Tokyo Tech

Clustering of primordial black holes with non-Gaussian initial fluctuations

Teruaki Suyama

(Tokyo Institute of Technology)

Ref.

1906.04958, TS and S.Yokoyama

Quy Nhon, Aug.11-17 2019

Review article on PBH

M.Sasaki, TS, T.Tanaka, S.Yokoyama, CQG 35, 063001(2018) [arXiv:1801.05235]

IOP Publishing   Classical and Quantum Gravity

Class. Quantum Grav. **35** (2018) 063001 (76pp)   https://doi.org/10.1088/1361-6382/aaa7b4

Topical Review

Primordial black holes—perspectives in gravitational wave astronomy

Misao Sasaki[1], Teruaki Suyama[2], Takahiro Tanaka[1,3] and Shuichiro Yokoyama[4,5]

[1] Center for Gravitational Physics, Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
[2] Research Center for the Early Universe (RESCEU), Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan
[3] Department of Physics, Kyoto University, Kyoto 606-8502, Japan
[4] Department of Physics, Rikkyo University, Tokyo 171-8501, Japan
[5] Kavli IPMU (WPI), UTIAS, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan

E-mail: misao@yukawa.kyoto-u.ac.jp

Received 5 November 2016, revised 11 January 2018
Accepted for publication 15 January 2018
Published 8 February 2018

CrossMark

Abstract

This article reviews current understanding of primordial black holes (PBHs), with particular focus on those massive examples ($\gtrsim 10^{15}$ g) which remain at the present epoch, not having evaporated through Hawking radiation. With the detection of gravitational waves by LIGO, we have gained a completely novel observational tool to search for PBHs, complementary to those using electromagnetic waves. Taking the perspective that gravitational-wave astronomy will make significant progress in the coming decades, the purpose of this article is to give a comprehensive review covering a wide range of topics on PBHs. After discussing PBH formation, as well as several inflation models leading to PBH production, we summarize various existing and future observational constraints. We then present topics on formation of PBH binaries, gravitational waves from PBH binaries, and various observational tests of PBHs using gravitational waves.

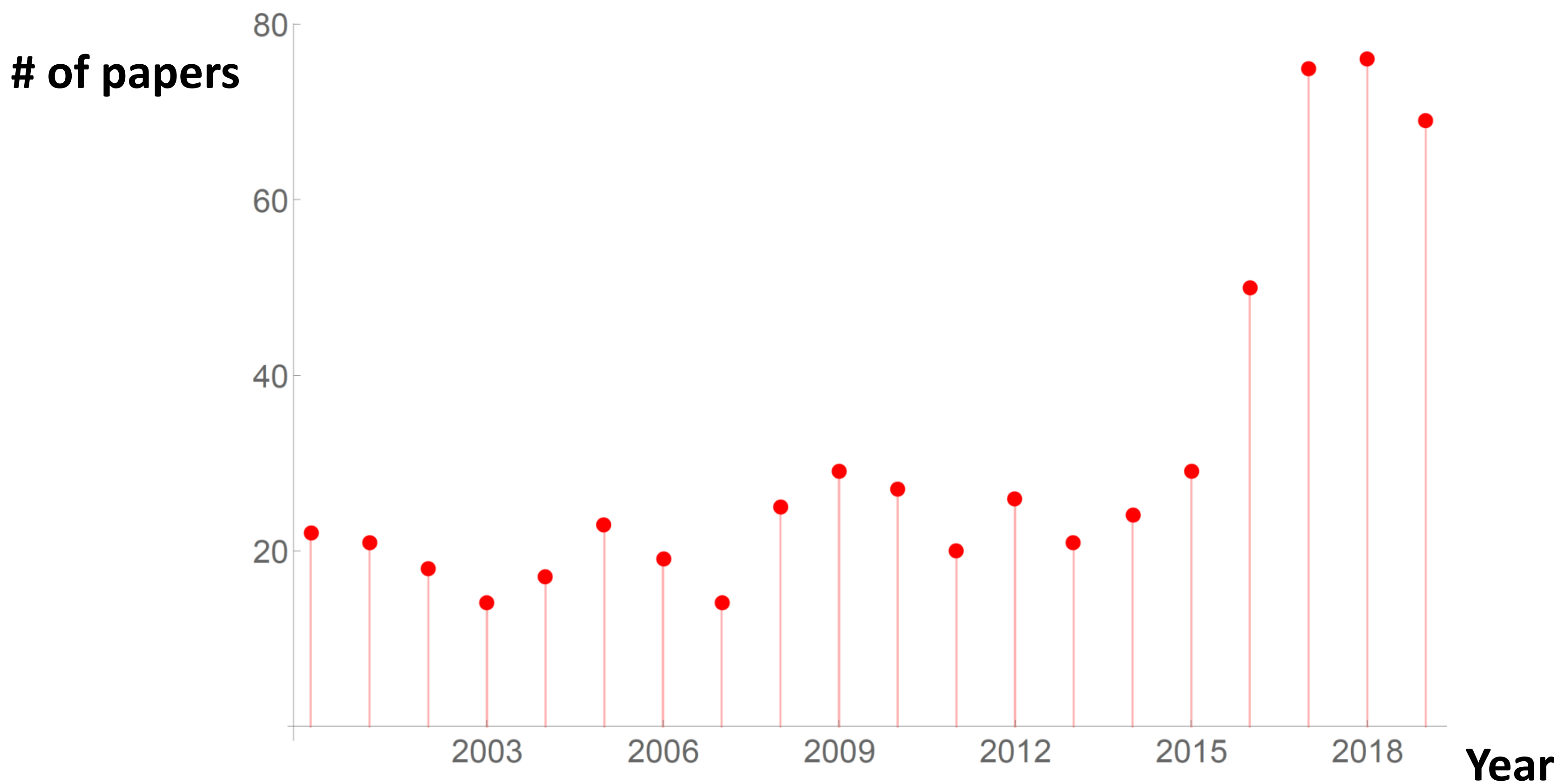
Keywords: black holes, gravitational waves, primordial perturbations

(Some figures may appear in colour only in the online journal)

Background of this work

Detections of BH-BH mergers by LIGO

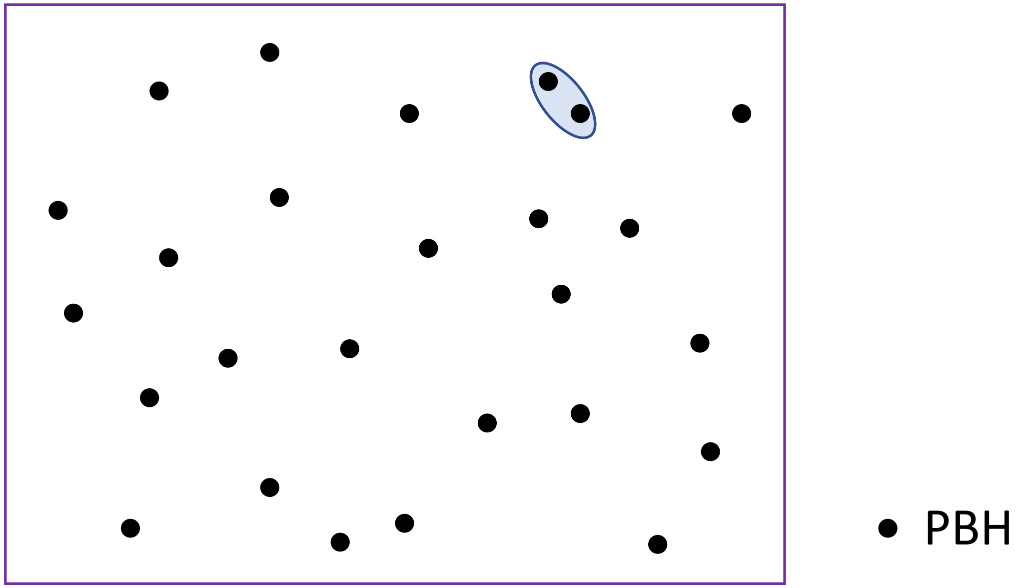
Renewed interest in PBHs



Formation of the PBH binaries

Nakamura+ 1997

- Spatial distribution of PBHs in the early universe -



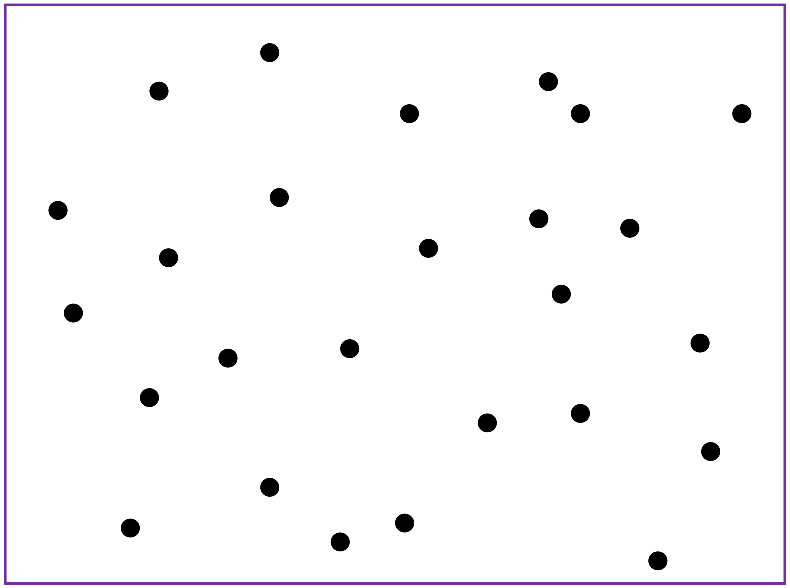
PBHs are initially at rest with respect to comoving coordinate.

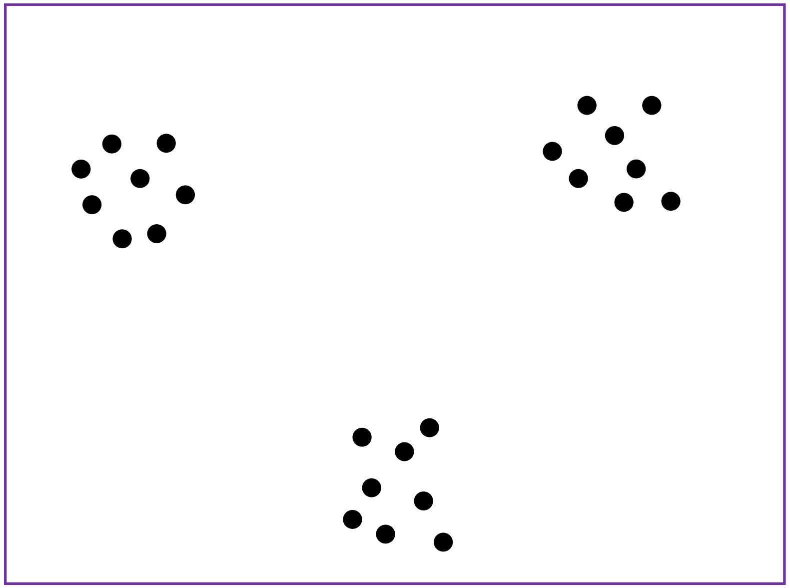
Some PBHs form binaries in the radiation dominated era.

PBH binaries merge at the present time.

LIGO observations → $f_{PBH} \sim 10^{-3}$

Clustering also determines the number of the binaries.





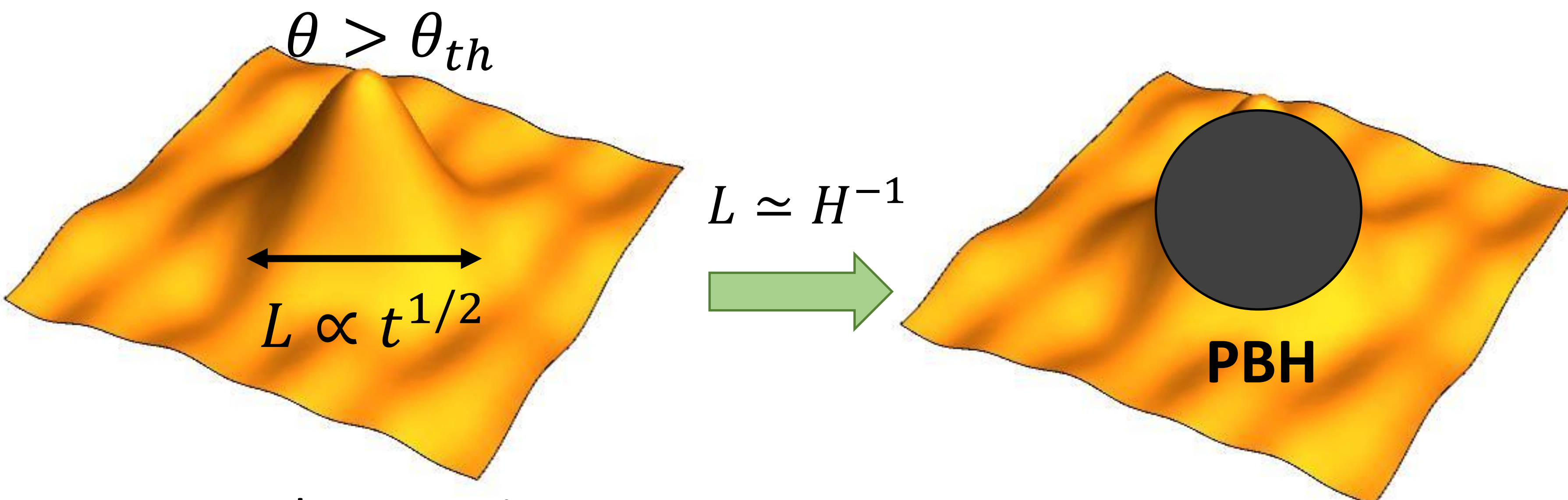
No clustering
smaller merger rate

clustering
larger merger rate

What determines the clustering of PBHs?

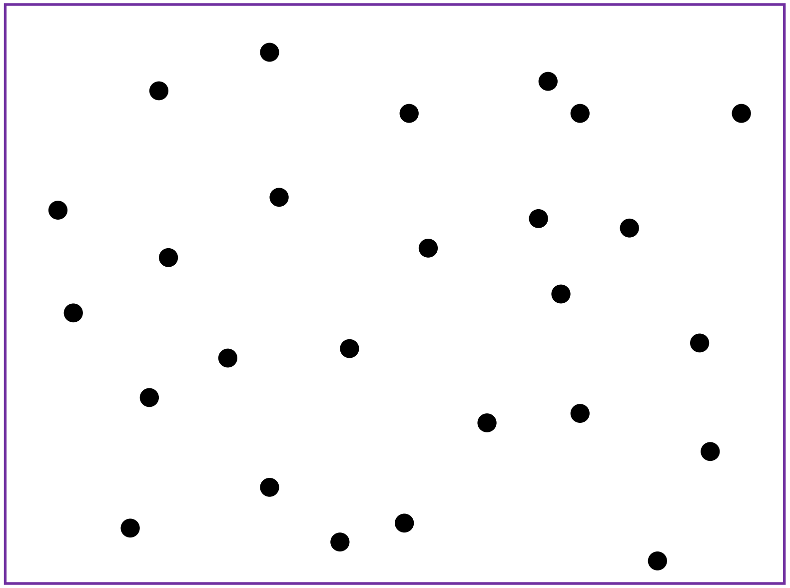
Setup

Super-Hubble scale perturbations θ generated by inflation



Overdense region
(Initially super-Hubble)

PBH is formed soon after the horizon reentry.
(Radiation dominated epoch)

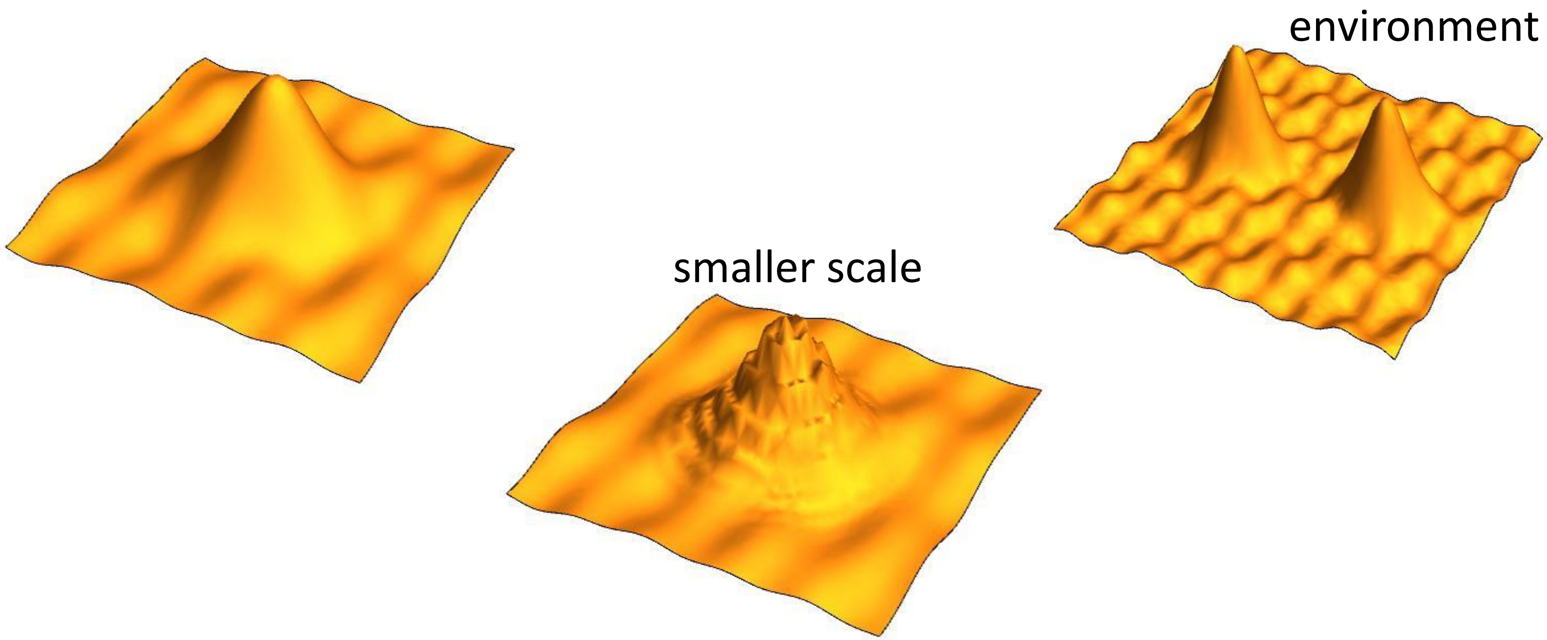


PBHs are rare initially (at formation time).

Only very high-σ fluctuations turn to PBHs.

PBHs are initially separated by super-Hubble distance.

Initial clustering = clustering on super-Hubble scales



Environment and smaller scale not relevant to PBH formation

PBH formation

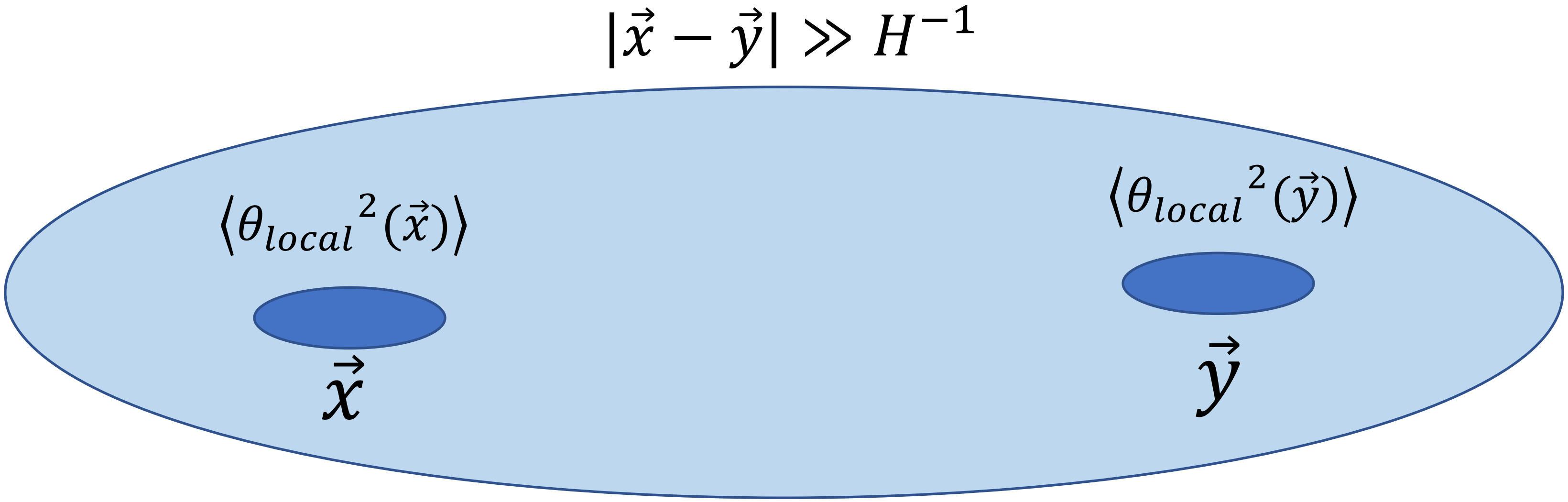
density contrast on the comoving slice

Young+ 2014

$$\theta_{local}(\vec{x}) = \int W(R, \vec{x} - \vec{y})\theta(\vec{y})d^3y$$

$$\theta_{local} > \theta_{th}$$

Clustering of PBHs



$$\langle \theta_{local}{}^2(\vec{x}) \theta_{local}{}^2(\vec{y}) \rangle \neq 0 \text{ for } |\vec{x} - \vec{y}| \gg H^{-1}$$

Super-Hubble scale clustering

Simple toy model

$$\mathcal{R}(\vec{x}) = \left(1 + \alpha\chi(\vec{x})\right)\phi(\vec{x})$$

Super-Hubble

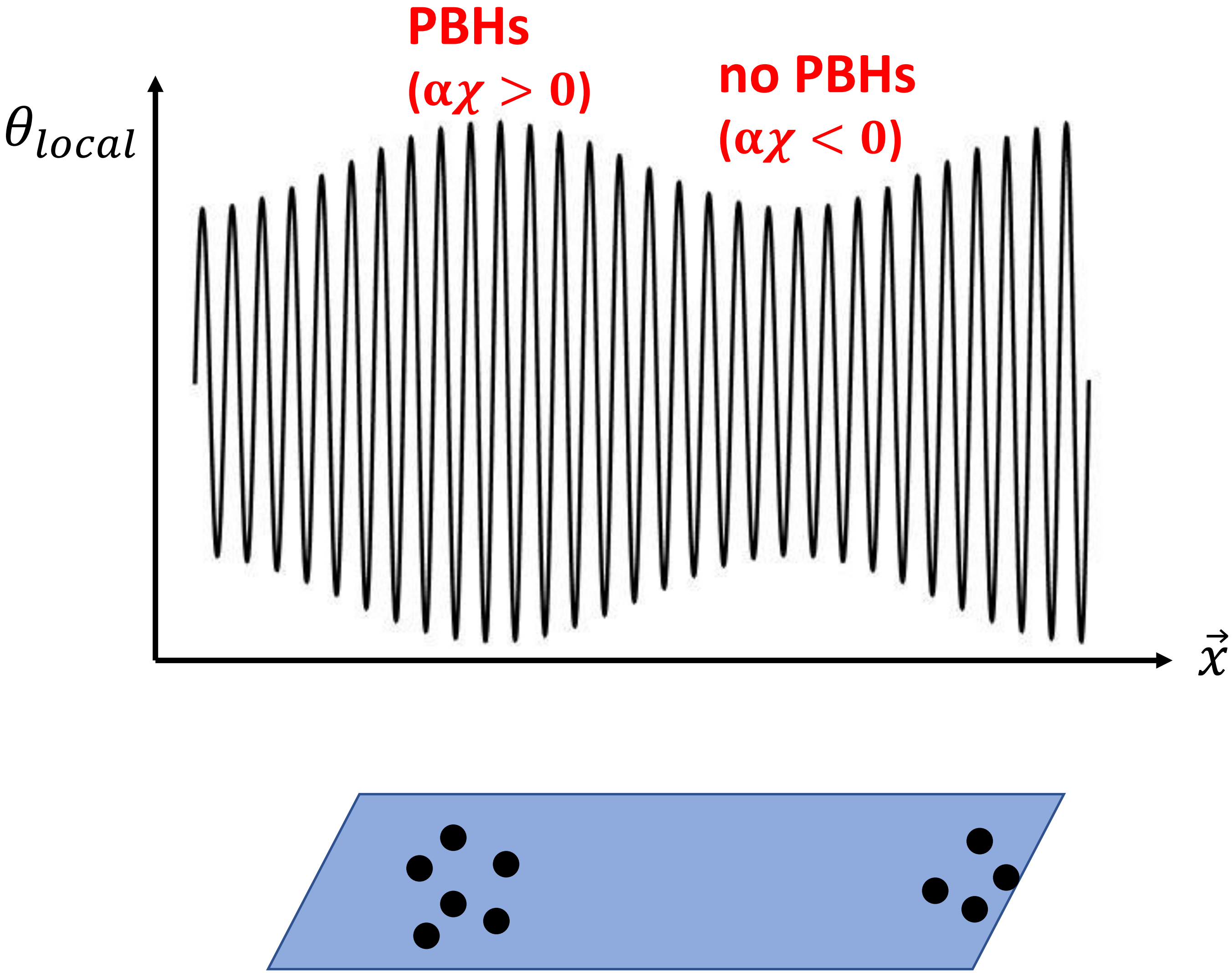
χ, ϕ: uncorrelated Gaussian variables

$$\frac{\langle \boldsymbol{\theta_{local}}^2(\vec{\boldsymbol{x}})\boldsymbol{\theta_{local}}^2(\vec{\boldsymbol{y}})\rangle}{\langle \boldsymbol{\theta_{local}}^2(\vec{\boldsymbol{x}})\rangle^2} - 1 \approx 4\alpha^2\langle\chi(\vec{x})\chi(\vec{y})\rangle + O(\alpha^4)$$

Super-Hubble correlation of the local variance is generated by the super-Hubble correlation of χ.

Clustering is characterized by the four-point function (trispectrum).

$$\mathcal{R}(\vec{x}) = \big(1 + \alpha\chi(\vec{x})\big)\phi(\vec{x})$$



$$P_\phi = P_\chi$$

$$\mathcal{R}(\vec{x}) = \big(1 + \alpha\chi(\vec{x})\big)\phi(\vec{x}) \quad \Rightarrow \quad \alpha^2 = \tau_{NL}, \qquad f_{NL} = 0$$

$$\mathcal{R}(\vec{x}) = \phi(\vec{x}) + \frac{3}{5} f_{NL}\phi^2(\vec{x})$$

ϕ : Gaussian

Presence of f_{NL} yields super-Hubble clustering of PBHs.

Tada&Yokoyama 2015, Young&Byrnes 2015

$\phi = \phi_l + \phi_s$ long mode: ϕ_l, short mode: ϕ_s

$$\Rightarrow \quad \mathcal{R} \approx \left(1 + \frac{6}{5} f_{NL}\phi_l\right)\phi_s \qquad \tau_{NL} = \frac{36}{25} f_{NL}^2$$

(τ_{NL} is non-zero)

There is no inconsistency.

PBH correlation function

Functional integral approach

e.g. Franciolini+ 2018

$P[\theta]$: probability density of θ

- Probability that point $\boldsymbol{x}$ becomes a PBH

$$P_1(\boldsymbol{x}) = \int [D\theta] P[\theta] \int_{\theta_{\rm th}}^{\infty} d\alpha\, \delta_D(\theta_{\rm local}(\boldsymbol{x}) - \alpha)$$

- Probability that points $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ becomes PBHs

$$P_2(\boldsymbol{x}_1,\, \boldsymbol{x}_2) = \int [D\theta] P[\theta] \int_{\theta_{\rm th}}^{\infty} d\alpha_1\, \delta_D(\theta_{\rm local}(\boldsymbol{x}_1) - \alpha_1) \int_{\theta_{\rm th}}^{\infty} d\alpha_2\, \delta_D(\theta_{\rm local}(\boldsymbol{x}_2) - \alpha_2)$$

- PBH correlation function

$$\xi_{\rm PBH}(\boldsymbol{x}_1, \boldsymbol{x}_2) := \frac{P_2(\boldsymbol{x}_1, \boldsymbol{x}_2)}{P_1^2} - 1$$

I assume θ is weakly (local type) non-Gaussian and expand ξ_{PBH} up to trispectrum ($f_{NL}, \tau_{NL}, g_{NL}$).

$$\langle \mathcal{R}_c(\boldsymbol{k}_1)\mathcal{R}_c(\boldsymbol{k}_2)\mathcal{R}_c(\boldsymbol{k}_3)\rangle := (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,\frac{6}{5}f_{\rm NL}[P_{\mathcal{R}_c}(k_1)P_{\mathcal{R}_c}(k_2)+2\ \text{perms.}$$

$$\begin{aligned}\langle \mathcal{R}_c(\boldsymbol{k}_1)\mathcal{R}_c(\boldsymbol{k}_2)\mathcal{R}_c(\boldsymbol{k}_3)\mathcal{R}_c(\boldsymbol{k}_4)\rangle := (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4)&\\ \times\Big\{\frac{54}{25}g_{\rm NL}[P_{\mathcal{R}_c}(k_1)P_{\mathcal{R}_c}(k_2)P_{\mathcal{R}_c}(k_3)+3\ \text{perms.}]&\\ +\tau_{\rm NL}[P_{\mathcal{R}_c}(k_1)P_{\mathcal{R}_c}(k_2)P_{\mathcal{R}_c}(|\boldsymbol{k}_1+\boldsymbol{k}_3|)+11\ \text{perms.}]\Big\}&\end{aligned}$$

$$P_1(\boldsymbol{x}) = \int [D\theta] P[\theta] \int_{\theta_{\rm th}}^{\infty} d\alpha \int_{-\infty}^{\infty} \frac{d\phi}{2\pi} \exp\left[i\phi \int d^3y \, W_{\rm local}(\boldsymbol{x}-\boldsymbol{y}) \, \theta(\boldsymbol{y}) - i\phi\,\alpha \right]$$

$$Z[J] := \int [D\theta] P[\theta] \exp\left[i \int d^3y J(\boldsymbol{y}) \theta(\boldsymbol{y}) \right]$$

$$\xi_{\theta(c)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n) := \frac{1}{i^n} \frac{\delta^n \log Z[J]}{\delta J(\boldsymbol{x}_1) \delta J(\boldsymbol{x}_2) \cdots \delta J(\boldsymbol{x}_n)} \bigg|_{J=0}$$

$$\begin{aligned} P_1(\boldsymbol{x}) &= \int_{\theta_{\rm th}}^{\infty} d\alpha \int_{-\infty}^{\infty} \frac{d\phi}{2\pi} e^{-i\phi\,\alpha} Z\left[\phi\, W_{\rm local}(\boldsymbol{x}-\boldsymbol{y})\right] \\ &= \int_{\theta_{\rm th}}^{\infty} d\alpha \int_{-\infty}^{\infty} \frac{d\phi}{2\pi} \exp\left[-i\phi\,\alpha\right] \exp\left[\sum_{n=2}^{\infty} \frac{i^n}{n!} \phi^n \, \underline{\xi^{(n)}_{\rm local(c)}} \right] \end{aligned}$$

Expand

$$
\begin{aligned}
P_{\rm PBH}(k) \simeq & \left(\frac{4\nu}{9\sigma_R}\right)^2 W_{\rm local}(k)^2 P_{\mathcal{R}_c}(k) \\
& + \frac{6}{5} f_{\rm NL} \left(\frac{4\nu}{9\sigma_R}\right)^3 W_{\rm local}(k) \\
& \times \int \frac{d^3p}{(2\pi)^3} W_{\rm local}(p) W_{\rm local}(|\boldsymbol{k}-\boldsymbol{p}|) \\
& \quad \times \left[2P_{\mathcal{R}_c}(p)P_{\mathcal{R}_c}(k) + P_{\mathcal{R}_c}(p)P_{\mathcal{R}_c}(|\boldsymbol{k}-\boldsymbol{p}|)\right] \\
& + \frac{18}{25} g_{\rm NL} \left(\frac{4\nu}{9\sigma_R}\right)^4 W_{\rm local}(k) \\
& \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\rm local}(p_1) W_{\rm local}(p_2) W_{\rm local}(|\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2|) \\
& \qquad \times \left[3P_{\mathcal{R}_c}(k)P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2) + P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2|)\right] \\
& + \frac{\tau_{\rm NL}}{3} \left(\frac{4\nu}{9\sigma_R}\right)^4 W_{\rm local}(k) \\
& \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\rm local}(p_1) W_{\rm local}(p_2) W_{\rm local}(|\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2|) \\
& \quad \times \left[6P_{\mathcal{R}_c}(k)P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(|\boldsymbol{p}_1+\boldsymbol{p}_2|) + 6P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}-\boldsymbol{p}_1|)\right] \\
& + \frac{54}{25} g_{\rm NL} \left(\frac{4\nu}{9\sigma_R}\right)^4 \\
& \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\rm local}(p_1) W_{\rm local}(p_2) W_{\rm local}(|\boldsymbol{k}+\boldsymbol{p}_1|) W_{\rm local}(|\boldsymbol{p}_2-\boldsymbol{k}|) \\
& \qquad\qquad \times P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}+\boldsymbol{p}_1|) \\
& + \frac{\tau_{\rm NL}}{4} \left(\frac{4\nu}{9\sigma_R}\right)^4 \\
& \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\rm local}(p_1) W_{\rm local}(p_2) W_{\rm local}(|\boldsymbol{k}+\boldsymbol{p}_1|) W_{\rm local}(|\boldsymbol{p}_2-\boldsymbol{k}|) \\
& \times \left[4P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(k)\right. \\
& \quad \left. +4P_{\mathcal{R}_c}(p_1)\left(P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}+\boldsymbol{p}_1-\boldsymbol{p}_2|) + P_{\mathcal{R}_c}(|\boldsymbol{k}+\boldsymbol{p}_1|)P_{\mathcal{R}_c}(|\boldsymbol{p}_1+\boldsymbol{p}_2|)\right)\right]
\end{aligned}
$$

In the super-Hubble ($k \to 0$) limit,

$$
\begin{aligned}
P_{\mathrm{PBH}}(k) \simeq & \left(\frac{4\nu}{9\sigma_R}\right)^2 W_{\mathrm{local}}(k)^2 P_{\mathcal{R}_c}(k) \\
& + \frac{6}{5} f_{\mathrm{NL}} \left(\frac{4\nu}{9\sigma_R}\right)^3 W_{\mathrm{local}}(k) \\
& \times \int \frac{d^3p}{(2\pi)^3} W_{\mathrm{local}}(p) W_{\mathrm{local}}(|\boldsymbol{k}-\boldsymbol{p}|) \\
& \quad \times \left[2P_{\mathcal{R}_c}(p)P_{\mathcal{R}_c}(k) + P_{\mathcal{R}_c}(p)P_{\mathcal{R}_c}(|\boldsymbol{k}-\boldsymbol{p}|)\right] \\
& + \frac{18}{25} g_{\mathrm{NL}} \left(\frac{4\nu}{9\sigma_R}\right)^4 W_{\mathrm{local}}(k) \\
& \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\mathrm{local}}(p_1) W_{\mathrm{local}}(p_2) W_{\mathrm{local}}(|\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2|) \\
& \quad \times \left[3P_{\mathcal{R}_c}(k)P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2) + P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2|)\right] \\
& + \frac{\tau_{\mathrm{NL}}}{3} \left(\frac{4\nu}{9\sigma_R}\right)^4 W_{\mathrm{local}}(k) \\
& \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\mathrm{local}}(p_1) W_{\mathrm{local}}(p_2) W_{\mathrm{local}}(|\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2|) \\
& \quad \times \left[6P_{\mathcal{R}_c}(k)P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(|\boldsymbol{p}_1+\boldsymbol{p}_2|) + 6P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}-\boldsymbol{p}_1|)\right] \\
& + \frac{54}{25} g_{\mathrm{NL}} \left(\frac{4\nu}{9\sigma_R}\right)^4 \\
& \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\mathrm{local}}(p_1) W_{\mathrm{local}}(p_2) W_{\mathrm{local}}(|\boldsymbol{k}+\boldsymbol{p}_1|) W_{\mathrm{local}}(|\boldsymbol{p}_2-\boldsymbol{k}|) \\
& \qquad \times P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}+\boldsymbol{p}_1|) \\
& + \frac{\tau_{\mathrm{NL}}}{4} \left(\frac{4\nu}{9\sigma_R}\right)^4 \\
& \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} W_{\mathrm{local}}(p_1) W_{\mathrm{local}}(p_2) W_{\mathrm{local}}(|\boldsymbol{k}+\boldsymbol{p}_1|) W_{\mathrm{local}}(|\boldsymbol{p}_2-\boldsymbol{k}|) \\
& \times \left[4P_{\mathcal{R}_c}(p_1)P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(k)\right. \\
& \quad \left. +4P_{\mathcal{R}_c}(p_1)\left(P_{\mathcal{R}_c}(p_2)P_{\mathcal{R}_c}(|\boldsymbol{k}+\boldsymbol{p}_1-\boldsymbol{p}_2|) + P_{\mathcal{R}_c}(|\boldsymbol{k}+\boldsymbol{p}_1|)P_{\mathcal{R}_c}(|\boldsymbol{p}_1+\boldsymbol{p}_2|)\right)\right]
\end{aligned}
$$

In the super-Hubble ($k \to 0$) limit,

Main result

$$\xi_{\rm PBH}^{(2)}(\boldsymbol{r}) = \tau_{\rm NL}\left(\frac{4\nu}{9}\right)^4 \xi_{\mathcal{R}_c}^{(2)}(\boldsymbol{r}) \qquad \nu = \frac{\theta_{th}}{\sigma}$$

PBH clusters produce dark matter isocurvature perturbations.

Tada&Yokoyama 2015, Young&Byrnes 2015

Isocurvature constraint

$$f_{PBH}^2 \tau_{NL} \nu^4 \leq O(10^{-2})$$

Summary

PBHs cluster on super-Hubble scale if the seed perturbation has local-type trispectrum parametrized by τ_{NL}.