


# Constraints on massive vector dark energy models from ISW-galaxy cross-correlations

Shintaro Nakamura (Tokyo University of Science)

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In collaboration with A. De Felice (YITP), R. Kase (TUS), and S. Tsujikawa (TUS)

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# Outline

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- Introduction to vector-tensor theories
- ISW effect as probe of dark energy models
- Observational constraints
- Summary

# Introduction

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- Our universe is accelerating now.
  - ⇒ The source of this acceleration is dubbed “dark energy (DE)”

GR

- $\Lambda$ CDM model ⇒ cosmological constant,  $H_0$  tension...

spin0

- Scalar-tensor theories ⇒ scalar field coupled to gravity  
(e.g.) Horndeski theories

spin1

- Vector-tensor theories ⇒ vector field coupled to gravity  
(e.g.) Generalized Proca theories

# Horndeski theories: a scalar field coupled to gravity

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The most general scalar-tensor theories with 2nd-order EoMs:

$$\begin{aligned}\mathcal{L}_H = & G_2(\pi, X) + G_3(\pi, X) \square\pi + G_4(\pi, X) R + G_{4,X}(\pi, X)[(\square\pi)^2 - \pi^{;\mu\nu}\pi_{;\mu\nu}] \\ & + G_5(\pi, X)G_{\mu\nu}\pi^{;\mu\nu} - \frac{1}{6}G_{5,X}(\pi, X)[(\square\pi)^3 - 3(\square\pi)\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\sigma}\pi^{;\nu}_{;\sigma}]\end{aligned}$$

where

$\pi$  : scalar field with kinetic energy  $X = -\nabla_\mu\pi\nabla^\mu\pi/2$ ,

$G_2, G_3, G_4, G_5$  : arbitrary function

$R$  : Ricci scalar,  $G_{\mu\nu}$  : Einstein tensor

DOFs: 2 tensors + 1 scalar



# Horndeski theories: a scalar field coupled to gravity

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The GW170817 event constrained the speed of gravitational waves to be very close to that of light.



Demanding that  $\mathcal{C}_T = \mathcal{C}$

$$\mathcal{L}_H = G_2(\pi, X) + G_3(\pi, X) \square\pi + G_4(\pi) R$$

In GR,  $G_4 = M_{\text{pl}}^2/2$

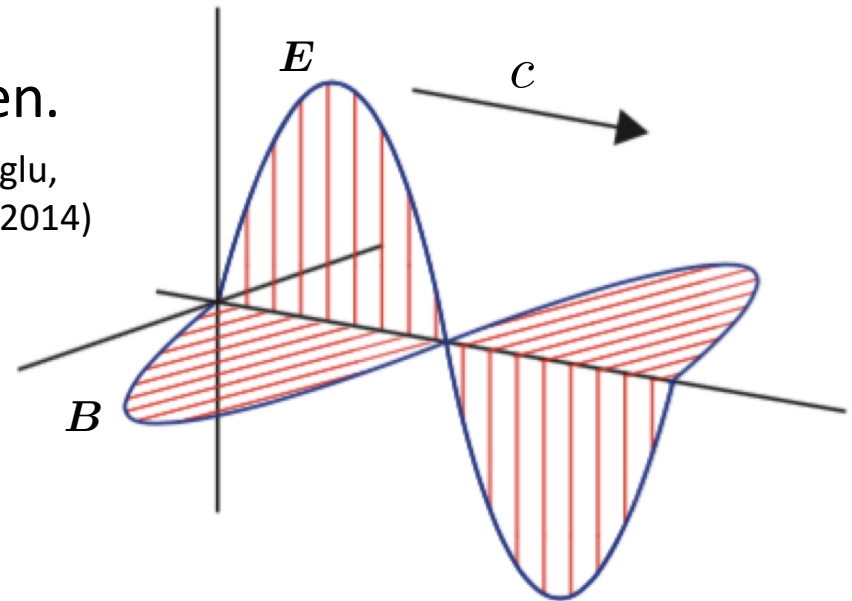
# Degrees of freedom for vector fields (Minkowski space-time)

- U(1) gauge field (massless)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$$

- ⇒ DOFs: 2 transverse polarizations.
- ⇒ If we keep U(1) gauge symmetry, Galileon-like interactions are forbidden.

C. Deffayet, A. E. Gumrukcuoglu,  
S. Mukohyama and Y. Wang (2014)



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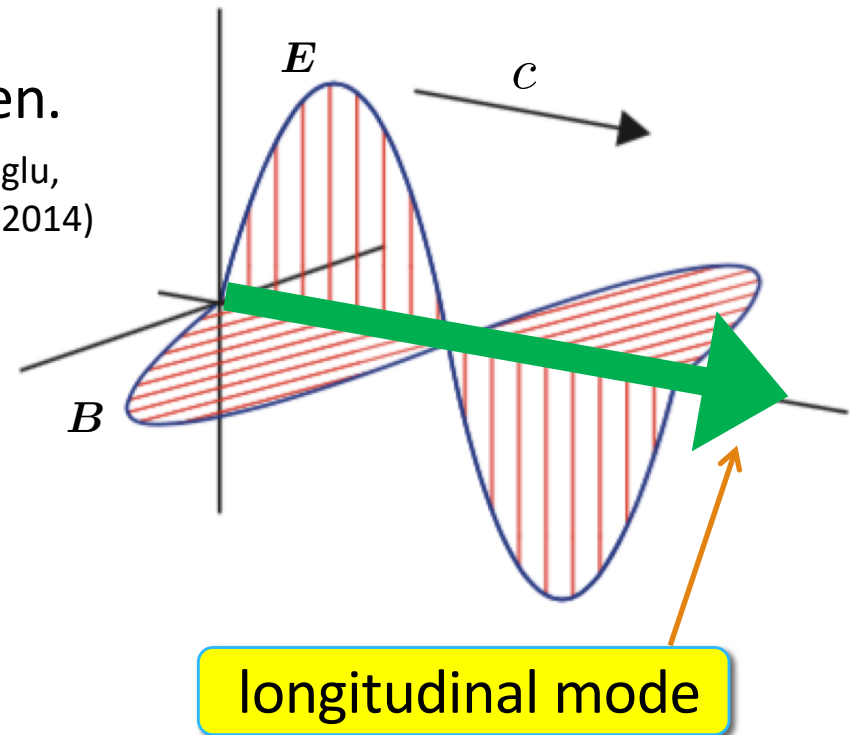
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- Proca field (massive) (3 DOFs)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu,$$

- ⇒ U(1) symmetry is broken.
- ⇒ DOFs: 2 transverse polarizations  
+ 1 longitudinal mode.



# Generalized Proca theories: a vector field coupled to gravity

G. Tasinato (2014), L. Heisenberg (2014), J. B. Jimenez and L. Heisenberg (2016)

$$\mathcal{L}_{\text{GP}} = \sum_{i=2}^6 \mathcal{L}_i,$$

Intrinsic vector mode

$$\mathcal{L}_2 = G_2(X, F, Y),$$

$$\mathcal{L}_3 = G_3(X) \nabla_\mu A^\mu,$$

The EOMs are second-order on general space-time with 5 DOFs.

$$\mathcal{L}_4 = G_4(X)R + G_{4,X}(X) [(\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho],$$

$$\mathcal{L}_5 = G_5(X)G_{\mu\nu} \nabla^\mu A^\nu - \frac{1}{6}G_{5,X}(X)[(\nabla_\mu A^\mu)^3 - 3\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma] - g_5(X) \tilde{F}^{\alpha\mu} \tilde{F}^\beta{}_\mu \nabla_\alpha A_\beta,$$

$$\mathcal{L}_6 = G_6(X)L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2}G_{6,X}(X) \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu,$$

Intrinsic vector modes

where  $X = -\frac{A_\mu A^\mu}{2}$ ,  $F = -\frac{F_{\mu\nu} F^{\mu\nu}}{4}$ ,  $Y = A^\mu A^\nu F_\mu{}^\alpha F_{\nu\alpha}$ ,  $\tilde{F}^{\mu\nu} = \frac{1}{2}\mathcal{E}^{\mu\nu\alpha\beta} F_{\alpha\beta}$ ,  $L^{\mu\nu\alpha\beta} = \frac{1}{4}\mathcal{E}^{\mu\nu\rho\sigma} \mathcal{E}^{\alpha\beta\gamma\delta} R_{\rho\sigma\gamma\delta}$ .

$\mathcal{E}^{\mu\nu\alpha\beta}$  : Levi-Civita tensor

- DOFs: 2 tensors + 2 vectors + 1 scalar
- In the limit ( $A_\mu \rightarrow \nabla_\mu \phi$ ),  $\mathcal{L}_{\text{GP}}$  reduces to the shift-symmetric Horndeski theories.

# Generalized Proca theories: a vector field coupled to gravity

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$$C_T = C$$

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~~$$-g_5(X) \tilde{F}^{\alpha\mu} \tilde{F}^\beta{}_\mu \nabla_\alpha A_\beta,$$~~

$$\mathcal{L}_6 = G_6(X) L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2} G_{6,X}(X) \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu,$$

Intrinsic vector modes

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# Cosmological background in Generalized Proca theories

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Action: 
$$\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{pl}}^2}{2} + G_2(X, F, Y) + G_3(X) \nabla_\mu A^\mu + \mathcal{L}_6 \right] + \mathcal{S}_M,$$

Metric: 
$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j.$$

$$c_T = c$$

A vector field with the perturbations:

$$X \equiv -A_\mu A^\mu / 2 = \phi^2 / 2$$

$$A^\mu = \left( \phi(t) + \delta\phi, \frac{1}{a^2} \delta^{ij} \partial_j \chi_V + E^i \right) \Leftrightarrow \text{From isotropy of the background}$$

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The background equations of motion:

$$\phi = \phi(H)$$

$$g_{00} : 3M_{\text{pl}}^2 H^2 = -G_2 + \rho_m + \rho_r, \quad A^\mu : \phi (G_{2,X} + 3G_{3,X} H \phi) = 0,$$

- **Intrinsic vector modes do not appear** at the background level.
- $\phi \neq 0 \Rightarrow$  There exist **de Sitter solutions** characterized by  $\phi = \text{constant}$  and  $H = \text{constant}$ .

# A simple dark energy model in vector-tensor theories

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$$\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{pl}}^2}{2} R + F + G_2(X) + G_3(X) \nabla_\mu A^\mu \right] + \mathcal{S}_M ,$$

where

$$G_2(X) = b_2 X^{p_2} , \quad G_3 = b_3 X^{p_3} , \quad b_2, b_3, p_2, p_3 : \text{constants.}$$

→ The cubic vector Galileon corresponds to the case with  $p_2 = p_3 = 1$ .

When  $\phi \neq 0$ , there are the solutions characterized by

$$\phi^p H = \text{constant} \quad \text{with} \quad p = 1 - 2p_2 + 2p_3 > 0$$

→  $\phi$  grows with the decrease of  $H$  to give rise to the late-time cosmic acceleration.



# Dark energy equation of state in our model

$$w_{\text{DE}} = -\frac{3(1+s) + s\Omega_r}{3(1+s\Omega_{\text{DE}})} \rightarrow$$

Radiation era :  $w_{\text{DE}} = -1 - 4s/3$

Matter era :  $w_{\text{DE}} = -1 - s$

de Sitter era :  $w_{\text{DE}} = -1$

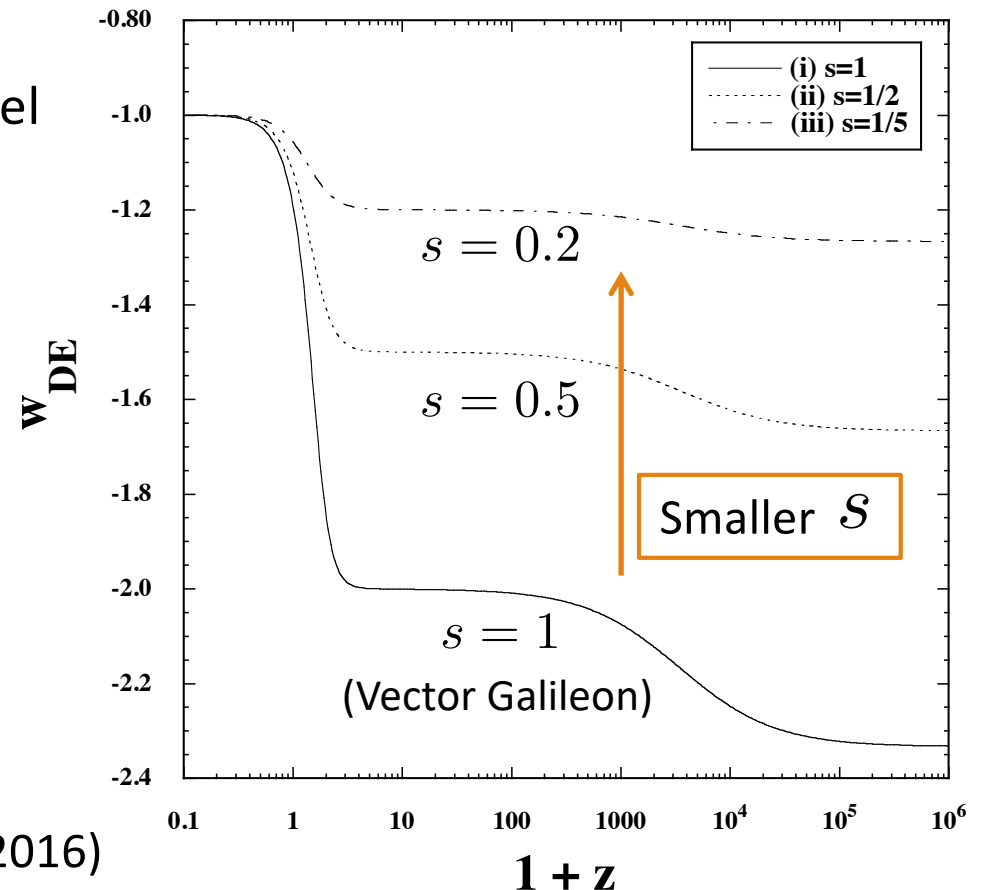
$s \equiv p_2/p$  : Deviation from  $\Lambda$ CDM model

$\Omega_r$  : Radiation density parameter

$\Omega_{\text{DE}}$  : DE density parameter

The  $\Lambda$ CDM limit:  $s = 0$

The solutions converge to a **de Sitter attractor**.



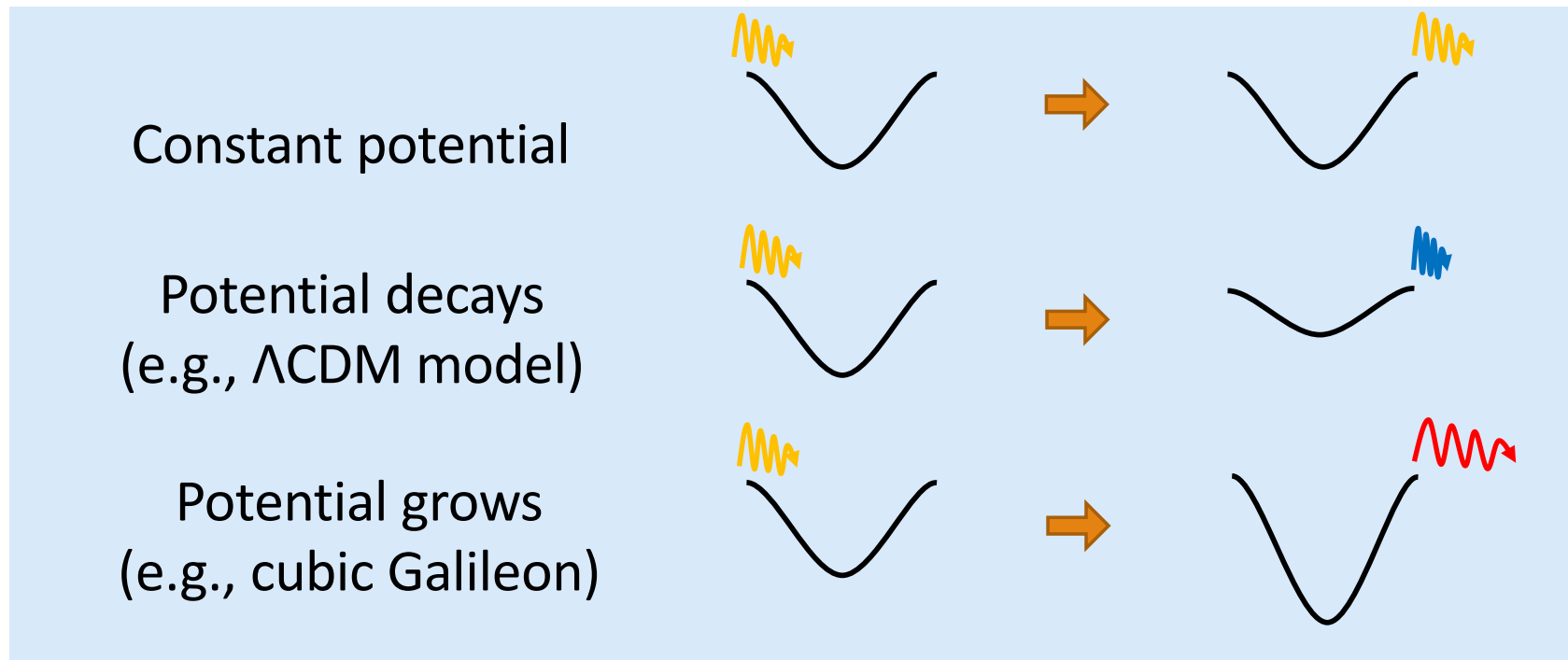
A. De Felice et al (2016)

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# Integrated Sachs-Wolfe (ISW) effect



- ➔ Detection of late-time ISW effect in a flat universe is **independent evidence for dark energy**.
- ➔ Taking the cross-correlation between the CMB anisotropy and the galaxy distributions, we can separate the ISW signal from the CMB anisotropy.

# Dark energy model with $c_T = c$

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The cross-correlation between ISW effect in CMB and galaxy distributions is given by

$$\left\langle \frac{\Delta T_{\text{ISW}}(\hat{n})}{T} \frac{\Delta N_{\text{Galaxy}}(\hat{n}')}{N} \right\rangle = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} C_l^{\text{IG}} \mathcal{P}_l(\cos \theta),$$

CMB temperature anisotropy      Galaxy number density fluctuations

This cross-correlation can distinguish the different dark energy models:

GR

•  $\Lambda$ CDM model → The ISW-galaxy cross-correlation is **positive**.

spin0

• Cubic-order scalar-tensor theories

# Example: Kinetic Gravity Braiding (scalar-tensor theories)

R. Kimura, T. Kobayashi and K. Yamamoto (2012)

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{pl}}^2}{2} R + K(\tilde{X}) - G(\tilde{X}) \square \varphi \right] + S_M$$

where

$$K(\tilde{X}) = -\tilde{X}, \quad G(\tilde{X}) \propto \tilde{X}^n$$

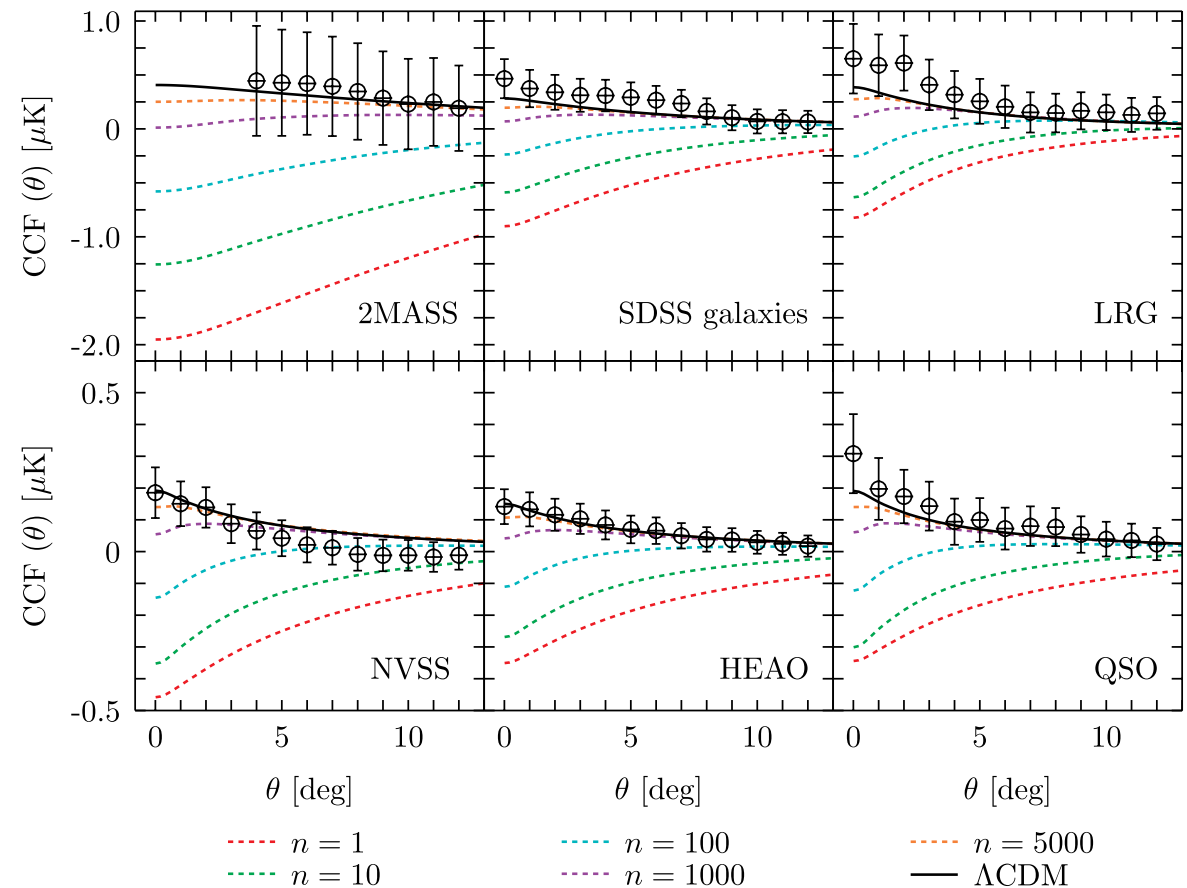
$\varphi$  : scalar field

$$\tilde{X} \equiv -\partial_\mu \varphi \partial^\mu \varphi / 2$$

The data of ISW-galaxy cross-correlation constrain the power in range

$$n \gtrsim \mathcal{O}(100)$$

→ Cubic Galileon ( $n = 1$ ) is excluded.



# Dark energy model with $c_T = c$

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This cross-correlation can distinguish the different dark energy models:

GR

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spin0

• Cubic-order scalar-tensor theories

→ The ISW-galaxy cross-correlation can be **negative**.

spin1

• Cubic-order vector-tensor theories → **positive or negative?**

# Evolution of matter density perturbations

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We introduce gauge-invariant gravitational potentials:  $\Psi$ ,  $\Phi$

In Fourier space with the comoving wave number  $k$ , these potentials are related with the matter density contrast  $\delta$ , as

$$\text{Newtonian potential: } \frac{k^2}{a^2} \Psi = -4\pi G \mu \rho_m \delta, \quad \mu \equiv G_{\text{eff}}/G$$

$$\text{Weak lensing potential: } \frac{k^2}{a^2} \psi_{\text{eff}} = 8\pi G \Sigma \rho_m \delta, \quad \psi_{\text{eff}} \equiv \Phi - \Psi$$

The density contrast obeys  $\ddot{\delta} + 2H\dot{\delta} - 4\pi G\mu\rho_m\delta \simeq 0$

In GR,  $\mu = \Sigma = 1$

# Cross-correlation amplitude

$$\left\langle \frac{\Delta T_{\text{ISW}}(\hat{n})}{T} \frac{\Delta N_{\text{Galaxy}}(\hat{n}')}{N} \right\rangle = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} C_l^{\text{IG}} \mathcal{P}_l(\cos \theta),$$

$$\frac{\Delta T_{\text{ISW}}(\hat{n})}{T} \equiv \int_{\eta_r}^{\eta_0} d\eta \frac{\partial}{\partial \eta} [\Psi - \Phi] = \int_0^{z_r} dz \frac{\partial \psi_{\text{eff}}}{\partial z}$$

Under the small angle approximation, the cross-correlation amplitude is given by

$$C_l^{\text{IG}} \simeq \frac{3\Omega_{m0}H_0^2}{l_{12}^2 D_0^2} \int d\mathcal{N} e^{-\mathcal{N}} H \mathcal{W} b_s D \Sigma \mathcal{F} P(l_{12})$$

$$\mathcal{N} \equiv \ln a(t)$$

$$\mathcal{F} \equiv 1 - \frac{D'}{D} - \frac{\Sigma'}{\Sigma} = 1 - (\ln D\Sigma)'$$

$$l_{12} \equiv l + 1/2$$

$\mathcal{W}$ : window function

$b_s$ : bias

$D$ : growth factor

$P$ : the matter power spectrum

This quantity determines the sign of the cross-correlation power spectrum.



# Gravitational coupling related to light bending

In our models, 
$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{pl}}^2}{2} R + F + b_2 X^{p_2} + b_3 X^{p_3} \nabla_\mu A^\mu \right] + S_M,$$

$$\Sigma = 1 + 1 / \left[ f(s, p, \Omega_{\text{DE}}) + \left( \frac{2}{3^{1/p}} \right)^{1/(1+s)} \frac{1}{\lambda_V} \frac{1}{\Omega_{\text{DE}}^{1/[p(1+s)]}} \right]$$

where  $\lambda_V$  is associated with the intrinsic vector mode such that

$$\lambda_V \equiv q_V \left[ \left( \frac{\phi}{M_{\text{pl}}} \right)^p \frac{H}{m} \right]^{2/[p(1+s)], \quad \phi \propto H^{-1/p}$$

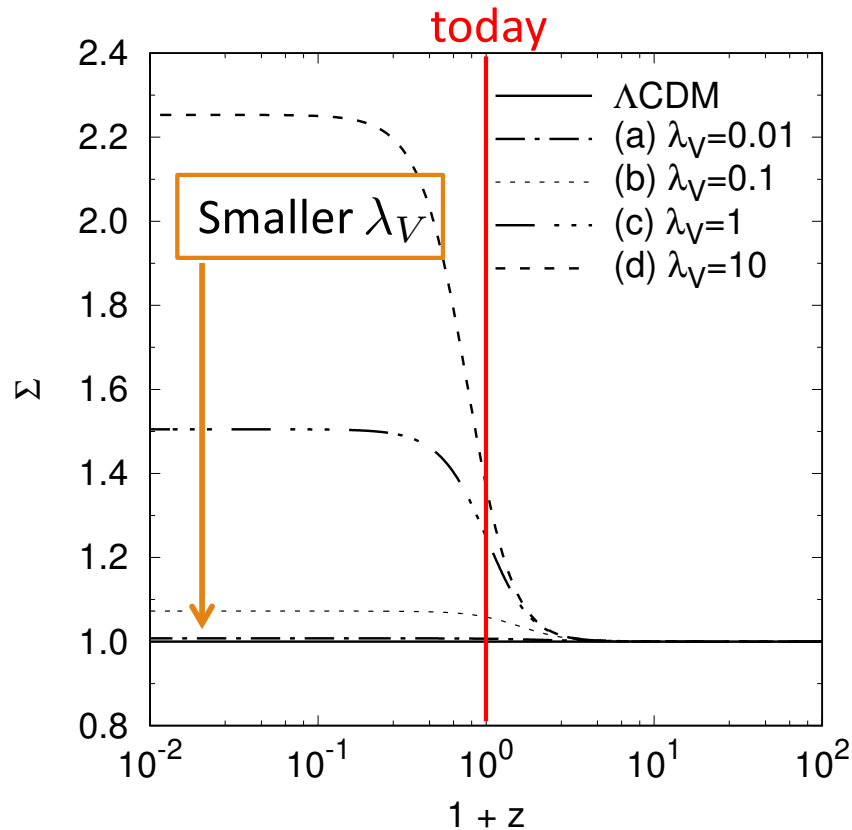
$q_V$  : coefficient of kinetic term of vector perturbation

- ➔ In the limit  $\lambda_V \rightarrow 0$ ,  
the evolution of perturbation is similar to that in  $\Lambda$ CDM.
- ➔ In the limit  $\lambda_V \rightarrow \infty$ ,  
this model reduces to a subclass of scalar-tensor theories.

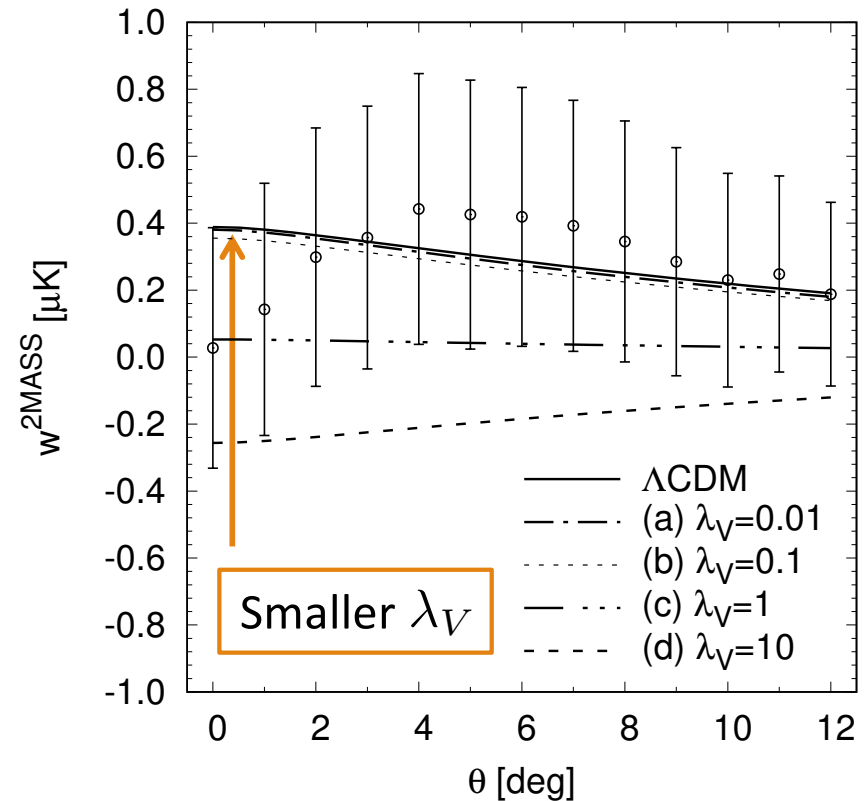
# ISW-galaxy cross-correlations in concrete models

SN, A. De Felice, R. Kase and S. Tsujikawa (2018)

$$\Sigma_{\text{dS}} = 1 + \left[ \frac{1 - ps}{ps} + \left( \frac{2}{3^{1/p}} \right)^{1/(1+s)} \frac{1}{\lambda_V} \right]^{-1}$$



## Cross-correlation function



The intrinsic vector mode can give rise to **positive** cross-correlations compatible with the data.

# Outline

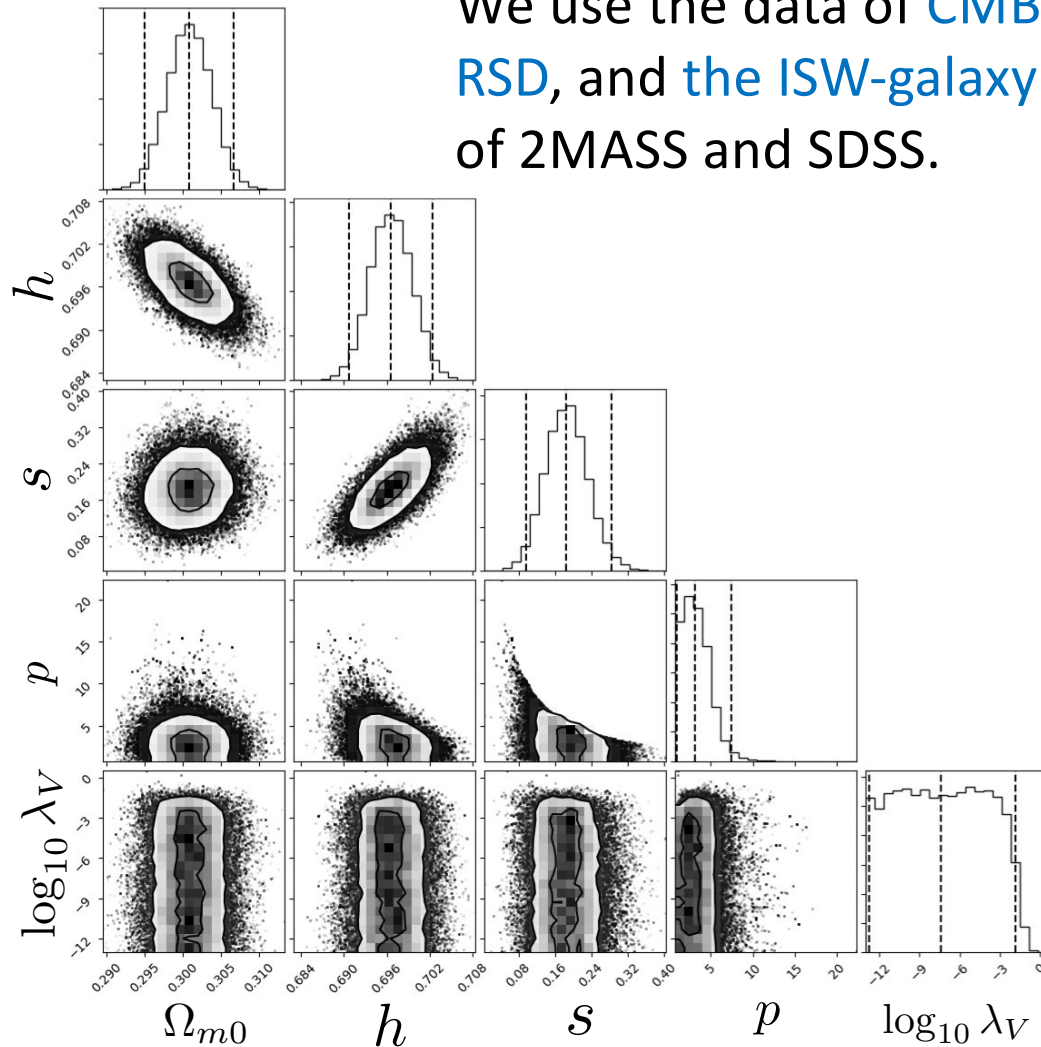
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# Observational constraints

SN, A. De Felice, R. Kase and S. Tsujikawa (2018)

We use the data of CMB, BAO, SN Ia, Hubble expansion rate, RSD, and the ISW-galaxy cross-correlations with the catalogues of 2MASS and SDSS.



$$\Omega_{m0} = 0.301^{+0.006}_{-0.006},$$

$$h = 0.697^{+0.006}_{-0.006},$$

$$s = 0.185^{+0.100}_{-0.089},$$

$$p = 3.078^{+4.317}_{-2.119}, \quad \phi \propto H^{-1/p}$$

$$\bar{\lambda}_V \leq \lambda_V < 0.015, \quad (95\% \text{ CL})$$

The model with  $s > 0$  still fits the data better than the  $\Lambda$ CDM model.

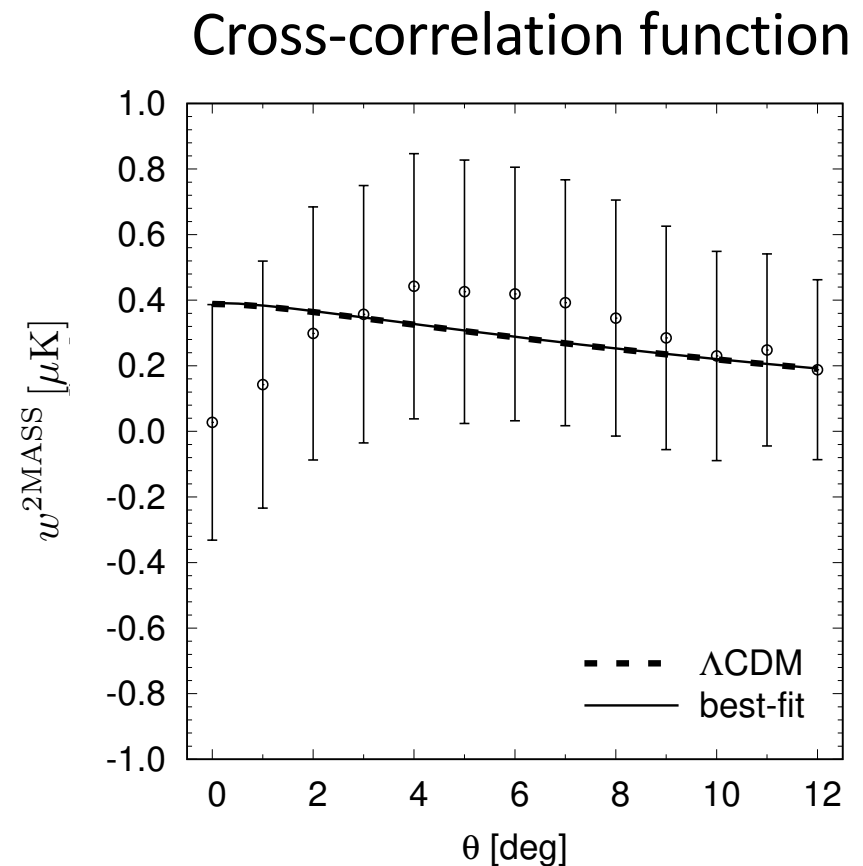
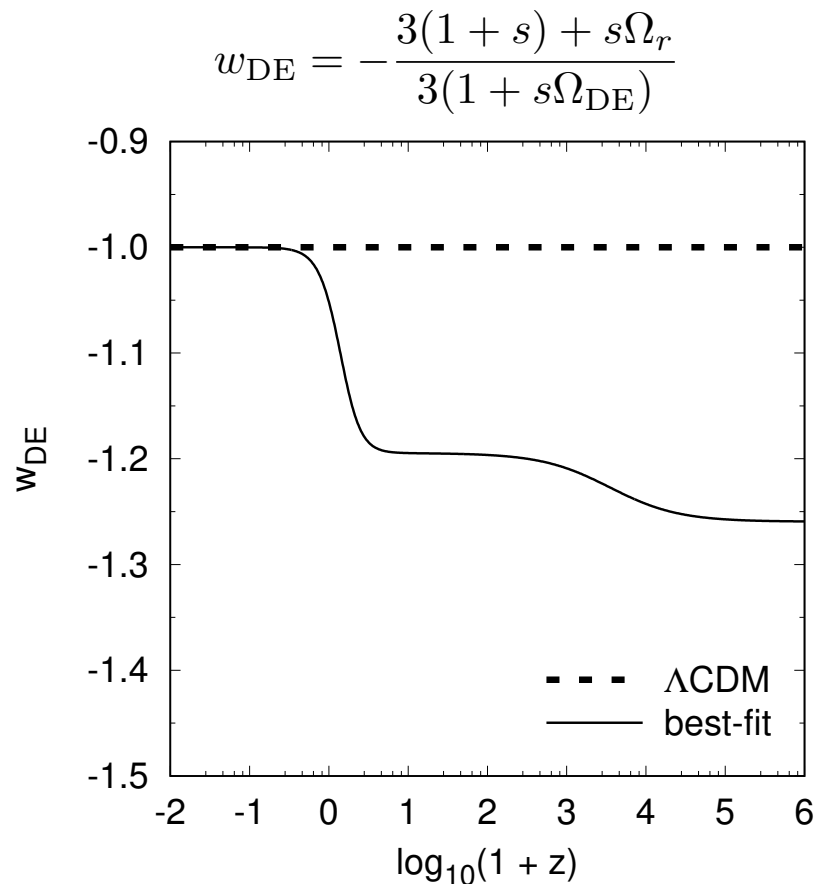
$$\text{Best-fit: BIC} = 651.2$$

$$\Lambda\text{CDM: BIC} = 655.6$$

# Best-fit case in massive vector dark energy model

$$\Omega_{m0} = 0.301, h = 0.697, s = 0.185, p = 3.078, \log_{10} \lambda_V = -7.359 \implies \text{BIC} = 651.2$$

The background dynamics in our model is different from that in  $\Lambda$ CDM, while the perturbation dynamics is almost the same as that in  $\Lambda$ CDM.



# Summary

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- We studied **observational constraints** on a dark energy model in **cubic-order generalized Proca theories** by using the data of CMB, BAO, SN Ia, RSD and **ISW-galaxy cross-correlation**.
- **Due to the existence of intrinsic vector mode**, the ISW-galaxy cross-correlation can be positive even for cubic interactions unlike that in scalar-tensor theories.
- **The model with  $s > 0$  still fits** the data better than the  $\Lambda$ CDM model even by including the ISW-galaxy cross-correlation data.
- It remains to be seen whether future high-precision observations show some evidence that the dark energy model in the vector-tensor theories is favored over the  $\Lambda$ CDM model.