Effect of NSI on non-local correlations in neutrino oscillations

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Outline

- Introduction
- Neutrino Oscillations
- Measures of Quantum Correlations
- Formalism
- Results and Discussions
- Conclusion

Introduction

What is the aim of particle physics?

- What are the fundamental particles in the universe?
- What are the fundamental interactions between particles?

- Standard model is a gauge theory which describe the interaction between fundamental particles in nature.

- Limitations of Standard model
- Neutrino as a main system to probe new physics

Neutrinos

Why neutrinos are so important?

- Travel without being deflected and absorbed
- Travel in straight line from their source
- Excellent messenger of information

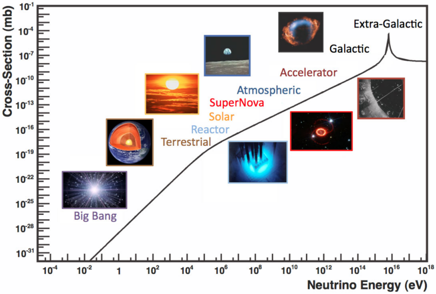

Figure: Sources of Neutrinos [Formaggio et. al., Rev. Mod. Phys. 84, 1307 (2012)]

Neutrino Oscillation

- Neutrino oscillation is a quantum mechanical phenomenon where a neutrino created with a specific lepton flavour (electron, muon , or tau) can later be measured to have a different flavour.
- Due to non-zero mass, they oscillate from one flavor to another which has been confirmed by many experiments [Super Kamiokande (1998), Sudbury Neutrino Observatory (2002)], [Nobel Prize:2015].
- Flavor of neutrino determined by superposition of mass eigenstates.
- For neutrinos flavor eigenstates different from mass eigenstates
  $\nu_e = \nu_1 cos\theta + \nu_2 sin\theta$
  $\nu_\mu = -\nu_1 sin\theta + \nu_2 cos\theta.$
- Fundamentally neutrino oscillations are three flavor oscillations but in some cases, it can be reduced to effective two flavor oscillations.

Non Standard Interaction(NSI)

- Neutrino oscillation experiments have strong evidence that neutrino oscillations occur.
- Neutrino oscillation is leading effect for neutrino flavor transitions.
- NSI comprises the effect beyond standard model [Wolfensteinn, Phys. Rev. D 17 (1978)].
- From neutrino oscillation experiments, we have received precision measurements for some of the neutrino parameters, i.e. Δm^2_{21}, $|\Delta m^2_{31}|$, θ_{12}, θ_{23} [Ohlsson, Rept. Prog. Phys. 76 (2013)].
- Other parameters are still completely unknown such as sign(Δm^2_{31}), CP phases and the absolute neutrino mass scale.
- We are entering the precision era, such subleading effects can be estimated with more accuracy

Local Realism

In 1964 John Bell formulated a mathematical statement in the form of inequalities which were based on following two assumptions [Bell, Physics 1 (1964)]

- Realism: A system has well defined values of an observable whether someone measures it or not.
- Locality: A measurement made on a system cannot influence other systems instantaneously.
- A system that can be described by a local realistic theory will satisfy this inequality.
- It turns out that nature experimentally invalidates that point of view and agreeing with quantum mechanics [Aspect et. al., Phys. Rev. Lett. 49 (1982)].

Measures of Quantum Correlations

Bell's Inequality

For a system consisting of two spin-1/2 particles A and B, the combined state with Hilbert space defined as $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, is expressed in terms of the density matrix (ρ) as follows [Horodecki et. al., Phys.Lett. A 200 (1995)]

$$\rho = \frac{1}{4}\left[I \otimes I + (r.\sigma) \otimes I + I \otimes (s.\sigma) + \sum_{A,B=1}^{3} T_{AB}(\sigma_A \otimes \sigma_B)\right].$$

- T is correlation matrix and elements of this matrix are $T_{AB} = Tr[\rho(\sigma_A \otimes \sigma_B)]$. $T^\dagger T$ having eigenvalues u_i ($i = 1, 2, 3$) out of which two largest positive eigenvalues are taken into account, denoted by u_i and u_j.Bell-CHSH inequality can be written as $M(\rho) = u_i + u_j \leq 1$.

NAQC(non-local advantage of quantum coherence)

Coherence of a system represented by the state ρ can be quantified by l_1 norm which in the eigen basis of Pauli spin matrix σ_i $(i = x, y, z)$ is defined as [Mondal et.al., Phys. Rev. A 95 (2017)]

$$C^i_{l_1}(\rho) = \sum_{i_1, i_2} |\langle i_1| \rho |i_2\rangle|, (i_1 \neq i_2).$$

Here $|i_1\rangle$ and $|i_2\rangle$ are the eigen vectors of σ_i.

Then the upper limit of the following quantity is given by

$$\sum_{i=x,y,z} C^i_{l_1}(\rho) \leq \sqrt{6} \approx 2.45.$$

Measures of Quantum Correlations

NAQC(non-local advantage of quantum coherence)

To understand NAQC, let us consider an entangled state, consisting of two subsystems A and B, expressed by the density matrix ρ . The violation of $C^i_{l_1}(\rho)$ infers the fact that the single system description of the coherence of the subsystem B is not feasible. Therefore NAQC of the state B is achieved by the condition [Ming et. al., Phys. Rev. A 98 (2018)]

$$N_{l_1}(\rho) = \frac{1}{2}\sum_{i,j,a} p(\rho_{B|\Pi^a_i})C^i_{l_1}(\rho_{B|\Pi^a_i}) > \sqrt{6}.$$

NAQC is a stronger measure of non-local correlation than Bell's inequality.

Mode Entanglement in Neutrinos

Neutrino oscillation requires the flavour eigenstates ν_α to be represented as a linear combination of mass eigenstates ν_i as follows

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle \,,$$

Time evolution of mass eigenstates is given by

$$|\nu_i(t)\rangle = e^{-\iota E_i t} |\nu_i\rangle \,,$$

In the relativistic limit, neutrino flavour states are considered to be individual modes. In the two flavour neutrino system, it can be expressed as [Blasone et. al., Eur. Phys. Lett. 85 (2009)]

$$|\nu_\alpha\rangle \equiv |1\rangle_\alpha |0\rangle_\beta \equiv |10\rangle_{\alpha\beta} \,, \qquad |\nu\rangle_\beta \equiv |0\rangle_\alpha |1\rangle_\beta \equiv |01\rangle_{\alpha\beta} \,.$$

Mode Entanglement in Neutrinos

The time evolution of flavor eigenstate can then be written as

$$|\nu_\alpha(t)\rangle = \bar{U}_{\alpha\alpha}(t)\,|1\rangle_\alpha\,|0\rangle_\beta + \bar{U}_{\alpha\beta}(t)\,|0\rangle_\alpha\,|1\rangle_\beta\,,$$

The density matrix corresponding to the state for above eq. is expressed as

$$\rho(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \left|\bar{U}_{\alpha\alpha}(t)\right|^2 & \bar{U}_{\alpha\alpha}(t)\bar{U}^*_{\alpha\beta}(t) & 0 \\ 0 & \bar{U}^*_{\alpha\alpha}(t)\bar{U}_{\alpha\beta}(t) & \left|\bar{U}_{\alpha\beta}(t)\right|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

Various measures of quantum correlations can now be determined using the density matrix $\rho_\alpha(t) = |\nu_\alpha(t)\rangle\,\langle\nu_\alpha(t)|$ as the parameters of the density matrix, mixing angle and mass squared difference [Alok et. al., Nucl. Phys. B 909 (2016)].

Neutrino oscillation in Matter

- While travelling through the matter neutrinos undergo charged current (CC) and neutral current (NC) interactions with matter particles.
- Earth matter is composed of only nucleons and electrons.

For an incoming ν_e traversing through Earth, the corresponding Hamiltonian is given by [Guinty et. al., Oxford university press (2007)]

$$\mathcal{H}_m = \mathcal{H}_{vac} + \mathcal{H}_{mat} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} + U^\dagger \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix} U,$$

The evolution operator in the mass eigen basis is represented as [Ohlsson et.al., J. Math. Phys. 41 (2000)]

$$U_m(L) = e^{-i\mathcal{H}_m L} = \phi \; e^{-iLT}$$

$$= \phi \sum_{a=1}^{2} e^{-iL\lambda_a} \frac{1}{2\lambda_a} \left(\lambda_a I + T\right).$$

NSI in Neutrino oscillation

In presence of NSI, the Hamiltonian is modified as follows [Ohlsson (2013)]

$$\mathcal{H}_{tot} = \mathcal{H}_{vac} + \mathcal{H}_{mat} + \mathcal{H}_{NSI} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

$$+U^{\dagger} A \begin{pmatrix} b + \epsilon_{\alpha\alpha}(x) & \epsilon_{\alpha\beta}(x) \\ \epsilon_{\beta\alpha}(x) & \epsilon_{\beta\beta}(x) \end{pmatrix} U.$$

$\epsilon_{\alpha\beta}(x)$ are the NSI parameters which are expressed as

$$\epsilon_{\alpha\beta}(x) = \sum_{f=e,u,d} \frac{N_f(x)}{N_e(x)} \epsilon^f_{\alpha\beta}.$$

The bounds on NSI parameters are extracted from global analysis of the data obtained from different oscillation and non-oscillation experiments [Esteban et.al., J. High Energy Phys., (2019), Coloma et. al., J. High Energy Phys. (2020)].

Quantum correaltion in Neutrino Oscillation

- The coherent time evolution of neutrino flavor eigenstates implies that there is a linear superposition between the mass eigenstates which make up a flavour state.
- Thus neutrino oscillations are related to the multi-mode entanglement of single-particle states which can be expressed in terms of flavor transition probabilities [Alok et. al., Nucl. Phys. B 909 (2016)].
- Hence neutrino is an interesting candidate for study of quantum correlations.

We are interested in studying measures of quantum correlations in neutrinos. In particular, we intend to study the measures of quantum correlations such as NAQC and Bell-Inequality in neutrino oscillation system.

Results and Discussions

The analytical expressions of the two parameters, $M(\rho)$ and $N_{l_1}(\rho)$, are obtained as [Yadav et. al., EPJC 82, no. 5, 1-10 (2022)]

$$M(\rho) = f_a(x,y,r) + f_b(x,y,z,r) + f_c(x,z,r),$$

$$N_{l_1}(\rho) = 2 + \sqrt{\frac{2f_b(x,y,z,r)}{3}},$$

where the form of quantities f_a, f_b and f_c are given as

$$f_a(x,y,r) = \frac{e^{\mathrm{Im}(4r)}[x^2 + y^2 + (x^2 - y^2)\cos(2r)]^2}{4x^4},$$

$$f_b(x,y,z,r) = \frac{3e^{\mathrm{Im}(4r)} z^2 \sin^2 r (x^2 + y^2 + (x^2 - y^2)\cos(2r))}{x^4},$$

$$f_c(x,z,r) = \frac{e^{\mathrm{Im}(4r)} z^4 \sin^4 r}{x^4},$$

with $r = \frac{Lx}{4E}$.

Results and Discussions

The quantities x, y and z are functions of NSI parameters and are given by

$$x = \sqrt{x_1 - x_2 + x_3},$$
$$y = \frac{-x_2}{2\,\Delta m^2\ \cos(2\theta)} + \Delta m^2\ \cos(2\theta),$$
$$z = \frac{x_3 - (\Delta m^2)^2 \cos(4\theta)}{2\,\Delta m^2\ \sin(2\theta)},$$

with

$$x_1 = 4A^2E^2(4\epsilon_{\alpha\beta}^2 + (\epsilon_{\alpha\alpha} + b - \epsilon_{\beta\beta})^2),$$
$$x_2 = 4\,A\,E\,(\epsilon_{\alpha\alpha} + b - \epsilon_{\beta\beta})\,\Delta m^2 \cos(2\theta),$$
$$x_3 = 8\,A\,E\,\epsilon_{\alpha\beta}\,\Delta m^2\ \sin(2\theta) + (\Delta m^2)^2.$$

Results and Discussions

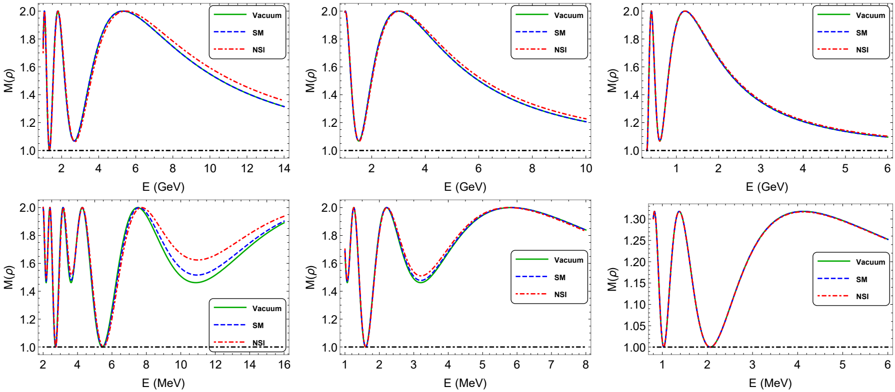

Figure: Variation of M(ρ) with energy (E) for the accelerator and reactor experiments. (a) Upper left: DUNE; (b) upper middle: MINOS; (c) upper right: T2K; (d) lower left: KamLAND; (e) lower middle: JUNO; and (f) lower right: Daya Bay Dotted (black) line represents the classical bound of M(ρ).

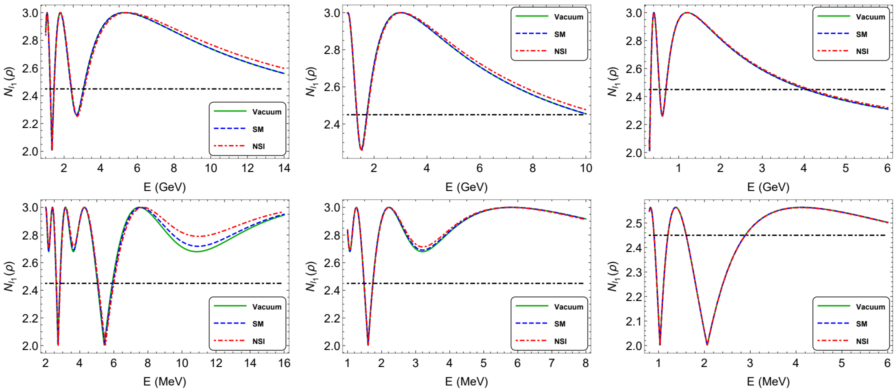

Figure: Variation of NAQC parameter with energy (E) for the accelerator and reactor experiments: DUNE, $L = 1300$ km, $E \approx 1 - 14$ GeV; MINOS, $L = 735$ km, $E \approx 1 - 10$ GeV; T2K, $L = 295$ km, $E \approx 0 - 6$ GeV; KamLAND, $L = 180$ km, $E \approx 1 - 16$ MeV; JUNO, $L = 53$ km, $E \approx 1 - 8$ MeV; and Daya Bay, $L = 2$ km, $E \approx 0.8 - 6$ MeV.

Results and Discussions

Expts.	Measure	% inc. w.r.t. vac	% inc. w.r.t. SM int.
DUNE	$M(\rho)$	4.3	4.3
	$N_{l_1}(\rho)$	4	4
MINOS	$M(\rho)$	2.4	2.4
	$N_{l_1}(\rho)$	2.3	2.3
T2K	$M(\rho)$	0.7	0.7
	$N_{l_1}(\rho)$	0.6	0.6
KamLAND	$M(\rho)$	16.5	11
	$N_{l_1}(\rho)$	11	7
JUNO	$M(\rho)$	5	3.3
	$N_{l_1}(\rho)$	3.5	2.4
Daya Bay	$M(\rho)$	≈ 0	≈ 0
	$N_{l_1}(\rho)$	≈ 0	≈ 0

Table: Percentage (%) increase in Bell's inequality parameter $M(\rho)$ and NAQC parameter $N_{l_1}(\rho)$ in presence of NSI in comparison to vacuum and SM interaction for six different experimental set-ups.

Results and Discussions

Experiments	$N_{l_1}(\rho)$	$M(\rho)$
DUNE (GeV)	1-1.25 ,1.5-2.45 ,3-14	1-14
MINOS (GeV)	1-1.3 ,1.75-10	1-10
T2K (GeV)	0.4-0.5 , 0.7-4	0-6
KamLAND (MeV)	1-2.5 , 3-5 ,6-16	1-16
JUNO (MeV)	1-1.45 , 1.8-8	1-8
Daya Bay (MeV)	0.8-0.9 ,1.2-1.6 ,2.9-6	0.8-6

Table: Energy regions showing the violation of $M\rho$ and NAQC for six different experimental set-ups. For accelerator experiments the energy range lies in GeV region, while for reactor experiments they are in MeV.

Conclusion

- Study the effect of NSI on NAQC parameter $N_{l_1}(\rho)$ as well as on Bell's inequality parameter $M(\rho)$ for two flavour neutrino oscillations in the context of different accelerator and reactor experimental set-ups.
- Observe that NSI can enhance the violation of $N_{l_1}(\rho)$ and $M(\rho)$ at higher energies in comparison to the SM and vacuum scenarios.
- In the case of LBL reactor experiment KamLAND, this distinction is significantly visible in specific energy regions.
- Possible enhancement in the violation of the Bell's inequality parameter over the standard model prediction can be up to 11% whereas for NAQC it is 7%.
- Thus NAQC is a comparatively stronger measure of nonclassicality, it shows lesser sensitivity to NSI effects in comparison to the Bell's inequality parameter.

Thank You