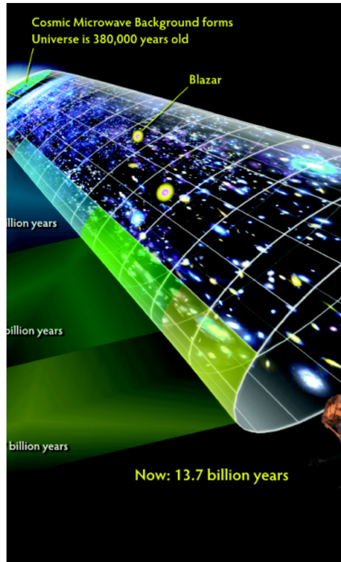
Institute of Cosmos Sciences

EXCELENCIA MARÍA DE MAEZTU



Albert Escrivà Mañas

Based on:
Albert Escrivà, Cristiano Germani and Ravi K. Sheth.
ArXiv:1907.13311

Universal threshold for primordial black hole formation

Institute of Cosmos Sciences

Universitat de Barcelona

15th Rencontres du Vietnam, Cosmology

International Centre for Interdisciplinary Science Education (ICISE)

August 14, 2019

Quy Nhon (Vietnam)

UNIVERSITAT DE BARCELONA

Motivation

- The existence of dark matter can be explained through compact objects like Black Holes that don't require new physics (!Controversy!)

- Primordial Black holes can be created after inflation through a variety of formation mechanisms, without requiring a stellar origin and still agreeing with constraints from the power spectrum

- Important parameter for estimation of PBH abundances, the threshold δ_c

- There are some analytical approximations, but are not profile dependent and not accurate

$$\delta_c = \frac{1}{3}$$

Jeans length aproximation

$$\delta_c \approx 0.41$$

Harada et. al.

→ **Only can be got Numerically with accuracy!**

(But until now!!)

Misner-Sharp equations

- The Misner-Sharp equations describes the motion of a relativistic fluid under a curved spacetime

- We consider a perfect fluid with an equation of state $p = \omega \rho$

$$T^{\mu\nu} = (\rho + p) u^{\mu} u^{\nu} + p g^{\mu\nu}$$

- Spherically symmetric spacetime

$$ds^2 = -A^2(r,t)dt^2 + B^2(r,t)dr^2 + R^2(r,t)d\Omega^2$$

- We can define an invariant quantities

$$D_t = u^{\mu}\partial_{\mu} = \frac{1}{A}\partial_t , D_r = v^{\mu}\partial_{\mu} = \frac{1}{B}\partial_r$$

Misner-Sharp equations

$$D_t U = -\frac{\Gamma}{\rho + p} D_r p - \frac{M}{R^2} - 4\pi R p$$

$$D_t \rho = -\frac{\rho + p}{\Gamma R^2} D_r (R^2 U)$$

$$M = \int_0^R 4\pi R^2 \rho \, dR$$

$$D_t R = U$$

$$D_t M = -4\pi R^2 U p$$

Hamiltonian constraint

$$D_r M = 4\pi R^2 \Gamma \rho$$

$$\Gamma^2 = 1 + U^2 - 2\frac{M}{R}$$

$$D_r R = \Gamma$$

$$D_r A = -\frac{A}{\rho + p} D_r p$$

Basics on primordial black hole formation

- PBHs could be formed by sufficiently large cosmological perturbations collapsing after re-entering the cosmological horizon. Assuming spherical symmetry, such regions can be described by the following approximate form of the metric at super-horizon scales

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-K(r)r^2} + r^2 d\Omega^2\right]$$

- The curvature profile K(r) characterize the cosmological perturbation $\frac{\delta\rho}{\rho}$

- The criteria of PBH formation is the compact function

$$\delta(r,t) := \frac{1}{V}\int_0^R 4\pi R^2 \frac{\rho-\rho_b}{\rho_b}\, dR$$

$$C = 2\frac{\delta M}{R}$$

Initial perturbations

$$C(r) = f(\omega)K(r)r^2$$

$$\delta(r,t) = \frac{1}{(aH)^2} f(\omega)K(r) \qquad \delta(r_m, t_H) = f(\omega)K(r_m)r_m^2$$

$$K(r_m) + \frac{r_m}{2}K'(r_m) = 0$$

- We set up the initial conditions for the Misner-Sharp evolution, long wavelength approximation

$$\epsilon(t) = \frac{1}{a(t)H(t)r_k}$$

$$A = 1 + \epsilon^2 \tilde{A}$$
$$\rho = \rho_b(t)(1 + \epsilon^2 \tilde{\rho})$$
$$U = H(t)R(1 + \epsilon^2 \tilde{U})$$
$$M = \frac{4\pi}{3}\rho_b(t)R^3(1 + \epsilon^2 \tilde{M})$$
$$R = a(t)r(1 + \epsilon^2 \tilde{R})$$

ArXiv:1809.02127. Ilia Musco

- We need some boundary conditions as well:

$$U(r=0,t) = 0, R(r=0,t) = 0, M(r=0,t) = 0$$

$$\rho'(r=0,t) = 0, \rho'(r=r_f,t) = 0$$

Numerical procedure

How can we solve this huge numerical problem???

- Until now: Using an hydrodynamic code , based on stellar/neutron collapse. Uses FD, 2 order accuracy in time and space + ADM + unnecessary thermodynamic variables.

- New procedure: using spectral methods

    1) **Computation of the threshold**.

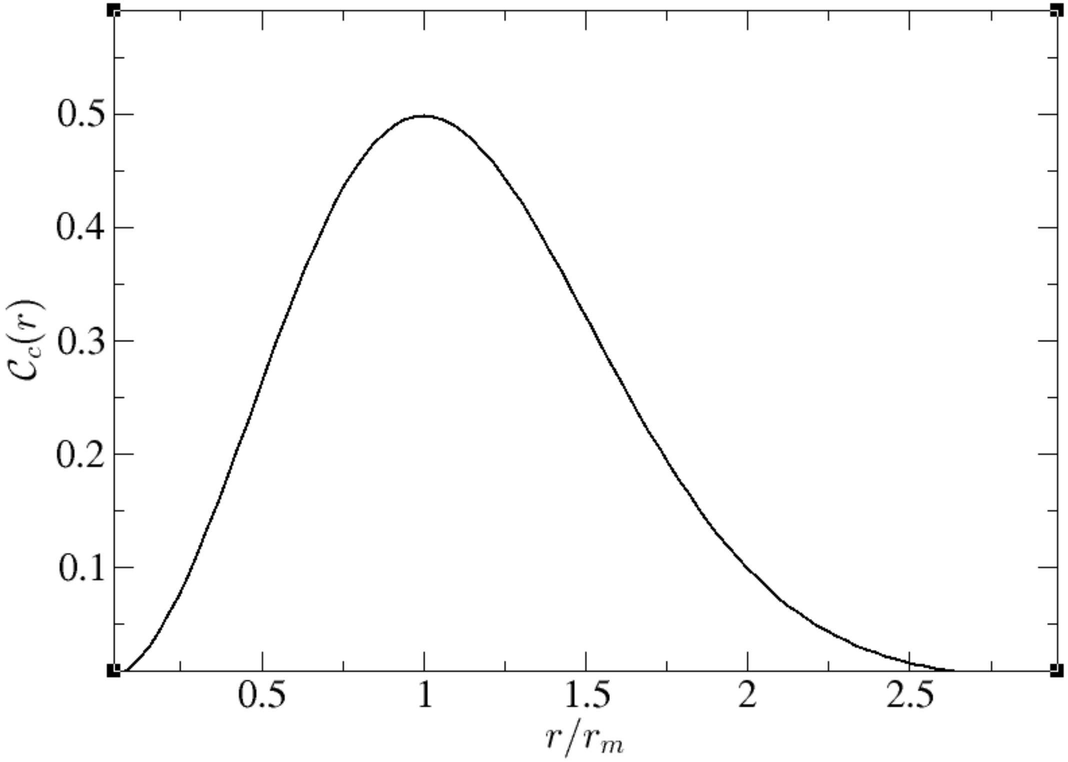
    2) Estimation of the mass of the black hole using an excision technique.

Albert Escrivà . *Simulation of primordial black hole formation using pseudo-spectral methods.* **arXiv:1907.13065**

The code is public avaiable!

Universal threshold

- **Idea!**: The threshold must be depend only on the curvature of the compaction function around its peak.



$$\mathcal{C}_c(r) = \delta_c \left(\frac{r}{r_m}\right)^2 e^{-\left(\frac{r}{r_m}\right)^2+1}$$

Gaussian curvature profile

Universal threshold for primordial black hole formation, Albert Escrivà

Universal threshold

- Use an exponential basis to parametrize the curvature profiles

$$q \equiv -\frac{\mathcal{C}''(r_m) r_m^2}{4\mathcal{C}(r_m)} \qquad K_b(r) = \frac{\mathcal{C}(r_m)}{f(\omega) r_m^2} e^{\frac{1}{q}\left(1-(r/r_m)^{2q}\right)}$$

Universal value!! Average of the compaction function

$$\bar{\mathcal{C}}_c = \frac{3}{r_m^3} \int_{r=0}^{r=r_m} \mathcal{C}_c(r) r^2 dr \qquad \bar{\mathcal{C}}_c = \frac{2}{5}$$

Universal within a deviation of 2% respect the numerical value

Universal threshold

$$K_b(r) = \frac{\mathcal{C}(r_m)}{f(\omega) r_m^2} e^{\frac{1}{q}\left(1-(r/r_m)^q\right)}$$

$$\rightarrow$$

$$\bar{\mathcal{C}}_c = \frac{3}{r_m^3}\int_{r=0}^{r=r_m} \mathcal{C}_c(r) r^2 dr$$

$$q \equiv -\frac{\mathcal{C}''(r_m) r_m^2}{4\mathcal{C}(r_m)}$$

Universal threshold formula

$$\delta_c = \frac{4}{15} e^{-1/q} \frac{q^{1-5/2q}}{\Gamma(5/2q) - \Gamma(5/2q, 1/q)}$$

Universal threshold: Numerical checks

- We test different curvature profiles to test our universal value

$$K_1 = \frac{\mathcal{A}_1}{1+\frac{2}{p-2}\left(\frac{r}{r_m}\right)^p},$$

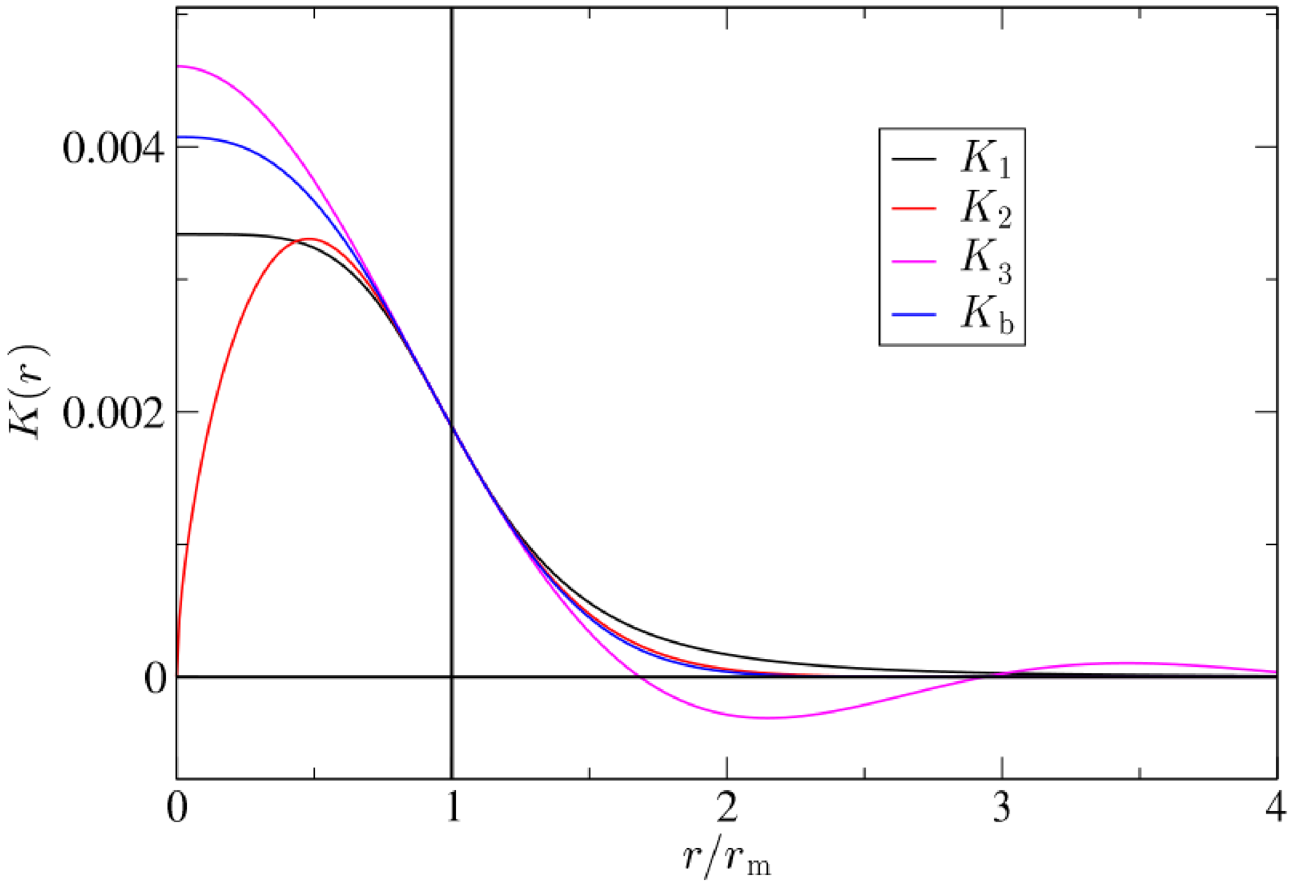
$$K_2 = \mathcal{A}_2 \left[\frac{2(\lambda+1)}{\alpha}\right]^{\lambda/\alpha} \left(\frac{r}{r_m}\right)^{2\lambda} e^{-\frac{(\lambda+1)}{\alpha}\left(\frac{r}{r_m}\right)^{2\alpha}},$$

$$K_3 = \mathcal{A}_3 \frac{3in}{2(k_p r)^3} \left[ik_p r \left\{E_{3+n}(-ik_p r) + E_{3+n}(ik_p r)\right\} + \left\{E_{4+n}(ik_p r) - E_{4+n}(-ik_p r)\right\}\right],$$

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

Universal threshold: Numerical checks

- For instance: same q for all the profiles, giving the same threshold

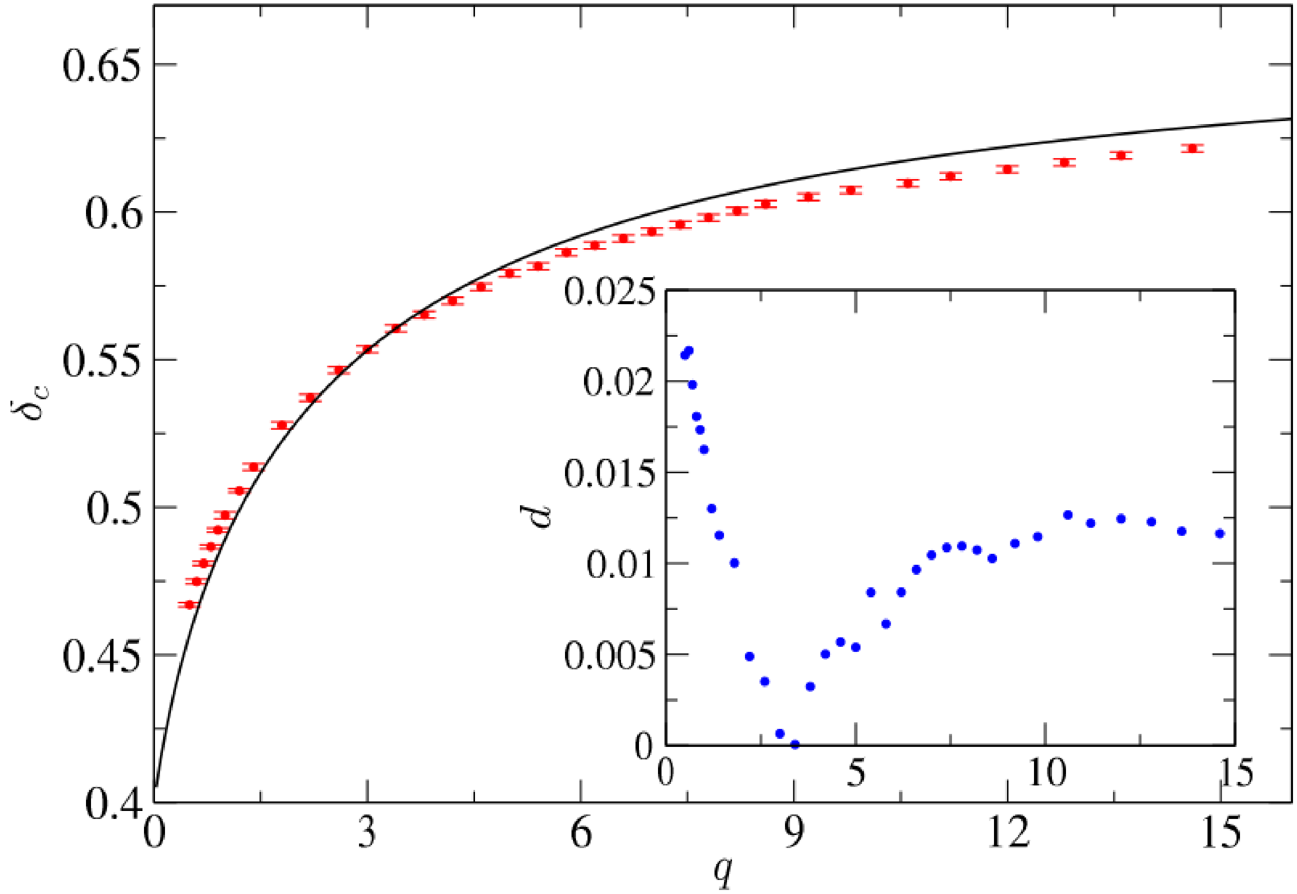


$$q = 1.3$$
$$\delta_c \approx 0.504$$

$$\lambda = 0.3$$
$$p = 4.6$$
$$n \approx 6.67$$

Universal threshold: Numerical checks

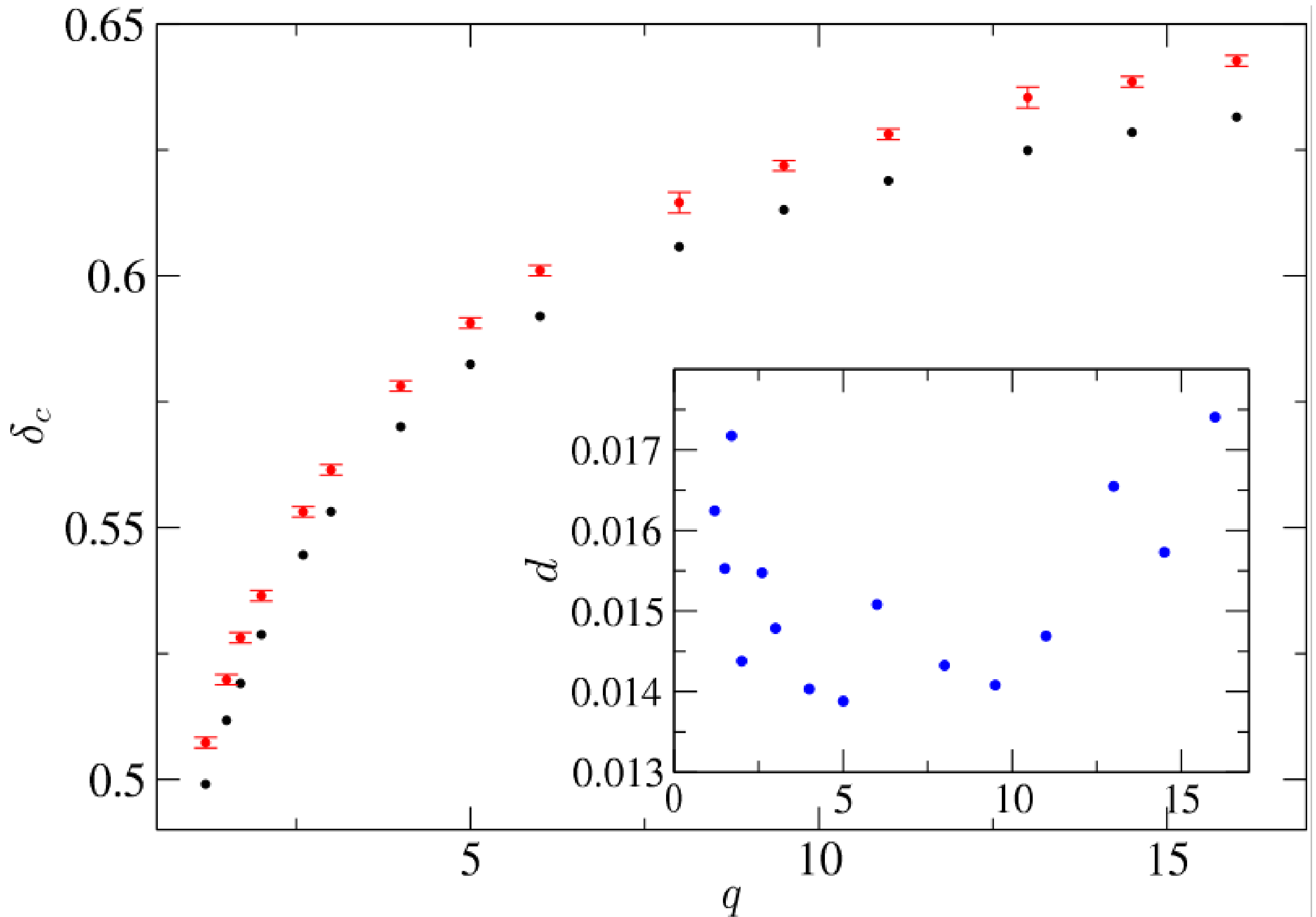
- Case with: K_b Exponential basis profile



ICCUB Institute of Cosmos Sciences
EXCELENCIA MARÍA DE MAEZTU

Universal threshold: Numerical checks

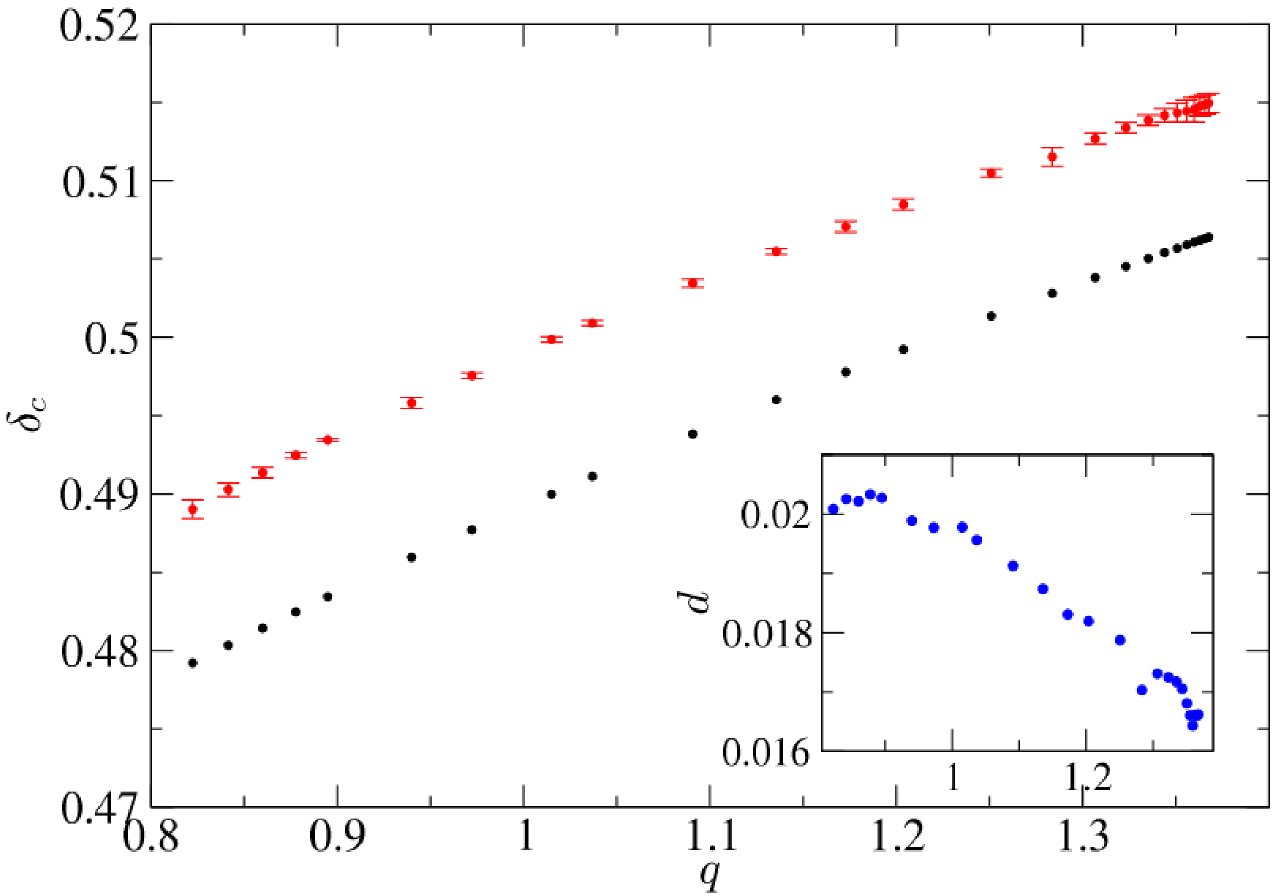
- Case with: $K_2, \quad \alpha = 1$ Non-centraly peaked profile



Universal threshold for primordial black hole formation, Albert Escrivà

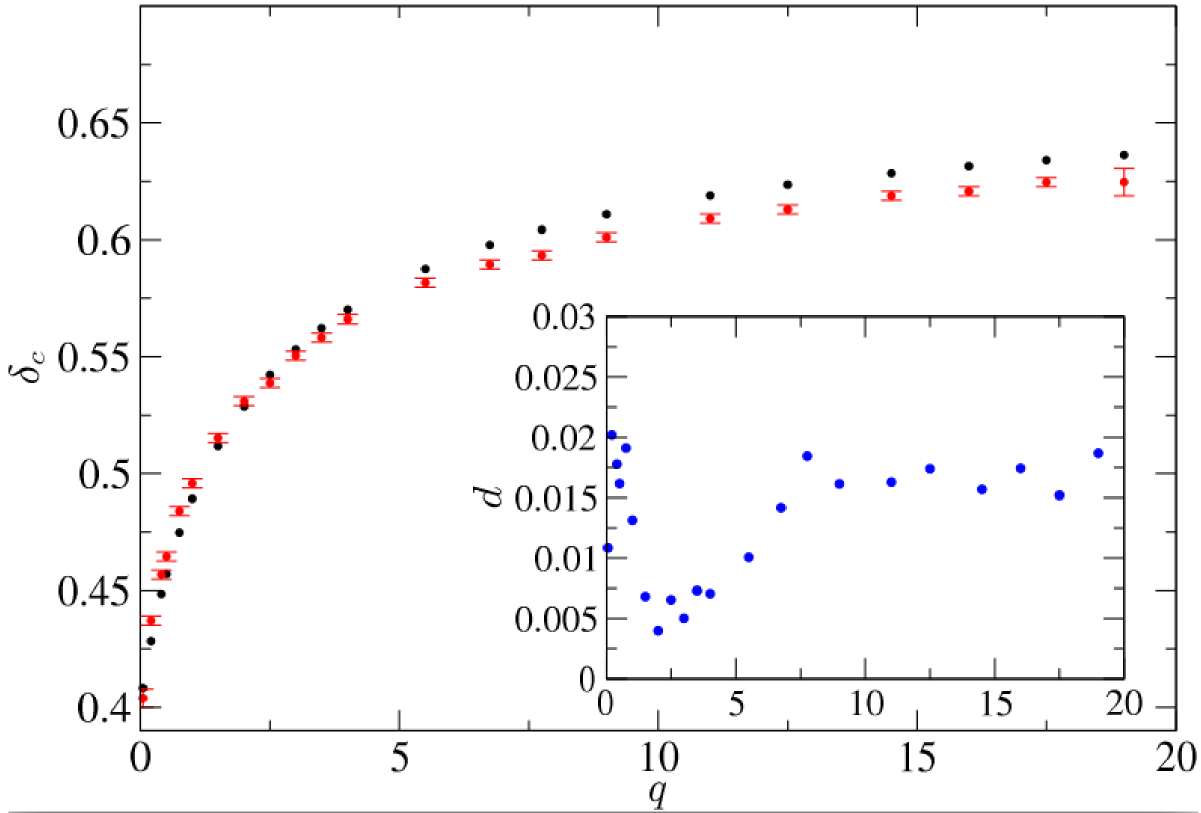
Universal threshold: Numerical checks

- Case with: K_3 Profile coming from the power spectrum



Universal threshold: Numerical checks

- Case with: K_1 Polynomial profile



Universal threshold: extra check

The effect of shape dispersion and non-Gaussianities in primordial black hole formation. ***Vicente Atal, Judith Cid, Albert Escrivà and Jaume Garriga.*** To appear soon in Arxiv.

↑

We have tested the universal threshold with the numerical values,
The deviation is within 2% again

Conclusions

- We have found a universal quantity: the average of the compaction function integrated up to its peak. Is used to get an analytical threshold formula. **We don’t need numerics anymore!!!**.

- The deviation lies within 2%.

Procedure to compute the threshold analytically:

1) From the given profile K(r), compute “q” using

2) Get the threshold introducing “q” in the formula

$$q \equiv -\frac{\mathcal{C}''(r_m) r_m^2}{4\mathcal{C}(r_m)}$$

$$\delta_c = \frac{4}{15} e^{-1/q} \frac{q^{1-5/2q}}{\Gamma(5/2q) - \Gamma(5/2q, 1/q)}$$

THANKS FOR YOUR ATTENTION!

$$\delta_c = \frac{4}{15} e^{-1/q} \frac{q^{1-5/2q}}{\Gamma(5/2q) - \Gamma(5/2q, 1/q)}$$

ICCUB Institute of Cosmos Sciences
EXCELENCIA MARÍA DE MAEZTU