Practical tests on the PMNS paradigm

Osamu Yasuda
Tokyo Metropolitan University

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3 neutrinos and beyond @ Quy Nhon, Vietnam
1. Introduction

2. Scenarios beyond the PMNS paradigm

3. Tests Before HK & DUNE

4. Tests at HK & DUNE

5. Conclusions
1. Introduction

Framework of 3 flavor $\nu$ oscillation

Mixing matrix

Functions of mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$, and CP phase $\delta$

All 3 mixing angles have been measured (2012):

$\nu_{\text{solar}} + \text{KamLAND (reactor)}$

$\nu_{\text{atm}} + \text{K2K, MINOS (accelerators)}$

$\text{DCHOOZ + Daya Bay + Reno (reactors), T2K + MINOS, others}$

$\theta_{12} \cong \frac{\pi}{6}, \Delta m_{21}^2 \cong 8 \times 10^{-5} \text{eV}^2$

$\theta_{23} \cong \frac{\pi}{4}, |\Delta m_{32}^2| \cong 2.5 \times 10^{-3} \text{eV}^2$

$\theta_{13} \cong \frac{\pi}{20}$
NH(NO), $\delta \sim 3\pi/2$ is preferred over IH(IO), $\delta = 0$. 

Lisi@Prospects of Neutrino Physics (IPMU, 2019/4)
Potential problems with the PMNS paradigm

- T2K appearance data may not be consistent with the PMNS paradigm

\[ N(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \]

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**T2K Run 1-9 preliminary**

- \( \sin^2 \theta_{23} = 0.50, 0.45, 0.55 \)
- \( \Delta m^2_{32} = 2.45 \times 10^{-3} \text{ eV}^2/\text{c}^4 \)
- \( \Delta m^2_{31} = -2.43 \times 10^{-3} \text{ eV}^2/\text{c}^4 \)
- \( \delta_{CP} = \pi \)
- \( \delta_{CP} = +\pi/2 \)
- \( \delta_{CP} = 0 \)
- \( \delta_{CP} = -\pi/2 \)

\( \phi \) Data (statistical errors only)

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Sakashita@Prospects of Neutrino Physics (IPMU, 2019/4)
Consistency on $\theta_{23}$ & $\delta$ between Nova & T2K has to be yet seen.
Tension between $\Delta m^2_{21}(\text{solar})$ & $\Delta m^2_{21}(\text{KamLAND})$

SK I - IV combined

$\sqrt{8}$ SK favors LMA solution > 3$\sigma$

$\sim$2$\sigma$ tension with KamLAND in $\Delta m^2_{21}$

$\sin^2 \theta_{12} = 0.316^{+0.034}_{-0.026}$

$\Delta m^2_{21} = 7.54^{+0.19}_{-0.18}$

$\sin^2 \theta_{12} = 0.337^{+0.027}_{-0.023}$

$\Delta m^2_{21} = 4.74^{+1.40}_{-0.80}$

$\sin \theta_{12}^2 = 0.326^{+0.022}_{-0.019}$

$\Delta m^2_{21} = 7.50^{+0.19}_{-0.17}$

The unit of $\Delta m^2_{21}$ is $10^{-5}$ eV$^2$

2$\sigma$ tension
Indication for $\Delta m^2 \sim O(1) eV^2$

- LSND anomaly \( \nu_\mu \rightarrow \nu_e \)
- MiniBooNE \( \nu_\mu \rightarrow \nu_e + \bar{\nu}_\mu \rightarrow \bar{\nu}_e \)
- Galium anomaly
- Reactor anomaly

$\nu_e \rightarrow \nu_e$
2. Scenarios beyond the PMNS paradigm

Just like at B factories, high precision measurements of $\nu$ oscillation in future experiments can be used to probe physics beyond SM by looking at deviation from SM+$m_\nu$ (beyond the PMNS paradigm).

→ Research on New Physics is important.

Test of the PMNS paradigm

Rather than looking for arbitrary possibilities of New Physics, here we discuss possible hints of the scenarios which have been discussed in the past.
### List of popular NP in $\nu$ oscillation phenomenology

<table>
<thead>
<tr>
<th>Scenario beyond SM+$m_\nu$</th>
<th>Experimental indication?</th>
<th>Phenomenological constraints on the magnitude of the effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light sterile $\nu$ ($\nu_s$)</td>
<td>Maybe</td>
<td>$O(10%)$</td>
</tr>
</tbody>
</table>
| Non Standard Interactions in propagation | Maybe                     | $e^-\tau$: $O(100\%)$
Others: $O(1\%)$ |
| Non Standard Interactions at production / detection | $\times$            | $O(1\%)$                                                     |
| Unitarity violation due to heavy particles | $\times$            | $O(0.1\%)$                                                   |

In this talk, we will focus on these two because of potential experimental hints.
### Features of light sterile $\nu (\nu_s)$

#### Interactions of active & sterile $\nu$

<table>
<thead>
<tr>
<th></th>
<th>$\nu_e$</th>
<th>$\nu_\mu, \nu_\tau$</th>
<th>$\nu_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CC</strong></td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td><strong>NC</strong></td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td><strong>$V$</strong></td>
<td>$A_e + A_n$</td>
<td>$A_n$</td>
<td>0</td>
</tr>
</tbody>
</table>

**Equations:**

- $A_e = \sqrt{2} G_F N_e$
- $A_n = -\left(\frac{1}{\sqrt{2}}\right) G_F N_n$
Matter effect in the presence of sterile $\nu$

The term which is proportional to identity can be ignored

$U \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$

$A_e \equiv \sqrt{2} G_F N_e$

$A_n \equiv -\frac{G_F N_n}{\sqrt{2}}$

$\nu_s$ has matter effect different from others
Phenomenological New Physics considered in this talk: 4-fermi Non Standard Interactions:

\[ \mathcal{L}_{\text{eff}} = G_{NP}^{\alpha \beta} \bar{\nu}_\alpha \gamma^\mu \nu_\beta \bar{f} \gamma_\mu f' \]

Modification of matter effect

\[ \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left[ U \text{ diag } (E_1, E_2, E_3) U^{-1} + A \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \]

\[ A \equiv \sqrt{2} G_F N_e \quad N_e \equiv \text{electron density} \]
3. Tests Before HK & DUNE

(1) $\nu_s$

To test the hypothesis of sterile $\nu$, (i) we get the constraints on the extra mixing angles $\theta_{14}, \theta_{24}, \theta_{34}$ or (ii) we get the constraints on different matter effect.

$$U = R_{34}(\theta_{34}, \delta_{34}) R_{24}(\theta_{24}, 0) R_{14}(\theta_{14}, \delta_{14}) R_{23}(\theta_{23}, 0) R_{13}(\theta_{13}, \delta_{13}) R_{12}(\theta_{12}, 0)$$

extra mixing angles

$$U_{e4} = e^{-i\delta_{14}} s_{14}$$
$$U_{\mu 4} = c_{14} s_{24}$$
$$U_{\tau 4} = c_{14} c_{24} e^{-i\delta_{34}} s_{34}$$
$$U_{s 4} = c_{14} c_{24} c_{34}$$

$3\nu$ mixing angles
(1-1) Neutral current interactions of $\pi^0$ at T2K

By measuring $\#(\pi^0 \rightarrow 2\gamma)$ due to NC, we can determine

$$P(\nu_\mu \rightarrow \nu_e) + P(\nu_\mu \rightarrow \nu_\mu) + P(\nu_\mu \rightarrow \nu_\tau) = 1 - P(\nu_\mu \rightarrow \nu_s)$$

Unfortunately, due to large statistical and systematic errors, precision is not so good:

$$P(\nu_\mu \rightarrow \nu_s) \sim \sin^2 2\theta_{24} < 0.4$$

Barger, Geer, Whisnant, New J. Phys. 6 (‘04) 135
(1-2) Appearance & disappearance probabilities at T2K

$\theta_{14} = 0$ is assumed

T2K, PRD99 (‘19) 071103
(1-3) $\nu_{\text{atm}}$ (SK, IceCUBE) is more sensitive to matter effect than T2K.

IceCUBE, PRD95 (‘17) 112002

SK, PRD91 (‘15) 052019

IceCUBE $\sim$ SK $>>$ T2K
(1-3) $\nu_{\text{atm}}$ (Antares)

Antares ~ IceCUBE ~ SK
Precise measurement of $\nu_\tau$ is in principle important to constrain $\theta_{24} \& \theta_{34}$, but it is challenging.

$P(\nu_{\mu} \rightarrow \nu_{\tau}) = 4\text{Re} \left[ U_{\mu 3} U_{\tau 3}^* (U_{\mu 3} U_{\tau 3} + U_{\mu 4} U_{\tau 4}) \right] \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) + \cdots$

$(1-4) \text{ SK } \nu_{\text{atm}} : \tau \text{ appearance}$

$U_{\mu 4} = c_{14} s_{24}$
$U_{\tau 4} = c_{14} c_{24} e^{-i\delta_{34}} s_{34}$

SK, PRD 98(‘18) 052006
(2) NSI in propagation

(2-1) SK $\nu_{\text{atm}}$: sensitive to matter effect

$\nu_{\text{atm}}$ is sensitive to matter effect. It is worth updating.

SK, PRD84 (‘11) 113008

[Graphs showing different values of $\epsilon_{ee}$, with shaded regions indicating 99%, 90%, and 68% C.L.]
It is worth extending this analysis to: $-4 < \alpha - 1 < +4$

This corresponds to NSI with $\varepsilon_{ee} \equiv \alpha - 1$ other $\varepsilon_{\alpha\beta} = 0$

$\Delta \chi^2$ vs $\alpha$

Inverted Hierarchy

Normal Hierarchy

$A_e \rightarrow \sqrt{2}G_F N_e$
4. Tests at HK & DUNE

(1) $\nu_s$  (1-1) Accelerator $\nu$ (T2HK, T2HKK, DUNE)

Combined accelerator $\nu$ can cover some of the LSND region @ 90%CL

$\Delta m^2_{41} = 1.7 \text{ (eV}^2\text{)}$

$P(\nu_\mu \rightarrow \nu_e) = 4\text{Re} \left[U_{e3}U_{\mu3}^*(U_{e3}U_{\mu3} + U_{e4}U_{\mu4})\right] \sin^2 \left(\frac{\Delta m^2_{31}L}{4E}\right) + \ldots$
(1-2) Accelerator $\nu$ (DUNE w/ $\nu_\tau$)

Ghoshal-Giarnetti-Meloni, arXiv:1906.06212v1

$\Delta m_{41}^2 = 1.0 \text{ eV}^2$

$\nu_\tau + \nu_\mu + \nu_e$

$\nu_\mu + \nu_e$

Accelerator $\nu$ has better (worse) sensitivity to $\theta_{24}$ ($\theta_{34}$) than $\nu_{\text{atm}}$
$|U_{\tau 4}|^2 = c_{14}^2 c_{24}^2 s_{34}^2$

$|U_{\mu 4}|^2 = c_{14}^2 s_{24}^2$

HK Sensitivity to $\theta_{34}$ ($\theta_{24}$) is (is not much) improved compared to SK:

90% CL (solid)
99% CL (dashed)
In solar $\nu$ analysis, $(\epsilon_D, \epsilon_N)$ was used:

$$U = R_{23}(\theta_{23}, 0)R_{13}(\theta_{13}, \delta_{13})R_{12}(\theta_{12}, 0)$$

$$(\epsilon'_{\alpha\beta}) \equiv R^{-1}_{13}(\theta_{13}, \delta_{13})R^{-1}_{23}(\theta_{23}, 0) (\epsilon_{\alpha\beta}) R_{23}(\theta_{23}, 0) R_{13}(\theta_{13}, \delta_{13})$$

$$\equiv \begin{pmatrix} 
\epsilon'_{11} & \epsilon'_{12} & \epsilon'_{13} \\
\epsilon'_{21} & \epsilon'_{22} & \epsilon'_{23} \\
\epsilon'_{31} & \epsilon'_{32} & \epsilon'_{33} 
\end{pmatrix} \equiv \begin{pmatrix} 
\frac{\epsilon'_{11} + \epsilon'_{22}}{2} - \epsilon_D & \epsilon_N & \epsilon'_{13} \\
\epsilon_N & \frac{\epsilon'_{11} + \epsilon'_{22}}{2} + \epsilon_D & \epsilon'_{23} \\
\epsilon'_{31} & \epsilon'_{32} & \epsilon'_{33} 
\end{pmatrix}$$

$\rightarrow$ Also for analysis of $\nu_{\text{atm}}$ & LBL, $(\epsilon_D, \epsilon_N)$ will be used instead of $\epsilon_{\alpha\beta}$.
HK $\nu_{\text{atm}}$ has sensitivity to some region of the $\nu_{\text{solar}}$ anomaly.

**Best fit point of solar & KamLAND for $f=u$: significance: 38σ**

$$\left(\epsilon^u_D, \epsilon^u_N\right) = (-0.22, -0.30)$$

**Best fit point of global analysis for $f=u$: significance: 5σ**

$$\left(\epsilon^u_D, \epsilon^u_N\right) = \left(-0.140, -0.030\right)$$

**Best fit point of global analysis for $f=d$: significance: 5σ**

$$\left(\epsilon^d_D, \epsilon^d_N\right) = \left(-0.145, -0.036\right)$$

**Best fit point of solar & KamLAND for $f=d$: significance: 11σ**

$$\left(\epsilon^d_D, \epsilon^d_N\right) = \left(-0.12, -0.16\right)$$
Excluded region by LBL is outside of the curve.

$\delta(\text{true}) = -90^\circ$
(2-3) Accelerator $\nu$ (DUNE)

$\delta(\text{true}) = -90^\circ$

Sensitivity of DUNE is slightly better than T2HKK

Ghosh & OY, arXiv:1709.08264
Comparison of sensitivity T2HKK, DUNE, $\nu_{\text{atm}@HK}$

In the case of NH, $\nu_{\text{atm}@HK}$ is the best

Best fit point of global analysis for $f=u$

Best fit point of global analysis for $f=d$

Ghosh & OY, arXiv:1709.08264
Comparison of sensitivity T2HKK, DUNE, $\nu_{atm}@HK$

In the case of IH, DUNE is the best

Ghosh & OY, arXiv:1709.08264
5. Conclusions (1)

- The 3 mixing angles and the two mass squared differences have been determined in the PMNS paradigm.
- We still have to determine:
  - Mass hierarchy (important to determine $\delta$)
  - $\theta_{23}-\pi/4$
  - $\delta$
- Currently there are several hints for deviation from the standard 3 flavor scenario. Research on sterile neutrinos and NSI is in progress and it may give us a hint for physics beyond the PMNS paradigm.
5. Conclusions (2)

- The most popular scenarios beyond the PMNS paradigm are $\nu_s$ & NSI in propagation. **Matter effect** is important to search for both.

- Before HK & DUNE, $\nu_{\text{atm}}$ at HK and/or IceCube are powerful in constraining the two scenarios.

- After HK & DUNE, T2HK+DUNE is expected to cover some of the LSND region @90%CL ($\nu_s$). $\nu_{\text{atm}}$@HK, T2HKK, DUNE have sensitivity to some region of NSI which was suggested by $\nu_{\text{solar}}$. 
Allowed regions by different groups almost agree

There may be slight difference in $\theta_{23}$-region
Motivation for Non Standard Interaction in $\nu$ propagation

- There seem to be tension between solar $\nu$ & KamLAND data.
  --> NSI may be necessary to explain data.

Best fit value of global fit

$$
\begin{align*}
(\epsilon^u_D, \epsilon^u_N) &= (-0.140, -0.030) \\
(\epsilon^d_D, \epsilon^d_N) &= (-0.145, -0.036)
\end{align*}
$$

Gonzalez-Garcia, Maltoni, JHEP 1309 (2013) 152
T2K results on $\nu_e$ appearance

Sakashita@Prospects of Neutrino Physics (IPMU, 2019/4)
IceCUBE, PRD95 ('17) 112002
Future exp. vs Mass Hierarchy

Significance ($\sigma$) of MH

Years:
- 2010
- 2020
- 2030
- 2040

Experiments:
- SK-atm
- PINGU
- Nova
- INO-atm
- DUNE
- HK-atm
- T2HK

Now
**Constraints on $\varepsilon_{\alpha\beta}$ for expts on Earth**

Davidson et al., JHEP 0303:011, 2003; Berezhiani, Rossi, PLB535 (‘02) 207; Barranco et al., PRD73 (‘06) 113001; Barranco et al., arXiv:0711.0698

- $|\varepsilon_{ee}| \lesssim 4 \times 10^0$
- $|\varepsilon_{e\mu}| \lesssim 3 \times 10^{-1}$
- $|\varepsilon_{\mu\mu}| \lesssim 7 \times 10^{-2}$
- $|\varepsilon_{e\tau}| \lesssim 3 \times 10^0$
- $|\varepsilon_{\mu\tau}| \lesssim 3 \times 10^{-1}$
- $|\varepsilon_{\tau\tau}| \lesssim 2 \times 10^1$

**Constraints are weak**
Behaviors of $\chi^2$ (IH) for multi-GeV: $\nu + \bar{\nu}$ vs individual $\nu$ & $\bar{\nu}$ for $\varepsilon_{e\tau} = 0$

Destructive phenomenon between $\nu$ & $\bar{\nu} \rightarrow$ Distinction between $\nu$ & $\bar{\nu}$ gives important information on $\varepsilon_{ee}$
Behaviors of $\chi^2$ (NH) for multi-GeV: Rate VS Spectrum for $\varepsilon_{e\tau} = 0$

Destructive phenomenon between Low & High energy bins → Information on energy spectrum is important
Behaviors of #(events) for multi-GeV: $\nu+\bar{\nu}$ vs individual $\nu$ & $\bar{\nu}$

Destructive phenomenon between $\nu$ & $\bar{\nu}$

Theoretical understanding in terms of oscillation probabilities is under study.
Sensitivity of HK: (1) Complex $|\varepsilon_N|$ for NH

- Best fit point of solar & KamLAND for $f=u$: significance $38\sigma$
  \[(\varepsilon_D^u, \varepsilon_N^u) = (-0.22, -0.30)\]

- Best fit point of solar & KamLAND for $f=d$: significance $11\sigma$
  \[(\varepsilon_D^d, \varepsilon_N^d) = (-0.12, -0.16)\]

- Best fit point of global analysis for $f=u$: significance $5\sigma$
  \[(\varepsilon_D^u, \varepsilon_N^u) = (-0.140, -0.030)\]

- Best fit point of global analysis for $f=d$: significance $5\sigma$
  \[(\varepsilon_D^d, \varepsilon_N^d) = (-0.145, -0.036)\]
Sensitivity of HK: (1) Complex $|\varepsilon_N|$ for IH

$\varepsilon_D^u = (\varepsilon_N^u) = (-0.22, -0.30)$

Best fit point of solar & KamLAND for $f=u$:
significance: $35\sigma$

$\varepsilon_D^d = (\varepsilon_N^d) = (-0.12, -0.16)$

Best fit point of solar & KamLAND for $f=d$:
significance: $8\sigma$

$\varepsilon_D^u = (\varepsilon_N^u) = (-0.140, -0.030)$

Best fit point of global analysis for $f=u$:
significance: $1.4\sigma$

$\varepsilon_D^d = (\varepsilon_N^d) = (-0.145, -0.036)$

Best fit point of global analysis for $f=d$:
significance: $1.5\sigma$
Sensitivity of HK: (2) Real $|\epsilon_N|$
Dependence of T2HKK on $\theta_{23}$(true) & $\delta$(true)

Ghosh & OY, arXiv:1709.08264
Dependence of DUNE on $\theta_{23}(\text{true})$ & $\delta(\text{true})$

Ghosh & OY, arXiv:1709.08264