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Quy Nhon
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Primordial Black Holes from Inflation and Gravitational Waves

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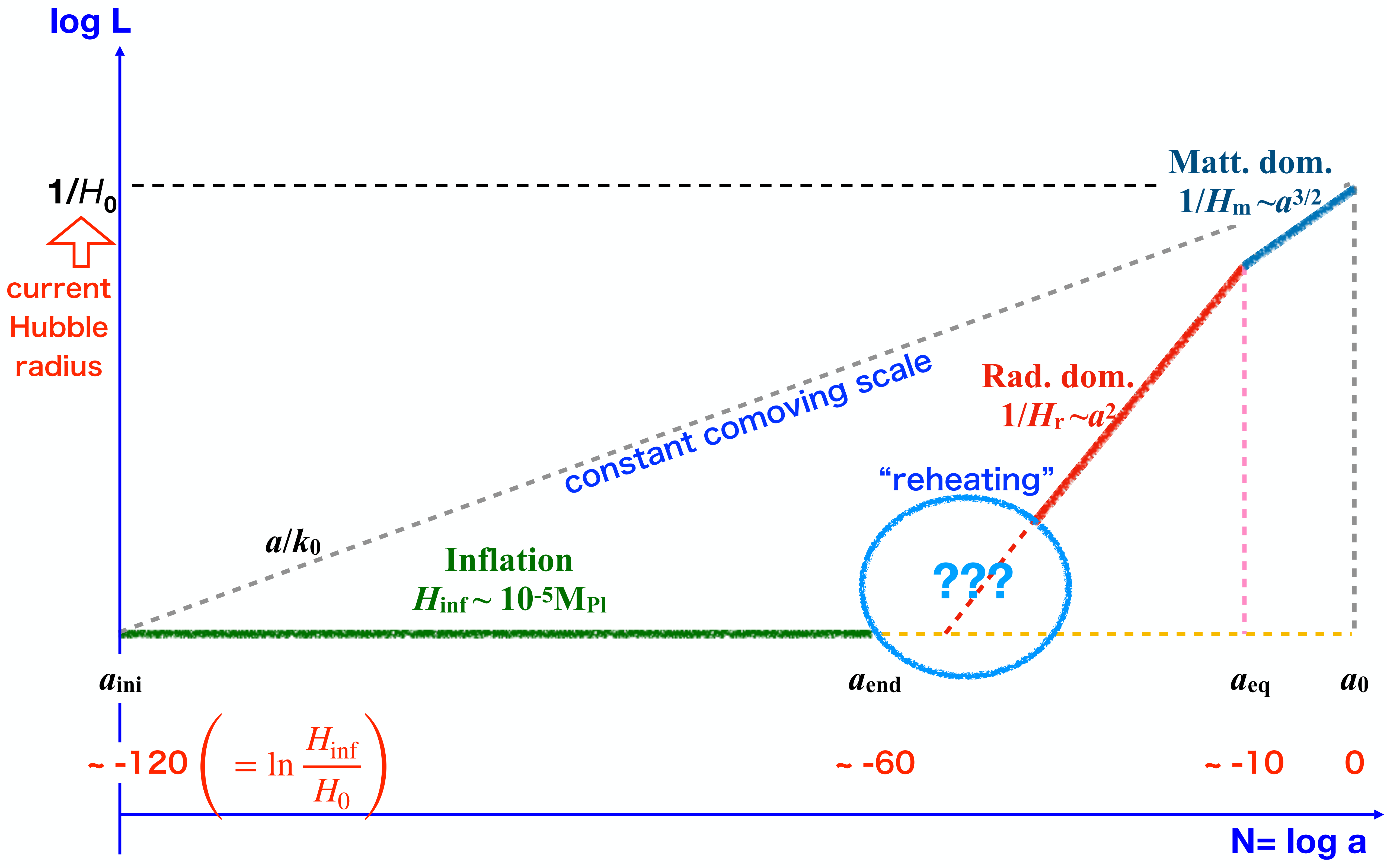
Based on

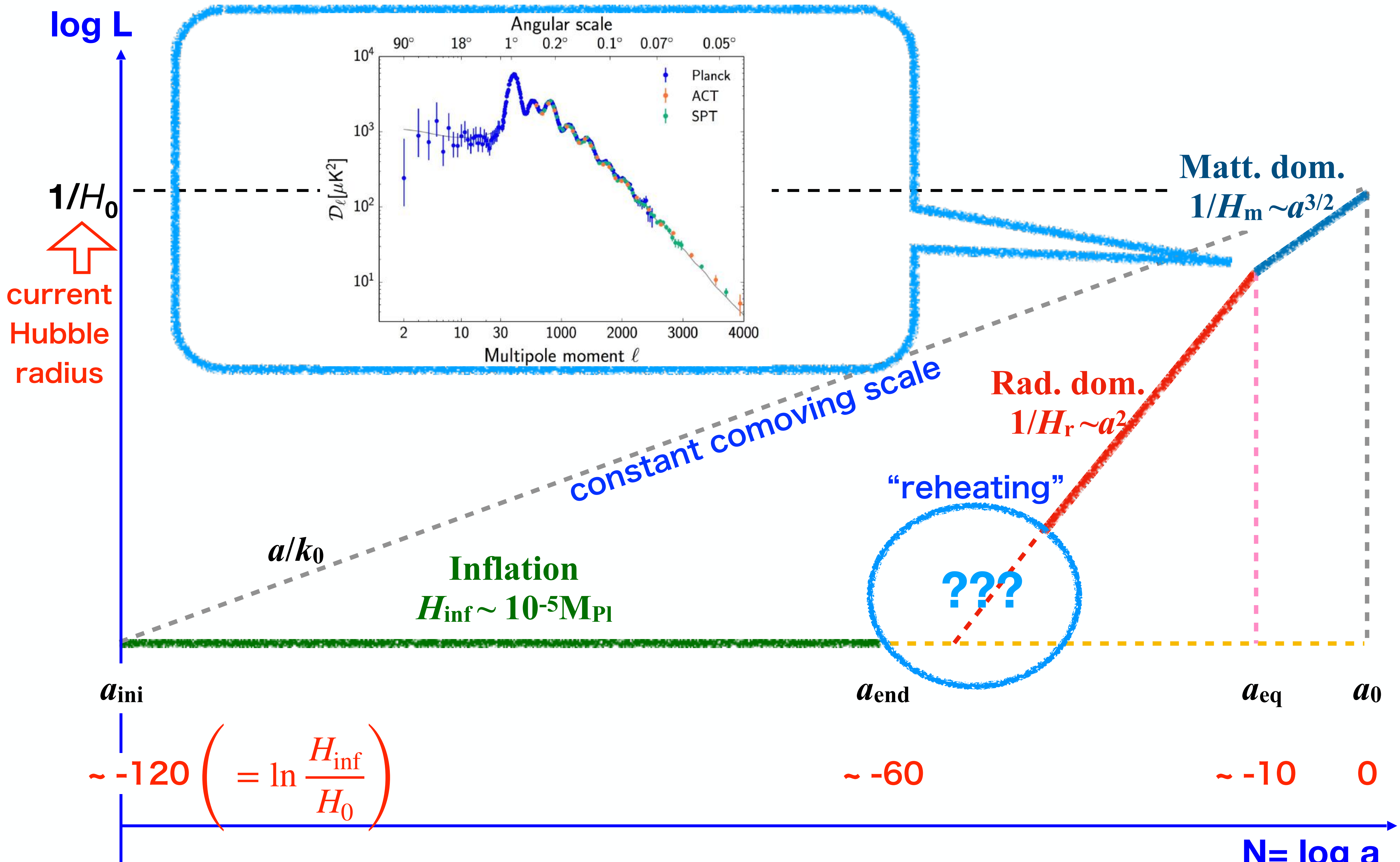
Ying-li Zhang, Qing-guo Huang, Shi Pi & MS, arXiv:1712.09896: JCAP 1805 (2018) 042
Rong-gen Cai, Shi Pi & MS, arXiv:1810.11000: PRL122 (2019) 201101

PBH Cosmology

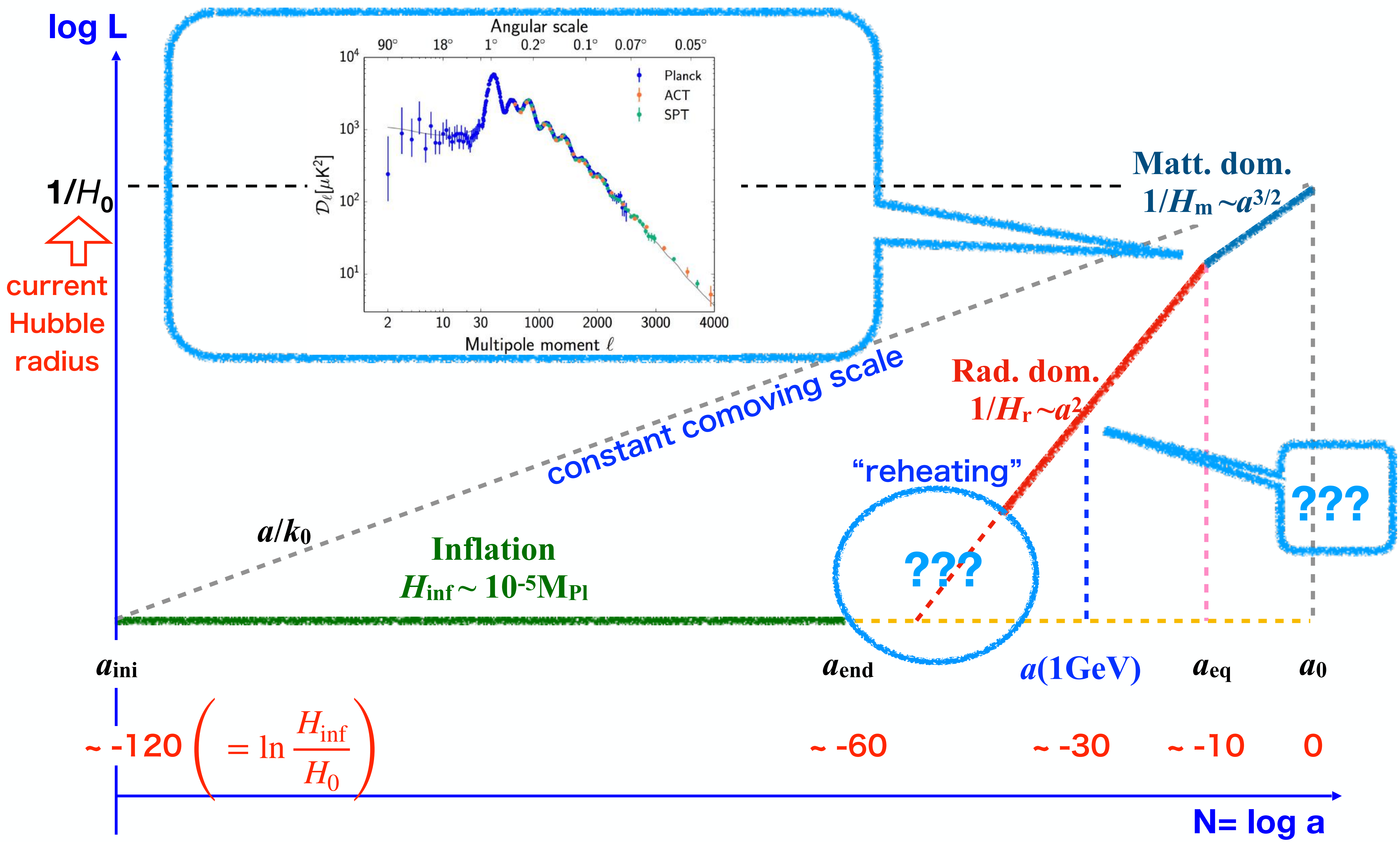
cosmic spacetime diagram



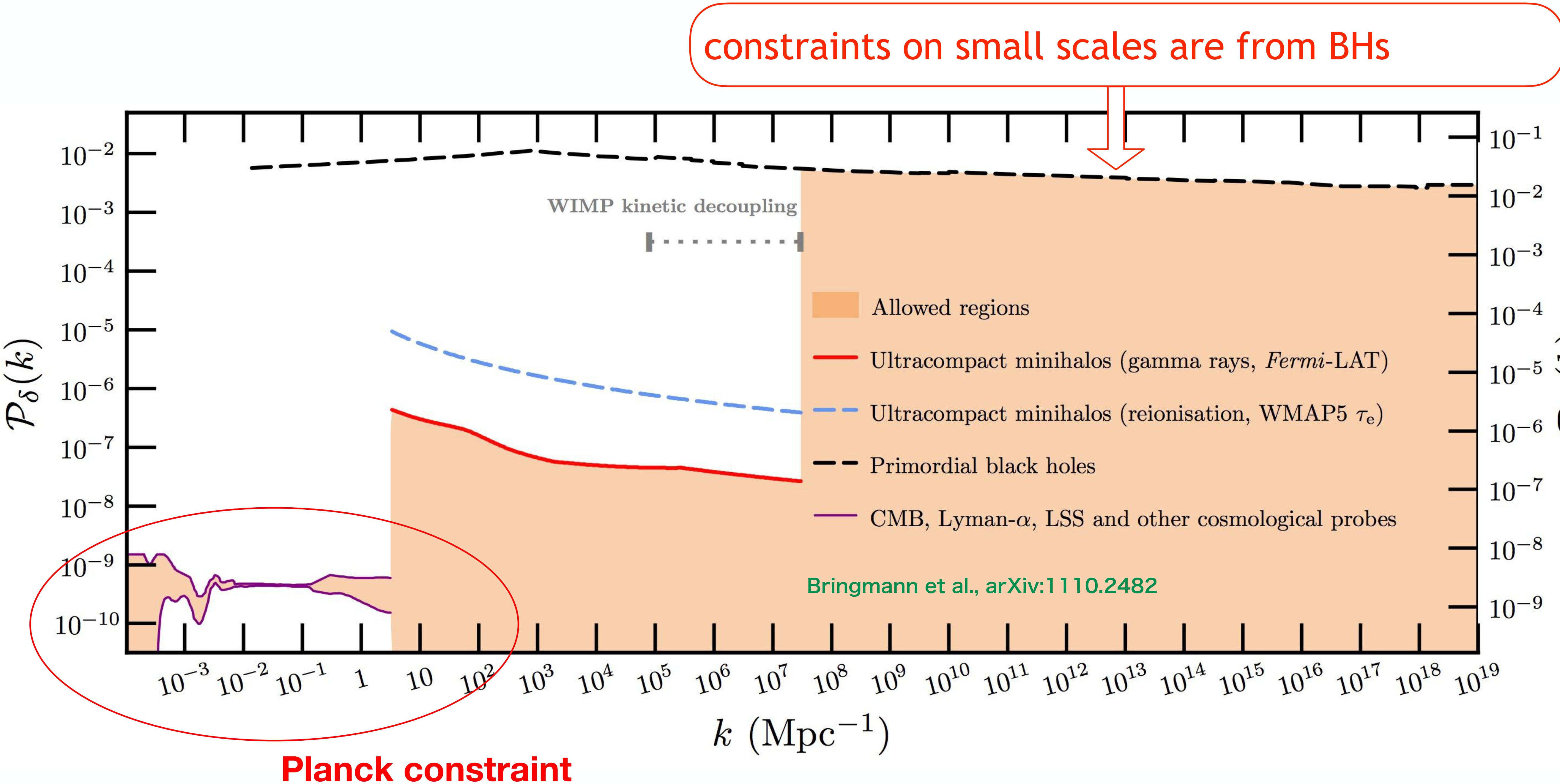
cosmic spacetime diagram



cosmic spacetime diagram

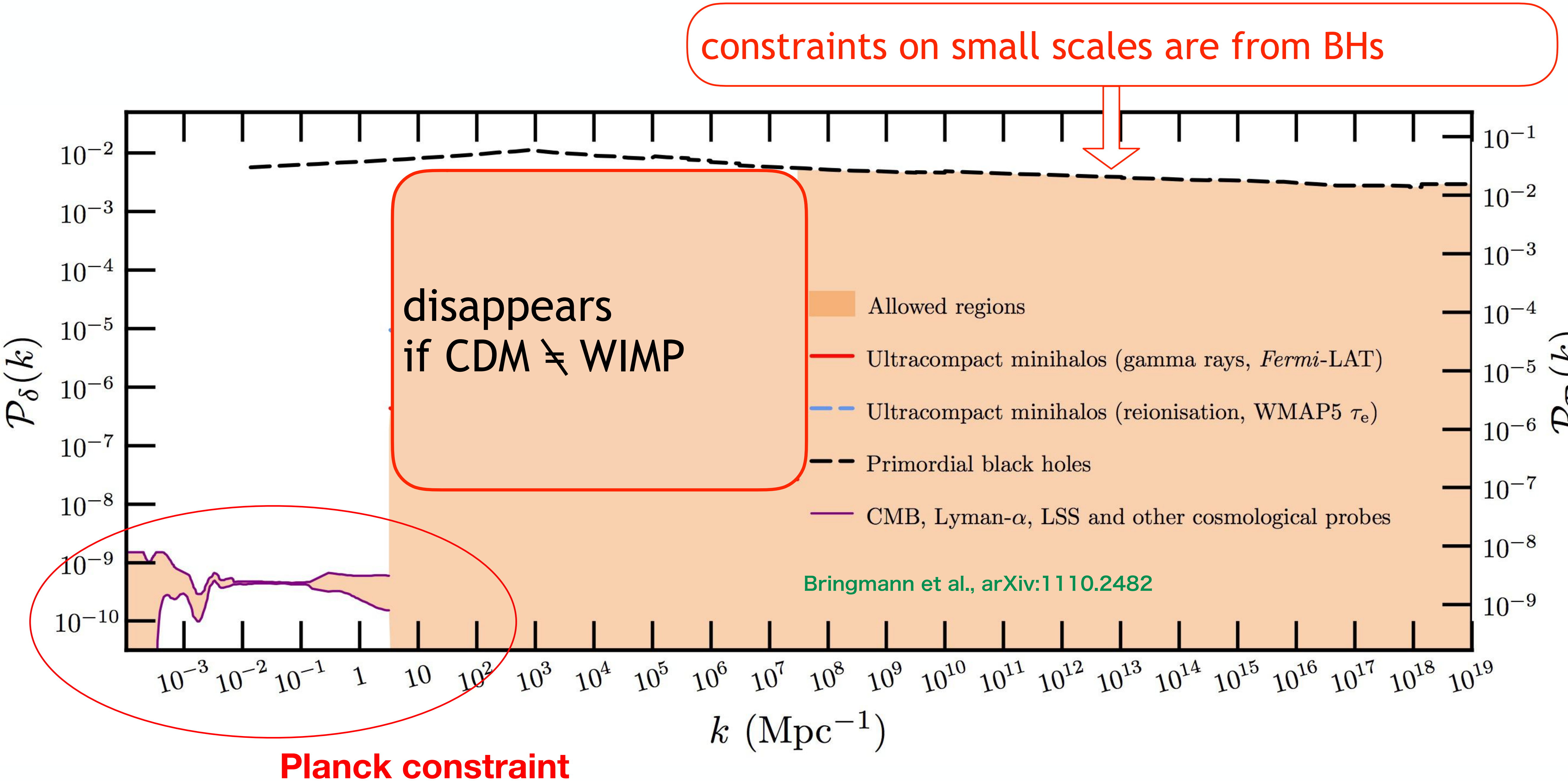


observational constraint on inflation



There are some constraints on small scales, but quite weak.

observational constraint on inflation



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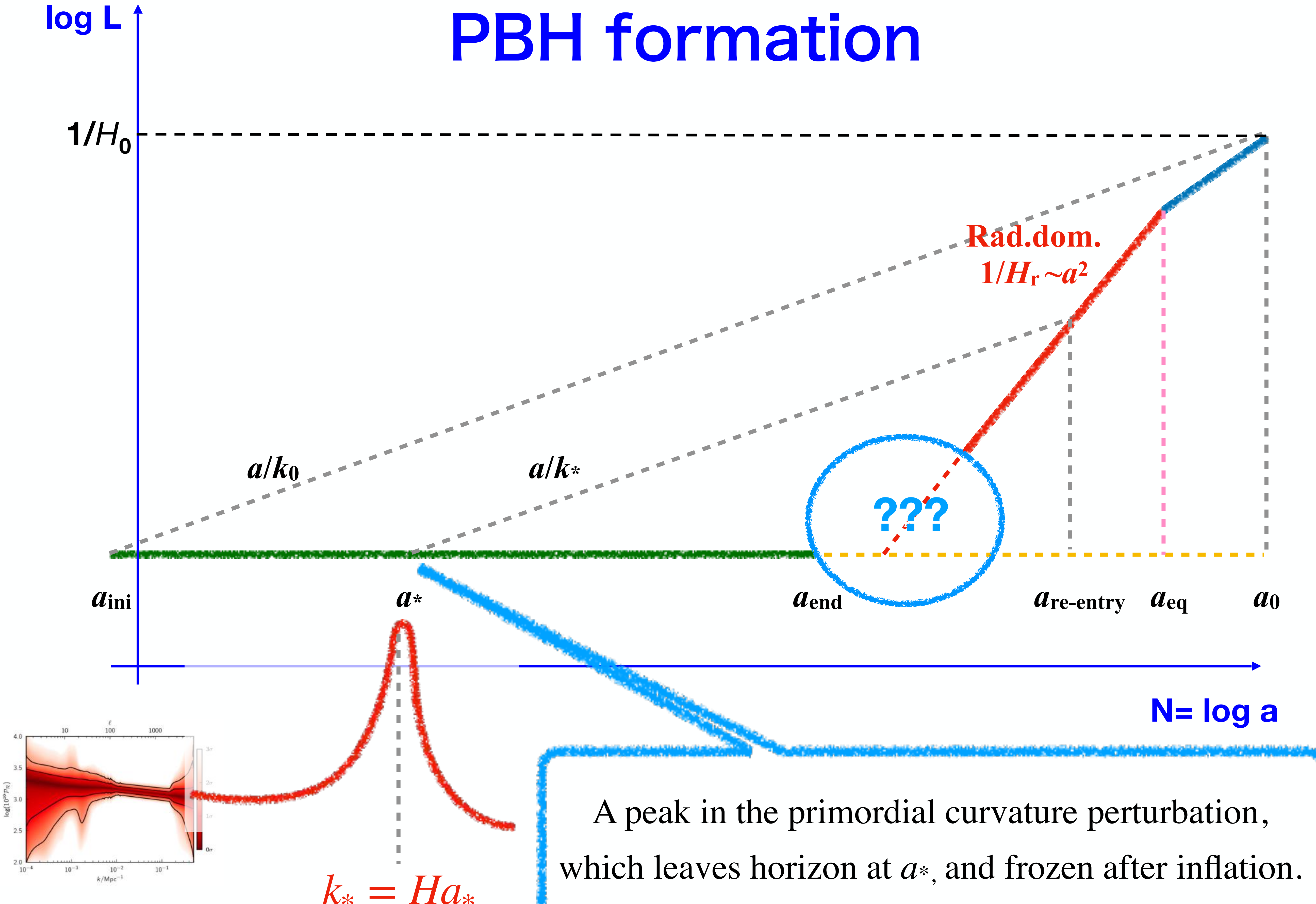
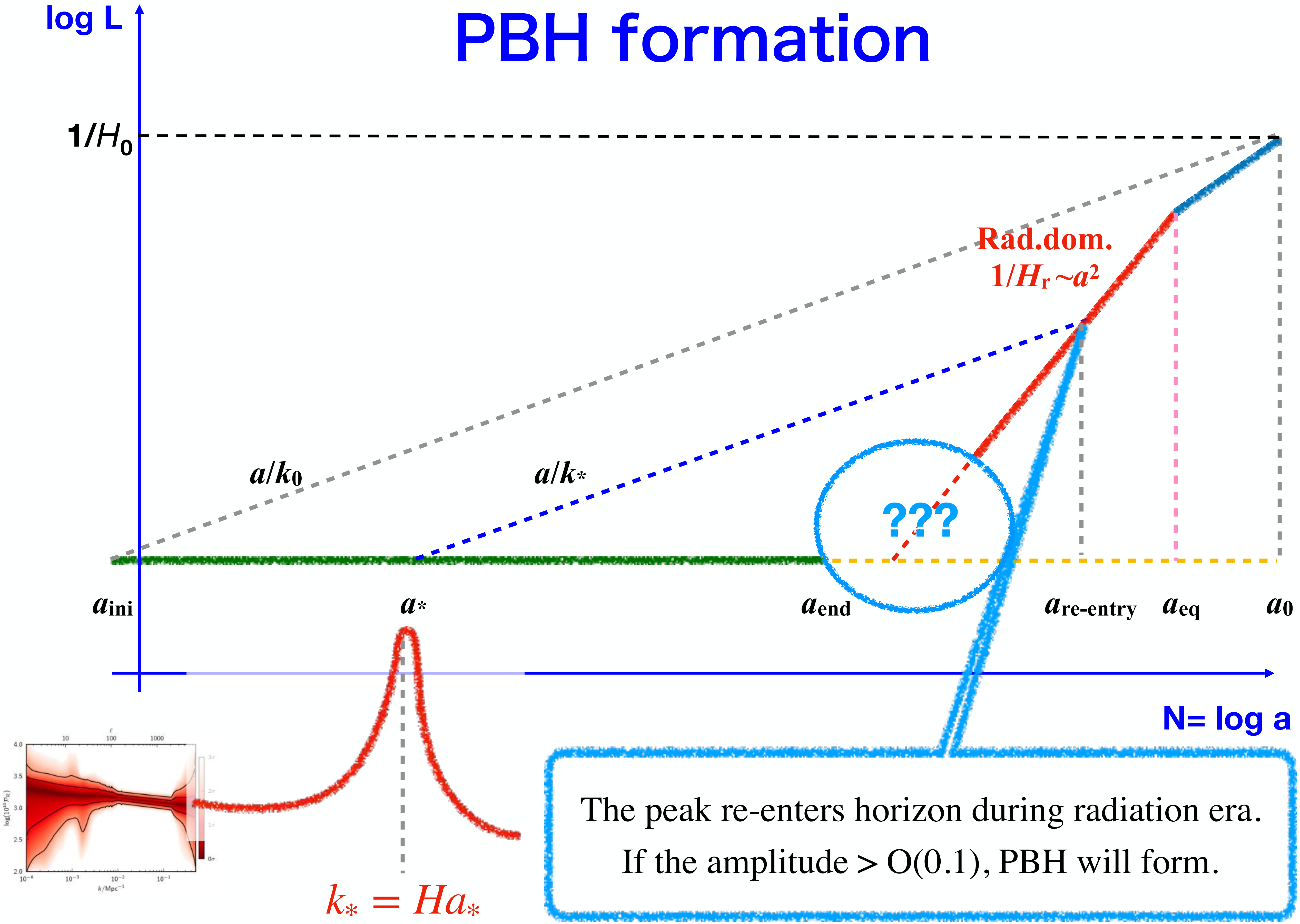
PBH formation



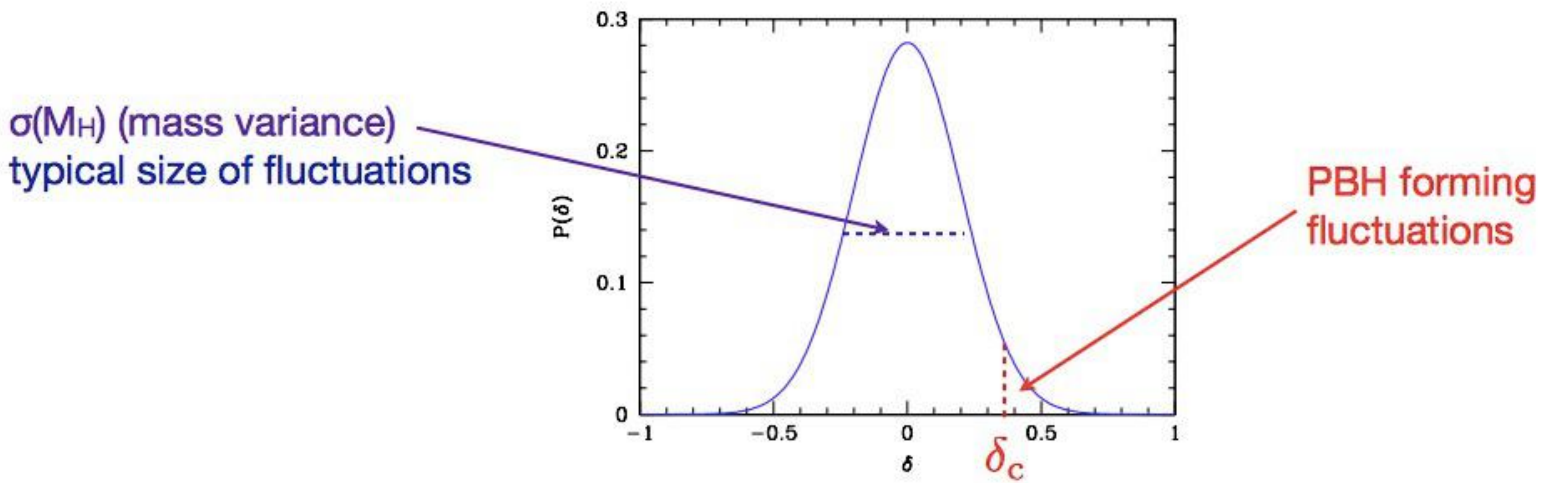
A peak in the primordial curvature perturbation, which leaves horizon at a_*, and frozen after inflation.

PBH formation



fraction β that turns into PBHs

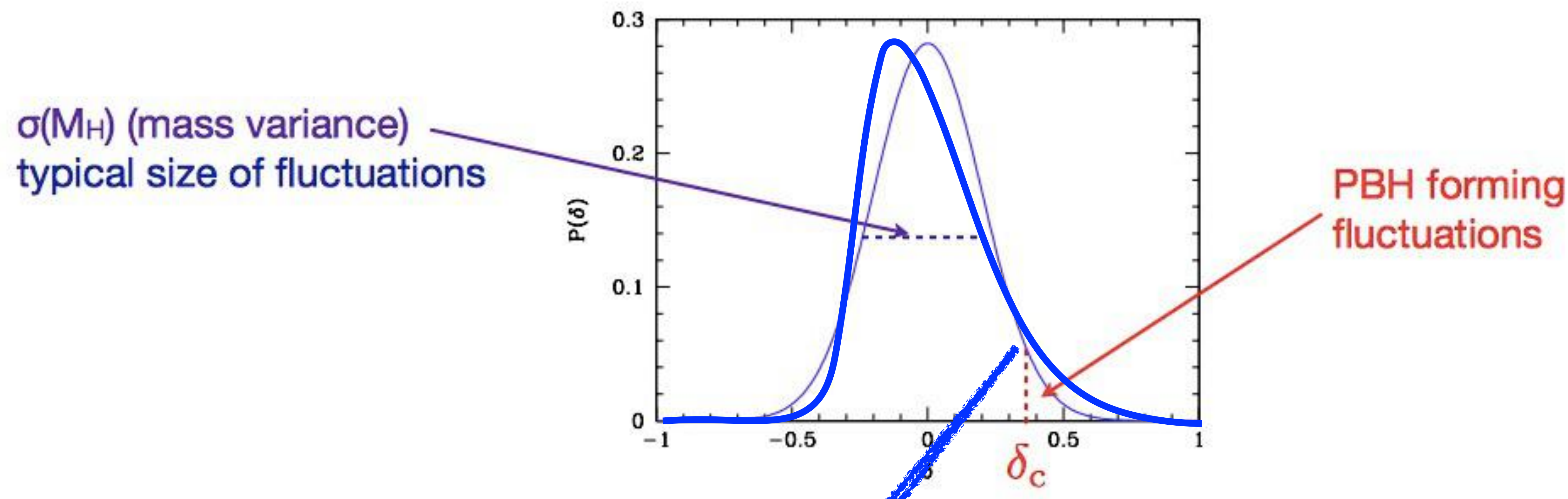
for Gaussian probability distribution



- When $\sigma_M << \delta_c$, β can be approximated by exponential:

$$\beta \approx \sqrt{\frac{2}{\pi}} \frac{\sigma_M}{\delta_c} \exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)$$

effect or non-Gaussianity



Non-Gaussianity can increase (f_{NL}>0) or decrease (f_{NL}<0) the PBH adundances, substantially if $\sigma(M_H) \ll 1$.

Young & Byrnes, 1307.4995

Formation of PBHs

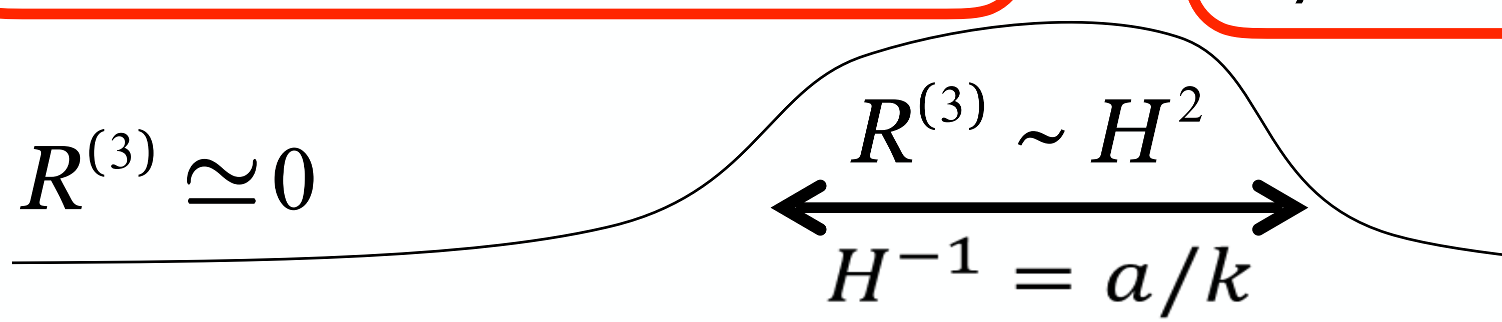
Curvature perturbation to PBH

➢ gradient expansion/separate universe approach

$$6H^2(t,x)+R^{(3)}(t,x)=16\pi G\rho(t,x)+\cdots$$ Hamiltonian constraint (Friedmann eq.)

$$R^{(3)} \approx -\frac{4}{a^2}\nabla^2\mathcal{R}_c \approx \frac{8\pi G}{3}\delta\rho_c$$

$$\frac{\delta\rho_c}{\rho} \approx \mathcal{R}_c \quad \text{at} \quad \frac{k^2}{a^2}=H^2$$



formation of a closed universe

➢If $R^{(3)} \sim H^2$ $(\Leftrightarrow \delta\rho_c/\rho \sim 1)$, it collapses to form BH

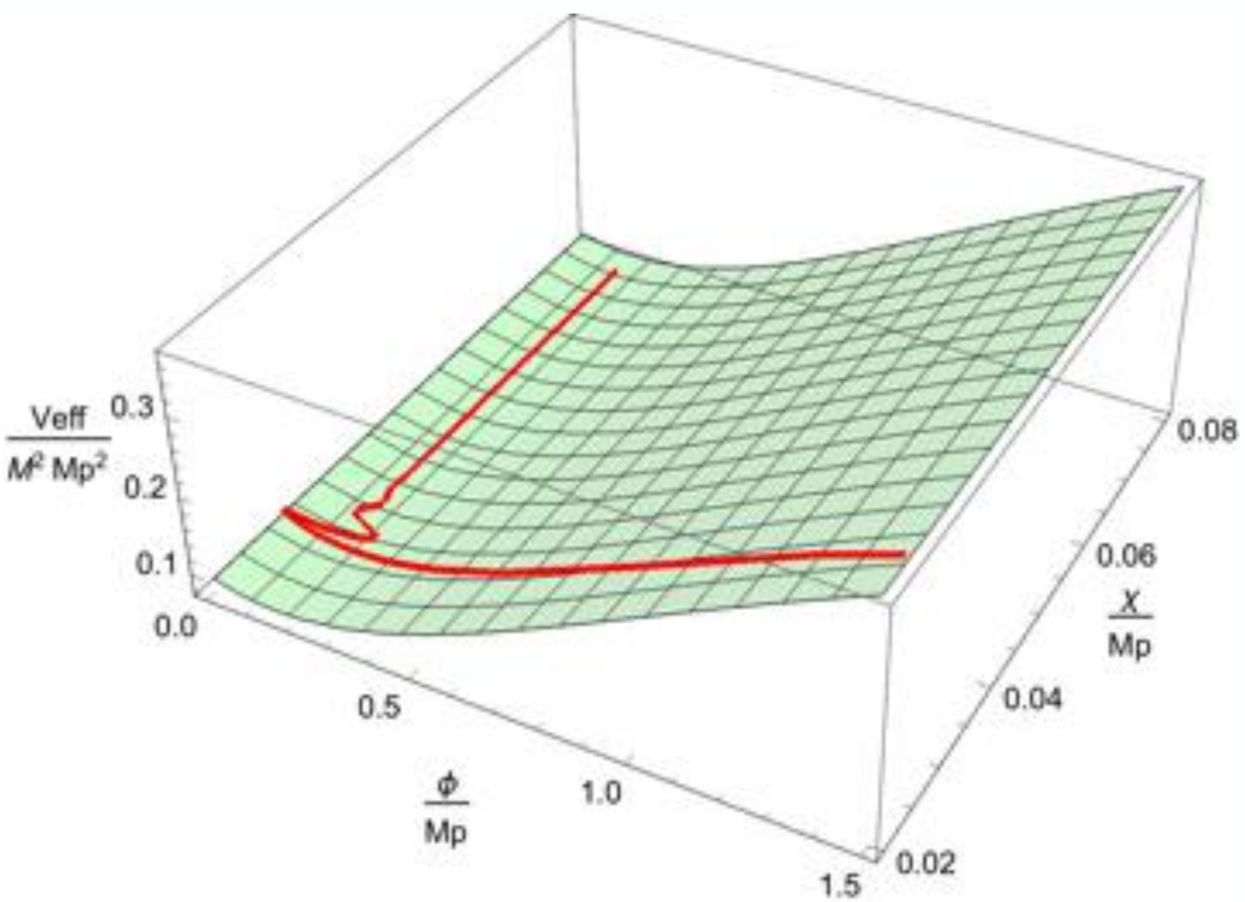
Young, Byrnes & MS '14

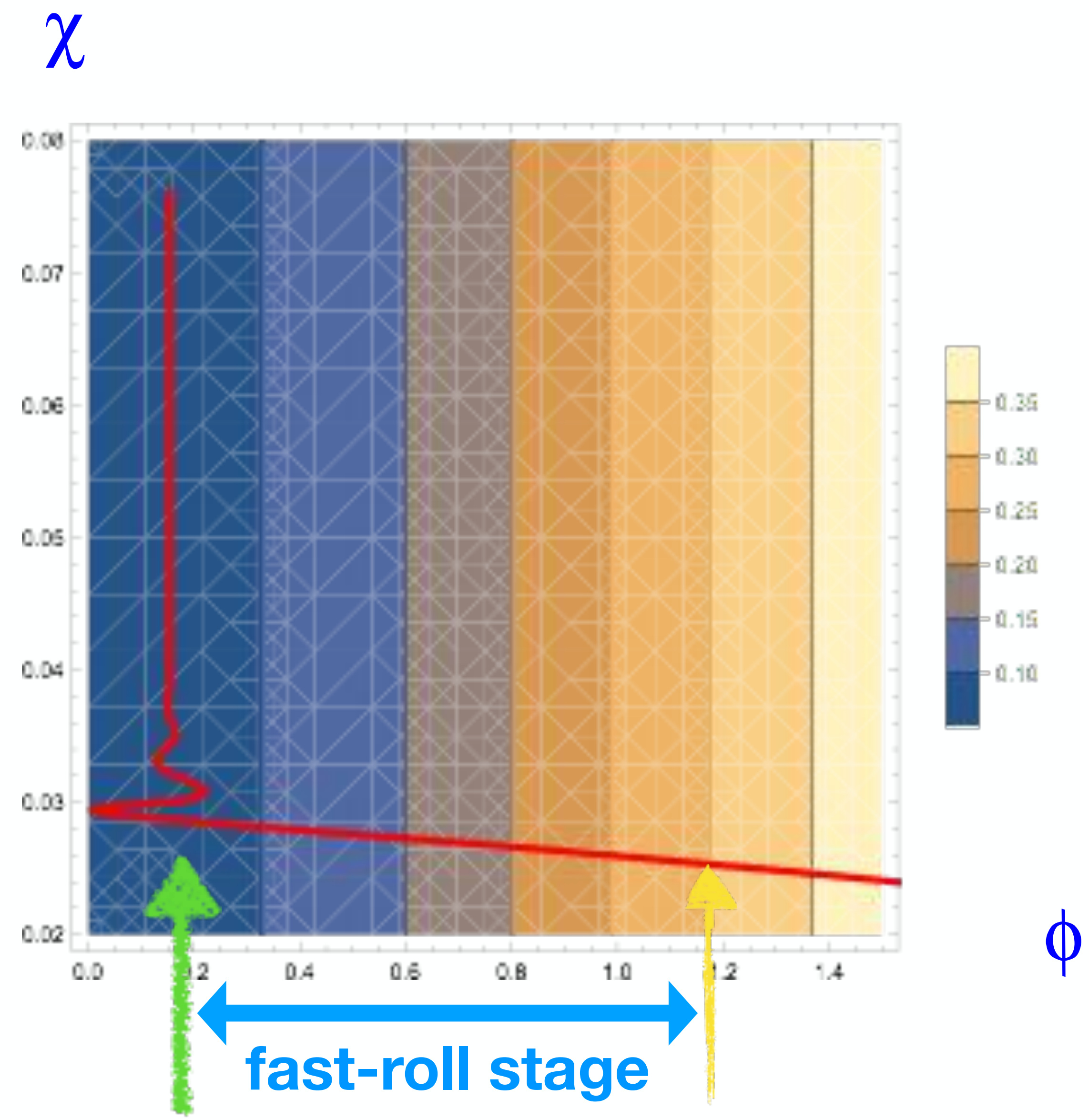
$$M_{\rm PBH} \sim \rho H^{-3} \sim 10^5 M_\odot \left(\frac{t}{1s}\right) \sim 20 M_\odot \left(\frac{k}{1\text{pc}^{-1}}\right)^{-2}$$

➢Spins of PBHs are expected to be very small

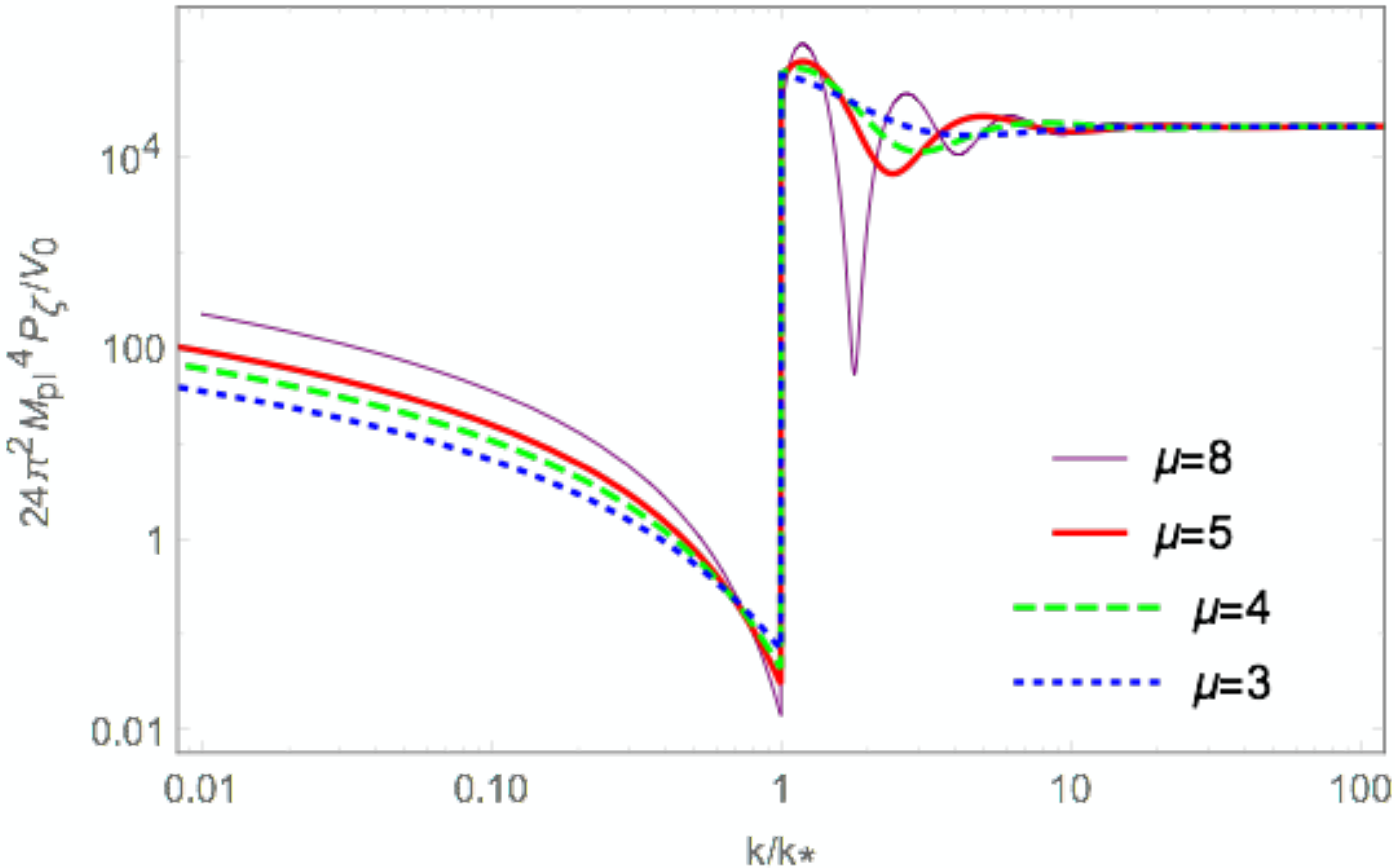
scalaron+x model

Pi, Zhang, Huang & MS ‘17

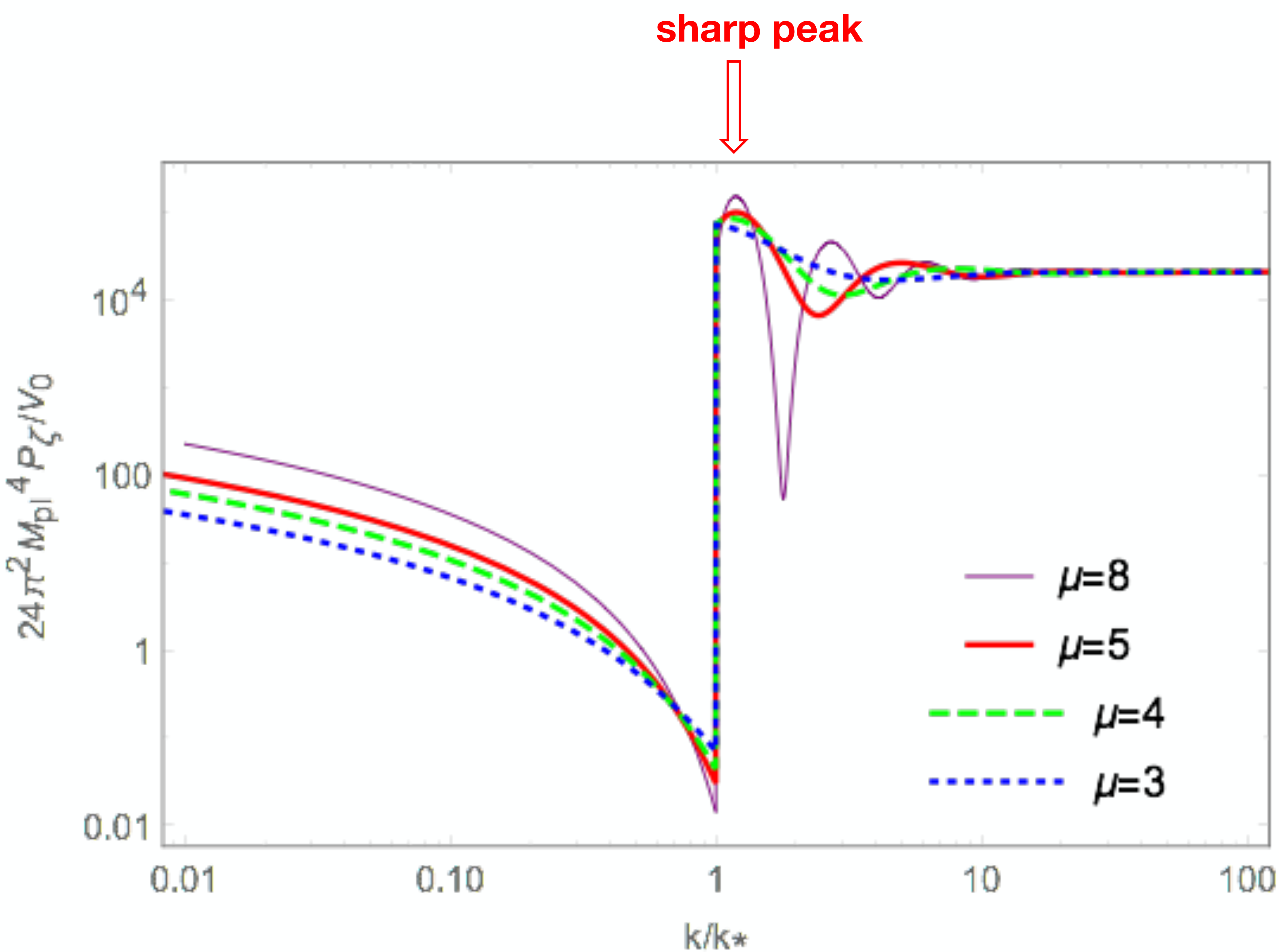




- Scaleron ϕ becomes massive at the end of the 1st stage.
- Field χ plays the role of inflaton at the 2nd stage.



can produce nearly
monochromatic PBH mass spectrum



can produce nearly
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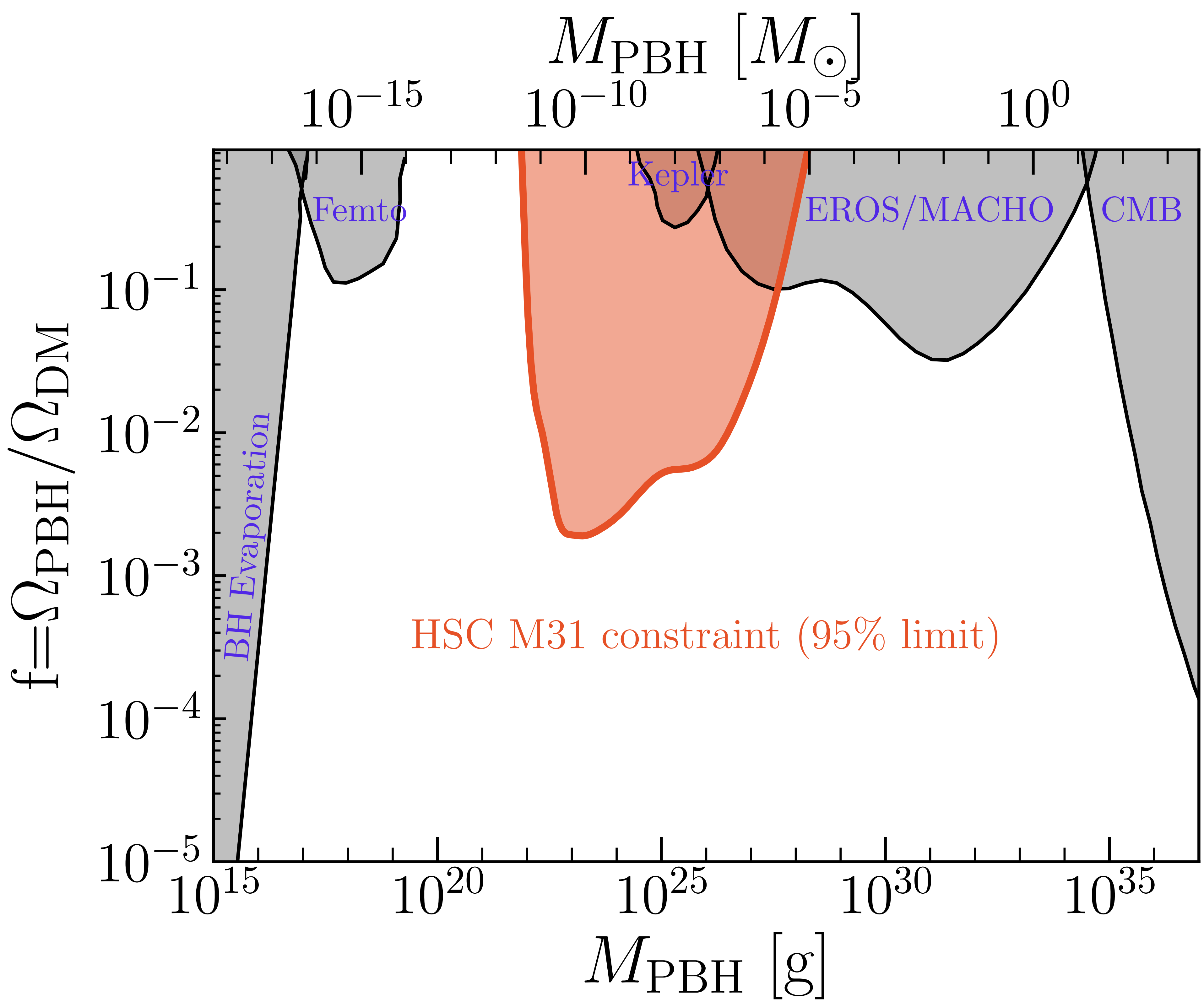
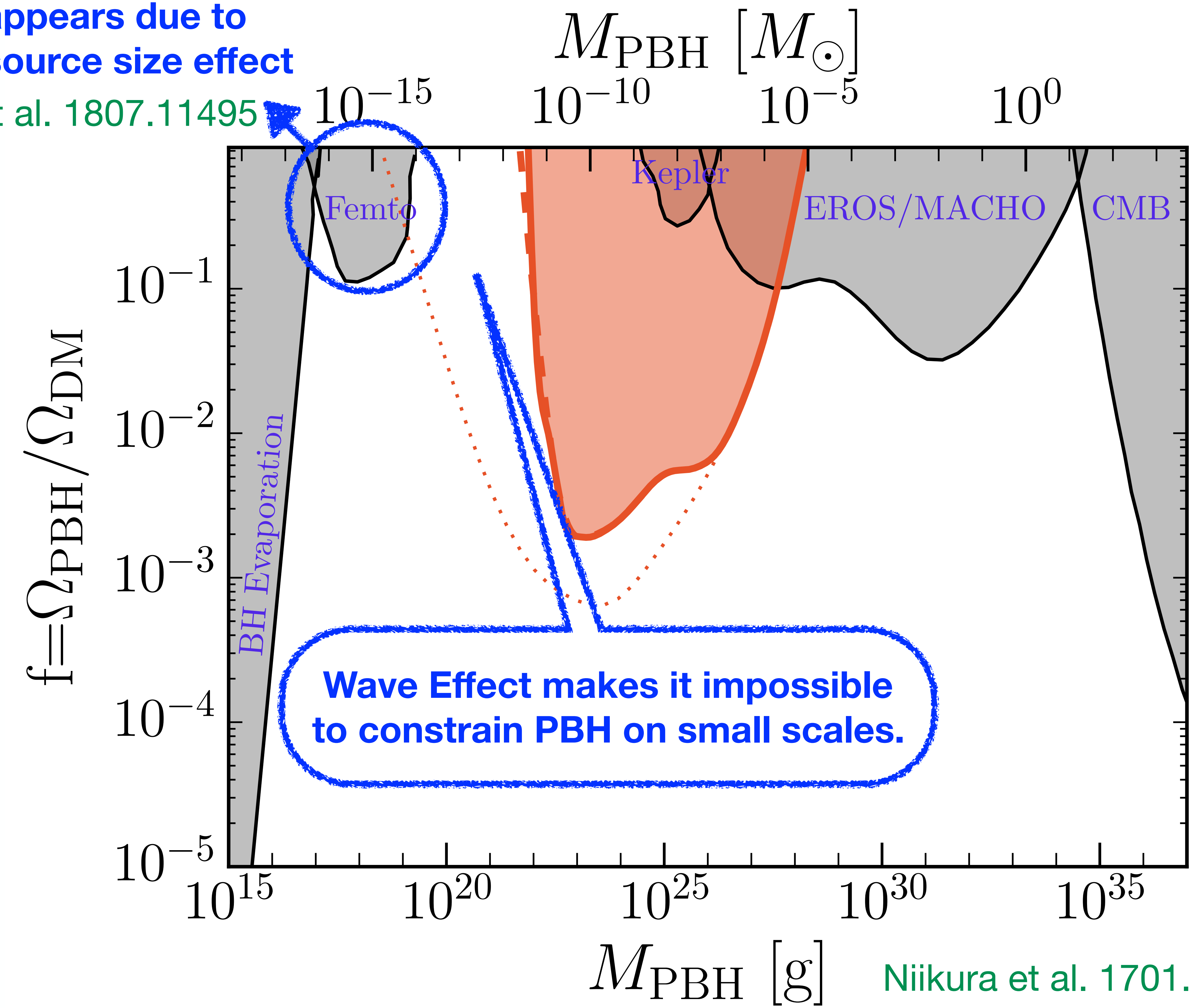
Constraints on PBH mass spectrum



Niikura et al. 1701.02151

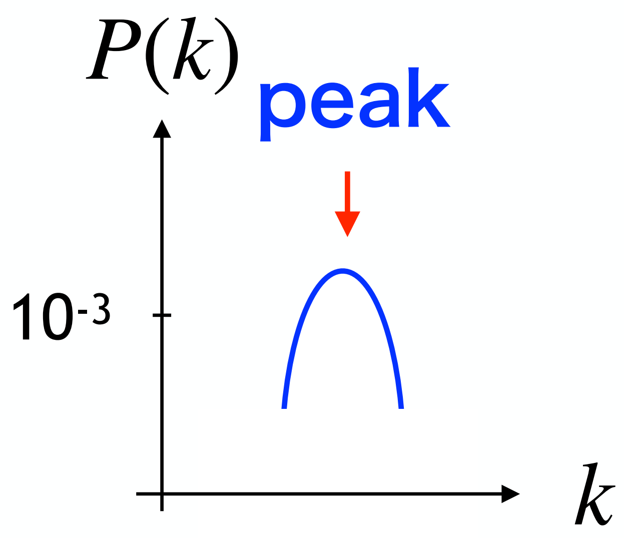
disappears due to finite source size effect

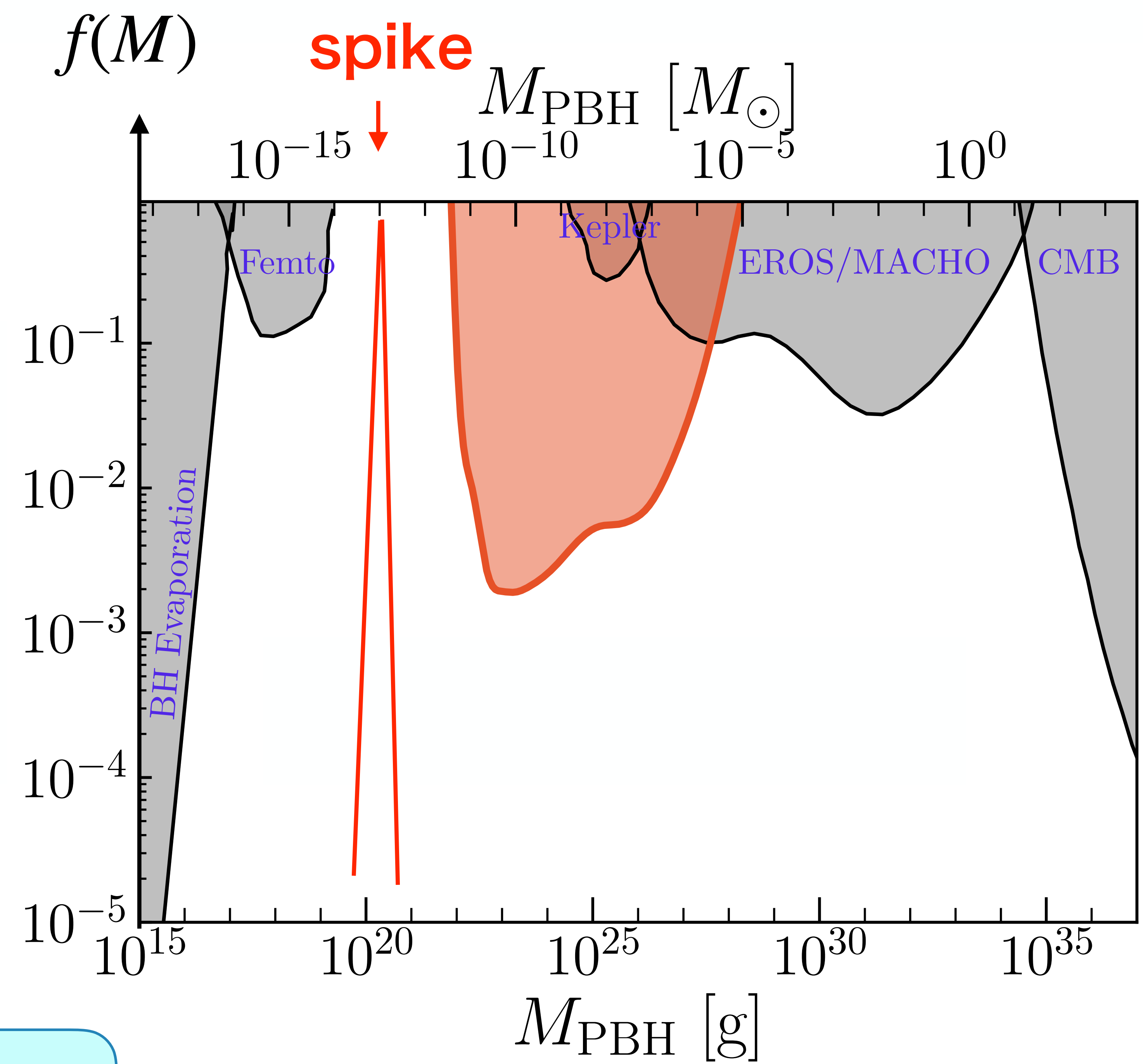
Katz et al. 1807.11495



Niikura et al. 1701.02151v3

PBHs as CMD





$$f(M) \equiv \frac{\Omega_{PBH}}{\Omega_{DM}} \propto \exp\left[-\frac{O(0.1)}{P(k)}\right]$$

a sharp peak in $P(k)$

⇩

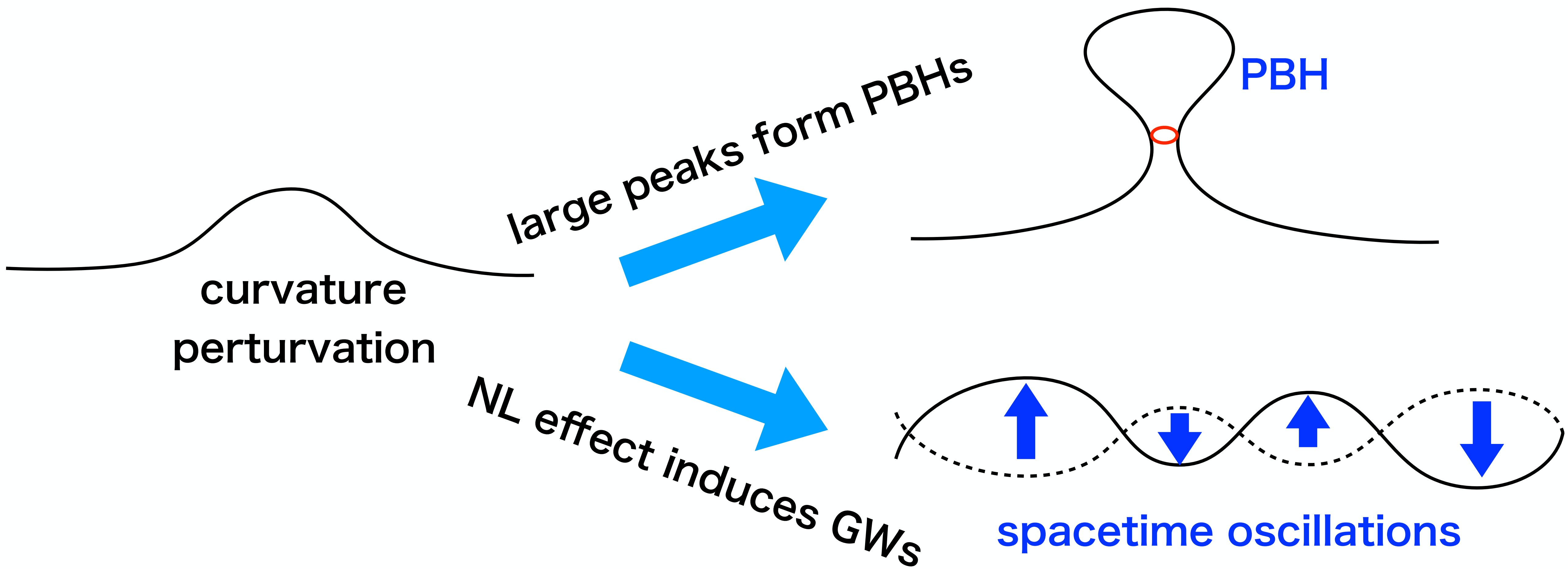
a spike in $f(M)$

⇩

monochromatic PBH mass fcn

GWs from Large Scalar Curvature Perturbation

GWs can capture PBHs!



PBHs = CDM with $M_{PBH} \sim 10^{21}$g
generates GWs with $f \sim 10^{-3}$ Hz

Background GWs
at LISA band

LIGO-Virgo :10 - 1000 Hz

Induced GWs

- The equation of motion for the tensor perturbation with source

$$h''_{\mathbf{k}} + 2\mathscr{H} h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = \mathscr{S}(\mathbf{k}, \eta) \sim \int d^3l \; l_i l_j \Phi_{\mathbf{l}}(\eta) \Phi_{\mathbf{k}-\mathbf{l}}(\eta)$$

- The quantity we want to calculate is

$$\Omega_{\rm GW}(k) \equiv \frac{1}{12}\left(\frac{k}{Ha}\right)^2 \frac{k^3}{\pi^2} \overline{\langle h_{\mathbf{k}}(\eta) h_{\mathbf{k}}(\eta)\rangle}.$$

$$\Rightarrow \quad \boldsymbol{\Omega}_{\rm GW} \sim \langle hh \rangle \sim \langle \mathscr{S}\mathscr{S} \rangle \sim \langle \boldsymbol{\Phi\Phi\Phi\Phi} \rangle \sim \mathscr{P}_{\boldsymbol{\Phi}}^2$$

- Φ may not be Gaussian. So consider a non-Gaussianity:

$$\mathscr{R}(\mathbf{x}) = \mathscr{R}_g(\mathbf{x}) + F_{\rm NL}\left[\mathscr{R}_g^2(\mathbf{x}) - \langle \mathscr{R}_g^2(\mathbf{x}) \rangle\right].$$

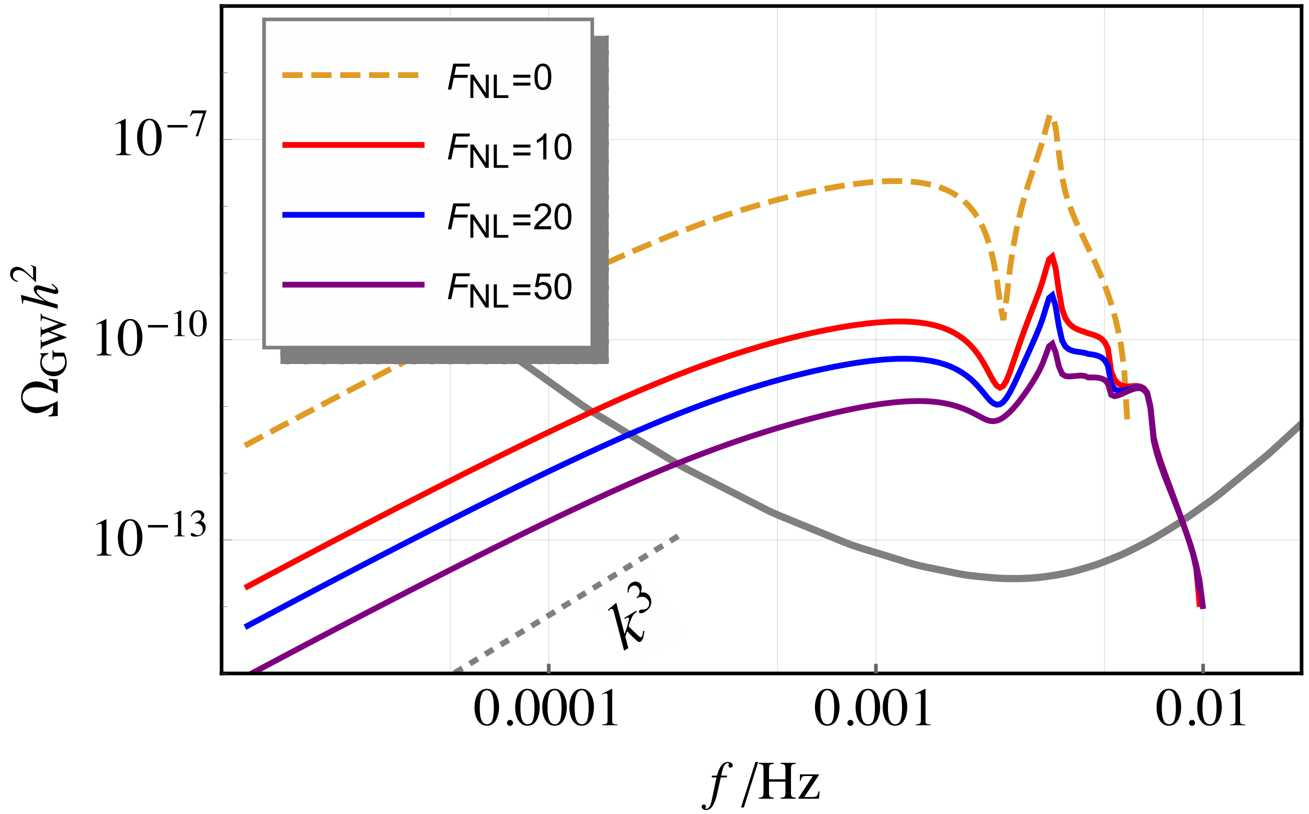
$$\boldsymbol{\Phi} = \frac{2}{3}\mathscr{R}$$

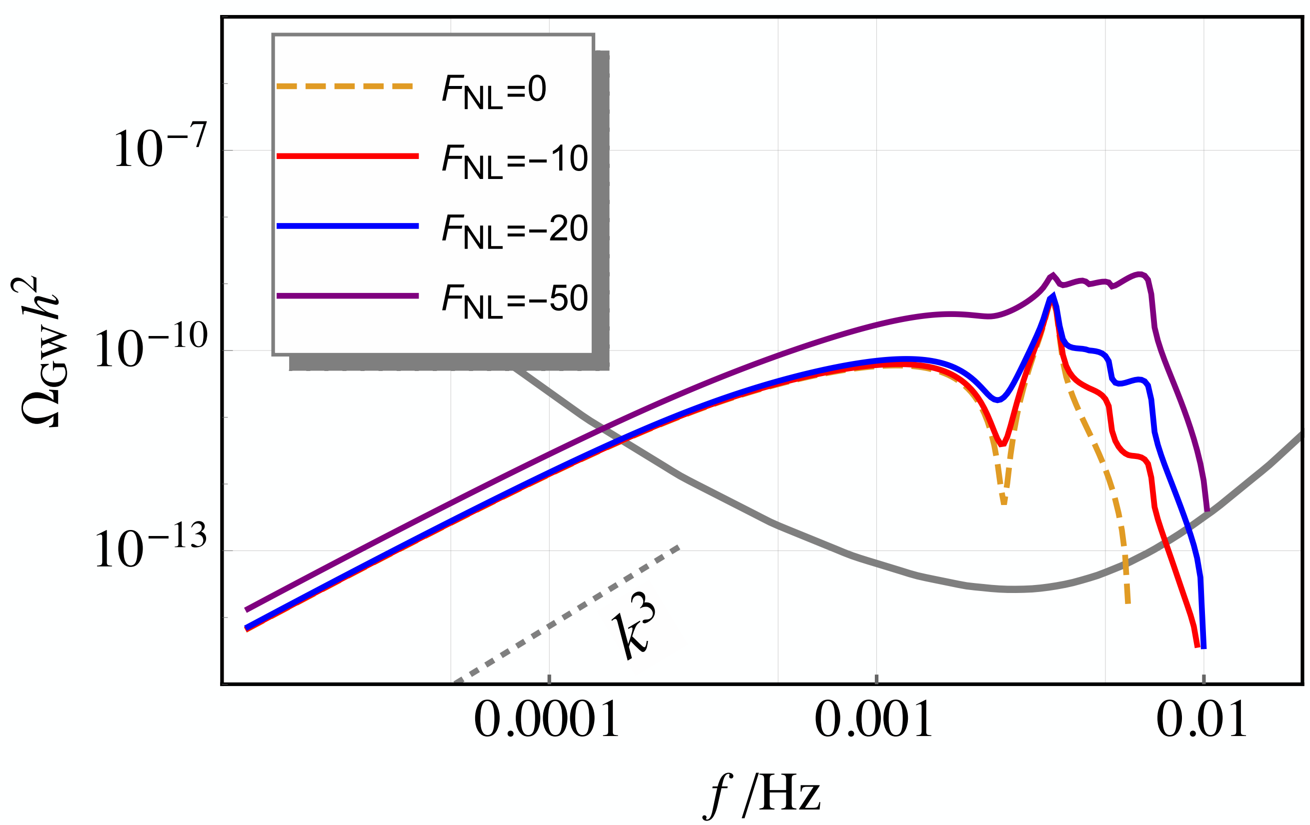
at radiation-dominated stage

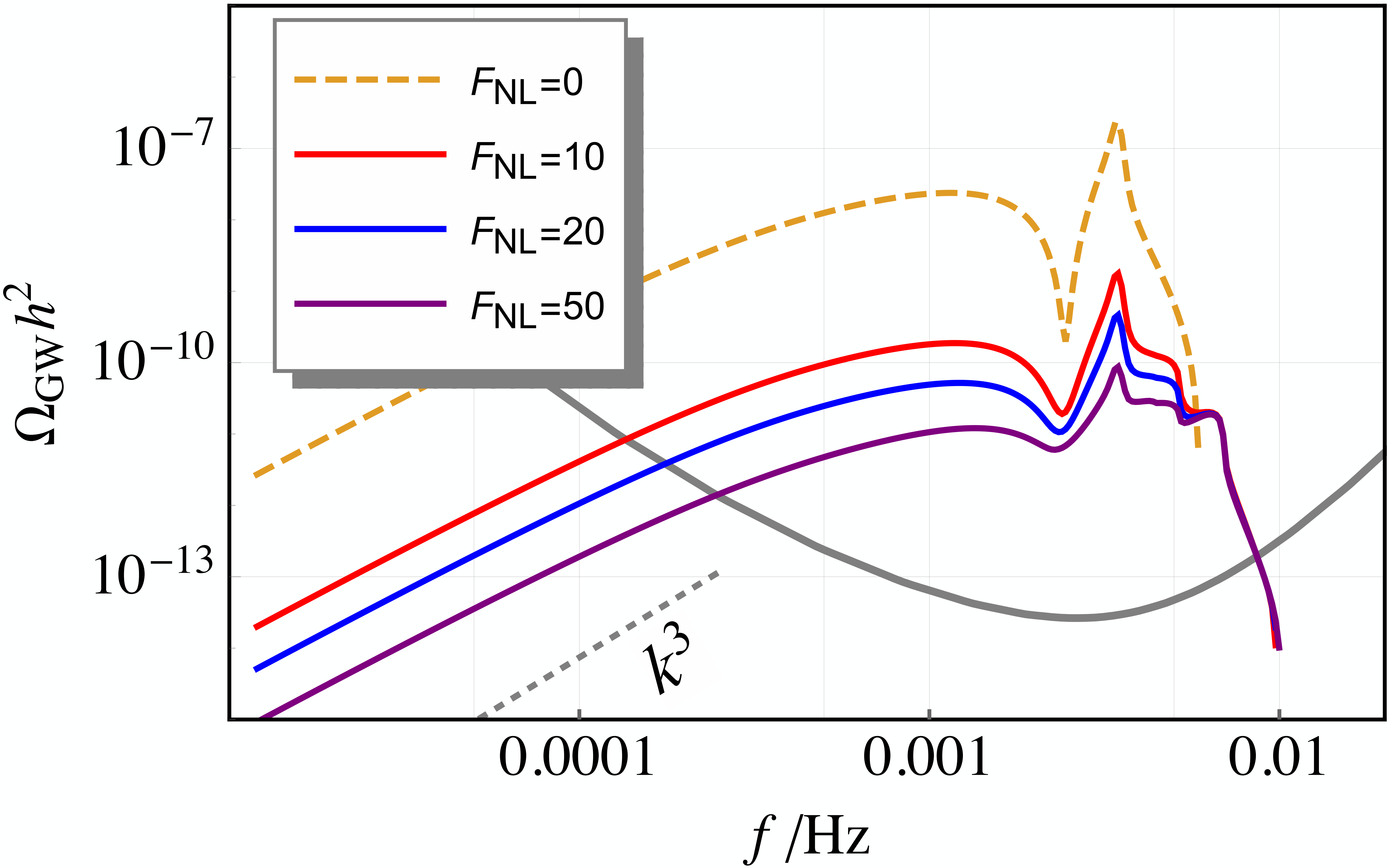
Effects of non-Gaussianity

Cai, Pi & MS, '18

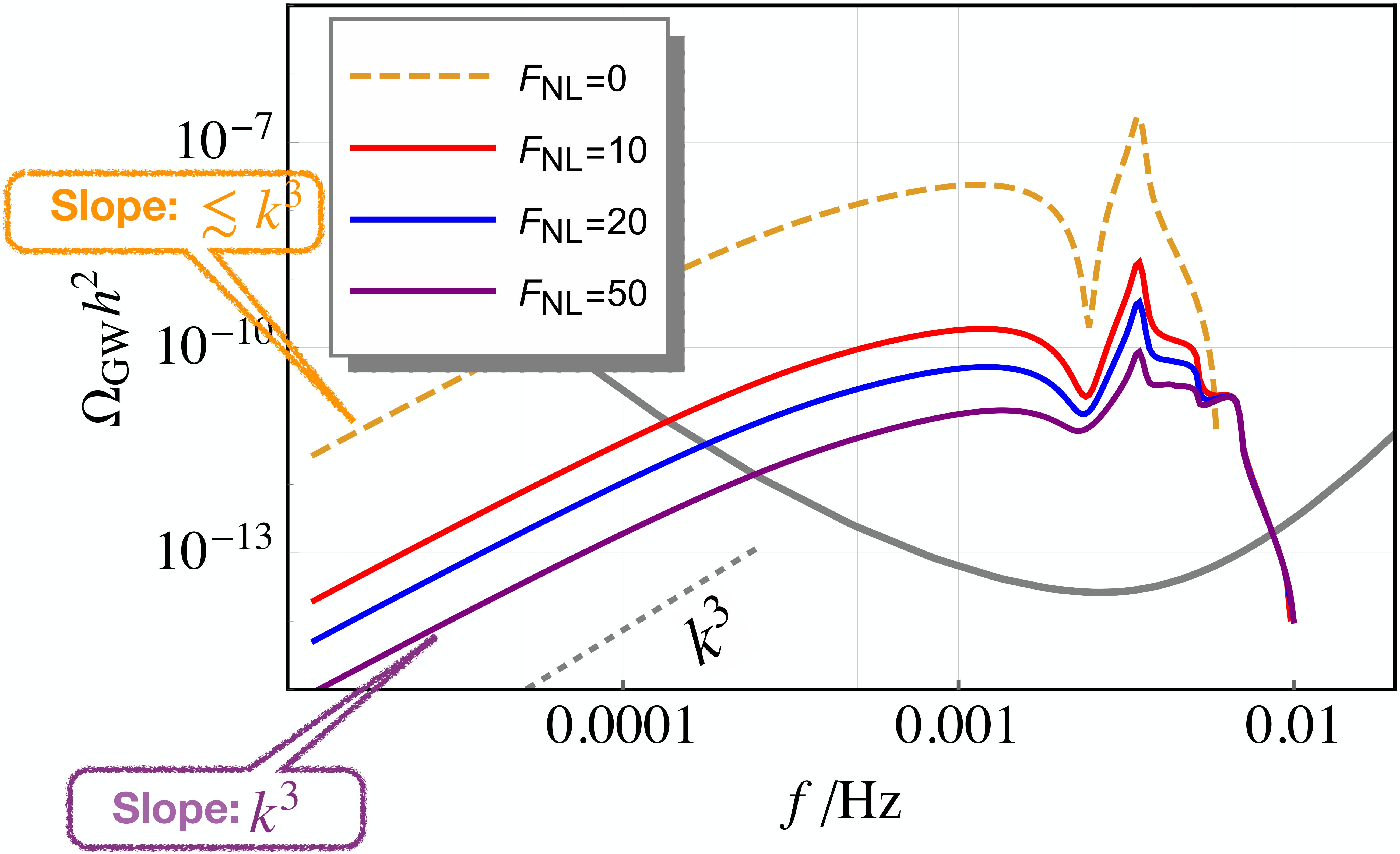
- Up: $F_{NL} > 0$, and we fix the PBH abundance to be 1.
- Down: $F_{NL} < 0$, and we fix the peak amplitude to be $\mathscr{A}_{\mathscr{R}} = 10^{-2}$
- Gray curve: LISA
- Frequency: PBH window <—> LISA band
- Coincidence, but fortunate for our universe.



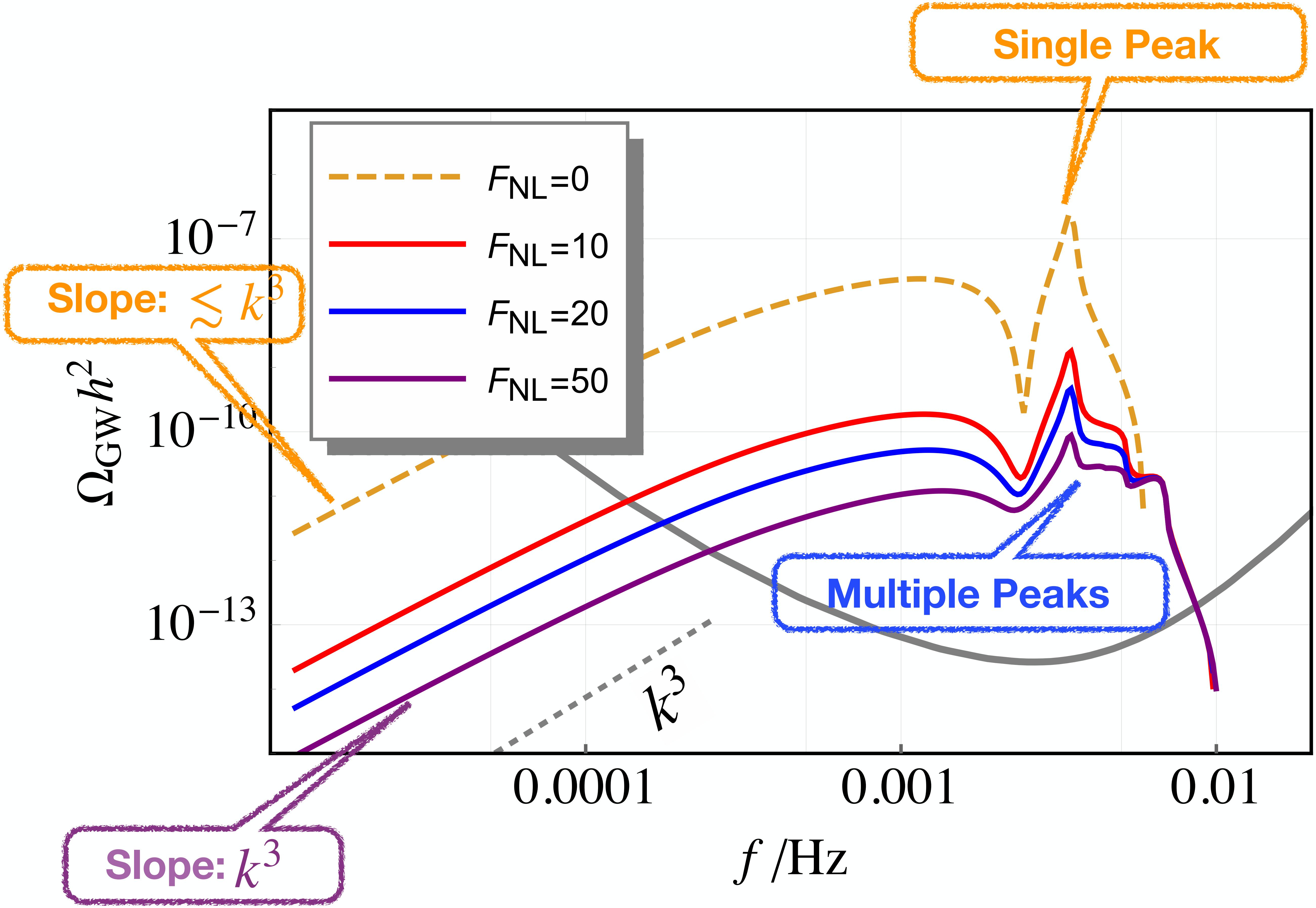




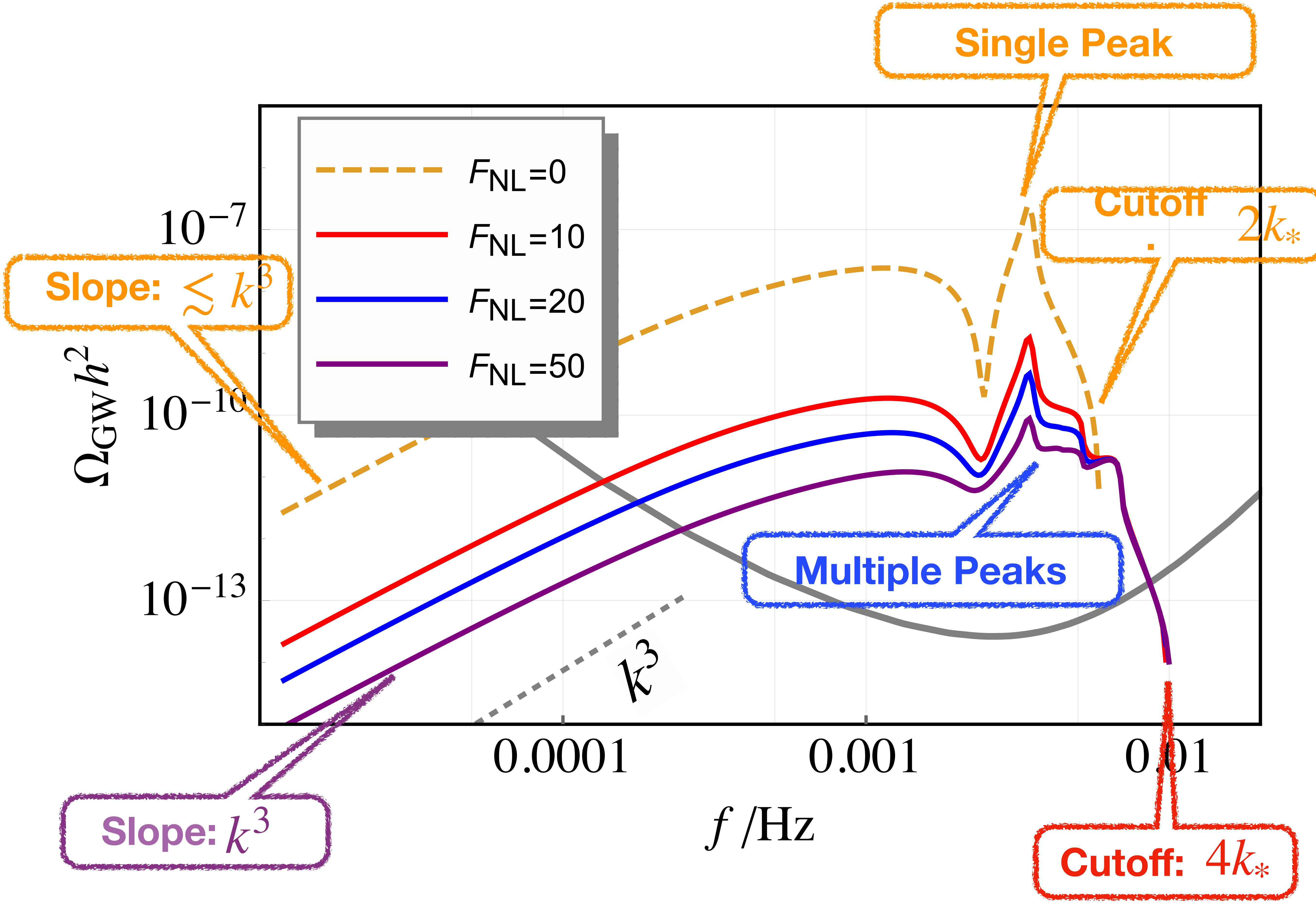
F_{NL}=0
F_{NL}=10
F_{NL}=20
F_{NL}=50
10^{-7}
10^{-10}
10^{-13}
$\Omega_{GW}h^2$
k^3
0.0001
0.001
0.01
f /Hz



F_{NL}=0
F_{NL}=10
F_{NL}=20
F_{NL}=50
Slope: $\lesssim k^3$
Slope: k^3
k^3
$\Omega_{GW}h^2$
10^{-7}
10^{-10}
10^{-13}
0.0001
0.001
0.01
f /Hz



Single Peak
$F_{\rm NL}$=0
$F_{\rm NL}$=10
$F_{\rm NL}$=20
$F_{\rm NL}$=50
10^{-7}
Slope: $\lesssim k^3$
$\Omega_{\rm GW} h^2$
10^{-10}
Multiple Peaks
10^{-13}
k^3
0.0001
0.001
0.01
Slope: k^3
f /Hz



Single Peak
Cutoff
$2k_*$
Slope: $\lesssim k^3$
F_{NL}=0
F_{NL}=10
F_{NL}=20
F_{NL}=50
10^{-7}
10^{-10}
10^{-13}
$\Omega_{GW} h^2$
Multiple Peaks
k^3
0.0001
0.001
0.01
f /Hz
Slope: k^3
Cutoff: $4k_*$

Summary

- 2-field inflation models can provide PBH-as-CDM scenario.

$N_1 \sim 35 - 40$ after CMB scale left the horizon

$\Leftrightarrow M_{\rm PBH} \sim 10^{19} - 10^{22}{\rm g}$

- GWs are generated from large scalar perturbations:

k^3 - slope, multiple peaks, cutoff

- If PBHs = CDM, induced GWs must be detectable by LISA, indep of non-Gaussianity $f_{\rm NL}$.

- Conversely if LISA doesn't detect the induced GWs, it constrains the PBH abundances on mass range $M_{\rm PBH} \sim 10^{19}$ -10^{22}g where no other experiment can explore.