Reconstructing the EFT of Inflation from Cosmological Data

based on arXiv:1904.00991

Amel Durakovic
in collaboration with Paul Hunt, Subodh Patil, Subir Sarkar

Niels Bohr International Academy and Discovery Center,
Niels Bohr Institute

Reconstructing the EFT of Inflation from Cosmological Data

or

Finding a *precise* dictionary between the parameters of the effective theory of inflation and their primordial power spectra

Amel Durakovic

in collaboration with Paul Hunt, Subodh Patil, Subir Sarkar

Niels Bohr International Academy and Discovery Center,
Niels Bohr Institute

Inflation basically

- An early stage of near-exponential expansion. Many multiplications of the scale factor a, $\times \sim e^{60}$.

Inflation basically

- An early stage of near-exponential expansion. Many multiplications of the scale factor a, $\times \sim e^{60}$.
- Proposed to solve the horizon problem, the flatness problem and also (historically) the monopole problem.

Inflation basically

- An early stage of near-exponential expansion. Many multiplications of the scale factor a, $\times \sim e^{60}$.
- Proposed to solve the horizon problem, the flatness problem and also (historically) the monopole problem.
- Typically driven by a scalar field ϕ with non-zero, almost-flat potential $V(\phi)$, slowly rolling.

Inflation basically

- An early stage of near-exponential expansion. Many multiplications of the scale factor a, $\times \sim e^{60}$.
- Proposed to solve the horizon problem, the flatness problem and also (historically) the monopole problem.
- Typically driven by a scalar field ϕ with non-zero, almost-flat potential $V(\phi)$, slowly rolling.
- Provides, in addition, quantitative predictions for the statistics of curvature perturbations $\mathcal{R}$, the seeds of later structure formation.

Inflation basically

- An early stage of near-exponential expansion. Many multiplications of the scale factor a, $\times \sim e^{60}$.
- Proposed to solve the horizon problem, the flatness problem and also (historically) the monopole problem.
- Typically driven by a scalar field ϕ with non-zero, almost-flat potential $V(\phi)$, slowly rolling.
- Provides, in addition, quantitative predictions for the statistics of curvature perturbations $\mathcal{R}$, the seeds of later structure formation.
- The scalar field fluctuates quantum mechanically, and, having energy-momentum, causes perturbations in curvature.

The primordial power spectrum

- The PPS $P(k)$ is the variance of the Fourier coefficients of curvature perturbation: $\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P(k)$.

The primordial power spectrum

- The PPS $P(k)$ is the variance of the Fourier coefficients of curvature perturbation: $\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P(k)$.
- Dimensionless: $\mathcal{P}(k) = k^3 P(k)/2\pi^2$.

The primordial power spectrum

- The PPS $P(k)$ is the variance of the Fourier coefficients of curvature perturbation: $\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P(k)$.
- Dimensionless: $\mathcal{P}(k) = k^3 P(k)/2\pi^2$.
- In simplest case $\mathcal{P}(k) = A(k/k_*)^{n_s - 1}$ where $n_s = 2\eta - 4\epsilon$ and $\epsilon = -\dot{H}/H^2 = \dot{\phi}^2/(2H^2)$.

The primordial power spectrum

- The PPS $P(k)$ is the variance of the Fourier coefficients of curvature perturbation: $\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P(k)$.
- Dimensionless: $\mathcal{P}(k) = k^3 P(k) / 2\pi^2$.
- In simplest case $\mathcal{P}(k) = A(k/k_*)^{n_s - 1}$ where $n_s = 2\eta - 4\epsilon$ and $\epsilon = -\dot{H}/H^2 = \dot{\phi}^2/(2H^2)$.
- In the simplest case: Gaussian statistics.

The primordial power spectrum

- The PPS $P(k)$ is the variance of the Fourier coefficients of curvature perturbation: $\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P(k)$.
- Dimensionless: $\mathcal{P}(k) = k^3 P(k) / 2\pi^2$.
- In simplest case $\mathcal{P}(k) = A(k/k_*)^{n_s - 1}$ where $n_s = 2\eta - 4\epsilon$ and $\epsilon = -\dot{H}/H^2 = \dot{\phi}^2/(2H^2)$.
- In the simplest case: Gaussian statistics.
- Leave imprint on the temperature fluctuations of the CMB: $\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell$ where $\Delta T(\hat{\mathbf{n}}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$.

The primordial power spectrum

- The PPS $P(k)$ is the variance of the Fourier coefficients of curvature perturbation: $\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P(k)$.
- Dimensionless: $\mathcal{P}(k) = k^3 P(k)/2\pi^2$.
- In simplest case $\mathcal{P}(k) = A(k/k_*)^{n_s - 1}$ where $n_s = 2\eta - 4\epsilon$ and $\epsilon = -\dot{H}/H^2 = \dot{\phi}^2/(2H^2)$.
- In the simplest case: Gaussian statistics.
- Leave imprint on the temperature fluctuations of the CMB: $\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{mm'} C_\ell$ where $\Delta T(\hat{\mathbf{n}}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$.
- Linear relation between $\mathcal{P}(k)$ and C_ℓ: $C_\ell = \int_0^\infty \mathrm{d} \log k \, \Delta_\ell^{TT}(k)^2 \mathcal{P}(k) \to \mathbf{d} = \mathbf{W}\mathbf{p}$.

The primordial power spectrum

- The PPS $P(k)$ is the variance of the Fourier coefficients of curvature perturbation: $\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P(k)$.
- Dimensionless: $\mathcal{P}(k) = k^3 P(k)/2\pi^2$.
- In simplest case $\mathcal{P}(k) = A(k/k_*)^{n_s - 1}$ where $n_s = 2\eta - 4\epsilon$ and $\epsilon = -\dot{H}/H^2 = \dot{\phi}^2/(2H^2)$.
- In the simplest case: Gaussian statistics.
- Leave imprint on the temperature fluctuations of the CMB: $\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell$ where $\Delta T(\hat{\mathbf{n}}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$.
- Linear relation between $\mathcal{P}(k)$ and C_ℓ: $C_\ell = \int_0^\infty \mathrm{d} \log k \, \Delta_\ell^{TT}(k)^2 \mathcal{P}(k) \rightarrow \mathbf{d} = \mathbf{W} \mathbf{p}$.
- Crucially, **W** depends on the cosmological parameters.

The effective field theory of inflation

- Contains only the curvature perturbation $\mathcal{R}$ field with two, in general, time-dependent coupling constants $c_s(\tau)$ and $\epsilon(\tau)$. ϵ is the expansion parameter of the EFT.
  $S_2 = M_{\mathrm{Pl}}^2 \int \mathrm{d}^3 x \int \mathrm{d}\tau \, a^2(\tau)\epsilon(\tau) \left(\mathcal{R}'^2/c_s(\tau)^2 - (\partial_i \mathcal{R})^2\right)$

The effective field theory of inflation

- Contains only the curvature perturbation $\mathcal{R}$ field with two, in general, time-dependent coupling constants $c_s(\tau)$ and $\epsilon(\tau)$. ϵ is the expansion parameter of the EFT.
  $S_2 = M_{\mathrm{Pl}}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau \, a^2(\tau)\epsilon(\tau) \left(\mathcal{R}'^2/c_s(\tau)^2 - (\partial_i \mathcal{R})^2\right)$
- A more complicated inflationary scenario is *shoehorned* into these time-dependent coupling constants.

The effective field theory of inflation

- Contains only the curvature perturbation $\mathcal{R}$ field with two, in general, time-dependent coupling constants $c_s(\tau)$ and $\epsilon(\tau)$. ϵ is the expansion parameter of the EFT.
  $S_2 = M_{\mathrm{Pl}}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau\, a^2(\tau)\epsilon(\tau)\left(\mathcal{R}'^2/c_s(\tau)^2 - (\partial_i\mathcal{R})^2\right)$
- A more complicated inflationary scenario is *shoehorned* into these time-dependent coupling constants.
- A time-dependence of $\epsilon(\tau)$ or $c_s(\tau)$ leads to characteristic scales, 'features', in $\langle\mathcal{R}_k\mathcal{R}_k\rangle \propto \mathcal{P}(k)$.

The effective field theory of inflation

- Contains only the curvature perturbation $\mathcal{R}$ field with two, in general, time-dependent coupling constants $c_s(\tau)$ and $\epsilon(\tau)$. ϵ is the expansion parameter of the EFT.
  $S_2 = M_{\mathrm{Pl}}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau \, a^2(\tau)\epsilon(\tau) \left(\mathcal{R}'^2/c_s(\tau)^2 - (\partial_i \mathcal{R})^2\right)$
- A more complicated inflationary scenario is *shoehorned* into these time-dependent coupling constants.
- A time-dependence of $\epsilon(\tau)$ or $c_s(\tau)$ leads to characteristic scales, 'features', in $\langle \mathcal{R}_k \mathcal{R}_k \rangle \propto \mathcal{P}(k)$.
- Fractional changes in PPS $\Delta\mathcal{P}/\mathcal{P} \propto \Delta\epsilon/\epsilon$ or $u(\tau) = 1/c_s^2 - 1$.

The effective field theory of inflation

- Contains only the curvature perturbation $\mathcal{R}$ field with two, in general, time-dependent coupling constants $c_s(\tau)$ and $\epsilon(\tau)$. ϵ is the expansion parameter of the EFT.
  $S_2 = M_{\mathrm{Pl}}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau \, a^2(\tau)\epsilon(\tau) \left(\mathcal{R}'^2/c_s(\tau)^2 - (\partial_i \mathcal{R})^2\right)$
- A more complicated inflationary scenario is *shoehorned* into these time-dependent coupling constants.
- A time-dependence of $\epsilon(\tau)$ or $c_s(\tau)$ leads to characteristic scales, 'features', in $\langle \mathcal{R}_k \mathcal{R}_k \rangle \propto \mathcal{P}(k)$.
- Fractional changes in PPS $\Delta\mathcal{P}/\mathcal{P} \propto \Delta\epsilon/\epsilon$ or $u(\tau) = 1/c_s^2 - 1$.
- Would like to *infer* $\Delta\epsilon/\epsilon(\tau)$ or $u(\tau)$ from estimates of $\Delta\mathcal{P}/\mathcal{P}$ itself estimated from *data* C_ℓ.

How large features?

- Compute corrections using perturbation theory.

How large features?

- Compute corrections using perturbation theory.
- Consider excursions from $\epsilon = \epsilon_0$ *or* from $c_s = 1$, $\Delta\epsilon/\epsilon(\tau) \equiv (\epsilon(\tau) - \epsilon_0)/\epsilon_0$ and $u(\tau) \equiv 1/c_s^2(\tau) - 1$.

How large features?

- Compute corrections using perturbation theory.
- Consider excursions from $\epsilon = \epsilon_0$ *or* from $c_s = 1$, $\Delta\epsilon/\epsilon(\tau) \equiv (\epsilon(\tau) - \epsilon_0)/\epsilon_0$ and $u(\tau) \equiv 1/c_s^2(\tau) - 1$.
- Split action S_2 into exactly solvable part with constant ϵ (or c_s) and an interacting part $S_{\rm int}$ proportional to $\Delta\epsilon/\epsilon$ or $u(\tau)$

How large features?

- Compute corrections using perturbation theory.
- Consider excursions from $\epsilon = \epsilon_0$ *or* from $c_s = 1$, $\Delta\epsilon/\epsilon(\tau) \equiv (\epsilon(\tau) - \epsilon_0)/\epsilon_0$ and $u(\tau) \equiv 1/c_s^2(\tau) - 1$.
- Split action S_2 into exactly solvable part with constant ϵ (or c_s) and an interacting part $S_{\rm int}$ proportional to $\Delta\epsilon/\epsilon$ or $u(\tau)$
- $S_2 = \epsilon_0 M_{\rm Pl}^2 \int {\rm d}^3x \int {\rm d}\tau a^2 ((\mathcal{R}')^2 - (\partial_i \mathcal{R})^2 + u(\tau)(\mathcal{R}')^2)$

How large features?

- Compute corrections using perturbation theory.
- Consider excursions from $\epsilon = \epsilon_0$ *or* from $c_s = 1$, $\Delta\epsilon/\epsilon(\tau) \equiv (\epsilon(\tau) - \epsilon_0)/\epsilon_0$ and $u(\tau) \equiv 1/c_s^2(\tau) - 1$.
- Split action S_2 into exactly solvable part with constant ϵ (or c_s) and an interacting part $S_{\rm int}$ proportional to $\Delta\epsilon/\epsilon$ or $u(\tau)$
- $S_2 = \epsilon_0 M_{\rm Pl}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau a^2((\mathcal{R}')^2 - (\partial_i\mathcal{R})^2 + u(\tau)(\mathcal{R}')^2)$
- $S_2 =$ $\epsilon_0 M_{\rm Pl}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau a^2((\mathcal{R}')^2 - (\partial_i\mathcal{R})^2 + \Delta\epsilon/\epsilon((\mathcal{R}')^2) - (\partial_i\mathcal{R})^2)$

How large features?

- Compute corrections using perturbation theory.
- Consider excursions from $\epsilon = \epsilon_0$ *or* from $c_s = 1$,
  $\Delta\epsilon/\epsilon(\tau) \equiv (\epsilon(\tau) - \epsilon_0)/\epsilon_0$ and $u(\tau) \equiv 1/c_s^2(\tau) - 1$.
- Split action S_2 into exactly solvable part with constant ϵ (or c_s) and an interacting part $S_{\rm int}$ proportional to $\Delta\epsilon/\epsilon$ or $u(\tau)$
- $S_2 = \epsilon_0 M_{\rm Pl}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau a^2((\mathcal{R}')^2 - (\partial_i\mathcal{R})^2 + u(\tau)(\mathcal{R}')^2)$
- $S_2 =$
  $\epsilon_0 M_{\rm Pl}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau a^2((\mathcal{R}')^2 - (\partial_i\mathcal{R})^2 + \Delta\epsilon/\epsilon((\mathcal{R}')^2) - (\partial_i\mathcal{R})^2)$
- For $\Delta\mathcal{P}/\mathcal{P} \sim 10\%$, corrections from 2nd order perturbation theory: $\sim (0.1)^2 = 1\%$.

How large features?

- Compute corrections using perturbation theory.
- Consider excursions from $\epsilon = \epsilon_0$ *or* from $c_s = 1$, $\Delta\epsilon/\epsilon(\tau) \equiv (\epsilon(\tau) - \epsilon_0)/\epsilon_0$ and $u(\tau) \equiv 1/c_s^2(\tau) - 1$.
- Split action S_2 into exactly solvable part with constant ϵ (or c_s) and an interacting part $S_{\rm int}$ proportional to $\Delta\epsilon/\epsilon$ or $u(\tau)$
- $S_2 = \epsilon_0 M_{\rm Pl}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau a^2((\mathcal{R}')^2 - (\partial_i\mathcal{R})^2 + u(\tau)(\mathcal{R}')^2)$
- $S_2 =$ $\epsilon_0 M_{\rm Pl}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau a^2((\mathcal{R}')^2 - (\partial_i\mathcal{R})^2 + \Delta\epsilon/\epsilon((\mathcal{R}')^2) - (\partial_i\mathcal{R})^2)$
- For $\Delta\mathcal{P}/\mathcal{P} \sim 10\%$, corrections from 2nd order perturbation theory: $\sim (0.1)^2 = 1\%$.
- For features $\Delta\mathcal{P}/\mathcal{P} \sim 20\%$, error from truncation 4% so can consider 2nd order perturbation theory, in which case error will be below $(0.2)^3 \sim 0.8\%$

Intermezzo. A no-Λ agenda: Subir's gambit

- Λ is small $\sim H_0^2/(8\pi G)$. If fundamental, difficult to justify why it should know about the expansion rate *today*.

Intermezzo. A no-Λ agenda: Subir's gambit

- Λ is small $\sim H_0^2/(8\pi G)$. If fundamental, difficult to justify why it should know about the expansion rate *today*.
- Let us instead retain $\Lambda = 0$ and see what we can get away with.

Intermezzo. A no-Λ agenda: Subir's gambit

- Λ is small $\sim H_0^2/(8\pi G)$. If fundamental, difficult to justify why it should know about the expansion rate *today*.
- Let us instead retain $\Lambda = 0$ and see what we can get away with.
- Effect on CMB (plotting $D_\ell \equiv \ell(\ell+1)/(2\pi)C_\ell$):

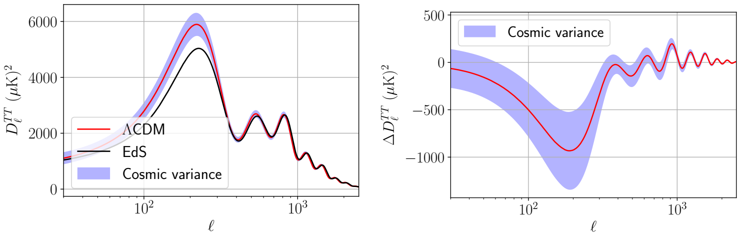


Figure: Using a power-law PPS for $\Omega_\Lambda = 0.67$ (red line) and $\Omega_\Lambda = 0$ (black line) but $H_0 = 44\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_b = 0.09$, $\Omega_{\mathrm{CDM}} = 0.8$.

What does data suggest?

- Take CMB data, C_ℓ, from Planck and find most likely $\mathcal{P}(k)$ subject to roughness penalty assuming different cosmological parameters.

What does data suggest?

- Take CMB data, C_ℓ, from Planck and find most likely $\mathcal{P}(k)$ subject to roughness penalty assuming different cosmological parameters.
- Roughness penalty necessary (regularisation) as $\mathbf{W}^{-1}$ does not exist, so no simple relation $\mathbf{p} = \mathbf{W}^{-1}\mathbf{d}$.

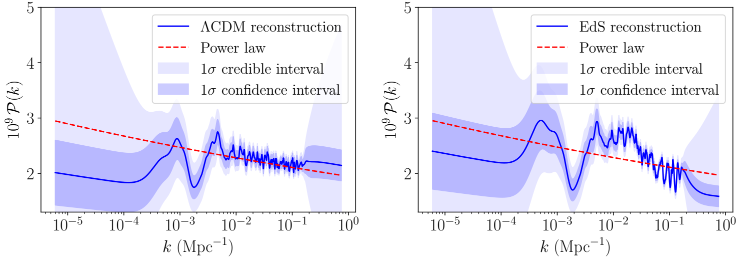


Figure: PPS reconstructing assuming different cosmological parameters.

Without Λ

- Features in the PPS: a *luxury* to ΛCDM, a *requirement* for a no-Λ (EdS) model.

Without Λ

- Features in the PPS: a *luxury* to ΛCDM, a *requirement* for a no-Λ (EdS) model.
- After all, the UV physics is the most speculative: too early to restrict to power-law form of PPS.

Without Λ

- Features in the PPS: a *luxury* to ΛCDM, a *requirement* for a no-Λ (EdS) model.
- After all, the UV physics is the most speculative: too early to restrict to power-law form of PPS.
- If unwilling to go this far: Do the acoustic peaks have an oscillatory primordial component?

Without Λ

- Features in the PPS: a *luxury* to ΛCDM, a *requirement* for a no-Λ (EdS) model.
- After all, the UV physics is the most speculative: too early to restrict to power-law form of PPS.
- If unwilling to go this far: Do the acoustic peaks have an oscillatory primordial component?
- Or can just appreciate the dictionary on its own.

Finding relations and their inverses

- Compute change to two-point function
  $\Delta\mathcal{P} \propto \Delta\langle\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \Delta\langle 0|\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}|0\rangle$

Finding relations and their inverses

- Compute change to two-point function
  $\Delta\mathcal{P} \propto \Delta\langle\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \Delta\langle 0|\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}|0\rangle$
- Expectation values in QFT. Use Schwinger-Keldysh formalism.

Finding relations and their inverses

- Compute change to two-point function
  $\Delta\mathcal{P} \propto \Delta\langle \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \Delta\langle 0|\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}|0\rangle$
- Expectation values in QFT. Use Schwinger-Keldysh formalism.
- Helped by Weinberg:

$$\langle \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \sum_{n=0} i^n \int_{-\infty}^{\tau_n} \mathrm{d}\tau_{n-1} \cdots \int_{-\infty}^{0} \mathrm{d}\tau_1$$

$$\langle 0|[H_{\mathrm{int}}(\tau_1), \ldots, [H_{\mathrm{int}}(\tau_{n-1}), [H_{\mathrm{int}}(\tau_n), \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}]]]|0\rangle$$

where $\mathcal{H}_{\mathrm{int}} = -\mathcal{L}_{\mathrm{int}}$ from $S_{\mathrm{int}} = \int \mathrm{d}\tau \int \mathrm{d}^3x\, \mathcal{L}_{\mathrm{int}}$.

Finding relations and their inverses

- Compute change to two-point function
  $\Delta\mathcal{P} \propto \Delta\langle\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \Delta\langle 0|\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}|0\rangle$
- Expectation values in QFT. Use Schwinger-Keldysh formalism.
- Helped by Weinberg:

$$\langle\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \sum_{n=0} i^n \int_{-\infty}^{\tau_n} \mathrm{d}\tau_{n-1} \cdots \int_{-\infty}^{0} \mathrm{d}\tau_1$$

$$\langle 0|[H_{\rm int}(\tau_1), \ldots, [H_{\rm int}(\tau_{n-1}), [H_{\rm int}(\tau_n), \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}]]]|0\rangle$$

where $\mathcal{H}_{\rm int} = -\mathcal{L}_{\rm int}$ from $S_{\rm int} = \int \mathrm{d}\tau \int \mathrm{d}^3 x\, \mathcal{L}_{\rm int}$.

Finding relations and their inverses

- Compute change to two-point function
  $\Delta \mathcal{P} \propto \Delta \langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = \Delta \langle 0 | \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} | 0 \rangle$
- Expectation values in QFT. Use Schwinger-Keldysh formalism.
- Helped by Weinberg:

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = \sum_{n=0} i^n \int_{-\infty}^{\tau_n} \mathrm{d}\tau_{n-1} \cdots \int_{-\infty}^{0} \mathrm{d}\tau_1$$
$$\langle 0 | [H_{\mathrm{int}}(\tau_1), \ldots, [H_{\mathrm{int}}(\tau_{n-1}), [H_{\mathrm{int}}(\tau_n), \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'}]]] | 0 \rangle$$

  where $\mathcal{H}_{\mathrm{int}} = -\mathcal{L}_{\mathrm{int}}$ from $S_{\mathrm{int}} = \int \mathrm{d}\tau \int \mathrm{d}^3 x \, \mathcal{L}_{\mathrm{int}}$.
- Fourier expand $\mathcal{R}(\tau) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} (\hat{a}_{\mathbf{k}} \mathcal{R}_k(\tau) e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}^\dagger_{\mathbf{k}} \mathcal{R}^*_k(\tau) e^{-i\mathbf{k}\cdot\mathbf{x}})$

Finding relations and their inverses

- Compute change to two-point function
  $\Delta\mathcal{P} \propto \Delta\langle \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \Delta\langle 0|\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}|0\rangle$
- Expectation values in QFT. Use Schwinger-Keldysh formalism.
- Helped by Weinberg:

$$\langle \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \sum_{n=0} i^n \int_{-\infty}^{\tau_n} \mathrm{d}\tau_{n-1} \cdots \int_{-\infty}^{0} \mathrm{d}\tau_1$$

$$\langle 0|[H_{\rm int}(\tau_1), \ldots, [H_{\rm int}(\tau_{n-1}), [H_{\rm int}(\tau_n), \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}]]]|0\rangle$$

  where $\mathcal{H}_{\rm int} = -\mathcal{L}_{\rm int}$ from $S_{\rm int} = \int \mathrm{d}\tau \int \mathrm{d}^3x\, \mathcal{L}_{\rm int}$.
- Fourier expand $\mathcal{R}(\tau) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}(\hat{a}_{\mathbf{k}}\mathcal{R}_k(\tau)e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}^\dagger_{\mathbf{k}}\mathcal{R}^*_k(\tau)e^{-i\mathbf{k}\cdot\mathbf{x}})$
- where $\mathcal{R}_k(\tau) = iH(1+ik\tau)e^{-ik\tau}/\sqrt{4\epsilon k^3}$

Finding relations and their inverses

- Compute change to two-point function $\Delta\mathcal{P} \propto \Delta\langle\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \Delta\langle 0|\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}|0\rangle$
- Expectation values in QFT. Use Schwinger-Keldysh formalism.
- Helped by Weinberg:

$$\langle\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \sum_{n=0} i^n \int_{-\infty}^{\tau_n} \mathrm{d}\tau_{n-1} \cdots \int_{-\infty}^{0} \mathrm{d}\tau_1$$

$$\langle 0|[H_{\mathrm{int}}(\tau_1), \ldots, [H_{\mathrm{int}}(\tau_{n-1}), [H_{\mathrm{int}}(\tau_n), \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}]]]|0\rangle$$

where $\mathcal{H}_{\mathrm{int}} = -\mathcal{L}_{\mathrm{int}}$ from $S_{\mathrm{int}} = \int \mathrm{d}\tau \int \mathrm{d}^3x\, \mathcal{L}_{\mathrm{int}}$.

- Fourier expand $\mathcal{R}(\tau) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}(\hat{a}_{\mathbf{k}}\mathcal{R}_k(\tau)e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}^\dagger_{\mathbf{k}}\mathcal{R}^*_k(\tau)e^{-i\mathbf{k}\cdot\mathbf{x}})$
- where $\mathcal{R}_k(\tau) = iH(1+ik\tau)e^{-ik\tau}/\sqrt{4\epsilon k^3}$
- and promote to ladder operators with $[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}] = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')$

Finding relations and their inverses

- Compute change to two-point function $\Delta\mathcal{P} \propto \Delta\langle \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \Delta\langle 0|\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}|0\rangle$
- Expectation values in QFT. Use Schwinger-Keldysh formalism.
- Helped by Weinberg:

$$\langle \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \sum_{n=0} i^n \int_{-\infty}^{\tau_n} \mathrm{d}\tau_{n-1} \cdots \int_{-\infty}^{0} \mathrm{d}\tau_1$$

$$\langle 0|[H_{\mathrm{int}}(\tau_1), \ldots, [H_{\mathrm{int}}(\tau_{n-1}), [H_{\mathrm{int}}(\tau_n), \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}]]]|0\rangle$$

  where $\mathcal{H}_{\mathrm{int}} = -\mathcal{L}_{\mathrm{int}}$ from $S_{\mathrm{int}} = \int \mathrm{d}\tau \int \mathrm{d}^3x\, \mathcal{L}_{\mathrm{int}}$.
- Fourier expand $\mathcal{R}(\tau) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}(\hat{a}_{\mathbf{k}}\mathcal{R}_k(\tau)e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}^{\dagger}_{\mathbf{k}}\mathcal{R}^*_k(\tau)e^{-i\mathbf{k}\cdot\mathbf{x}})$
- where $\mathcal{R}_k(\tau) = iH(1 + ik\tau)e^{-ik\tau}/\sqrt{4\epsilon k^3}$
- and promote to ladder operators with $[a_{\mathbf{k}}, a^{\dagger}_{\mathbf{k}'}] = (2\pi)^3\delta^{(3)}(\mathbf{k} + \mathbf{k}')$
- Then use Wick's theorem.

The relations and inverse

- We find the dictionary: $\frac{\Delta\mathcal{P}}{\mathcal{P}}(k) = -k\int_{-\infty}^{0} \mathrm{d}\tau\, u(\tau)\sin(2k\tau)$ inverting to $u(\tau) = \frac{4}{\pi}\int_0^{\infty}\frac{\mathrm{d}k}{k}\frac{\Delta\mathcal{P}}{\mathcal{P}}(k)\sin(-2k\tau)$

The relations and inverse

- We find the dictionary: $\frac{\Delta\mathcal{P}}{\mathcal{P}}(k) = -k\int_{-\infty}^{0} \mathrm{d}\tau\, u(\tau)\sin(2k\tau)$ inverting to $u(\tau) = \frac{4}{\pi}\int_0^\infty \frac{\mathrm{d}k}{k}\frac{\Delta\mathcal{P}}{\mathcal{P}}(k)\sin(-2k\tau)$
- $\Delta_1\mathcal{P}/\mathcal{P}(k) =$ $\frac{1}{k}\int_{-\infty}^{0} \frac{\mathrm{d}\tau}{\tau^2}\Delta\epsilon/\epsilon(\tau)((1-2k^2\tau^2)\sin(2k\tau) - 2k\tau\cos(2k\tau))$ inverting to $\Delta\epsilon/\epsilon(\tau) = \frac{2}{\pi}\int_0^\infty \frac{\mathrm{d}k}{k}\frac{\Delta_1\mathcal{P}}{\mathcal{P}}(k)\left(\frac{2\sin^2(k\tau)}{k\tau} - \sin(2k\tau)\right)$

The relations and inverse

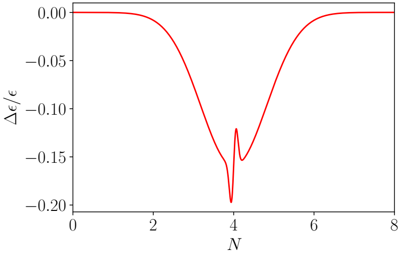
- We find the dictionary: $\frac{\Delta\mathcal{P}}{\mathcal{P}}(k) = -k\int_{-\infty}^{0} \mathrm{d}\tau\, u(\tau)\, \sin(2k\tau)$ inverting to $u(\tau) = \frac{4}{\pi}\int_0^{\infty} \frac{\mathrm{d}k}{k} \frac{\Delta\mathcal{P}}{\mathcal{P}}(k)\, \sin(-2k\tau)$
- $\Delta_1\mathcal{P}/\mathcal{P}(k) =$
  $\frac{1}{k}\int_{-\infty}^{0} \frac{\mathrm{d}\tau}{\tau^2} \Delta\epsilon/\epsilon(\tau)((1-2k^2\tau^2)\sin(2k\tau) - 2k\tau\cos(2k\tau))$
  inverting to
  $\Delta\epsilon/\epsilon(\tau) = \frac{2}{\pi}\int_0^{\infty} \frac{\mathrm{d}k}{k} \frac{\Delta_1\mathcal{P}}{\mathcal{P}}(k) \left(\frac{2\sin^2(k\tau)}{k\tau} - \sin(2k\tau)\right)$
- Find at 2nd order that correction is the square of the 1st order correction: $\Delta\mathcal{P}_{\mathrm{rec}}/\mathcal{P}_{\mathrm{rec}}(k) = \Delta_1\mathcal{P}/\mathcal{P}(k) + (\Delta_1\mathcal{P}/\mathcal{P}(k))^2$

The relations and inverse

- We find the dictionary: $\frac{\Delta\mathcal{P}}{\mathcal{P}}(k) = -k \int_{-\infty}^{0} \mathrm{d}\tau \, u(\tau) \sin(2k\tau)$ inverting to $u(\tau) = \frac{4}{\pi} \int_0^{\infty} \frac{\mathrm{d}k}{k} \frac{\Delta\mathcal{P}}{\mathcal{P}}(k) \sin(-2k\tau)$
- $\Delta_1\mathcal{P}/\mathcal{P}(k) =$
  $\frac{1}{k} \int_{-\infty}^{0} \frac{\mathrm{d}\tau}{\tau^2} \Delta\epsilon/\epsilon(\tau) ((1 - 2k^2\tau^2) \sin(2k\tau) - 2k\tau \cos(2k\tau))$
  inverting to
  $\Delta\epsilon/\epsilon(\tau) = \frac{2}{\pi} \int_0^{\infty} \frac{\mathrm{d}k}{k} \frac{\Delta_1\mathcal{P}}{\mathcal{P}}(k) \left(\frac{2\sin^2(k\tau)}{k\tau} - \sin(2k\tau) \right)$
- Find at 2nd order that correction is the square of the 1st order correction: $\Delta\mathcal{P}_{\mathrm{rec}}/\mathcal{P}_{\mathrm{rec}}(k) = \Delta_1\mathcal{P}/\mathcal{P}(k) + (\Delta_1\mathcal{P}/\mathcal{P}(k))^2$
- Quadratic equation! So
  $\Delta_1\mathcal{P}/\mathcal{P}(k) = \frac{1}{2} \left(-1 + \sqrt{1 + 4\frac{\Delta\mathcal{P}_{\mathrm{rec}}}{\mathcal{P}_{\mathrm{rec}}}} \right) \equiv \Delta\mathcal{P}_{\mathrm{eff}}/\mathcal{P}_{\mathrm{eff}}(k)$ and we know how $\Delta_1\mathcal{P}/\mathcal{P}(k)$ relates to c_s or ϵ so can isolate c_s or ϵ (by inverse transform).

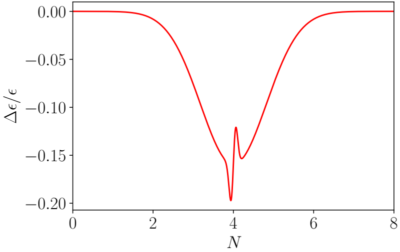
Toy model

- Model with localised feature at N_0 with a fast (σ_2) and slow component (σ_1) and amplitudes c_1,c_2:
$\Delta\epsilon/\epsilon(N) = c_1 e^{-(N-N_0)^2/\sigma_1^2} + c_2(N - N_0)e^{-(N-N_0)^2/\sigma_2^2}$.



Toy model

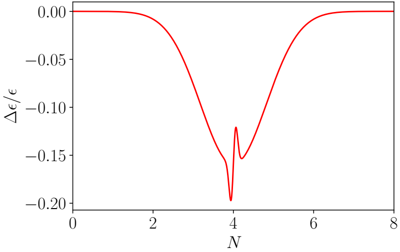
- Model with localised feature at N_0 with a fast (σ_2) and slow component (σ_1) and amplitudes c_1,c_2:
  $\Delta\epsilon/\epsilon(N) = c_1 e^{-(N-N_0)^2/\sigma_1^2} + c_2(N - N_0)e^{-(N-N_0)^2/\sigma_2^2}$.



- Can find the resulting change in the PPS using the dictionary.

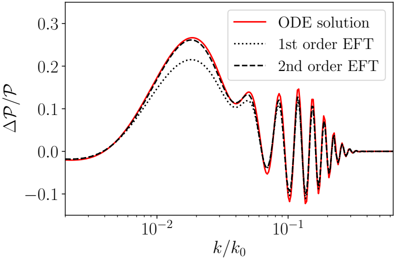
Toy model

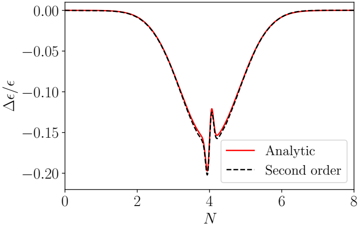
- Model with localised feature at N_0 with a fast (σ_2) and slow component (σ_1) and amplitudes c_1,c_2:
$\Delta\epsilon/\epsilon(N) = c_1 e^{-(N-N_0)^2/\sigma_1^2} + c_2(N - N_0)e^{-(N-N_0)^2/\sigma_2^2}$.



- Can find the resulting change in the PPS using the dictionary.
- Can also solve the Mukhanov-Sasaki equation numerically assuming this change.
$\frac{\mathrm{d}^2\mathcal{R}_k}{\mathrm{d}N^2} + \left(3 - \epsilon(N) + \frac{\epsilon'(N)}{\epsilon(N)}\right)\frac{\mathrm{d}\mathcal{R}_k}{\mathrm{d}N} + \left(\frac{k}{aH}\right)^2 \mathcal{R}_k = 0$

Toy model checks





The potential

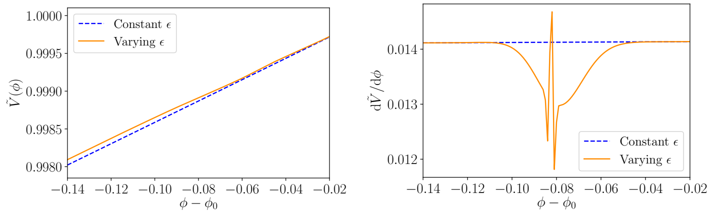
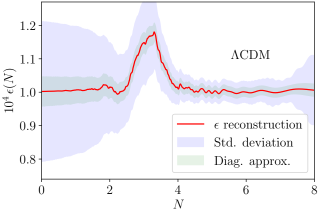
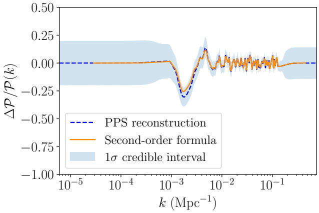


Figure: Scalar field potential (left) and derivative of potential (right) for constant $\epsilon = \epsilon_0$ and $\epsilon(\tau)$ with features.

From Planck data

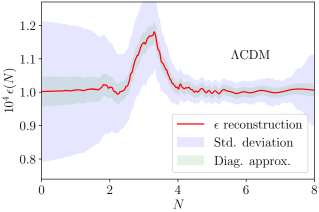
- For ΛCDM:

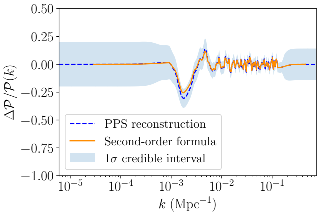




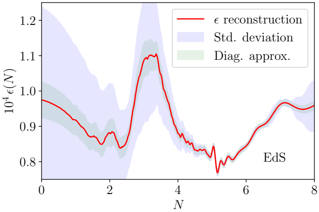
From Planck data

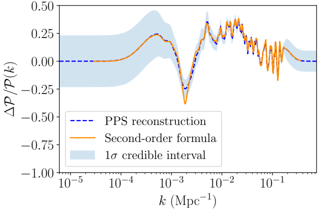
- For ΛCDM:





- For EdS:





Note on degeneracy

- Features can come from a combined change in c_s and ϵ.

Note on degeneracy

- Features can come from a combined change in c_s and ϵ.
- Not possible to invert unless feature is exclusively from one of the EFT parameters.

Note on degeneracy

- Features can come from a combined change in c_s and ϵ.
- Not possible to invert unless feature is exclusively from one of the EFT parameters.
- However, theoretically c_s should not exceed 1, hence $u(\tau) \equiv 1/c_s^2 - 1 > 0$, so it may not be possible to generate *any* given PPS subject to this constraint.

Note on degeneracy

- Features can come from a combined change in c_s and ϵ.
- Not possible to invert unless feature is exclusively from one of the EFT parameters.
- However, theoretically c_s should not exceed 1, hence $u(\tau) \equiv 1/c_s^2 - 1 > 0$, so it may not be possible to generate *any* given PPS subject to this constraint.
- Contributions to *n*-point functions from changes in ϵ and c_s do differ, so one way to disentangle.

Note on degeneracy

- Features can come from a combined change in c_s and ϵ.
- Not possible to invert unless feature is exclusively from one of the EFT parameters.
- However, theoretically c_s should not exceed 1, hence $u(\tau) \equiv 1/c_s^2 - 1 > 0$, so it may not be possible to generate *any* given PPS subject to this constraint.
- Contributions to *n*-point functions from changes in ϵ and c_s do differ, so one way to disentangle.
- Complicated by the fact that new (Wilson) functions appear at higher order that may reintroduce degeneracy.

Summary

- Features: a luxury for ΛCDM. Necessary for other cosmological models.

Summary

- Features: a luxury for ΛCDM. Necessary for other cosmological models.
- Computed corrections to PPS due to changes in ϵ and c_s, parameters in the EFTI.

Summary

- Features: a luxury for ΛCDM. Necessary for other cosmological models.
- Computed corrections to PPS due to changes in ϵ and c_s, parameters in the EFTI.
- Inverted those relations to get the EFT parameters as transforms of the desired change in the PPS.

Summary

- Features: a luxury for ΛCDM. Necessary for other cosmological models.
- Computed corrections to PPS due to changes in ϵ and c_s, parameters in the EFTI.
- Inverted those relations to get the EFT parameters as transforms of the desired change in the PPS.
- Performed the inversion to second order in EFT parameters.

Summary

- Features: a luxury for ΛCDM. Necessary for other cosmological models.
- Computed corrections to PPS due to changes in ϵ and c_s, parameters in the EFTI.
- Inverted those relations to get the EFT parameters as transforms of the desired change in the PPS.
- Performed the inversion to second order in EFT parameters.
- Inversion precise to $\sim 1\%$ even when the features are $\sim 20\%$.

Summary

- Features: a luxury for ΛCDM. Necessary for other cosmological models.
- Computed corrections to PPS due to changes in ϵ and c_s, parameters in the EFTI.
- Inverted those relations to get the EFT parameters as transforms of the desired change in the PPS.
- Performed the inversion to second order in EFT parameters.
- Inversion precise to $\sim 1\%$ even when the features are $\sim 20\%$.
- Can find simple realisations by reconstructing potential from the EFT parameter ϵ.

Summary

- Features: a luxury for ΛCDM. Necessary for other cosmological models.
- Computed corrections to PPS due to changes in ϵ and c_s, parameters in the EFTI.
- Inverted those relations to get the EFT parameters as transforms of the desired change in the PPS.
- Performed the inversion to second order in EFT parameters.
- Inversion precise to $\sim 1\%$ even when the features are $\sim 20\%$.
- Can find simple realisations by reconstructing potential from the EFT parameter ϵ.
- Can reconstruct these parameters from cosmological data sets.

Outlook: future directions

- Combine constraints from CMB and LSS. Currently considering the matter power spectrum from SDSS (DR12).

Outlook: future directions

- Combine constraints from CMB and LSS. Currently considering the matter power spectrum from SDSS (DR12).
- Embrace the fine-tuning.

Outlook: future directions

- Combine constraints from CMB and LSS. Currently considering the matter power spectrum from SDSS (DR12).
- Embrace the fine-tuning.
- The third-order action S_3 of the curvature perturbation $\mathcal{R}$ depends on $c_s(\tau)$ and $\epsilon(\tau)$.

Outlook: future directions

- Combine constraints from CMB and LSS. Currently considering the matter power spectrum from SDSS (DR12).
- Embrace the fine-tuning.
- The third-order action S_3 of the curvature perturbation $\mathcal{R}$ depends on $c_s(\tau)$ and $\epsilon(\tau)$.
- In very non-standard cosmological models these EFT parameters are highly constrained.

Outlook: future directions

- Combine constraints from CMB and LSS. Currently considering the matter power spectrum from SDSS (DR12).
- Embrace the fine-tuning.
- The third-order action S_3 of the curvature perturbation $\mathcal{R}$ depends on $c_s(\tau)$ and $\epsilon(\tau)$.
- In very non-standard cosmological models these EFT parameters are highly constrained.
- Hence the non-Gaussianity should be very specific: easy to look for. May face that the non-Gaussianity is still too weak for Planck.

Outlook: future directions

- Combine constraints from CMB and LSS. Currently considering the matter power spectrum from SDSS (DR12).
- Embrace the fine-tuning.
- The third-order action S_3 of the curvature perturbation $\mathcal{R}$ depends on $c_s(\tau)$ and $\epsilon(\tau)$.
- In very non-standard cosmological models these EFT parameters are highly constrained.
- Hence the non-Gaussianity should be very specific: easy to look for. May face that the non-Gaussianity is still too weak for Planck.
- Strictly, it is not necessary to use estimates of the PPS to get EFT parameters.

Outlook: future directions

- Combine constraints from CMB and LSS. Currently considering the matter power spectrum from SDSS (DR12).
- Embrace the fine-tuning.
- The third-order action S_3 of the curvature perturbation $\mathcal{R}$ depends on $c_s(\tau)$ and $\epsilon(\tau)$.
- In very non-standard cosmological models these EFT parameters are highly constrained.
- Hence the non-Gaussianity should be very specific: easy to look for. May face that the non-Gaussianity is still too weak for Planck.
- Strictly, it is not necessary to use estimates of the PPS to get EFT parameters.
- Can go from CMB data, C_ℓ, directly to the EFT parameters $c_s(\tau)$ and $\epsilon(\tau)$. There is a linear relation ($\mathbf{W}_{\ell k}$) between $\mathcal{P}(k)$ and C_ℓ, and a linear relation between EFT parameters and $\mathcal{P}(k)$ ($\mathbf{W}_{k\tau}$) . So can just multiply matrices.

Potential reconstruction

- When writing ϵ in terms of *e*-folds: $\epsilon(N) = (\mathrm{d}\phi/\mathrm{d}N)^2/2$

Potential reconstruction

- When writing ϵ in terms of *e*-folds: $\epsilon(N) = (\mathrm{d}\phi/\mathrm{d}N)^2/2$
- Solution: $\phi(N) = \phi_0 \pm \int_{N_0}^{N} \mathrm{d}N' \sqrt{2\epsilon(N')}$.

Potential reconstruction

- When writing ϵ in terms of *e*-folds: $\epsilon(N) = (\mathrm{d}\phi/\mathrm{d}N)^2/2$
- Solution: $\phi(N) = \phi_0 \pm \int_{N_0}^{N} \mathrm{d}N' \sqrt{2\epsilon(N')}$.
- Recall that $\epsilon(N) = -\mathrm{d}\log H/\mathrm{d}N$

Potential reconstruction

- When writing ϵ in terms of e-folds: $\epsilon(N) = (\mathrm{d}\phi/\mathrm{d}N)^2/2$
- Solution: $\phi(N) = \phi_0 \pm \int_{N_0}^{N} \mathrm{d}N' \sqrt{2\epsilon(N')}$.
- Recall that $\epsilon(N) = -\mathrm{d}\log H/\mathrm{d}N$
- Hence, $H(N) = H_0 \exp(-\int_{N_0}^{N} \mathrm{d}N' \epsilon(N'))$

Potential reconstruction

- When writing ϵ in terms of *e*-folds: $\epsilon(N) = (\mathrm{d}\phi/\mathrm{d}N)^2/2$
- Solution: $\phi(N) = \phi_0 \pm \int_{N_0}^{N} \mathrm{d}N' \sqrt{2\epsilon(N')}$.
- Recall that $\epsilon(N) = -\mathrm{d}\log H/\mathrm{d}N$
- Hence, $H(N) = H_0 \exp(-\int_{N_0}^{N} \mathrm{d}N' \epsilon(N'))$
- Considering the Friedmann equation $H^2 \approx V/3$ we have $V(N) = H_0^2 \exp(-2\int_{N_0}^{N} \mathrm{d}N' \epsilon(N'))/3$

Potential reconstruction

- When writing ϵ in terms of e-folds: $\epsilon(N) = (\mathrm{d}\phi/\mathrm{d}N)^2/2$
- Solution: $\phi(N) = \phi_0 \pm \int_{N_0}^{N} \mathrm{d}N' \sqrt{2\epsilon(N')}$.
- Recall that $\epsilon(N) = -\mathrm{d}\log H/\mathrm{d}N$
- Hence, $H(N) = H_0 \exp(-\int_{N_0}^{N} \mathrm{d}N' \epsilon(N'))$
- Considering the Friedmann equation $H^2 \approx V/3$ we have $V(N) = H_0^2 \exp(-2\int_{N_0}^{N} \mathrm{d}N' \epsilon(N'))/3$
- Now we have $(\phi(N), V(N))$. Can also find $N = N(\phi)$ and then calculate $V(N(\phi)) = V(\phi)$