Cosmology 2019 session of the Rencontres du Vietnam

Parameterized Post-Friedmann Framework for Interacting Dark Energy Model

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Outline

- Interacting dark energy (IDE) model and the large-scale instability
- Parameterized Post-Friedmann (PPF) framework for IDE Scenario
- System divergence when w closes to -1 in PPF framework
- Conclusion

Interacting dark energy (IDE) model and the large-scale instability

- A brief review of cosmology perturbation theory [Ma & Bertschinger 1995]

When consider the background evolution

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

FRW matrix

stress-energy tensor

Interacting dark energy (IDE) model and the large-scale instability

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FRW matrix | stress-energy tensor

Then we have the Friedmann equations which describe the evolution of scale factor (a flat universe),

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \,, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \,. \longrightarrow \dot{\rho} + 3H(\rho + P) = 0$$

Three variables and only two independent equations. We need another equation

$$w = P/\rho$$

Interacting dark energy (IDE) model and the large-scale instability

- A brief review of cosmology perturbation theory

The same as background evolution, cosmology perturbation evolution equations have the same process but more complex.
We focus on scalar perturbation in the IDE model. The perturbed metric can be expressed as

$$\delta g_{00} = -2a^2 A,$$

$$\delta g_{0i} = -a^2 B_{,i},$$

$$\delta g_{ij} = a^2(2H_L \delta_{ij} + 2D_{ij} H_T),$$

Interacting dark energy (IDE) model and the large-scale instability

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The perturbation of the stress-energy tensor can be expressed by another four variables.

$$\delta T^0_{\ 0} = -\delta\rho,$$

$$\delta T^i_{\ 0} = -(\bar{\rho} + \bar{p})\partial^i v,$$

$$\delta T^i_{\ j} = \delta p \delta^i_{\ j} + \Pi^i_{\ j}.$$

Interacting dark energy (IDE) model and the large-scale instability

➢ A brief review of cosmology perturbation theory

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$$

Then we should be careful to take the above perturbations into perturbed Einstein equation,

$$\nabla^2 H_L + 3\mathcal{H}(\mathcal{H}A - H_L') - \mathcal{H}\nabla^2 B - \frac{1}{2}\partial_j\partial^i D_i^j H_T = -4\pi G a^2 \delta\rho ,$$

$$\mathcal{H}\nabla^2 A - \nabla^2 H_L' + \frac{1}{2}\partial_j\partial^i D_i^j H_T' + 4\pi G a^2 (P+\rho)(\theta - kB) = 0 ,$$

$$-\partial^i\partial_j[H_L + A - 2\mathcal{H}(B + H_T') - (B + H_T')'] + \frac{1}{3}\nabla^2 D_j^i H_T = 8\pi G a^2 \Pi_j^i ,$$

$$\mathcal{H}A' + (2\mathcal{H}' + \mathcal{H}^2)A + \frac{1}{3}\nabla^2(A + H_L) - 2\mathcal{H}H_L' - H_L'' - \frac{2}{3}\mathcal{H}\nabla^2 B - \frac{1}{3}\nabla^2 B' - \frac{1}{6}\partial^i\partial_j D_i^j H_L = 4\pi G a^2 \delta P .$$

Interacting dark energy (IDE) model and the large-scale instability

- A brief review of cosmology perturbation theory

We can also derive the perturbation evolutions using conservation law $\nabla_\mu T^\mu_\nu = 0$

Interacting dark energy (IDE) model and the large-scale instability

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When ν=0, we have continuity equation

$$\delta\rho' + 3\mathcal{H}(\delta\rho + \delta P) + (\rho + P)(3H_L' + kv) = 0$$

When ν=i, we have Euler equation

$$[(\rho + P)(v - B)]' + 4\mathcal{H}(\rho + P)(v - B) - k\delta P + \frac{2}{3}k\Pi - (\rho + P)kA = 0\,.$$

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A, B, H_L, H_T two of them are independent

$\delta\rho$, δP, v, Π

} 6 variables

four Einstein equations and the above two equations
but only four of them are independent

} 4 equations

Interacting dark energy (IDE) model and the large-scale instability

- Interacting dark energy model

Why Interacting? Cosmic coincidence problem

How to consider interaction? transfer rate: Q $\nabla_\nu T_I^{\mu\nu} = Q_I^\mu, \; \sum_I Q_I^\mu = 0$

Interacting dark energy (IDE) model and the large-scale instability

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A general form $Q_\mu^I = a(-Q_I(1 + A) - \delta Q_I, [f_I + Q_I(v - B)]_{,i})$ [Kodama & Sasaki 1984]

The energy and momentum perturbation equations are modified.

$$\delta\rho_I' + 3\mathcal{H}(\delta\rho_I + \delta P_I) + (\rho_I + P_I)(3H_L' + \theta_I) = a(\delta Q_I + AQ_I)\,,$$

$$[(\rho_I + P_I)(v_I - B)]' + 4\mathcal{H}(\rho_I + P_I)(v_I - B) - k[\delta P_I + (\rho_I + P_I)A] + \frac{2}{3}k\Pi = a(f_I + Q_I(v_I - B))\,.$$

Einstein equations are unchanged.

Interacting dark energy (IDE) model and the large-scale instability

- How to complete the equations? Traditional method

1. For dark energy, $\Pi_{de}=0$

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$$c_s^2 = \left.\frac{\delta p}{\delta\rho}\right|_{rf} \qquad c_a^2 = \frac{p'}{\rho'} = w_I + \frac{w_I'}{\rho_I'/\rho_I}$$

$$\delta p = c_a^2\delta\rho + (c_s^2 - c_a^2)\left(\delta\rho - \bar{\rho}'\frac{v - B}{k}\right)$$

Interacting dark energy (IDE) model and the large-scale instability

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$$\delta p = c_a^2 \delta\rho + (c_s^2 - c_a^2)\left(\delta\rho - \bar{\rho}'\frac{v-B}{k}\right)$$

Then the energy and momentum equations become

$$\delta_I' + 3\mathcal{H}(c_{s,I}^2 - w_I)\delta_I + (1+w_I)kv_I + 9\mathcal{H}^2(c_{s,I}^2 - c_{a,I}^2)(1+w_I)\frac{v_I - B}{k} + 3(1+w_I)H_L'$$
$$= \frac{aQ_I}{\rho_I}\left[A - \delta_I + 3\mathcal{H}(c_{s,I}^2 - c_{a,I}^2)\frac{v_I - B}{k}\right] + \frac{a}{\rho_I}\delta Q_I$$

$$(v_I - B)' + \mathcal{H}(1 - 3c_{s,I}^2)(v_I - B)) - \frac{c_{s,I}^2}{1+w_I}k\delta_I - kA$$
$$= \frac{aQ_I}{(1+w_I)\rho_I}\left[v - B - (1 + c_{s,I}^2)(v - B)\right] + \frac{af_I}{(1+w_I)\rho_I}$$

Interacting dark energy (IDE) model and the large-scale instability

- Large-scale instability [Väliviita et al. 2008]

In Newtonian gauge and early radiation era, with a phenomenological model

$$Q_c^\nu = -Q_{de}^\nu = -3\beta H \rho_c u_c^\nu$$

Non-adiabatic initial conditions

$$\Phi = A_\Phi (k\eta)^{n_\Phi},\ \Psi = A_\Psi (k\eta)^{n_\Psi},$$
$$\delta_I = B_I (k\eta)^{n_I},\ kv_I = C_I (k\eta)^{s_I}.$$

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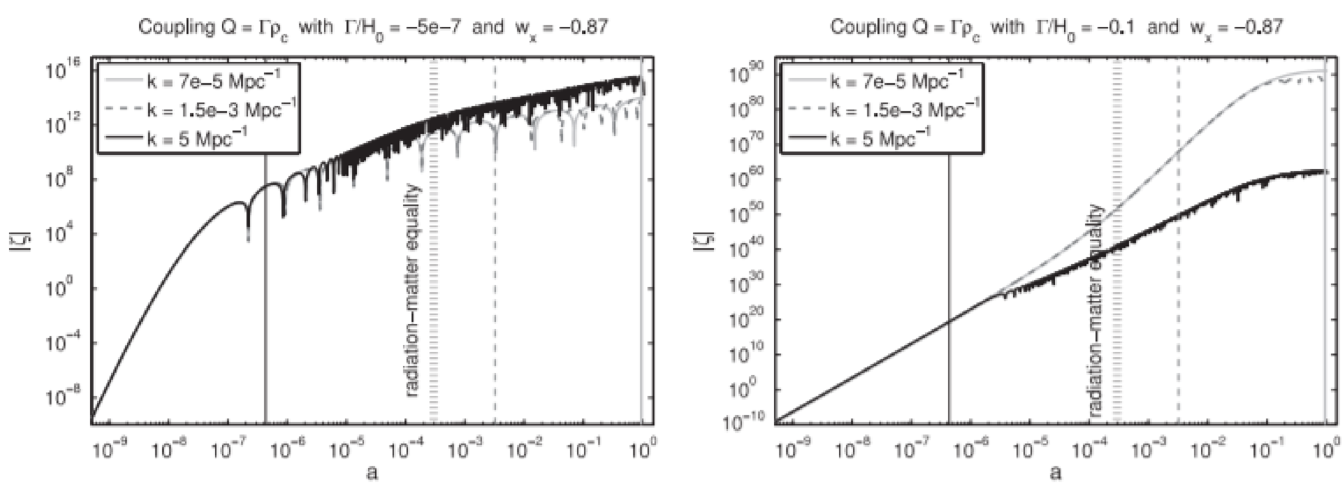
$$\delta_I = B_I(k\eta)^{n_I},\ kv_I = C_I(k\eta)^{s_I}.$$

Take them into the former equations, we get

$$n_{\Phi} = \frac{-(1+2w) \pm \sqrt{3w^2-2}}{1+w} \quad \text{when } -1 < w < -\sqrt{\frac{2}{3}},\ n_{\Phi} > 0$$



Väliviita et al. 2008

Parameterized Post-Friedmann (PPF) framework for IDE Scenario

A parallel treatment of parameterized dark energy.

- How to complete the equations? ~~fluid c_a and c_s~~ PPF approach [Hu 2008]

Under comoving gauge $\rho\Delta \equiv \delta\rho$, and the subscript T is used to refer to all the components without dark energy

larger scales: Parameterization of effective dark energy

$$\lim_{k_H \ll 1} \frac{4\pi G a^2}{\mathcal{H}^2}(\rho_{de} + p_{de})\frac{V_{de} - V_T}{k_H} = -\frac{1}{3} f_\zeta(a) k_H V_T$$

smaller scales: Poisson-like equation

$$\Phi = 4\pi G a^2 \rho_T \Delta_T / k^2$$

Use a dynamical variable Γ to meet the two conditions.

$$\Phi + \Gamma = \frac{4\pi G a^2}{k^2}\rho_T \Delta_T$$

Parameterized Post-Friedmann (PPF) framework for IDE Scenario

➢ How to obtain the differential equation of Γ?

1. $k_H \gg 1 \quad \Gamma \to 0$

2. $k_H \ll 1$ take the derivative of both sides of $\Phi + \Gamma = \frac{4\pi G a^2}{k^2}\rho_T \Delta_T$

$$\lim_{k_H \ll 1} \Gamma' = S - \mathcal{H}\Gamma$$

[Hu 2008, Li et al. 2014]

$$S = \frac{4\pi G a^2}{k^2}\left\{[(\rho_{de} + P_{de}) - f_\zeta(\rho_T + P_T)]kV_T + \frac{3a}{k_H}[Q_c(V - V_T) + f_c] + a(\Delta Q_c + \xi Q_c)\right\}$$

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$$S = \frac{4\pi G a^2}{k^2}\left\{[(\rho_{de} + P_{de}) - f_\zeta(\rho_T + P_T)]kV_T + \frac{3a}{k_H}[Q_c(V - V_T) + f_c] + a(\Delta Q_c + \xi Q_c)\right\}$$

So we can obtain the differential equation of Γ

$$(1 + c_\Gamma^2 k_H^2)[\Gamma' + \mathcal{H}\Gamma + c_\Gamma^2 k_H^2 \mathcal{H}\Gamma] = S$$

The energy density and velocity perturbations of dark energy are

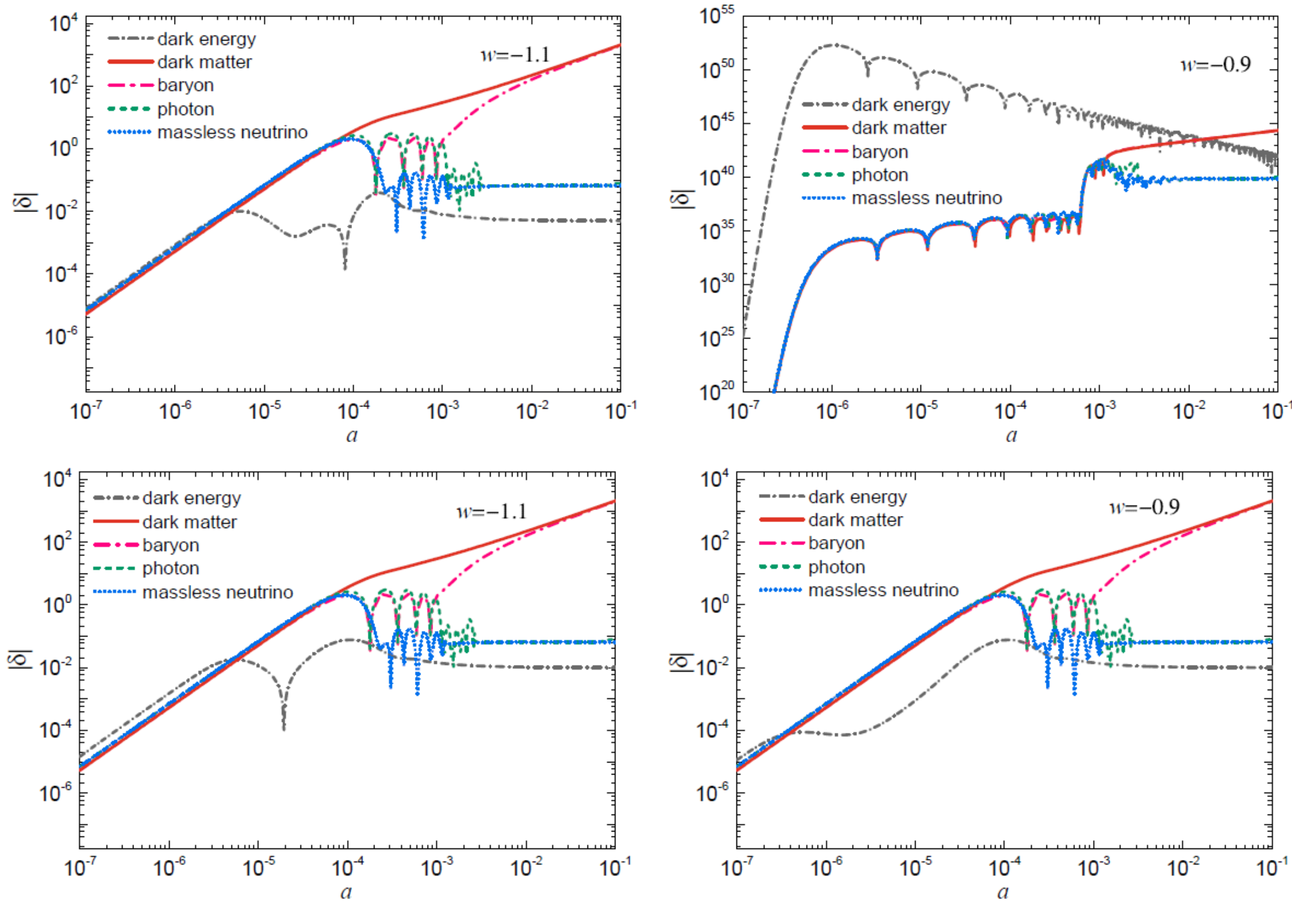
$$\rho_{de}\Delta_{de} = -3(\rho_{de} + P_{de})\frac{V_{de} - V_T}{k_H} - \frac{k^2\Gamma}{4\pi G a^2}$$

$$V_{de} - V_T = \frac{-k}{4\pi G a^2(\rho_{de} + P_{de})F}\left[S - \Gamma' - \mathcal{H}\Gamma + f_\zeta \frac{4\pi G a^2(\rho_{de} + P_{de})}{k}V_T\right]$$

Parameterized Post-Friedmann (PPF) framework for IDE Scenario

PPF framework can avoid the large-scale instability mentioned above.

$$Q_c^{\nu} = -Q_{de}^{\nu} = -3\beta H \rho_c u_c^{\nu}$$



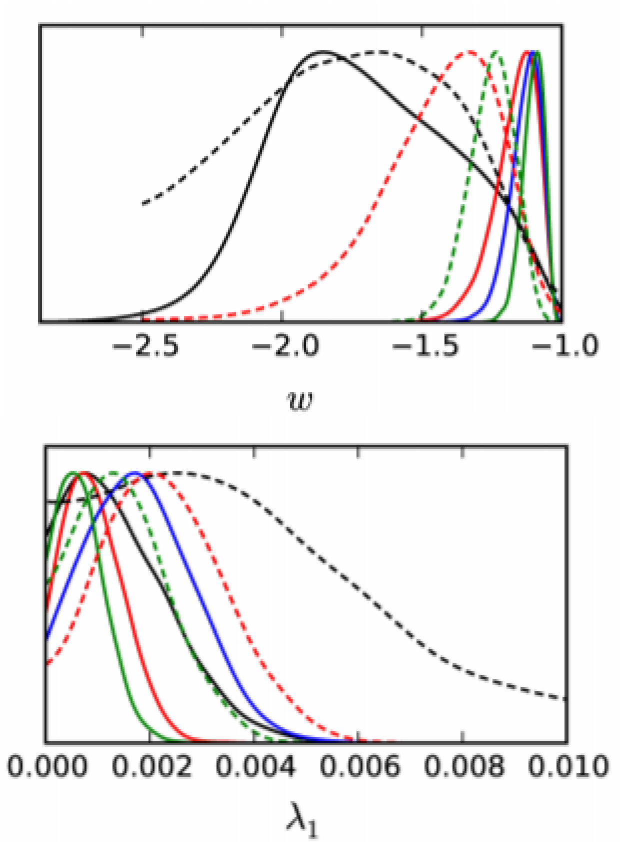
Non-adiabatic fluid

PPF framework

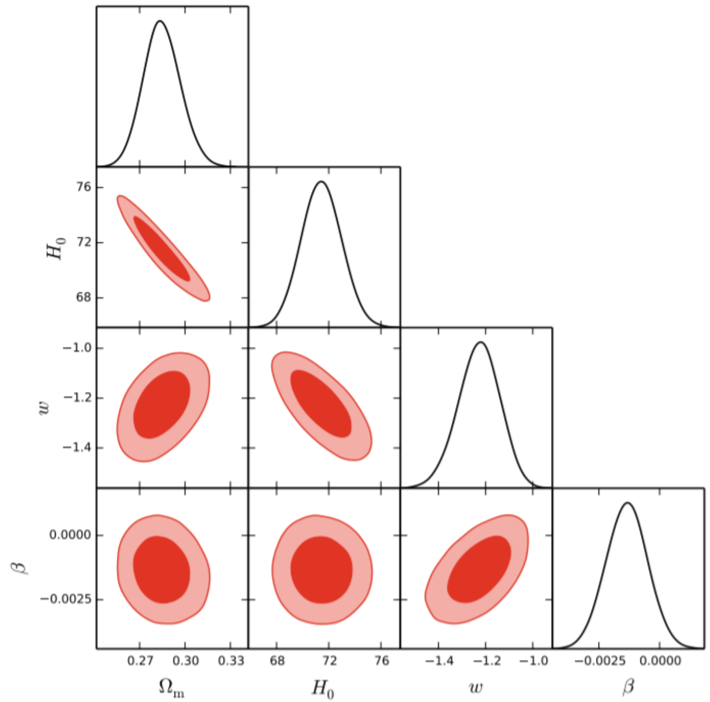
[Li et al. 2014]

Parameterized Post-Friedmann (PPF) framework for IDE Scenario

➢ The advantage of PPF — constrain *w* globally



[Costa et al. 2014]



[Li et al. 2014]

System divergence when w closes to -1 in PPF framework

Now consider the perturbation equations of dark energy again

$$\rho_{de}\Delta_{de} = -3(\rho_{de} + P_{de})\frac{V_{de} - V_T}{k_H} - \frac{k^2\Gamma}{4\pi G a^2}$$

$$V_{de} - V_T = \frac{-k}{4\pi G a^2(\rho_{de} + P_{de})F}\left[S - \Gamma' - \mathcal{H}\Gamma + f_\zeta \frac{4\pi G a^2(\rho_{de} + P_{de})}{k} V_T\right]$$

$\rho_{de}(1+w)$!!! Diverge when w closes to -1

System divergence when w closes to -1 in PPF framework

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$\rho_{de}(1+w)$!!! Diverge when w closes to -1

Dose this divergence really matters?

$\lim_{k_H \ll 1} \Gamma' = S - \mathcal{H}\Gamma$, so it is safe when $k_H \ll 1$

In our analysis, $f_\zeta = 0$ and $\Gamma \to 0$ when $k_H \gg 1$

$$S - \Gamma' - \mathcal{H}\Gamma = S$$

$$S = \frac{4\pi G a^2}{k^2}\left\{[(\rho_{de} + P_{de}) - f_\zeta(\rho_T + P_T)]kV_T + \frac{3a}{k_H}[Q_c(V - V_T) + f_c] + a(\Delta Q_c + \xi Q_c)\right\}$$

System divergence when w closes to -1 in PPF framework

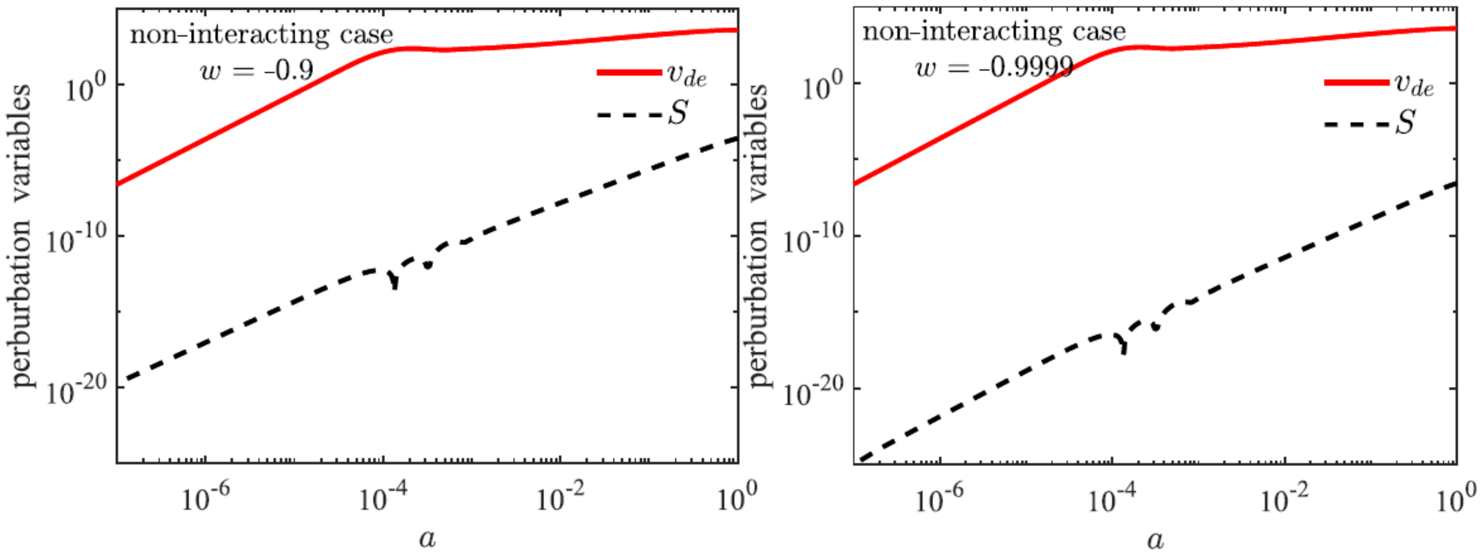
$k_H \ll 1 \qquad S - \Gamma' - \mathcal{H}\Gamma = 0$

$k_H \gg 1 \qquad S - \Gamma' - \mathcal{H}\Gamma = S$

$$V_{de} - V_T = \frac{-k}{4\pi G a^2(\rho_{de} + P_{de})F}\left[S - \Gamma' - \mathcal{H}\Gamma + f_\zeta \frac{4\pi G a^2(\rho_{de} + P_{de})}{k} V_T\right]$$

Non-interacting case

$$S = \frac{4\pi G a^2}{k^2}\{[(\rho_{de} + p_{de}) - f_\zeta(\rho_T + p_T)]kV_T\}$$ proportional to $\rho_{de}(1 + w)$



k=0.1 Mpc^{-1}

The system are not divergence

$k_H \ll 1 \qquad S - \Gamma' - \mathcal{H}\Gamma = 0$

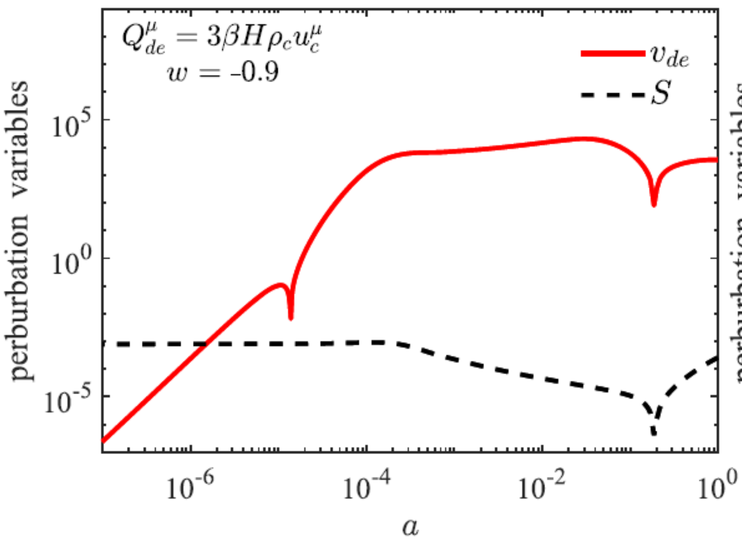
$k_H \gg 1 \qquad S - \Gamma' - \mathcal{H}\Gamma = S$

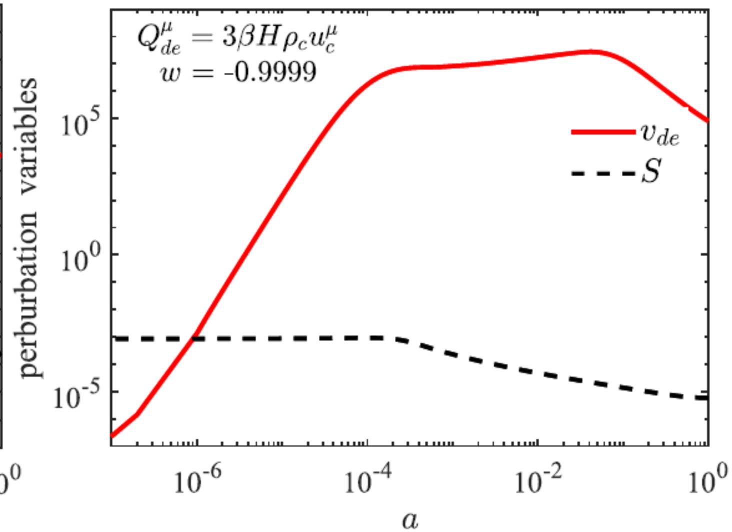
$$V_{de} - V_T = \frac{-k}{4\pi G a^2(\rho_{de} + P_{de})F}\left[S - \Gamma' - \mathcal{H}\Gamma + f_\zeta \frac{4\pi G a^2(\rho_{de} + P_{de})}{k} V_T\right]$$

Interacting case

May cause divergence

$$S = \frac{4\pi G a^2}{k^2}\left\{[(\rho_{de} + P_{de}) - f_\zeta(\rho_T + P_T)]kV_T + \boxed{\frac{3a}{k_H}[Q_c(V - V_T) + f_c] + a(\Delta Q_c + \xi Q_c)}\right\}$$





System divergence when w closes to -1 in PPF framework

- The effects of divergence of $v_{\rm de}$

When consider a interacting model $Q^{\mu}_{de} = 3\beta H\rho_c u^{\mu}_c$
perturbed continuity and Euler equations of dark matter are

$$\delta'_c + kv_c + 3H'_L = -3aH\beta A,$$
$$(v_c - B)' + \mathcal{H}(v_c - B) - kA = 0$$

But for another model $Q^{\mu}_{de} = 3\beta H\rho_c u^{\mu}_{de}$

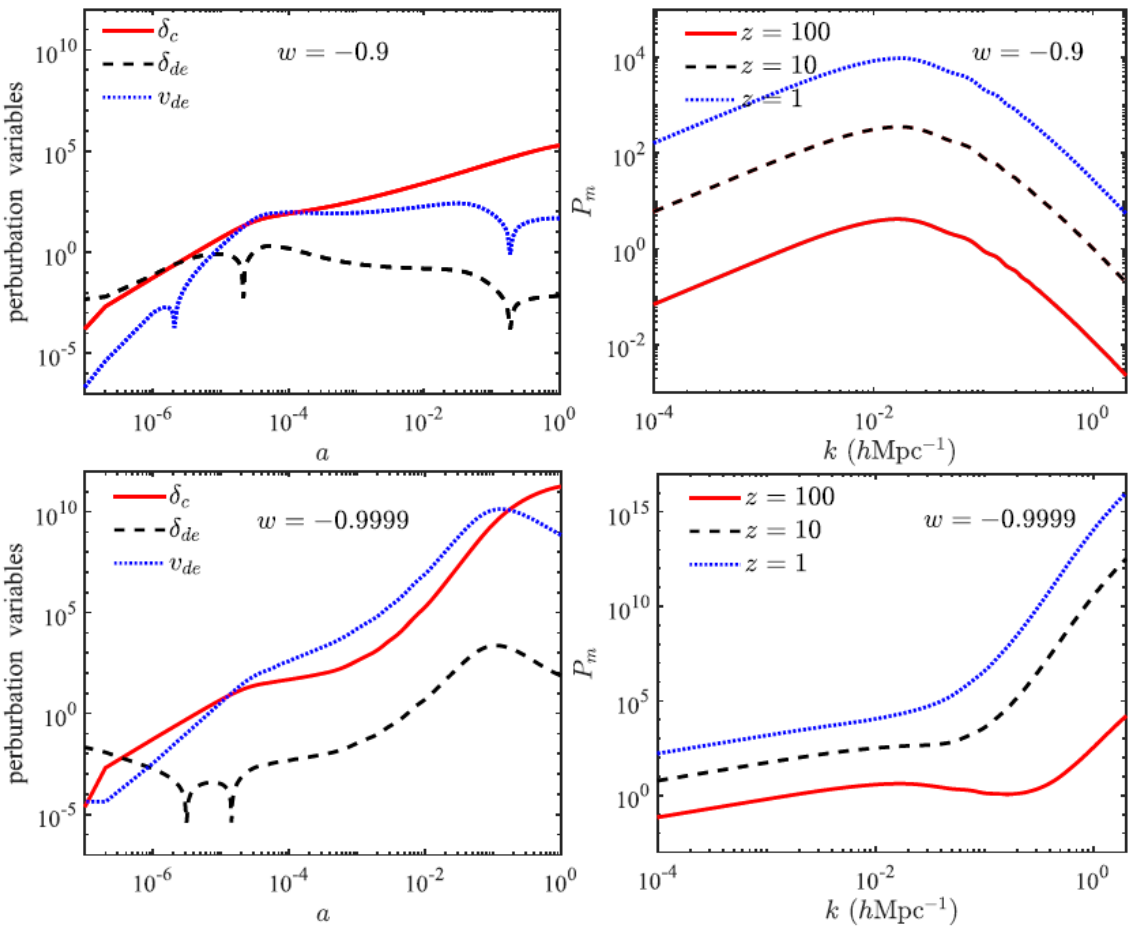
$$\delta'_c + \boxed{kv_c} + 3H'_L = -3aH\beta A,$$
$$(v_c - B)' + \mathcal{H}(v_c - B) - kA = 3aH\beta(v_c - \boxed{v_{de}})$$

The divergence of $v_{\rm de}$ may affect the evolution of dark matter, which may cause the divergence of density perturbation of dark matter

This condition occurs when Q is proportional to $u_{\rm de}$

System divergence when *w* closes to −1 in PPF framework

$$Q_{de}^{\mu} = 3\beta H \rho_c u_{de}^{\mu}$$



Perturbation variables Matter power spectrum

System divergence when w closes to -1 in PPF framework

- How to avoid this divergence?

First check the widely used IDE model

$$Q_c^\nu = -Q_{de}^\nu = -3\beta H\rho_c u_c^\nu \qquad Q_\mu^I = a(-Q_I(1+A) - \delta Q_I, [f_I + Q_I(v-B)]_{,i})$$

the energy and momentum transfer perturbation are

$$Q_{de} = -Q_c = 3\beta H\rho_c,$$
$$\delta Q_{de} = -\delta Q_c = 3\beta H\rho_c\delta_c,$$
$$f_{de} = -f_c = 3\beta H\rho_c(v_c - v).$$

The main reason of diverging

System divergence when w closes to -1 in PPF framework

- How to avoid this divergence?

First check the widely used IDE model

$$Q_c^\nu = -Q_{de}^\nu = -3\beta H\rho_c u_c^\nu \qquad Q_\mu^I = a(-Q_I(1+A) - \delta Q_I, [f_I + Q_I(v-B)]_{,i})$$

the energy and momentum transfer perturbation are

$$Q_{de} = -Q_c = 3\beta H\rho_c,$$
$$\delta Q_{de} = -\delta Q_c = 3\beta H\rho_c\delta_c,$$
$$f_{de} = -f_c = 3\beta H\rho_c(\boxed{v_c} - v).$$

The main reason of diverging

Can we construct a model that momentum transfer perturbation is proportional to $(1+w)v$?
If w closes to -1, $(1+w)v$ will not diverge.

System divergence when w closes to -1 in PPF framework

- How to avoid this divergence? $Q^I_\mu = a(-Q_I(1+A) - \delta Q_I, [f_I + Q_I(v-B)]_{,i})$

energy transfer perturbation is proportional to $\rho_J \delta_J$, which is $\delta T^0_{\ 0,J}$

$$Q_I \mathrm{A} + \delta Q_I \propto \rho_J \delta_J + \rho_J A$$

momentum transfer perturbation is proportional to $(\rho_J + p_J) v_J$, which is $\delta T^i_{\ 0,J}$

$$f_I + Q_I(v - B) \propto (\rho_J + p_J)(v_J - B)$$

System divergence when w closes to -1 in PPF framework

- How to avoid this divergence? $Q^I_\mu = a(-Q_I(1+A) - \delta Q_I, [f_I + Q_I(v-B)]_{,i})$

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$$f_I + Q_I(v-B) \propto (\rho_J + p_J)(v_J - B)$$

The final formats are

$$Q_{de} = -Q_c = 3\beta H\rho_x$$

$$\delta Q_{de} = -\delta Q_c = 3\beta H\delta\rho_x$$

$$Q_{de}(v-B) + f_{de} = -Q_c(v-B) - f_c = 3\beta H(\rho_x + p_x)(v_x - B)$$

If x=c, the above results are same as the widely used case.
If x=de, the only difference is momentum transfer perturbation, which is proportional to $(1+w)v_{\rm de}$ rather than $v_{\rm de}$

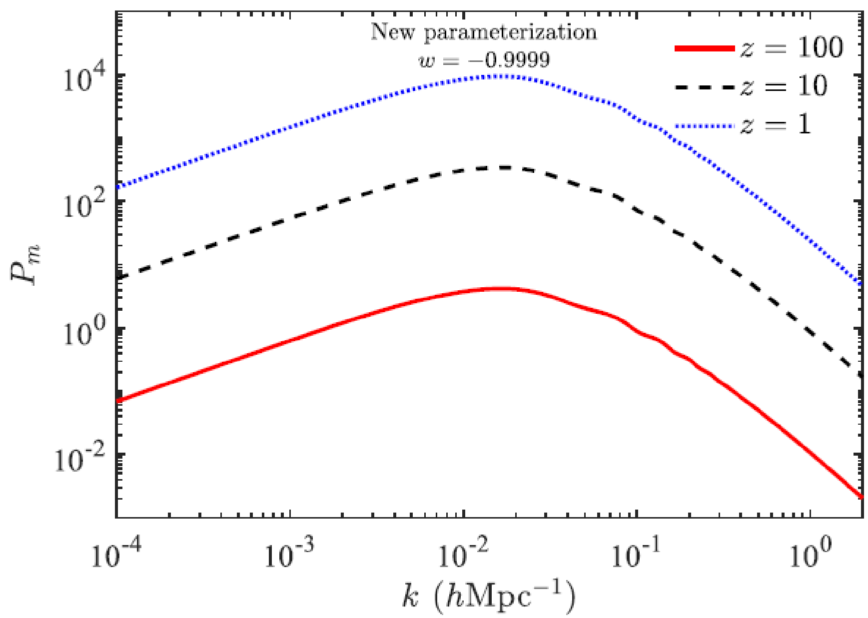
System divergence when w closes to -1 in PPF framework

- How to avoid this divergence?

If x=de, the perturbed continuity and Euler equations will be

$$\delta_c' + kv_c + 3H_L' = -\frac{3a\beta H\rho_{de}A}{\rho_c}$$

$$(v_c - B)' + \mathcal{H}(v_c - B) - kA$$
$$= \frac{3a\beta H\rho_{de}}{\rho_c}(v_c - B) - \frac{3(1+w)a\beta H\rho_{de}}{\rho_c}(v_{de} - B)$$



System divergence when w closes to -1 in PPF framework

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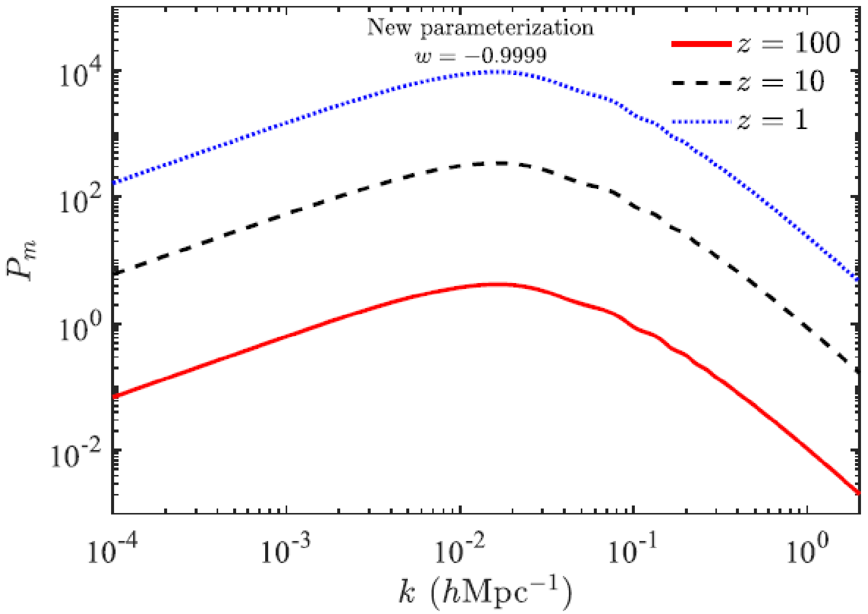
$$(v_c - B)' + \mathcal{H}(v_c - B) - kA = \frac{3a\beta H\rho_{de}}{\rho_c}(v_c - B) - \frac{3(1+w)a\beta H\rho_{de}}{\rho_c}(v_{de} - B)$$



A general parameterization

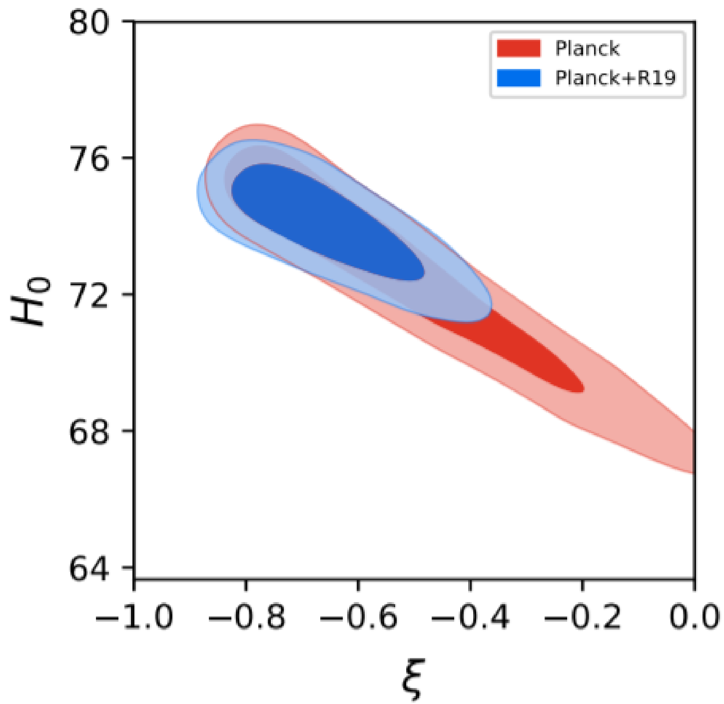
$$Q_{de} = -Q_c = C_1\rho_c + C_2\rho_{de},$$

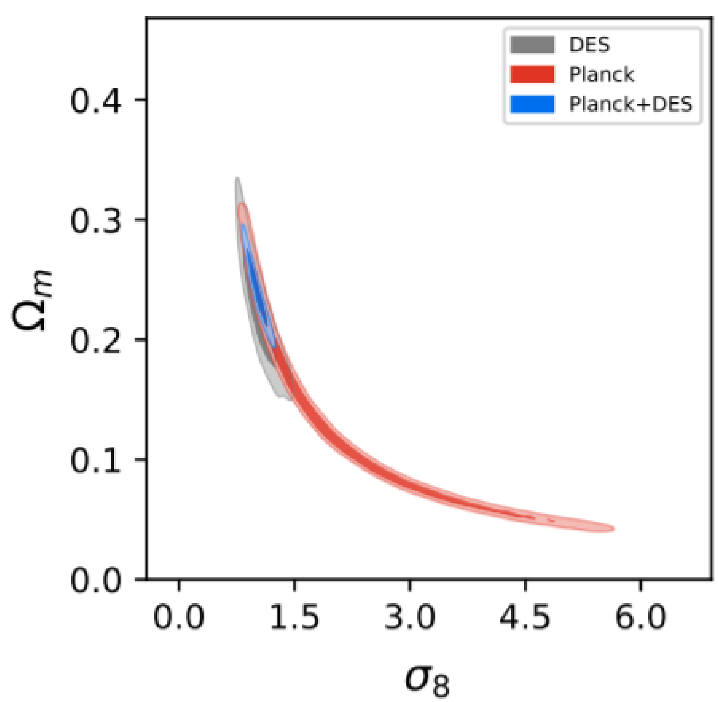
$$\delta Q_{de} = -\delta Q_c = D_1\delta\rho_c + D_2\delta\rho_{de},$$

$$Q_{de}(v - B) + f_{de} = -Q_c(v - B) - f_c = E_1(\rho_c + p_c)(v_c - B) + E_2(\rho_{de} + p_{de})(v_{de} - B)$$

Interacting dark energy after the latest *Planck*, *DES*, and H_0 measurements: an excellent solution to the H_0 and cosmic shear tensions

Eleonora Di Valentino,[1,*] Alessandro Melchiorri,[2,3,†] Olga Mena,[4,‡] and Sunny Vagnozzi[5,6,7,§]





Conclusion

- As the dominant components of the universe, dark energy may interact with cold dark matter in a direct, non-gravitational way, which can help overcome several theoretical problems.

- Dark energy is usually considered as a non-adiabatic fluid. The perturbation equations are completed by defining the sound speed and adiabatic sound speed. However this may cause the early time large-scale instability.

- Under PPF framework, there is no need to calculate the density and velocity perturbations, which can avoid the instability successfully.

- However under PPF framework, when Q is proportional to u_{de}, the system may be diverging when w close to -1. We propose a general parameterization which can be used safely. The interaction between dark matter and dark energy may be further explored by using the general parameterization in future work.

Thank you!

References

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