No small hairs in anisotropic power-law Gauss-Bonnet inflation

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Contents

Motivations

Understanding the nature of our universe's space
is always very important but not straightforward **observationally**

Featured in Physics   Editors' Suggestion

How Isotropic is the Universe?

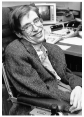
Daniela Saadeh, Stephen M. Feeney, Andrew Pontzen, Hiranya V. Peiris, and Jason D. McEwen
Phys. Rev. Lett. **117**, 131302 – Published 21 September 2016

Physics See Synopsis: Anisotropy Limits for the Universe

Motivations

and theoretically !

- Cosmological (Copernican) principle: our universe's space is just homogeneous and isotropic such as the flat FLRW (or de Sitter) spacetime.
- Cosmic no-hair conjecture proposed by Hawking and his colleagues claims that all classical hairs of the early universe, i.e., anisotropy and/or inhomogeneity, will disappear at the late time [Gibbons & Hawking, PRD15(1977)2738; Hawking & Moss, PLB110(1982)35] $\rightarrow$ the cosmic no-hair conjecture seems to be more general than the cosmological principle since it allows the early universe take arbitrary state.

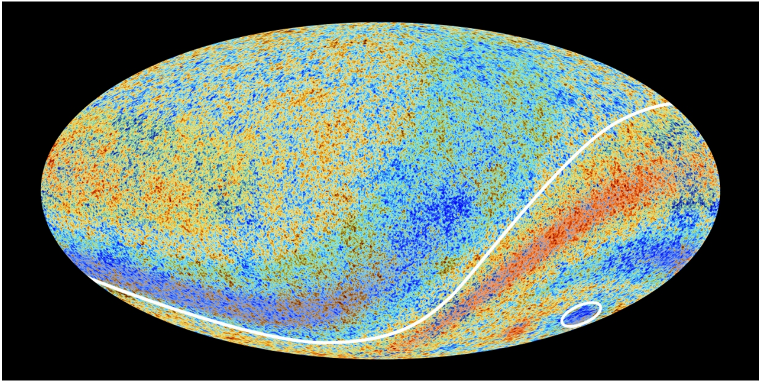
  

From left to right: S. W. Hawking, G. W. Gibbons, and I. G. Moss (Source: Internet).

Motivations

- This conjecture was partially proven by Wald, PRD28(1983)2118, for the Bianchi spacetimes, which are homogeneous but anisotropic, using energy conditions approach.
- Kitada & Maeda, PRD45(1992)1416, proved the cosmic no-hair conjecture for Bianchi models in power-law inflation.
- Kleban & Senatore, JCAP10(2016)022; East, Kleban, Linde & Senatore, JCAP09(2016)010: try to extend the Wald's proof to inhomogeneous and anisotropic spacetimes.
- Carroll & Chatwin-Davies, PRD97(2018)046012: try to prove the conjecture in a difference approach using the idea of maximum entropy of de Sitter spacetime.
- There are several claimed (Bianchi) counterexamples to the cosmic no-hair conjecture, e.g., Kaloper, PRD44(1991)2380; Barrow & Hervik, PRD73(2006)023007, PRD81(2010)023513; Kanno, Soda & Watanabe (KSW), PRL102(2009)191302, JCAP12(2010)024.
- Some claimed counterexamples, i.e., Kaloper, PRD44(1991)2380; Barrow & Hervik, PRD73(2006)023007, PRD81(2010)023513, have been shown to be unstable by stability analysis Kao & Lin, JCAP01(2009)022, PRD79(2009)043001, PRD83(2011)063004; Chang, Kao & Lin, PRD84(2011)063014 $\rightarrow$ they do not really violate the cosmic no-hair conjecture.

Motivations



Two CMB anomalous features, the hemispherical asymmetry and the Cold Spot, hinted by Planck's predecessor, NASA's WMAP, are confirmed in the new high precision data from Planck, **both are not predicted by standard inflationary models based on the cosmological principle**. (Information source and picture credit: ESA and the Planck Collaboration).

Motivations

- The mentioned anomalies imply that the early universe might be slightly anisotropic $\rightarrow$ We might need a cosmological model which violates the cosmological principle $\rightarrow$ Seek Bianchi spacetimes, which are homogeneous but anisotropic metrics, in cosmological models.
- What is the state of our current universe ? Is it isotropic or still slightly anisotropic ? We might need the help of the cosmic no-hair conjecture $\rightarrow$ Need to check the validity of the cosmic no-hair conjecture in the studied cosmological model.

Kanno-Soda-Watanabe model

- It seems to be the first valid counterexample to the cosmic no-hair conjecture
  Kanno, Soda & Watanabe, PRL102(2009)191302, JCAP12(2010)024

$$S_{\rm KSW} = \int d^4x\sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right],$$

  with $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ the field strength of the electromagnetic (Maxwell) field A_μ.
- Bianchi type I metric (BI):

$$ds^2 = -dt^2 + \exp\left[2\alpha(t) - 4\sigma(t)\right]dx^2 + \exp\left[2\alpha(t) + 2\sigma(t)\right]\left(dy^2 + dz^2\right).$$

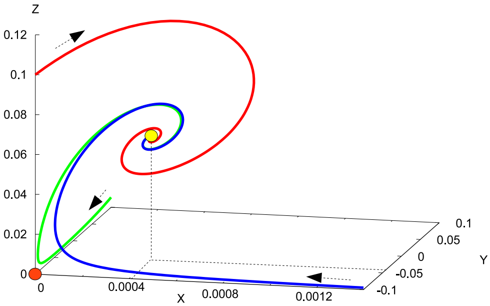
- $\sigma(t)$ a deviation from the isotropy determined by $\alpha(t)$, i.e., $\sigma(t) \ll \alpha(t)$.
- Vector and scalar fields: $A_\mu = (0, A_x(t), 0, 0)$ and $\phi = \phi(t)$.
- Exponential potentials: $f(\phi) = f_0\exp[\rho\phi]$ and $V(\phi) = V_0\exp[\lambda\phi]$.
- Ansatz: $\alpha = \zeta\log t$; $\sigma = \eta\log t$; $\phi = \xi\log t + \phi_0$, which lead to power-law forms of scale factors

$$\exp[\alpha(t) + \sigma(t)] = t^{\zeta+\eta};\ \exp[\alpha(t) - 2\sigma(t)] = t^{\zeta-2\eta}.$$

Kanno-Soda-Watanabe model

Dynamical variables:

$$X = \frac{\dot{\sigma}}{\dot{\alpha}}; \;\; Y = \frac{\dot{\phi}}{\dot{\alpha}}; \;\; Z = \frac{p_A f_0^{-1}}{\dot{\alpha}} \exp\left[-\rho\phi - 2\alpha - 2\sigma\right].$$



Attractor behavior of anisotropic fixed point, which is equivalent to anisotropic power-law solution [taken from JCAP12(2010)024].

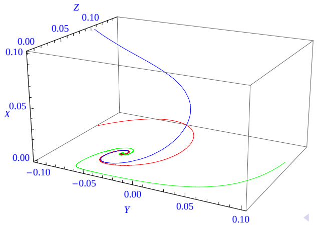
Noncanonical extensions of KSW model: DBI model Do & Kao

PRD84(2011)123009

- Canonical kinetic term of scalar field is replaced by the noncanonical Dirac-Born-Infeld term Silverstein & Tong, PRD70(2004)103505; Alishahiha, Silverstein & Tong, PRD70(2004)123505

$$S_{\rm DBI} = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \frac{1}{\tilde{f}(\phi)}\frac{\gamma-1}{\gamma} - V(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right].$$

- The Lorentz factor $\gamma = 1/\sqrt{1+\tilde{f}(\phi)\,\partial_\mu\phi\partial^\mu\phi} \geq 1$.
- Attractor behavior of anisotropic fixed point



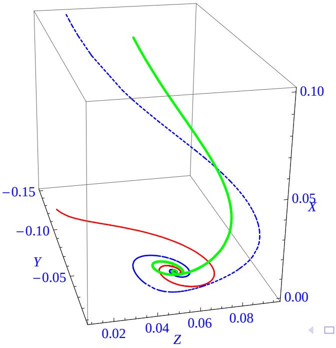
Noncanonical extensions of KSW model: SDBI model Do & Kao, CQG33(2016)085009

- Canonical kinetic term of scalar field is replaced by the noncanonical supersymmetric DBI (SDBI) term Sasaki, Yamaguchi & Yokoyama, PLB718(2012)1

$$S_{\mathrm{SDBI}} = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \frac{1}{\tilde{f}(\phi)}\frac{\gamma-1}{\gamma} - \Sigma_0^2\, V(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right],$$

$$\Sigma_0(\gamma) = [(\gamma+1)/(2\gamma)]^{1/3} \leq 1.$$

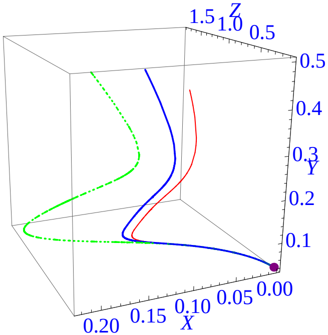
- Attractor behavior of anisotropic fixed point



Noncanonical extensions of KSW model: covariant Galileon model Do & Kao, PRD96(2017)023529

Canonical kinetic term and potential of scalar field are replaced by the noncanonical covariant Galileon terms Deffayet, Esposito-Farese & Vikman, PRD79(2009)084003; Kobayashi, Yamaguchi & Yokoyama, PRL105(2010)231302

$$S_{\rm Galileon} = \int d^4x\sqrt{-g}\left\{\frac{R}{2} + k_0 \exp\left[\tau\phi\right] X - g_0 \exp\left[\lambda\phi\right] X\Box\phi - \frac{f_0^2}{4}\exp\left[-2\rho\phi\right] F_{\mu\nu}F^{\mu\nu}\right\}.$$



Attractor behavior of anisotropic fixed point

Anisotropic power-law scalar-Gauss-Bonnet inflation

- The cosmic no-hair has been shown to be violated generally in the KSW model due to the existence of the unusual coupling $f^2(\phi)F^{\mu\nu}F_{\mu\nu}$.
- What if the cosmic no-hair conjecture is extensively violated in other cosmological models, which do not include $f^2(\phi)F^{\mu\nu}F_{\mu\nu}$?

Anisotropic power-law scalar-Gauss-Bonnet inflation

- We propose to study the scalar-Gauss-Bonnet gravity without $V(\phi)$ Kanti, Gannouji, & Dadhich, PRD92(2015)041302(R); PRD92(2015) 083524

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{\omega}{2}\partial_\mu\phi\partial^\mu\phi - \frac{h(\phi)}{8}G\right],$$

with

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma};\ \omega = \pm 1.$$

- The Gauss-Bonnet term $\sqrt{-g}G$ acts as a total derivative in four dimensions $\rightarrow$ the existence of $h(\phi)$ is necessary. Choose the exponential function

$$h(\phi) = h_0 \exp\left[\frac{\lambda\phi}{M_p}\right].$$

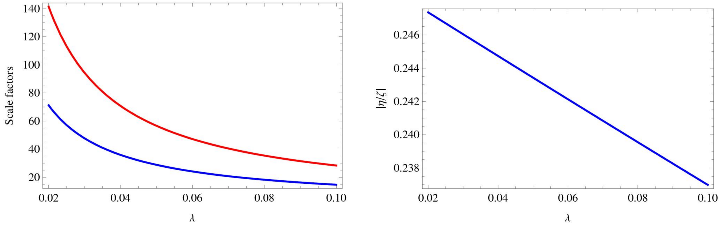
- The corresponding physical solution with $\omega = -1$:

$$\zeta = \frac{1}{3} + \frac{2}{3\lambda}\sqrt{\lambda^2+8};\ \eta = \frac{1}{6} - \frac{1}{6\lambda}\sqrt{\lambda^2+8}.$$

Anisotropic power-law scalar-Gauss-Bonnet inflation

- This solution will represent an inflationary solution if $\lambda \ll 1$.
- The anisotropy during the inflationary phase:

$$\zeta \simeq \frac{4\sqrt{2}}{3\lambda};\ \eta \simeq -\frac{\sqrt{2}}{3\lambda} \to \left|\frac{\eta}{\zeta}\right| \simeq 0.25.$$



(Left) $\zeta - 2\eta$ (upper red curve) and $\zeta + \eta$ (lower blue curve) as functions of λ. (Right) $|\eta/\zeta|$ as a function of λ.

- The inflationary solution is highly anisotropic $\to$ non-viable model

Anisotropic power-law scalar-vector-Gauss-Bonnet inflation

- We include the coupling $f^2(\phi)F^{\mu\nu}F_{\mu\nu}$ with the expectation that the anisotropy will be reduced significantly

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{\omega}{2}\partial_\mu\phi\partial^\mu\phi - \frac{h(\phi)}{8}G - \frac{f^2(\phi)}{4}F_{\mu\nu}F^{\mu\nu}\right].$$

- The exponential function: $f(\phi) = f_0 \exp\left[-\frac{\rho\phi}{M_p}\right]$.
- The corresponding physical solution:

$$\eta = 1/2 + \rho/\lambda - \zeta$$

along with the equation of ζ:

$$F(\zeta) \equiv A\zeta^3 + B\zeta^2 + C\zeta + D = 0,$$

where

$$A = 54\lambda^4 + 108\lambda^3\rho,$$
$$B = -81\lambda^4 - 252\lambda^3\rho - 180\lambda^2\rho^2 + 144\omega\lambda^2,$$
$$C = 30\lambda^4 + 156\lambda^3\rho + 288\lambda^2\rho^2 - 48\omega\lambda^2 + 192\lambda\rho^3,$$
$$D = -3\lambda^4 - 24\lambda^3\rho - 84\lambda^2\rho^2 - 144\lambda\rho^3 - 32\omega\lambda\rho - 96\rho^4 - 64\omega\rho^2.$$

Anisotropic power-law scalar-vector-Gauss-Bonnet inflation

- The inflationary solution will exist if $\rho \gg \lambda \to F(\zeta)$ approximately becomes

$$F(\zeta) \simeq \tilde{F}(\zeta) = 12\lambda^3\rho \left(9\zeta^3 - 15\frac{\rho}{\lambda}\zeta^2 + 16\frac{\rho^2}{\lambda^2}\zeta - 8\frac{\rho^3}{\lambda^3}\right).$$

- The following discriminant $\Delta = -26784\frac{\rho^6}{\lambda^6} < 0 \to \tilde{F}(\zeta) = 0$ admits one real root and two complex ones.
- Approximated inflationary solution (no matter the sign of ω):

$$\zeta = \frac{1}{9}\frac{\rho}{\lambda}\left(5 + \sqrt[3]{18\sqrt{62}+89} - \frac{23}{\sqrt[3]{18\sqrt{62}+89}}\right) \simeq 0.82\frac{\rho}{\lambda} \gg 1,$$

$$\eta \simeq 0.18\frac{\rho}{\lambda} \to \frac{\eta}{\zeta} \simeq 0.22.$$

- The magnitude of the ration $\frac{\eta}{\zeta}$ is of the same order as that obtained in the scalar-Gauss-Bonnet model, in contrast to our expectation that its magnitude would be reduced to a small number due to the existence of vector field $\to$ non-viable model, too.

Conclusions

- The scalar-Gauss-Bonnet gravity model with the absence of potential of scalar field might not be suitable to produce a small anisotropic hair during the inflationary phase, even when the coupling $f^2(\phi)F^{\mu\nu}F_{\mu\nu}$ is involved $\rightarrow$ The existence of the potential $V(\phi)$ might be necessary Lahiri, JCAP09(2016)025
- Might we have small anisotropy in another subclass of $f(G)$ gravity ?

Further remarks

- The cosmic no-hair conjecture is still a conjecture.
- It is not easy to have the de Sitter space in String theory $\rightarrow$ Any indication to the validity of the cosmic no-hair conjecture ?
- Anisotropic inflation might be viable if its imprints on the CMB correlations were confirmed by future observations.

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Imprints of the anisotropic inflation on the cosmic microwave background

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JCAP08(2014)027

The TT, TB, EB and BB correlations in anisotropic inflation

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+ Do, Kao, & Lin, work in progress.

Thank you all for your attention !