Understanding the large-scale structure of the Universe

Sergey Sibiryakov

w/ D.Blas, M.Garny, M.Ivanov, A.Kaurov, A.Vasudevan, J.Lesgourgues

Rencontres du Vietnam, Quy Nhon, August 2019

What we want to understand ?

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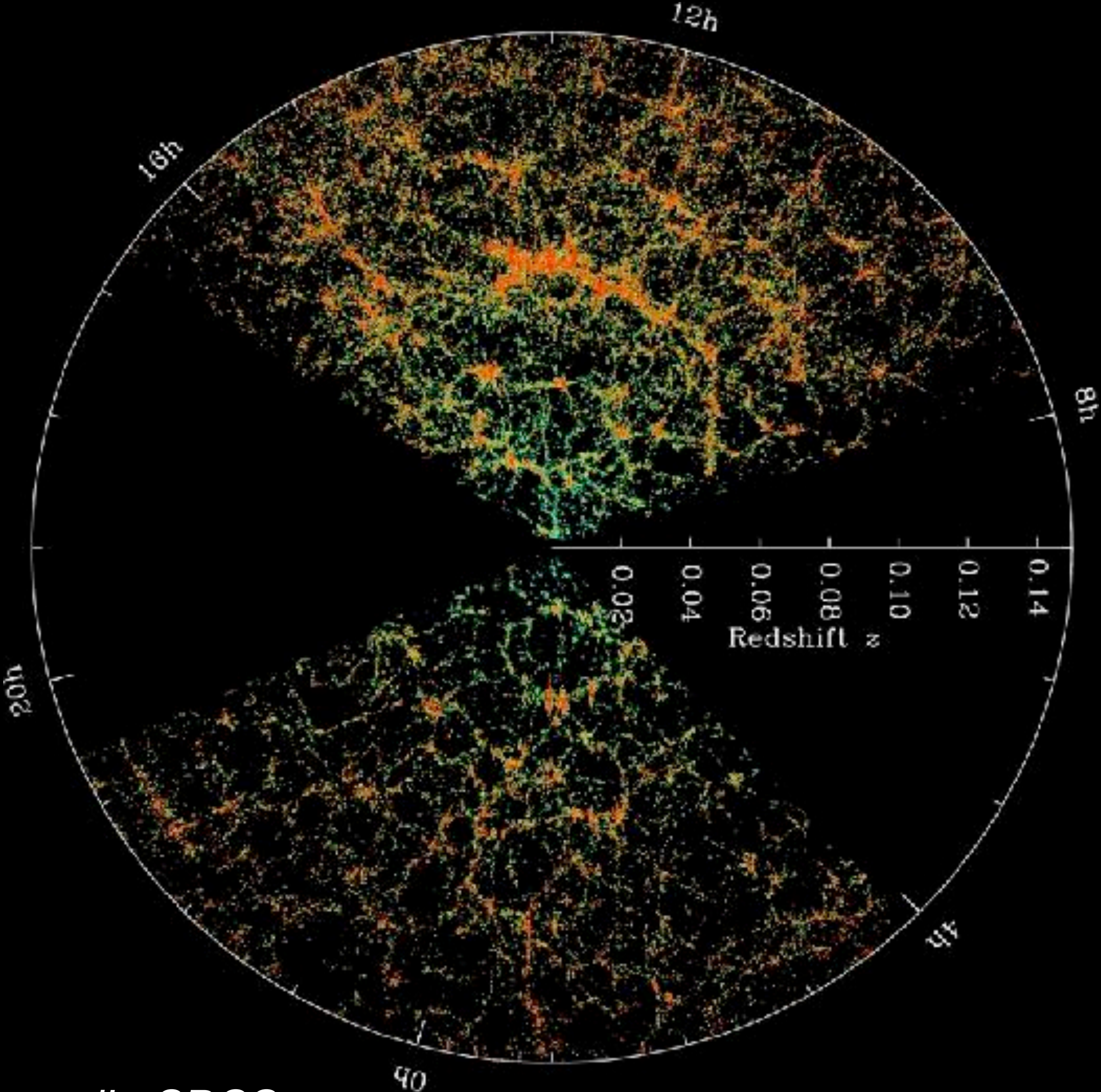

credit: SDSS

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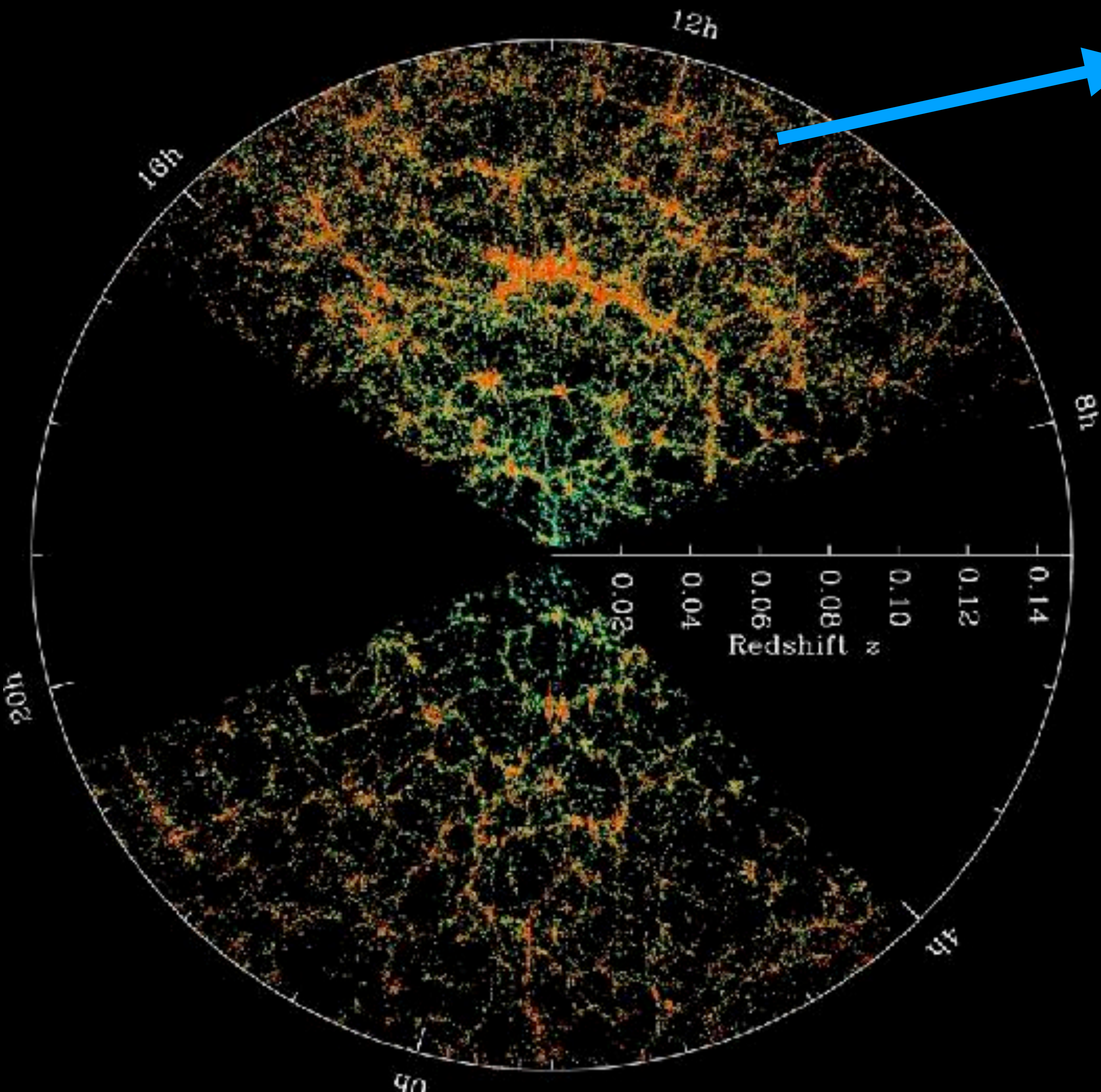

$$\frac{\delta\rho(x)}{\rho} \equiv \delta(x)$$

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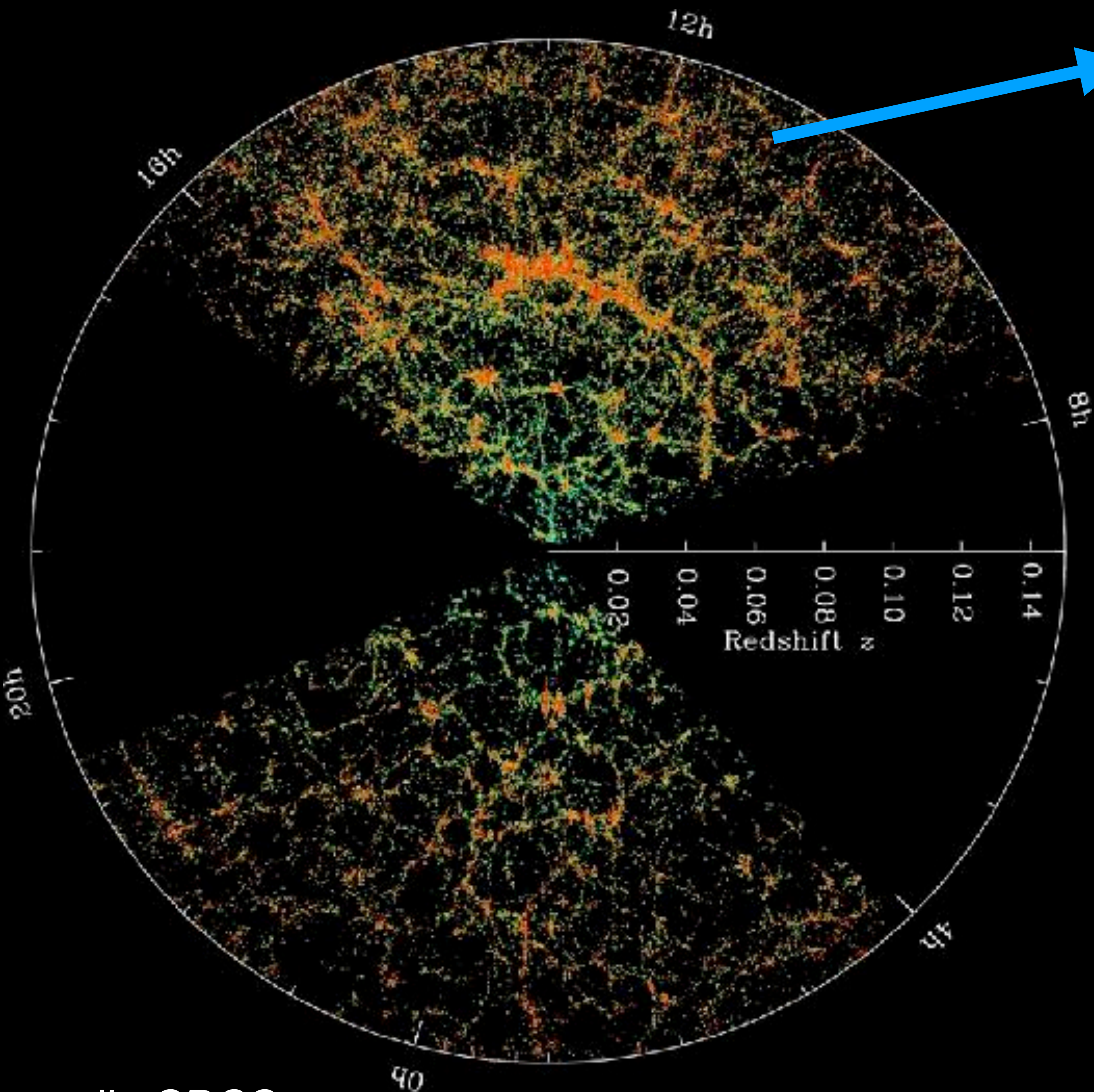

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statistical properties

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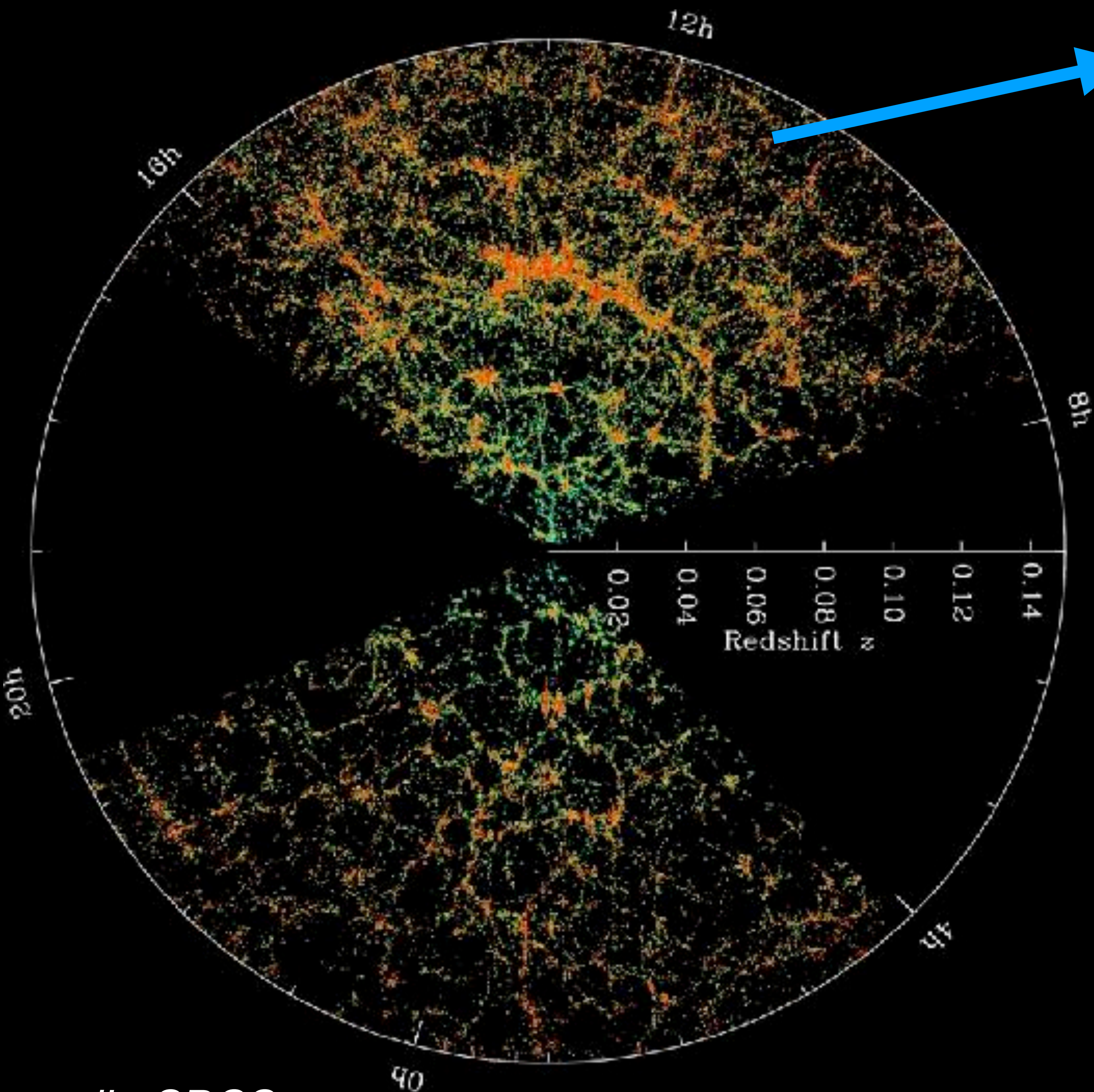

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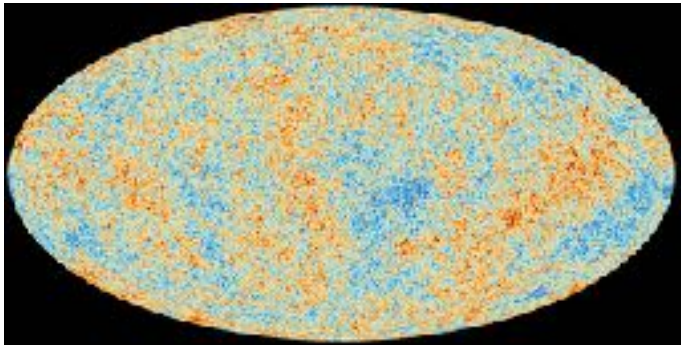
$$\frac{\delta\rho(x)}{\rho} \equiv \delta(x)$$

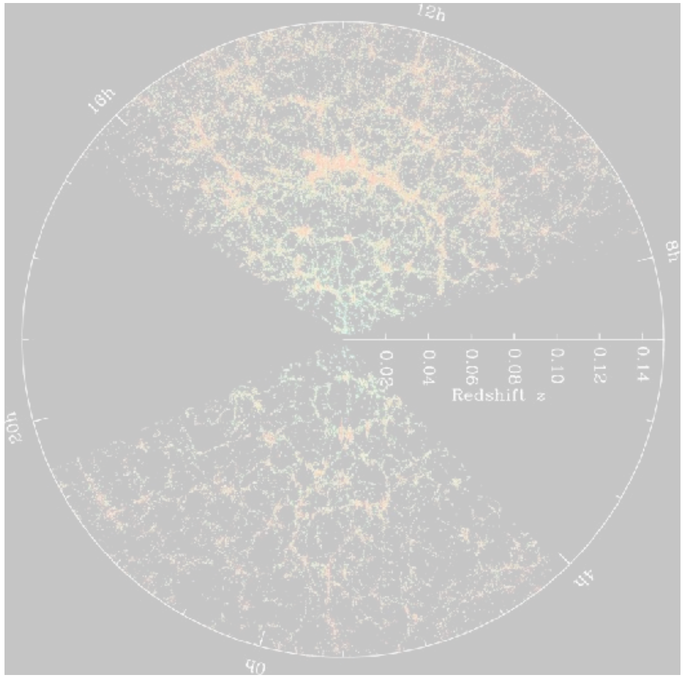
statistical properties

- cosmological parameters
- properties of DM, DE
- initial conditions e.g. primordial non-Gaussianity

CMB vs. LSS



- 2d map: information $\propto (l_{\max})^2$
- T saturated ➡ polarization
- linear theory



- 3d: information $\propto (k_{\max})^3$
- will dominate cosmo data in the coming years
- non-linear gravitational clustering at $k > k_{NL}(z)$

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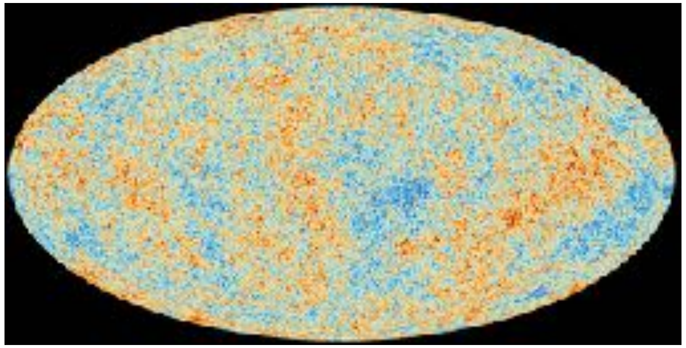

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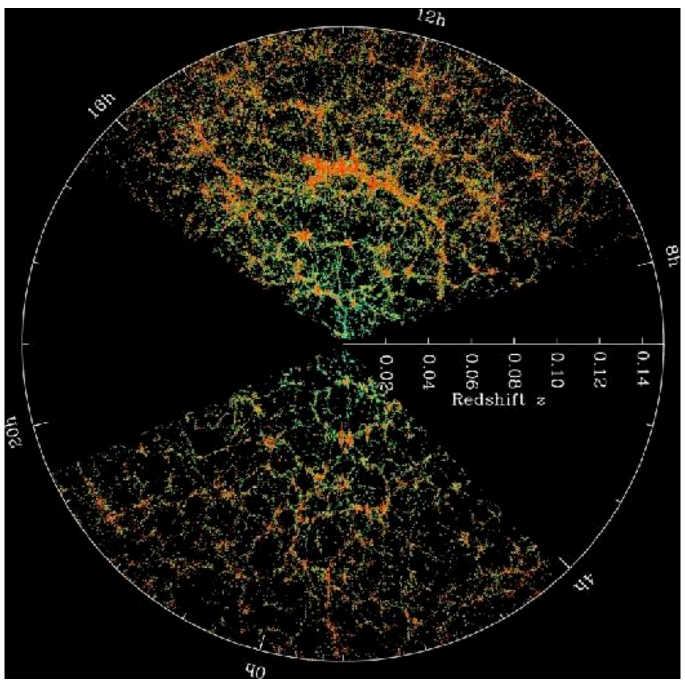

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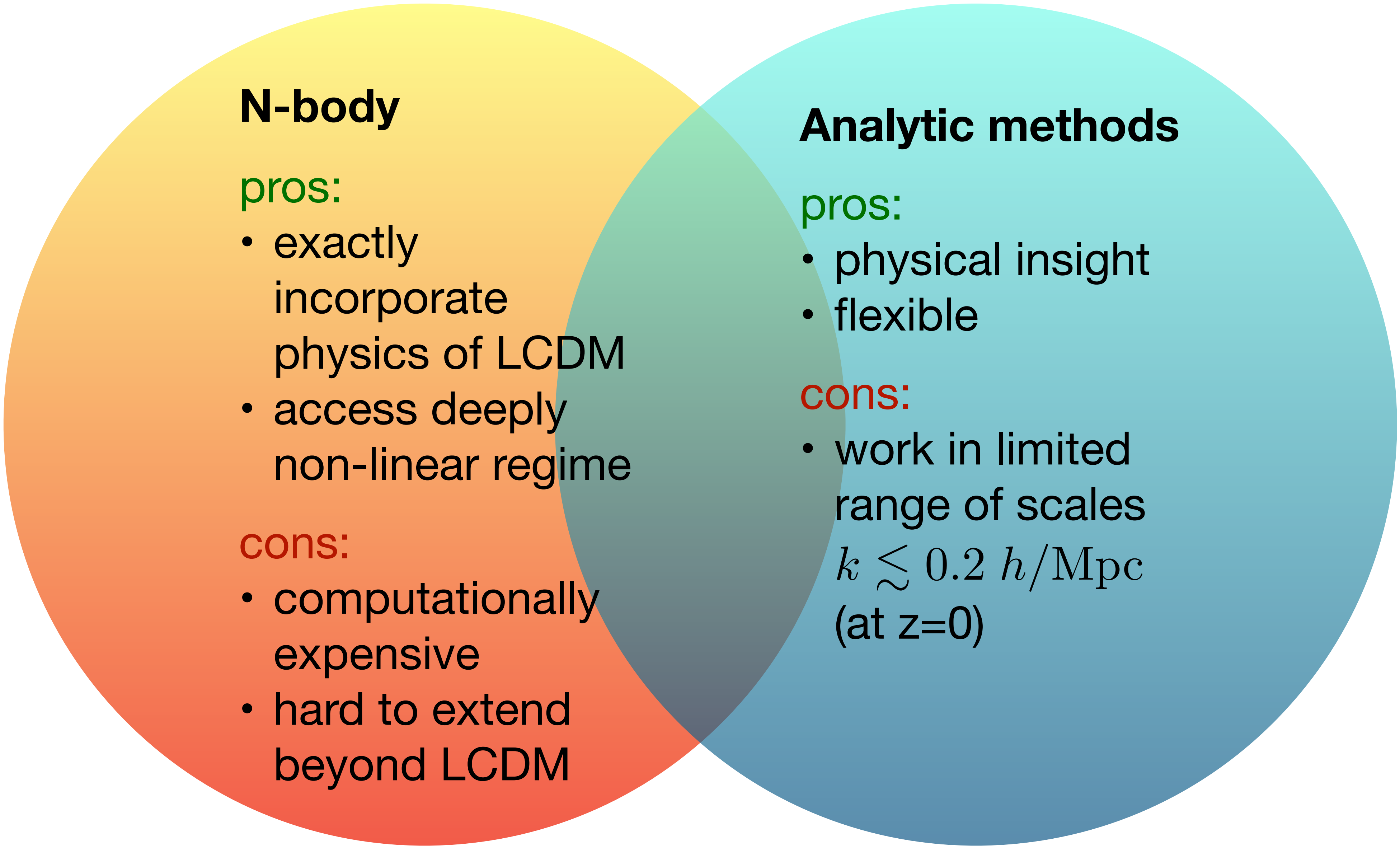

N-body
pros:
• exactly incorporate physics of LCDM
• access deeply non-linear regime
cons:
• computationally expensive
• hard to extend beyond LCDM
Analytic methods
pros:
• physical insight
• flexible
cons:
• work in limited range of scales $k \lesssim 0.2\ h/\mathrm{Mpc}$ (at z=0)

It is an old subject

Zeldovich, Peebles,… (1960+)

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XXI century challenge: controlled accuracy at ~1% level

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Progress due to insights from QFT

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Progress due to insights from QFT

$$Z[J,t] = \int [\mathcal{D}\delta_L(x)] \exp\left\{ -\int \frac{|\delta_L(k)|^2}{2P_L(k,t)} + \int J(x)\delta(x) \right\}$$

functional of δ_L

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IR resummation of
BAO / primordial features

non-perturbative
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Counts-in-cells

Perturbative schemes

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Standard Perturbation Theory *Bernardeau et al. (2001)*

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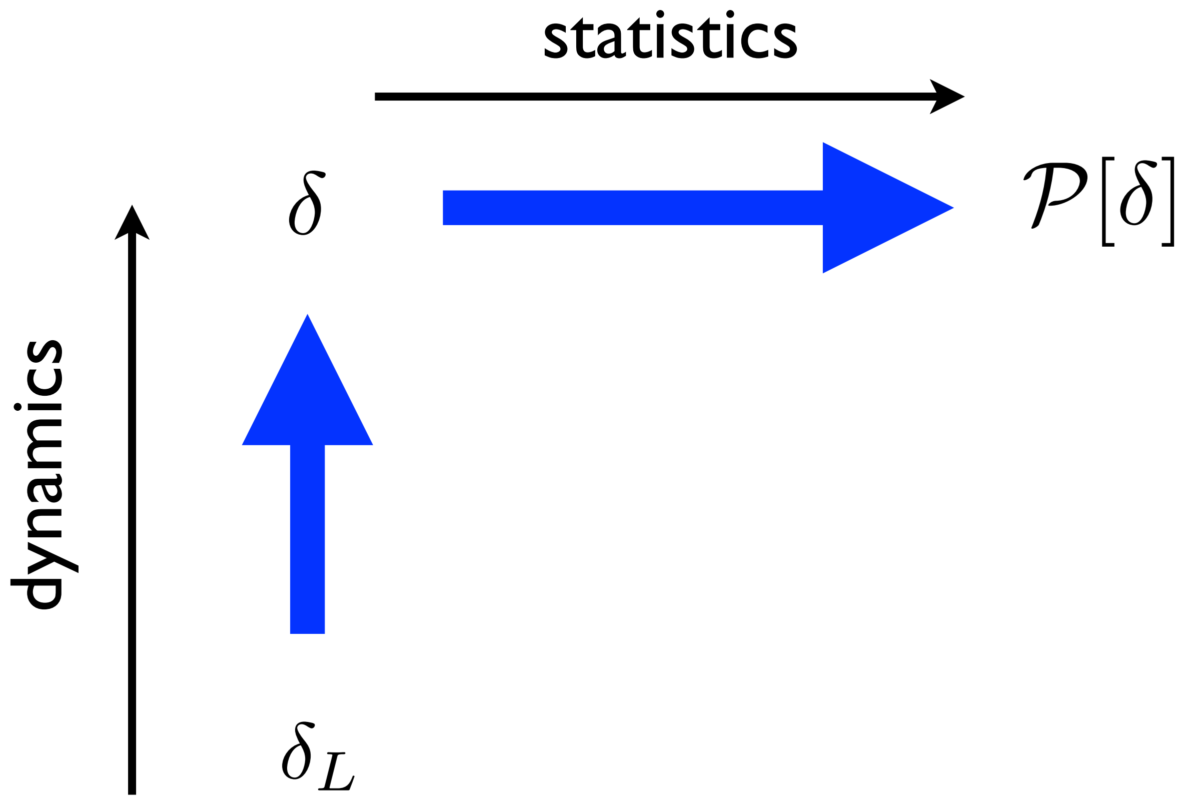

Perturbative schemes

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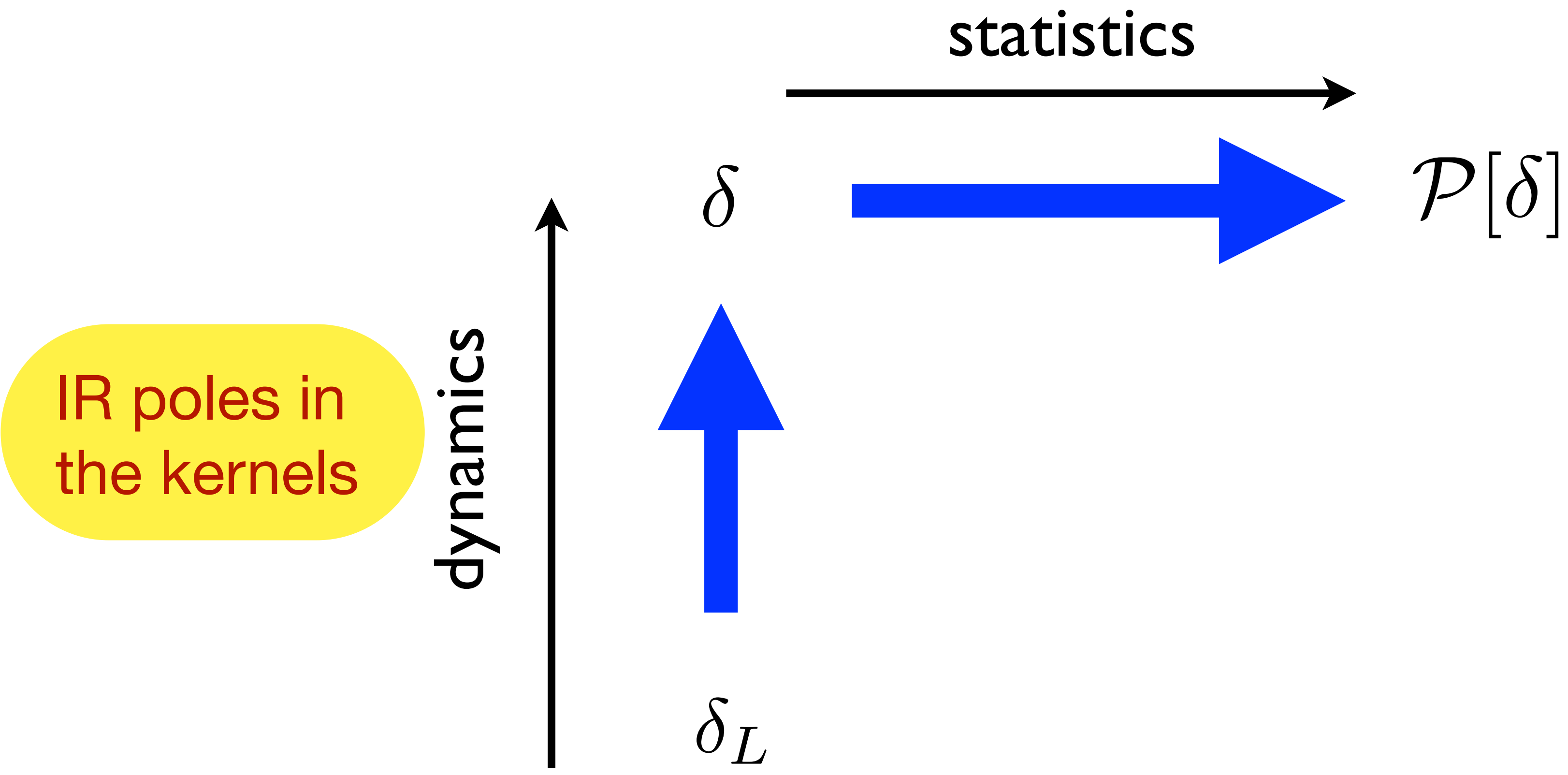

Perturbative schemes

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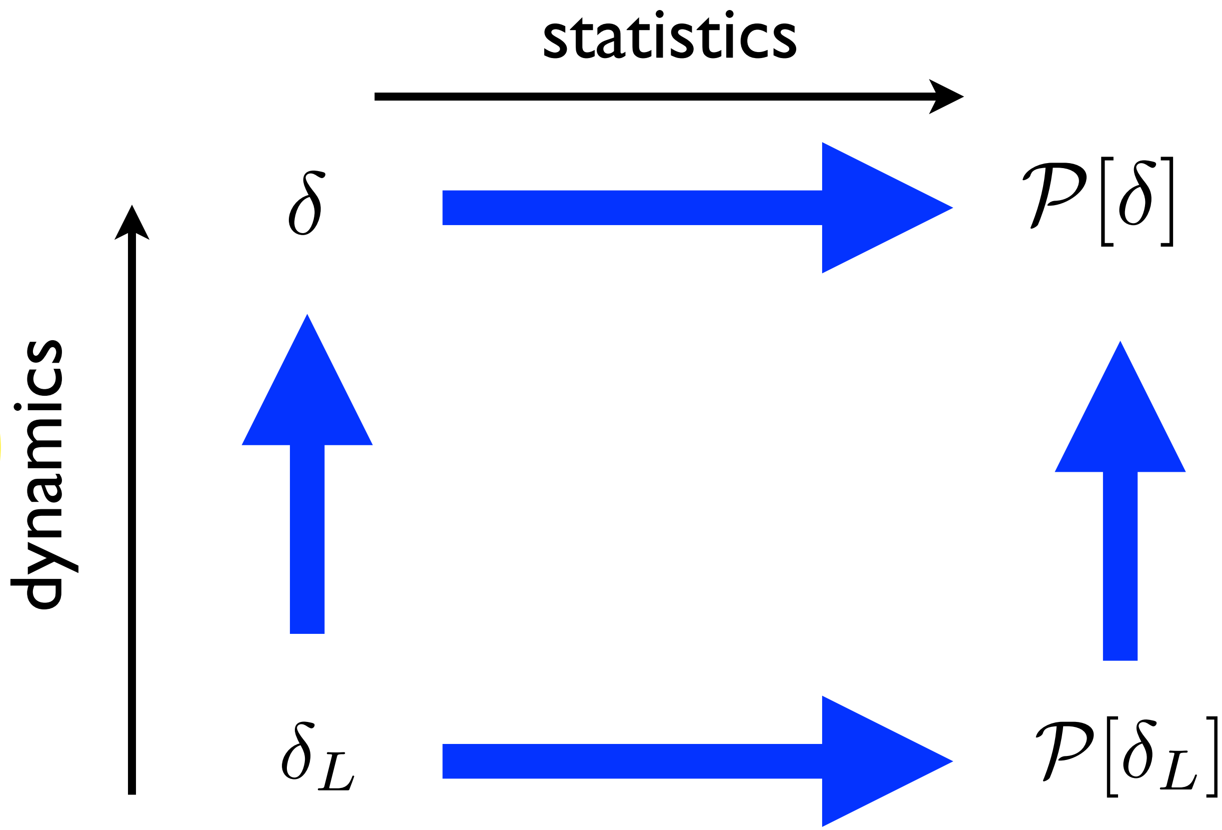
IR poles in the kernels



Time-Sliced Perturbation Theory

Valageas (2004)

Blas, Garny, Ivanov, S.S. (2015,2016)

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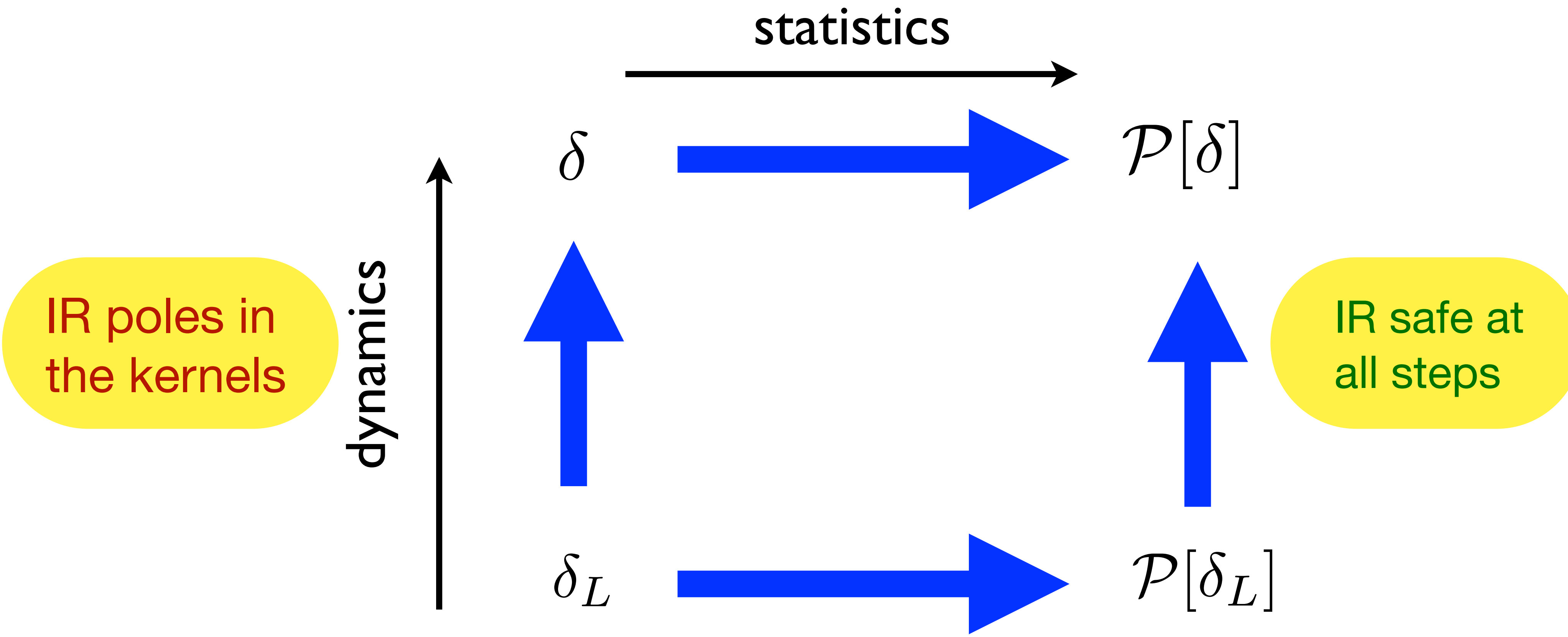

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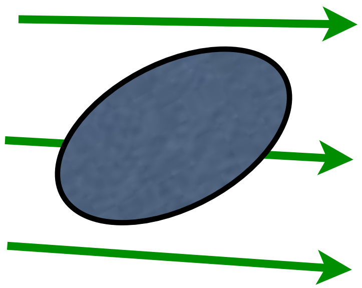
Spurious IR divergences from large bulk flows



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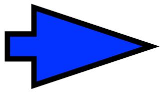

secular
growth

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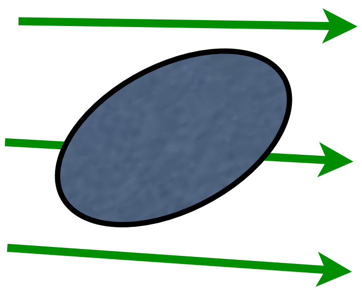
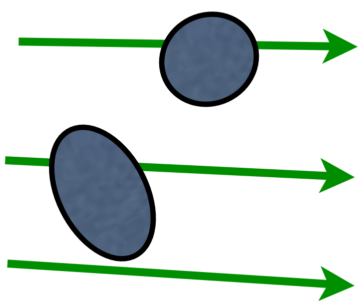


overdensity is moved by an almost homogeneous flow ⇒ secular growth



two overdensities will move (almost) identically ⇒ cancellation in equal-time statistics

More on the generating functional

$$Z[J] = \int [d\delta] \exp\left\{ -\frac{1}{g^2} \sum_n \frac{1}{n!} \bar{\Gamma}_n * \delta^n + J * \delta \right\}$$

linear growth factor $g(z)$ ➡ formal expansion parameter = effective coupling constant; true small parameter:

$$\sigma_d^2(k_*) = g^2 \int_{k<k_*} d^3k \bar{P}_L(k)$$

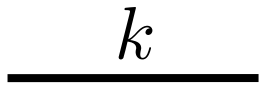
$$\bar{\Gamma}_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{(2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2)}{\bar{P}_L(k)}$$

➡ $\bar{\Gamma}_n$ obtained by recursion relations

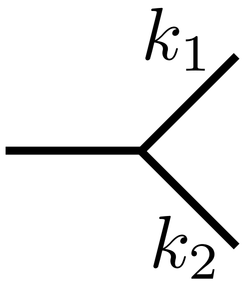
IR safety:

$$\lim_{\epsilon \longrightarrow 0} \bar{\Gamma}_{n+m}(\epsilon q_1, \ldots, \epsilon q_m, k_1, \ldots, k_n) < \infty$$

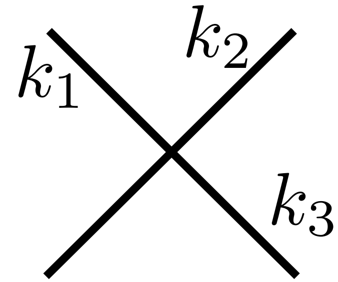
Diagrammar



$$= g^2 \bar{P}_L(k)$$

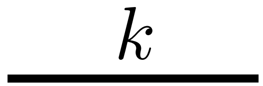


$$= \frac{1}{g^2}\bar{\Gamma}_3(k_1, k_2)$$



$$= \frac{1}{g^2}\bar{\Gamma}_4(k_1, k_2, k_3)$$

Diagrammar



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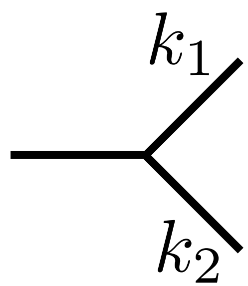

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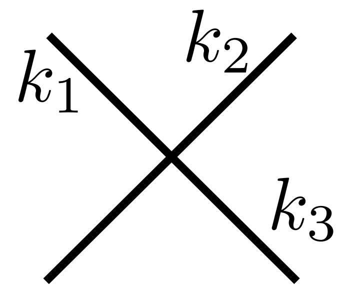

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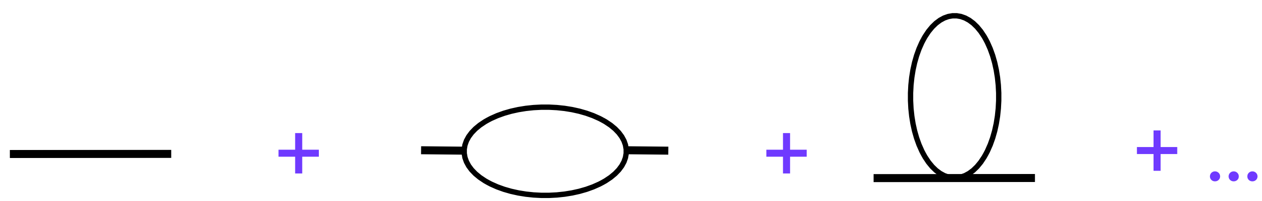

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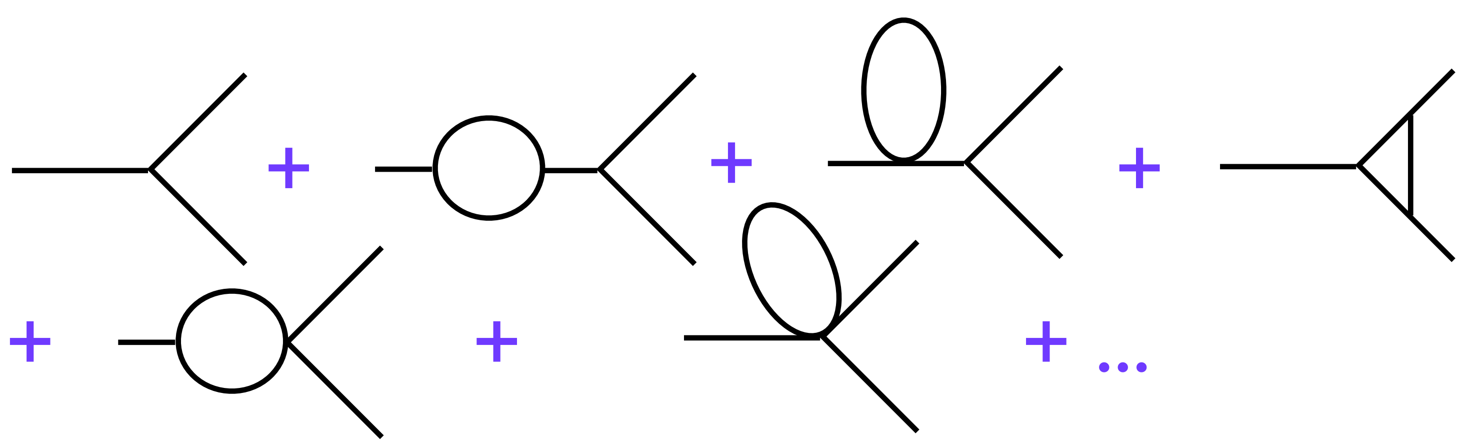
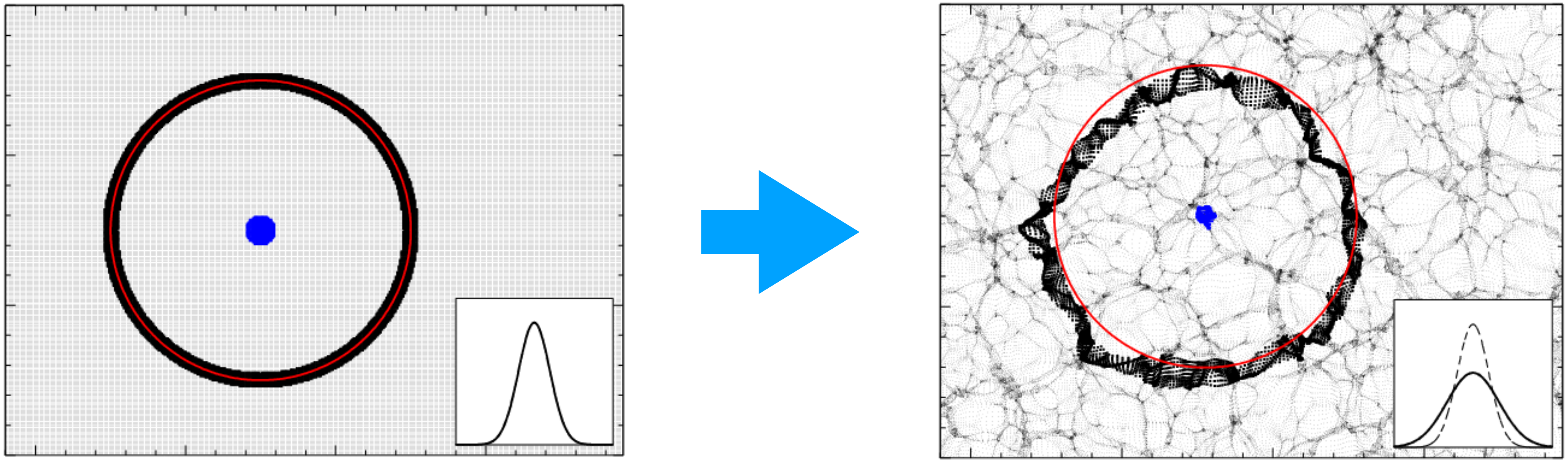


Application: Bulk flows vs. features



from Padmanabhan et al. (2012)

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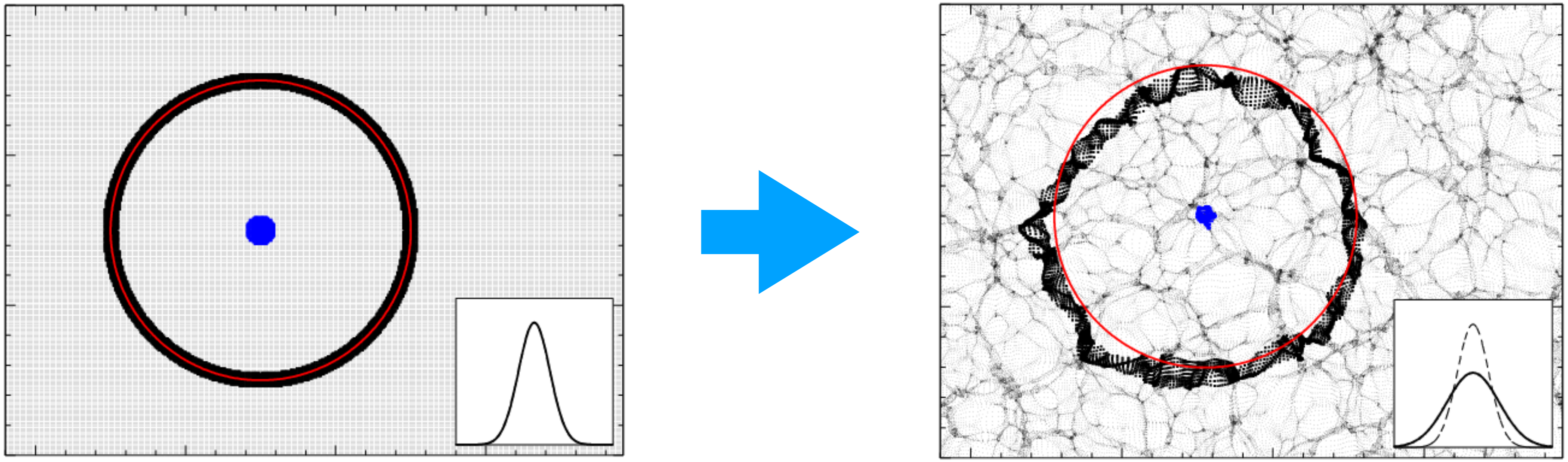

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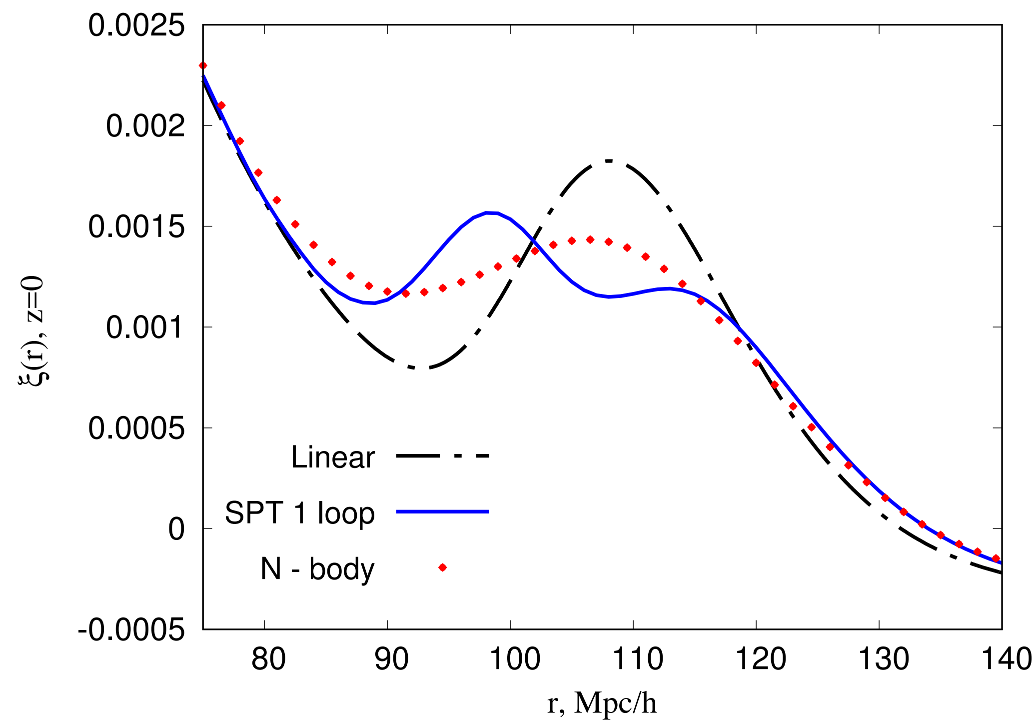
Simple 1-loop calculation gets it wrong:

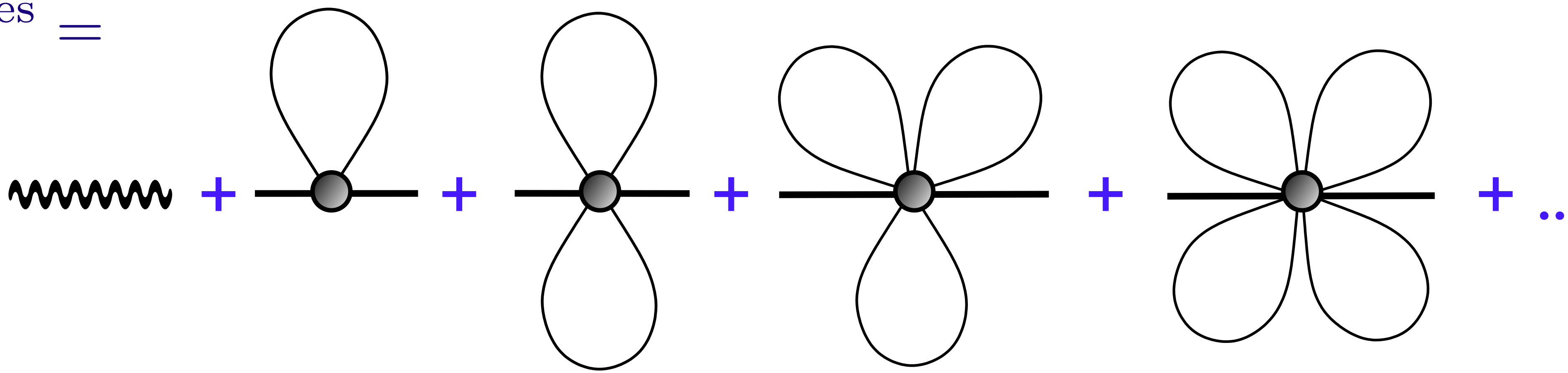


IR resummation

Blas, Garny, Ivanov, S.S. (2016)

Wiggly part of PS gets dressed with soft loops

$P_w^{\rm res} =$



$g^2 \frac{k^2}{q^2}$ $\quad \left(g^2 \frac{k^2}{q^2}\right)^2$ $\quad \left(g^2 \frac{k^2}{q^2}\right)^3$ $\quad \left(g^2 \frac{k^2}{q^2}\right)^4$

loop momentum

$= {\rm e}^{-k^2 \Sigma^2} P_{L\,w}$

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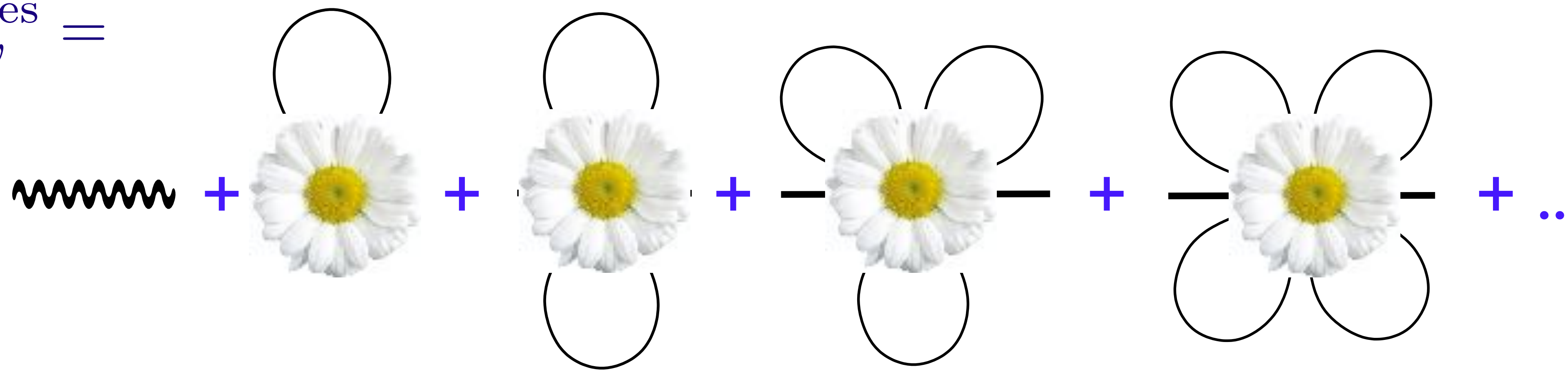

$$g^2 \frac{k^2}{q^2} \qquad \left(g^2 \frac{k^2}{q^2}\right)^2 \qquad \left(g^2 \frac{k^2}{q^2}\right)^3 \qquad \left(g^2 \frac{k^2}{q^2}\right)^4$$

loop momentum

$$= {\rm e}^{-k^2\Sigma^2} P_{L\,w}$$

$$\Sigma^2 = \frac{4\pi}{3} \int_0^{k_S} dq\, P_{Ls}(q) \big(1 - j_0(q/k_{osc}) + 2 j_2(q/k_{osc})\big)$$

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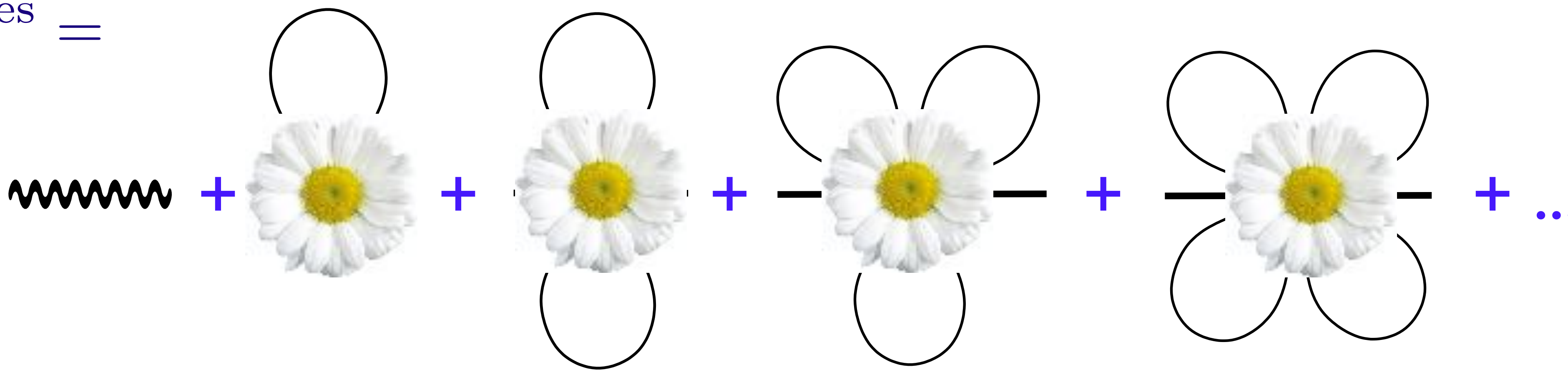

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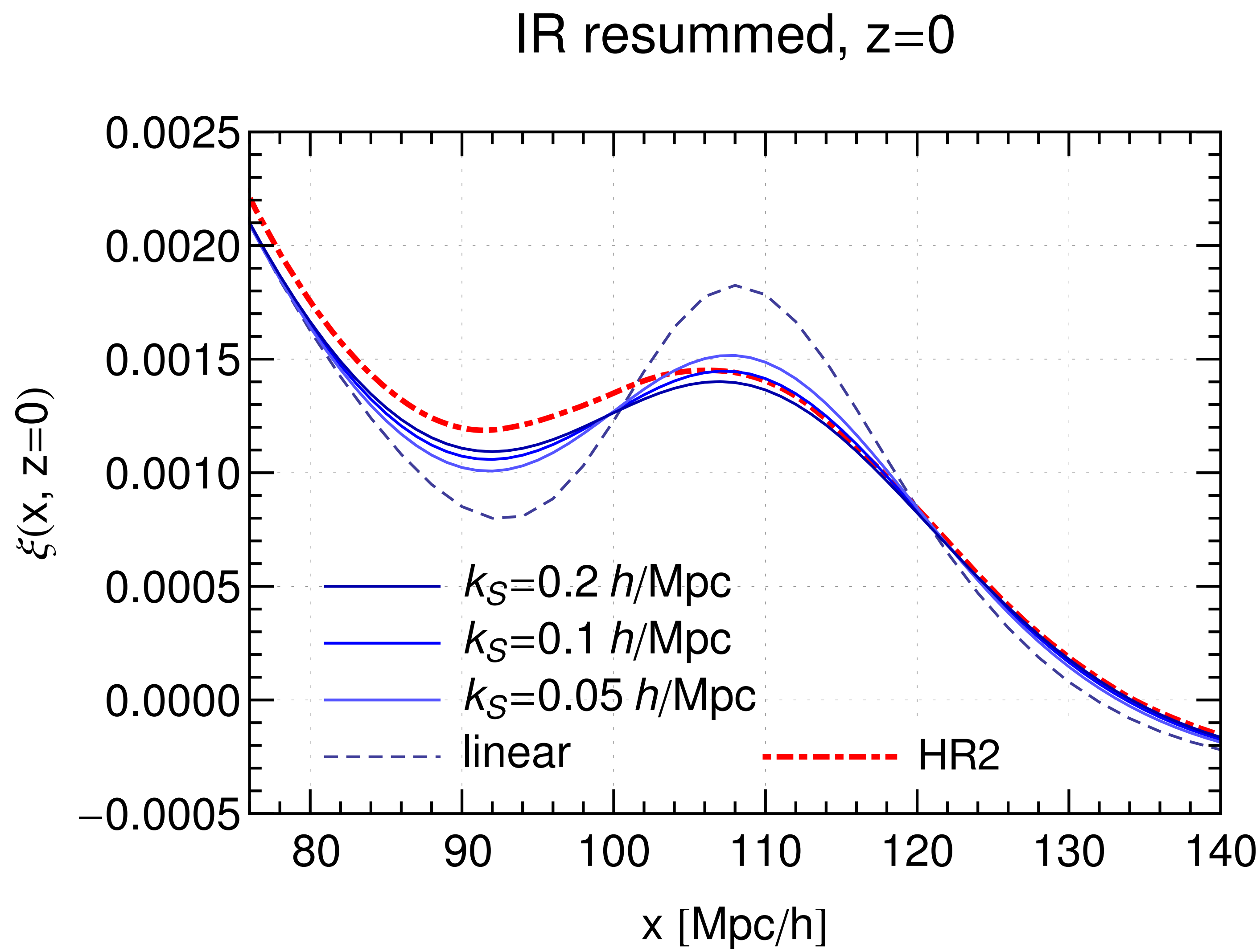
$$= \mathrm{e}^{-k^2\Sigma^2} P_{L\,w}$$

separation between hard and soft momenta

loop momentum

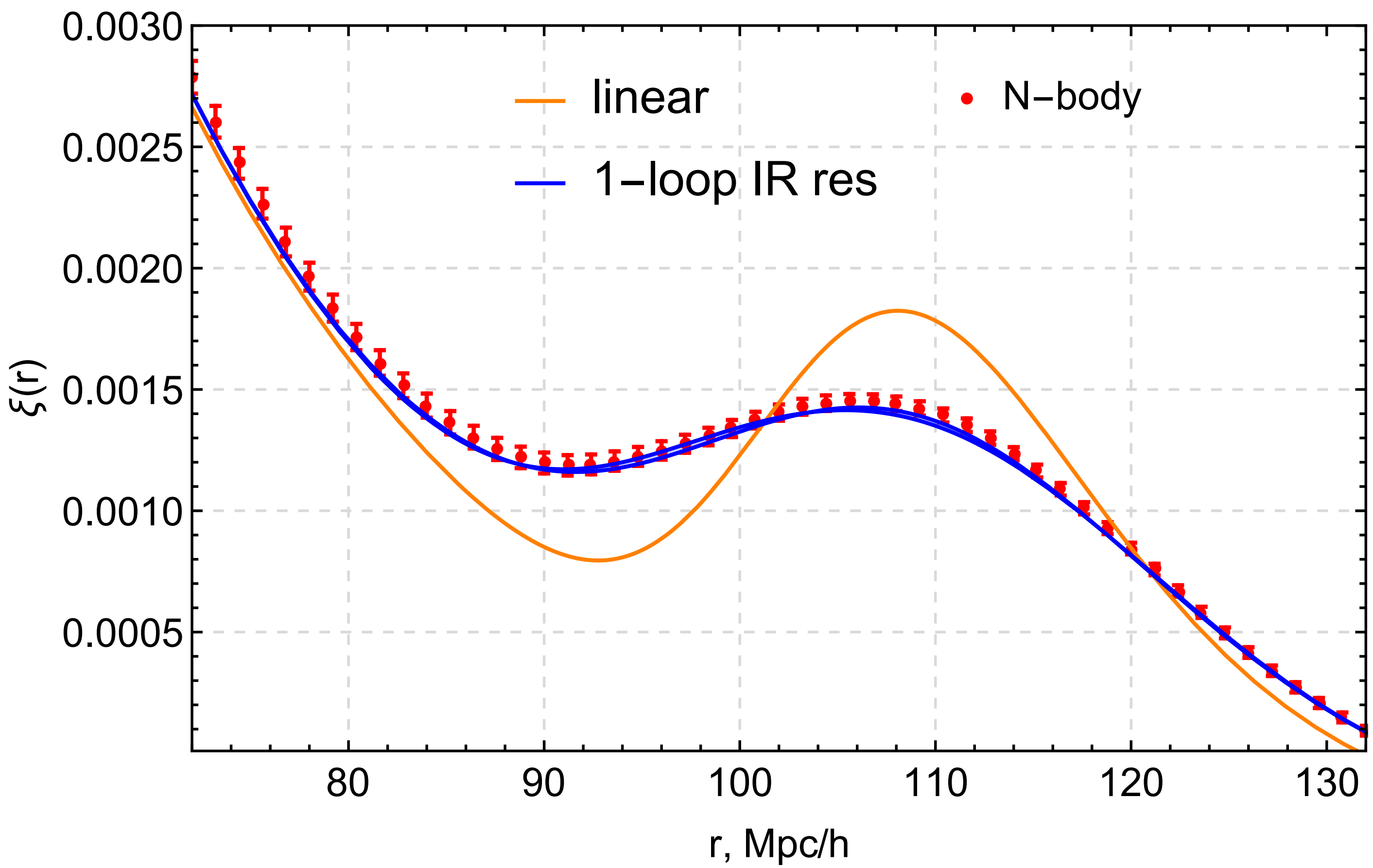
$$\Sigma^2 = \frac{4\pi}{3}\int_0^{k_S} dq\, P_{Ls}(q)\big(1 - j_0(q/k_{osc}) + 2j_2(q/k_{osc})\big)$$

Sensitivity to the IR separation scale: LO



- dependence on k_S gives an estimate of the error due to neglecting higher loops

(In)sensitivity to the IR separation scale: NLO



dependence on k_S decreases with the loop order

NB. EFT counterterm included to account for the failure of the fluid approximation

Baumann et al. (2010); Carrasco, Hertzberg, Senatore (2012); Pajer, Zaldarriaga (2013)

NB. Theoretical uncertainty included to account for higher orders

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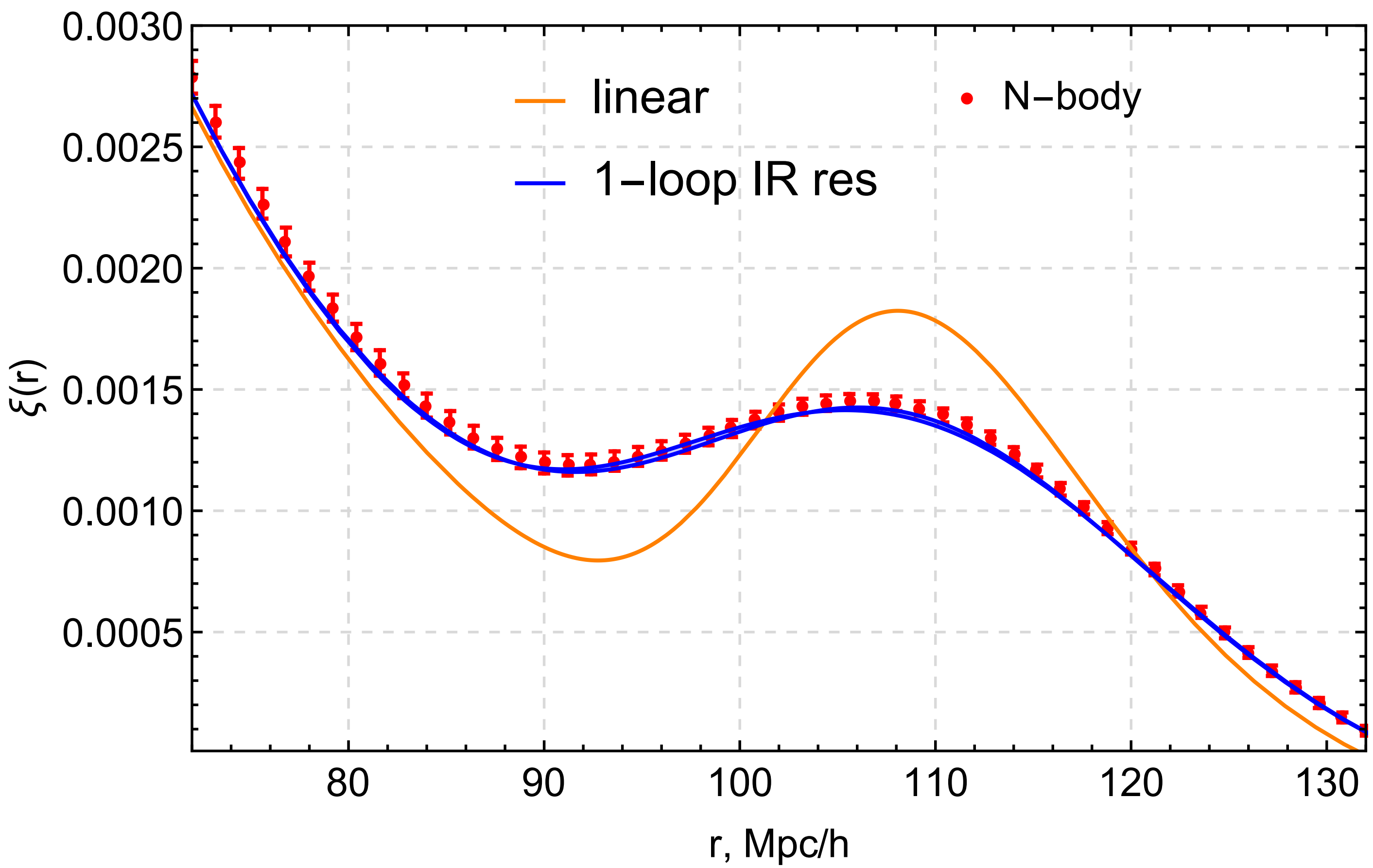

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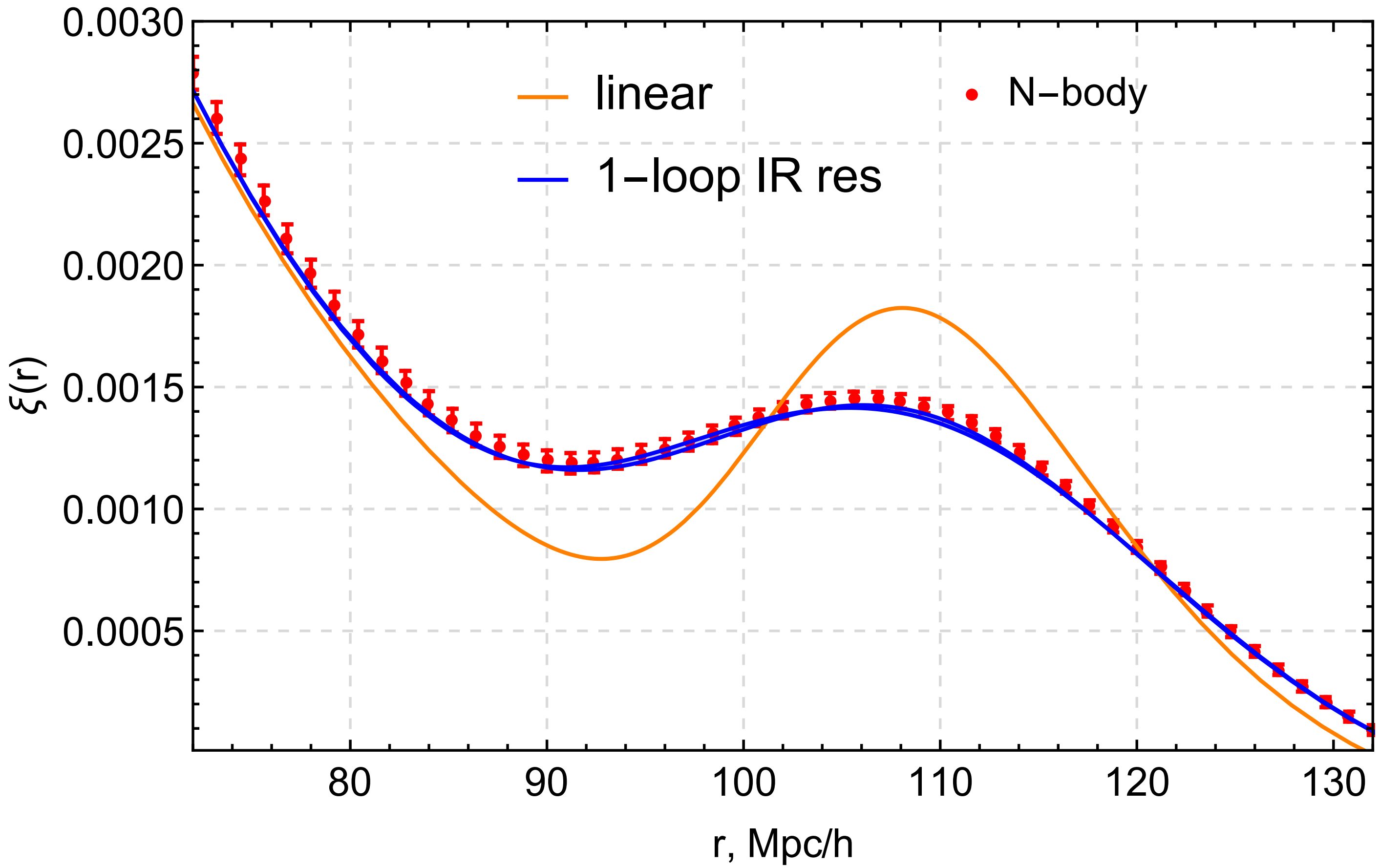

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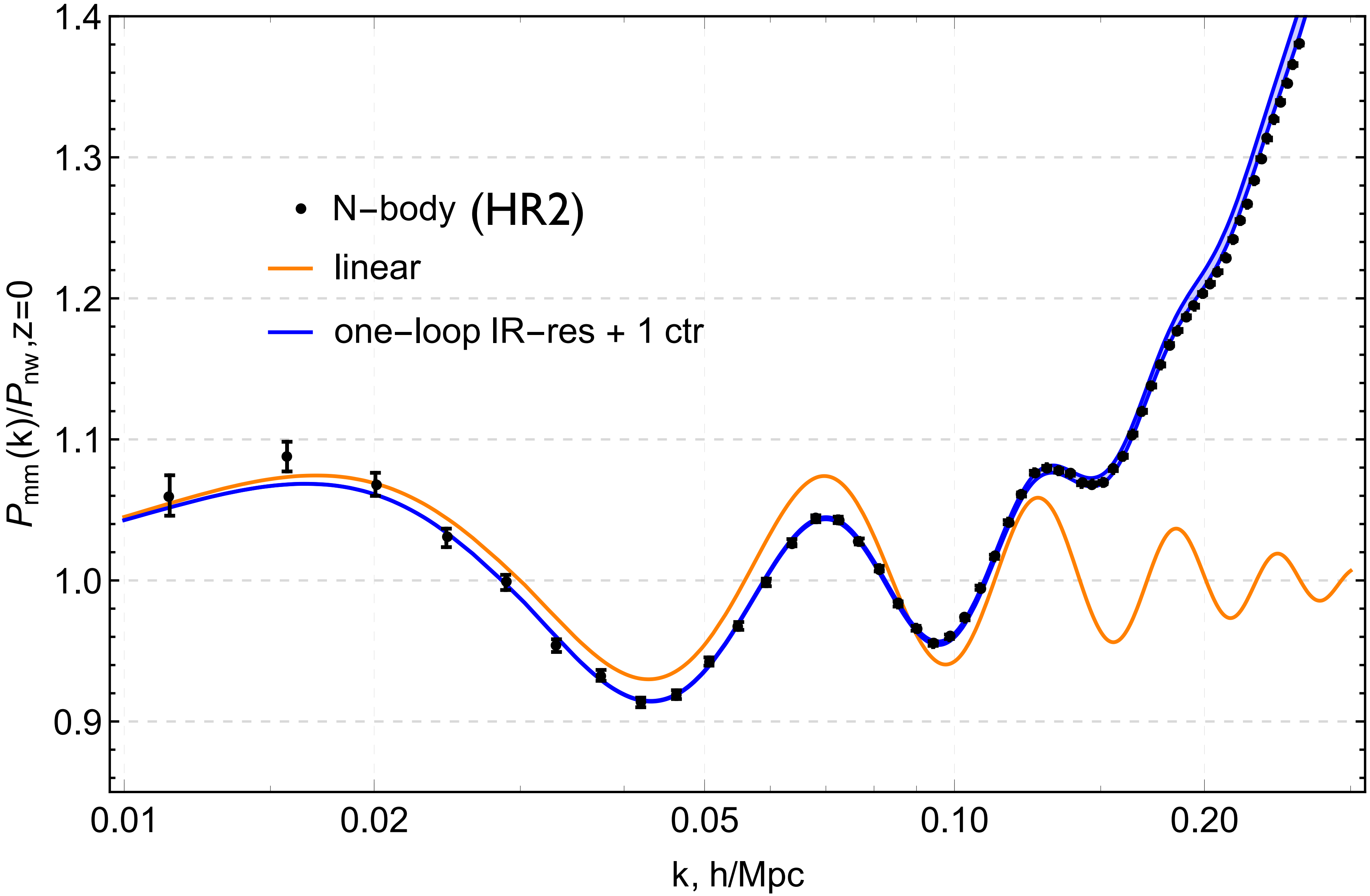
One-loop resummed PS

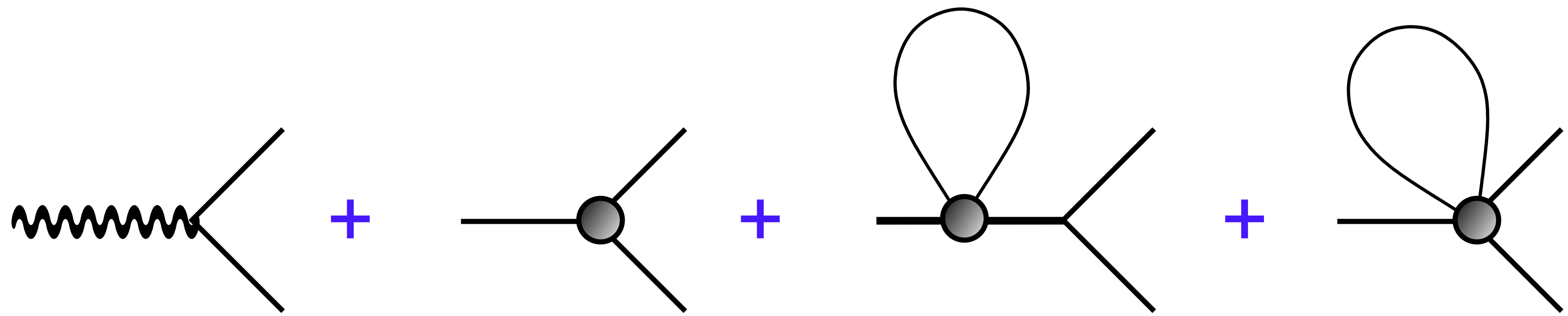


(Tentative) conclusion: precision $O(10^{-3})$ up to $k \sim 0.1$, sub-percent up to $k \sim 0.2$ (at z=0).

Resummation: general scheme

To get the leading IR diagrams dress the wiggly vertices with soft petals

$$B_{LO,w} =$$



$$= e^{-\mathcal{S}} B_w^{\text{tree}}$$

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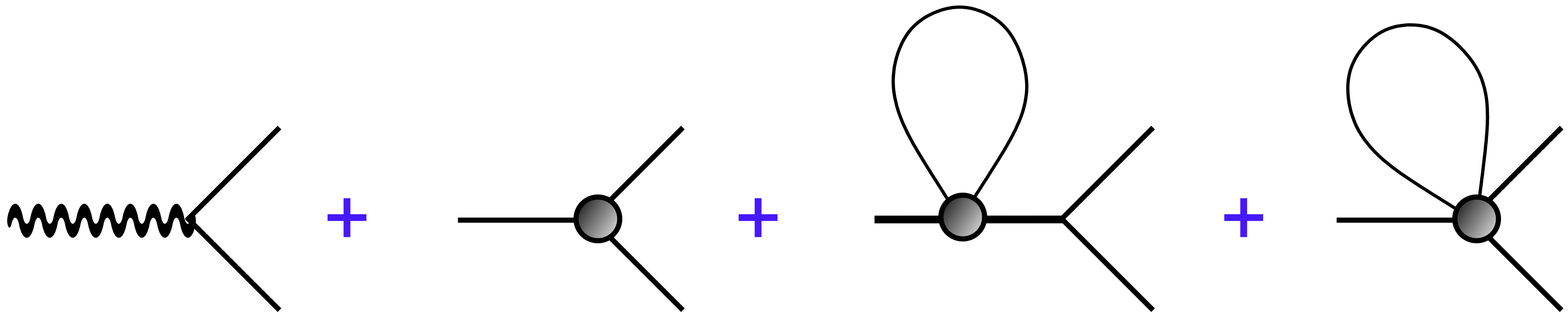

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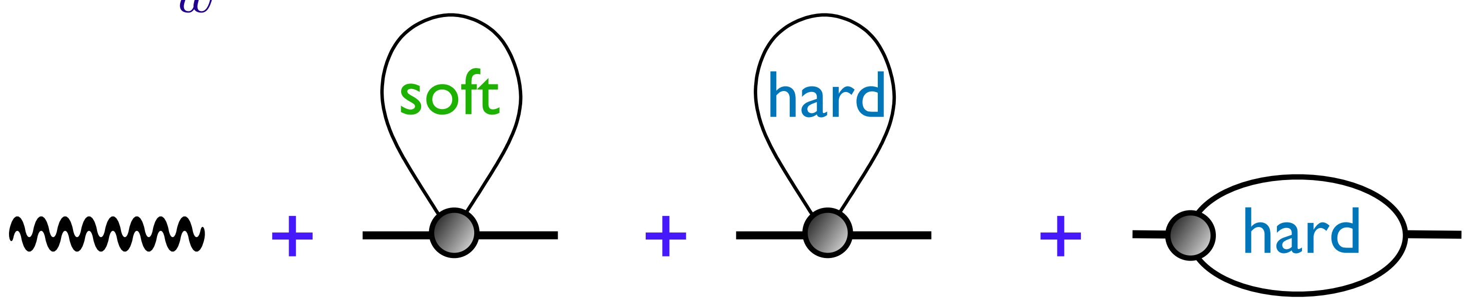

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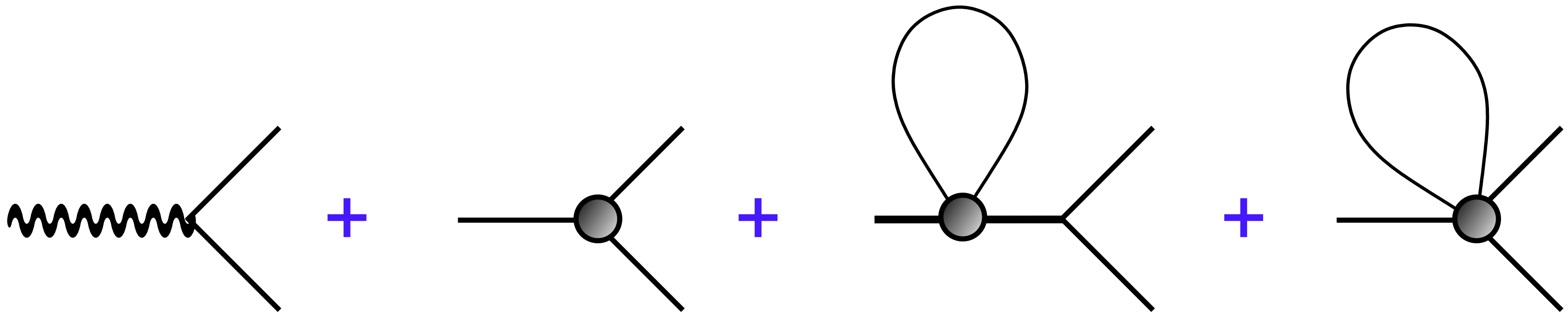

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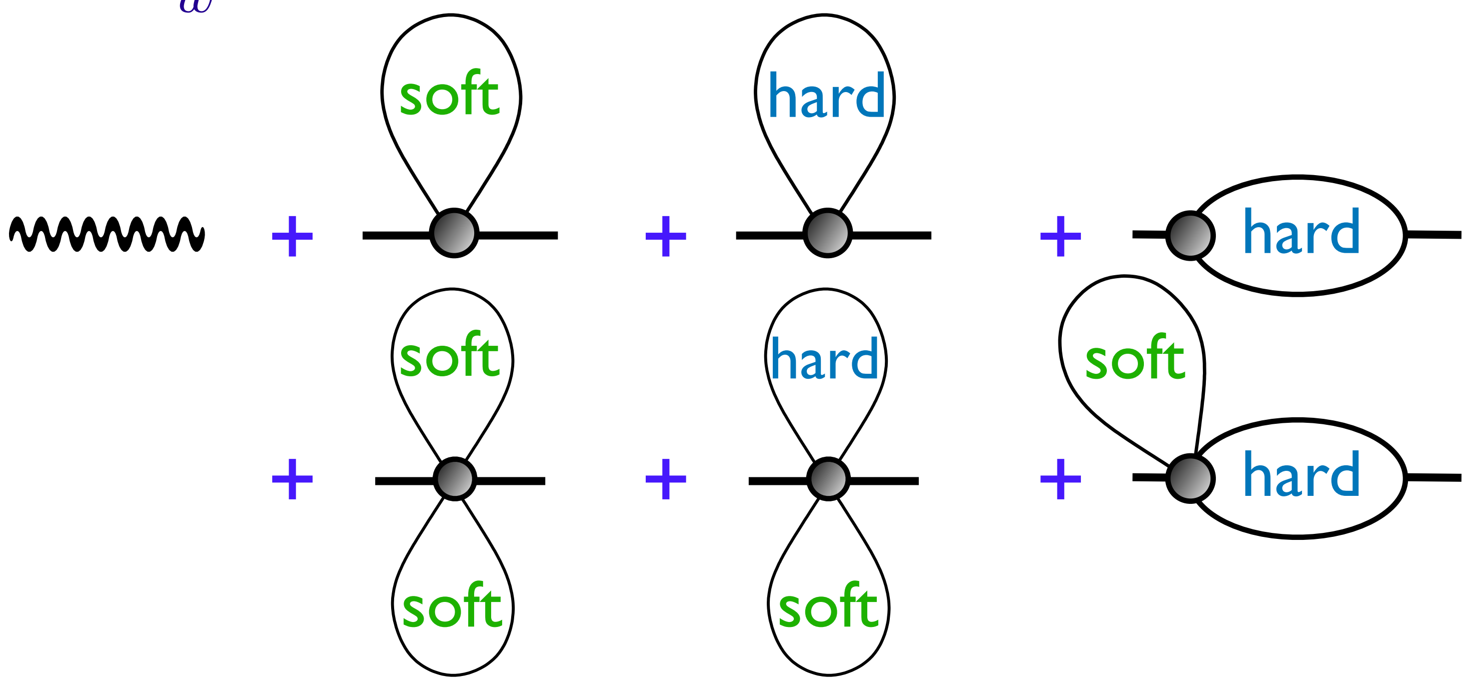

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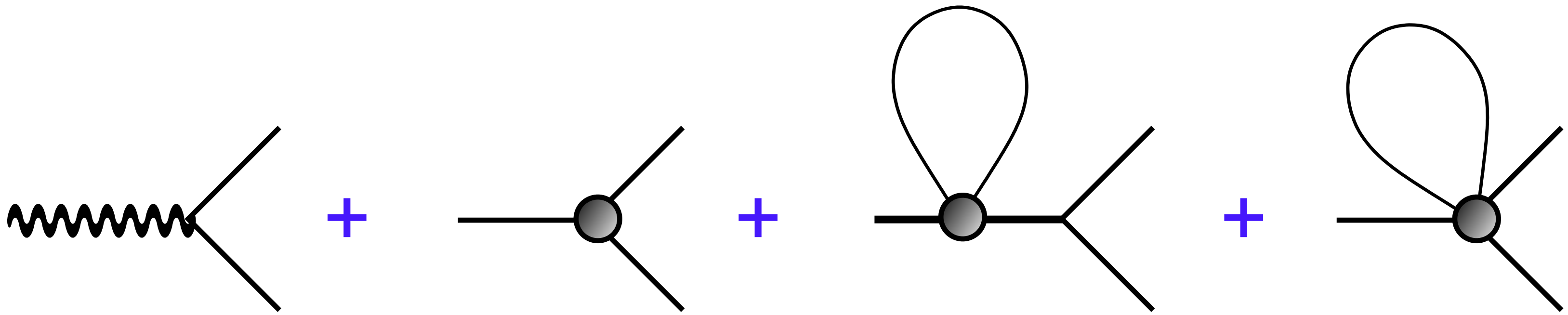

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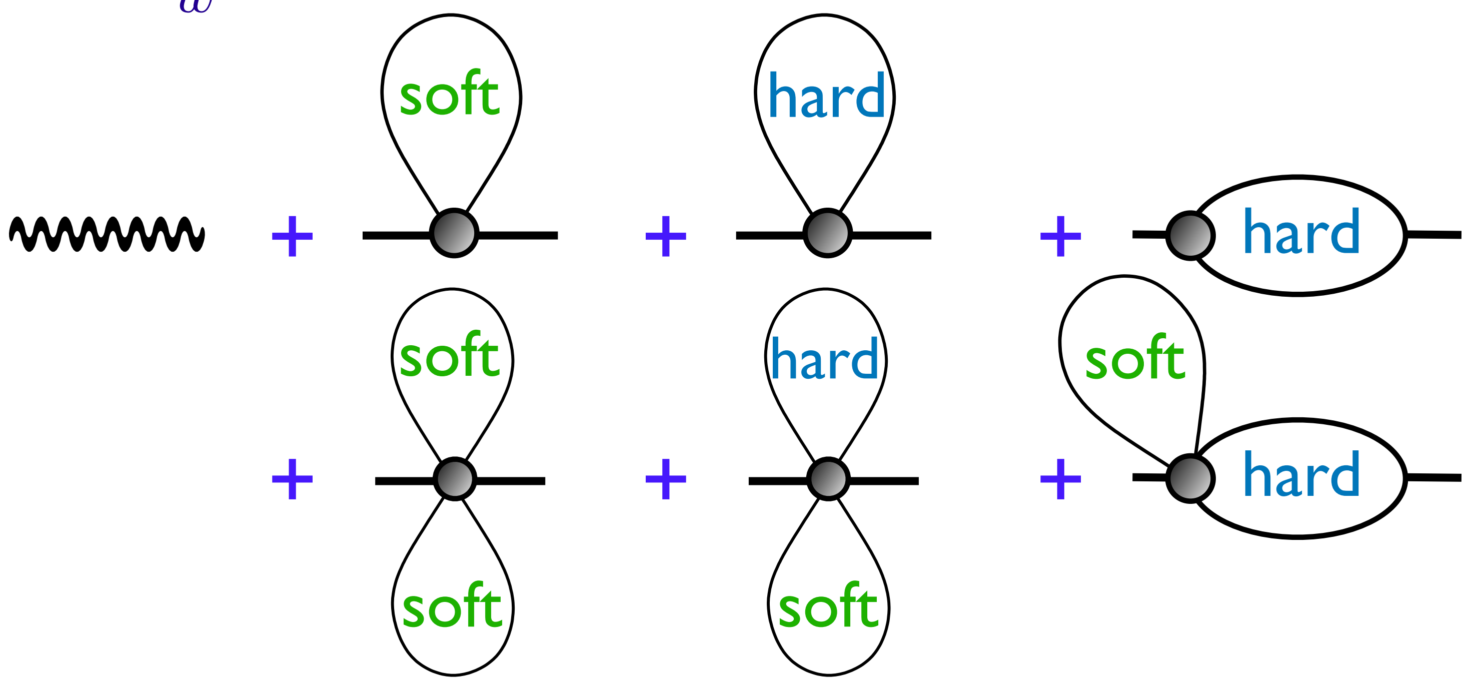

$$= e^{-\mathcal{S}} P_{Lw} + e^{-\mathcal{S}} P_w^{1-\text{loop}} + \mathcal{S} P_{Lw}$$

IR resummation summary (with RSD and bias)

Ivanov, S.S. (2018)

smooth

wiggly

$$P_L(k) = P_{Ls}(k) + P_{Lw}(k)$$

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Ivanov, S.S. (2018)

smooth → P_{Ls}, wiggly → P_{Lw}

$$P_L(k) = P_{Ls}(k) + P_{Lw}(k)$$

$$\Rightarrow \quad P^{\text{res}}_{LO}(\mathbf{k}) = P_{Ls}(k) + P_{Lw}(k)e^{-k^2\Sigma^2(\mu;k_S)}$$

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Ivanov, S.S. (2018)

smooth → P_{Ls}, wiggly → P_{Lw}

$$P_L(k) = P_{Ls}(k) + P_{Lw}(k)$$

$$\Rightarrow \quad P_{LO}^{\rm res}(\mathbf{k}) = P_{Ls}(k) + P_{Lw}(k)e^{-k^2\Sigma^2(\mu;k_S)}$$

Tree level: $\mathcal{C}_{LO} = \mathcal{C}^{\rm tree}[P_{LO}^{\rm res}]$

example: $B_{ggg}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = Z_2(\mathbf{k}_1, \mathbf{k}_2)P_{LO}^{\rm res}(\mathbf{k}_1)P_{LO}^{\rm res}(\mathbf{k}_2) + \text{ perm.}$

IR resummation summary (with RSD and bias)

Ivanov, S.S. (2018)

smooth → P_{Ls}, wiggly → P_{Lw}

$$P_L(k) = P_{Ls}(k) + P_{Lw}(k)$$

$$\Rightarrow \quad P_{LO}^{\rm res}(\mathbf{k}) = P_{Ls}(k) + P_{Lw}(k)e^{-k^2\Sigma^2(\mu;k_S)}$$

Tree level: $\mathcal{C}_{LO} = \mathcal{C}^{\rm tree}[P_{LO}^{\rm res}]$

example: $B_{ggg}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = Z_2(\mathbf{k}_1,\mathbf{k}_2)P_{LO}^{\rm res}(\mathbf{k}_1)P_{LO}^{\rm res}(\mathbf{k}_2) + \text{ perm.}$

1-loop:

$$\mathcal{C}_{NLO} = \mathcal{C}^{\rm tree}[P_{Ls} + P_{Lw}e^{-k^2\Sigma^2}(1+k^2\Sigma^2)] + \mathcal{C}^{1-{\rm loop}}[P_{LO}^{\rm res}]$$

etc.

An example of new physics: features in primordial statistics

$$P_L(k) = P_{L,\Lambda CDM} + A_{\rm lin}\cos(\omega k) + A_{\rm log}\cos(\gamma \log k/k_*)$$

$\omega \gg 1/k$

$\gamma \gg 1$ ➡ $k_{osc} \sim k/\gamma \ll k$

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Damping is given by resummation of daisy diagrams

Vasudevan, Ivanov, S.S., Lesgourgues (2019)

$$\Sigma^2(k) = \frac{4\pi}{3}\int_{q<k_S} dq\, P_{Ls}(q)\big(1 - j_0(\gamma q/k) + 2j_2(\gamma q/k)\big)$$

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Important: LSS is more sensitive to features than CMB

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Works similarly for **oscillating primordial bispectrum**, with the damping factor depending on the shape

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- Per cent accuracy in LSS statistics at least up to k ~ 0.2 h/Mpc (z=0). Bigger reach at high redshifts
- Internal estimate of the error budget
- Flexible to play with physics beyond LambdaCDM. Many scenarios to explore
- Complementary to N-body

To do:

- Efficient codes to implement high order PT (2-loop PS, 1-loop BS, RSD, bias)
- More non-pert. spatistics: CiC in redshift space, joint PDF for multiple cells, PDF for convergence and shear, …
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