

# Understanding the large-scale structure of the Universe

Sergey Sibiryakov

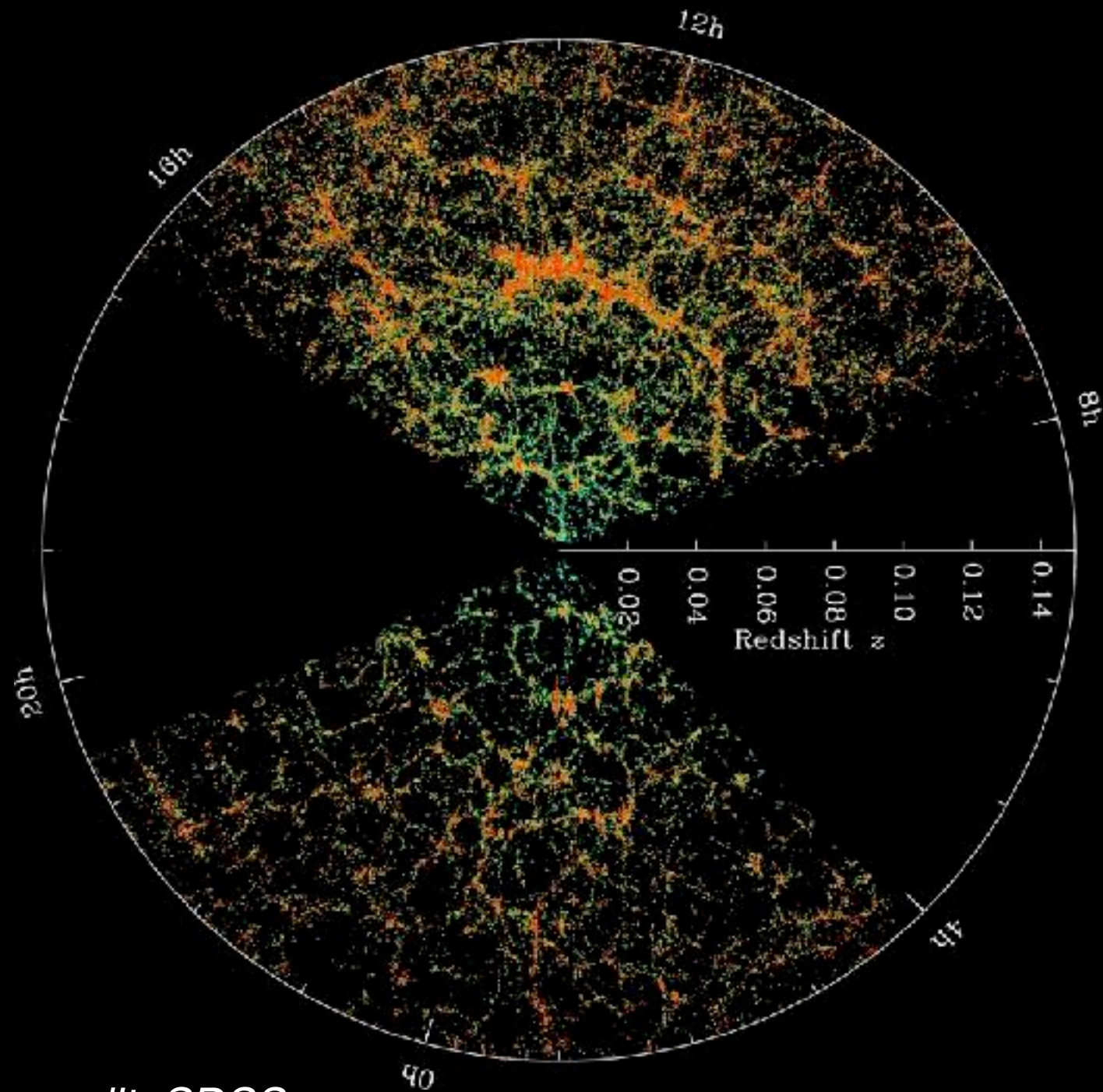


w/ D.Blas, M.Garny, M.Ivanov, A.Kaurov,  
A.Vasudevan, J.Lesgourgues

Rencontres du Vietnam, Quy Nhon, August 2019

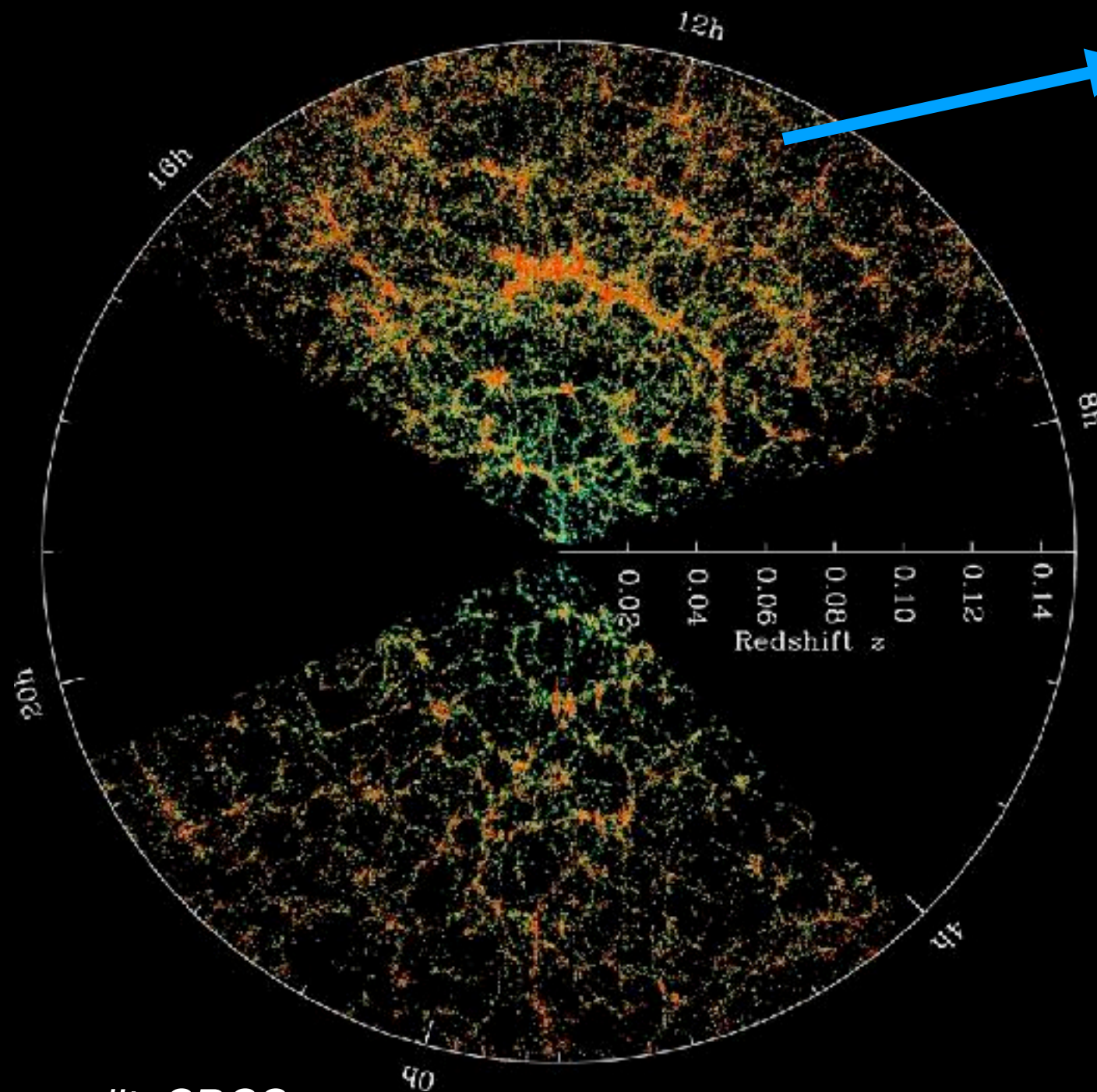
**What we want to understand ?**

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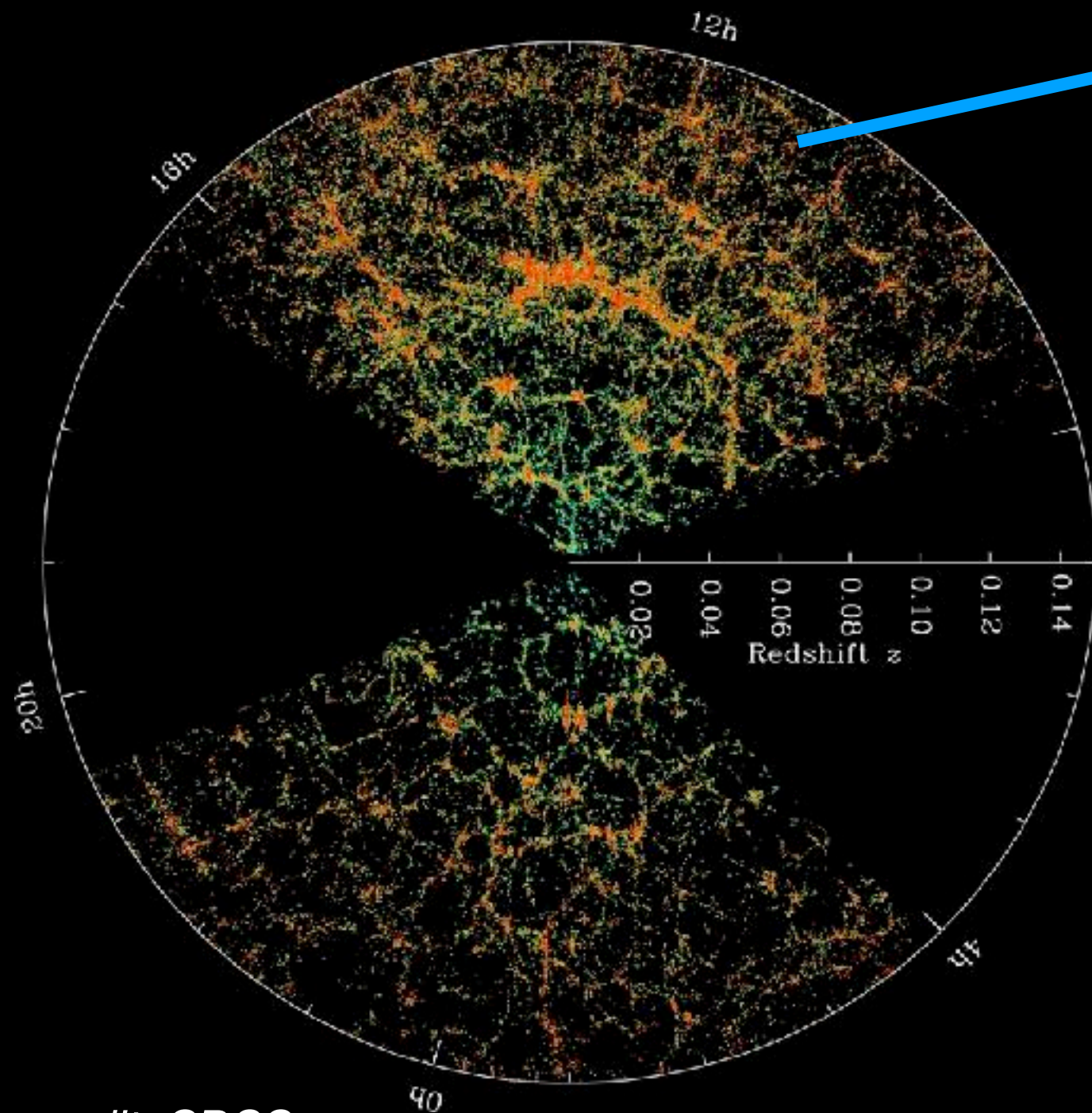
credit: SDSS

# What we want to understand ?



$$\frac{\delta\rho(x)}{\rho} \equiv \delta(x)$$

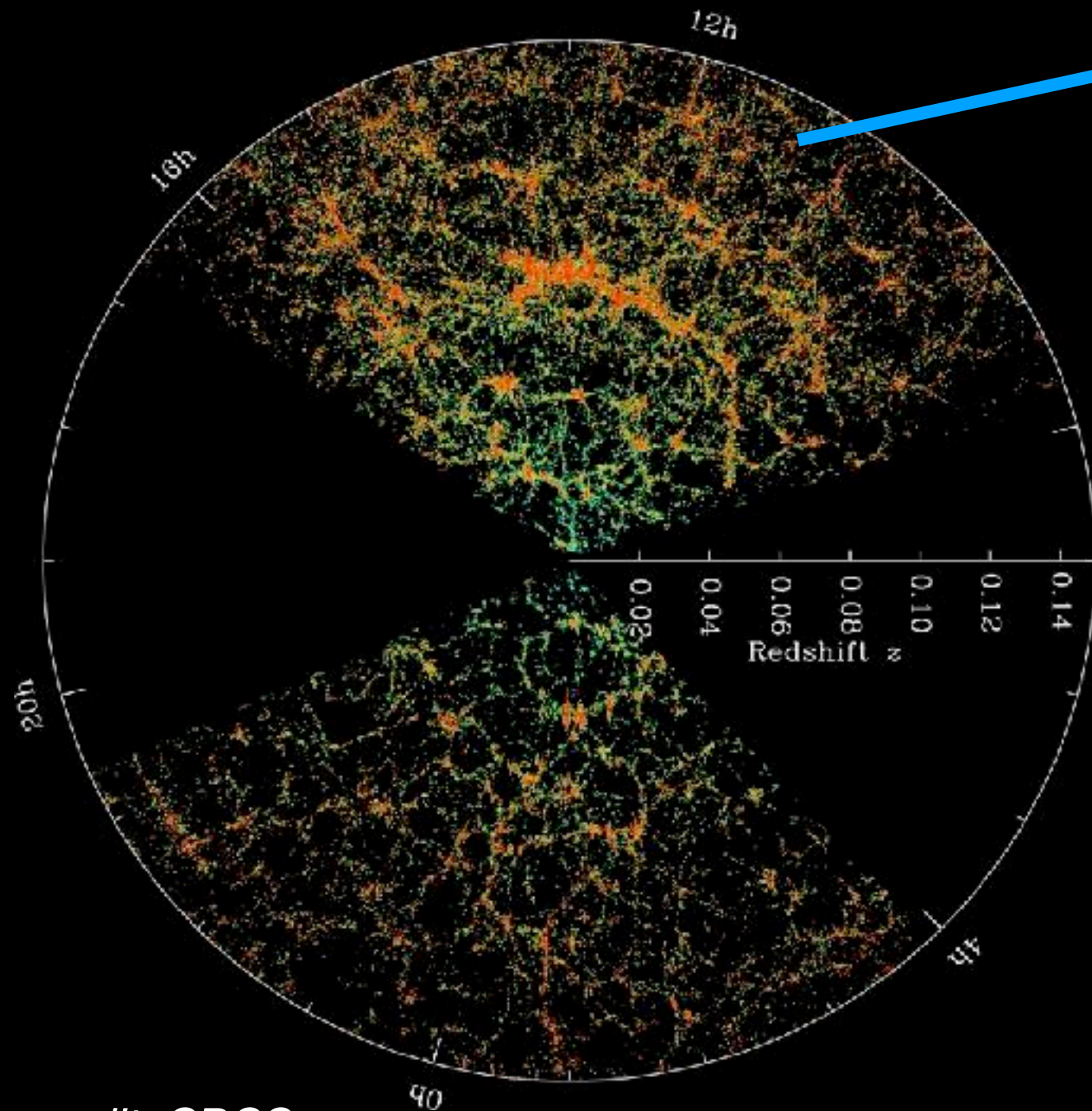
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statistical properties

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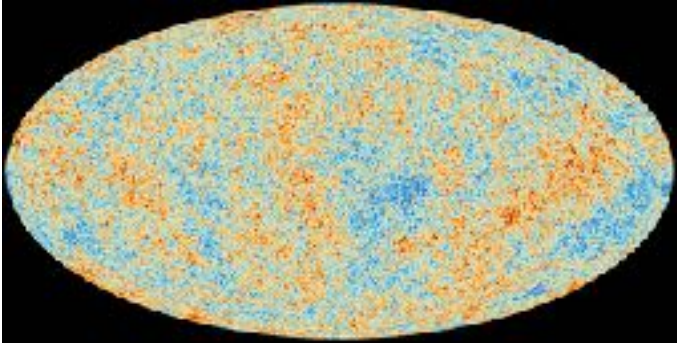


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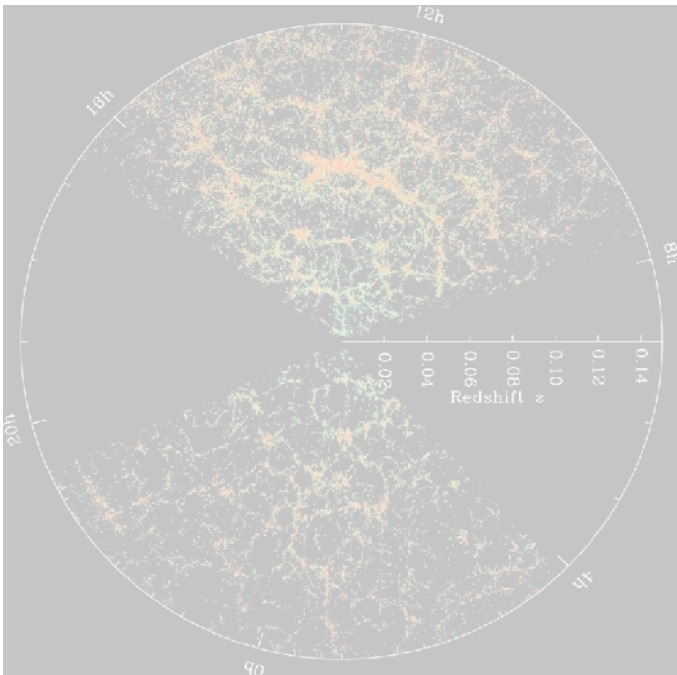
statistical properties

- cosmological parameters
- properties of DM, DE
- initial conditions e.g. primordial non-Gaussianity

# CMB vs. LSS

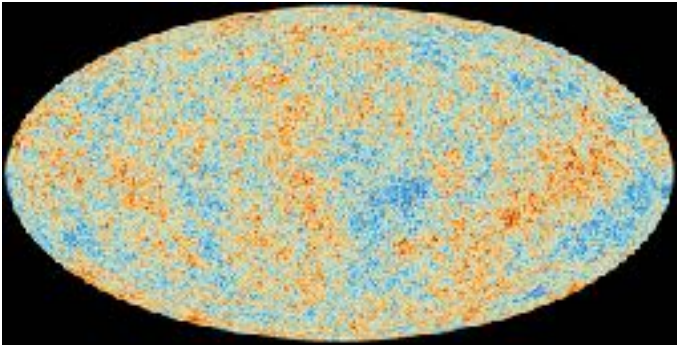


- 2d map: information  $\propto (l_{\max})^2$
- T saturated  $\rightarrow$  polarization
- linear theory

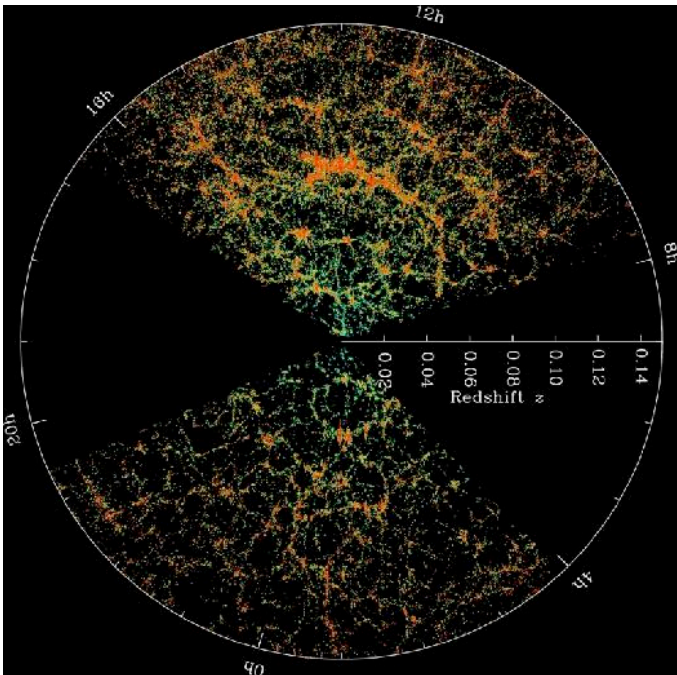


- 3d: information  $\propto (k_{\max})^3$
- will dominate cosmo data in the coming years
- non-linear gravitational clustering at  $k > k_{NL}(z)$

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- will dominate cosmo data in the coming years
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## N-body

### pros:

- exactly incorporate physics of LCDM
- access deeply non-linear regime

### cons:

- computationally expensive
- hard to extend beyond LCDM

## Analytic methods

### pros:

- physical insight
- flexible

### cons:

- work in limited range of scales  
 $k \lesssim 0.2 h/\text{Mpc}$   
(at  $z=0$ )

It is an old subject

*Zeldovich, Peebles,...* (1960+)

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XXI century challenge: controlled accuracy at ~1% level

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Progress due to insights from QFT

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Progress due to insights from QFT

$$Z[J, t] = \int [\mathcal{D}\delta_L(x)] \exp \left\{ - \int \frac{|\delta_L(k)|^2}{2P_L(k, t)} + \int J(x)\delta(x) \right\}$$



functional of  $\delta_L$

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**perturbative**  
diagrammatic  
technique

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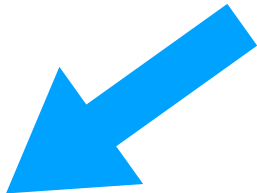
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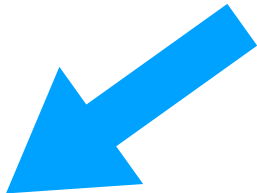
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IR resummation of  
BAO / primordial features



**non-perturbative**  
“semiclassical”

Counts-in-cells



# Perturbative schemes

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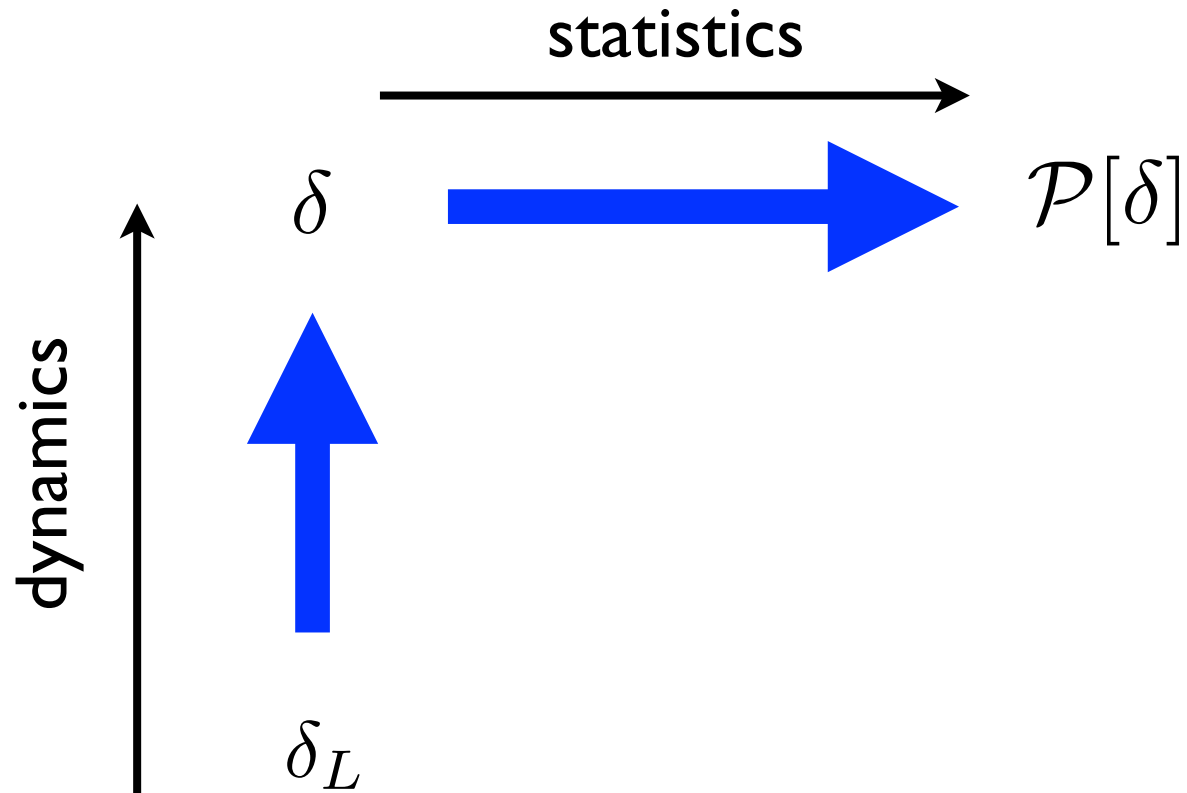
Standard Perturbation Theory

*Bernardeau et al. (2001)*

# Perturbative schemes

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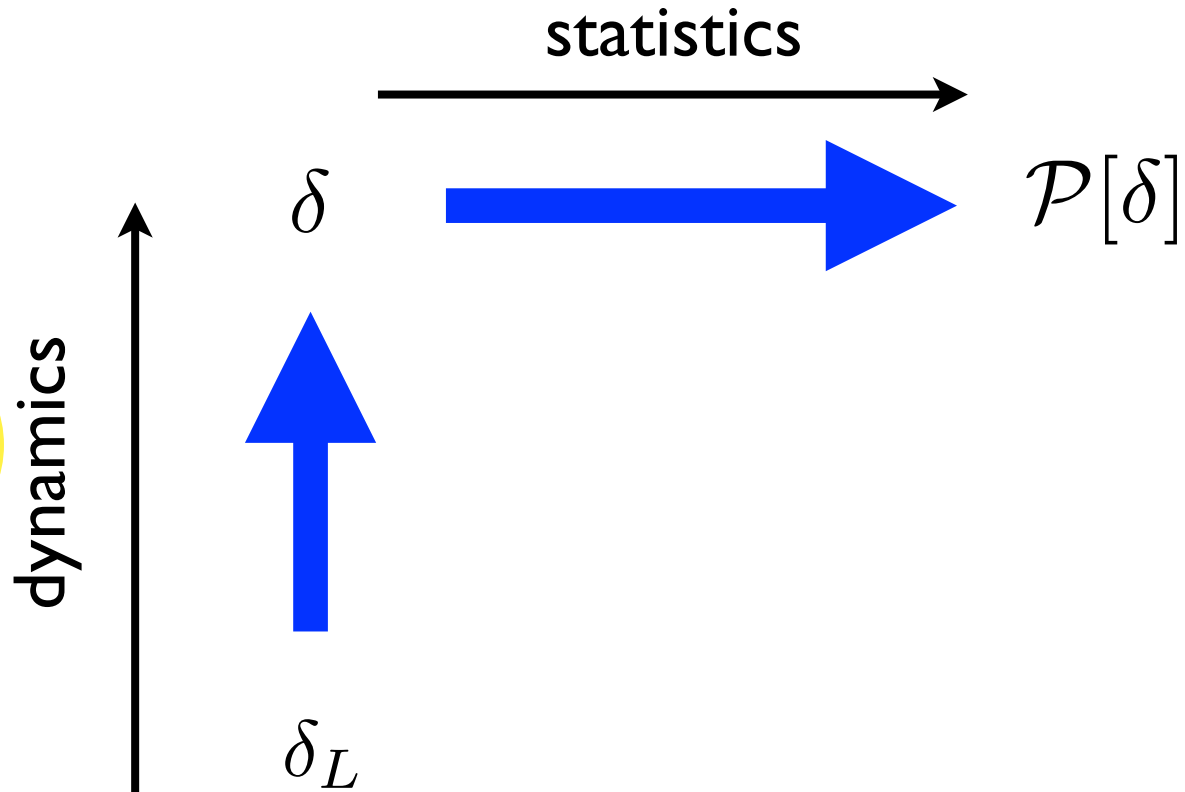
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# Perturbative schemes

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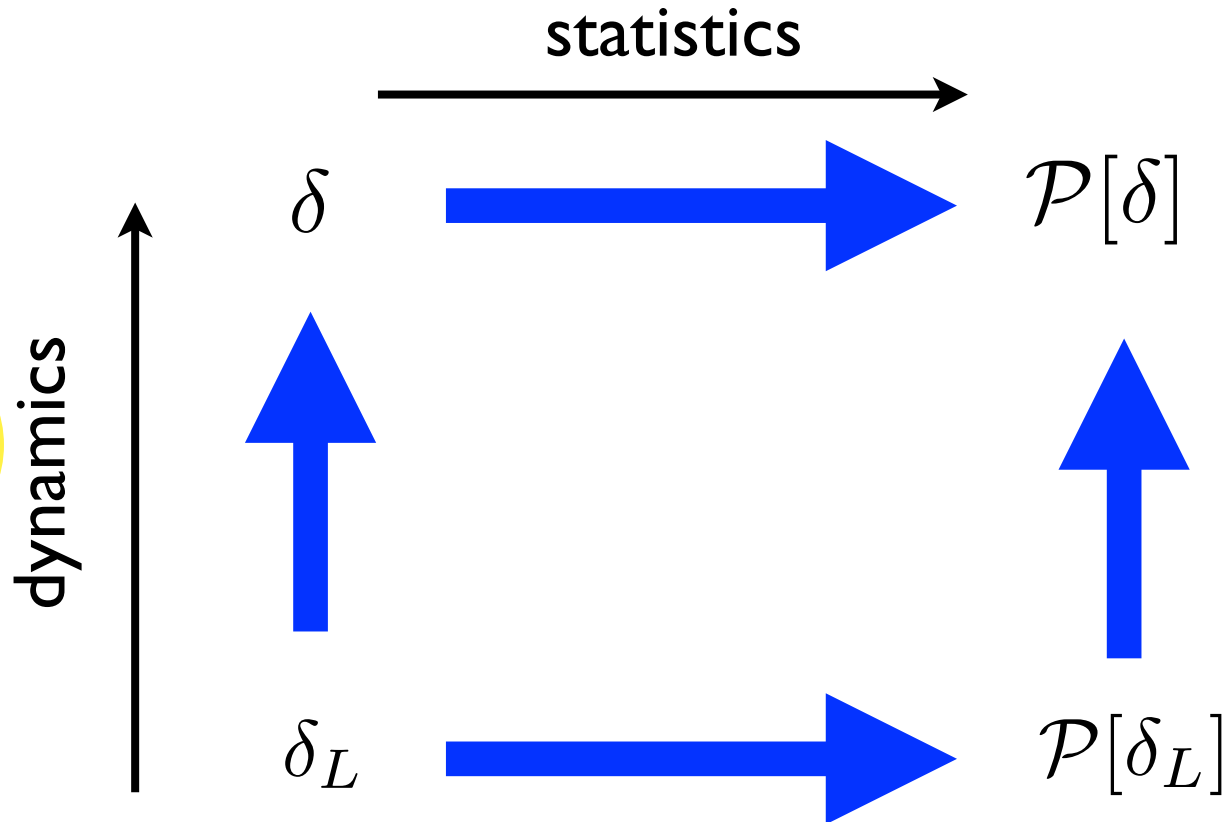


IR poles in  
the kernels

# Perturbative schemes

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IR poles in the kernels

Time-Sliced Perturbation Theory

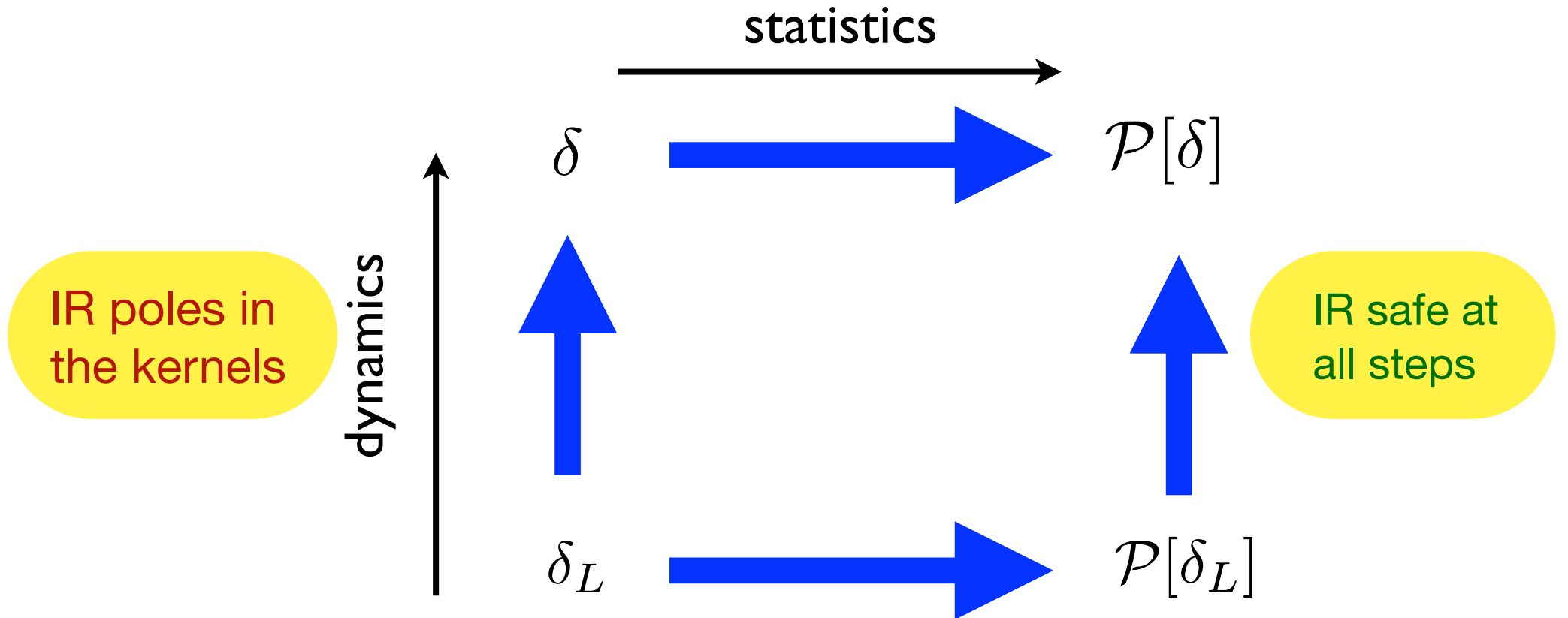
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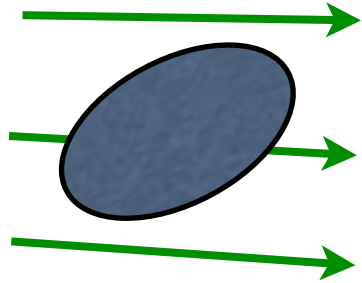


Time-Sliced Perturbation Theory

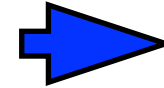
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# Spurious IR divergences from large bulk flows

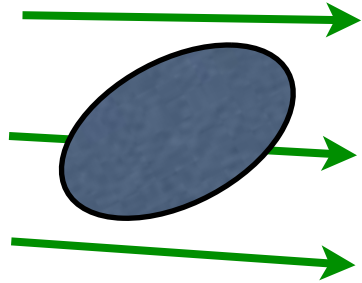


overdensity is  
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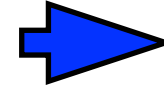


secular  
growth

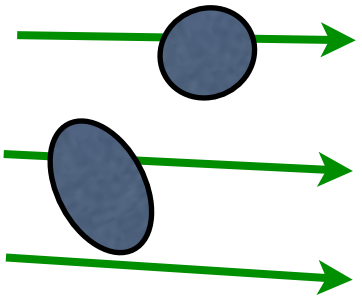
# Spurious IR divergences from large bulk flows



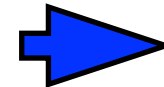
overdensity is  
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secular  
growth



two overdensities  
will move (almost)  
identically



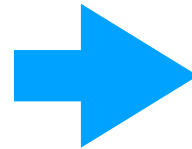
cancellation in  
equal-time  
statistics



# More on the generating functional

$$Z[J] = \int [d\delta] \exp \left\{ -\frac{1}{g^2} \sum_n \frac{1}{n!} \bar{\Gamma}_n * \delta^n + J * \delta \right\}$$

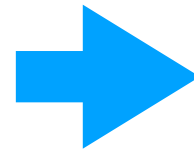
*linear growth factor*  $g(z)$



formal expansion parameter  
= effective coupling constant;  
true small parameter:

$$\sigma_d^2(k_*) = g^2 \int_{k < k_*} d^3 k \bar{P}_L(k)$$

$$\bar{\Gamma}_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{(2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2)}{\bar{P}_L(k)}$$



$\bar{\Gamma}_n$  obtained by recursion relations

**IR safety:**

$$\lim_{\epsilon \rightarrow 0} \bar{\Gamma}_{n+m}(\epsilon q_1, \dots, \epsilon q_m, k_1, \dots, k_n) < \infty$$

# Diagrammar

$$\text{---} \overset{k}{\text{---}} = g^2 \bar{P}_L(k)$$

$$\text{---} \begin{array}{l} \nearrow \overset{k_1}{\text{---}} \\ \searrow \underset{k_2}{\text{---}} \end{array} = \frac{1}{g^2} \bar{\Gamma}_3(k_1, k_2)$$

$$\begin{array}{l} \nearrow \overset{k_2}{\text{---}} \\ \searrow \underset{k_3}{\text{---}} \end{array} \begin{array}{l} \nwarrow \underset{k_1}{\text{---}} \\ \nearrow \text{---} \end{array} = \frac{1}{g^2} \bar{\Gamma}_4(k_1, k_2, k_3)$$



# Diagrammar

$$\text{---}^k = g^2 \bar{P}_L(k)$$

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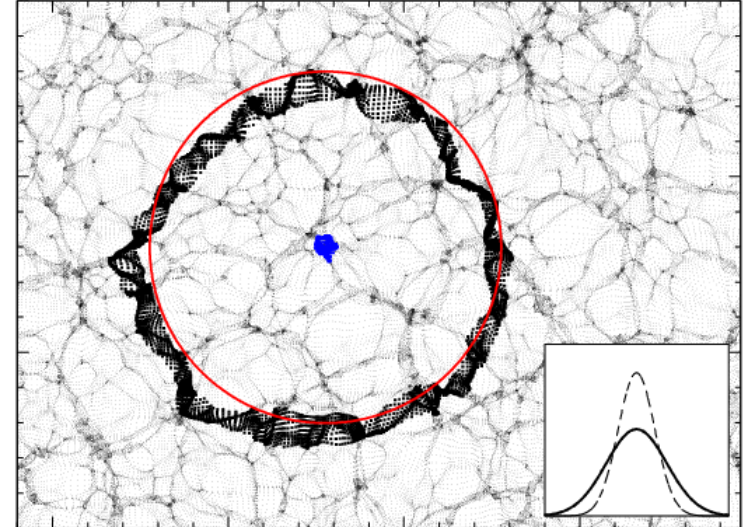
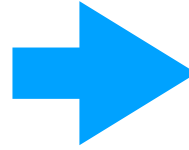
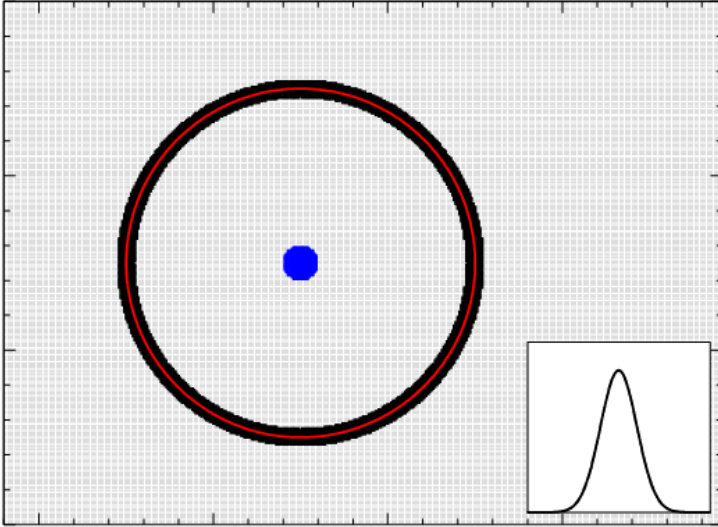
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$$\langle \delta \delta \rangle = \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \dots$$

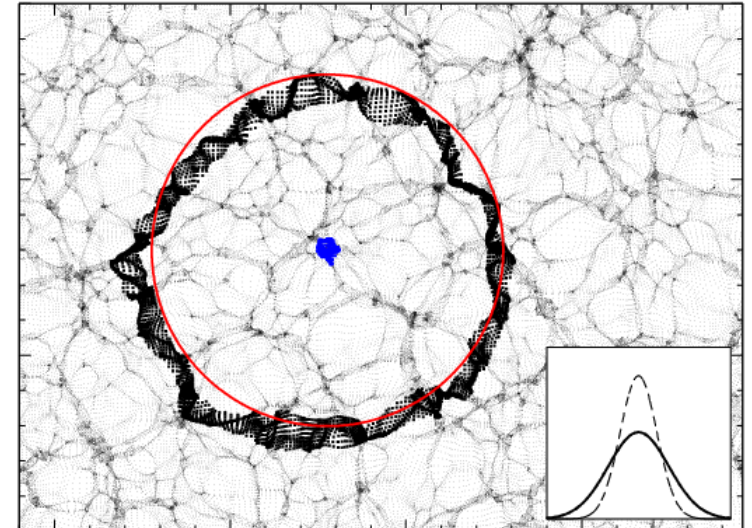
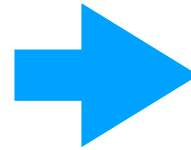
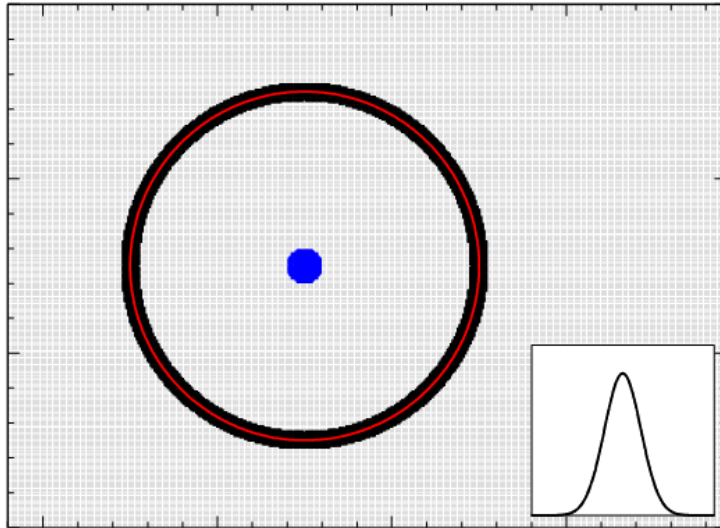
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# Application: Bulk flows vs. features



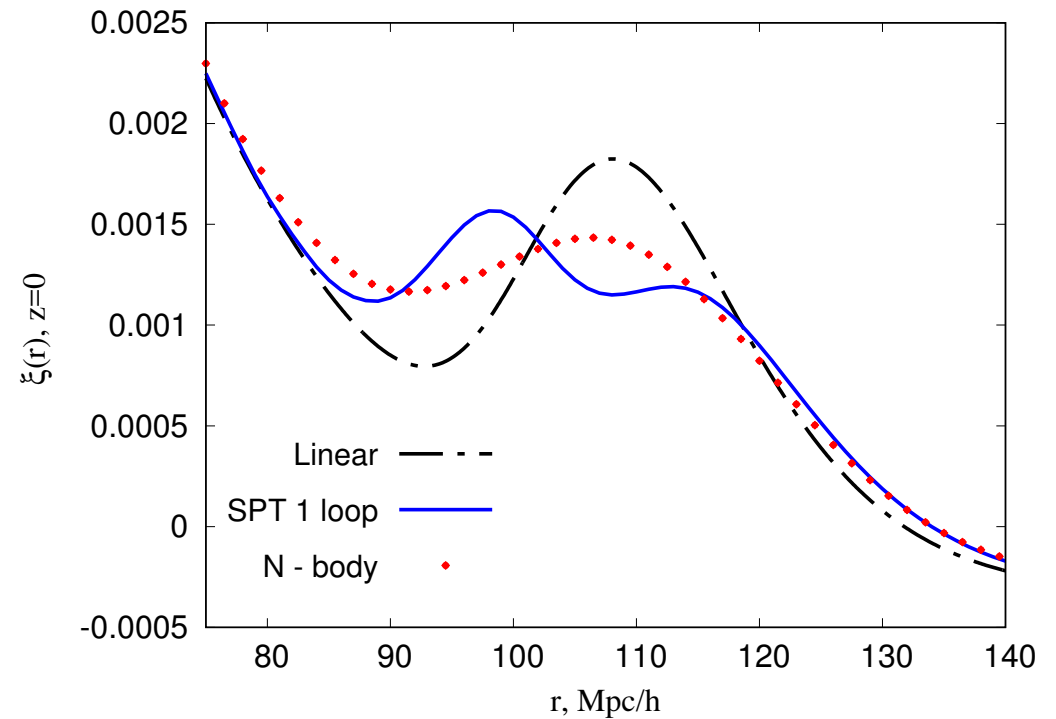
*from Padmanabhan et al. (2012)*

# Application: Bulk flows vs. features



*from Padmanabhan et al. (2012)*

Simple 1-loop calculation gets it wrong:



# IR resummation

*Blas, Garny, Ivanov, S.S. (2016)*

Wiggly part of PS gets dressed with soft loops

$$P_w^{\text{res}} =$$

$g^2 \frac{k^2}{q^2}$     $\left(g^2 \frac{k^2}{q^2}\right)^2$     $\left(g^2 \frac{k^2}{q^2}\right)^3$     $\left(g^2 \frac{k^2}{q^2}\right)^4$    ...

$= e^{-k^2 \Sigma^2} P_{Lw}$

loop momentum

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$$+ \left( g^2 \frac{k^2}{q^2} \right) + \left( g^2 \frac{k^2}{q^2} \right)^2 + \left( g^2 \frac{k^2}{q^2} \right)^3 + \left( g^2 \frac{k^2}{q^2} \right)^4 + \dots$$

$$= e^{-k^2 \Sigma^2} P_{Lw}$$

loop momentum

$$\Sigma^2 = \frac{4\pi}{3} \int_0^{k_S} dq P_{Ls}(q) (1 - j_0(q/k_{osc}) + 2j_2(q/k_{osc}))$$

# IR resummation

*Blas, Garny, Ivanov, S.S. (2016)*

Wiggly part of PS gets dressed with soft loops

$$P_w^{\text{res}} =$$

$$\begin{aligned}
 & \text{wavy line} + \text{1-loop flower} + \text{2-loop flower} + \text{3-loop flower} + \text{4-loop flower} + \dots \\
 & \qquad \qquad \qquad g^2 \frac{k^2}{q^2} \qquad \left(g^2 \frac{k^2}{q^2}\right)^2 \qquad \left(g^2 \frac{k^2}{q^2}\right)^3 \qquad \left(g^2 \frac{k^2}{q^2}\right)^4
 \end{aligned}$$

$$= e^{-k^2 \Sigma^2} P_{Lw}$$

separation between hard and soft momenta

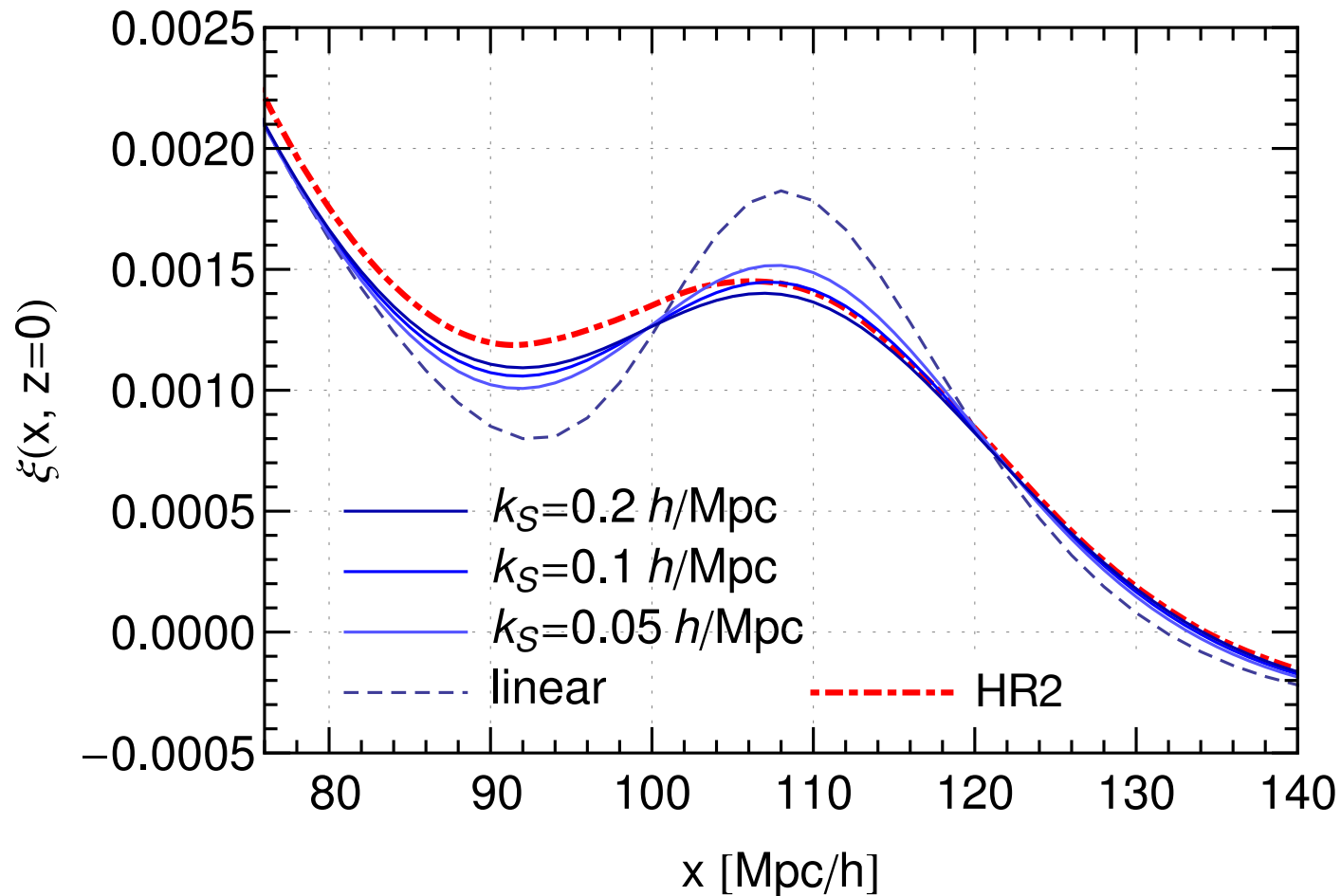
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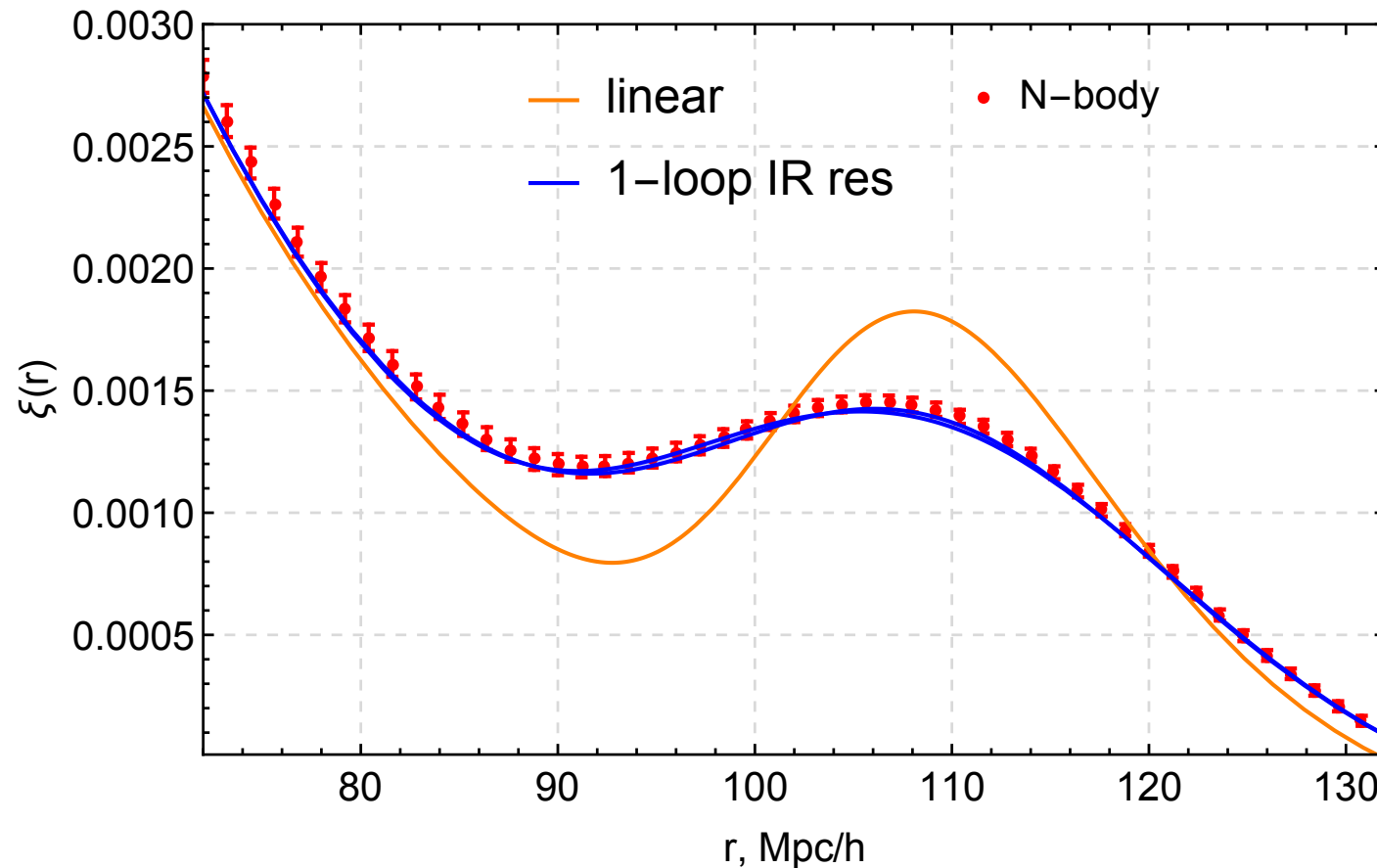
# Sensitivity to the IR separation scale: LO

IR resummed,  $z=0$



- dependence on  $k_S$  gives an estimate of the error due to neglecting higher loops

# (In)sensitivity to the IR separation scale: NLO



dependence on  $k_S$  decreases with the loop order

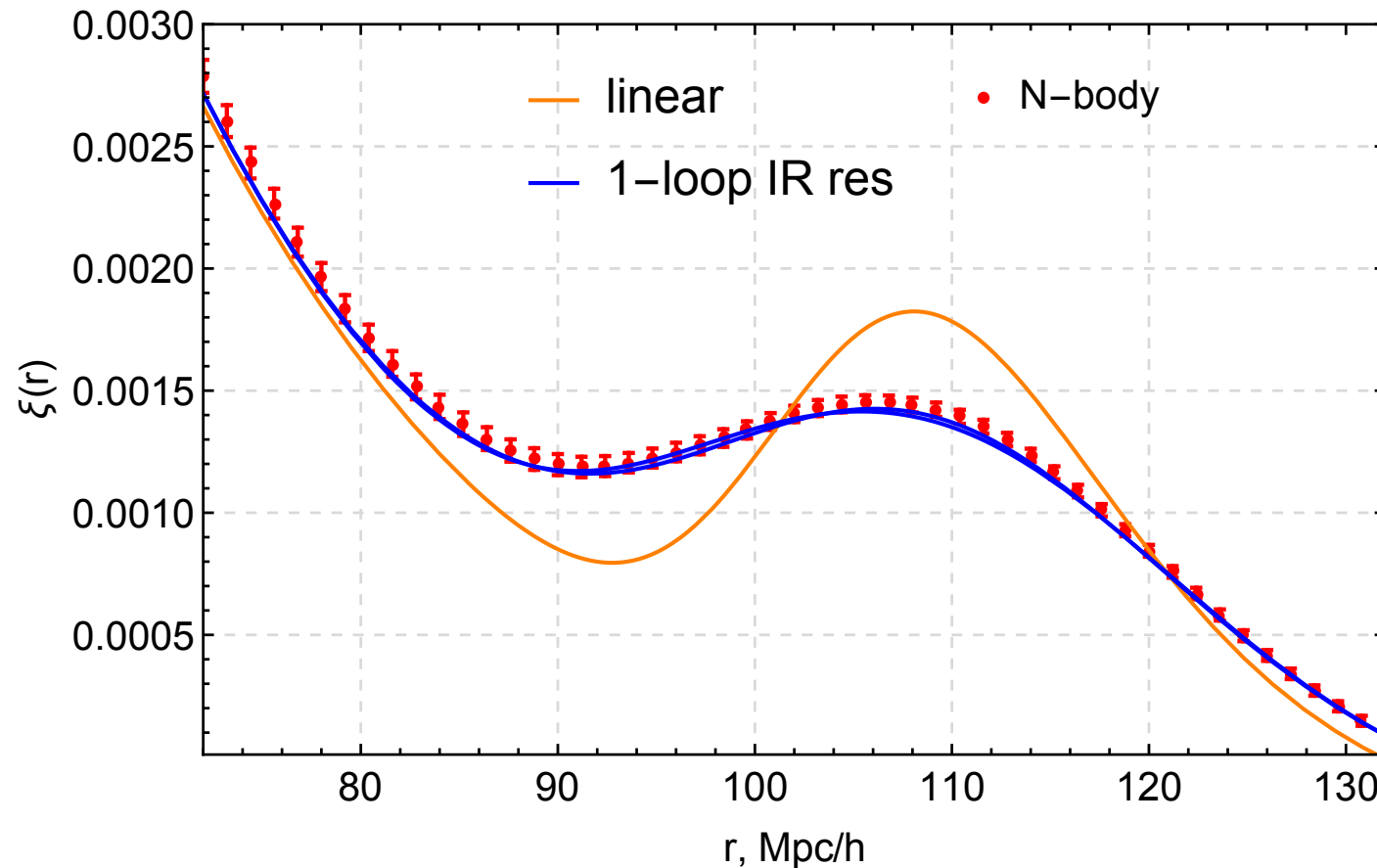
**NB.** EFT counterterm included to account for the failure of the fluid approximation

*Baumann et al. (2010); Carrasco, Hertzberg, Senatore (2012); Pajer, Zaldarriaga (2013)*

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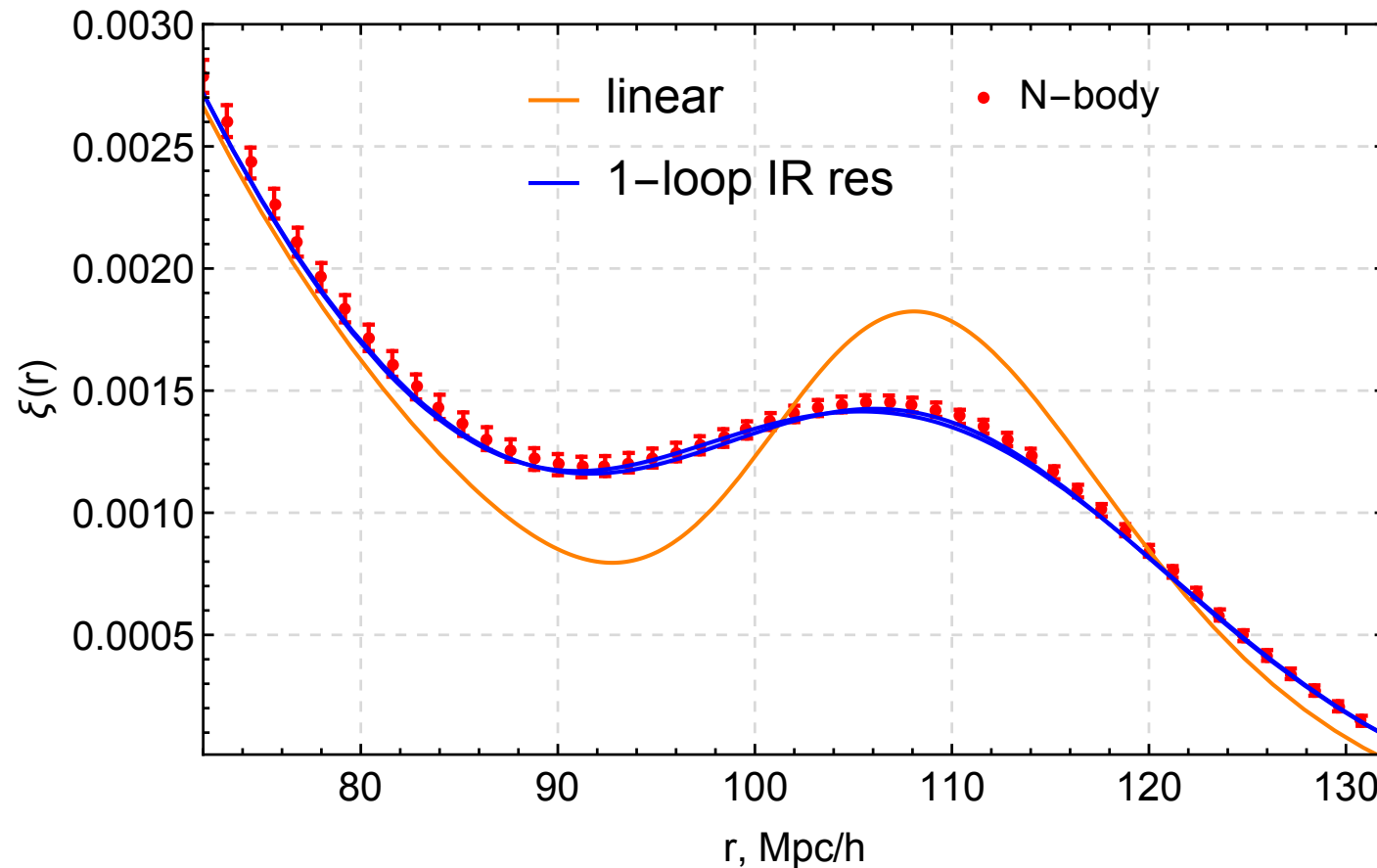
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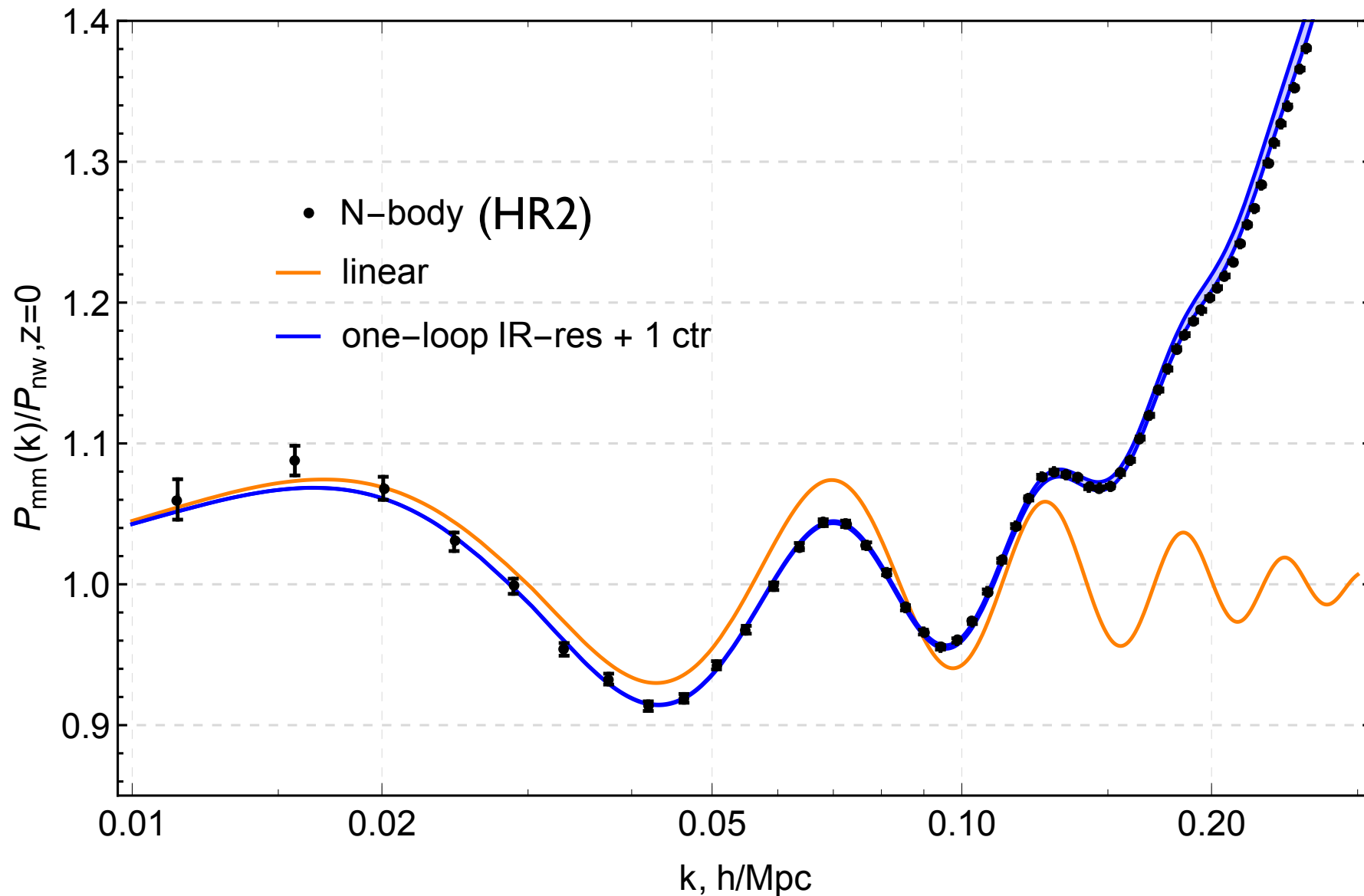
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# One-loop resummed PS



(Tentative) conclusion: precision  $O(10^{-3})$  up to  $k \sim 0.1$ ,  
sub-percent up to  $k \sim 0.2$  (at  $z=0$ ).

# Resummation: general scheme

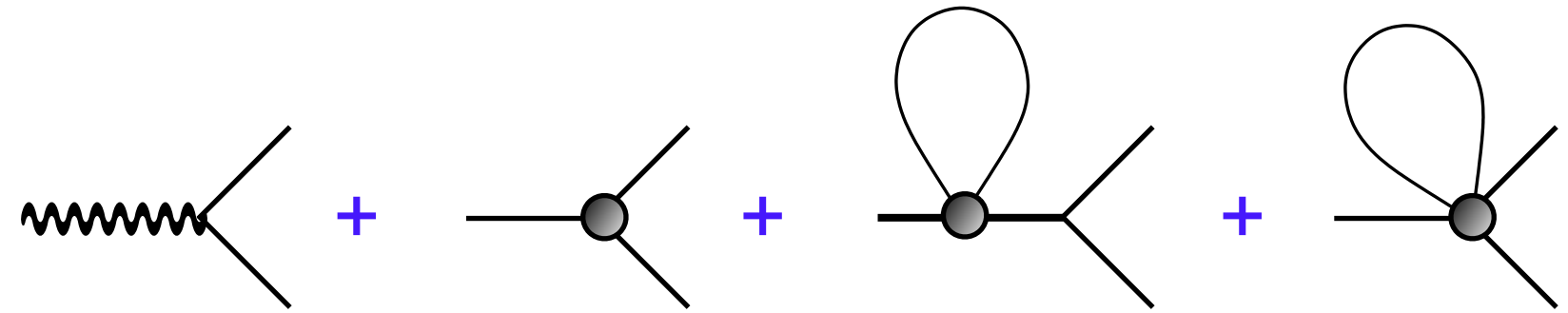
To get the leading IR diagrams dress the wiggly vertices with soft petals

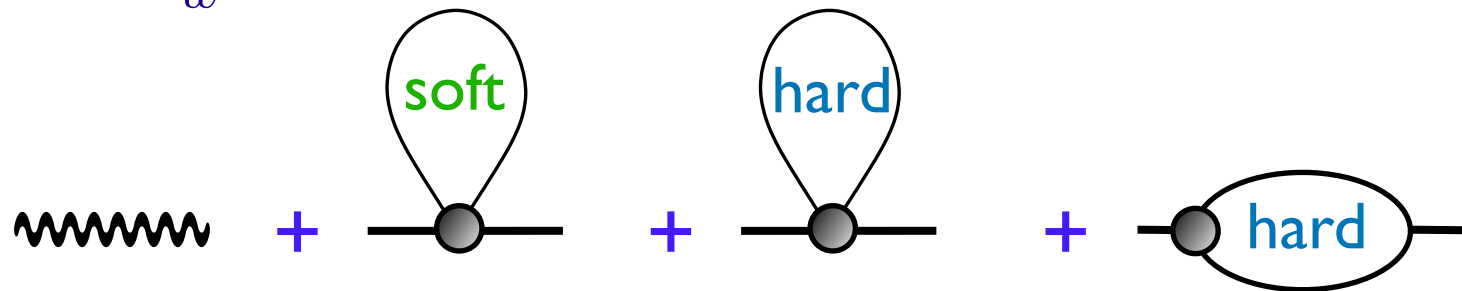
$$B_{LO,w} = \text{wavy line vertex} + \text{black dot vertex} + \text{black dot vertex with soft petal} + \text{black dot vertex with soft petal} \\ = e^{-S} B_w^{\text{tree}}$$

The diagrammatic equation shows the resummation of leading-order diagrams. On the left, the expression is  $B_{LO,w} = e^{-S} B_w^{\text{tree}}$ . On the right, the same expression is represented as a sum of four Feynman diagrams. The first diagram is a wavy line (representing a soft gluon) attached to a vertex from which two lines emerge. The second diagram is a black dot (representing a soft gluon) attached to a vertex from which two lines emerge. The third diagram is a black dot with a soft petal (a loop) attached to a vertex from which two lines emerge. The fourth diagram is a black dot with a soft petal attached to a vertex from which two lines emerge, with an additional line extending from the vertex.

# Resummation: general scheme

To get the leading IR diagrams dress the wiggly vertices with soft petals

$$B_{LO,w} = \text{wavy line vertex} + \text{tree vertex} + \text{soft loop} + \text{hard loop}$$
$$= e^{-S} B_w^{\text{tree}}$$
The diagrammatic expansion of  $B_{LO,w}$  consists of four terms separated by plus signs. The first term is a wavy line connected to a vertex with two outgoing lines. The second term is a solid line connected to a vertex with two outgoing lines. The third term is a solid line connected to a vertex with two outgoing lines, with a loop (petal) attached to the vertex. The fourth term is a solid line connected to a vertex with two outgoing lines, with a loop (petal) attached to the vertex.

$$P_{NLO,w} = \text{wavy line} + \text{soft loop} + \text{hard loop} + \text{hard loop}$$
The diagrammatic expansion of  $P_{NLO,w}$  consists of four terms separated by plus signs. The first term is a wavy line. The second term is a solid line with a loop (petal) attached to the vertex, labeled "soft" in green. The third term is a solid line with a loop (petal) attached to the vertex, labeled "hard" in blue. The fourth term is a solid line with a loop (petal) attached to the vertex, labeled "hard" in blue.

# Resummation: general scheme

To get the leading IR diagrams dress the wiggly vertices with soft petals

$$\begin{aligned}
 B_{LO,w} &= \text{wiggly vertex} + \text{tree vertex} + \text{soft loop} + \text{hard loop} \\
 &= e^{-S} B_w^{\text{tree}}
 \end{aligned}$$

$$\begin{aligned}
 P_{NLO,w} &= \text{wiggly vertex} + \text{soft loop} + \text{hard loop} + \text{hard loop} \\
 &\quad + \text{soft loop} + \text{hard loop} + \text{soft loop} + \text{soft loop} + \text{soft loop} + \text{hard loop} + \text{hard loop}
 \end{aligned}$$



# Resummation: general scheme

To get the leading IR diagrams dress the wiggly vertices with soft petals

$$\begin{aligned}
 B_{LO,w} &= \text{wiggly vertex} + \text{vertex} + \text{loop vertex} + \text{loop vertex} \\
 &= e^{-S} B_w^{\text{tree}}
 \end{aligned}$$

$$\begin{aligned}
 P_{NLO,w} &= \text{wiggly vertex} + \text{soft loop} + \text{hard loop} + \text{hard loop} \\
 &\quad + \text{soft loop} + \text{hard loop} + \text{soft loop} + \text{hard loop} \\
 &= e^{-S} P_{Lw} + e^{-S} P_w^{1\text{-loop}} + \mathcal{S} P_{Lw}
 \end{aligned}$$

# IR resummation summary (with RSD and bias)

*Ivanov, S.S. (2018)*

smooth

wiggly



$$P_L(k) = P_{L_s}(k) + P_{L_w}(k)$$

# IR resummation summary (with RSD and bias)

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smooth

wiggly

$$P_L(k) = P_{L_s}(k) + P_{L_w}(k)$$

$$\Rightarrow P_{LO}^{\text{res}}(\mathbf{k}) = P_{L_s}(k) + P_{L_w}(k)e^{-k^2 \Sigma^2(\mu; k_S)}$$

# IR resummation summary (with RSD and bias)

*Ivanov, S.S. (2018)*

smooth

wiggly

$$P_L(k) = P_{Ls}(k) + P_{Lw}(k)$$

$$\longrightarrow P_{LO}^{\text{res}}(\mathbf{k}) = P_{Ls}(k) + P_{Lw}(k)e^{-k^2\Sigma^2(\mu;k_S)}$$

**Tree level:**  $\mathcal{C}_{LO} = \mathcal{C}^{\text{tree}}[P_{LO}^{\text{res}}]$

**example:**  $B_{ggg}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = Z_2(\mathbf{k}_1, \mathbf{k}_2)P_{LO}^{\text{res}}(\mathbf{k}_1)P_{LO}^{\text{res}}(\mathbf{k}_2) + \text{perm.}$

# IR resummation summary (with RSD and bias)

Ivanov, S.S. (2018)

smooth

wiggly

$$P_L(k) = P_{Ls}(k) + P_{Lw}(k)$$

$$\Rightarrow P_{LO}^{\text{res}}(\mathbf{k}) = P_{Ls}(k) + P_{Lw}(k)e^{-k^2\Sigma^2(\mu;k_S)}$$

**Tree level:**  $\mathcal{C}_{LO} = \mathcal{C}^{\text{tree}}[P_{LO}^{\text{res}}]$

**example:**  $B_{ggg}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = Z_2(\mathbf{k}_1, \mathbf{k}_2)P_{LO}^{\text{res}}(\mathbf{k}_1)P_{LO}^{\text{res}}(\mathbf{k}_2) + \text{perm.}$

**1-loop:**

$$\mathcal{C}_{NLO} = \mathcal{C}^{\text{tree}}[P_{Ls} + P_{Lw}e^{-k^2\Sigma^2}(1 + k^2\Sigma^2)] + \mathcal{C}^{1\text{-loop}}[P_{LO}^{\text{res}}]$$

**etc.**

# An example of new physics: features in primordial statistics

$$P_L(k) = P_{L,\Lambda\text{CDM}} + A_{\text{lin}} \cos(\omega k) + A_{\text{log}} \cos(\gamma \log k/k_*)$$

$$\omega \gg 1/k$$


$$\gamma \gg 1 \quad \Rightarrow \quad k_{\text{osc}} \sim k/\gamma \ll k$$


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Works similarly for **oscillating primordial bispectrum**, with the damping factor depending on the shape

# Summary and Outlook

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- Per cent accuracy in LSS statistics at least up to  $k \sim 0.2 h/\text{Mpc}$  ( $z=0$ ).  
Bigger reach at high redshifts
- Internal estimate of the error budget
- Flexible to play with physics beyond LambdaCDM. Many scenarios to explore
- Complementary to N-body

## To do:

- Efficient codes to implement high order PT (2-loop PS, 1-loop BS, RSD, bias)
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