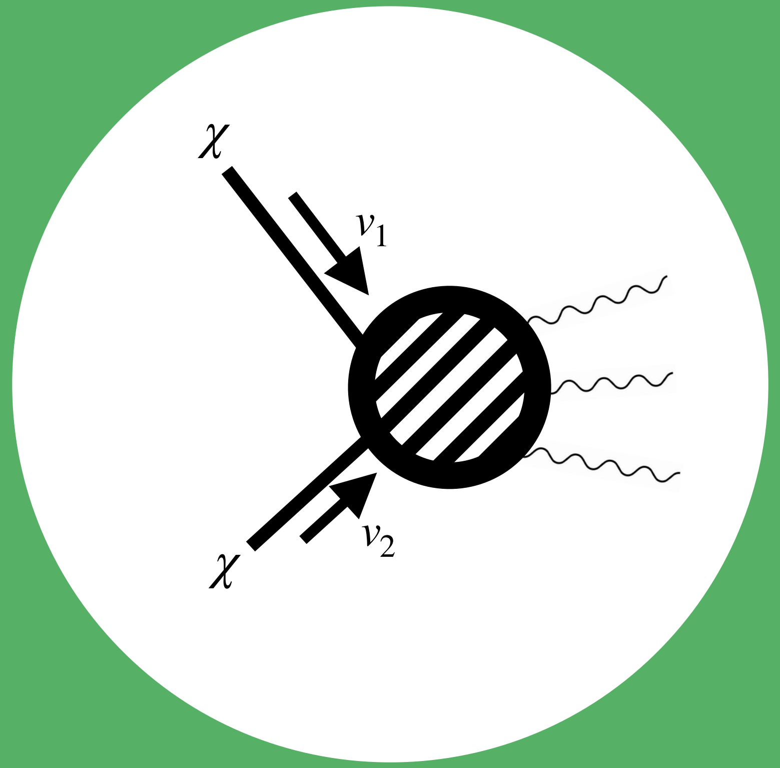
We fit the subhalo mass function at z=0

The constraint fixes the low-mass behavior

$$\frac{\mathrm{d}N_1}{\mathrm{d}m} \sim \gamma m^{-\alpha} \quad \text{with} \quad \alpha \sim 1.95$$

Analytical models are fast and easily adaptable to any host, at any redshift and to any cosmology

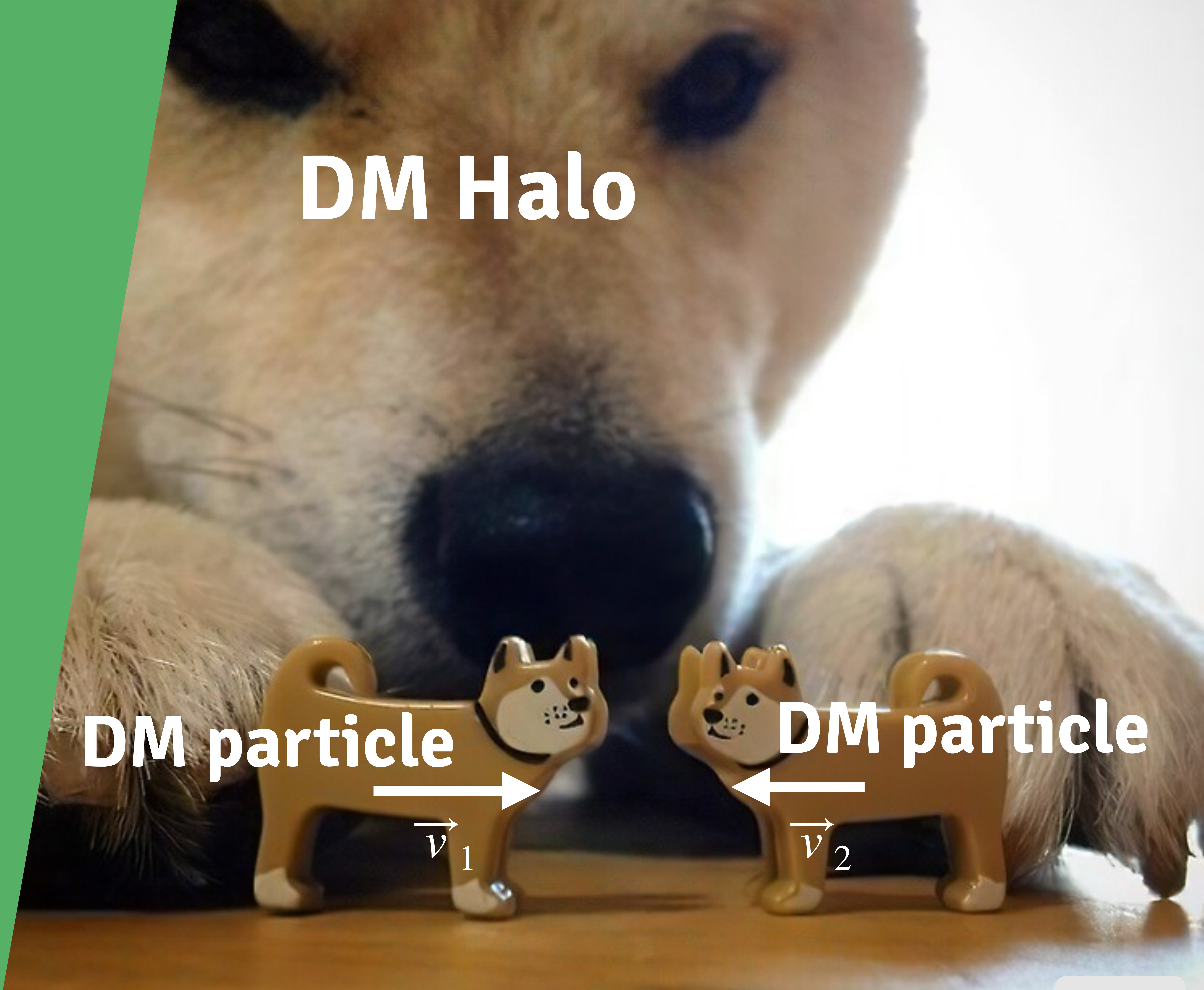
Almost the perfect tool to compute boost factors!
(drawback: theoretical uncertainties)



Part 2:

Sommerfeld enhanced and subhalo boosted J-factors

[2007.10392, 2203.16440]



We want

to classify astrophysical targets
In terms of their potential of DM detection
(using our analytical model)...

... in scenarios with
Sommerfeld enhancement
and accounting for the subhalo boost

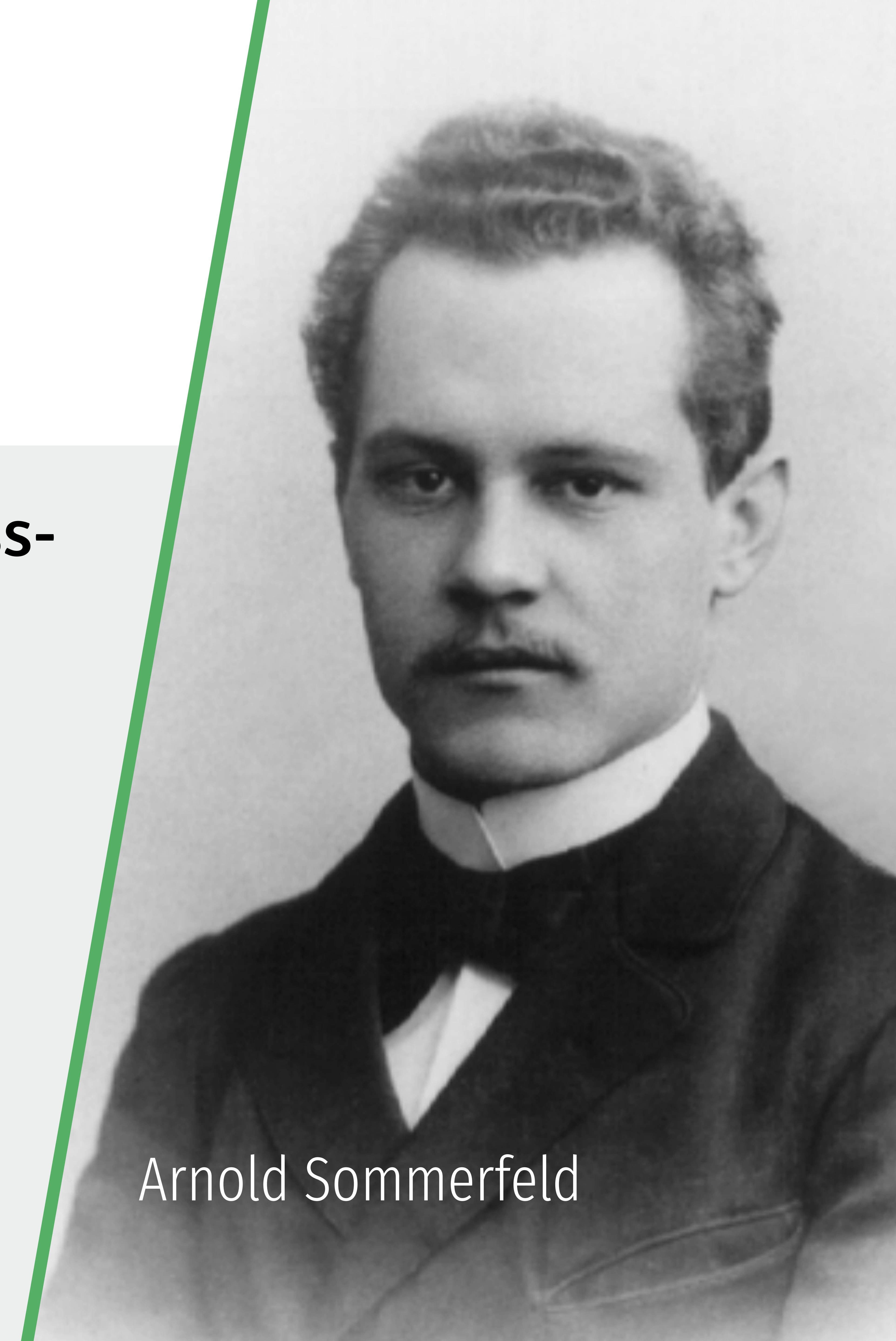
Enhancement of the cross-section

- **Light mediators ϕ can modify the cross-section in the non-relativistic limit**

Need to solve Schrodinger's equation:

$$\left[-\frac{1}{2mr}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{l(l+1)}{2mr^2}+V(r)\right]\psi_l(r)=E\psi_l(r)$$

An attractive (Yukawa) potential enhances the cross-section



Enhancement of the cross-section

$$\sigma_{\text{ann}} v_{\text{rel}} = \sigma_0 + \sigma_1 v_{\text{rel}}^2$$

$$\sigma_{\text{ann}} v_{\text{rel}} = S_0(v_{\text{rel}})\sigma_0 + S_1(v_{\text{rel}})\sigma_1 v_{\text{rel}}^2$$

- To obtain analytic results approximate the Yukawa potential by the Hulthen potential:

$$V_H(r) = -\frac{\alpha m_* e^{-m_* r}}{1 - e^{-m_* r}} \rightarrow S_l(\epsilon_\phi, \epsilon_v) \qquad m_* = \frac{\pi^2}{6} m_\phi$$

Enhancement of the cross-section

- **Two important parameters:**

$$\epsilon_\phi = \frac{m_\phi}{\alpha_\chi m_\chi}$$ Particle physics

$$\epsilon_v = \frac{v_{\rm rel}}{2\alpha_\chi}$$ Astrophysics

With the coupling strength: $\alpha_\chi \equiv \frac{g_\chi^2}{4\pi}$

Arnold Sommerfeld

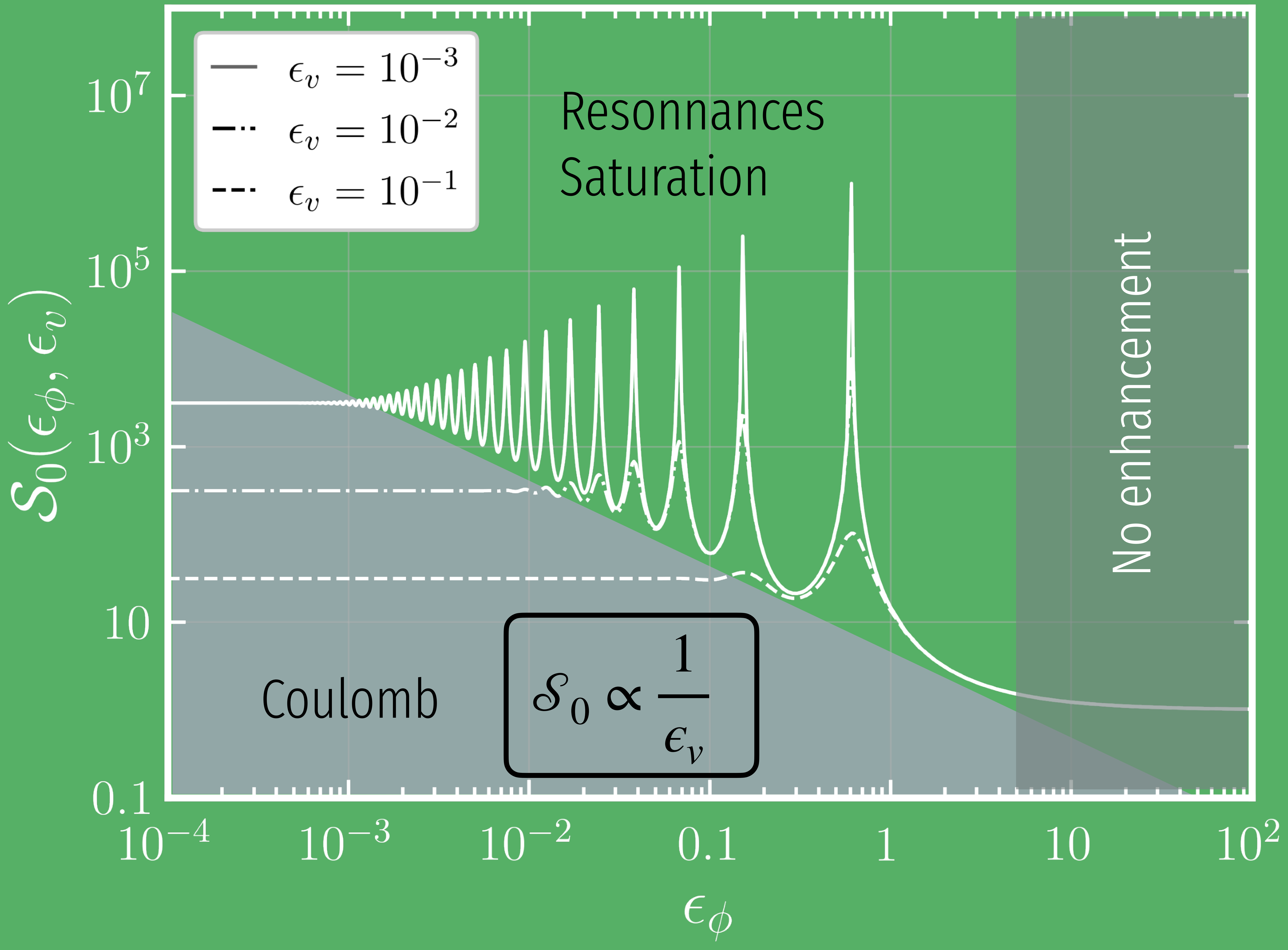
Regroup velocity dependent terms:

$$\mathcal{S}_l(\epsilon_\phi, v_{\rm rel}) \equiv v_{\rm rel}^{2l} S_l(\epsilon_\phi, \epsilon_v)$$

$$\boxed{\sigma_{\rm ann} v_{\rm rel} = \sum_{l=1,2,\ldots} \sigma_l \mathcal{S}_l(\epsilon_\phi, v_{\rm rel})}$$

$$\epsilon_\phi = \frac{m_\phi}{\alpha_\chi m_\chi}$$

$$\epsilon_v = \frac{v_{\rm rel}}{2\alpha_\chi}$$



$$\epsilon_\phi = \frac{m_\phi}{\alpha_\chi m_\chi}$$

$$\epsilon_v = \frac{v_{\rm rel}}{2\alpha_\chi}$$

What are the possible interesting targets?

What are the possible interesting targets?

Dwarf spheroidal galaxies (dSphs)

$D \sim 10$ - 100 kpc and $m \sim 10^7$ - 10^9 $M_\odot$

Sculptor
[ESO/Digitized Sky Survey 2]

What are the possible interesting targets?

Dwarf irregular galaxies (dIrrs)

$D \sim 1$ Mpc and $m \sim 10^7$ - 10^{10} $M_\odot$

What are the possible interesting **targets**?

Galaxy clusters

$D > 10$ Mpc and $m \sim 10^{14}$ - 10^{15} $M_\odot$

dSphs

[Bonnivard+15]

- Reticulum 2 (ultra-faint)
- Sculptor (classical)
- Draco (classical)

dIrrs

[Gammaldi+21]

	D [kpc]	m (=M_{200}) [10^{10} $M_\odot$]
NGC6822	480	3.16
IC10	790	3.98
WLM	970	0.40

Clusters

[Sánchez-Conde+11]

	D [Mpc]	m (=M_{200}) [10^{14} $M_\odot$]
Coma	102.18	8.77-13.16
Fornax	20.35	0.51-0.61
Perseus	80.69	5.14-7.71

From the mass distribution to the phase-space/velocity distribution

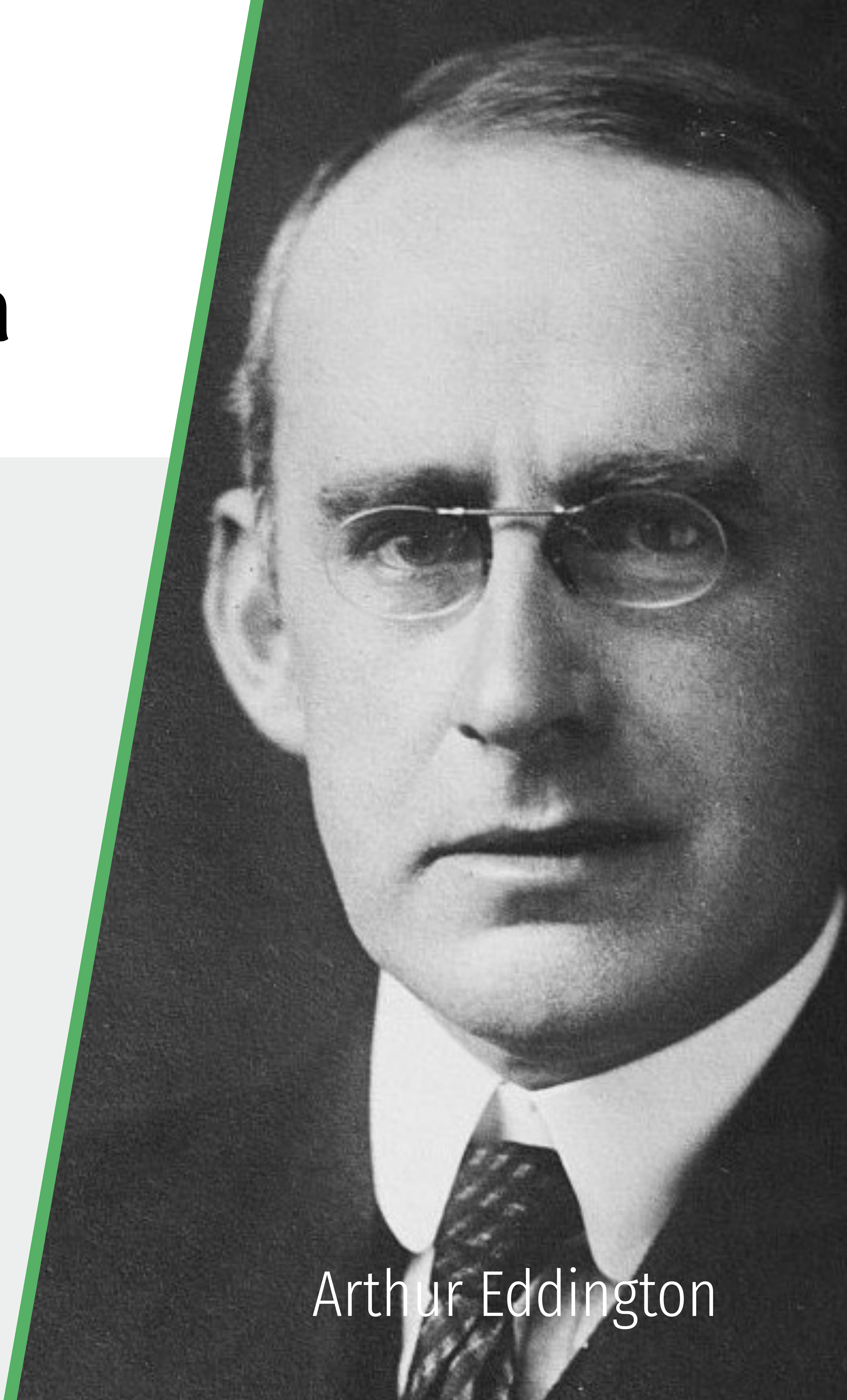
We rely on Eddington's inversion method

Assuming isotropy & spherical symmetry

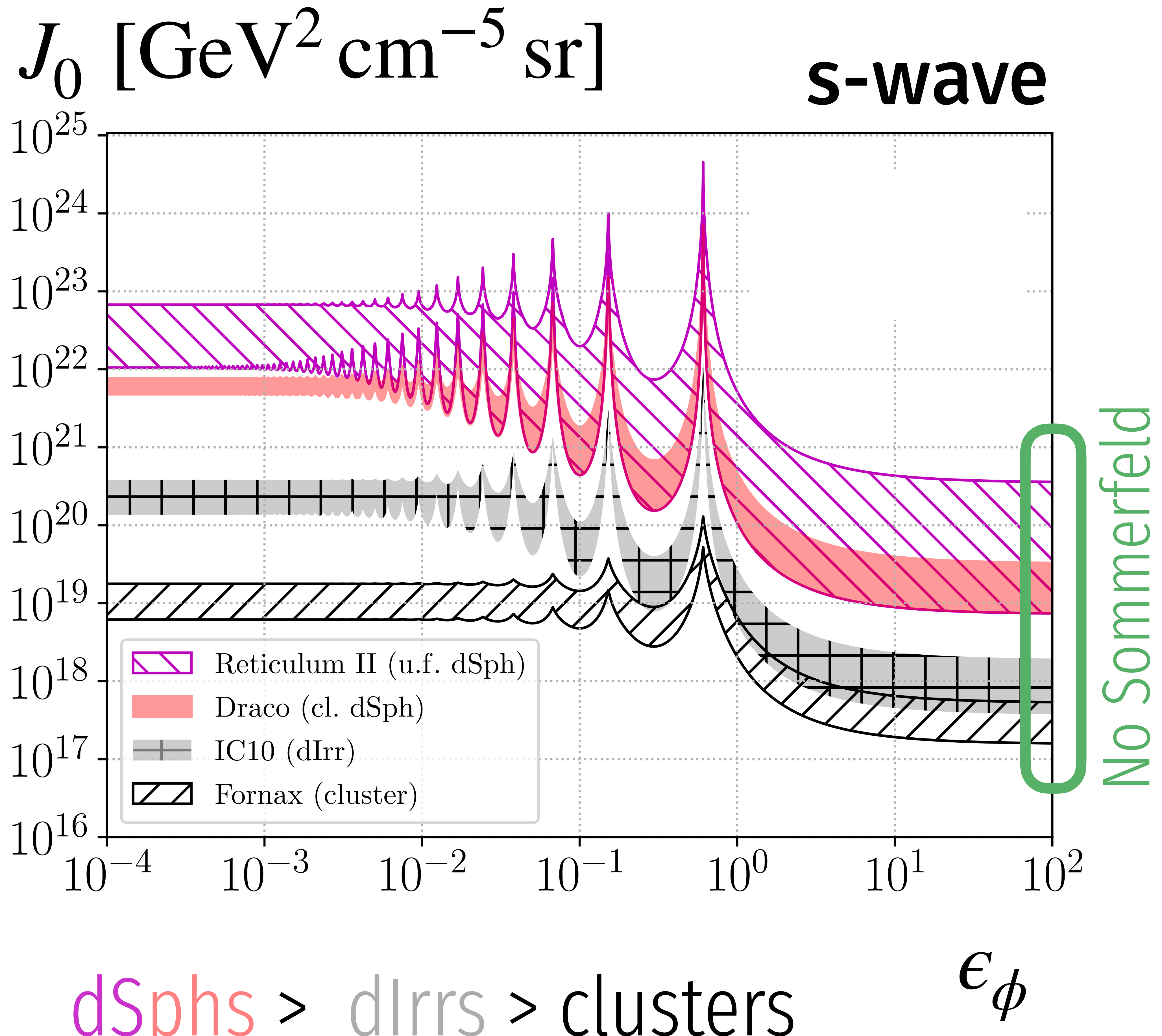
$$f(r,v) = \frac{1}{2\sqrt{2}\pi^2}\frac{\mathrm{d}}{\mathrm{d}\mathcal{E}}\int_0^{\mathcal{E}}\frac{\mathrm{d}\rho}{\mathrm{d}\Psi'}\frac{1}{\sqrt{\mathcal{E}-\Psi'}}\mathrm{d}\Psi'$$

$$\mathcal{E}(r,v) = \Psi(r) - v^2/2$$

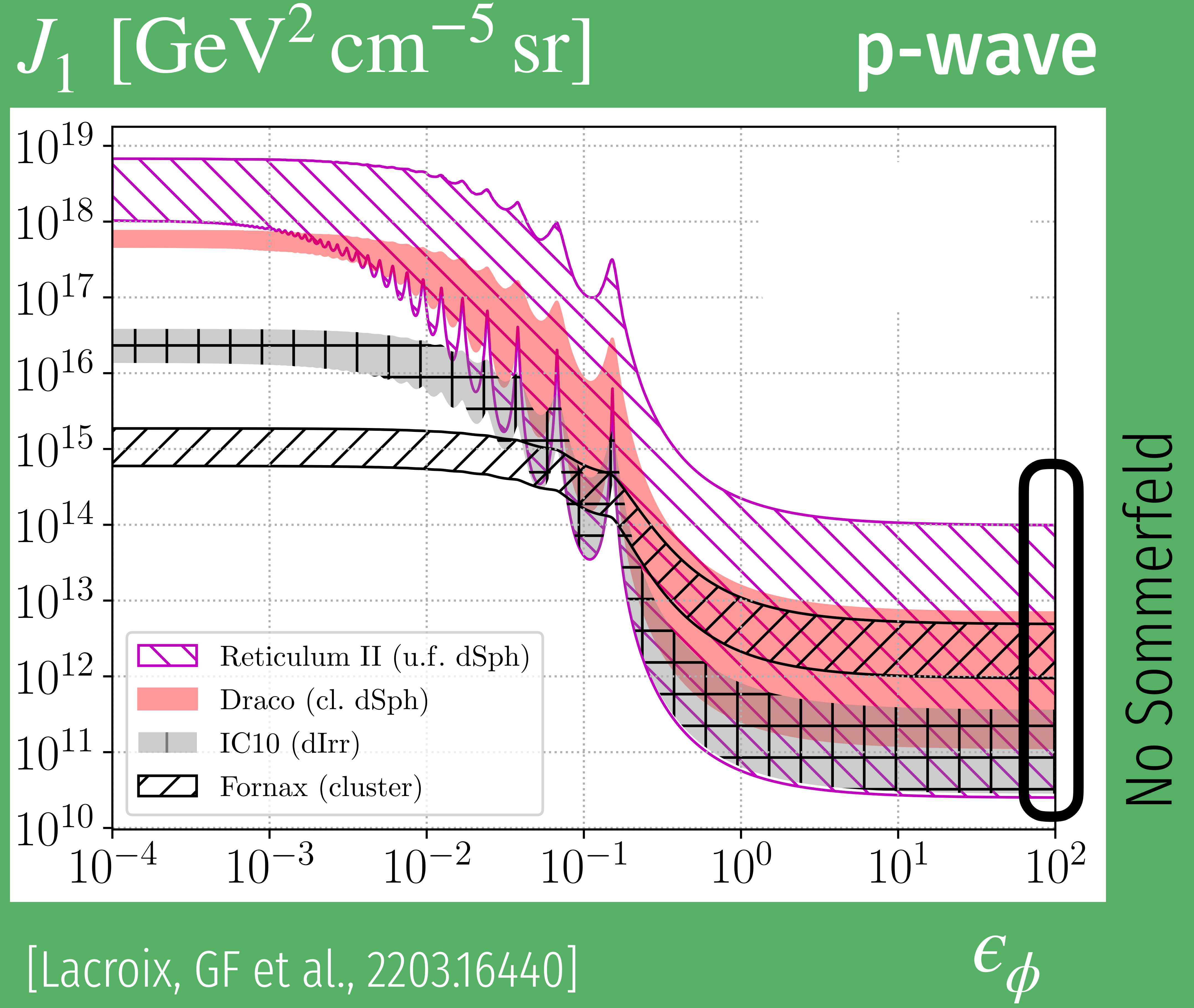
$$\Delta\Psi = -4\pi G_N \rho$$



Results for
smooth halos



dSphs > dIrrs > clusters



[Lacroix, GF et al., 2203.16440]

the smaller
the structure,
the lower
the velocity

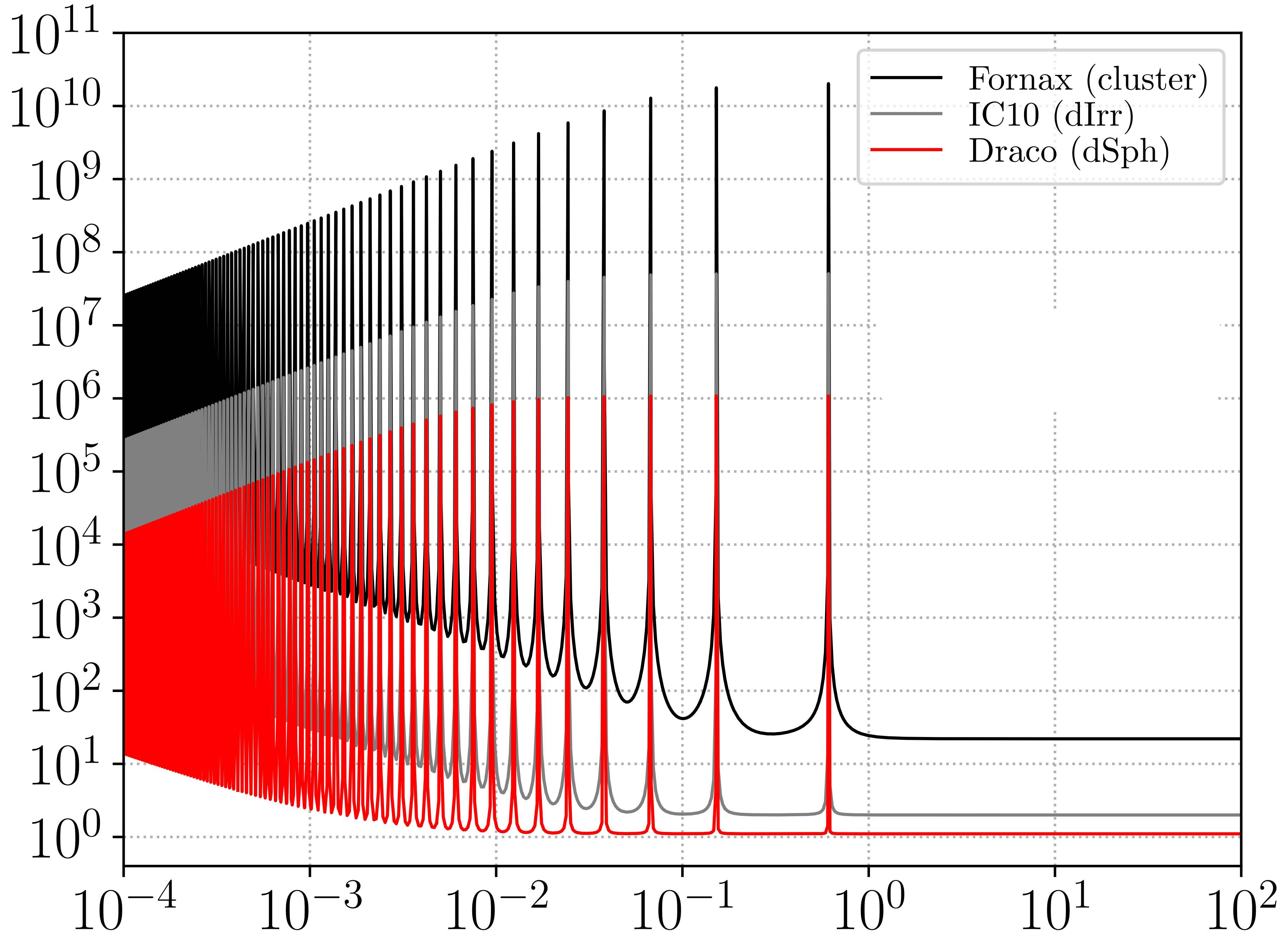
the impact of subhalos
can be very important

Boost factors

largest subhalo boost for clusters,
lower subhalo boost for dSphs

cluster
dwarf spheroidal
dwarf irregular

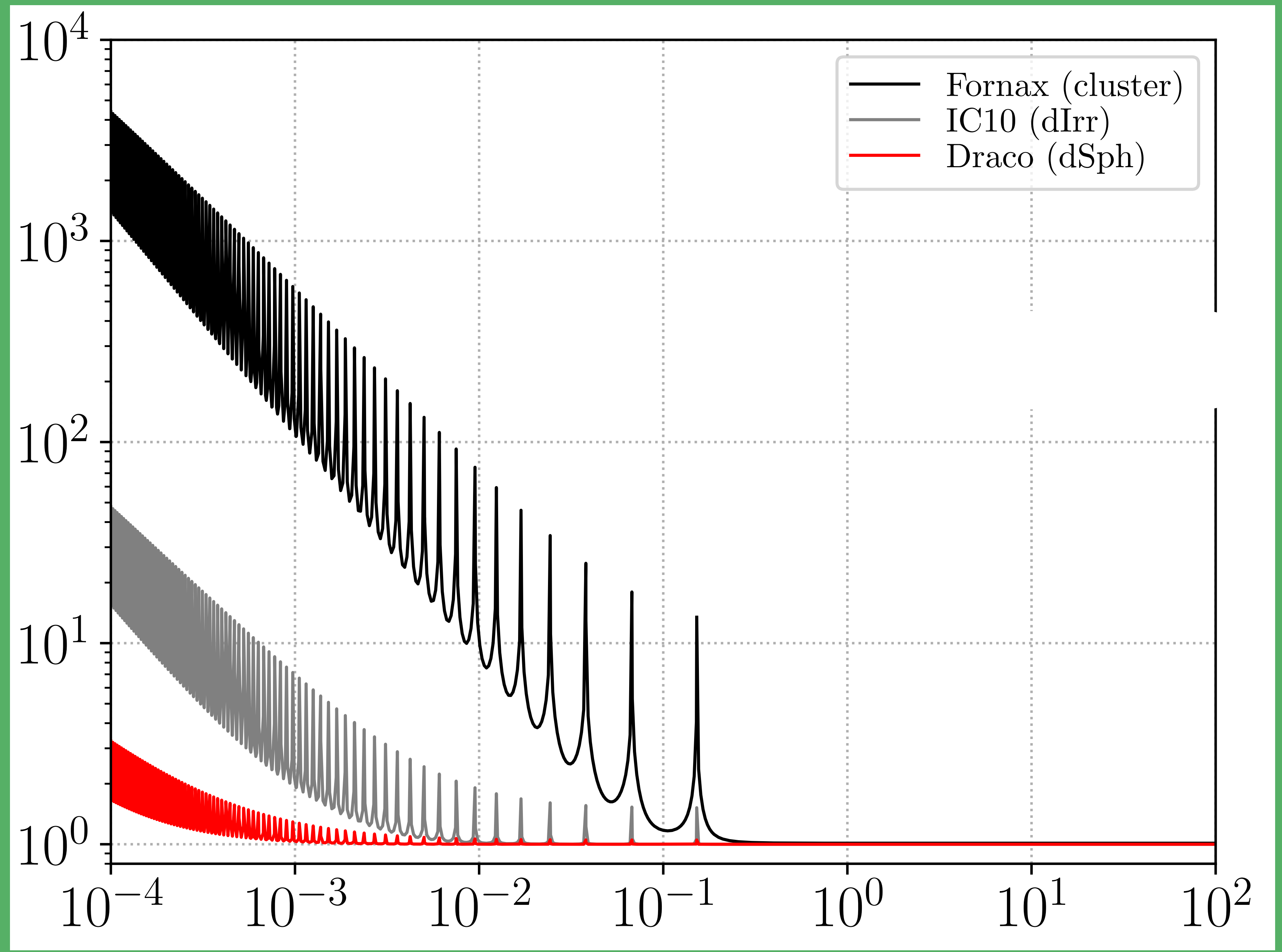
Boost **s-wave**



[Lacroix, GF et al., 2203.16440] ϵ_ϕ

p-wave

Boost



[Lacroix, GF et al., 2203.16440] ϵ_ϕ

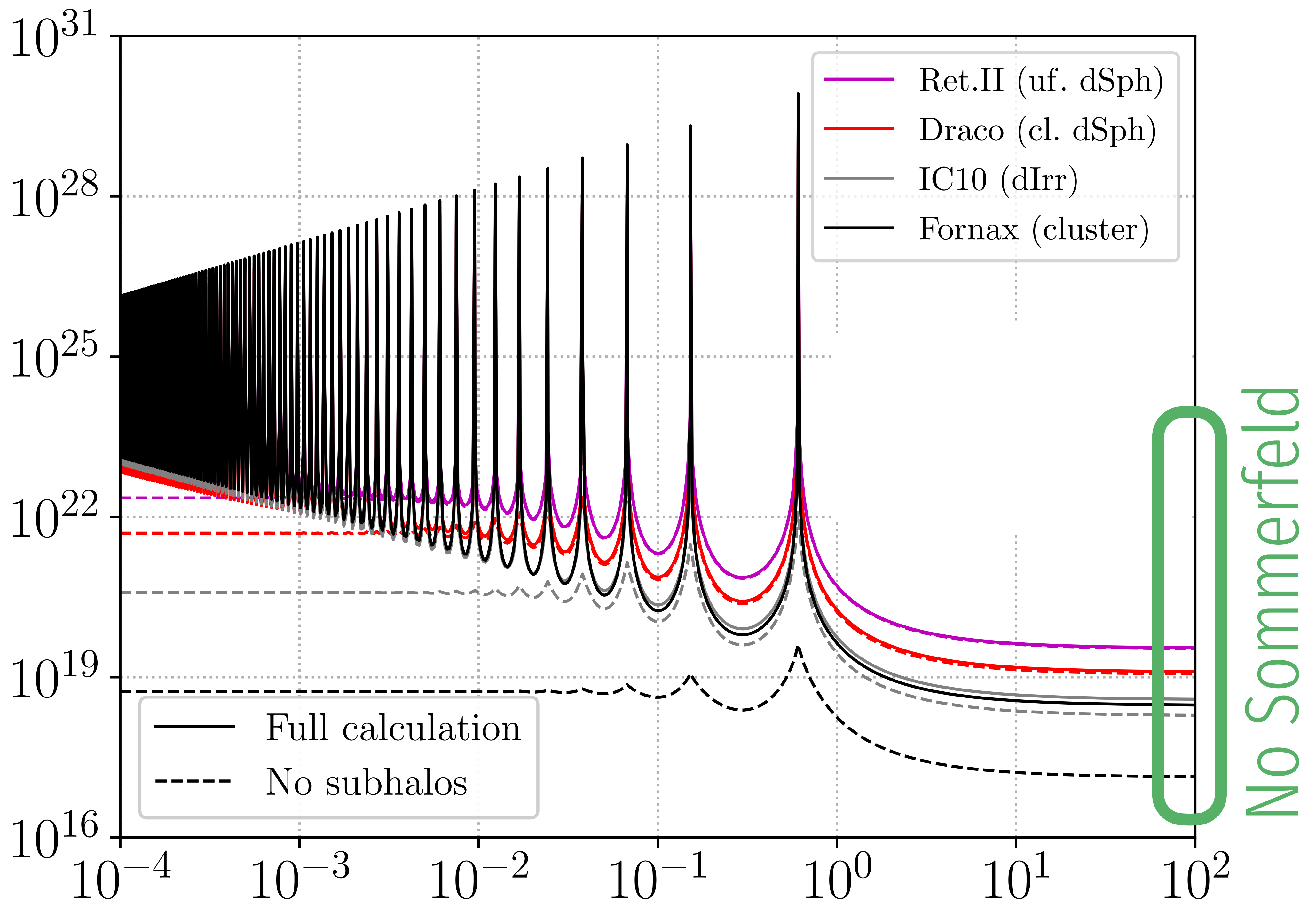
Results for full halos (with subhalos)

cluster

dwarf spheroidal

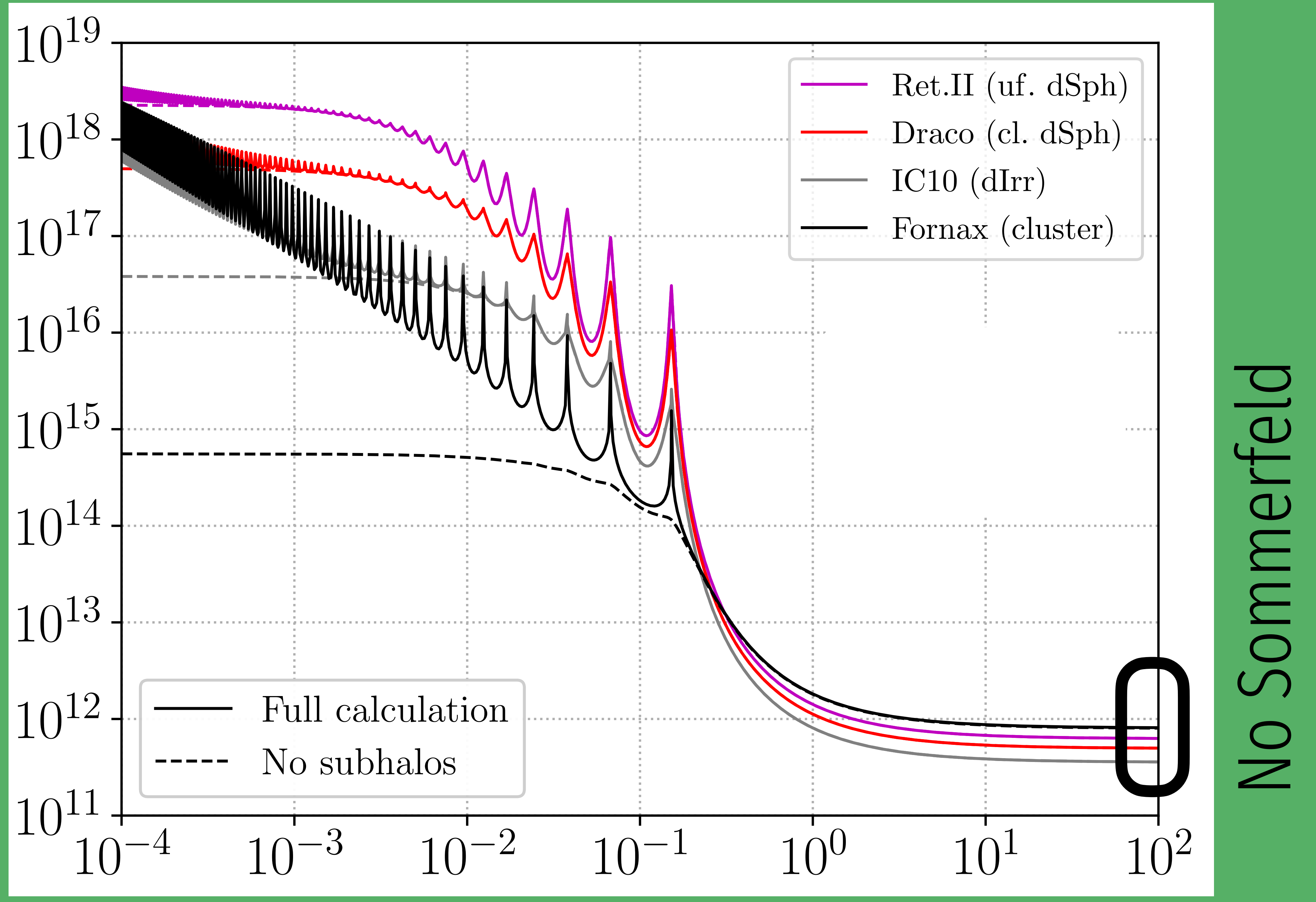
dwarf irregular

J_0 [GeV^2 cm^{-5} sr] **s-wave**



dSphs > dIrrs ~ clusters ϵ_ϕ

J_1 [GeV^2 cm^{-5} sr] **p-wave**



[Lacroix, GF et al., 2203.16440] ϵ_ϕ

Results for full halos (with subhalos)

cluster

dwarf spheroidal

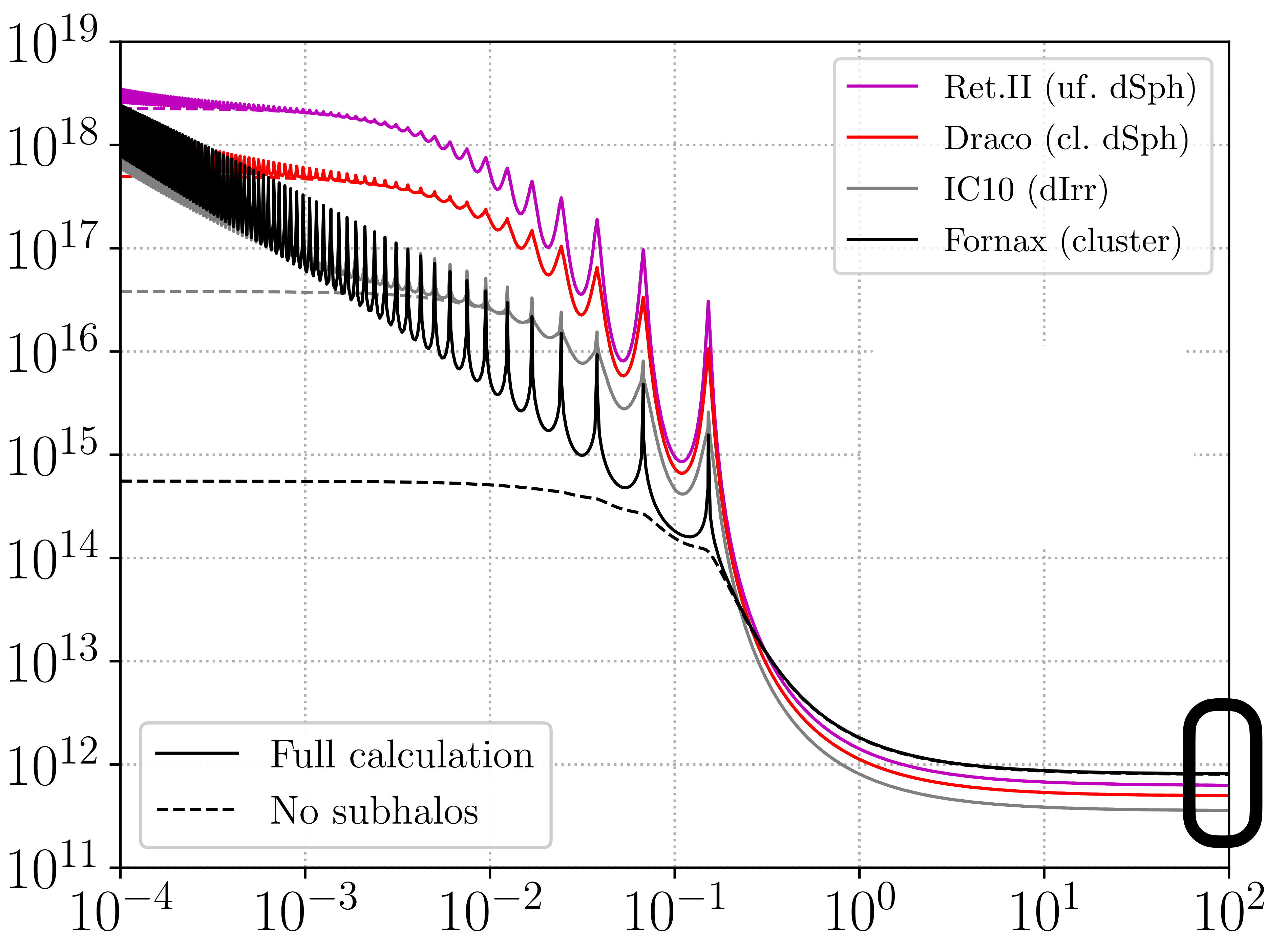
dwarf irregular

p-wave:

Different hierarchies depending on ϵ_ϕ

J_1 [GeV^2 cm^{-5} sr]

p-wave

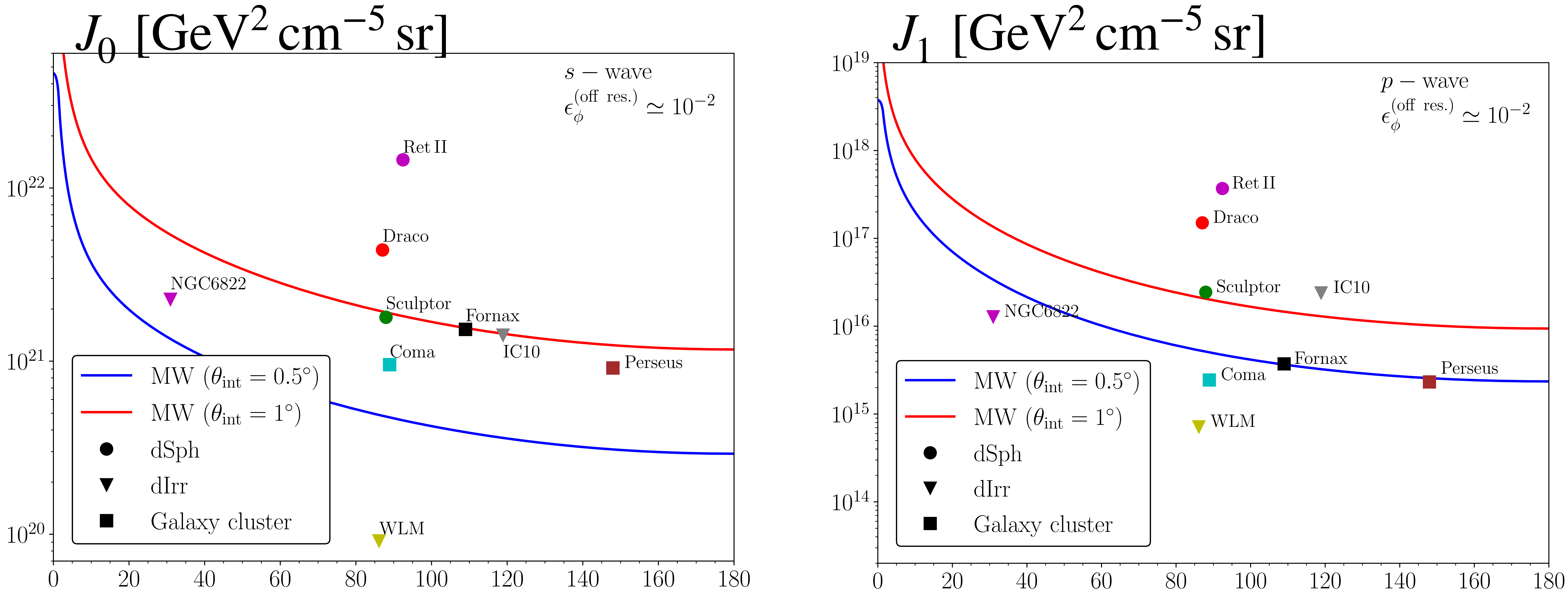


No Sommerfeld

[Lacroix, GF et al., 2203.16440]

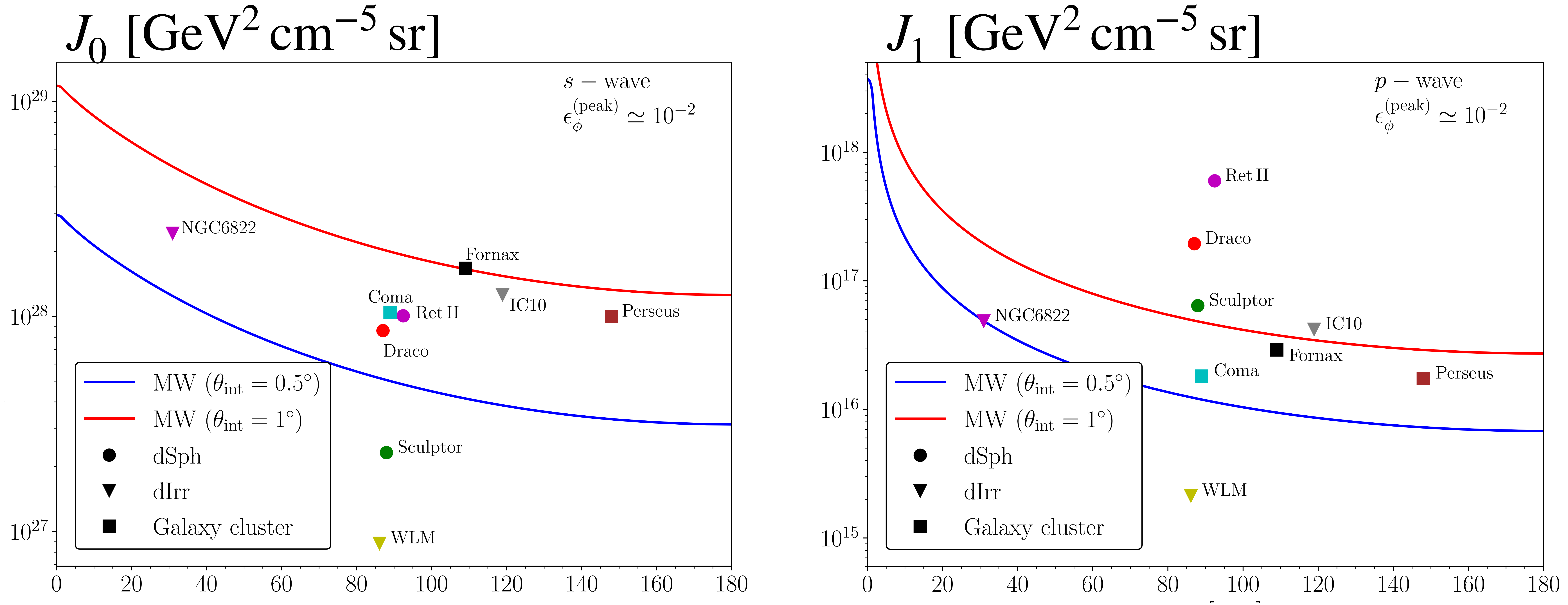
ϵ_ϕ

Comparison to the Milky-way background/foreground



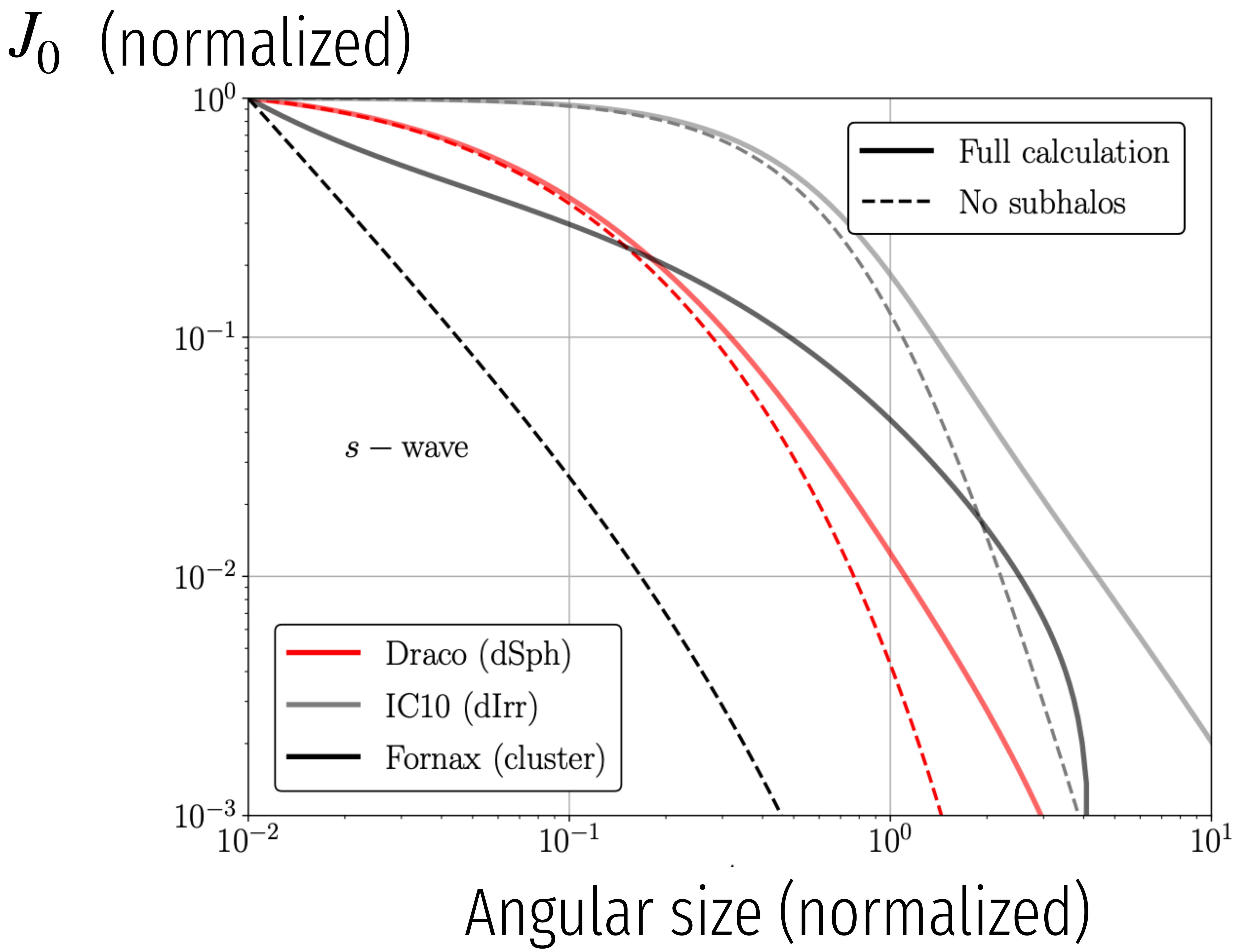
Angle from the Galactic center [deg]

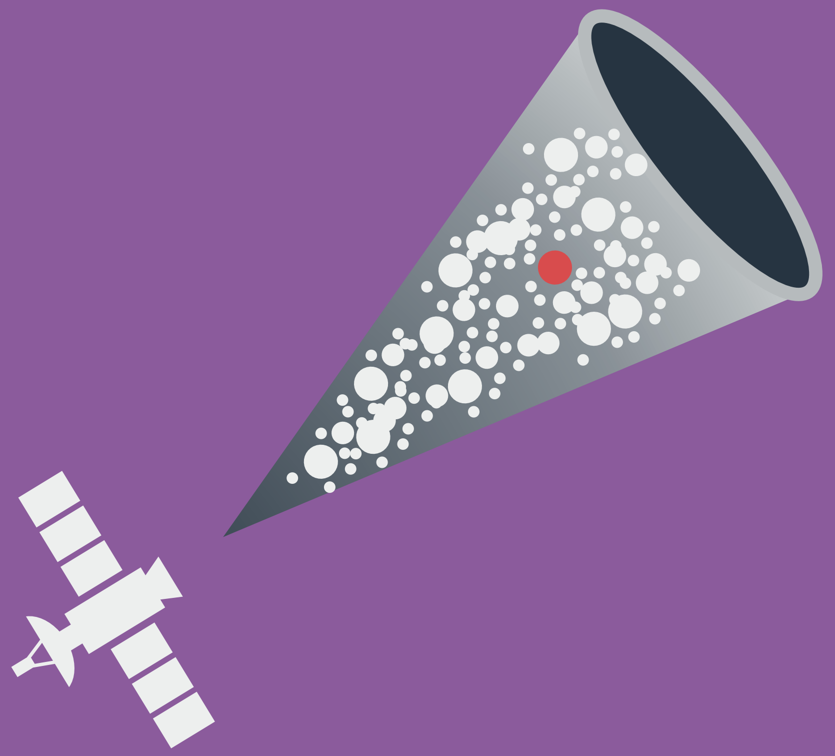
Comparison to the Milky-way background/foreground



Angle from the Galactic center [deg]

Subhalos can also enlarge the apparent size of the source



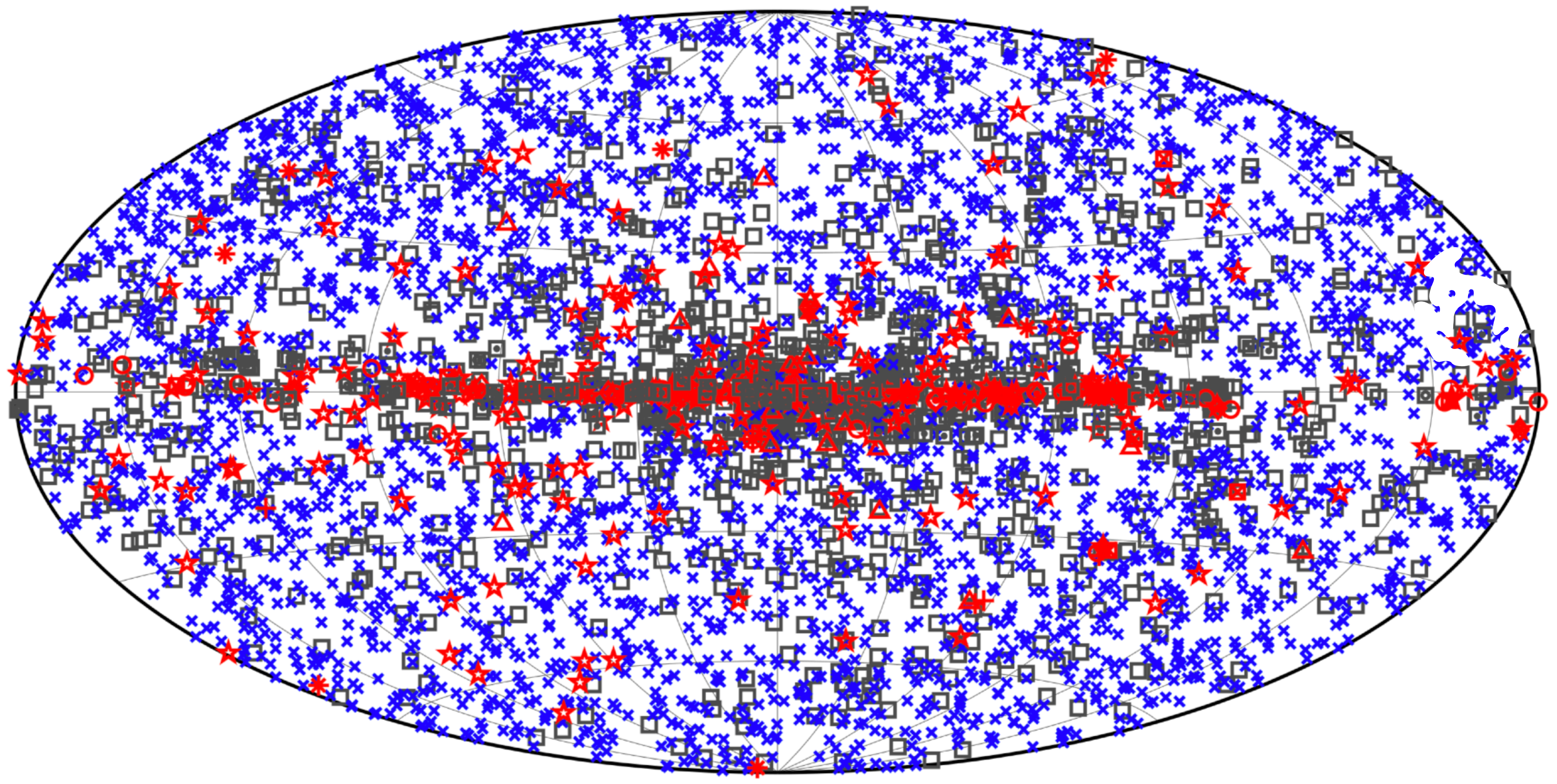


Part 3:

Subhalos as point sources in the Milky Way

[2007.1039]

Can dark matter subhalos be amongst the Fermi-LAT point sources?



[Fermi-LAT collaboration 19]

- **1525** **unassociated point sources in Fermi-LAT 4th catalog (4FGL)** [Fermi-LAT collaboration 19]
- **With our subhalo model + foreground/background model:**
  Can some of these sources be DM halos?
  Could we detect them before the diffuse Galactic component?

With our model we compute probabilities for the J-factors

- Probability to find a point-like subhalo with a J-factor above a threshold

$$\mathbb{P}\left(> J, \psi, \delta\Omega \right) = \frac{\delta\Omega}{N_{\rm sub}} \iiint_{pt\text{-}like} \mathrm{d}m_{\rm t}\mathrm{d}c\mathrm{d}s \left. \frac{\partial^2 n(m_{\rm t}, c, s)}{\partial m_{\rm t}\partial c} \right|_{\rm f} \Theta(J_i(m_{\rm t}, c, s) - J)$$

- Average number of visible subhalos:

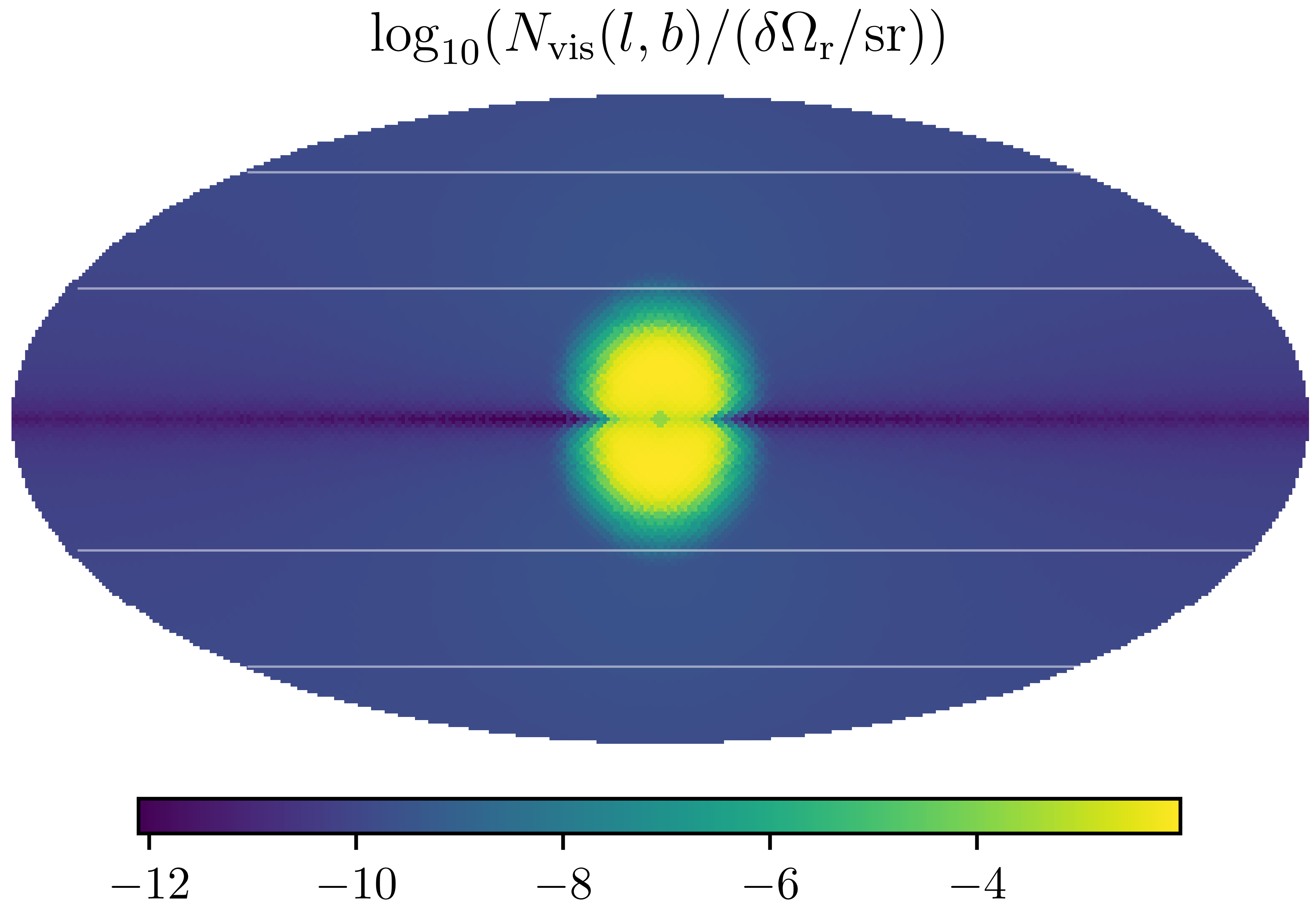
$$\langle N_{\rm vis} \rangle = N_{\rm sub} \mathbb{P}_J \left(> J_{\rm min}, \psi, \delta\Omega \right)$$

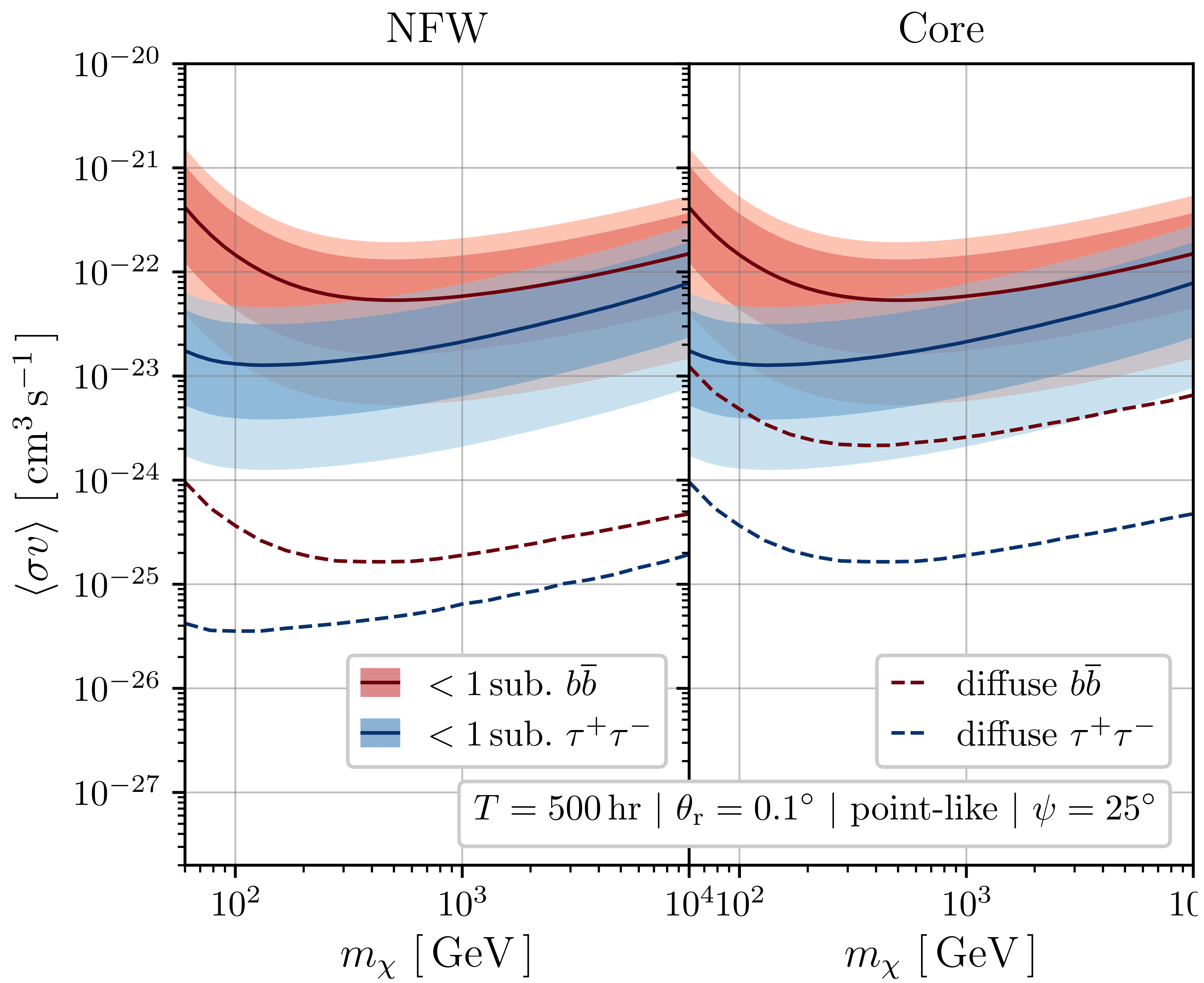
We add a background and perform a likelihood analysis

- Background model compatible with the baryonic distribution contributing to tidal stripping of the subhalos

- Likelihood analysis and mock data to find the sensitivity to the diffuse halo and to subhalos (for Fermi-LAT and CTA)

Most « visible » sources are around the galactic center





For CTA and Fermi-LAT it is improbable to detect a subhalo before the diffuse emission

(better chances if the MW halo is cored)

Conclusions

- Subhalos modify/boost possible signals from dark matter annihilation
- Subhalos give a very significant boost with Sommerfeld enhancement (because smaller typical velocity)
- Subhalos could be searched as individual sources (even if hard to detect)

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