Infrared universality of ζ in asymptotically FLRW Universe

Yuko Urakawa (Bielefeld U., NagoyaU.)

Tanaka & Y.U. JCAP 1606 (16) 020

Tanaka & Y.U. JHEP 1710 (17)127

Tanaka & Y.U. In progress

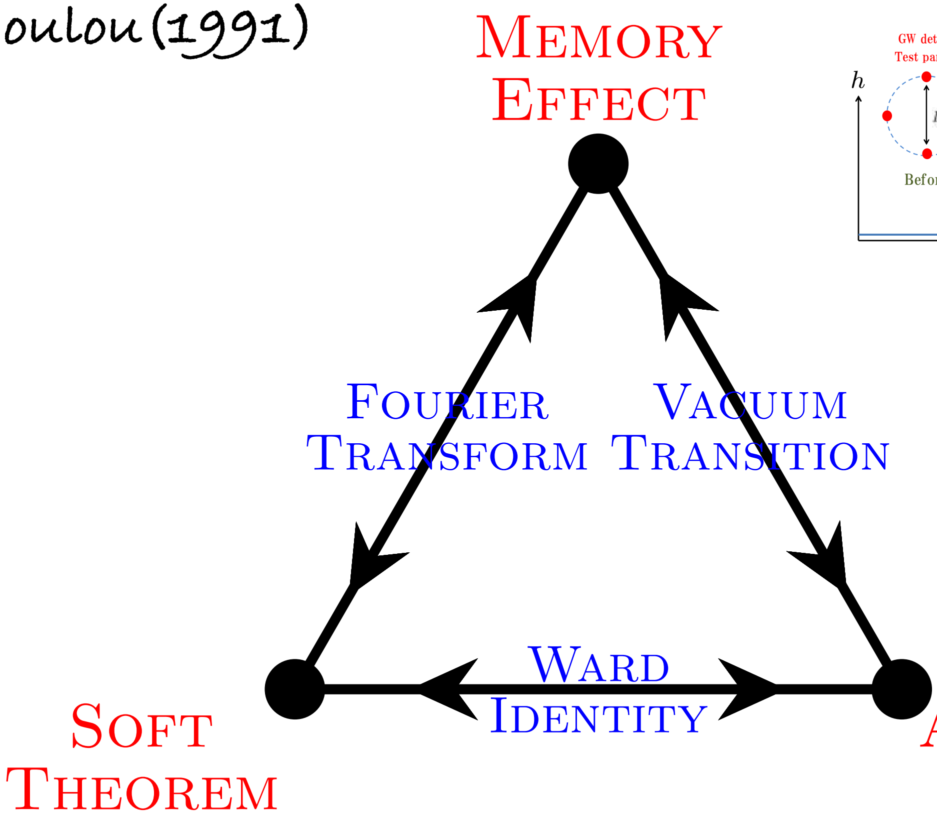
with Takahiro Tanaka (Kyoto U.)

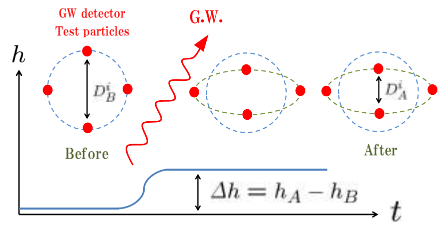
Infrared triangle

Strominger+ (13,14, ...)

review 1703.05448

Christodoulou (1991)





from talk slide of A.Ishibashi

(approaches to M_4 in $r \to \infty$)

Bloch & Nordsieck (1937)

Weinberg (1965)

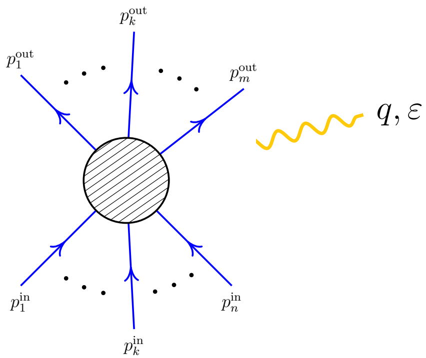
Bondi, Metzner, Sachs (1962)

Soft theorem

for QED Bloch & Nordsieck (1937)

for gravitons Weinberg (1965)

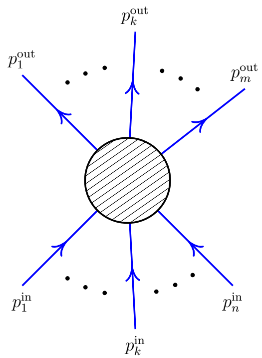
S matrix w/soft leg



=

universal factor

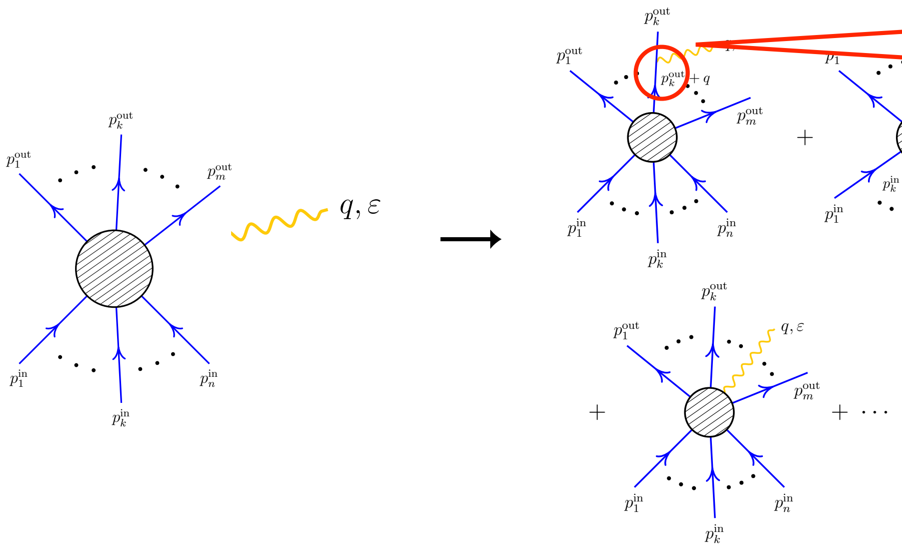
x



S matrix
(n→m scattering)

Soft photon theorem

$$\langle \text{out}|a_+^{\text{out}}(\vec{q})\mathcal{S}|\text{in}\rangle \;= e\left[\sum_{k=1}^{m}\frac{Q_k^{\text{out}}p_k^{\text{out}}\cdot\varepsilon^+}{p_k^{\text{out}}\cdot q}-\sum_{k=1}^{n}\frac{Q_k^{\text{in}}p_k^{\text{in}}\cdot\varepsilon^+}{p_k^{\text{in}}\cdot q}\right]\langle\text{out}|\mathcal{S}|\text{in}\rangle+\mathcal{O}(q^0)$$



propagator

$$\frac{-i}{(p+q)^2+m^2}=\frac{-i}{p^2+2p\cdot q+q^2+m^2}=\frac{-i}{2p\cdot q}$$

vertex factor

$$ie\varepsilon^{\mu}2Qp_{\mu}$$

from Strominger 1703.05448

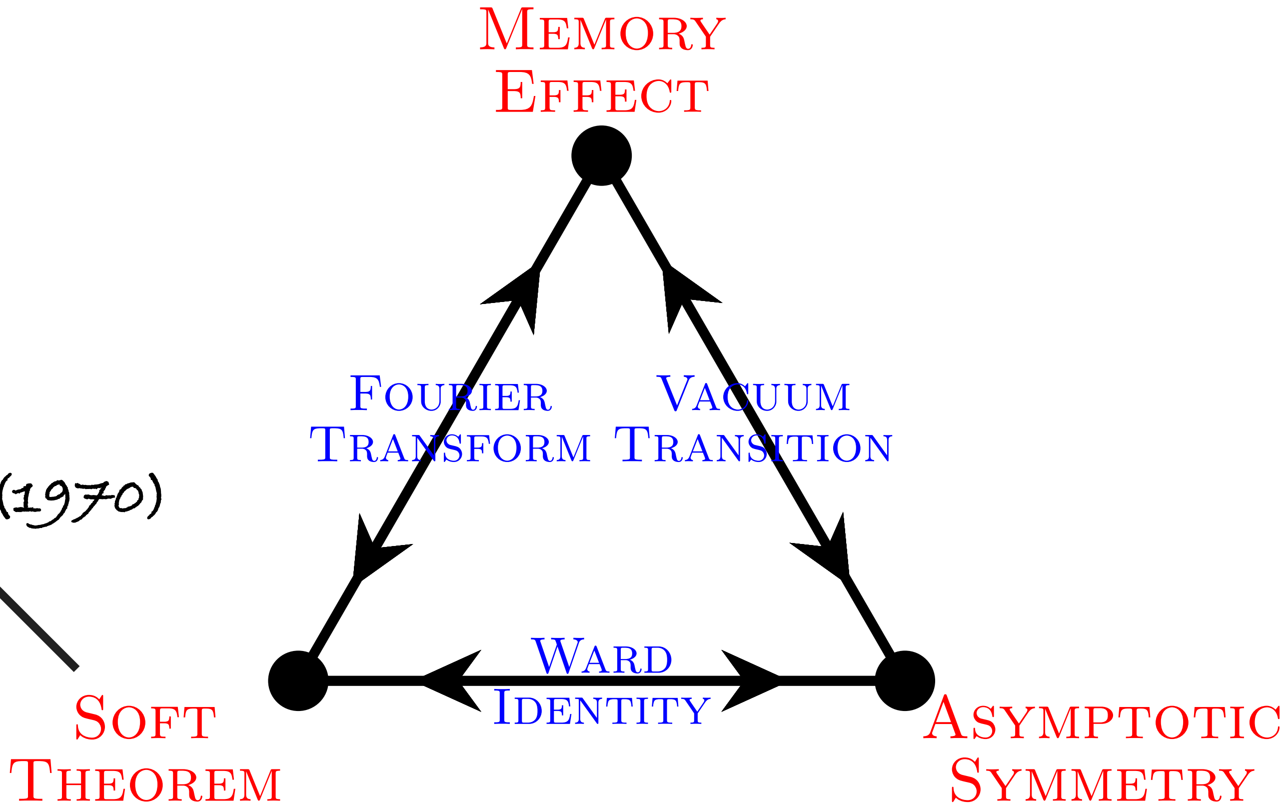
from Strominger 1703.05448

Infrared triangle

Strominger+ (13,14, ...)

review 1703.05448

Christodoulou (1991)



Cancellation of IR divergence

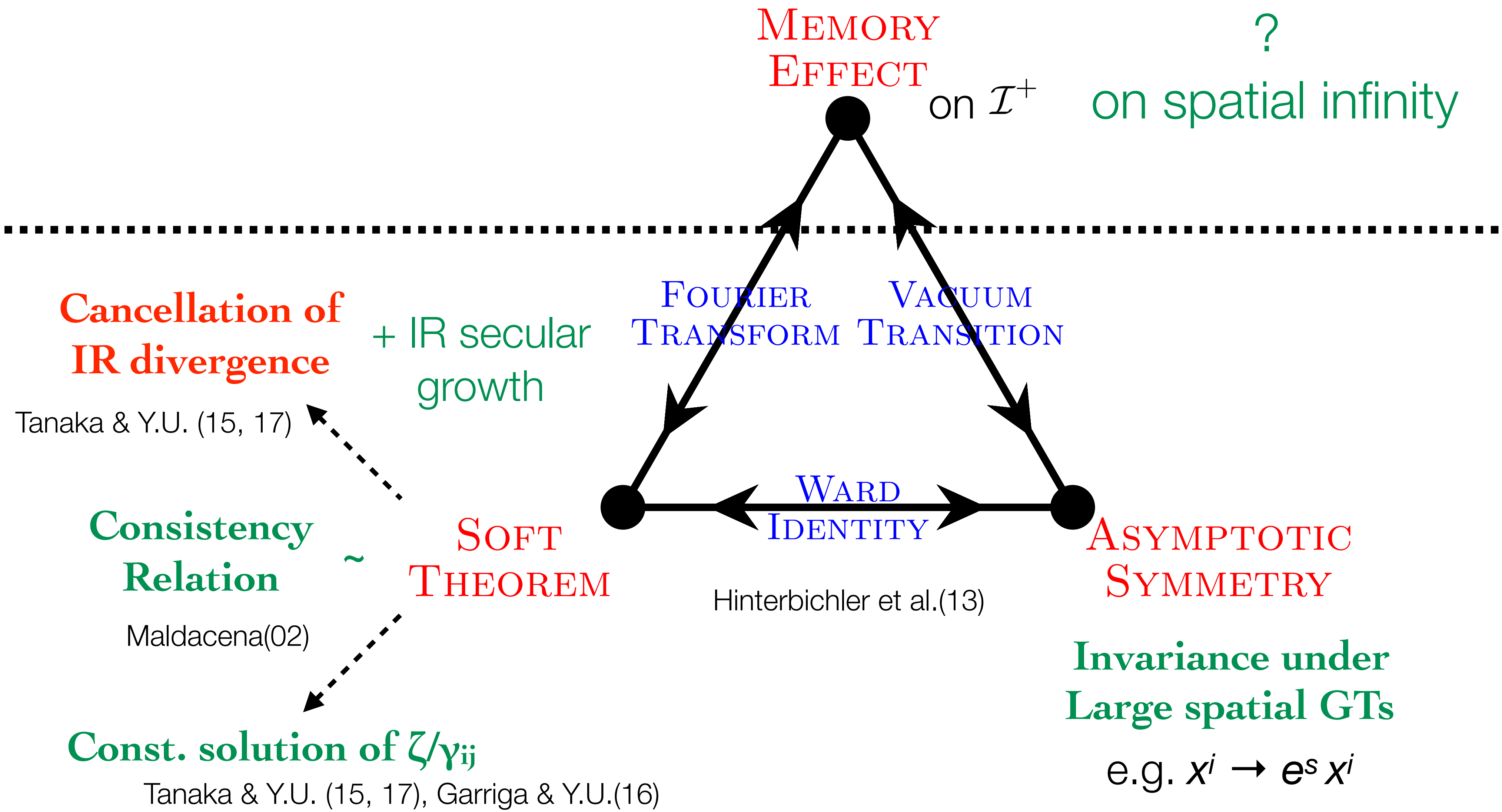
Faddeev-Kulish (1970)

Bloch & Nordsieck (1937)

Weinberg (1965)

Bondi, Metzner, Sachs (1962)

Infrared physics in cosmology



Questions

- Gauge theories in asymptotically flat spacetimes,

  Conditions on asym. geometry + Gauge invariance
  (symmetry group therein)

  ⟶ IR universal properties: Soft th., Memory effect, Canc. of IR div.

Q1. Does the same argument apply to ζ?

Questions

- Gauge theories in asymptotically flat spacetimes,

Conditions on asym. geometry + Gauge invariance

(symmetry group therein)

⟶ IR universal properties: Soft th., Memory effect, Canc. of IR div.

Q1. Does the same argument apply to ζ?

Conditions on asym. geometry + Gauge invariance

asym. FLRW spatial Diff (including large GTs)

⟶ IR universal properties: Soft th., const sol. of ζ, Canc. of IR div.

Questions

- Gauge theories in asymptotically flat spacetimes,

Conditions on asym. geometry + Gauge invariance

(symmetry group therein)

⟶ IR universal properties: Soft th., Memory effect, Canc. of IR div.

Q1. Does the same argument apply to ζ?

Conditions on asym. geometry + Gauge invariance

asym. FLRW   spatial Diff (including large GTs)

⟶ IR universal properties: Soft th., const sol. of ζ, Canc. of IR div.

Q2. Does it apply just w/ spatial Diff but w/o 4D Diff?

IR universal properties hold e.g., for Horava-Lifshitz gravity

Outline

1) Exercise in classical theory (Mostly review)

- Definition of asym. FLRW, following gradient expansion

- IR properties: Const. solution of ζ (a.k.a.Weinberg's adiabatic mode)

2) Extension to quantum theory

- Stochastic approach + Gradient expansion

Basic assumptions

【 Asymptotically FLRW spacetime】

【Spatial Diff. invariance】 $x^i \to \tilde{x}^i(t, x^i)$

Including dilatation inv. $x^i \to e^{-s} x^i$

Asymptotic symmetry which is compatible w/ asym FLRW

【 Locality】 (of the original Lagrangian) * before solving constraints

$$\mathcal{L}(x) = \mathcal{L}\left[\Phi(x)\right]$$

Gradient expansion

Gradient expansion

Salopek & Bond (1990), Shibata & Sasaki (1999)

Physical scale of inhomogeneity of our focus *L* >> 1/*H*

(or coarse graining scale)

$$\epsilon \equiv 1/(HL)$$

In local theory, we identity ϵ exp. as derivative exp.

(d+1)-dim line element & Gauge choice

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \qquad g_{ij} = a^2 e^{2\zeta} \gamma_{ij} \qquad \det[\gamma] = 1$$

- spatial coordinates: Transverse $\quad {\gamma_{ij}}^{|j} = 0$

Asymptotically FLRW spacetime

Asymp FLRW condition

$$\left| e^{-\bar{\zeta}} \frac{N_i}{aN} \right| = \mathcal{O}(\epsilon)$$

spatial average at reference time $t_\star$

$$\bar{\zeta} \equiv \frac{\int d^d\boldsymbol{x}\, \zeta(t_\star, \boldsymbol{x})}{\int d^d\boldsymbol{x}}$$

$$x^i \to e^{-s} x^i \qquad \bar{\zeta} \to \bar{\zeta} - s$$

* Do not impose condition on time derivative of γ_{ij}

Lyth, Malik & Sasaki (2004)

* At linear order, 【AsympFLRW】 is a condition on shear. $k\sigma_{\rm g} \sim \partial_i N_i / a$

* $\bar{\zeta}$ is introduced for the dilatation invariance.

(* For (d+1)-Diff, the above condition is imposed on K=dH slicing.)

Classical Lagrangian

【Spatial Diff. invariance】 & 【Locality】

$$S = \int d^{d+1}x\sqrt{-g}\,[\mathcal{L}_{\rm g} + \mathcal{L}_{\rm matter}]$$

with

$$\mathcal{L}_{\rm g} = \frac{1}{16\pi G}\left[\left(K^i{}_j K^j{}_i - \beta_1 K^2\right) + \beta_2 {}^sR + \mathcal{O}(\epsilon^2 \text{ w/o } N_i, \epsilon^3 \text{ w/} N_i)\right]$$

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - D_i N_j - D_j N_i) \qquad\qquad K \equiv \gamma^{ij} K_{ij}$$

* Condition on matter sector needs a bit more inspection.

Matter sector ∋ Integer spin fields w/sDiff + locality

Bottom-line argument

Tanaka & Y.U. (in progress)

(d+1)-dim Diff H-const & M-const $\longrightarrow$ $$\frac{\dot{\zeta}}{H} = \mathcal{J} + \mathcal{O}(\epsilon)$$

$\mathcal{J}$ is described only by other independent dynamical fields.

c.f. projectable HL M-const

non-projectable HL H&M-const $\longrightarrow$ $$\frac{\dot{\zeta}}{H} = \mathcal{J} + \mathcal{O}(\epsilon)$$

Izumi&Mukohyama (11)
Gumrukcuoglu et al. (11)

Armendariz-Picon et al (10)
Arai, Sibiryakov, Y.U. (18)

(ex) Single scalar field, Linear perturbation

H-const $$\frac{\dot{\zeta}}{H} = \frac{2(d-1)H^2}{16\pi G\dot{\bar{\phi}}^2}\frac{\partial_k N_k}{a^2 H} + \mathcal{O}(\epsilon^2) \; = \mathcal{O}(\epsilon^2)$$ 【AsympFLRW】

$$k\sigma_{\rm g} \sim \partial_i N_i/a \;\propto 1/a^{d-1} \qquad \zeta^{\rm (L),\,dec} \propto \int dt \frac{H^2}{a^d \dot{\bar{\phi}}^2}$$

ultra-SR Kinney (05)

see also Leach et al. (01), Y.Tanaka&Sasaki (06)

Existence of const. solution w/ (d+1)-dim Diff

Choosing time slicing $K = dH$ ⟶ $N = 1 + \frac{\dot{\zeta}}{H} + \mathcal{O}(\epsilon^2)$

recall δN formalism Starobinsky(82), Sasaki-Stewardt(95)

H-const $\frac{\delta\rho_{\rm tot}}{\bar{\rho}} = \mathcal{O}(\epsilon)$ $\delta\rho_{\rm total} \equiv \delta\rho + \delta\rho_{\rm TT}$ $\delta\rho_{\rm TT} \equiv \frac{A^i{}_j A^j{}_i}{16\pi G}$

M-const Remove one field e.g. scalar field system $\phi^1 = \phi^1(\phi^2, \phi^3, \cdots)$

Tanaka & Y.U. (in progress)

ζ w/o $\mathcal{O}(\epsilon^2)$ appears only through *N* (or ***δN***) in $\frac{\delta\rho_{\rm tot}}{\bar{\rho}} = \mathcal{O}(\epsilon)$

【Spatial Diff.】 & 【Asymp FLRW】

⟶ $\frac{\dot{\zeta}}{H} = \mathcal{J} + \mathcal{O}(\epsilon)$ also for inhomogeneous ζ 【locality】

Outline

1) Exercise in classical theory (Mostly review)

- Definition of asym. FLRW, following gradient expansion

- IR properties: Cost. solution of ζ (a.k.a.Weinberg's adiabatic mode)

2) Extension to quantum theory

- Stochastic approach + Gradient expansion

Repeat the same argument, using effective action.

Dilatation invariance

$|\Psi\rangle$: Quantum state for $g_{\mu\nu}$, φ^α

$$\varphi^{\alpha s}(t,\,\boldsymbol{x}_s) = e^{-\mathcal{S}_\alpha s}\varphi^\alpha(t,\,\boldsymbol{x}) \qquad \boldsymbol{x}_s = e^s\boldsymbol{x}$$

including higher spin particles

Arkani-Hamed & Maldacena (15)

Noether charge of the dilatation

$$Q_\zeta \equiv \frac{1}{2}\int d^3\boldsymbol{x}\left[\Delta_s\zeta(t,\,\boldsymbol{x})\pi(t,\,\boldsymbol{x}) + \sum_\alpha \Delta_s\varphi^\alpha(t,\,\boldsymbol{x})\pi_\alpha(t,\,\boldsymbol{x}) \;+\; \ldots. \;+ (\mathrm{h.c.})\right]$$

Dilatation invariance of the quantum system

$$Q_\zeta|\,\Psi\rangle = 0$$

Locality

Effective action is in general non-local.

Assuming that a quantum state can be expressed as

$$|\,\Psi\rangle = U(\delta g,\, \varphi^{\alpha})|\,\Psi_{\mathrm{adi}}\rangle \qquad\qquad U^{\dagger}U = 1$$

$|\,\Psi_{\mathrm{adi}}\rangle$: adiabatic vacuum or Euclidean vacuum

$$Q_{\zeta}\,|\,\Psi_{\mathrm{adi}}\rangle = 0$$

e.g. non-adiabatic vac. $|\,\Psi'\rangle = \prod_{\boldsymbol{k}} S_{\boldsymbol{k}}(\theta_k,\, \phi_k)|\,\Psi_{\mathrm{adi}}\rangle$ $\qquad S_{\boldsymbol{k}}(\theta_k,\, \phi_k) \equiv \exp\left[i\theta_k\left(a_{\boldsymbol{k}}a_{-\boldsymbol{k}}e^{i\phi_k} + a^{\dagger}_{-\boldsymbol{k}}a^{\dagger}_{\boldsymbol{k}}e^{-i\phi_k}\right)\right]$

【locality】

$S_{\mathrm{tot}} = S + \delta S$ is local. $\qquad U \sim e^{i\delta S}$

Integrating out short modes

$\boldsymbol{\varphi} \equiv \{\delta g,\, \varphi^\alpha\}$ metric perturbations & matter fields

short modes $$\boldsymbol{\varphi}^{(\mathrm{S})}(t,\,\boldsymbol{x}) \equiv \int \frac{d^d\boldsymbol{k}}{(2\pi)^{\frac{d}{2}}}\theta(k-k_c(a))e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,\boldsymbol{\varphi}(t,\,\boldsymbol{k})$$

long modes $$\boldsymbol{\varphi}^{(\mathrm{L})}(t,\,\boldsymbol{x}) \equiv \boldsymbol{\varphi}(t,\,\boldsymbol{x}) - \boldsymbol{\varphi}^{(\mathrm{S})}(t,\,\boldsymbol{x})$$

Stochastic inflation
Starobinsky (86)

Smeared field in gradient exp. corresponds to $\boldsymbol{\varphi}^{(\mathrm{L})}$

Feynman & Vernon (63)
Feynman & Hibbs (65)

Influence functional

$$iS_{\mathrm{eff}}\left[\boldsymbol{\varphi}_+^{(\mathrm{L})},\,\boldsymbol{\varphi}_-^{(\mathrm{L})}\right] \equiv \ln\left[\int D^g\boldsymbol{\varphi}_+^{(\mathrm{S})}\int D^g\boldsymbol{\varphi}_-^{(\mathrm{S})}\,e^{iS_{\mathrm{tot}}[\boldsymbol{\varphi}_+^{(\mathrm{L})},\boldsymbol{\varphi}_+^{(\mathrm{S})}]-iS_{\mathrm{tot}}[\boldsymbol{\varphi}_-^{(\mathrm{L})},\boldsymbol{\varphi}_-^{(\mathrm{S})}]}\right]$$

effective action w/ influence of $\boldsymbol{\varphi}^{(\mathrm{S})}$

Existence of constant solution

【Spatial Diff. invariance】

including dilatation inv. $x^i \rightarrow e^s x^i$ $Q_\zeta | \Psi \rangle = 0$

【 Asymptotically FLRW spacetime】

$\left| e^{-\bar{\zeta}} \frac{N_i}{aN} \right| = \mathcal{O}(\epsilon)$ N, Ni, ζ satisfy the eoms derived from effective action

【 Locality】 $S_{\rm tot} = S + \delta S$ is local. δS: excitation from adiabatic vac.

$$\longrightarrow \quad \frac{\dot{\zeta}^{(\rm L)}}{H} = \underline{\mathcal{J}_{\rm cl}} + \underline{\mathcal{J}_{\rm IF}} + \mathcal{O}(\epsilon)$$

long modes of isocurvature

integrated out short modes

WT identity for dilatation

dilatation preserves asymp FLRW

$$\bigstar = \frac{\int D^g \boldsymbol{\varphi}_+^{(\mathrm{S})} \int D^g \boldsymbol{\varphi}_-^{(\mathrm{S})} \;\; \boldsymbol{\varphi}_\pm^{(\mathrm{S})}(x_1)\cdots\;\; \boldsymbol{\varphi}_\pm^{(\mathrm{S})}(x_n)\, e^{i(S_{\mathrm{int}}[\boldsymbol{\varphi}_+^{(\mathrm{L})},\boldsymbol{\varphi}_+^{(\mathrm{S})}]-iS_{\mathrm{int}}[\boldsymbol{\varphi}_-^{(\mathrm{L})},\boldsymbol{\varphi}_-^{(\mathrm{S})}])}}{\int D^g\boldsymbol{\varphi}_+ \int D^g \boldsymbol{\varphi}_-\, e^{i(S_{\mathrm{int}}[\boldsymbol{\varphi}_+^{(\mathrm{L})},\boldsymbol{\varphi}_+^{(\mathrm{S})}]-iS_{\mathrm{int}}[\boldsymbol{\varphi}_-^{(\mathrm{L})},\boldsymbol{\varphi}_-^{(\mathrm{S})}])}}$$

$$S_{\mathrm{int}}[\boldsymbol{\varphi}^{(\mathrm{L})},\,\boldsymbol{\varphi}^{(\mathrm{S})}] \equiv S_{\mathrm{tot}}[\boldsymbol{\varphi}^{(\mathrm{L})},\,\boldsymbol{\varphi}^{(\mathrm{S})}] - S[\boldsymbol{\varphi}^{(\mathrm{L})}]$$

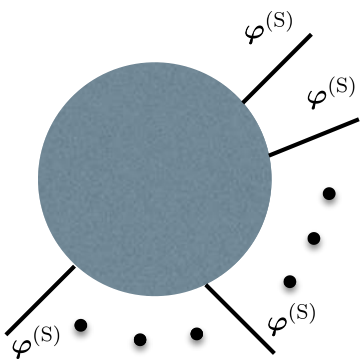
WT $\quad\bigstar$ before dilatation = $\bigstar$ after dilatation

$$\left(\sum_{i=1}^{n}\boldsymbol{x}_i\cdot\partial_{\boldsymbol{x}_i}+\sum_{\alpha}\mathcal{S}_\alpha\right)\langle\boldsymbol{\varphi}_\pm^{(\mathrm{S})}(x_1)\cdots\boldsymbol{\varphi}_\pm^{(\mathrm{S})}(x_n)\rangle_{\boldsymbol{\varphi}^{(\mathrm{S})}}$$

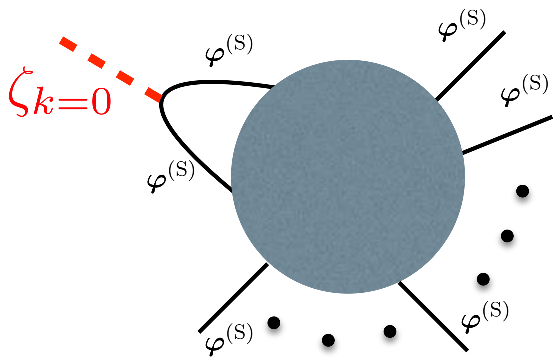
$$S_{\mathrm{int}}[\boldsymbol{\varphi}^{(\mathrm{L})},\boldsymbol{\varphi}^{(\mathrm{L}),s}] = S_{\mathrm{int}}[\boldsymbol{\varphi}^{(\mathrm{L})}-\delta\boldsymbol{\varphi}^{(\mathrm{L})},\boldsymbol{\varphi}^{(\mathrm{L})}]$$

$$= \int d^{d+1}x\left\langle \boldsymbol{\varphi}_\pm^{(\mathrm{S})}(x_1)\cdots\boldsymbol{\varphi}_\pm^{(\mathrm{S})}(x_n)\frac{\delta iS_{\mathrm{int}}[\boldsymbol{\varphi}_+^{(\mathrm{L})},\,\boldsymbol{\varphi}_+^{(\mathrm{S})}]}{\delta\zeta_+^{(\mathrm{L})}(x)}\right|_{\boldsymbol{\varphi}_+^{(\mathrm{L})}=0}\Bigg\rangle_{\boldsymbol{\varphi}^{(\mathrm{S})}} - \int d^{d+1}x\left\langle \boldsymbol{\varphi}_\pm^{(\mathrm{S})}(x_1)\cdots\boldsymbol{\varphi}_\pm^{(\mathrm{S})}(x_n)\frac{\delta iS_{\mathrm{int}}[\boldsymbol{\varphi}_-^{(\mathrm{L})},\,\boldsymbol{\varphi}_-^{(\mathrm{S})}]}{\delta\zeta_-^{(\mathrm{L})}(x)}\right|_{\boldsymbol{\varphi}_-^{(\mathrm{L})}=0}\Bigg\rangle_{\boldsymbol{\varphi}^{(\mathrm{S})}}$$

$$\left(\sum_{i=1}^{n}\boldsymbol{x}_i\cdot\partial_{\boldsymbol{x}_i}+\sum_{\alpha}\mathcal{S}_\alpha\right)$$



$=$



amputated

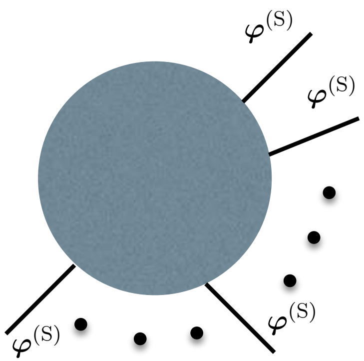
spin-0 Tanaka & Y.U. (2015)

arbitrary integer spin Tanaka & Y.U. (in progress)

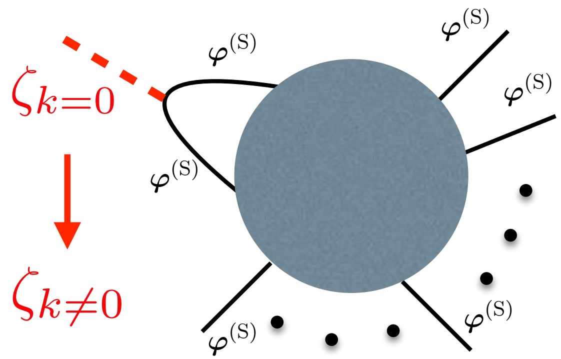
Soft theorem

Soft theorem: WT for dilatation + 【locality】

$$\left(\sum_{i=1}^{n} \boldsymbol{x}_i \cdot \partial_{\boldsymbol{x}_i} + \sum_{\alpha} \mathcal{S}_\alpha\right)$$



=



amputated

$$\left(-\sum_{i=1}^{n} \partial_{\boldsymbol{k}_i}\boldsymbol{k}_i + \sum_{\alpha}\mathcal{S}_\alpha\right)\langle \varphi_\pm^{(S)}(t_1,\,\boldsymbol{k}_1)\cdots\varphi_\pm^{(S)}(t_n,\,\boldsymbol{k}_n)\rangle_{\varphi^{(S)}}$$

$$= \int dt \left\langle \varphi_\pm^{(S)}(t_1,\,\boldsymbol{k}_1)\cdots\varphi_\pm^{(S)}(t_n,\,\boldsymbol{k}_n)\frac{\delta i S_{\rm int}[\varphi_+^{(L)},\,\varphi_+^{(S)}]}{\delta\zeta_+^{(L)}(t,\,\boldsymbol{k}_{\rm L})}\bigg|_{\varphi_+^{(L)}=0}\right\rangle_{\varphi^{(S)}} - \int dt \left\langle \varphi_\pm^{(S)}(t_1,\,\boldsymbol{k}_1)\cdots\varphi_\pm^{(S)}(t_n,\,\boldsymbol{k}_n)\frac{\delta i S_{\rm int}[\varphi_-^{(L)},\,\varphi_-^{(S)}]}{\delta\zeta_-^{(L)}(t,\,\boldsymbol{k}_{\rm L})}\bigg|_{\varphi_-^{(L)}=0}\right\rangle_{\varphi^{(S)}}$$

Remark

Soft theorem $D\varphi^{(S)}$ $\neq$ Consistency relation $D\varphi^{(S)}D\varphi^{(L)}$

Soft theorem → Consistency relation

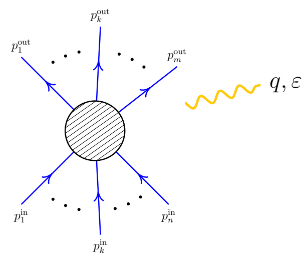
Soft theorem $D^g \boldsymbol{\varphi}^{(\mathrm{S})}$ = Consistency relation $D^g \boldsymbol{\varphi}^{(\mathrm{S})} D^g \boldsymbol{\varphi}^{(\mathrm{L})}$

only when the contribution of $\zeta_{k\mathrm{L}}$ is factorizable.

$$\zeta^{(\mathrm{L})}(t,\, \boldsymbol{k}_{\mathrm{L}}) = \zeta^{(ad)}(\boldsymbol{k}_{\mathrm{L}}) + \int^t dt' H \left[\mathcal{J}_{\mathrm{cl}}(t',\, \boldsymbol{k}_{\mathrm{L}}) + \mathcal{J}_{\mathrm{IF}}(t',\, \boldsymbol{k}_{\mathrm{L}})\right] + \mathcal{O}(\epsilon)$$

CR
$$\left(\sum_{i=2}^{n} \boldsymbol{k}_i \cdot \partial_{\boldsymbol{k}_i} + (n-1)d - \sum_{\alpha} \mathcal{S}_\alpha\right) \langle T \boldsymbol{\varphi}^{(\mathrm{S})}(t_1,\, \boldsymbol{k}_1) \cdots \boldsymbol{\varphi}^{(\mathrm{S})}(t_n,\, \boldsymbol{k}_n)\rangle$$
$$= -\frac{(2\pi)^{\frac{d}{2}}}{P_\zeta(k_{\mathrm{L}})} \left\langle T\, \zeta^{(\mathrm{L})}(\boldsymbol{k}_{\mathrm{L}}) \boldsymbol{\varphi}^{(\mathrm{S})}(t_1,\, \boldsymbol{k}_1) \cdots \boldsymbol{\varphi}^{(\mathrm{S})}(t_n,\, \boldsymbol{k}_n)\right\rangle .$$

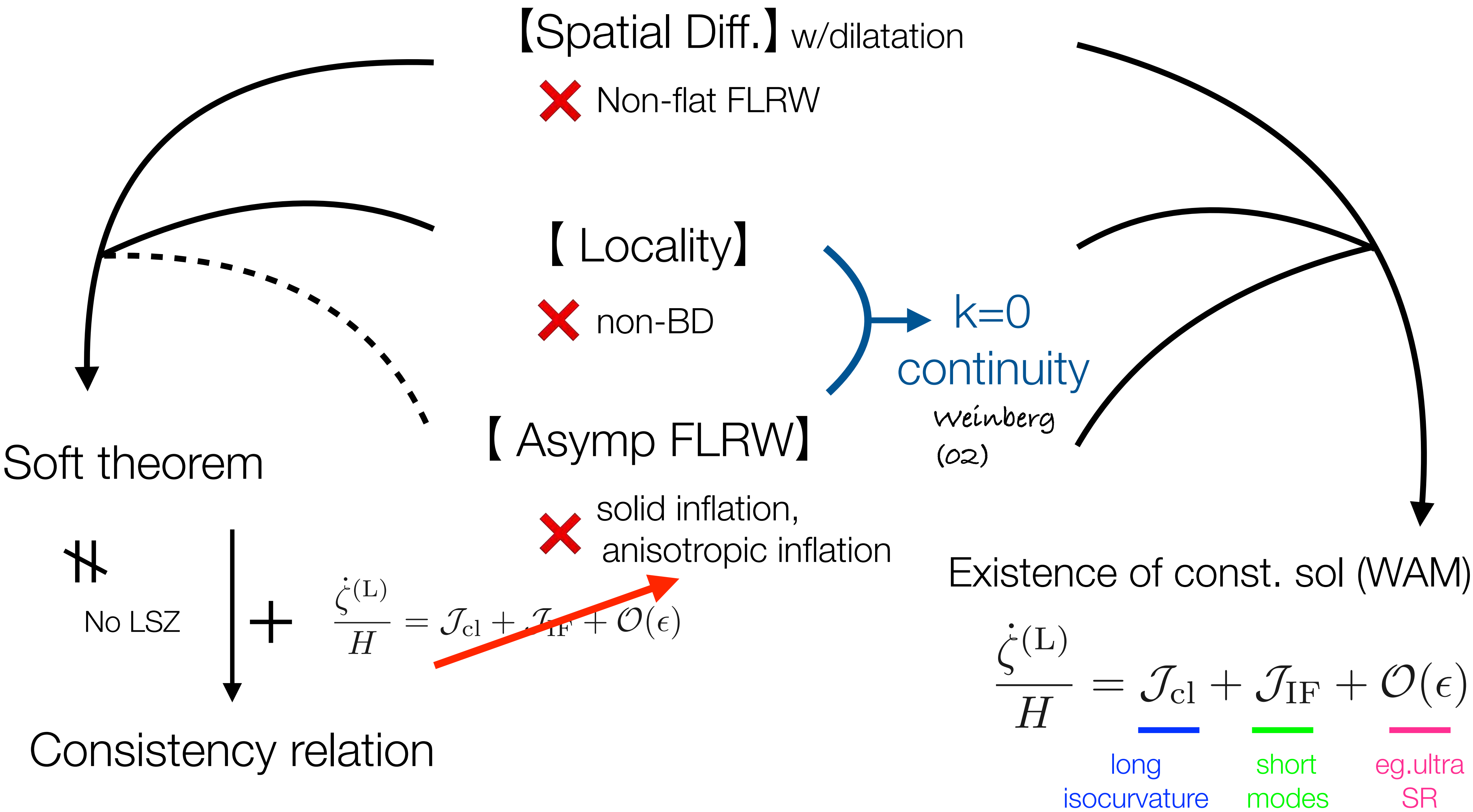
No additional requirement for gauge theories in asymp. flat sp.



LSZ reduction formula (, Lorentz symmetry)

Inserting soft legs. $\frac{-i}{(p+q)^2+m^2} = \frac{-i}{p^2+2p\cdot q+q^2+m^2} = \frac{-i}{2p\cdot q}$ & $ie\varepsilon^\mu 2Qp_\mu$

Summary


【Spatial Diff.】 w/dilatation
Non-flat FLRW
【 Locality】
non-BD
k=0
continuity
Weinberg
(02)
【 Asymp FLRW】
solid inflation,
anisotropic inflation
Soft theorem
No LSZ
$+ \quad \frac{\dot{\zeta}^{(\mathrm{L})}}{H} = \mathcal{J}_{\mathrm{cl}} + \mathcal{J}_{\mathrm{IF}} + \mathcal{O}(\epsilon)$
Consistency relation
Existence of const. sol (WAM)
$\frac{\dot{\zeta}^{(\mathrm{L})}}{H} = \mathcal{J}_{\mathrm{cl}} + \mathcal{J}_{\mathrm{IF}} + \mathcal{O}(\epsilon)$
long
isocurvature
short
modes
eg.ultra
SR