Weyl symmetry for Dark Matter and Dark Energy

Alexander Vikman

12.08.2019

FZU Institute of Physics of the Czech Academy of Sciences
CEICO

EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research, Development and Education
MINISTRY OF EDUCATION, YOUTH AND SPORTS

Simplest Fluid-Like DM*

Simplest Fluid-Like DM*

$$S_{DM}\left[g,\phi,\rho\right]=\frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2-1\right]$$

Brown, Kuchař (1994)

Simplest Fluid-Like DM*

$$S_{DM}\left[g,\phi,\rho\right] = \frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2 - 1\right]$$

Brown, Kuchař (1994)

$$T_{\mu\nu} = \rho\,\phi_{,\mu}\phi_{,\nu}$$

Simplest Fluid-Like DM*

$$S_{DM}\left[g,\phi,\rho\right]=\frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2-1\right]$$

Brown, Kuchař (1994)

$$T_{\mu\nu}=\rho\,\phi_{,\mu}\phi_{,\nu} \qquad\qquad u_{\mu}=\phi_{,\mu}$$

Simplest Fluid-Like DM*

$$S_{DM}\left[g,\phi,\rho\right] = \frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2 - 1\right]$$

Brown, Kuchař (1994)

$$T_{\mu\nu} = \rho\,\phi_{,\mu}\phi_{,\nu} \qquad\qquad u_\mu = \phi_{,\mu}$$

- Scale invariance, as there is no fixed scale in the action

Simplest Fluid-Like DM*

$$S_{DM}\left[g,\phi,\rho\right] = \frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2 - 1\right]$$

Brown, Kuchař (1994)

$$T_{\mu\nu} = \rho\,\phi_{,\mu}\phi_{,\nu} \qquad\qquad u_\mu = \phi_{,\mu}$$

- Scale invariance, as there is no fixed scale in the action
- Global symmetry $\phi \to \phi + c$

Simplest Fluid-Like DM*

$$S_{DM}\left[g,\phi,\rho\right]=\frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2-1\right]$$

Brown, Kuchař (1994)

$$T_{\mu\nu}=\rho\,\phi_{,\mu}\phi_{,\nu} \qquad\qquad u_\mu=\phi_{,\mu}$$

- Scale invariance, as there is no fixed scale in the action
- Global symmetry $\phi\rightarrow\phi+c$
- “Fake” violation of the Lorenz-symmetry, $\langle u^\mu\rangle\neq 0$

Simplest Fluid-Like DM*

$$S_{DM}\left[g,\phi,\rho\right]=\frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2-1\right]$$

Brown, Kuchař (1994)

$$T_{\mu\nu}=\rho\,\phi_{,\mu}\phi_{,\nu} \qquad\qquad u_\mu=\phi_{,\mu}$$

- Scale invariance, as there is no fixed scale in the action
- Global symmetry $\phi\to\phi+c$
- "Fake" violation of the Lorenz-symmetry, $\langle u^\mu\rangle\neq 0$
- DM energy density ρ as a Lagrange multiplier

Fluid-Like DM and Ostrogradsky

Fluid-Like DM and Ostrogradsky

$$S_{DM}\left[g,\phi,\rho\right] = \frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2 - 1\right]$$

Fluid-Like DM and Ostrogradsky

$$S_{DM}\left[g,\phi,\rho\right]=\frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2-1\right]$$

- Similarly to the Ostrogradsky Hamiltonian for systems with higher derivatives, the system is *linear* in canonical momentum π, i.e. in the energy density of DM, ρ

Fluid-Like DM and Ostrogradsky

$$S_{DM}\left[g,\phi,\rho\right]=\frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2-1\right]$$

- Similarly to the Ostrogradsky Hamiltonian for systems with higher derivatives, the system is *linear* in canonical momentum π, i.e. in the energy density of DM, ρ

$$\dot{\phi}=u^{\mu}\partial_{\mu}\phi=\left(\partial\phi\right)^2$$

Fluid-Like DM and Ostrogradsky

$$S_{DM}\left[g,\phi,\rho\right] = \frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2 - 1\right]$$

- Similarly to the Ostrogradsky Hamiltonian for systems with higher derivatives, the system is *linear* in canonical momentum π, i.e. in the energy density of DM, ρ

$$\dot{\phi} = u^{\mu}\partial_{\mu}\phi = \left(\partial\phi\right)^2 \qquad \pi = \frac{\delta L}{\delta\dot{\phi}} = \frac{\sqrt{-g}}{2}\rho$$

Fluid-Like DM and Ostrogradsky

$$S_{DM}\left[g,\phi,\rho\right] = \frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2 - 1\right]$$

- Similarly to the Ostrogradsky Hamiltonian for systems with higher derivatives, the system is *linear* in canonical momentum π, i.e. in the energy density of DM, ρ

$$\dot{\phi} = u^\mu\partial_\mu\phi = \left(\partial\phi\right)^2 \qquad\qquad \pi = \frac{\delta L}{\delta\dot{\phi}} = \frac{\sqrt{-g}}{2}\rho$$

This is very sketchy, for a detailed analysis see e.g. Ganz, Karmakar, Matarrese, Sorokin (2018)

Fluid-Like DM and Ostrogradsky

$$S_{DM}\left[g,\phi,\rho\right] = \frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2 - 1\right]$$

- Similarly to the Ostrogradsky Hamiltonian for systems with higher derivatives, the system is *linear* in canonical momentum π, i.e. in the energy density of DM, ρ

$$\dot{\phi} = u^\mu\partial_\mu\phi = \left(\partial\phi\right)^2 \qquad \pi = \frac{\delta L}{\delta\dot{\phi}} = \frac{\sqrt{-g}}{2}\rho$$

This is very sketchy, for a detailed analysis see e.g. Ganz, Karmakar, Matarrese, Sorokin (2018)

- However, this canonical momentum cannot change sign!

Fluid-Like DM and Ostrogradsky

$$S_{DM}\left[g,\phi,\rho\right] = \frac{1}{2}\int d^4x\sqrt{-g}\,\rho\left[\left(\partial\phi\right)^2 - 1\right]$$

- Similarly to the Ostrogradsky Hamiltonian for systems with higher derivatives, the system is *linear* in canonical momentum π, i.e. in the energy density of DM, ρ

$$\dot{\phi} = u^\mu\partial_\mu\phi = \left(\partial\phi\right)^2 \qquad\qquad \pi = \frac{\delta L}{\delta\dot{\phi}} = \frac{\sqrt{-g}}{2}\rho$$

This is very sketchy, for a detailed analysis see e.g. Ganz, Karmakar, Matarrese, Sorokin (2018)

- However, this canonical momentum cannot change sign!

Indeed $\dot{\rho} = -\theta\rho$ and $\ddot{\rho} = -\dot{\theta}\rho - \theta\dot{\rho}$ etc

here $\theta = \nabla_\mu u^\mu$

- Can one promote the global scale-invariance to the local Weyl-invariance?

$$g_{\mu\nu} \rightarrow \Omega^2(x)\, g_{\mu\nu}$$

- Can one promote the global scale-invariance to the local Weyl-invariance?

$$g_{\mu\nu} \rightarrow \Omega^2(x)\, g_{\mu\nu}$$

- Is there a higher-derivative scalar-tensor theory for this fluid-like dust?

- Can one promote the global scale-invariance to the local Weyl-invariance?

$$g_{\mu\nu} \to \Omega^2(x)\, g_{\mu\nu}$$

- Is there a higher-derivative scalar-tensor theory for this fluid-like dust?

- New highly symmetric theories with useful physical content and quantum properties potentially different from "fluid dust"

Mimetic Matter

Chamseddine, Mukhanov (2013)

Mimetic Matter

Chamseddine, Mukhanov (2013)

- One can *encode* the conformal part of the *physical* metric in a scalar field:

Mimetic Matter

Chamseddine, Mukhanov (2013)

- One can *encode* the conformal part of the *physical* metric in a scalar field:

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} \cdot \left(\tilde{g}^{\alpha\beta} \partial_\alpha \phi \, \partial_\beta \phi \right)$$

Mimetic Matter

Chamseddine, Mukhanov (2013)

- One can *encode* the conformal part of the *physical* metric in a scalar field:

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} \cdot \left(\tilde{g}^{\alpha\beta} \partial_\alpha \phi \, \partial_\beta \phi \right)$$

physical metric of free fall

Mimetic Matter

Chamseddine, Mukhanov (2013)

- One can *encode* the conformal part of the *physical* metric in a scalar field:

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} \cdot \left(\tilde{g}^{\alpha\beta} \partial_\alpha \phi \, \partial_\beta \phi \right)$$

physical metric of free fall *auxiliary metric, dynamical variable*

Mimetic Matter

Chamseddine, Mukhanov (2013)

- One can *encode* the conformal part of the *physical* metric in a scalar field:

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} \cdot \left(\tilde{g}^{\alpha\beta} \partial_\alpha \phi \, \partial_\beta \phi \right)$$

physical metric of free fall *auxiliary metric, dynamical variable*

$$S\left[\tilde{g}_{\mu\nu}, \phi, \Phi_m\right] = \int \mathrm{d}^4 x \left[\sqrt{-g} \left(-\frac{1}{2} R\left(g\right) + \boxed{\mathscr{L}\left(g, \Phi_m\right)} \right) \right]_{g_{\mu\nu} = g_{\mu\nu}(\tilde{g}, \phi)}$$

"matter"

Mimetic Matter

Chamseddine, Mukhanov (2013)

Mimetic Matter

Chamseddine, Mukhanov (2013)

- $$g_{\mu\nu} = \tilde{g}_{\mu\nu} \left(\tilde{g}^{\alpha\beta}\, \partial_\alpha \phi\, \partial_\beta \phi\right)$$

physical metric of free fall *auxiliary metric, dynamical variable*

Mimetic Matter

Chamseddine, Mukhanov (2013)

- $$g_{\mu\nu} = \tilde{g}_{\mu\nu} \left(\tilde{g}^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi\right)$$

physical metric of free fall *auxiliary metric, dynamical variable*

- The new theory becomes invariant with respect to Weyl transformations:

$$\tilde{g}_{\mu\nu} \to \Omega^2\,(x)\,\tilde{g}_{\mu\nu}$$

Mimetic Matter

Chamseddine, Mukhanov (2013)

- $$g_{\mu\nu} = \tilde{g}_{\mu\nu} \left(\tilde{g}^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi\right)$$

physical metric of free fall *auxiliary metric, dynamical variable*

- The new theory becomes invariant with respect to Weyl transformations:

$$\tilde{g}_{\mu\nu} \rightarrow \Omega^2(x)\,\tilde{g}_{\mu\nu}$$

- The scalar field obeys the relativistic Hamilton-Jacobi equation:

$$g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi = 1$$

Mimetic Matter

Chamseddine, Mukhanov (2013)

- $$g_{\mu\nu} = \tilde{g}_{\mu\nu} \left(\tilde{g}^{\alpha\beta}\, \partial_\alpha \phi\, \partial_\beta \phi\right)$$

physical metric of free fall *auxiliary metric, dynamical variable*

- The new theory becomes invariant with respect to Weyl transformations:

$$\tilde{g}_{\mu\nu} \rightarrow \Omega^2 (x)\, \tilde{g}_{\mu\nu}$$

- The scalar field obeys the relativistic Hamilton-Jacobi equation:

$$g^{\mu\nu}\, \partial_\mu \phi\, \partial_\nu \phi = 1$$

$$g^{\mu\nu} = \tilde{g}^{\mu\nu} \left(\tilde{g}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi\right)^{-1}$$

Lagrange Multiplier and Weyl-Invariance

Hammer, Vikman (2015)

Lagrange Multiplier and Weyl-Invariance

Hammer, Vikman (2015)

$$S_0\left[\tilde{g},\varphi,X,\lambda\right] = -\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right)+\frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{\beta}}{X}+\lambda\left(X-\frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\right]$$

Lagrange Multiplier and Weyl-Invariance

Hammer, Vikman (2015)

$$S_0\left[\tilde{g},\varphi,X,\lambda\right]=-\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right)+\frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{\beta}}{X}+\lambda\left(X-\frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\right]$$

Weyl-invariance:

$$\tilde{g}_{\mu\nu}\rightarrow\Omega^2\left(x\right)\tilde{g}_{\mu\nu}\,,$$

$$\phi\rightarrow\phi\,,$$

$$X\rightarrow\Omega^{-2}\left(x\right)X\,,$$

$$\lambda\rightarrow\Omega^{-2}\left(x\right)\lambda\,,$$

Lagrange Multiplier and Weyl-Invariance

Hammer, Vikman (2015)

$$S_0\left[\tilde{g},\varphi,X,\lambda\right]=-\int\mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right)+\frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{\beta}}{X}+\lambda\left(X-\frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\right]$$

Weyl-invariance:

$$\tilde{g}_{\mu\nu}\rightarrow\Omega^2\left(x\right)\tilde{g}_{\mu\nu}\,,$$

$$\phi\rightarrow\phi\,,$$

$$X\rightarrow\Omega^{-2}\left(x\right)X\,,$$

$$\lambda\rightarrow\Omega^{-2}\left(x\right)\lambda\,,$$

gauge invariant variables **(A)**

$$\tilde{g}_{\mu\nu}=\left(2X\right)^{-1}g_{\mu\nu}\,,$$

$$\varphi=\varphi\,,$$

$$X=X\,,$$

$$\lambda=2X\,\rho\,,$$

Lagrange Multiplier and Weyl-Invariance

Hammer, Vikman (2015)

$$S_0\left[\tilde{g},\varphi,X,\lambda\right] = -\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right) + \frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{\beta}}{X} + \lambda\left(X - \frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\right]$$

Weyl-invariance:

$$\tilde{g}_{\mu\nu} \rightarrow \Omega^2\left(x\right)\tilde{g}_{\mu\nu}\,,$$
$$\phi \rightarrow \phi\,,$$
$$X \rightarrow \Omega^{-2}\left(x\right)X\,,$$
$$\lambda \rightarrow \Omega^{-2}\left(x\right)\lambda\,,$$

gauge invariant variables **(A)**

$$\tilde{g}_{\mu\nu} = \left(2X\right)^{-1}g_{\mu\nu}\,,$$
$$\varphi = \varphi\,,$$
$$X = X\,,$$
$$\lambda = 2X\,\rho\,,$$

$$S_0\left[g,\varphi,\rho\right] = \int \mathrm{d}^4x\,\sqrt{-g}\left[-\frac{1}{2}R\left(g\right) + \frac{\rho}{2}\left(g^{\alpha\beta}\varphi_{,\alpha}\varphi_{,\beta} - 1\right)\right]\,.$$

Lagrange Multiplier and Weyl-Invariance

Hammer, Vikman (2015)

$$S_0\left[\tilde{g},\varphi,X,\lambda\right]=-\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right)+\frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{\beta}}{X}+\lambda\left(X-\frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\right]$$

Weyl-invariance:

$$\tilde{g}_{\mu\nu}\rightarrow\Omega^2\left(x\right)\tilde{g}_{\mu\nu}\,,$$
$$\phi\rightarrow\phi\,,$$
$$X\rightarrow\Omega^{-2}\left(x\right)X\,,$$
$$\lambda\rightarrow\Omega^{-2}\left(x\right)\lambda\,,$$

gauge invariant variables **(A)**

$$\tilde{g}_{\mu\nu}=\left(2X\right)^{-1}g_{\mu\nu}\,,$$
$$\varphi=\varphi\,,$$
$$X=X\,,$$
$$\lambda=2X\,\rho\,,$$

$$S_0\left[g,\varphi,\rho\right]=\int \mathrm{d}^4x\,\sqrt{-g}\left[-\frac{1}{2}R\left(g\right)+\frac{\rho}{2}\left(g^{\alpha\beta}\varphi_{,\alpha}\varphi_{,\beta}-1\right)\right].$$

cf Golovnev (2013)

“Canonical” Normalisation

$$S_0\left[\tilde{g},\varphi,X,\lambda\right] = -\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right) + \frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{\beta}}{X} + \lambda\left(X - \frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\right]$$

“Canonical” Normalisation

$$S_0\left[\tilde{g}, \varphi, X, \lambda\right] = -\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right) + \frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{\beta}}{X} + \lambda\left(X - \frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\right]$$

new scalar field of usual conformal weight one $X = \dfrac{\chi^2}{12}$

“Canonical” Normalisation

$$S_0\left[\tilde{g},\varphi,X,\lambda\right]=-\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right)+\frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{\beta}}{X}+\lambda\left(X-\frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\right]$$

new scalar field of usual conformal weight one $\quad X=\dfrac{\chi^2}{12}$

$$\tilde{g}_{\mu\nu}\rightarrow\Omega^2(x)\,\tilde{g}_{\mu\nu}\qquad\qquad \chi\rightarrow\Omega^{-1}(x)\,\chi$$

“Canonical” Normalisation

$$S_0\left[\tilde{g},\varphi,X,\lambda\right]=-\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right)+\frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{\beta}}{X}+\lambda\left(X-\frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\right]$$

new scalar field of usual conformal weight one $\quad X=\dfrac{\chi^2}{12}$

$$\tilde{g}_{\mu\nu}\rightarrow\Omega^2\left(x\right)\tilde{g}_{\mu\nu}\qquad\qquad \chi\rightarrow\Omega^{-1}\left(x\right)\chi$$

$$S\left[\tilde{g},\chi,\lambda,\phi\right]=\int d^4x\sqrt{-\tilde{g}}\left(-\frac{1}{2}\left(\partial\chi\right)^2-\frac{1}{12}\chi^2R-\frac{\lambda}{12}\chi^2+\frac{1}{2}\lambda\left(\partial\phi\right)^2\right)$$

“Canonical” Normalisation

$$S_0\left[\tilde{g},\varphi,X,\lambda\right] = -\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right) + \frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{\beta}}{X} + \lambda\left(X - \frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\right]$$

new scalar field of usual conformal weight one $X = \dfrac{\chi^2}{12}$

$$\tilde{g}_{\mu\nu} \to \Omega^2\left(x\right)\tilde{g}_{\mu\nu} \qquad \chi \to \Omega^{-1}\left(x\right)\chi$$

$$S\left[\tilde{g},\chi,\lambda,\phi\right] = \int d^4x\sqrt{-\tilde{g}}\left(\boxed{-}\frac{1}{2}\left(\partial\chi\right)^2 - \frac{1}{12}\chi^2 R - \frac{\lambda}{12}\chi^2 + \frac{1}{2}\lambda\left(\partial\phi\right)^2\right)$$

wrong sign

“Canonical” Normalisation

$$S_0\left[\tilde{g},\varphi,X,\lambda\right] = -\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right) + \frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{\beta}}{X} + \lambda\left(X - \frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\right]$$

new scalar field of usual conformal weight one $\quad X = \dfrac{\chi^2}{12}$

$$\tilde{g}_{\mu\nu} \to \Omega^2(x)\,\tilde{g}_{\mu\nu} \qquad\qquad \chi \to \Omega^{-1}(x)\,\chi$$

$$S\left[\tilde{g},\chi,\lambda,\phi\right] = \int d^4x\sqrt{-\tilde{g}}\left(\boxed{-}\frac{1}{2}\left(\partial\chi\right)^2 - \frac{1}{12}\chi^2 R - \frac{\boxed{\lambda}}{12}\chi^2 + \frac{1}{2}\boxed{\lambda}\left(\partial\phi\right)^2\right)$$

wrong sign $\qquad\qquad\qquad\qquad$ Lagrange multiplier

Two faces of dust

Two faces of dust

$$S\left[\tilde{g},\chi,\lambda,\phi\right]=\int d^4x\sqrt{-\tilde{g}}\left(-\frac{1}{2}\left(\partial\chi\right)^2-\frac{1}{12}\chi^2R-\frac{\lambda}{12}\chi^2+\frac{1}{2}\lambda\left(\partial\phi\right)^2\right)$$

Two faces of dust

$$S\left[\tilde{g},\chi,\lambda,\phi\right] = \int d^4x\sqrt{-\tilde{g}}\left(-\frac{1}{2}\left(\partial\chi\right)^2 - \frac{1}{12}\chi^2 R - \frac{\lambda}{12}\chi^2 + \frac{1}{2}\lambda\left(\partial\phi\right)^2\right)$$

gauge invariant variables **(B) :** $\tilde{g}_{\mu\nu} = \lambda^{-1}h_{\mu\nu} \qquad \chi = \lambda^{1/2}\theta$

Two faces of dust

$$S\left[\tilde{g},\chi,\lambda,\phi\right]=\int d^4x\sqrt{-\tilde{g}}\left(-\frac{1}{2}\left(\partial\chi\right)^2-\frac{1}{12}\chi^2R-\frac{\lambda}{12}\chi^2+\frac{1}{2}\lambda\left(\partial\phi\right)^2\right)$$

gauge invariant variables **(B) :** $\tilde{g}_{\mu\nu}=\lambda^{-1}h_{\mu\nu} \qquad \chi=\lambda^{1/2}\theta$

$$S^{\mathbf{B}}_{dust}\left[h,\theta,\phi\right]=\int d^4x\sqrt{-h}\left(-\frac{1}{2}(\partial\theta)^2-\frac{1}{12}\theta^2R-\frac{1}{12}\theta^2+\frac{1}{2}\left(\partial\phi\right)^2\right)$$

Two faces of dust

$$S\left[\tilde{g},\chi,\lambda,\phi\right] = \int d^4x\sqrt{-\tilde{g}}\left(-\frac{1}{2}\left(\partial\chi\right)^2 - \frac{1}{12}\chi^2 R - \frac{\lambda}{12}\chi^2 + \frac{1}{2}\lambda\left(\partial\phi\right)^2\right)$$

gauge invariant variables **(B) :** $\tilde{g}_{\mu\nu} = \lambda^{-1}h_{\mu\nu}$ $\quad \chi = \lambda^{1/2}\theta$

$$S^{\mathbf{B}}_{dust}\left[h,\theta,\phi\right] = \int d^4x\sqrt{-h}\left(-\frac{1}{2}(\partial\theta)^2 - \frac{1}{12}\theta^2 R - \frac{1}{12}\theta^2 + \frac{1}{2}\left(\partial\phi\right)^2\right)$$

$$\Updownarrow$$

$$S^{\mathbf{A}}_{dust}\left[h,\rho,\phi\right] = \int d^4x\sqrt{-h}\left[-\frac{1}{2}R + \frac{\rho}{2}\left(\left(\partial\phi\right)^2 - 1\right)\right]$$

Two faces of dust

$$S\left[\tilde{g},\chi,\lambda,\phi\right]=\int d^4x\sqrt{-\tilde{g}}\left(-\frac{1}{2}\left(\partial\chi\right)^2-\frac{1}{12}\chi^2R-\frac{\lambda}{12}\chi^2+\frac{1}{2}\lambda\left(\partial\phi\right)^2\right)$$

gauge invariant variables **(B) :** $\tilde{g}_{\mu\nu}=\lambda^{-1}h_{\mu\nu}$ $\quad$ $\chi=\lambda^{1/2}\theta$

$$S^{\mathbf{B}}_{dust}\left[h,\theta,\phi\right]=\int d^4x\sqrt{-h}\left(-\frac{1}{2}(\partial\theta)^2-\frac{1}{12}\theta^2R-\frac{1}{12}\theta^2+\frac{1}{2}\left(\partial\phi\right)^2\right)$$

Other Matter couples $\quad g_{\mu\nu}=\frac{\theta^2}{6}\cdot h_{\mu\nu}$

$$S^{\mathbf{A}}_{dust}\left[h,\rho,\phi\right]=\int d^4x\sqrt{-h}\left[-\frac{1}{2}R+\frac{\rho}{2}\left(\left(\partial\phi\right)^2-1\right)\right]$$

Calculating vacuum energy we are in trouble…

$$T_{\mu\nu}^{vac} = \Lambda g_{\mu\nu}$$

Calculating vacuum energy we are in trouble...

$$T^{vac}_{\mu\nu} = \Lambda g_{\mu\nu} \qquad \Lambda = \frac{1}{2}\sum_k \hbar\omega_k / V = \hbar \int dk\, k^2 \sqrt{k^2 + m^2}$$

Calculating vacuum energy we are in trouble...

$$T_{\mu\nu}^{vac} = \Lambda g_{\mu\nu} \qquad \Lambda = \frac{1}{2}\sum_k \hbar\omega_k / V = \hbar \int dk\, k^2 \sqrt{k^2 + m^2}$$

$$m^4 \log\left(\frac{m}{M}\right) \qquad \text{or} \qquad M^4$$

Calculating vacuum energy we are in trouble...

$$T^{vac}_{\mu\nu} = \Lambda g_{\mu\nu} \qquad \Lambda = \frac{1}{2}\sum_k \hbar\omega_k/V = \hbar\int dk\, k^2\sqrt{k^2+m^2}$$

$$m^4 \log\left(\frac{m}{M}\right) \qquad \text{or} \qquad M^4$$

observations: $\Lambda \simeq 10^{-29}\mathrm{g/cm^3}$

Calculating vacuum energy we are in trouble…

$$T_{\mu\nu}^{vac} = \Lambda g_{\mu\nu} \qquad \Lambda = \frac{1}{2}\sum_k \hbar\omega_k / V = \hbar \int dk\, k^2 \sqrt{k^2 + m^2}$$

$$m^4 \log\left(\frac{m}{M}\right) \qquad \text{or} \qquad M^4$$

observations: $\Lambda \simeq 10^{-29} \mathrm{g/cm}^3$

$$\left(10^{-3}\mathrm{eV}\right)^4$$

Calculating vacuum energy we are in trouble...

$$T^{vac}_{\mu\nu} = \Lambda g_{\mu\nu} \qquad \Lambda = \frac{1}{2}\sum_k \hbar\omega_k/V = \hbar\int dk\, k^2\sqrt{k^2+m^2}$$

$$m^4 \log\left(\frac{m}{M}\right) \qquad \text{or} \qquad M^4$$

observations: $\Lambda \simeq 10^{-29}\mathrm{g/cm}^3$

$$\left(10^{-3}\mathrm{eV}\right)^4$$

$$\left(10^{-9} m_e\right)^4$$

Calculating vacuum energy we are in trouble...

$$T_{\mu\nu}^{vac} = \Lambda g_{\mu\nu} \qquad \Lambda = \frac{1}{2}\sum_k \hbar\omega_k/V = \hbar\int dk\, k^2\sqrt{k^2+m^2}$$

$$m^4 \log\left(\frac{m}{M}\right) \qquad \text{or} \qquad M^4$$

observations: $\Lambda \simeq 10^{-29}\mathrm{g/cm}^3$

$$\left(10^{-3}\mathrm{eV}\right)^4$$

$$\left(10^{-9}m_e\right)^4$$

$$\left(10^{-15}E_{\mathrm{LHC}}\right)^4$$

Calculating vacuum energy we are in trouble...

$$T_{\mu\nu}^{vac} = \Lambda g_{\mu\nu} \qquad \Lambda = \frac{1}{2}\sum_k \hbar\omega_k / V = \hbar \int dk\, k^2 \sqrt{k^2 + m^2}$$

$$m^4 \log\left(\frac{m}{M}\right) \qquad \text{or} \qquad M^4$$

observations: $\Lambda \simeq 10^{-29} \mathrm{g/cm}^3$

$$\left(10^{-3}\mathrm{eV}\right)^4$$

$$\left(10^{-9} m_e\right)^4$$

$$\left(10^{-15} E_{\mathrm{LHC}}\right)^4$$

Should vacuum really weigh?

Traceless
Einstein Equations?

Traceless Einstein Equations?

$$G_{\mu\nu} - T_{\mu\nu} = 0$$

Traceless Einstein Equations?

$$G_{\mu\nu} - T_{\mu\nu} = 0$$

$$\Downarrow$$

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

Traceless Einstein Equations?

$$G_{\mu\nu} - T_{\mu\nu} = 0$$

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

Bianchi identity + energy-momentum conservation

$$\nabla_\mu G^{\mu\nu} = 0 \quad + \quad \nabla_\mu T^{\mu\nu} = 0$$

Traceless Einstein Equations?

$$G_{\mu\nu} - T_{\mu\nu} = 0$$

$$\Downarrow$$

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

Bianchi identity + energy-momentum conservation

$$\nabla_\mu G^{\mu\nu} = 0 \quad + \quad \nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \partial_\mu (G - T) = 0$$

Traceless Einstein Equations?

$$G_{\mu\nu} - T_{\mu\nu} = 0$$

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

Bianchi identity + energy-momentum conservation

$$\nabla_\mu G^{\mu\nu} = 0 \quad + \quad \nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \partial_\mu (G - T) = 0$$

$$G_{\mu\nu} - T_{\mu\nu} - \Lambda g_{\mu\nu} = 0$$

Traceless Einstein Equations?

$$G_{\mu\nu} - T_{\mu\nu} = 0$$

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

§ 2. Die skalarfreien Feldgleichungen.

Die dargelegten Schwierigkeiten werden dadurch beseitigt, daß man an die Stelle der Feldgleichungen (1) die Feldgleichungen

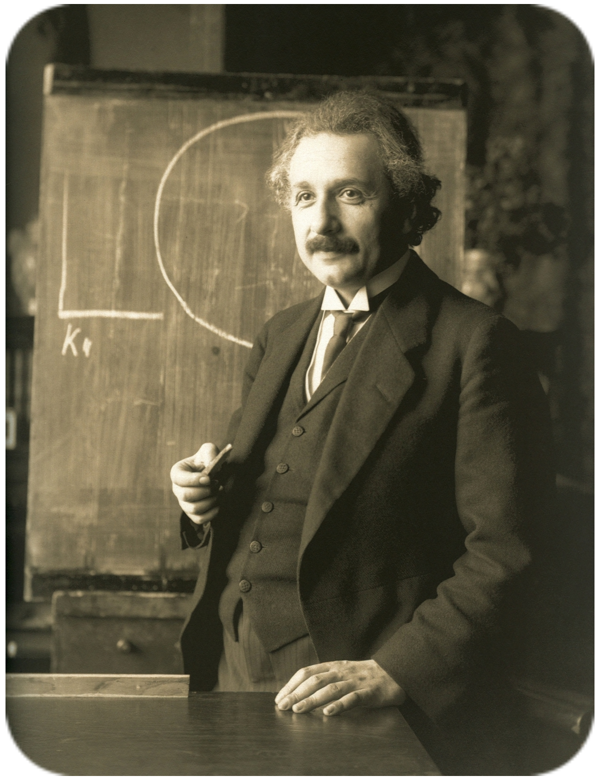
$$R_{i\kappa} - \frac{1}{4} g_{i\kappa} R = -\kappa T_{i\kappa} \qquad (1a)$$

setzt, wobei $(T_{i\kappa})$ den durch (3) gegebenen Energietensor des elektromagnetischen Feldes bedeutet.

Bianchi identity + energy-momentum conservation

$$\nabla_\mu G^{\mu\nu} = 0 \quad + \quad \nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \partial_\mu (G - T) = 0$$

$$G_{\mu\nu} - T_{\mu\nu} - \Lambda g_{\mu\nu} = 0$$



Traceless Einstein Equations?

$$G_{\mu\nu} - T_{\mu\nu} = 0$$

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

§ 2. Die skalarfreien Feldgleichungen.

Die dargelegten Schwierigkeiten werden dadurch beseitigt, daß man an die Stelle der Feldgleichungen (1) die Feldgleichungen

$$R_{i\kappa} - \frac{1}{4} g_{i\kappa} R = -\kappa T_{i\kappa} \tag{1a}$$

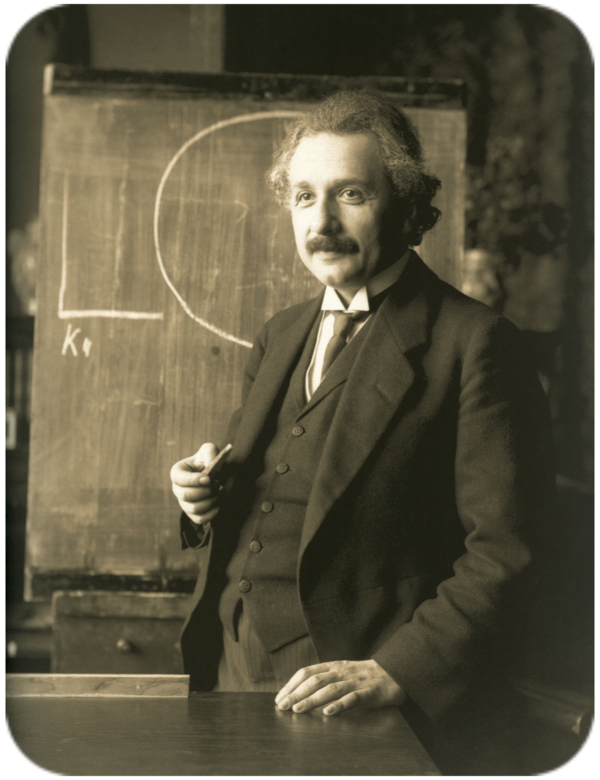
setzt, wobei ($T_{i\kappa}$) den durch (3) gegebenen Energietensor des elektromagnetischen Feldes bedeutet.

Bianchi identity + energy-momentum conservation

$$\nabla_\mu G^{\mu\nu} = 0 \quad + \quad \nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \partial_\mu (G - T) = 0$$

$$G_{\mu\nu} - T_{\mu\nu} - \Lambda g_{\mu\nu} = 0$$

Traceless Einstein Equations?



$$G_{\mu\nu} - T_{\mu\nu} = 0$$

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

EINSTEIN: Gravitationsfelder im Aufbau der materiellen Elementarteilchen 349

IN: *Königlich Preußische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte* (1919): 349–356.

Spielen Gravitationsfelder im Aufbau der materiellen Elementarteilchen eine wesentliche Rolle?

Von A. EINSTEIN.

Weder die NEWTONsche noch die relativistische Gravitationstheorie hat bisher der Theorie von der Konstitution der Materie einen Fortschritt gebracht. Demgegenüber soll im folgenden gezeigt werden, daß Anhaltspunkte für die Auffassung vorhanden sind, daß die die Bausteine der Atome bildenden elektrischen Elementargebilde durch Gravitationskräfte zusammengehalten werden.

§ 1. Mängel der gegenwärtigen Auffassung.

Die Theoretiker haben sich viel bemüht, eine Theorie zu ersinnen, welche von dem Gleichgewicht der das Elektron konstituierenden Elektrizität Rechenschaft gibt. Insbesondere G. MIE hat dieser Frage tiefgehende Unternehmungen gewidmet. Seine Theorie, welche bei den Fachgenossen vielfach Zustimmung gefunden hat, beruht im wesentlichen darauf, daß außer den Energietermen der MAXWELL-LORENTZschen Theorie des elektromagnetischen Feldes von den Komponenten des elektrodyna-

§ 2. Die skalarfreien Feldgleichungen.

Die dargelegten Schwierigkeiten werden dadurch beseitigt, daß man an die Stelle der Feldgleichungen (1) die Feldgleichungen

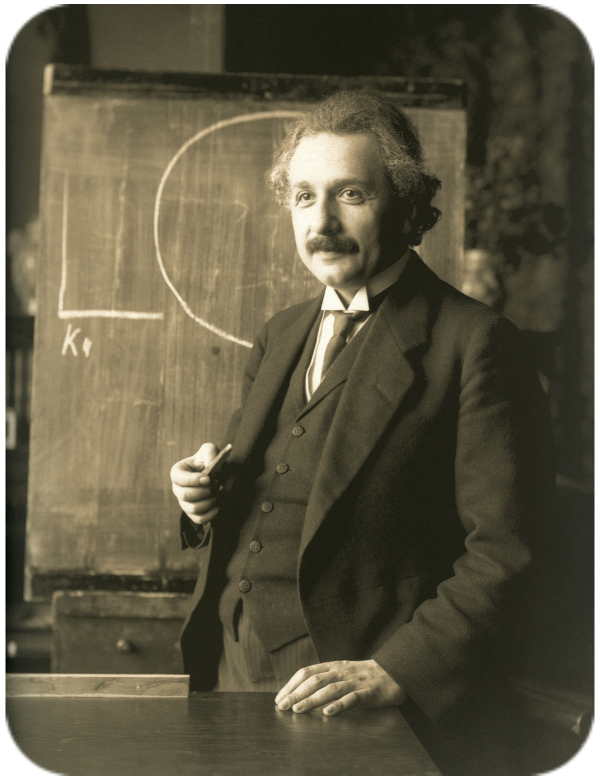
$$R_{i\kappa} - \frac{1}{4} g_{i\kappa} R = -\kappa T_{i\kappa} \qquad (1a)$$

setzt, wobei ($T_{i\kappa}$) den durch (3) gegebenen Energietensor des elektromagnetischen Feldes bedeutet.

Bianchi identity + energy-momentum conservation

$$\nabla_\mu G^{\mu\nu} = 0 \quad + \quad \nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \partial_\mu (G - T) = 0$$

$$G_{\mu\nu} - T_{\mu\nu} - \Lambda g_{\mu\nu} = 0$$



Traceless Einstein Equations?

$$G_{\mu\nu} - T_{\mu\nu} = 0$$

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu}(G - T) = 0$$

EINSTEIN: Gravitationsfelder im Aufbau der materiellen Elementarteilchen 349

IN: *Königlich Preußische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte* (1919): 349–356.

Spielen Gravitationsfelder im Aufbau der materiellen Elementarteilchen eine wesentliche Rolle?

Von A. EINSTEIN.

Weder die NEWTONsche noch die relativistische Gravitationstheorie hat bisher der Theorie von der Konstitution der Materie einen Fortschritt gebracht. Demgegenüber soll im folgenden gezeigt werden, daß Anhaltspunkte für die Auffassung vorhanden sind, daß die die Bausteine der Atome bildenden elektrischen Elementargebilde durch Gravitationskräfte zusammengehalten werden.

§ 1. Mängel der gegenwärtigen Auffassung.

Die Theoretiker haben sich viel bemüht, eine Theorie zu ersinnen, welche von dem Gleichgewicht der das Elektron konstituierenden Elektrizität Rechenschaft gibt. Insbesondere G. MIE hat dieser Frage tiefgehende Unternehmungen gewidmet. Seine Theorie, welche bei den Fachgenossen vielfach Zustimmung gefunden hat, beruht im wesentlichen darauf, daß außer den Energietermen der MAXWELL-LORENTZschen Theorie des elektromagnetischen Feldes von den Komponenten des elektrodyna-

§ 2. Die skalarfreien Feldgleichungen.

Die dargelegten Schwierigkeiten werden dadurch beseitigt, daß man an die Stelle der Feldgleichungen (1) die Feldgleichungen

$$R_{i\kappa} - \frac{1}{4} g_{i\kappa} R = -\kappa T_{i\kappa} \qquad (1a)$$

setzt, wobei $(T_{i\kappa})$ den durch (3) gegebenen Energietensor des elektromagnetischen Feldes bedeutet.

Bianchi identity + energy-momentum conservation

$$\nabla_\mu G^{\mu\nu} = 0 \quad + \quad \nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \partial_\mu (G - T) = 0$$

neue universelle Konstante λ eingeführt werden mußte, die zu der Gesamtmasse der Welt (bzw. zu der Gleichgewichtsdichte der Materie) in fester Beziehung steht. Hierin liegt ein besonders schwerwiegender Schönheitsfehler der Theorie.

$$G_{\mu\nu} - T_{\mu\nu} - \Lambda g_{\mu\nu} = 0$$

Decoupling vacuum energy from spacetime curvature

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

Decoupling vacuum energy from spacetime curvature

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

invariant under *vacuum shifts* of energy-momentum tensor

$$T_{\mu\nu} \to T_{\mu\nu} + \Lambda g_{\mu\nu}$$

What is the action for the *traceless* Einstein field equations ?

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

What is the action for the *traceless* Einstein field equations ?

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

No Trace

What is the action for
the *traceless* Einstein field equations ?

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

No Trace

Weyl invariance?

What is the action for the *traceless* Einstein field equations ?

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

No Trace

$$g_{\mu\nu} = \Omega^2 \cdot h_{\mu\nu}$$

$$\frac{\delta S}{\delta \Omega^2} = \frac{g^{\mu\nu}}{\Omega^2} \cdot \frac{\delta S}{\delta g^{\mu\nu}} = 0$$

Weyl invariance?

Simplest Dark Energy (theory of *all* dS and adS)

Simplest Dark Energy (theory of *all* dS and adS)

$$S_{DE}\left[g, W, \Lambda\right] = \int d^4x\sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

Henneaux, Teitelboim(Bunster)(1989)

Simplest Dark Energy (theory of *all* dS and adS)

$$S_{DE}\left[g, W, \Lambda\right] = \int d^4x\sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

Henneaux, Teitelboim(Bunster)(1989)

$$T_{\mu\nu} = \Lambda g_{\mu\nu}$$

Simplest Dark Energy (theory of *all* dS and adS)

$$S_{DE}\left[g, W, \Lambda\right] = \int d^4x\sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

Henneaux, Teitelboim(Bunster)(1989)

$$T_{\mu\nu} = \Lambda g_{\mu\nu}$$

- Scale invariance, as there is no fixed scale in the action

Simplest Dark Energy (theory of *all* dS and adS)

$$S_{DE}\left[g, W, \Lambda\right] = \int d^4x\sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

Henneaux, Teitelboim(Bunster)(1989)

$$T_{\mu\nu} = \Lambda g_{\mu\nu}$$

- Scale invariance, as there is no fixed scale in the action
- Gauge degeneracy $W^\mu \to W^\mu + \epsilon^\mu$ where $\nabla_\mu \epsilon^\mu = 0$

Simplest Dark Energy (theory of *all* dS and adS)

$$S_{DE}\left[g, W, \Lambda\right] = \int d^4x\sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

Henneaux, Teitelboim(Bunster)(1989)

$$T_{\mu\nu} = \Lambda g_{\mu\nu}$$

- Scale invariance, as there is no fixed scale in the action
- Gauge degeneracy $W^\mu \rightarrow W^\mu + \epsilon^\mu$ where $\nabla_\mu \epsilon^\mu = 0$
- Fake violation of the Lorenz-symmetry, $\langle W^\mu \rangle \neq 0$

Simplest Dark Energy (theory of *all* dS and adS)

$$S_{DE}\left[g, W, \Lambda\right] = \int d^4x\sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

Henneaux, Teitelboim(Bunster)(1989)

$$T_{\mu\nu} = \Lambda g_{\mu\nu}$$

- Scale invariance, as there is no fixed scale in the action
- Gauge degeneracy $W^\mu \rightarrow W^\mu + \epsilon^\mu$ where $\nabla_\mu \epsilon^\mu = 0$
- Fake violation of the Lorenz-symmetry, $\langle W^\mu \rangle \neq 0$
- DE / CC energy density as a Lagrange multiplier

Simplest Dark Energy (theory of *all* dS and adS)

$$S_{DE}\left[g, W, \Lambda\right] = \int d^4x\sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

Henneaux, Teitelboim(Bunster)(1989)

$$T_{\mu\nu} = \Lambda g_{\mu\nu}$$

- Scale invariance, as there is no fixed scale in the action
- Gauge degeneracy $W^\mu \to W^\mu + \epsilon^\mu$ where $\nabla_\mu \epsilon^\mu = 0$
- Fake violation of the Lorenz-symmetry, $\langle W^\mu \rangle \neq 0$
- DE / CC energy density as a Lagrange multiplier
- Similarly to the Ostrogradsky Hamiltonian, the system is *linear* in canonical momentum π, i.e. in energy density of DE, Λ

Simplest Dark Energy (theory of *all* dS and adS)

$$S_{DE}\left[g, W, \Lambda\right] = \int d^4x\sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

Henneaux, Teitelboim(Bunster)(1989)

$$T_{\mu\nu} = \Lambda g_{\mu\nu}$$

- Scale invariance, as there is no fixed scale in the action
- Gauge degeneracy $W^\mu \to W^\mu + \epsilon^\mu$ where $\nabla_\mu \epsilon^\mu = 0$
- Fake violation of the Lorenz-symmetry, $\langle W^\mu \rangle \neq 0$
- DE / CC energy density as a Lagrange multiplier
- Similarly to the Ostrogradsky Hamiltonian, the system is *linear* in canonical momentum π, i.e. in energy density of DE, Λ

$$\pi = \frac{\delta L}{\delta \dot{W}^t} = \sqrt{-g}\,\Lambda$$

Simplest Dark Energy (theory of *all* dS and adS)

$$S_{DE}\left[g, W, \Lambda\right] = \int d^4x\sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

Henneaux, Teitelboim(Bunster)(1989)

$$T_{\mu\nu} = \Lambda g_{\mu\nu}$$

- Scale invariance, as there is no fixed scale in the action
- Gauge degeneracy $W^\mu \to W^\mu + \epsilon^\mu$ where $\nabla_\mu \epsilon^\mu = 0$
- Fake violation of the Lorenz-symmetry, $\langle W^\mu \rangle \neq 0$
- DE / CC energy density as a Lagrange multiplier
- Similarly to the Ostrogradsky Hamiltonian, the system is *linear* in canonical momentum π, i.e. in energy density of DE, Λ

$$\pi = \frac{\delta L}{\delta \dot{W}^t} = \sqrt{-g}\,\Lambda \qquad \text{However,} \qquad \partial_\mu \Lambda = 0$$

- Can one promote the global scale-invariance to the local Weyl-invariance?

- Can one promote the global scale-invariance to the local Weyl-invariance?
- Is there a higher-derivative scalar-tensor generally covariant theory for this simplest DE i.e.

- Can one promote the global scale-invariance to the local Weyl-invariance?

- Is there a higher-derivative scalar-tensor generally covariant theory for this simplest DE i.e.

- New highly symmetric theories with useful physical content and quantum properties potentially different from "unimodular" gravity?

Mimetic vector-tensor theory

Jiroušek, Vikman (2018)

Mimetic vector-tensor theory

Jiroušek, Vikman(2018)

Ansatz : $$g_{\mu\nu} = h_{\mu\nu} \cdot \left(\nabla^{h)}_{\alpha} V^{\alpha} \right)^{1/2}$$

Mimetic vector-tensor theory

Jiroušek, Vikman(2018)

Ansatz : $$g_{\mu\nu} = h_{\mu\nu} \cdot \left(\nabla_{\alpha}^{h)} V^{\alpha} \right)^{1/2}$$

$$\nabla_{\alpha}^{h)} h_{\mu\nu} = 0$$

Mimetic vector-tensor theory

Jiroušek, Vikman (2018)

Ansatz :
$$g_{\mu\nu} = h_{\mu\nu} \cdot \left(\nabla^{h)}_{\alpha} V^{\alpha} \right)^{1/2}$$

$$\nabla^{h)}_{\alpha} h_{\mu\nu} = 0$$

- Weyl invariance for the *vector field of* ***conformal weight 4***

Mimetic vector-tensor theory

Jiroušek, Vikman (2018)

Ansatz : $$g_{\mu\nu} = h_{\mu\nu} \cdot \left(\nabla^{h)}_{\alpha} V^{\alpha} \right)^{1/2}$$

$$\nabla^{h)}_{\alpha} h_{\mu\nu} = 0$$

- Weyl invariance for the *vector field of* ***conformal weight 4***

$$h_{\mu\nu} = \Omega^2(x)\, h'_{\mu\nu}$$

Mimetic vector-tensor theory

Jiroušek, Vikman(2018)

Ansatz : $$g_{\mu\nu} = h_{\mu\nu} \cdot \left(\nabla_{\alpha}^{h)} V^{\alpha} \right)^{1/2}$$

$$\nabla_{\alpha}^{h)} h_{\mu\nu} = 0$$

- Weyl invariance for the *vector field of* ***conformal weight 4***

$$h_{\mu\nu} = \Omega^2(x)\, h'_{\mu\nu} \qquad V^{\mu} = \Omega^{-4}(x)\, V'^{\mu}$$

Mimetic vector-tensor theory

Jiroušek, Vikman (2018)

Ansatz : $$g_{\mu\nu} = h_{\mu\nu} \cdot \left(\overset{h)}{\nabla}_{\alpha} V^{\alpha} \right)^{1/2}$$

$$\overset{h)}{\nabla}_{\alpha} h_{\mu\nu} = 0$$

- Weyl invariance for the *vector field of* ***conformal weight 4***

$$h_{\mu\nu} = \Omega^2(x)\, h'_{\mu\nu} \qquad V^{\mu} = \Omega^{-4}(x)\, V'^{\mu}$$

Mimetic vector-tensor theory

Jiroušek, Vikman(2018)

Ansatz : $$g_{\mu\nu} = h_{\mu\nu} \cdot \left(\overset{h)}{\nabla}_{\alpha} V^{\alpha} \right)^{1/2}$$

$$\overset{h)}{\nabla}_{\alpha} h_{\mu\nu} = 0$$

- Weyl invariance for the *vector field of* ***conformal weight 4***

$$h_{\mu\nu} = \Omega^{2}(x)\, h'_{\mu\nu} \qquad V^{\mu} = \Omega^{-4}(x)\, V'^{\mu}$$

- Gauge degeneracy $V^{\mu} \rightarrow V^{\mu} + \epsilon^{\mu}$ where $\overset{h)}{\nabla}_{\mu} \epsilon^{\mu} = 0$

Action for a vector-tensor theory beyond Horndeski

$$S_g\left[h, V\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(\nabla^{(h)}_{\alpha}V^{\alpha}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla^{(h)}_{\mu}\nabla^{(h)}_{\alpha}V^{\alpha}\right)^{2}}{\left(\nabla^{(h)}_{\sigma}V^{\sigma}\right)^{3/2}}\right] .$$

Action for a vector-tensor theory beyond Horndeski

$$S_g\left[h, V\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(\nabla^{h)}_{\alpha}V^{\alpha}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla^{h)}_{\mu}\nabla^{h)}_{\alpha}V^{\alpha}\right)^{2}}{\left(\nabla^{h)}_{\sigma}V^{\sigma}\right)^{3/2}}\right] .$$

Equations of motion

Action for a vector-tensor theory beyond Horndeski

$$S_g\left[h,V\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(\nabla_\alpha^{h)}V^\alpha\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla_\mu^{h)}\nabla_\alpha^{h)}V^\alpha\right)^2}{\left(\nabla_\sigma^{h)}V^\sigma\right)^{3/2}}\right].$$

Equations of motion

$$\frac{1}{\sqrt{-h}}\cdot\frac{\delta S}{\delta V^\mu} = \frac{1}{4}\,\partial_\mu\left(T-G\right) = 0$$

Action for a vector-tensor theory beyond Horndeski

$$S_g\left[h, V\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(\nabla_\alpha^{h)} V^\alpha\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla_\mu^{h)}\nabla_\alpha^{h)} V^\alpha\right)^2}{\left(\nabla_\sigma^{h)} V^\sigma\right)^{3/2}}\right].$$

Equations of motion

$$\frac{1}{\sqrt{-h}}\cdot\frac{\delta S}{\delta V^\mu} = \frac{1}{4}\,\partial_\mu\left(T - G\right) = 0$$

$$\frac{1}{\sqrt{-h}}\cdot\frac{\delta S}{\delta h^{\alpha\beta}} = \frac{\sqrt{\nabla_\alpha^{h)} V^\alpha}}{2}\left[T_{\alpha\beta} - G_{\alpha\beta} - \frac{1}{4}g_{\alpha\beta}\left(T - G - \frac{1}{\nabla_\alpha^{h)} V^\alpha}V^\lambda\partial_\lambda\left(T - G\right)\right)\right] = 0$$

Action for a vector-tensor theory beyond Horndeski

$$S_g\left[h,V\right]=-\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(\nabla_\alpha^{h)}V^\alpha\right)^{1/2}R\left(h\right)+\frac{3}{8}\cdot\frac{\left(\nabla_\mu^{h)}\nabla_\alpha^{h)}V^\alpha\right)^2}{\left(\nabla_\sigma^{h)}V^\sigma\right)^{3/2}}\right].$$

Equations of motion

$$\frac{1}{\sqrt{-h}}\cdot\frac{\delta S}{\delta V^\mu}=\frac{1}{4}\partial_\mu\left(T-G\right)=0$$

$$\frac{1}{\sqrt{-h}}\cdot\frac{\delta S}{\delta h^{\alpha\beta}}=\frac{\sqrt{\nabla_\alpha^{h)}V^\alpha}}{2}\left[T_{\alpha\beta}-G_{\alpha\beta}-\frac{1}{4}g_{\alpha\beta}\left(T-G-\frac{1}{\nabla_\alpha^{h)}V^\alpha}V^\lambda\partial_\lambda\left(T-G\right)\right)\right]=0$$

$$G_{\mu\nu}-T_{\mu\nu}-\frac{1}{4}g_{\mu\nu}\left(G-T\right)=0$$

More Familiar Action

$$S\left[h,\varphi,V,\lambda\right]=\int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2-\frac{1}{12}\varphi^2\,R\left(h\right)-\frac{\lambda}{72}\varphi^4+\frac{\lambda}{2}\cdot\nabla_\alpha^{h)}V^\alpha\right]$$

More Familiar Action

$$S\left[h,\varphi,V,\lambda\right]=\int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2-\frac{1}{12}\varphi^2\,R\left(h\right)-\frac{\lambda}{72}\varphi^4+\frac{\lambda}{2}\cdot\nabla^{h)}_{\alpha}V^{\alpha}\right]$$

Lagrange multiplier

More Familiar Action

$$S\left[h,\varphi,V,\lambda\right]=\int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2-\frac{1}{12}\varphi^2 R\left(h\right)-\frac{\lambda}{72}\varphi^4+\frac{\lambda}{2}\cdot\nabla_\alpha^{h)}V^\alpha\right]$$

Lagrange multiplier

$$\nabla_\alpha^{h)}V^\alpha=\left(\frac{\varphi^2}{6}\right)^2$$

More Familiar Action

$$S\left[h,\varphi,V,\lambda\right]=\int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2-\frac{1}{12}\varphi^2R\left(h\right)-\frac{\lambda}{72}\varphi^4+\frac{\lambda}{2}\cdot\nabla^{h)}_{\alpha}V^{\alpha}\right]$$

Lagrange multiplier

$$\nabla^{h)}_{\alpha}V^{\alpha}=\left(\frac{\varphi^2}{6}\right)^2$$

More Familiar Action

$$S\left[h,\varphi,V,\lambda\right] = \int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2 - \frac{1}{12}\varphi^2 R\left(h\right) - \frac{\lambda}{72}\varphi^4 + \frac{\lambda}{2}\cdot\nabla_\alpha^{h)} V^\alpha\right]$$

wrong sign

Lagrange multiplier

$$\nabla_\alpha^{h)} V^\alpha = \left(\frac{\varphi^2}{6}\right)^2$$

More Familiar Action

$$S\left[h,\varphi,V,\lambda\right]=\int d^4x\sqrt{-h}\left[\boxed{-}\frac{1}{2}\left(\partial\varphi\right)^2-\frac{1}{12}\varphi^2 R\left(h\right)-\frac{\boxed{\lambda}}{72}\varphi^4+\frac{\boxed{\lambda}}{2}\cdot\nabla^{h)}_{\alpha}V^{\alpha}\right]$$

wrong sign

Lagrange multiplier

$$\nabla^{h)}_{\alpha}V^{\alpha}=\left(\frac{\varphi^2}{6}\right)^2$$

V^{μ} Stückelberg Freiherr von Breidenbach zu Breidenstein und Melsbach field

More Familiar Action

$$S\left[h,\varphi,V,\lambda\right]=\int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2-\frac{1}{12}\varphi^2\,R\left(h\right)-\frac{\lambda}{72}\varphi^4+\frac{\lambda}{2}\cdot\nabla^{h)}_{\alpha}V^{\alpha}\right]$$

wrong sign

Lagrange multiplier

$$\nabla^{h)}_{\alpha}V^{\alpha}=\left(\frac{\varphi^2}{6}\right)^2$$

V^{μ} Stückelberg Freiherr von Breidenbach zu Breidenstein und Melsbach field

Weyl transformations

$$h_{\mu\nu}=\Omega^2\left(x\right)h'_{\mu\nu}$$

$$\varphi=\Omega^{-1}\left(x\right)\varphi'$$

$$V^{\mu}=\Omega^{-4}\left(x\right)V'^{\mu}$$

$$\lambda=\lambda'$$

More Familiar Action

$$S\left[h,\varphi,V,\lambda\right]=\int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2-\frac{1}{12}\varphi^2R\left(h\right)-\frac{\lambda}{72}\varphi^4+\frac{\lambda}{2}\cdot\nabla_\alpha^{h)}V^\alpha\right]$$

wrong sign

Lagrange multiplier

$$\nabla_\alpha^{h)}V^\alpha=\left(\frac{\varphi^2}{6}\right)^2$$

V^μ Stückelberg Freiherr von Breidenbach zu Breidenstein und Melsbach field

Weyl transformations

$$h_{\mu\nu}=\Omega^2(x)\,h'_{\mu\nu}$$

$$\varphi=\Omega^{-1}(x)\,\varphi'$$

$$V^\mu=\Omega^{-4}(x)\,V'^\mu$$

$$\lambda=\lambda'$$

Weyl-invariant variables

$$g_{\mu\nu}=\frac{\varphi^2}{6}h_{\mu\nu}$$

$$\varphi=\varphi$$

$$W^\mu=\left(\frac{\varphi^2}{6}\right)^{-2}V^\mu$$

$$\Lambda=\frac{\lambda}{2}$$

$$S\left[g, W, \Lambda, \Phi_m\right] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R\left(g\right) + \Lambda\left(\nabla_\mu^{g)} W^\mu - 1\right)\right] + S_m\left[g, \Phi_m\right]$$

$$S\left[g, W, \Lambda, \Phi_m\right] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R\left(g\right) + \Lambda\left(\nabla^{g)}_{\mu} W^{\mu} - 1\right)\right] + S_m\left[g, \Phi_m\right]$$

Henneaux–Teitelboim
"unimodular" gravity (1989)

$$S\left[g, W, \Lambda, \Phi_m\right] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R\left(g\right) + \Lambda\left(\nabla_\mu^{g)} W^\mu - 1\right)\right] + S_m\left[g, \Phi_m\right]$$

Henneaux–Teitelboim
"unimodular" gravity (1989)

Global degree of freedom

$$\mathscr{T}\left(t\right) = \int d^3\mathbf{x}\sqrt{-g}\; W^t\left(t, \mathbf{x}\right)$$

$$S\left[g, W, \Lambda, \Phi_m\right] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R\left(g\right) + \Lambda\left(\nabla^{g)}_{\mu} W^{\mu} - 1\right)\right] + S_m\left[g, \Phi_m\right]$$

Henneaux–Teitelboim "unimodular" gravity (1989)

Global degree of freedom

$$\mathcal{T}\left(t\right) = \int d^3\mathbf{x}\sqrt{-g}\, W^t\left(t, \mathbf{x}\right)$$

gauge invariance $W^{\mu} \to W^{\mu} + \epsilon^{\mu}$ generates global shift-symmetry $\mathcal{T} \to \mathcal{T} + c$

$$S\left[g, W, \Lambda, \Phi_m\right] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R\left(g\right) + \Lambda\left(\nabla_\mu^{g)}W^\mu - 1\right)\right] + S_m\left[g, \Phi_m\right]$$

Henneaux–Teitelboim "unimodular" gravity (1989)

Global degree of freedom $\mathscr{T}(t) = \int d^3\mathbf{x}\sqrt{-g}\,W^t(t,\mathbf{x})$

gauge invariance $W^\mu \to W^\mu + \epsilon^\mu$ generates global shift-symmetry $\mathscr{T} \to \mathscr{T} + c$

invariant $\mathscr{T}\left(t_2\right) - \mathscr{T}\left(t_1\right) = \int_{t_1}^{t_2} dt \int d^3\mathbf{x}\sqrt{-g}$

$$S\left[g, W, \Lambda, \Phi_m\right] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R\left(g\right) + \Lambda\left(\nabla^{(g)}_{\mu} W^{\mu} - 1\right)\right] + S_m\left[g, \Phi_m\right]$$

Henneaux–Teitelboim "unimodular" gravity (1989)

Global degree of freedom $\mathscr{T}\left(t\right) = \int d^3\mathbf{x}\sqrt{-g}\, W^t\left(t, \mathbf{x}\right)$

gauge invariance $W^\mu \rightarrow W^\mu + \epsilon^\mu$ generates global shift-symmetry $\mathscr{T} \rightarrow \mathscr{T} + c$

invariant $\mathscr{T}\left(t_2\right) - \mathscr{T}\left(t_1\right) = \int_{t_1}^{t_2} dt \int d^3\mathbf{x}\sqrt{-g}$

shift-symmetry in coordinate

conservation of momentum, $\Lambda = conts$

Global degree of freedom

$$\mathcal{T}(t) = \int d^3\mathbf{x}\sqrt{-g}\, W^t(t,\mathbf{x})$$

Global degree of freedom

$$\mathcal{T}(t) = \int d^3\mathbf{x}\sqrt{-g}\, W^t(t,\mathbf{x}) = \int d^3\mathbf{x}\sqrt{-h}\, V^t(t,\mathbf{x})$$

Global degree of freedom

$$\mathscr{T}(t) = \int d^3\mathbf{x}\sqrt{-g}\, W^t(t,\mathbf{x}) = \int d^3\mathbf{x}\sqrt{-h}\, V^t(t,\mathbf{x})$$

For the action

$$S_g[h,V] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(\nabla^{h)}_{\alpha} V^{\alpha}\right)^{1/2} R(h) + \frac{3}{8}\cdot\frac{\left(\nabla^{h)}_{\mu}\nabla^{h)}_{\alpha} V^{\alpha}\right)^2}{\left(\nabla^{h)}_{\sigma} V^{\sigma}\right)^{3/2}}\right] .$$

Global degree of freedom

$$\mathcal{T}(t) = \int d^3\mathbf{x}\sqrt{-g}\, W^t(t,\mathbf{x}) = \int d^3\mathbf{x}\sqrt{-h}\, V^t(t,\mathbf{x})$$

For the action

$$S_g[h,V] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(\nabla^{h)}_{\alpha}V^{\alpha}\right)^{1/2} R(h) + \frac{3}{8}\cdot\frac{\left(\nabla^{h)}_{\mu}\nabla^{h)}_{\alpha}V^{\alpha}\right)^2}{\left(\nabla^{h)}_{\sigma}V^{\sigma}\right)^{3/2}}\right].$$

The conjugated canonical momentum of

$$\Lambda$$

Conformal weight four is unusual for a vector field

Conformal weight four is unusual
for a vector field

Can one find a more usual construction?

Axionic Cosmological Constant

Jiroušek, Vikman(2019) in preparation

$$g_{\mu\nu} = h_{\mu\nu} \cdot \sqrt{F_{\alpha\beta}\widetilde{F}^{\alpha\beta}}$$

Axionic Cosmological Constant

Jiroušek, Vikman(2019) in preparation

$$g_{\mu\nu} = h_{\mu\nu} \cdot \sqrt{F_{\alpha\beta}\widetilde{F}^{\alpha\beta}}$$

$$\widetilde{F}^{\alpha\beta} = \frac{1}{2} \cdot \frac{\epsilon^{\alpha\beta\mu\nu}}{\sqrt{-h}} \cdot F_{\mu\nu}$$

Axionic Cosmological Constant

Jiroušek, Vikman(2019) in preparation

$$g_{\mu\nu} = h_{\mu\nu} \cdot \sqrt{F_{\alpha\beta} \widetilde{F}^{\alpha\beta}}$$

$$\widetilde{F}^{\alpha\beta} = \frac{1}{2} \cdot \frac{\epsilon^{\alpha\beta\mu\nu}}{\sqrt{-h}} \cdot F_{\mu\nu}$$

Weyl-Invariance for $h_{\mu\nu} = \Omega^2(x)\, h'_{\mu\nu}$

Axionic Cosmological Constant

Jiroušek, Vikman(2019) in preparation

$$g_{\mu\nu} = h_{\mu\nu} \cdot \sqrt{F_{\alpha\beta}\widetilde{F}^{\alpha\beta}}$$

$$\widetilde{F}^{\alpha\beta} = \frac{1}{2} \cdot \frac{\epsilon^{\alpha\beta\mu\nu}}{\sqrt{-h}} \cdot F_{\mu\nu}$$

Weyl-Invariance for $h_{\mu\nu} = \Omega^2(x)\, h'_{\mu\nu}$

$$g_{\mu\nu} = \frac{h_{\mu\nu}}{\left(-h\right)^{1/4}} \cdot \sqrt{\mathscr{P}}$$

Pontryagin Density

Axionic Cosmological Constant

Jiroušek, Vikman(2019) in preparation

$$g_{\mu\nu} = h_{\mu\nu} \cdot \sqrt{F_{\alpha\beta}\widetilde{F}^{\alpha\beta}}$$

$$\widetilde{F}^{\alpha\beta} = \frac{1}{2} \cdot \frac{\epsilon^{\alpha\beta\mu\nu}}{\sqrt{-h}} \cdot F_{\mu\nu}$$

Weyl-Invariance for $h_{\mu\nu} = \Omega^2(x)\, h'_{\mu\nu}$

$$g_{\mu\nu} = \frac{h_{\mu\nu}}{(-h)^{1/4}} \cdot \sqrt{\mathscr{P}}$$

Pontryagin Density

$$g_{\mu\nu} = h_{\mu\nu} \cdot \sqrt{F_{\alpha\beta}F^{\alpha\beta}}$$

Mukohyama et all (2018)

$$F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \nabla_{\alpha}^{h)} C^{\alpha}$$

$$F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \nabla^{h)}_{\alpha} C^{\alpha}$$

Chern-Simons Current

$$C^{\alpha} = \mathrm{tr}\frac{\varepsilon^{\alpha\beta\gamma\delta}}{\sqrt{-h}}\left(\boldsymbol{F}_{\beta\gamma}\boldsymbol{A}_{\delta} - \frac{2}{3}ig\boldsymbol{A}_{\beta}\boldsymbol{A}_{\gamma}\boldsymbol{A}_{\delta}\right)$$

$$F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \nabla^{h)}_{\alpha} C^{\alpha}$$

Chern-Simons Current

$$C^{\alpha} = \mathrm{tr}\frac{\varepsilon^{\alpha\beta\gamma\delta}}{\sqrt{-h}}\left(\boldsymbol{F}_{\beta\gamma}\boldsymbol{A}_{\delta} - \frac{2}{3}ig\boldsymbol{A}_{\beta}\boldsymbol{A}_{\gamma}\boldsymbol{A}_{\delta}\right)$$

composite vector variable
of conformal weight four!

$$F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \nabla^{h)}_{\alpha} C^{\alpha}$$

Chern-Simons Current

$$C^{\alpha} = \mathrm{tr}\frac{\varepsilon^{\alpha\beta\gamma\delta}}{\sqrt{-h}}\left(\boldsymbol{F}_{\beta\gamma}\boldsymbol{A}_{\delta} - \frac{2}{3}ig\boldsymbol{A}_{\beta}\boldsymbol{A}_{\gamma}\boldsymbol{A}_{\delta}\right)$$

composite vector variable
of conformal weight four!

gauge transformations $\boldsymbol{A}_{\mu} \rightarrow U\boldsymbol{A}_{\mu}U^{-1} + \frac{i}{g}\left(\partial_{\mu}U\right)U^{-1}$

$$F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \nabla_{\alpha}^{h)} C^{\alpha}$$

Chern-Simons Current

$$C^{\alpha} = \mathrm{tr}\frac{\varepsilon^{\alpha\beta\gamma\delta}}{\sqrt{-h}}\left(\boldsymbol{F}_{\beta\gamma}\boldsymbol{A}_{\delta} - \frac{2}{3}ig\boldsymbol{A}_{\beta}\boldsymbol{A}_{\gamma}\boldsymbol{A}_{\delta}\right)$$

composite vector variable
of conformal weight four!

gauge transformations $\quad \boldsymbol{A}_{\mu} \to U\boldsymbol{A}_{\mu}U^{-1} + \frac{i}{g}\left(\partial_{\mu}U\right)U^{-1}$

introduce the shifts $\quad C^{\mu} \to C^{\mu} + \epsilon^{\mu} \qquad \nabla_{\mu}^{h)}\epsilon^{\mu} = 0$

$$S_g\left[h, A\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla_{\mu}^{h)}\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)\right)^2}{\left(F_{\sigma\rho}\widetilde{F}^{\sigma\rho}\right)^{3/2}}\right]$$

$$S_g\left[h,A\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla_\mu^{h)}\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)\right)^2}{\left(F_{\sigma\rho}\widetilde{F}^{\sigma\rho}\right)^{3/2}}\right]$$

matter couples to $g_{\mu\nu} = \frac{\varphi^2}{6}\cdot h_{\mu\nu}$ where $F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \left(\frac{\varphi^2}{6}\right)^2$

$$S_g\left[h, A\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla_{\mu}^{h)}\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)\right)^2}{\left(F_{\sigma\rho}\widetilde{F}^{\sigma\rho}\right)^{3/2}}\right]$$

matter couples to $g_{\mu\nu} = \frac{\varphi^2}{6}\cdot h_{\mu\nu}$ **where** $F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \left(\frac{\varphi^2}{6}\right)^2$

$$\frac{1}{\sqrt{-h}}\cdot\frac{\delta S}{\delta A_\nu} = \widetilde{F}^{\mu\nu}\partial_\mu\left(T - G\right) = 0$$

$$\frac{1}{\sqrt{-h}}\cdot\frac{\delta S}{\delta h^{\alpha\beta}} = \frac{\varphi^2}{12}\left[T_{\alpha\beta} - G_{\alpha\beta} - \frac{1}{4}\left(T - G\right)g_{\alpha\beta}\right] = 0$$

Benign Higher Derivatives, beyond Horndeski

Benign Higher Derivatives, beyond Horndeski

$$S_g\left[h,A\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla^{h)}_{\mu}\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)\right)^2}{\left(F_{\sigma\rho}\widetilde{F}^{\sigma\rho}\right)^{3/2}}\right]$$

Benign Higher Derivatives, beyond Horndeski

$$S_g\left[h,A\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla_\mu^{h)}\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)\right)^2}{\left(F_{\sigma\rho}\widetilde{F}^{\sigma\rho}\right)^{3/2}}\right]$$

$$F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \left(\frac{\varphi^2}{6}\right)^2$$

Benign Higher Derivatives, beyond Horndeski

$$S_g\left[h,A\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla_\mu^{h)}\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)\right)^2}{\left(F_{\sigma\rho}\widetilde{F}^{\sigma\rho}\right)^{3/2}}\right]$$

$$F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \left(\frac{\varphi^2}{6}\right)^2$$

$$\Downarrow$$

$$S\left[h,\varphi,A,\lambda\right] = \int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2 - \frac{1}{12}\varphi^2 R\left(h\right) - \frac{\lambda}{72}\varphi^4 + \frac{\lambda}{2}\cdot F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right]$$

Benign Higher Derivatives, beyond Horndeski

$$S_g\left[h,A\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla_\mu^{h)}\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)\right)^2}{\left(F_{\sigma\rho}\widetilde{F}^{\sigma\rho}\right)^{3/2}}\right]$$

$$F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \left(\frac{\varphi^2}{6}\right)^2$$

$$S\left[h,\varphi,A,\lambda\right] = \int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2 - \frac{1}{12}\varphi^2 R\left(h\right) - \frac{\boxed{\lambda}}{72}\varphi^4 + \frac{\boxed{\lambda}}{2}\cdot F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right]$$

Lagrange multiplier

Benign Higher Derivatives, beyond Horndeski

$$S_g\left[h,A\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla_\mu^{h)}\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)\right)^2}{\left(F_{\sigma\rho}\widetilde{F}^{\sigma\rho}\right)^{3/2}}\right]$$

$$F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \left(\frac{\varphi^2}{6}\right)^2$$

$$S\left[h,\varphi,A,\lambda\right] = \int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2 - \frac{1}{12}\varphi^2 R\left(h\right) - \frac{\lambda}{72}\varphi^4 + \frac{\lambda}{2}\cdot F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right]$$

Lagrange multiplier

Benign Higher Derivatives, beyond Horndeski

$$S_g\left[h, A\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla_\mu^{h)}\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)\right)^2}{\left(F_{\sigma\rho}\widetilde{F}^{\sigma\rho}\right)^{3/2}}\right]$$

$$F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \left(\frac{\varphi^2}{6}\right)^2$$

$$S\left[h, \varphi, A, \lambda\right] = \int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2 - \frac{1}{12}\varphi^2 R\left(h\right) - \frac{\lambda}{72}\varphi^4 + \frac{\lambda}{2}\cdot F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right]$$

wrong sign

Lagrange multiplier

Axionic Cosmological Constant?

Axionic Cosmological Constant?

Weyl-invariant variables

$$g_{\mu\nu} = \frac{\varphi^2}{6} h_{\mu\nu} \qquad \Lambda = \frac{\lambda}{2} \qquad A_\mu$$

Axionic Cosmological Constant?

Weyl-invariant variables

$$g_{\mu\nu} = \frac{\varphi^2}{6} h_{\mu\nu} \qquad \Lambda = \frac{\lambda}{2} \qquad A_\mu$$

$$S\left[g, A, \Lambda\right] = \int d^4x \sqrt{-g}\left[-\frac{1}{2}R\left(g\right) + \Lambda\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta} - 1\right)\right]$$

Axionic Cosmological Constant?

Weyl-invariant variables

$$g_{\mu\nu} = \frac{\varphi^2}{6} h_{\mu\nu} \qquad \Lambda = \frac{\lambda}{2} \qquad A_\mu$$

$$S\left[g, A, \Lambda\right] = \int d^4x \sqrt{-g} \left[-\frac{1}{2} R\left(g\right) + \Lambda \left(F_{\alpha\beta} \widetilde{F}^{\alpha\beta} - 1 \right) \right]$$

PHYSICS REPORTS (Review Section of Physics Letters) 104, Nos. 2–4 (1984) 143–157. North-Holland, Amsterdam

Foundations and Working Pictures in Microphysical Cosmology

Frank WILCZEK

I would like to briefly mention one idea in this regard, that I am now exploring. It is to do something for the Λ-parameter very similar to what the axion does for the θ-parameter in QCD, another otherwise mysteriously tiny quantity. The basic idea is to promote these parameters to dynamical variables, and then see if perhaps small values will be chosen dynamically. In the case of the

Another way of thoughts

Another way of thoughts

Cleaning up the cosmological constant

2012

Ian Kimpton and Antonio Padilla

School of Physics and Astronomy, University of Nottingham, Nottingham NG7 2RD, UK

Another way of thoughts

Cleaning up the cosmological constant

2012

Ian Kimpton and Antonio Padilla

School of Physics and Astronomy, University of Nottingham, Nottingham NG7 2RD, UK

We now observe that the vacuum energy coming from particle physics enters the action via a term of the form $-2\Lambda \int d^4x\sqrt{-\tilde{g}}$. This has no effect on the dynamics provided

$$\frac{\delta}{\delta\phi_i}\int d^4x\sqrt{-\tilde{g}} = 0. \tag{2.1}$$

This is only possible when $\tilde{g}_{ab}$ is a composite field, for which $\sqrt{-\tilde{g}}$ is the integrand of a topological invariant, and/or a total derivative. Note that our method is distinct from unimodular gravity in which the metric determinant is *constrained* to be unity [13].

Conclusions

Conclusions

- There are Weyl-invariant and general covariant actions with higher derivatives for DE and DM

Conclusions

- There are Weyl-invariant and general covariant actions with higher derivatives for DE and DM
- In “Unimodular” Gravity vacuum does not weigh

Conclusions

- There are Weyl-invariant and general covariant actions with higher derivatives for DE and DM
- In “Unimodular” Gravity vacuum does not weigh
- Cosmological constant as an Axionic field

Conclusions

- There are Weyl-invariant and general covariant actions with higher derivatives for DE and DM
- In “Unimodular” Gravity vacuum does not weigh
- Cosmological constant as an Axionic field

Thanks a lot for attention!