ALP Dark Matter

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N. Fonseca, E. Morgante, RS, G. Servant, 1911.08472, JHEP 04 (2020) 010
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E. Morgante, W. Ratzinger, RS, B.A. Stefanek, 2109.13823, JHEP 12 (2021) 037
C. Eröncel, RS, G. Servant, P. Sørensen, 2206.14259, JCAP 10 (2022) 053

2023. 1. 10 @ 19th Rencontres du Vietnam : TMEX-2023

Axion fragmentation

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ALP : Axion-like particle

Axion field : ϕ

- Shift symmetry (NG boson) + $\phi G_{\mu\nu}\tilde{G}^{\mu\nu}$-type coupling w/ gauge fields

$$\phi \rightarrow \phi + \delta\phi$$

$$\frac{1}{f}\phi G_{\mu\nu}\widetilde{G^{\mu\nu}}$$

Photon,
Gluon,
Hidden gauge bosons.,
…

⇩

- Shift symmetry breaking
  by strong dynamics

$$V(\phi) = \Lambda_b^4 \cos\frac{\phi}{f}$$

Axion-like particle

- Light and stable spin-0 particle is predicted from $\Lambda_b \ll f$.

ALP mass $$m_a = \sqrt{V''} = \frac{\Lambda_b^2}{f}$$

ALP lifetime $$\tau_a \propto \frac{f^2}{m_a^3} = \frac{f^5}{\Lambda_b^6}$$

Axion-like particle & cosmology

Theoretical motivation, interesting phenomenology, …

- Strong CP problem, QCD axion
- Naturalness of electroweak scale, Relaxion
- Axion monodromy
- Axion inflation
- …

Dynamics of axion field is interesting

- Axion & ALP dark mattter
- Relaxion : dynamical expanation of electroweak scale
- …

⇩

Solving EOM $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$ with some initial condition

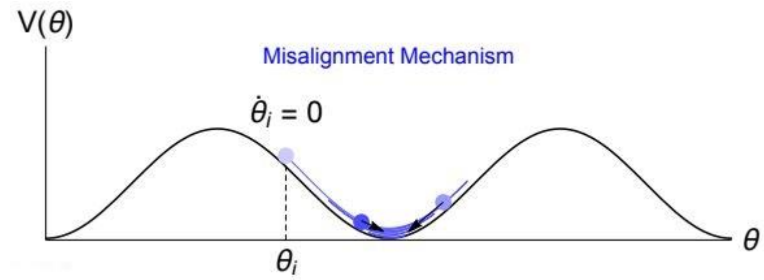
ex) Axion-like particle DM scenario

- Misalignment mechanism

[Preskill, Wise, Wilczek (1983)]
[Abbott, Sikivie (1983)]
[Dine, Fischler (1983)]

Initial condition
$$\phi = \phi_0 \neq 0$$
$$\dot{\phi} = 0$$

EOM
$$\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f} \sin\frac{\phi}{f} = 0$$



[taken from Co, Hall, Harigaya (2019)]

The axion starts to oscillate when $3H(T) \sim m(T)$

$$\Rightarrow \quad \rho_{DM} \sim m_a \times \left(\frac{a(T_{osc})}{a_0}\right)^3 \times \frac{\Lambda_b(T_{osc})^4 \theta_i^2}{m_a(T_{osc})}$$

w/ $m_a(T_{osc}) \sim 3H(T_{osc})$

mass   Dilution factor   Number density at $T = T_{osc}$

ex) Axion-like particle DM scenario

- Misalignment mechanism

[Preskill, Wise, Wilczek (1983)]
[Abbott, Sikivie (1983)]
[Dine, Fischler (1983)]

Initial condition
$$\phi = \phi_0 \neq 0$$
$$\dot{\phi} = 0$$

What happens if $\dot{\phi} > \Lambda_b^2$?

[taken from Co, Hall, Harigaya (2019)]

EOM
$$\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f}\sin\frac{\phi}{f} = 0$$

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w/ $m_a(T_{osc}) \sim 3H(T_{osc})$

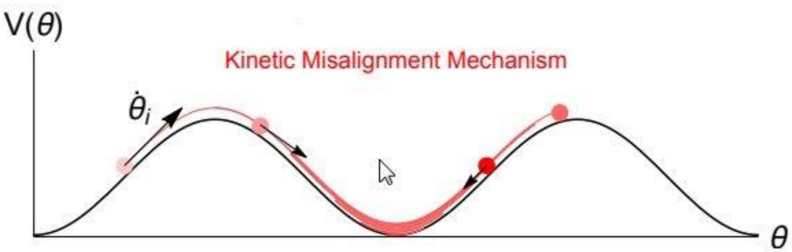
mass   Dilution factor   Number density at $T = T_{osc}$

ex) Axion-like particle DM scenario

- Kinetic Misalignment mechanism [Co, Hall, Harigaya (2019)] [Chang, Cui (2019)]

Initial condition $\dot{\phi} > \Lambda_b^2$

EOM $\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f} \sin\frac{\phi}{f} = 0$



[taken from Co, Hall, Harigaya (2019)]

The axion starts to oscillate when $\dot{\phi}^2(T) \sim \Lambda_b^4(T)$

$$\rho_{DM} \sim m_a \times \left(\frac{a(T_{osc})}{a_0}\right)^3 \times \frac{\Lambda_b(T_{osc})^4}{m_a(T_{osc})}$$

w/ $\dot{\phi}^2(T_{osc}) \sim \Lambda_b^4(T_{osc})$

mass — Dilution factor — Number density at $T = T_{osc}$

Delay of onset of oscillation → larger ρ_{DM}

Axion fluctuation?

What people usually do

Solving EOM for spatially homogeneous field : $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

However…

Even we start from (almost) homogeneous field configuration,
fluctuations can grow later.

Velocity as U(1) charge

Velocity $\dot{\phi}$ is U(1) charge : $\rho_{\text{shift}} = f \dfrac{\partial L}{\partial_0 \phi} = f \dot{\phi}$

$\phi \to \phi + f\,\delta$

Shift transf.

Explicit breaking of U(1) : $V(\phi) = \Lambda_b^4 \cos \dfrac{\phi}{f} + \cdots$

⇒ U(1) charge will be lost = energy dissipation

Axion fragmentation

[Fonseca, Morgante, RS, Servant (2019)]

For related earlier works, see
[Green, Kofman, Starobinsky (1998)]
[Flauger, McAllister, Pajer, Westphal, Xu (2009)]
[Jaeckel, Mehta, Witkowski (2016)]
[Arvanitaki, Dimopoulos, Galanis, Lehner, Thompson, Van Tilburg (2019)]

1. Introduction
2. Perturbative analysis
3. Non-perturbative analysis
4. Application

EOM of axion

Let us investigate the simplest case.

- $H = 0$ (no cosmic expansion)
- $V(\phi) = \Lambda_b^4 \cos(\phi/f)$

We have only three parameters :

$\dot{\phi}_0$: initial velocity
f : decay constant
Λ_b^4 : height of barrier

EOM of axion :

$$\frac{d^2\phi}{dt^2} - \nabla^2\phi - \frac{\Lambda_b^4}{f}\sin\frac{\phi}{f} = 0$$

EOM of axion

We decompose $\phi(\vec{x},t) = \bar{\phi}(t) + \left[\int \frac{d^3k}{(2\pi)^3}\delta\phi_k(t)e^{ikx} + h.c.\right]$

EOM of axion :

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At the leading order of $\delta\phi_k$,

$$\frac{d^2\bar{\phi}}{dt^2} - \frac{\Lambda_b^4}{f} \sin\frac{\bar{\phi}}{f} = \underbrace{\frac{1}{2}\frac{\Lambda_b^4}{f^3} \sin\frac{\bar{\phi}}{f} \int \frac{d^3x}{V_{vol}} \langle \delta\phi(x) \rangle^2}_{\text{Back reaction}}$$

$$\frac{d^2\delta\phi}{dt^2} - \nabla^2\delta\phi - \frac{\Lambda_b^4}{f^2} \cos\frac{\bar{\phi}}{f} \delta\phi = 0$$

EOM of axion

We decompose $\phi(\vec{x},t) = \bar{\phi}(t) + \left[\int \frac{d^3k}{(2\pi)^3}\delta\phi_k(t)e^{ikx} + h.c.\right]$

At the leading order of $\delta\phi_k$,

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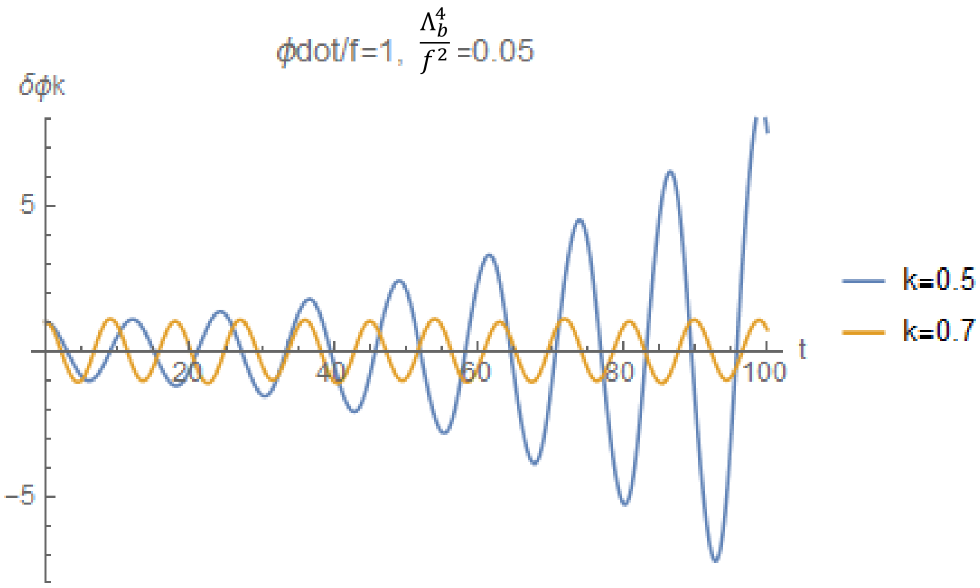
$$\frac{d^2\delta\phi_k}{dt^2} + \left(k^2 - \frac{\Lambda_b^4}{f^2}\cos\frac{\dot{\bar{\phi}}t}{f}\right)\delta\phi_k = 0$$

Mathieu equation

EOM of axion



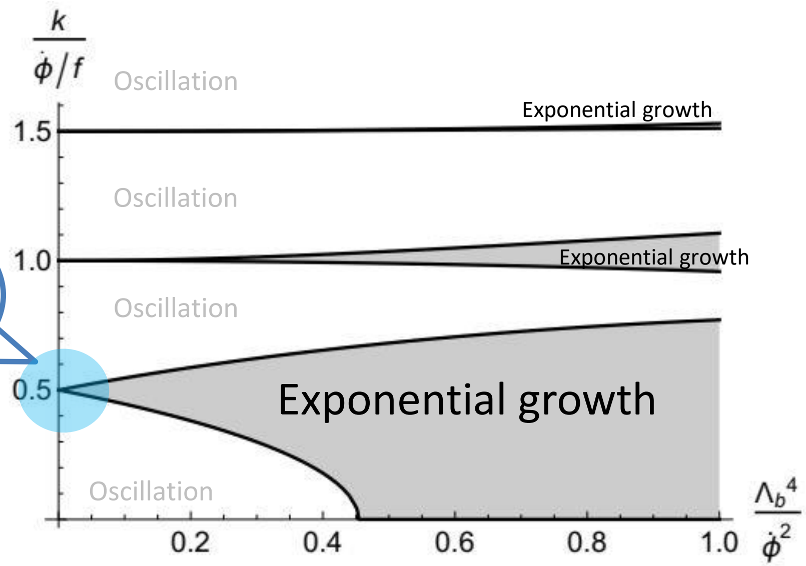
There exist resonant solutions for this.
It's like a swing!

$$\frac{d^2\delta\phi_k}{dt^2}+\left(k^2-\frac{\Lambda_b^4}{f^2}\cos\frac{\dot{\phi}t}{f}\right)\delta\phi_k = 0$$

Mathieu equation

EOM of axion

$$\left|k - \frac{\dot{\phi}}{2f}\right| < \frac{\Lambda_b^4}{2f\dot{\phi}}$$



Large velocity ← → Small velocity

There exist resonant solutions for this.
It's like a swing!

$$\frac{d^2\delta\phi_k}{dt^2} + \left(k^2 - \frac{\Lambda_b^4}{f^2}\cos\frac{\dot{\bar{\phi}}t}{f}\right)\delta\phi_k = 0$$

Mathieu equation

Growth of fluctuation

⇩

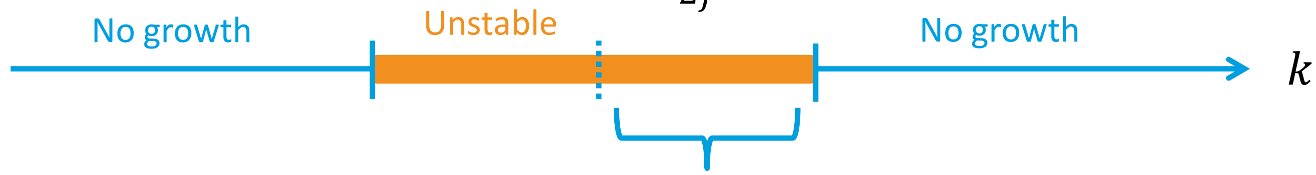
Back reaction to zeromode

Naïve estimation on back reaction

As long as $\dot{\phi}$ is constant, $\quad \delta\phi_k \sim \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right) \quad$ for $\quad \left|k - \frac{\dot{\phi}}{2f}\right| < \frac{\Lambda_b^4}{2f\dot{\phi}}$

$t = 0$

$k_{cr} = \frac{\dot{\phi}}{2f}$



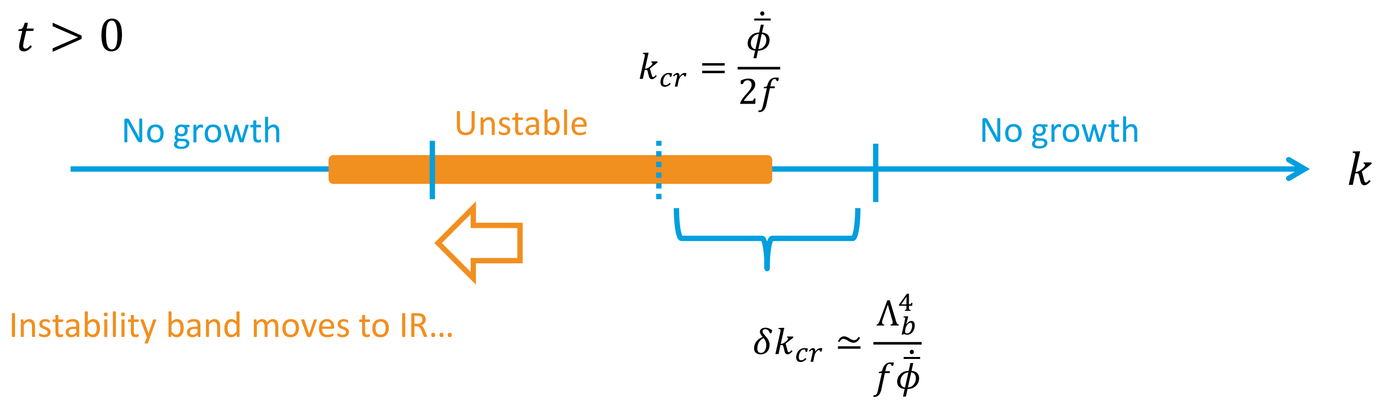
$\delta k_{cr} \simeq \frac{\Lambda_b^4}{f\dot{\phi}}$

By using dimensional analysis

$$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$$

Naïve estimation on back reaction

As long as $\dot{\phi}$ is constant, $\delta\phi_k \sim \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$ for $\left|k - \frac{\dot{\phi}}{2f}\right| < \frac{\Lambda_b^4}{2f\dot{\phi}}$



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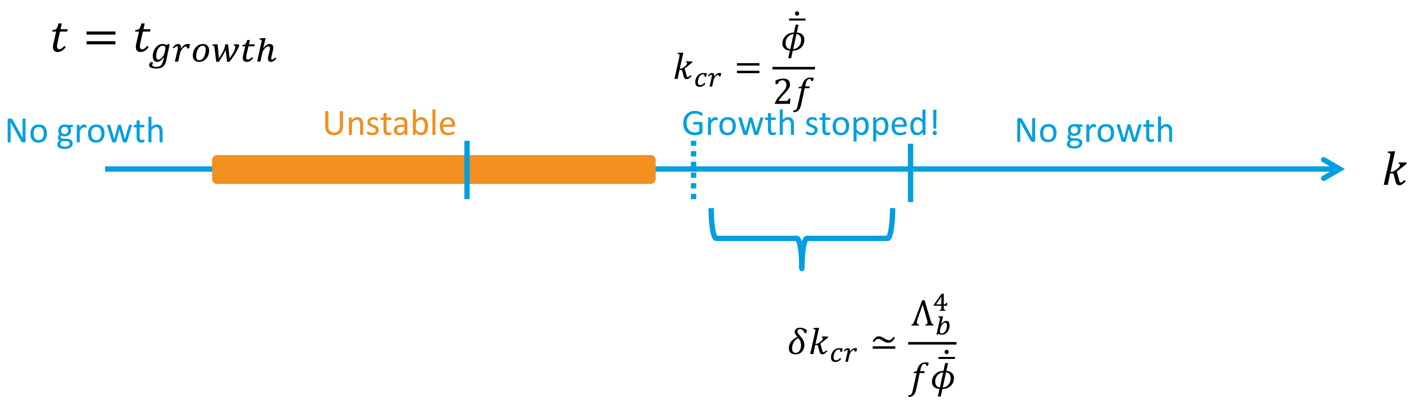
$t = t_{growth}$



By using dimensional analysis

$$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$$

The growth stops when (of mode with k=kcr)

$$\rho_{fluc}(t_{growth}) \sim \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\left(\dot{\phi} - 2f\delta k_{cr}\right)^2$$

Naïve estimation on back reaction

As long as $\dot{\phi}$ is constant, $\delta\phi_k \sim \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$ for $\left|k - \frac{\dot{\phi}}{2f}\right| < \frac{\Lambda_b^4}{2f\dot{\phi}}$

1. ϕ rolls
2. Fluctuation grows & takes energy from ϕ
3. Instability band moves to IR
4. Back to 1. with smaller ϕ

$t = t_{growth}$ $k_{cr} = \frac{\dot{\phi}}{2f}$

No growth

k

$\delta k_{cr} \simeq \frac{\Lambda_b^4}{f\dot{\phi}}$

This process repeats until ϕ loses its kinetic energy!

By using dimensional analysis

$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$

The growth stops when (of mode with k=kcr)

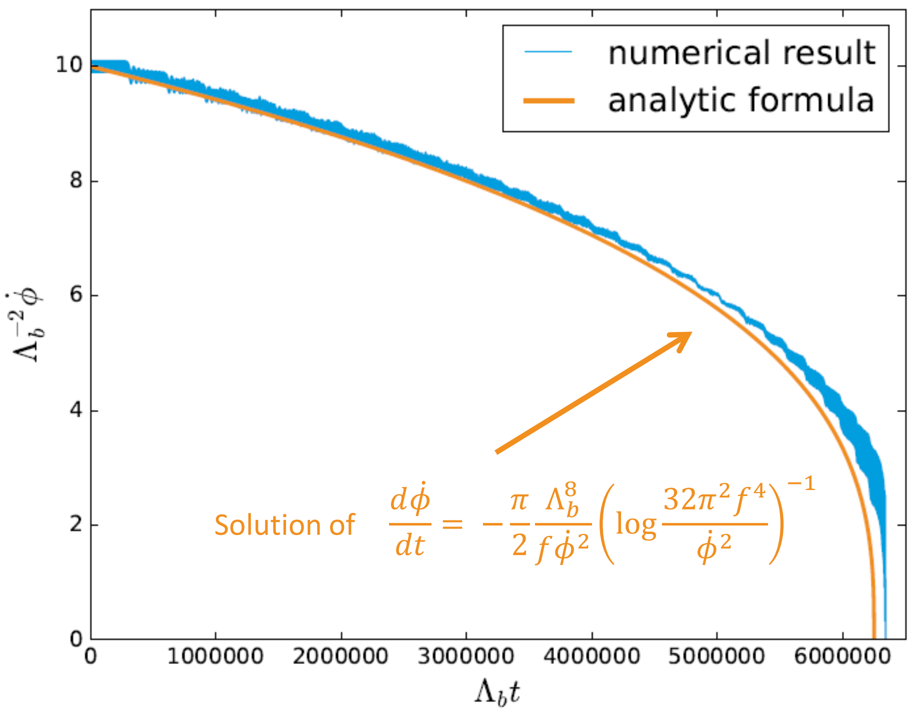
$\rho_{fluc}(t_{growth}) \sim \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\left(\dot{\phi} - 2f\delta k_{cr}\right)^2$

Result in perturbative analysis

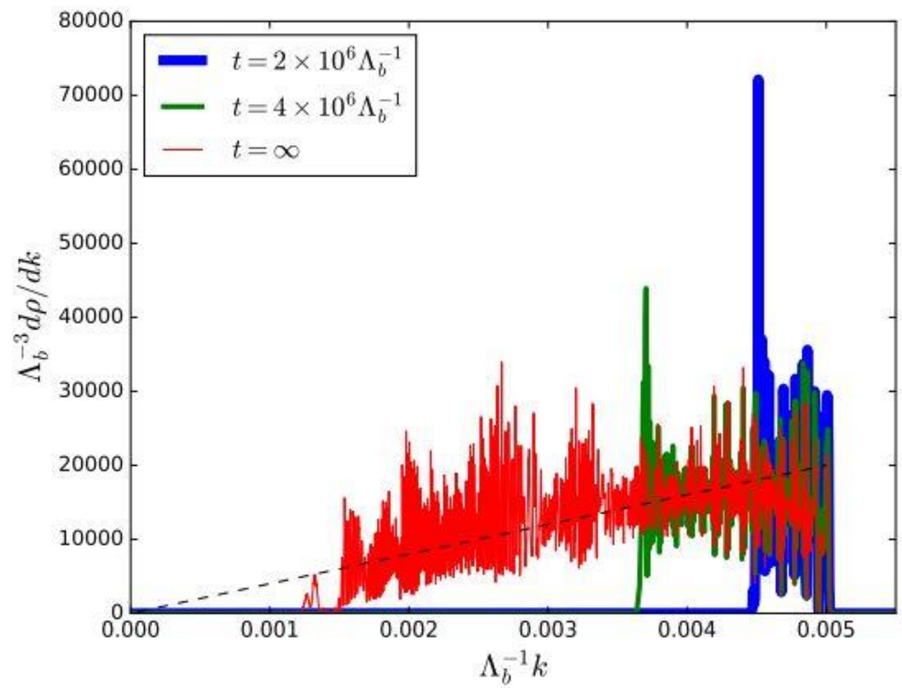
It works!

$$\frac{f}{\Lambda_b} = 1000$$

Time evolution of zeromode velocity



Fluctuation spectrum



[Fonseca, Morgante, RS, Servant (2019)]

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Necessity of non-linear analysis

Perturbative analysis gives intuition.
But reliable?

Initial kinetic energy : $\dot{\phi}_0^2/2$

Typical wavenumber : $\dot{\phi}_0/f$

Energy conservation : $(\delta\phi)^2 \times \left(\dot{\phi}_0/f\right)^2 \sim \dot{\phi}_0{}^2$

⇒ Typical field variation : $\delta\phi \sim f$ **NOT SMALL !!**

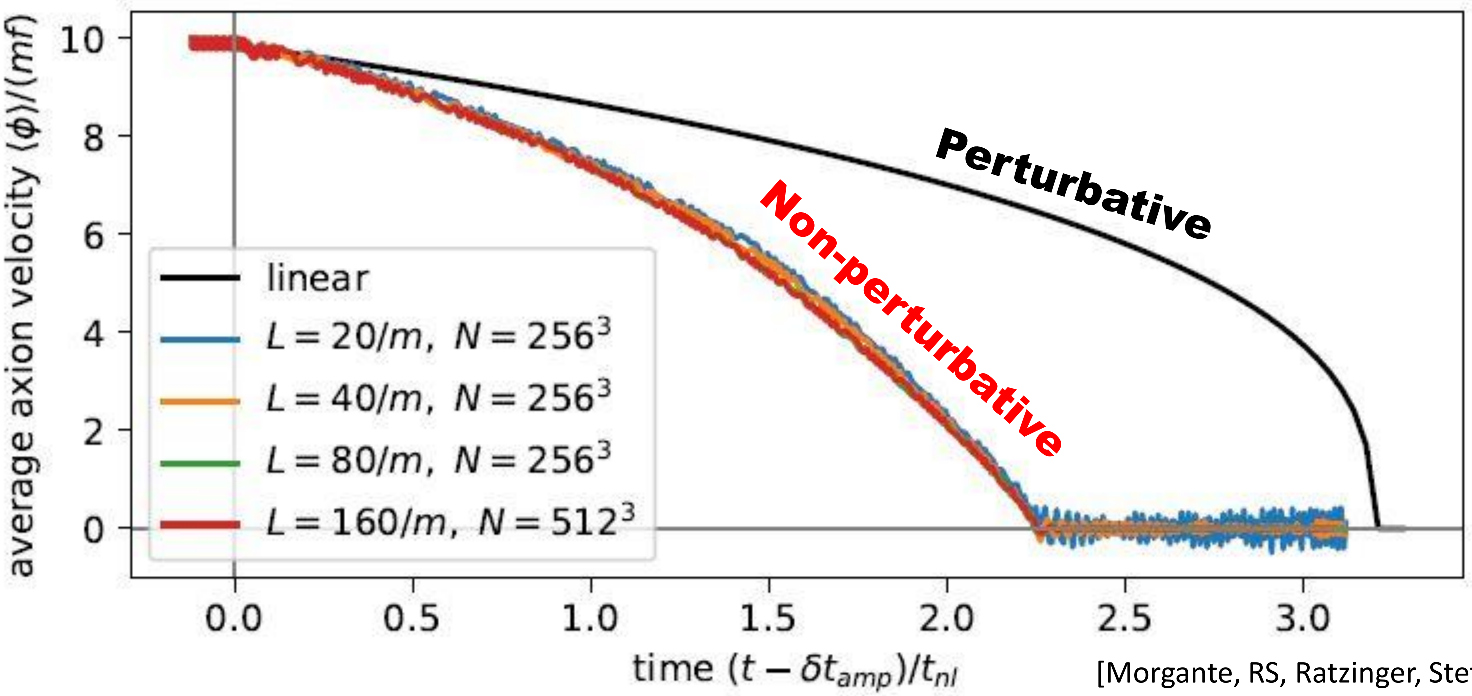
Classical lattice simulation

$$\ddot{\phi} = \nabla^2\phi + \frac{\Lambda_b^4}{f}\sin\frac{\phi}{f}$$

⇒

$$\begin{aligned}\frac{d^2\phi_{i,j,k}}{dt^2} =& \frac{1}{a^2}\left(\phi_{i+1,j,k} - 2\phi_{i,j,k} + \phi_{i-1,j,k}\right)\\ &+ \frac{1}{a^2}\left(\phi_{i,j+1,k} - 2\phi_{i,j,k} + \phi_{i,j-1,k}\right)\\ &+ \frac{1}{a^2}\left(\phi_{i,j,k+1} - 2\phi_{i,j,k} + \phi_{i,j,k-1}\right)\\ &+ \frac{\Lambda_b^4}{f}\sin\frac{\phi_{i,j,k}}{f}.\end{aligned}$$

Velocity of zeromode

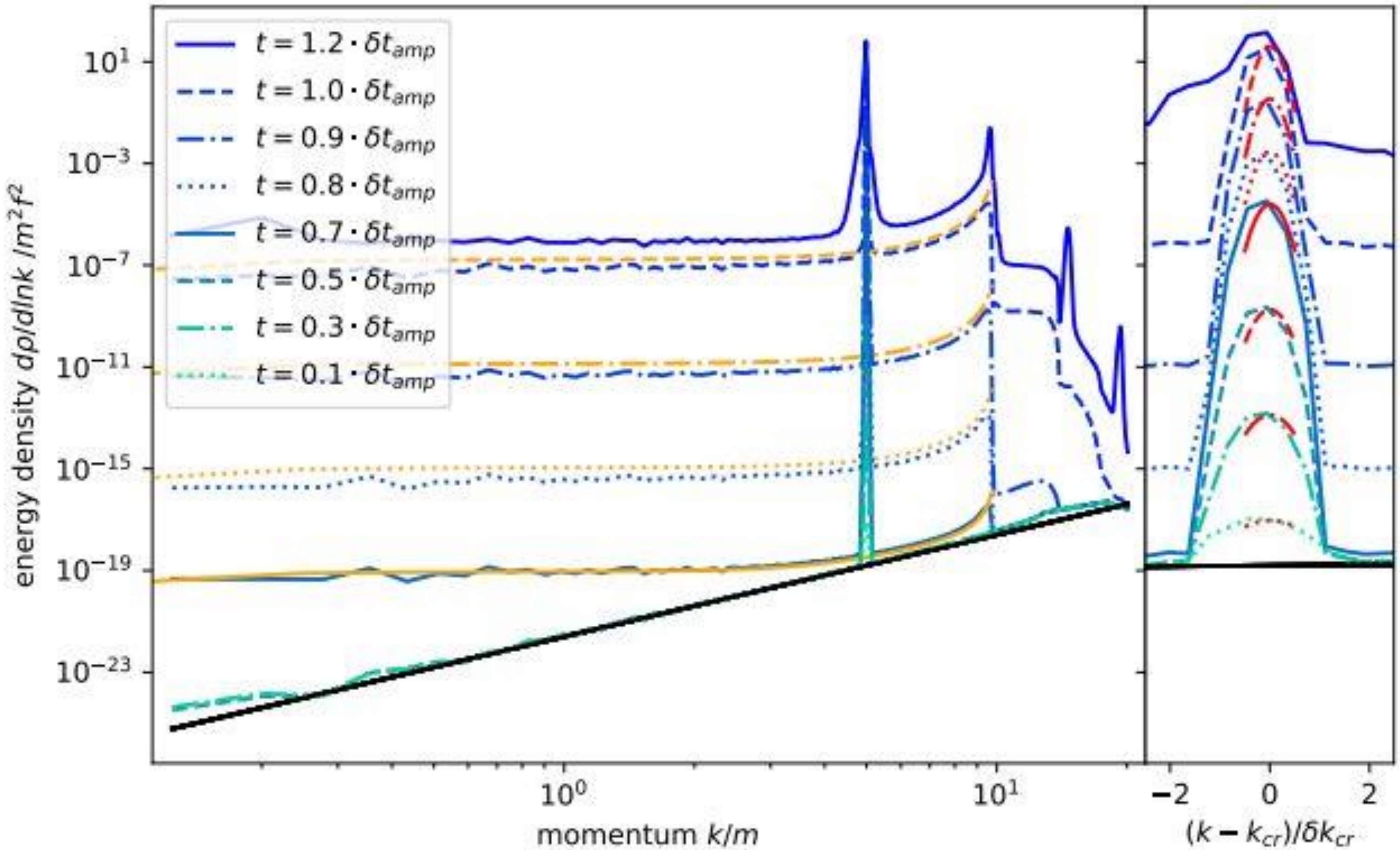


[Morgante, RS, Ratzinger, Stefanek (2021)]

$$\left(t_{nl}=\frac{f\dot{\phi}_0^3}{\Lambda_b^8}\right)$$

- Confirmed energy dissipation in non-perturbative calculation.
- Dissipation effect is stronger than perturbative analysis.

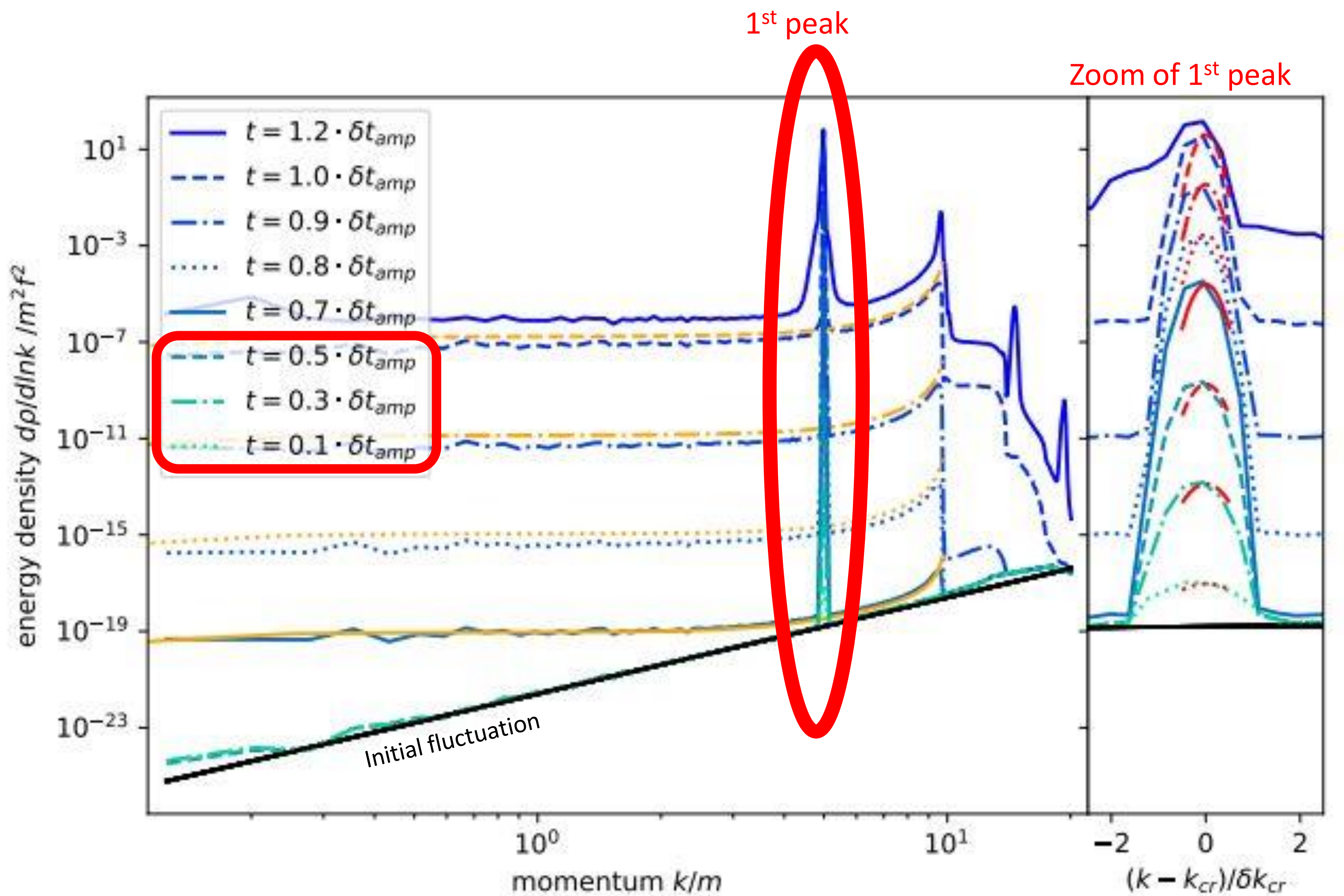
Growth of spectrum (early stage)



[Morgante, RS, Ratzinger, Stefanek (2021)]

$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4} \log \frac{16 f^4}{\dot{\phi}^2}$$

Growth of spectrum (early stage)



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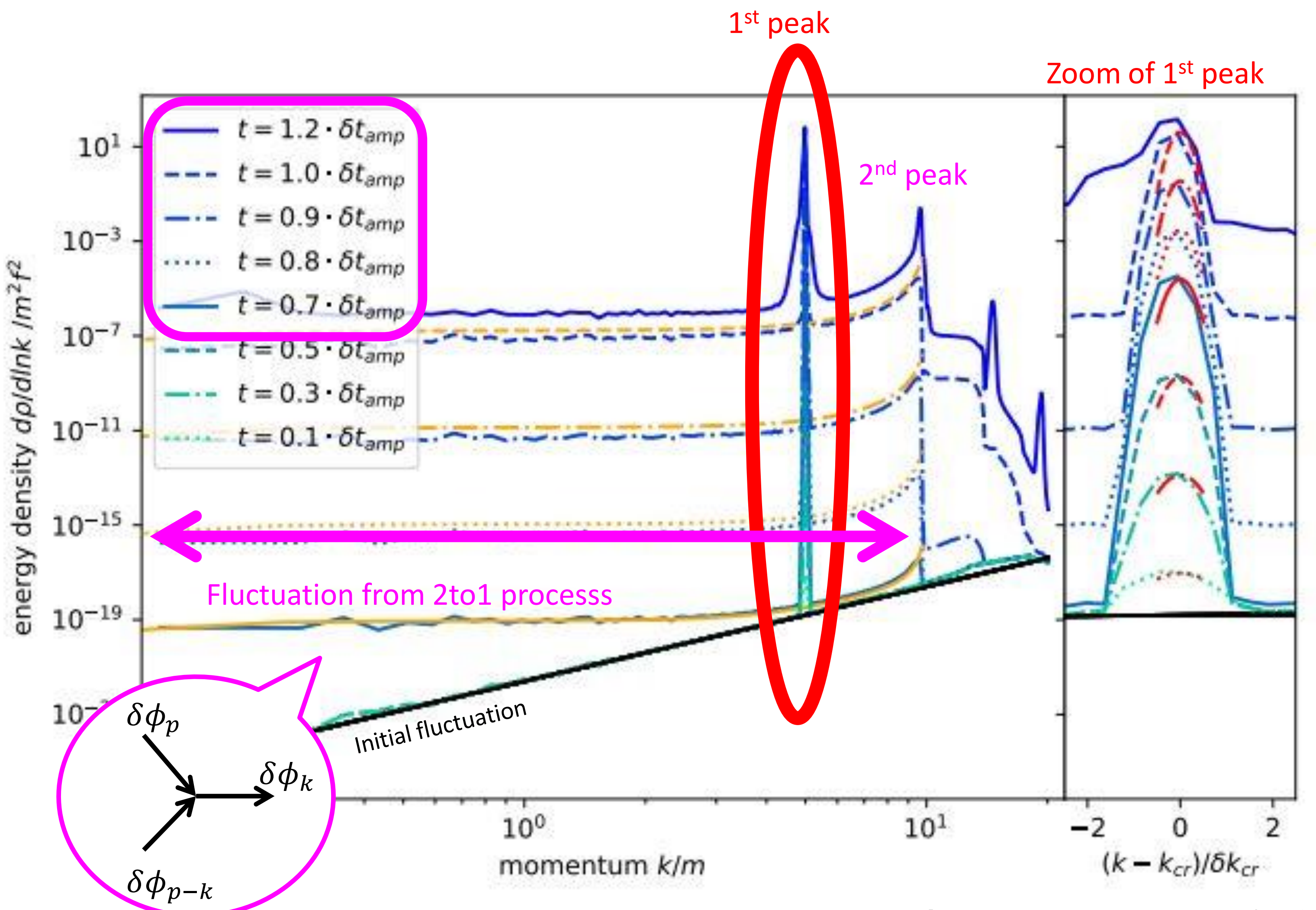

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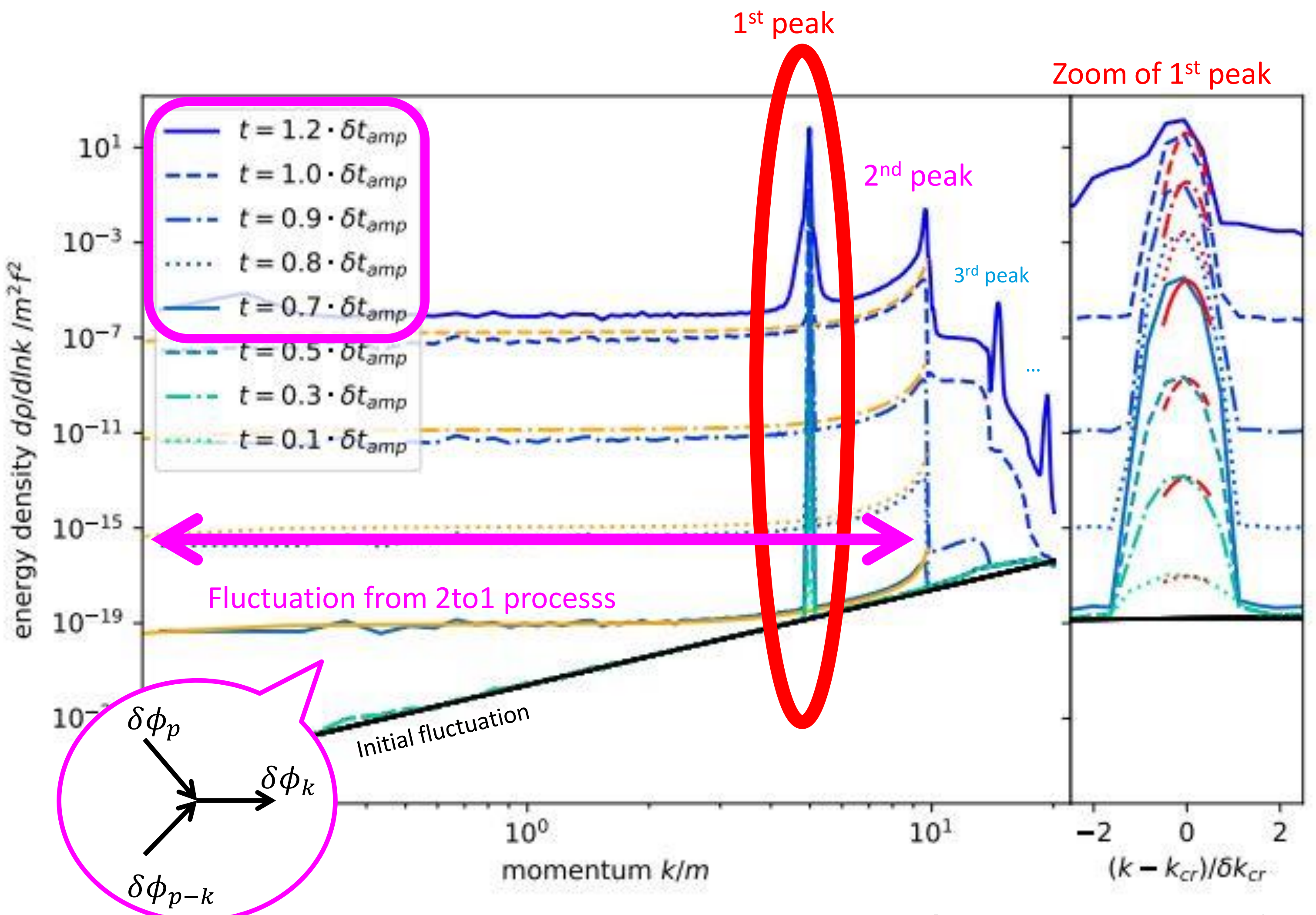
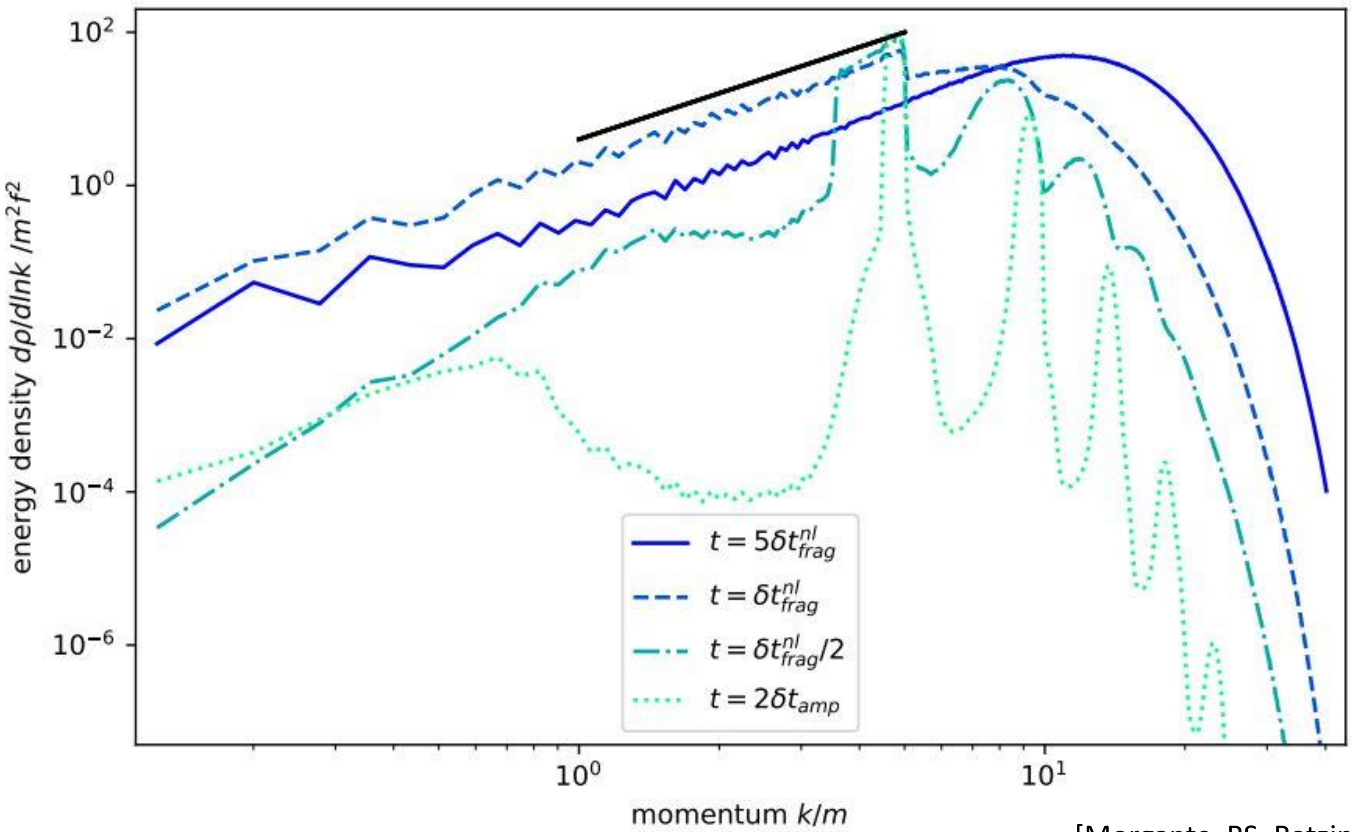


[Morgante, RS, Ratzinger, Stefanek (2021)]

$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4} \log \frac{16 f^4}{\dot{\phi}^2}$$

Growth of spectrum (late stage)



$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4}\log\frac{16f^4}{\dot{\phi}^2}$$

[Morgante, RS, Ratzinger, Stefanek (2021)]

- We can see peak-like structure in the early stage
- The spectrum becomes broad
- Cascading towards UV (early stage of thermalization)

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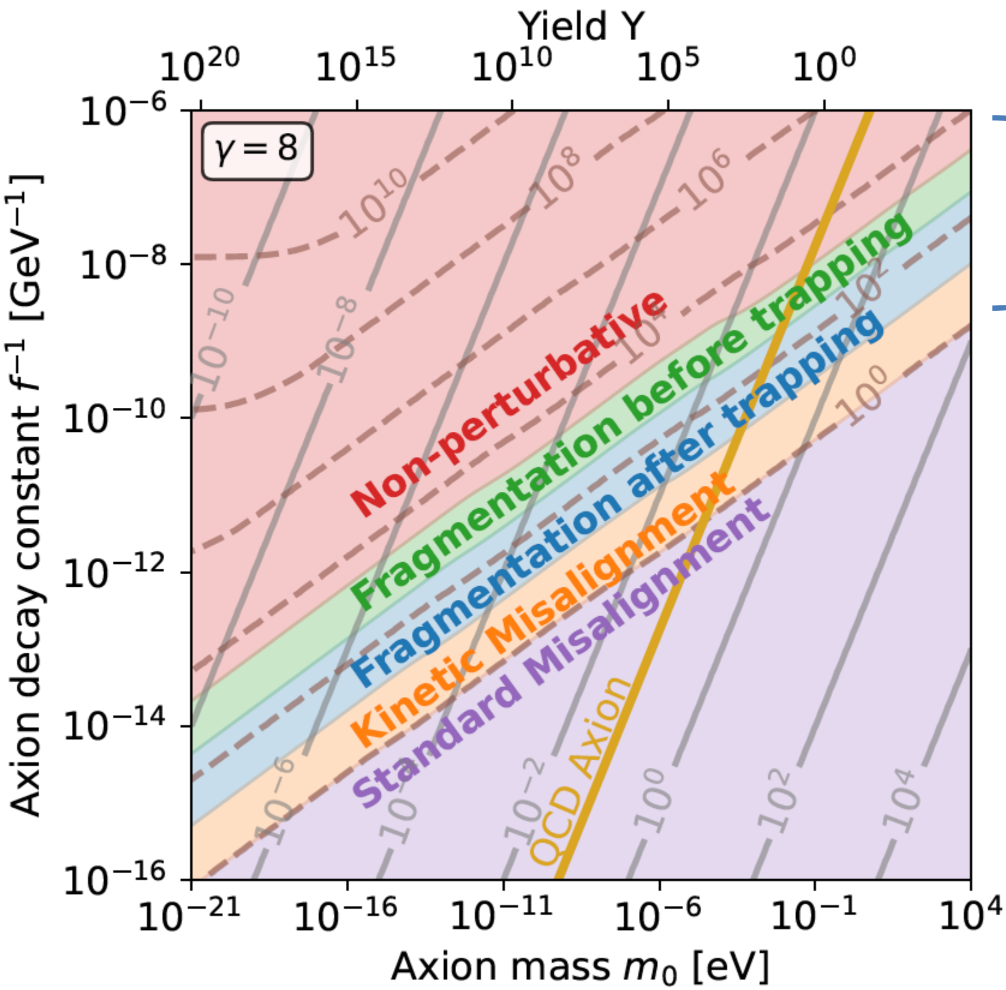
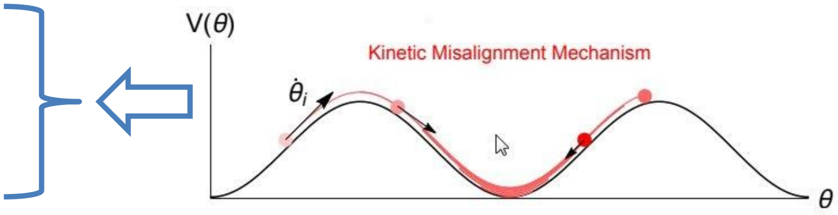
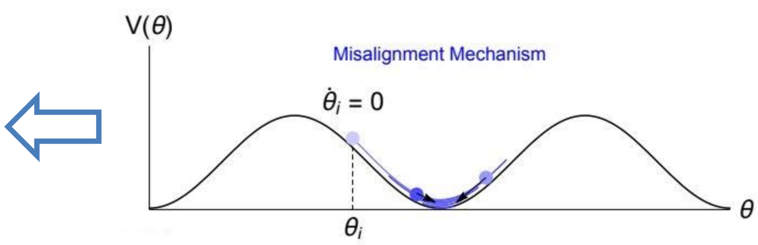
Implication to ALP dark matter

ALP dark matter : Fragmentation could happen before axion starts to oscillate







[Eröncel, RS, Sørensen, Servant (2022)]

Possible signals

- Axion mini-cluster

See Eröncel-Servant (2207.10111)

- Gravitational Wave (tensor perturbation in metric)

$$\nu \sim \frac{k}{a_{emit}} \frac{a_{emit}}{a_0} \quad \text{(Typically, } k \sim m\text{)}$$

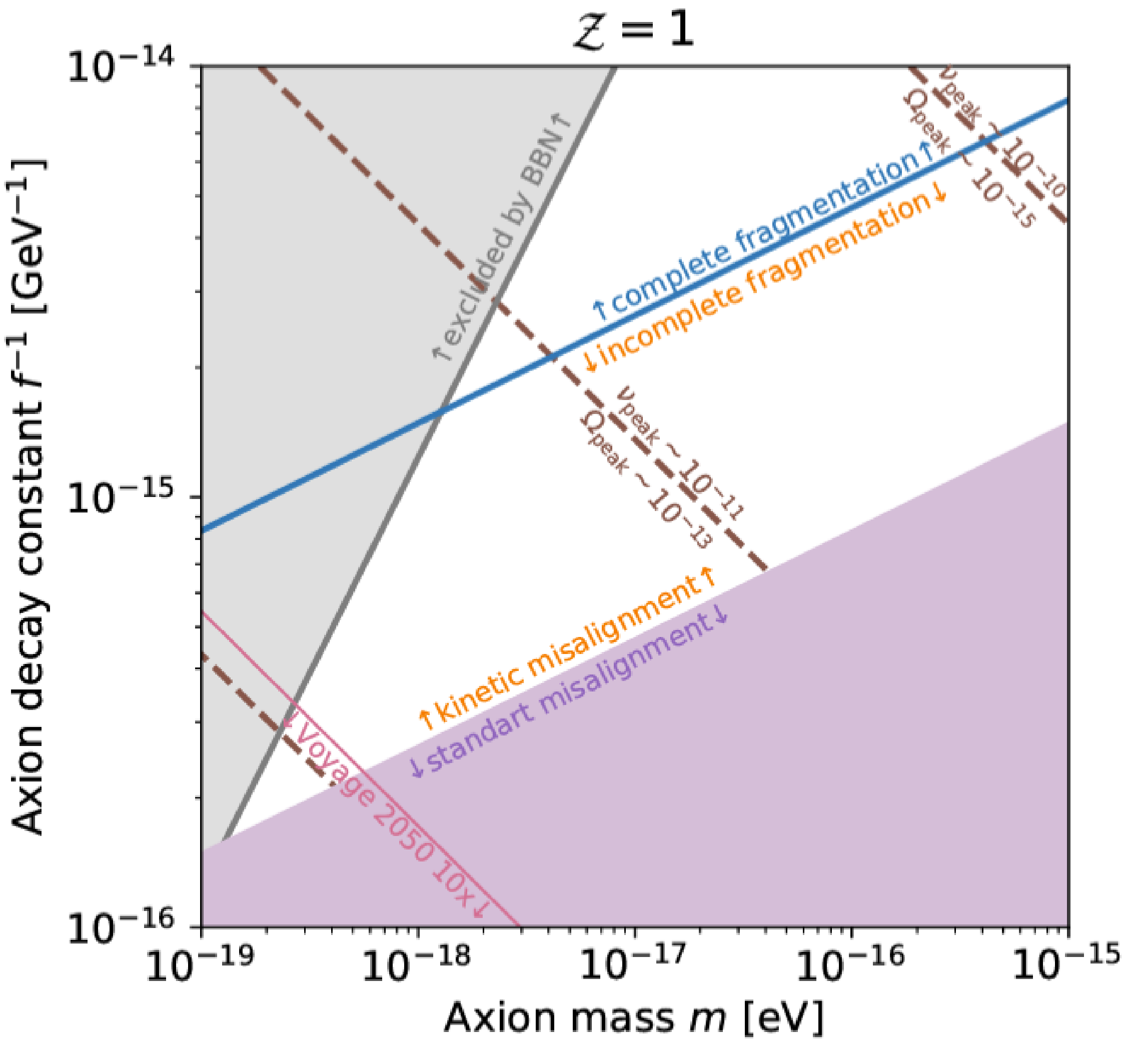
Wave number at emission — Redshift

$$\Omega_{GW}^{peak} \sim \frac{64\pi^2}{3M_{pl}^4 H_{emit}^2} \frac{\rho_{\theta,emit}^2}{\left(k_{peak}/a_{emit}\right)^2} \frac{\alpha^2}{\beta} \quad \text{(Typically, } \alpha < 1,\ \beta > 1\text{)}$$

See [Chatrchyan, Jaeckel (2020)]

c.f.) $\ddot{h} + 3H\dot{h} \sim \frac{1}{M_{pl}^2}\rho_\phi, \quad \rho_{GW} \sim M_{pl}^2 \dot{h}^2$

Possible signals : gravitational waves



[Eröncel, RS, Sørensen, Servant (2022)]

Detailed analysis is future work

Summary

- Large axion velocity → growth of fluctuation
- Zeromode kinetic energy dissipates into fluctuations
- Generic phenomena w/ periodic potential and large velocity
- Applications
  - ALP dark matter
  - Relaxion scenario (1911.08473, Fonseca-Morgante-Sato-Servant)
    Relaxion fragmentation can be a source of friction to stop relaxion.
  - Any other interesting application?